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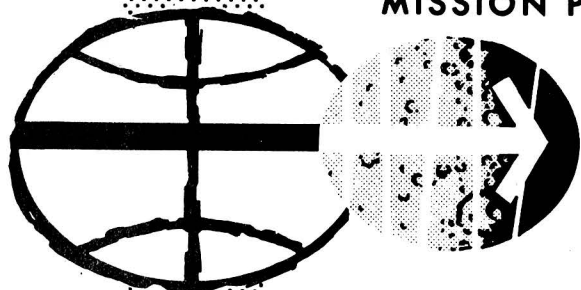
APOLLO 13 (MISSION H-2)  
SPACECRAFT DISPERSION ANALYSIS

VOLUME IV

DESCENT AND ASCENT  
DISPERSION ANALYSES

Landing Analysis Branch

MISSION PLANNING AND ANALYSIS DIVISION



MANNED SPACECRAFT CENTER  
HOUSTON, TEXAS



PROJECT APOLLO

REVISION 1 TO THE APOLLO 13 (MISSION H-2) SPACECRAFT  
DISPERSION ANALYSIS  
VOLUME IV - DESCENT AND ASCENT DISPERSION ANALYSES

By Gilbert L. Carman, Moises N. Montez, and James V. West  
Landing Analysis Branch

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MANNED SPACECRAFT CENTER  
HOUSTON, TEXAS

Approved:   
Floyd V. Bennett, Chief  
Landing Analysis Branch

Approved:   
John P. Mayer, Chief  
Mission Planning and Analysis Division





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# REVISION 1 TO THE APOLLO 13 (MISSION H-2)

## SPACECRAFT DISPERSION ANALYSIS

### VOLUME IV - DESCENT AND ASCENT DISPERSION ANALYSES

By Gilbert L. Carman, Moises N. Montez,  
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#### PART I - LUNAR DESCENT

##### 1.0 SUMMARY

The results of the Apollo 13 (Mission H-2) dispersion analysis for the LM descent are presented in the form of statistical tables and 99-percent, 90-percent, and 50-percent landing dispersion ellipses. The study is based on a nonlinear Monte Carlo analysis, and 100 descents were run.

In summary, the 99-percent ellipse has been reduced from 1.5 n. mi. by 0.8 n. mi. as given in the Apollo 13 SC Dispersion Analysis, Volume IV (MSC IN 70-FM-12) to 0.54 n. mi. by 0.73 n. mi. as a result of a reduction in flight-path angle uncertainty at PDI, a reduction in the random bias error, and exclusion of noise effects in computing  $\Delta$ RLS. Also, in the past, if the  $\Delta$ RLS was between  $\pm 2000$  feet, it was not incorporated. Now any  $\Delta$ RLS with a magnitude between  $\pm 30\ 000$  feet is incorporated.

##### 2.0 INTRODUCTION

The purpose of this document is to present the technical approach assumptions and the results of the final dispersion analysis performed for the lunar descent phase of Apollo 13 (Mission H-2) as defined in reference 1. The purpose of the dispersion analysis is to assess the effects of navigation, terrain, and systems uncertainties on the probabilities for mission success and flight safety.

A nonlinear Monte Carlo analysis of the mission was performed to permit inclusion of nonlinear effects so that the resultant non-Gaussian sample statistic could be computed. The covariance matrices sampled by the Monte Carlo simulation describe the expected navigation accuracies for Apollo 13. These matrices were generated by the Mathematical Physics Branch (ref. 2) and the Guidance and Performance Branch.

### 3.0 ABBREVIATIONS

APS	ascent propulsion system
DAMP	descent ascent Monte Carlo program
DOI	descent orbit insertion
DPS	descent propulsion system
FTP	fixed throttle point
GAC	Grumman Aerospace Corporation
G&N	guidance and navigation
LAB	Landing Analysis Branch
LGC	LM guidance computer
LM	lunar module
LPD	landing point designator
LR	landing radar
PDI	powered descent initiation
PGNCS	primary guidance and navigation control subsystem
RCS	reaction control subsystem
TLAND	nominal landing time
$t_{GO}$	time to go
$t_{IG}$	time of ignition

VCO	velocity compensating oscillator
$\Delta V$	change in velocity
$\Delta RLS$ (NOUN 69)	landing site vector displacement to correct for down-range or up-range errors in the state vector

#### 4.0 ANALYSIS

Dispersion data for this Monte Carlo analysis were generated by the LAB DAMP program. This study considered the effects of propulsion uncertainties, landing radar errors, terrain, and terrain slope on the lunar landing trajectory from lunar orbit. Initial systems errors for each Monte Carlo cycle were selected by a random number generator. Navigation uncertainties were incorporated into the initial state vector for each cycle by sampling a covariance matrix. Statistics were generated for the dispersion data at eight points in the trajectory. A detailed description of the major points which constituted this Monte Carlo analysis is given in subsections 4.1 through 4.3.

##### 4.1 Descent Trajectory and Guidance

The latest descent targeting philosophy was used to generate the desired conditions used in this study; these conditions are documented in reference 1. The descent burn will be the only burn to be done with the DPS because the CSM will place the LM into an 8-by 60-n. mi. orbit.

During the powered descent, a  $\Delta RLS$  will be incorporated if the down-range (or up-range) error is less than 30 000 feet. The following equation will be used to determine the  $\Delta RLS$  magnitude.

$$\Delta RLS = \Delta UDOT * \frac{|\bar{R}|}{|\bar{V}|}$$

where

$\Delta UDOT$  = the difference between the PGNCs velocity vector and the powered flight processor velocity vector projected onto the PGNCs position vector

$\bar{R}$  = PGNCs position vector

$\bar{V}$  = PGNCs velocity vector

A  $\Delta RLS$  will be incorporated at PDI plus 2 minutes if  $\Delta UDOT$  is less than 30 fps. A  $\Delta UDOT$  of 30 fps is equivalent to a  $\Delta RLS$  of 30 000 feet because  $|R|/|V|$  is approximately 1000 seconds.

The simulation was initialized over the landing site one revolution prior to landing. The sequence of events was as follows.

- a. Sample a covariance matrix to initialize the state vector.
- b. Coast with an ENCKE program to TLAND - 830 sec
- c. Execute the ignition algorithm.
- d. Perform the powered descent.

All the navigation and guidance routines used in the simulation were acquired from reference 3 except for the ENCKE and the DOI targeting routine.

#### 4.2 Error Sources

The PGNC errors considered in this analysis were accelerometer bias, scale factor, gyro bias drift, misalignment, and acceleration sensitive gyro drift for both the input axis as well as the spin reference axis. The  $3\sigma$  values for these errors were obtained from reference 4 and are listed in table I.

The terrain variation model for Fra Mauro (ref. 5) was used with a  $3\sigma$  uncertainty of  $\pm 1^\circ$  in terrain slope superimposed on the terrain for ranges of less than 40 kilometers. Elevation uncertainties are computed in the following manner.

If range < 40 km, use  $-TS * Range$

If range  $\geq$  40 km, use  $-TS *(40 \text{ km})$

where TS is the tangent of the uncertainty in the terrain slope. A positive slope is defined as one which slopes up (from the LM) toward the landing site.

The LM DPS model (Victory 7) used in this analysis is described in reference 6. It is a linear DPS engine model that used basic acceptance test data and adjusts them to flight interface conditions. Interface conditions calculated in Victory 7 are functions of vehicle longitudinal acceleration and propellant flow rates. Engine characterization data and class coefficients which were used in this analysis are documented in reference 7.



Navigation uncertainties for a position and velocity vector one revolution prior to landing were used in the form of a covariance matrix. Matrix (a) in table II was generated by the Guidance and Performance Branch and represents the uncertainties of the actual from reference. Matrix (b) was generated by the Mathematical Physics Branch and describes the uncertainties of the estimated about the actual.

Entries in the matrix are given in a UVW system which is defined as follows.

$$U = \frac{\underline{r}}{|\underline{r}|}, \text{ vertical}$$

$$V = W \times U, \text{ down range}$$

$$W = \frac{\underline{r} \times \underline{v}}{|\underline{r} \times \underline{v}|}$$

where  $\underline{r}$  is the position vector and  $\underline{v}$  is the inertial velocity in selenocentric coordinates.

The landing radar model used in the studies for the report was the GAC mathematical model of reference 8. This model includes terrain effects and detailed track logic based on signal to noise ratio calculations. A detailed error model is also included which introduces the following error sources into the range and velocity measurements.

- a. Deterministic errors
  1. Terrain bias error
  2. Preamplifier slope error
  3. VCO drift error
  4. Dynamic lag error
  5. Doppler compensation error
- b. Random bias errors
  1. Boresight misalignment error
  2. Installation misalignment error
  3. Modulation rate error

c. Random fluctuation errors

1. Spread spectrum error
2. Doppler compensation error
3. Quantization error

The R2 potential function was modified as suggested in reference 9 to obtain the L1 potential function which was used to update the actual state vector.

All the parameters which are randomized during each Monte Carlo cycle are listed in table III.

4.3 Statistical Output

Covariance matrices were generated at landing and at powered descent initiation. The format for these matrices is the following.

$$\left[ \begin{array}{ccc}
 \overbrace{6 \times 6}^{AR} & & \\
 & \overbrace{6 \times 6}^{EA} & \\
 & & \overbrace{1 \times 1}^W
 \end{array} \right] \begin{array}{l} \\ \text{(covariance terms)} \\ \end{array}$$

13 x 13

where  $\overbrace{6 \times 6}^{AR}$  describes the uncertainty of the actual about the reference  
 $\overbrace{6 \times 6}^{EA}$  describes the uncertainty of the estimated about the actual  
 $\overbrace{1 \times 1}^T$  describes the uncertainty in weight

The state vector uncertainties in these matrices are given in the PGNCS stable member coordinate system (ref. 3).

Statistics were computed for the parameters defined in table IV at each of the following points in the descent trajectory.

- a. PDI
- b. High gate

- c. Ranges of 20 000, 10 000, 5000, and 2000 feet
- d. Vertical descent initiation
- e. Landing

Statistics were generated only for cases which did not violate any of the following abort criteria.

- a. Propellant depletion prior to lunar landing
- b. Failure to achieve landing radar altitude update initiation prior to high gate
- c. Lunar landing prior to the vertical descent phase
- d. Violation of the 4-second APS abort boundary (altitude versus altitude rate constraint) prior to 600-foot altitude above lunar surface
- e. Throttle recovery later than 15 seconds into P64

## 5.0 DISCUSSION AND RESULTS

One hundred descents were run, and none of these violated the abort criteria defined in the preceding section. The results for the 100 successful landings are presented in the form of statistical tables and landing dispersion ellipses. This discussion includes dispersions at PDI; landing radar acquisition; throttle recovery; dispersions at high gate, glide path, and low gate; dispersions at vertical descent; pitch dispersions at landing and DPS burn time; landing dispersion ellipses, landing velocity;  $\Delta V$  and propellant dispersions; and time histories with associated  $3\sigma$  dispersions for selected parameters.

The summary for the statistical parameters is presented in table V while the covariance matrices which were generated in this study are presented in table VI.

### 5.1 Dispersion at PDI

No convergence problems caused by dispersions at PDI were encountered in the ignition computation. Because of navigation uncertainties, the  $3\sigma$  dispersion in perilune altitude is 7800 feet. The  $3\sigma$  dispersions in vertical and horizontal velocity (at PDI) are  $\pm 7.0$  fps and  $\pm 11.0$  fps, respectively. Dispersions of this magnitude can be corrected by the G&N system and will result in a safe landing.

## 5.2 Landing Radar Acquisition

Landing radar acquisitions of altitude and velocity data nominally occur at altitudes of 35 100 feet and 17 600 feet, respectively. The  $3\sigma$  high and low altitudes for landing radar altitude acquisition are 37 300 feet and 32 900 feet, respectively. The  $3\sigma$  high and low altitudes for velocity acquisition are 19 250 feet and 15 950 feet, respectively.

## 5.3 Throttle Recovery

Throttle recovery nominally occurs at  $t_{GO} = 182$  seconds (approximately 120 sec prior to high gate because target switch occurs when  $t_{GO} = 62$  sec). A  $3\sigma$  high value for  $t_{GO}$  at throttle recovery is 228 seconds, and a  $3\sigma$  low value would be 132 seconds. The shutoff valve malfunction, which would reduce the  $t_{GO}$  at throttle recovery by 60 seconds, is not incorporated in these values. These statistics indicate that without a shutoff valve malfunction there will be at least a 70-second period of throttle recovery in which to meet the target conditions at high gate. A minimum throttle recovery time of 30 seconds has been shown to be adequate in previous studies.

## 5.4 High-gate Dispersions

Nominally, high gate occurs at an altitude of 6900 feet with a flight-path angle of  $-22^\circ$  and a vertical velocity of  $-178$  fps. The  $3\sigma$  high values for altitude, flight-path angle, and vertical velocity are 7900 feet,  $-26^\circ$ , and  $-210$  fps, respectively. The  $3\sigma$  low values, in the same order, are 5900 feet,  $-18^\circ$ , and  $-148$  fps. These dispersions do not indicate a flight safety problem at high gate.

## 5.5 Glide Path

The  $3\sigma$  altitude dispersions at ranges of 20 000 feet, 10 000 feet, 5000 feet, and 2000 feet are  $\pm 900$  feet,  $\pm 510$  feet,  $\pm 330$  feet, and  $\pm 200$  feet, respectively. The  $3\sigma$  approach corridor for Apollo 13 is presented in figure 1(a). The  $10^\circ$  angle indicated on figure 1(a) is the nominal sun elevation angle at landing for an April launch (ref. 10). Altitude versus range for the nominal trajectory as well as the  $3\sigma$  altitude dispersions are presented in figure 1(b) for ranges less than 70 000 feet.

## 5.6 Low Gate (range of 2000 ft)

Low gate occurs nominally at an altitude of 500 feet. The approach trajectory has been designed to provide a more efficient automatic trajectory and an improved redesignation capability at low gate. The statistics presented in this section were computed at a range of 2000 feet, which is approximately the range at which low gate occurs. The nominal values for horizontal velocity, vertical velocity, and pitch are 84 fps, -16 fps, and  $23^\circ$ , respectively. The  $3\sigma$  dispersion of these parameters are  $\pm 3.3$  fps,  $\pm 6$  fps, and  $\pm 1.0^\circ$ , respectively. These dispersions are considered to be within the capability of the pilot to perform his takeover function.

## 5.7 Vertical Descent

The nominal altitude at vertical descent initiation is 100 feet (no terrain). A  $3\sigma$  high altitude at vertical descent initiation is 113 feet, and a  $3\sigma$  low altitude is 89 feet. These altitudes include terrain effects and are for completely automatic landings. Terminal propellant dispersions are extremely sensitive to altitude dispersions at vertical descent initiation because of the small vertical rate. For each second of time, approximately 9 pounds of propellant is expended, and altitude is reduced by only 3 feet. Therefore, propellant dispersions caused by this effect are 3 lb/ft.

## 5.8 Pitch at Landing (Landing Gear Probe Contact)

The nominal pitch (from local vertical) at landing is  $0^\circ$ . The  $3\sigma$  low and high samples are  $-0.2^\circ$  and  $0.2^\circ$ , respectively. These values were well within the  $6^\circ$  constraint of reference 11.

## 5.9 Powered Descent Burn Time

The nominal DPS burn time is 684 seconds, and there is a 99-percent probability that the time will be less than 702 seconds. The smallest sample was 666 seconds. These dispersions indicate that there is no conflict with the DPS burn time constraint, which is approximately 910 seconds.

### 5.10 Landing Dispersion Ellipses

The 99 percent ellipse has been reduced from 1.5 n. mi. by 0.8 n. mi. to .54 n. mi. by .73 n. mi. as a result of a reduction in flight-path angle uncertainty at PDI, a reduction in the random bias error, and exclusion of noise effects in computing  $\Delta$ RLS. Also, in the past, if the  $\Delta$ RLS was between  $\pm 2000$  feet, it was not incorporated. Now any  $\Delta$ RLS with a magnitude between  $\pm 30\,000$  feet is incorporated.

Table VII presents the 99-percent, 90-percent, and 50-percent probability ellipses, and the 99-percent ellipse is plotted in figure 2.

### 5.11 Landing (Landing Gear Probe Contact) Velocity

Nominal horizontal and vertical velocities at landing are 0.008 fps and -3 fps, respectively. The  $3\sigma$  dispersions, in the same order, are  $\pm 0.8$  fps and  $\pm 0.4$  fps. No free-fall effects are reflected in these velocity uncertainties. The  $3\sigma$  velocity error ellipse is compared in figure 3 with the landing gear constraints at landing specified in reference 11.

### 5.12 $\Delta V$ and Propellant Summary

The  $\Delta V$  and propellants are summarized in this document for an automatically guided descent only; that is, manual translation and landing site redesignation (manually input) are not considered. The  $\Delta V$  and propellants required are tabulated at major event points during the descent in table VIII(a). The fuel, oxidizer, and total propellant results are derived by use of the linear propulsion system model (ref. 6). The propellant status for the total descent is presented in table VIII(b), which includes loaded propellant, propellant consumed, and usable remaining propellant. Usable remaining propellant is, by definition, the propellant remaining at landing which can be used for  $\Delta V$ . The usable remaining propellant values do not allow for system malfunctions, biases, or contingency situations (redline low-level sensor, engine valve malfunction).

### 5.13 Time Histories and Dispersions for Selected Parameters

Time histories for selected parameters which are significant during the powered descent are shown in figure 4. Each plot contains time histories for 70 randomly selected trajectories. The outer traces approximate the  $3\sigma$  cases.

## 6.0 CONCLUSIONS

There were no violations of known systems or trajectory constraints. All 100 descents were successful. The 99 percent ellipse has been reduced from 1.5 n. mi. by 0.8 n. mi. to .54 n. mi. by .73 n. mi. as a result of a reduction in flight-path angle uncertainty at PDI, a reduction in the random bias error, and exclusion of noise effects in computing  $\Delta$ RLS. Also, in the past, if the  $\Delta$ RLS was between  $\pm 2000$  feet, it was not incorporated. Now any  $\Delta$ RLS with a magnitude between  $\pm 30\ 000$  feet is incorporated.

TABLE I.- IMU ERRORS

Component	3 $\sigma$ error	Equivalent errors in program units
Gyro misalignment	0.057 deg	0.000948 rad
Gyro bias drift	.09 deg/hr	.436332 $\times 10^{-6}$ rad/sec
Acceleration sensitive gyro drift, input axis	24 meru/g	5.42E-8 $\frac{\text{rad-ft}}{\text{sec}}$
Acceleration sensitive gyro drift, spin reference axis	15 meru/g	3.39E-8 $\frac{\text{rad-ft}}{\text{sec}}$
Accelerometer bias	.2 cm/sec <sup>2</sup>	.0065616 ft/sec <sup>2</sup>
Accelerometer scale factor	300/10 <sup>6</sup>	.003



TABLE II.- COVARIANCE MATRIX OF NAVIGATION UNCERTAINTIES<sup>a</sup>

(a) Actual uncertainties about nominal

3.9608452+06	-6.3288579+07	-2.9949724+06	7.8945224+04	-2.3658249+03	9.0879535+02
-8.3288579+07	7.6536655+09	7.2834423+07	-7.2381841+06	1.1942167+05	-2.1870028+04
-8.9949724+06	7.2834423+07	9.8746867+07	-6.8807236+04	1.4031897+03	-2.6784535+04
7.8945224+04	-7.2381841+06	-6.8807236+04	6.8476855+03	-1.1341236+02	2.0389369+01
-8.3658249+03	1.1942167+05	1.4031897+03	-1.1341236+02	6.7632415+00	-3.7188923+01
8.0879535+02	-2.1870028+04	-2.6784535+04	2.0389369+01	-3.7188923+01	8.1339897+00

(b) Estimated uncertainties about the actual

1.3000000+09	0.0000000	0.0000000	0.0000000	-1.1452330+02	0.0000000
0.0000000	3.4996000+06	0.0000000	-3.8828670+03	0.0000000	0.0000000
0.0000000	0.0000000	1.0400000+06	0.0000000	0.0000000	0.0000000
0.0000000	-3.0429670+03	0.0000000	2.9339080+00	0.0000000	0.0000000
-1.1452330+02	0.0000000	0.0000000	0.0000000	1.0088910+01	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	1.7437620+01
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

<sup>a</sup> Format for covariance matrix

- $\sigma_{xx}$     $\sigma_{xy}$     $\sigma_{xz}$     $\sigma_{xx}$     $\sigma_{xy}$     $\sigma_{xz}$
- $\sigma_{yy}$     $\sigma_{yz}$     $\sigma_{yy}$     $\sigma_{yx}$     $\sigma_{yy}$     $\sigma_{yz}$
- $\sigma_{zz}$     $\sigma_{zx}$     $\sigma_{zy}$     $\sigma_{zx}$     $\sigma_{zy}$     $\sigma_{zz}$
- $\sigma_{xx}$     $\sigma_{xy}$     $\sigma_{xz}$
- $\sigma_{yy}$     $\sigma_{yz}$     $\sigma_{zz}$

TABLE III.- PARAMETERS RANDOMIZED DURING EACH MONTE CARLO CYCLE

## (a) IMU

Gyro misalignment  
 Coasting flight gyro drift  
 Powered flight gyro drift  
 Accelerometer bias  
 Accelerometer scale factor

## (b) LR

Boresight misalignment error  
 Installation misalignment error  
 Modulation rate error  
 Spread spectrum error  
 Doppler compensation error  
 Quantization error

## (c) Engine model and propulsion system

Thrust  
 Specific impulse  
 Mixture ratio  
 Pressure drop ratio across the injector  
 Loaded fuel  
 Loaded oxidizer  
 Dry weight of LM + APS propellant

## (d) State vector

Navigated position and velocity vectors

## (e) Terrain

Terrain slope

TABLE IV.- DEFINITION OF SYMBOLS

FPAG	Estimated flight-path angle, deg
FPAGT	True flight-path angle, deg
HORV	Estimated horizontal velocity, fps
HORVT	True horizontal velocity, fps
HDOT	Estimated vertical velocity, fps
VERVT	True vertical velocity, fps
PVRT	Estimated pitch from vertical, deg
PVRTT	True pitch from vertical, deg
HEST	Estimated altitude, ft
HTRU	Altitude above the local terrain, ft
RANGE	Z component of LM position vector in the guidance coordinate system, ft
HVIS	True altitude at landing sight visibility acquisition, ft
HORZ	True altitude at horizon visibility acquisition. ft
LATLM	Latitude of LM, deg
LONLM	Longitude of LM, deg
HLRA	True altitude at landing radar altitude acquisition, ft
HLRV	True altitude at landing radar velocity acquisition, ft
ELANG	Glide angle, deg
ELAS	LPD angle, navigated, deg
HPER	LM perilune altitude, ft
HAPO	LM apolune altitude, ft
ECC	Eccentricity, n.d.

TABLE IV.- DEFINITION OF SYMBOLS - Concluded

DEVMT	Total actual accumulated $\Delta V$ , fps
TFIG	Total DPS burn time, sec
BURN	Burn time at FTP (fixed throttle point), sec
TFXA	TGO at throttle down time, sec
WT	Weight deviation from nominal, lb
TTVIS	Total landing sight visibility time, sec
WPR	DPS propellant remaining, lb
WF	DPS fuel remaining, lb
WOX	DPS oxidizer remaining, lb
FC	Thrust commanded by the guidance, lb
THF	DPS thrust, lb
PIO	Oxidizer interface pressure, lb
PIF	Fuel interface pressure, lb
DWO	Oxidizer flow rate, lb/sec
DWF	Fuel flow rate, lb/sec
RE	Mixture ratio, n.d.
REE	Effective mixture ratio, n.d.
CISP	Specific impulse, sec
EISP	Effective specific impulse, sec
RESVG	Error in flight-path angle, deg

TABLE V. - SUMMARY OF STATISTICAL PARAMETERS

Powered descent initiation

STANDARD DEVIATION

FPAG	2.324854e+02	HPER	2.6235853e+03	HAPO	1.124967e+04	ECC	8.463729e+04	EIAS	6.7913355e+01	PVRT	5.593444e+01
MORV	3.6055513e+00	HDOT	2.2661077e+00	RANGE	1.2240423e+03	ELANG	9.2285637e+02	MTRU	2.6784932e+03	HEST	2.6039948e+03
WPR	1.2561481e+01	WF	5.8309519e+00	WOX	1.0583005e+01	DEVMT	0.0000000	TFIC	0.0000000	TTVTS	0.0000000
HVIS	0.0000000	BURN	0.0000000	TFXA	0.0000000	FC	6.6199125e+01	VERV	2.1822982e+00	MORV	3.8403722e+00
FPAGT	2.238497e+02	PVRTT	5.6226516e+01	LATLM	1.0458028e+01	LONLM	6.644759e+02	MIRA	0.0000000	MORV	0.0000000
MORZ	0.0000000	RESVG	3.8702319e+03	THF	0.0000000	PIO	0.0000000	PIF	0.0000000	DNO	0.0000000
DWF	0.0000000	RE	0.0000000	REE	0.0000000	CISP	0.0000000	ETSP	0.0000000		

NOMINAL VALUES

FPAG	-7.9921688e+02	HPER	5.0008687e+04	HAPO	3.6453837e+05	ECC	2.6662515e+02	EIAS	1.0248071e+02	PVRT	9.2203818e+01
MORV	3.5792822e+03	HDOT	-7.7435780e+00	RANGE	1.4489168e+00	ELANG	7.2482698e+00	MTRU	5.0401873e+04	HEST	5.0223362e+04
WPR	1.8324000e+04	WF	7.0334000e+03	WOX	1.1290600e+04	DEVMT	0.0000000	TFIC	-3.0000000e+01	TTVTS	0.0000000
HVIS	0.0000000	BURN	0.0000000	TFXA	0.0000000	FC	9.4221470e+01	VERV	-7.4933848e+00	MORV	5.5793885e+03
FPAGT	-7.62519e+02	PVRTT	9.2451460e+01	LATLM	-2.2682618e+00	LONLM	1.2252343e+01	MIRA	0.0000000	MORV	0.0000000
MORZ	0.0000000	RESVG	2.3877811e+03	THF	0.0000000	PIO	0.0000000	PIF	0.0000000	DNO	0.0000000
DWF	0.0000000	RE	0.0000000	REE	0.0000000	CISP	0.0000000	ETSP	0.0000000		

MEAN VALUES

FPAG	-8.6929771e+02	HPER	5.0928224e+04	HAPO	3.7080253e+05	ECC	2.7099259e+02	EIAS	1.0305669e+02	PVRT	9.2328288e+01
MORV	3.5500260e+03	HDOT	-6.4869519e+00	RANGE	1.4492514e+06	ELANG	7.2217820e+00	MTRU	5.1378609e+04	HEST	5.1127978e+04
WPR	1.8323402e+04	WF	7.0332226e+03	WOX	1.1290780e+04	DEVMT	0.0000000	TFIC	-3.0000000e+01	TTVTS	0.0000000
HVIS	0.0000000	BURN	0.0000000	TFXA	0.0000000	FC	9.4433085e+01	VERV	-6.7233858e+00	MORV	5.5800752e+03
FPAGT	-8.4539959e+02	PVRTT	9.2451460e+01	LATLM	-2.2511475e+00	LONLM	1.2360859e+01	MIRA	0.0000000	MORV	0.0000000
MORZ	0.0000000	RESVG	3.8702319e+03	THF	0.0000000	PIO	0.0000000	PIF	0.0000000	DNO	0.0000000
DWF	0.0000000	RE	0.0000000	REE	0.0000000	CISP	0.0000000	ETSP	0.0000000		

High gate

STANDARD DEVIATION

FPAG	1.1242571e+00	HPER	2.8963894e+03	HAPO	6.0898372e+02	ECC	5.2010428e+04	EIAS	1.0202069e+00	PVRT	6.0003089e+01
MORV	4.9513255e+00	HDOT	9.9566310e+00	RANGE	2.4308023e+02	ELANG	6.4429337e+01	MTRU	3.3799591e+02	HEST	3.1101122e+02
WPR	6.4321458e+01	WF	2.8711387e+01	WOX	4.1733022e+01	DEVMT	1.3125722e+01	TFIC	5.2574335e+00	TTVTS	0.0000000
HVIS	0.0000000	BURN	9.4484821e+00	TFXA	1.4441101e+01	FC	1.0268593e+00	VERV	9.7272211e+00	MORV	5.1729844e+00
FPAGT	1.1021476e+00	PVRTT	6.0384105e+01	LATLM	1.5421496e+02	LONLM	1.4887353e+02	MIRA	1.4778038e+03	MORV	1.0299619e+02
MORZ	1.251091e+03	RESVG	3.8609191e+03	THF	1.3332854e+02	PIO	3.5971451e+01	PIF	3.1535533e+01	DNO	2.4453139e+01
DWF	1.5236639e+01	RE	9.9579132e+03	REE	5.8771499e+03	CISP	2.5435657e+00	ETSP	1.5884263e+00		

NOMINAL VALUES

FPAG	-2.1902029e+01	HPER	-5.6742374e+06	HAPO	9.5090000e+03	ECC	9.9410988e+01	EIAS	7.1074898e+01	PVRT	5.5127206e+01
MORV	4.3928465e+02	HDOT	-1.7353173e+02	RANGE	2.3979420e+04	ELANG	1.5678601e+01	MTRU	6.6888320e+03	HEST	6.7800793e+03
WPR	3.9138933e+03	WF	1.4902225e+03	WOX	2.4236611e+03	DEVMT	2.4236611e+03	TFIC	5.1180000e+02	TTVTS	0.0000000
HVIS	0.0000000	BURN	3.6600000e+02	TFXA	1.7895912e+02	FC	7.0731556e+01	VERV	-1.7381708e+02	MORV	4.3928095e+02
FPAGT	-2.1532730e+01	PVRTT	5.3152985e+01	LATLM	-3.6014461e+01	LONLM	-1.7313123e+01	MIRA	3.7288250e+04	MORV	1.6728600e+04
MORZ	1.6728604e+04	RESVG	2.5697481e+03	THF	6.2286838e+03	PIO	2.3434467e+02	PIF	2.3434467e+02	DNO	1.2455271e+01
DWF	7.9221922e+00	RE	1.5971233e+06	REE	1.5995161e+00	CISP	3.0273201e+02	ETSP	3.0138927e+02		

MEAN VALUES

FPAG	-2.1935369e+01	HPER	-5.6742269e+06	HAPO	9.6408149e+03	ECC	9.9410643e+01	EIAS	7.1437618e+01	PVRT	5.5030538e+01
MORV	4.3404541e+02	HDOT	-1.7355555e+02	RANGE	2.3956103e+04	ELANG	1.5628824e+01	MTRU	6.7607720e+03	HEST	6.8419113e+03
WPR	3.9130805e+03	WF	1.4889578e+03	WOX	2.4243004e+03	DEVMT	5.4288447e+01	TFIC	5.1180000e+02	TTVTS	0.0000000
HVIS	0.0000000	BURN	3.6622000e+02	TFXA	1.7849823e+02	FC	7.0995067e+01	VERV	-1.7467885e+02	MORV	4.3835508e+02
FPAGT	-2.1567877e+01	PVRTT	5.3600178e+01	LATLM	-3.5989592e+00	LONLM	-1.7313123e+01	MIRA	3.7293819e+04	MORV	1.6743882e+04
MORZ	1.682637e+04	RESVG	5.9292099e+03	THF	6.2553127e+03	PIO	2.3444051e+02	PIF	2.3444051e+02	DNO	1.2495624e+01
DWF	7.99337808e+00	RE	1.5992063e+06	REE	1.5990165e+00	CISP	3.0293666e+02	ETSP	3.0169498e+02		

TABLE V. - SUMMARY OF STATISTICAL PARAMETERS - Continued

Range to go = 20 000 ft

STANDARD DEVIATION

FPAG	1.053208*00	HPER	2.9412161*01	HAPO	4.9725647*02	ECC	5.3202246*04	ELAS	4.7596979*01	PVRT	8.10955742*01
HORV	1.8048507*00	HOOT	8.0144482*00	RANGE	2.3599152*02	ELANG	5.8381778*01	HTRU	2.4121101*02	HEST	2.3593715*02
WPR	1.3483265*01	WF	2.7728032*01	WOX	4.2256656*01	DEVMT	6.4051780*01	TEIG	5.2940087*00	TTVIS	9.5993962*01
HVIS	3.2231971*02	BURN	9.4484621*00	TFXA	1.4441110*01	FC	1.2500000*03	VERVT	7.8507656*00	HORV	3.0131377*00
FPACT	1.0212944*00	PVRTT	8.6913749*01	LATLM	1.5409896*02	LONLM	1.2955585*02	MIRA	1.4778038*03	MLRV	5.1028179*02
HORZ	1.2351081*03	RESVG	3.9803919*01	THF	1.0817578*02	PIO	4.4028813*01	PIF	4.3582228*01	DWO	2.0138141*01
DWF	1.2960901*01	RE	1.10449675*02	REE	5.1922612*03	CISP	2.5483604*00	ETSP	1.5497622*00		

NOMINAL VALUES

FPAG	-2.0168845*01	HPER	-5.6782680*06	HAPO	6.9541250*03	ECC	9.9351447*01	ELAS	4.2703491*01	PVRT	2.8343785*01
HORV	4.9849609*02	HOOT	1.4121448*02	RANGE	1.9879490*04	ELANG	1.4460341*01	HTRU	4.7758625*03	HEST	5.0504402*03
WPR	3.1728763*03	WF	1.4128372*03	WOX	2.1300039*03	DEVMT	5.2200000*03	TEIG	8.0000000*00	TTVIS	3.8440176*02
HVIS	6.2561668*03	BURN	3.6600000*02	TFXA	1.7893823*02	FC	4.2420116*01	VERVT	-1.4746357*02	HORV	1.4728164*04
FPACT	-2.1024102*01	PVRTT	2.8368963*01	LATLM	-3.6042185*00	LONLM	-1.7358588*01	MIRA	3.7584952*04	MLRV	1.4963487*01
HORZ	1.6728604*00	RESVG	2.8697483*01	THF	5.8858607*03	PIO	2.3526870*02	PIF	2.3588849*02	DWO	
DWF	7.4889725*00	RE	1.5974804*00	REE	1.5995833*02	CISP	3.0154853*02	ETSP	3.0156869*02		

MEAN VALUES

FPAG	-2.0258401*01	HPER	-5.6784998*06	HAPO	6.8612189*03	ECC	9.9359587*01	ELAS	4.2463481*01	PVRT	2.8189066*01
HORV	3.807217*02	HOOT	-1.405061*02	RANGE	1.9594549*04	ELANG	1.4076902*01	HTRU	4.6997857*03	HEST	4.6483633*03
WPR	3.6976192*03	WF	1.4057823*03	WOX	2.2918368*03	DEVMT	5.579875*03	TEIG	8.6999999*00	TTVIS	3.4101931*02
HVIS	6.3398661*03	BURN	3.6622000*02	TFXA	1.7849823*02	FC	4.2440116*01	VERVT	-1.4778055*02	HORV	1.4945428*04
FPACT	-2.10404085*01	PVRTT	2.8206607*01	LATLM	-3.6020020*00	LONLM	-1.7355035*01	MIRA	3.7283612*04	MLRV	1.4974411*01
HORZ	1.6822663*04	RESVG	3.9292999*01	THF	5.8753129*03	PIO	2.3568188*02	PIF	2.3667491*02	DWO	
DWF	7.5004624*00	RE	1.5963970*00	REE	1.5989771*03	CISP	3.0160830*02	ETSP	3.0167490*02		

Range to go = 10 000 ft

STANDARD DEVIATION

FPAG	6.8227540*01	HPER	3.6108988*03	HAPO	1.9279571*02	ECC	5.5269685*04	ELAS	1.1965055*00	PVRT	5.1302809*03
HORV	6.6107933*00	HOOT	2.1891942*00	RANGE	1.1480431*02	ELANG	8.9957946*01	HTRU	1.8538610*02	HEST	1.8722945*02
WPR	3.12231971*02	WF	2.19328059*01	WOX	4.3563486*01	DEVMT	1.3327749*01	TEIG	5.2681976*00	TTVIS	4.7499100*01
HVIS	6.1903269*01	BURN	9.4484621*00	TFXA	1.4441110*01	FC	1.1151471*00	VERVT	3.2481684*00	HORV	2.8770487*00
FPACT	1.2691607*01	PVRTT	5.3258833*00	LATLM	3.5483233*02	LONLM	3.4322479*02	MIRA	3.2780386*03	MLRV	5.1028179*02
HORZ	1.2451081*03	RESVG	3.8050918*01	THF	7.2289349*01	PIO	3.6932286*01	PIF	3.6744900*01	DWO	1.4948866*01
DWF	9.17281997*02	RE	1.3537213*02	REE	6.11569923*03	CISP	2.9821189*00	ETSP	1.6348888*00		

NOMINAL VALUES

FPAG	-1.2683383*01	HPER	-5.6861354*06	HAPO	2.3311250*03	ECC	9.9826892*01	ELAS	3.9161979*01	PVRT	2.91444837*01
HORV	2.1448029*02	HOOT	-5.1510310*01	RANGE	1.9494410*03	ELANG	9.9480086*00	HTRU	1.7931199*03	HEST	1.7540981*03
WPR	3.1428666*03	WF	1.1934280*01	WOX	1.9494410*03	DEVMT	5.8264913*03	TEIG	5.5400000*02	TTVIS	4.0120000*01
HVIS	6.3398661*03	BURN	3.6600000*02	TFXA	1.7898392*02	FC	4.1193019*01	VERVT	-5.5111829*01	HORV	2.4430930*02
FPACT	1.2691607*01	PVRTT	2.9159184*01	LATLM	-3.6099411*00	LONLM	-1.7359188*01	MIRA	3.2844558*03	MLRV	1.4948866*01
HORZ	1.6728604*01	RESVG	2.8697483*01	THF	4.6329341*00	PIO	2.3599227*02	PIF	2.3849779*02	DWO	
DWF	5.9793328*00	RE	1.5984412*00	REE	1.5995207*00	CISP	2.9818195*02	ETSP	3.0138870*02		

MEAN VALUES

FPAG	-1.2633870*01	HPER	-5.6861482*06	HAPO	2.2852337*03	ECC	9.9827957*01	ELAS	3.8881366*01	PVRT	2.8710795*01
HORV	2.1441502*02	HOOT	-5.1495679*01	RANGE	1.9431390*03	ELANG	9.8269310*00	HTRU	1.6606623*03	HEST	1.6132406*03
WPR	3.1135344*03	WF	1.1895395*01	WOX	1.9462945*03	DEVMT	5.8264913*03	TEIG	5.5392000*02	TTVIS	4.0120000*01
HVIS	6.3398661*03	BURN	3.6622000*02	TFXA	1.7849823*02	FC	4.0581621*01	VERVT	-5.5119269*01	HORV	2.4430930*02
FPACT	-1.2705125*01	PVRTT	2.8733827*01	LATLM	-3.6089083*00	LONLM	-1.7353198*01	MIRA	3.2783612*04	MLRV	1.4948866*01
HORZ	1.6822663*04	RESVG	3.9292099*01	THF	4.6336022*03	PIO	2.3830588*02	PIF	2.3840086*02	DWO	9.6514171*00
DWF	6.1044847*00	RE	1.5967176*00	REE	1.5988863*00	CISP	2.9839915*02	ETSP	3.0150353*02		

TABLE V. - SUMMARY OF STATISTICAL PARAMETERS - Continued

Range to go = 5000 ft.

STANDARD DEVIATION

FPAC	9.15976680+01	HPER	2.6106980+03	HAPO	1.1611437+02	ECC	5.1790047+04	EIAS	1.39556742+00	PVRT	3.0526787+01
HRV	1.9433390+00	HDOT	2.6400329+00	RANGE	9.0033327+01	ELANG	1.2699124+00	HTRU	1.0733410+00	HESY	1.0851154+02
WPR	6.8095061+01	WF	3.0914448+01	WOX	4.4840007+01	DEVMT	1.4124447+01	TEIG	5.2526035+00	TTVIS	7.0113009+01
HVTS	3.2231971+02	BURN	9.4486821+00	TPXA	1.4441101+01	FC	1.3871888+00	VERVT	2.5779104+00	HORVT	2.4088887+00
FPAGT	9.11958436+01	PVRTT	3.0248837+03	LATLM	1.5240890+02	LONLM	1.4874957+02	HLRA	1.4777838+01	MLRV	5.1828619+02
HORZ	1.2310781+03	RESVG	3.9889839+03	THF	3.0116441+01	PIO	1.1717777+01	PIF	1.4058611+01	DWO	6.3321135+02
DWF	5.16066982+02	RE	2.0361240+02	REE	6.4130495+03	CISP	2.4824449+00	EISP	1.6644842+00		

NOMINAL VALUES

FPAC	-9.16200221+00	HPER	-5.6892635+06	HAPO	1.1433125+03	ECC	9.9938539+01	EIAS	3.9950595+01	PVRT	2.1044659+01
HRV	1.5198433+02	HDOT	-2.591215+03	RANGE	4.7340108+03	ELANG	1.0198447+01	HTRU	1.0112197+03	HESY	9.1903126+01
WPR	2.7744923+03	WF	1.0516648+03	WOX	1.7328022+03	DEVMT	6.014510+03	TEIG	5.0000000+02	TTVIS	6.1080000+01
HVTS	6.2581668+03	BURN	3.6600000+02	TPXA	1.7898392+02	FC	4.1143498+01	VERVT	-2.3734429+01	HORVT	1.1811396+02
FPAGT	-9.1621080+00	PVRTT	2.8130388+01	LATLM	-3.6144540+00	LONLM	-1.7910629+01	HLRA	3.7284250+04	MLRV	1.0728604+04
HORZ	1.6728004+04	RESVG	2.3497481+03	THF	3.7903038+03	PIO	2.4002913+02	PIF	2.4011245+02	DWO	7.0942590+00
DWF	4.1913020+00	RE	1.9986660+00	REE	1.5994936+00	CISP	2.9887499+02	EISP	3.0111049+02		

MEAN VALUES

FPAC	-9.1089412+00	HPER	-5.6882915+06	HAPO	1.1058744+03	ECC	9.9937338+01	EIAS	3.9934074+01	PVRT	2.0011402+01
HRV	1.5288452+02	HDOT	-2.4459920+01	RANGE	4.8012134+03	ELANG	1.0892334+01	HTRU	9.5298753+02	HESY	9.1610159+02
WPR	2.1770892+03	WF	1.0488624+03	WOX	1.7220029+03	DEVMT	6.0168490+03	TEIG	5.7944000+02	TTVIS	6.8660000+01
HVTS	6.3398661+03	BURN	3.6622000+02	TPXA	1.7788982+02	FC	4.0849011+01	VERVT	-2.2438749+01	HORVT	1.1308070+02
FPAGT	-9.1066869+00	PVRTT	2.8020672+01	LATLM	-3.6129569+00	LONLM	-1.7803930+01	HLRA	3.7283612+04	MLRV	1.0948920+04
HORZ	1.6828637+04	RESVG	3.9292099+03	THF	3.8093087+03	PIO	2.4000937+02	PIF	2.4008553+02	DWO	7.0885135+00
DWF	4.1940047+00	RE	1.5963771+00	REE	1.5988031+00	CISP	2.9702411+02	EISP	3.0113017+02		

Range to go = 2000 ft.

STANDARD DEVIATION

FPAC	1.13541381+00	HPER	2.6604300+03	HAPO	7.0864888+01	ECC	5.4591505+04	EIAS	1.9965989+00	PVRT	3.3306720+01
HRV	1.9204493+01	HDOT	2.0913586+00	RANGE	2.5473568+01	ELANG	1.5313833+00	HTRU	3.9294946+01	HESY	3.2404432+01
WPR	9.1483991+01	WF	3.1625371+01	WOX	4.6239883+01	DEVMT	1.4044800+01	TEIG	3.2578067+00	TTVIS	0.0000000+01
HVTS	3.2231971+02	BURN	9.1448462+00	TPXA	1.4441101+01	FC	1.7807488+00	VERVT	1.9812163+00	HORVT	1.0902807+00
FPAGT	1.2247695+01	PVRTT	3.0295481+01	LATLM	1.5524332+02	LONLM	1.7278328+01	HLRA	3.4778328+01	MLRV	5.1924479+02
HORZ	1.2351081+03	RESVG	3.9605919+03	THF	3.1974599+01	PIO	3.1058601+01	PIF	3.4058611+01	DWO	7.2224438+02
DWF	5.16222292+02	RE	2.3768942+02	REE	6.17282782+03	CISP	2.6858643+00	EISP	1.6929888+00		

NOMINAL VALUES

FPAC	-1.0695529+01	HPER	-5.6906248+06	HAPO	6.2112500+02	ECC	9.9841144+01	EIAS	3.9792730+01	PVRT	2.3298872+01
HRV	1.4594204+01	HDOT	-1.5912954+01	RANGE	1.9252403+03	ELANG	1.6742053+01	HTRU	6.8439997+01	HESY	5.3277416+02
WPR	2.1884688+03	WF	1.0516648+03	WOX	1.5485200+03	DEVMT	6.1924845+01	TEIG	6.0400000+02	TTVIS	9.0000000+01
HVTS	6.2581668+03	BURN	3.6600000+02	TPXA	1.7898392+02	FC	4.0900242+01	VERVT	-1.5993333+01	HORVT	8.4329572+01
FPAGT	-1.0713942+01	PVRTT	2.3324583+01	LATLM	-3.6159471+00	LONLM	-1.7938789+01	HLRA	3.7284250+04	MLRV	1.0728604+04
HORZ	1.6728004+04	RESVG	2.3497481+03	THF	3.3771528+03	PIO	2.4073807+02	PIF	2.4081356+02	DWO	7.0081249+00
DWF	4.1938314+00	RE	1.9986660+00	REE	1.5994936+00	CISP	2.9887499+02	EISP	3.0099241+02		

MEAN VALUES

FPAC	-1.0082605+01	HPER	-5.6906197+06	HAPO	6.1345062+02	ECC	9.9839987+01	EIAS	3.9731128+01	PVRT	2.3369523+01
HRV	1.4502333+01	HDOT	-1.5033960+01	RANGE	1.9503666+03	ELANG	1.5923345+01	HTRU	5.6401697+02	HESY	5.5435549+02
WPR	2.1770892+03	WF	1.0516648+03	WOX	1.5435332+03	DEVMT	6.1924845+01	TEIG	6.0380000+02	TTVIS	8.4329572+01
HVTS	6.2581668+03	BURN	3.6622000+02	TPXA	1.7849823+02	FC	4.0504031+01	VERVT	-1.4731604+01	HORVT	8.0000000+01
FPAGT	-9.1050552+00	PVRTT	2.3388486+01	LATLM	-3.6141669+00	LONLM	-1.7952587+01	HLRA	3.7283612+04	MLRV	1.0728604+04
HORZ	1.6728004+04	RESVG	2.3497481+03	THF	3.3663031+03	PIO	2.4078000+02	PIF	2.4081356+02	DWO	6.9215861+00
DWF	4.1987952+00	RE	1.9986660+00	REE	1.5987811+00	CISP	2.9856653+02	EISP	3.0111277+02		



TABLE V. - SUMMARY OF STATISTICAL PARAMETERS - Concluded

Vertical descent initiation

STANDARD DEVIATION

HORV	4.482729+01	HDOT	5.994208+01	RANGE	3.305721+00	ELANG	1.2380779+00	HTRU	3.608761+00	HST	3.434282+00
WPR	7.0575978+01	WF	3.309220+01	WOX	4.7581349+01	DEVMT	1.3360339+01	TFIG	5.0237716+00	TTVIS	1.4261611+00
HVIS	3.2231971+02	BURN	9.4484621+00	TPXA	1.4441110+01	FC	1.0661119+00	VERVT	5.9038009+01	HORVT	5.9038009+01
FPACT	4.1229964+00	PVRTT	3.8577062+01	LATLM	1.1550262+02	LONLM	1.4745770+02	HLRA	1.4778038+03	HLRV	5.1028619+02
HORZ	1.2351081+03	RESVG	3.9608919+03	THF	4.3737855+01	PIO	3.0935922+03	PIF	3.2173584+03	DWO	9.5833831+02
DWF	7.12214469+02	RE	2.6676204+02	REE	7.1318112+03	CISP	2.7431644+00	EISP	1.7388005+00		

NOMINAL VALUES

HORV	5.7480612+00	HDOT	3.4231244+00	RANGE	4.7598800+01	ELANG	6.4782775+01	HTRU	1.0148179+02	HST	1.0108617+02
WPR	1.1492022+03	WF	7.3277818+02	WOX	1.2130424+03	DEVMT	6.4598911+03	TFIG	6.5400000+02	TTVIS	1.3000000+02
HVIS	6.12561668+03	BURN	3.6600000+02	TPXA	1.7898392+02	FC	7.4213544+01	VERVT	3.4898987+00	HORVT	5.6667902+00
FPACT	3.1480024+01	PVRTT	7.1887074+00	LATLM	3.6176324+00	LONLM	-1.7175914+01	HLRA	3.7284250+04	HLRV	1.6728604+04
HORZ	1.6728604+04	RESVG	2.5697481+03	THF	2.9348745+03	PIO	2.4130327+02	PIF	2.4137777+02	DWO	6.1042262+00
DWF	3.1819311+00	RE	1.5983811+00	REE	1.5994544+00	CISP	2.9525552+02	EISP	3.0061774+02		

MEAN VALUES

HORV	5.3737864+00	HDOT	4.0464866+00	RANGE	4.4581817+01	ELANG	6.6317628+01	HTRU	1.0563549+02	HST	1.0295071+02
WPR	1.9368532+03	WF	7.12747308+02	WOX	1.2091798+03	DEVMT	6.4853679+03	TFIG	6.5400000+02	TTVIS	1.3092000+02
HVIS	6.3398661+03	BURN	3.6622000+02	TPXA	1.7849823+02	FC	7.4389487+01	VERVT	3.8582289+00	HORVT	5.4975911+00
FPACT	3.5073051+01	PVRTT	7.1228423+00	LATLM	-3.6162010+00	LONLM	-1.7551852+01	HLRA	3.7283612+04	HLRV	1.6945828+04
HORZ	1.6826637+04	RESVG	5.9292099+03	THF	2.9726977+03	PIO	2.4128988+02	PIF	2.41335849+02	DWO	6.1760897+00
DWF	3.18712529+00	RE	1.5954668+00	REE	1.5926673+00	CISP	2.9586773+02	EISP	3.0073319+02		

Landing (probe contact)

STANDARD DEVIATION

FPAG	1.0389450+00	HPER	3.4346004+03	HAPO	1.5734159+01	ECC	4.5674529+04	EIAS	6.1108534+00	PVRT	6.676197+02
HORV	1.4071035+02	HDOT	1.9049228+02	RANGE	1.2283765+00	ELANG	6.0649580+00	HTRU	1.3779256+00	HST	1.7818183+00
WPR	5.7298005+01	WF	3.4061468+01	WOX	4.9696265+01	DEVMT	1.5297050+01	TFIG	5.2600000+00	TTVIS	1.4261611+00
HVIS	3.2231971+02	BURN	9.4484621+00	TPXA	1.4441110+01	FC	5.3900701+00	VERVT	1.2826843+01	HORVT	2.6371924+01
FPACT	5.139.23+00	PVRTT	5.7878330+02	LATLM	1.552164+02	LONLM	-1.5002238+02	HLRA	1.4278038+03	HLRV	5.1028619+02
HORZ	1.2351081+03	RESVG	3.9608919+03	THF	1.2730377+01	PIO	3.1326027+01	PIF	3.0618682+01	DWO	5.0921909+02
DWF	4.1598270+02	RE	2.7799353+02	REE	7.6836450+03	CISP	2.7629312+00	EISP	1.7485632+00		

NOMINAL VALUES

FPAG	1.6254933+01	HPER	5.6910549+06	HAPO	-2.3125000+00	ECC	9.9999286+01	EIAS	6.2356230+01	PVRT	1.0283552+01
HORV	1.9432643+01	HDOT	2.9687661+00	RANGE	7.4290006+00	ELANG	6.2160323+01	HTRU	1.3772203+01	HST	1.4034805+01
WPR	1.1749559+03	WF	6.2816558+02	WOX	1.0486707+03	DEVMT	6.6102717+03	TFIG	6.8200000+02	TTVIS	1.3000000+02
HVIS	6.2561668+03	BURN	3.6600000+02	TPXA	1.7898392+02	FC	6.2343395+01	VERVT	3.0627441+00	HORVT	6.0387149+02
FPACT	8.1848317+01	PVRTT	2.1233949+01	LATLM	-3.1617659+00	LONLM	-1.7358339+01	HLRA	3.7284250+04	HLRV	1.6728604+04
HORZ	1.6728604+04	RESVG	2.5697481+03	THF	2.8320708+03	PIO	2.4137209+02	PIF	2.4164228+02	DWO	9.5833831+02
DWF	1.6900342+00	RE	1.5982613+00	REE	1.5994357+00	CISP	2.9530667+02	EISP	3.0041689+02		

MEAN VALUES

FPAG	1.6727348+01	HPER	5.6910553+06	HAPO	4.3369375+00	ECC	9.9999242+01	EIAS	6.0345890+01	PVRT	1.7467398+01
HORV	1.6335566+01	HDOT	2.9578290+00	RANGE	6.0220962+00	ELANG	6.0396926+00	HTRU	1.2198216+01	HST	1.4034805+01
WPR	1.6389523+03	WF	6.1240941+02	WOX	1.0256427+03	DEVMT	6.3081384+03	TFIG	6.8200000+02	TTVIS	1.3092000+02
HVIS	6.3398661+03	BURN	3.6622000+02	TPXA	1.7849823+02	FC	6.3288737+01	VERVT	2.8875492+00	HORVT	4.4228639+01
FPACT	8.1203815+01	PVRTT	1.7978914+01	LATLM	-3.1618296+00	LONLM	-1.7522254+01	HLRA	3.7283612+04	HLRV	1.6945828+04
HORZ	1.6826637+04	RESVG	5.9292099+03	THF	2.8262066+03	PIO	2.4159738+02	PIF	2.4166311+02	DWO	5.0921909+02
DWF	1.6666508+00	RE	1.5951374+00	REE	1.5986150+00	CISP	2.9545838+02	EISP	3.0051905+02		



TABLE VI. - COVARIANCE MATRICES

(a) Powered descent initiation

9.1022842+06	2.1922943+09	1.2728848+07	-1.4868910+04	-2.4727373+02	1.9182411+03	-4.3637705+00
2.1528943+06	1.1335080+08	-5.9599165+06	1.5679057+03	2.8231400+03	1.5778965+03	1.5199105+00
1.2748899+07	-5.5639165+06	4.0735910+07	-3.6983119+04	-5.0167760+02	9.0778231+03	-1.2601439+07
1.4804810+04	1.5679057+03	-5.6883119+04	3.8912749+01	5.2897491+01	-9.9954089+00	1.1814411+04
2.4727373+02	2.8231400+03	-5.0167760+02	9.0778231+01	8.5892264+01	1.6378002+01	1.9712008+02
1.9182411+03	1.5778965+03	9.0778231+03	-9.9954089+00	1.4376028+01	7.9533315+00	-3.5026040+03
4.3637705+00	1.5199105+00	-1.2601439+07	1.1814411+04	1.9712008+02	-3.5026040+03	4.2212896+06
1.5022842+06	3.2228942+06	-4.0288893+07	-4.2288893+07	-2.2317873+02	-3.1774637+02	-2.0688888+04
1.5088032+07	3.6117436+06	4.0288893+07	3.7807089+04	4.2288893+02	-1.1008499+04	1.2293181+07
1.5008674+04	3.1782473+03	3.7807089+04	-3.5034673+01	-4.3460731+01	1.0237739+01	-5.2104420+04
1.5022842+06	3.2228942+06	-4.0288893+07	-4.2288893+07	-2.2317873+02	-3.1774637+02	-2.0688888+04
1.5101970+03	1.7783936+02	1.0047920+04	9.4872231+00	5.5246361+02	-2.6581482+00	3.1144811+03
1.5169954+03	-6.0167760+02	2.7999073+04	-1.4103010+01	5.4088165+01	1.3478724+01	-9.9712872+03
4.1122842+06	1.3884032+07	3.5030424+04	3.5030424+03	-3.5017970+03	1.5168854+03	-2.8799339+02
3.2228942+06	3.1611743+06	-3.7588478+03	-3.7588478+03	7.7783936+02	-6.0167760+02	-1.5168854+03
2.0005922+05	4.0288893+07	3.1723105+04	2.3093796+03	-1.0067928+04	2.7999073+04	-1.5168854+03
4.1528826+01	3.1780708+04	-3.5037946+01	-2.1022326+00	9.4872231+00	-1.6105016+01	1.6105016+01
2.2317373+02	4.2232909+02	4.0368607+01	4.0368607+01	5.5246361+02	5.4886485+01	5.4886485+01
-5.7714397+02	-1.1008409+04	1.0237373+01	1.0237373+01	-6.6581682+00	1.3478724+01	1.3478724+01
-8.0934848+04	1.2931811+07	-1.1113820+03	-5.4061387+03	3.1177799+02	-9.9712872+03	-9.9712872+03
3.2892473+04	1.0398386+09	-1.0833306+02	-5.9882254+03	1.1177799+02	3.1468169+03	3.1468169+03
1.8978008+09	4.1067379+07	3.6118083+04	3.6118083+03	-1.0114394+04	-2.9735493+04	-2.9735493+04
1.0833306+02	-2.1467388+03	2.1467388+01	2.1467388+00	-8.0888979+01	-7.4146888+00	-7.4146888+00
1.4347759+02	1.0234994+04	2.0879999+01	-1.8931999+01	-7.4146888+00	1.0031999+03	1.0031999+03
1.4889182+03	-8.9375993+04	2.0879999+01	-1.8931999+01	-7.4146888+00	1.0031999+03	1.0031999+03

(b) Landing

2.5812399+02	8.9399831+02	1.9883017+04	-4.9351738+02	9.7840028+01	-8.6373397+01	-2.8799339+02
6.6399121+02	1.0949973+06	-9.1429701+04	-2.9842443+01	-1.3974600+01	1.5047095+01	1.6241399+02
1.5823171+04	-7.1328231+06	1.3282311+06	1.3282311+06	-8.7536807+01	4.7228995+01	1.6179871+04
6.3517381+02	-2.1812463+01	4.3248563+00	1.7090131+02	7.7951192+03	6.4808928+02	6.4808928+02
9.7490229+01	1.1378700+01	6.1737882+01	6.1737882+01	-1.8787070+03	-6.7180753+03	-6.7180753+03
-8.6873197+01	1.1930470+01	4.7229292+01	7.7951192+03	-6.7180753+03	1.1015069+01	1.6009386+01
-8.5709135+02	1.1922784+06	1.6817987+04	6.9888828+02	-8.7412078+01	1.0669388+00	-2.8008753+02
5.4960289+02	1.1922784+06	9.6437070+04	2.9439981+01	1.3411207+01	1.0669388+00	-2.8008753+02
8.2384672+03	2.2153970+05	7.7544955+05	1.9079933+02	-2.7383627+02	1.4887713+02	3.5030424+02
8.6547843+02	2.1734370+05	2.1734370+05	1.4817506+02	6.6112600+03	-2.7322199+03	3.6689755+02
8.4713364+01	7.1848889+00	6.1039732+01	6.1039732+01	-1.0366992+03	5.8789386+03	8.3138989+01
5.2527265+01	-2.5274850+00	-4.1744398+01	-8.6145243+03	6.6547682+03	-9.5719871+02	-8.3138989+01
1.0986010+02	5.1166331+03	1.1252533+04	-2.3335998+00	-3.8262568+00	-3.8913460+00	5.2834076+01
5.4960289+02	8.1238467+03	-2.6547843+02	-5.4713364+01	9.3872698+01	1.0366992+03	-1.0366992+03
4.19622784+06	2.1215397+05	2.1734370+05	7.1648866+00	-5.8759159+00	5.1664391+03	5.1664391+03
9.6437070+04	-7.3445551+05	2.1734370+05	6.1039732+01	-1.1784399+01	1.2322351+04	1.2322351+04
2.9439981+01	1.6075833+02	-1.4847306+02	1.4847306+02	-6.5066992+03	-6.6145243+03	-2.3335998+03
1.3212737+01	-2.1734370+05	6.1039732+01	6.1039732+01	-1.0366992+03	6.0488782+03	-1.6288998+00
1.6247070+01	1.4287155+03	-2.7921999+03	5.8789386+03	-5.8789386+03	-5.8789386+03	-5.8789386+03
1.5277184+02	3.5030424+02	3.6657755+02	8.3138989+01	-8.3138989+01	5.2834076+01	5.2834076+01
1.9898775+06	-2.1492328+05	2.1492328+05	2.1492328+05	9.3864230+00	-5.1096219+03	-5.1096219+03
2.4923288+05	4.0776174+07	3.9161572+01	2.1551213+02	-3.1200287+03	-9.4439659+04	-9.4439659+04
-2.7107254+01	-5.9161572+01	1.3333597+02	-5.1221566+03	4.0067119+03	1.7486324+00	1.7486324+00
6.6360010+00	2.1551213+02	-5.1221566+03	8.2434361+02	-8.2434361+02	5.2104420+04	5.2104420+04
3.3844238+00	1.2500287+03	4.0067119+03	-8.2434361+03	6.4113959+02	3.6862769+00	3.6862769+00
5.10982179+03	891439653+04	1.1748632+00	1.2911457+00	3.8862769+00	7.2489985+03	7.2489985+03

TABLE VII.- CURRENT ESTIMATE OF LM LANDING DISPERSION ELLIPSE

Accelerometer bias ( $3\sigma$ ), cm/sec <sup>2</sup>	Percent ellipse	Down-range semimajor axis, n. mi.	Cross-range semiminor axis, n. mi.
0.2	99	0.54	0.73
.2	90	.39	.52
.2	50	.21	.28

TABLE VIII.- ΔV AND PROPELLANT SUMMARY

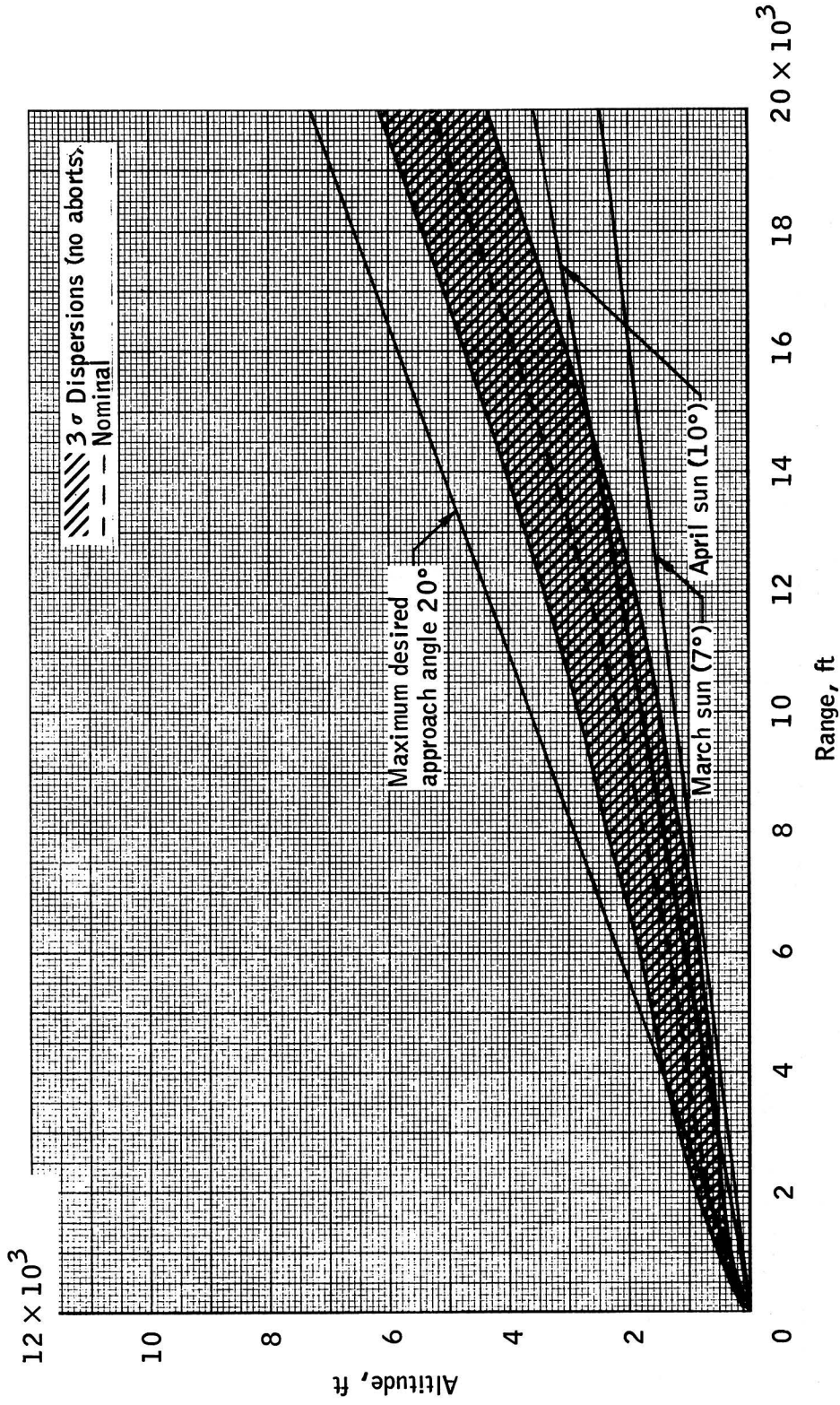
(a) Major event points<sup>a</sup>

Event	ΔV, fps	Propellant, lb	Oxidizer, lb	Fuel, lb
Braking (high gate)	5428 ± 13	14 410 ± 65	8866 ± 42	5543 ± 28
Approach (P64 terminus)	6460 ± 13	16 378 ± 72	10 077 ± 47	6300 ± 33
Landing	6630 ± 15	16 685 ± 73	10 264 ± 49	6420 ± 33

(b) Total descent

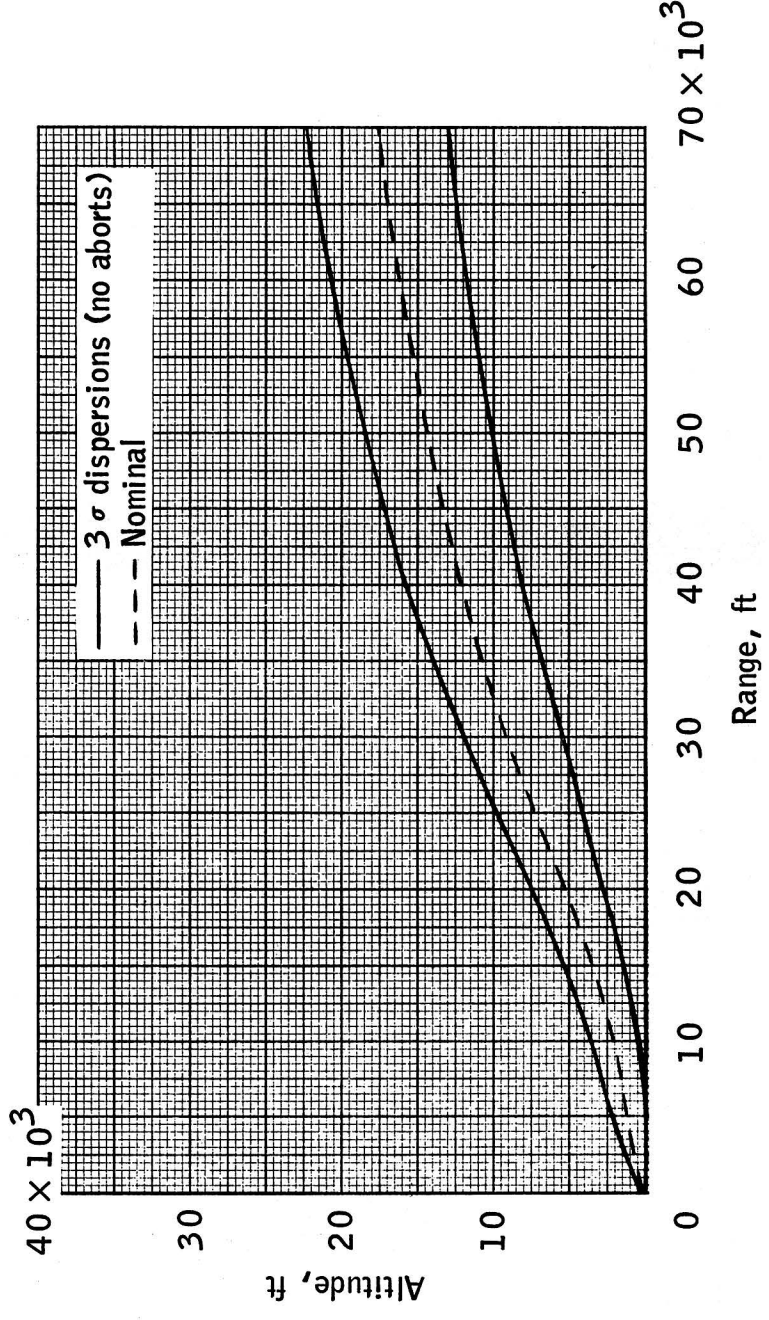
Item	Propellant, lb	Oxidizer, lb	Fuel, lb
Loaded (tank)	18 324.0	11 290.6 ± 10	7033.2 ± 6.3
Required	16 685.0 ± 73	10 264.0 ± 48	6420.0 ± 33
Usable remaining	1540.0 ± 73	948.0 ± 49	592.8 ± 31

<sup>a</sup>All uncertainties presented are 1σ.



(a) Range 20 000 feet to 0.

Figure 1. - Altitude dispersion corridor.



(b) Range 70 000 feet to 0.

Figure 1. - Concluded.

# FRA MAURO

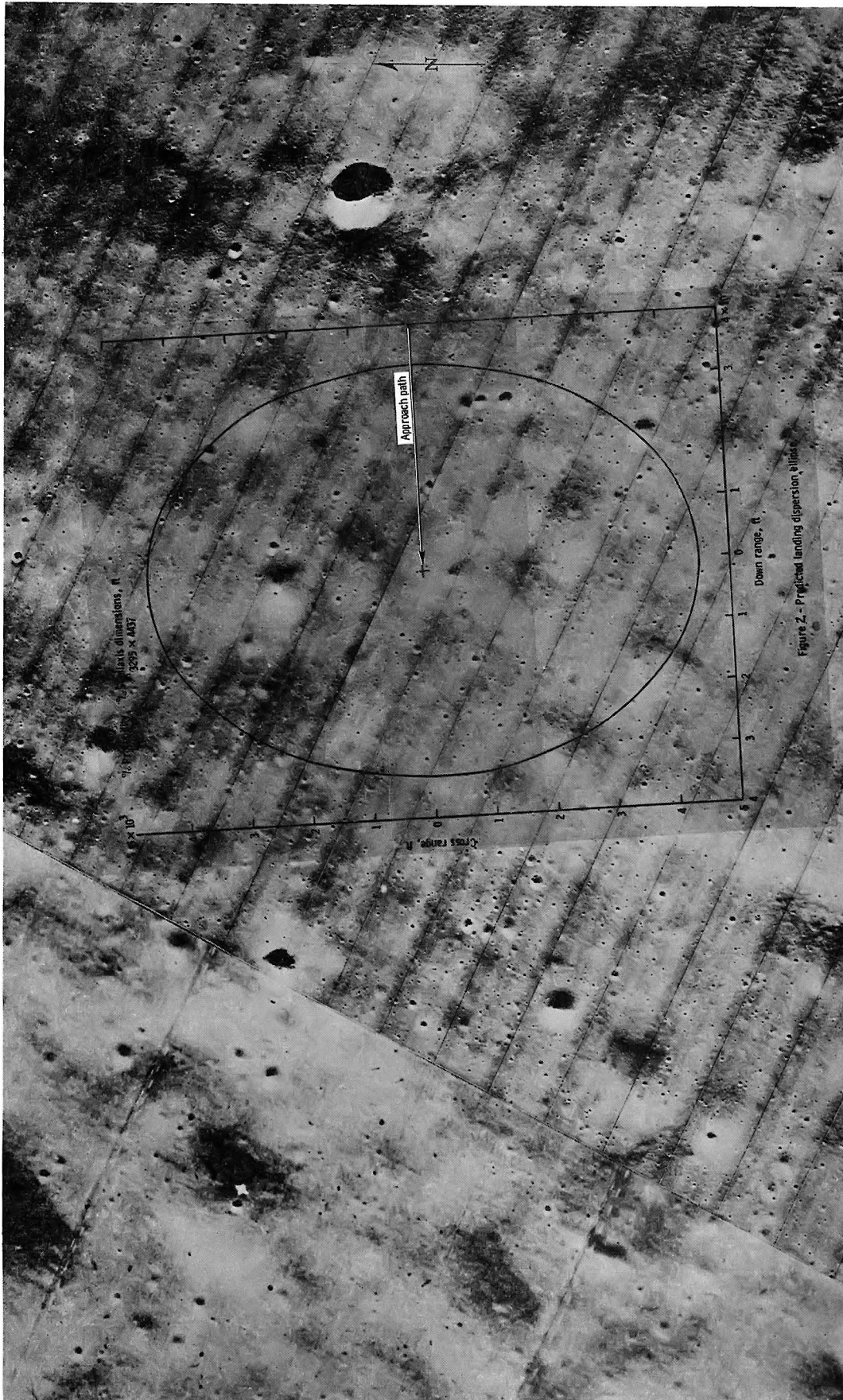


Figure 2 - Predicted landing dispersion ellipse



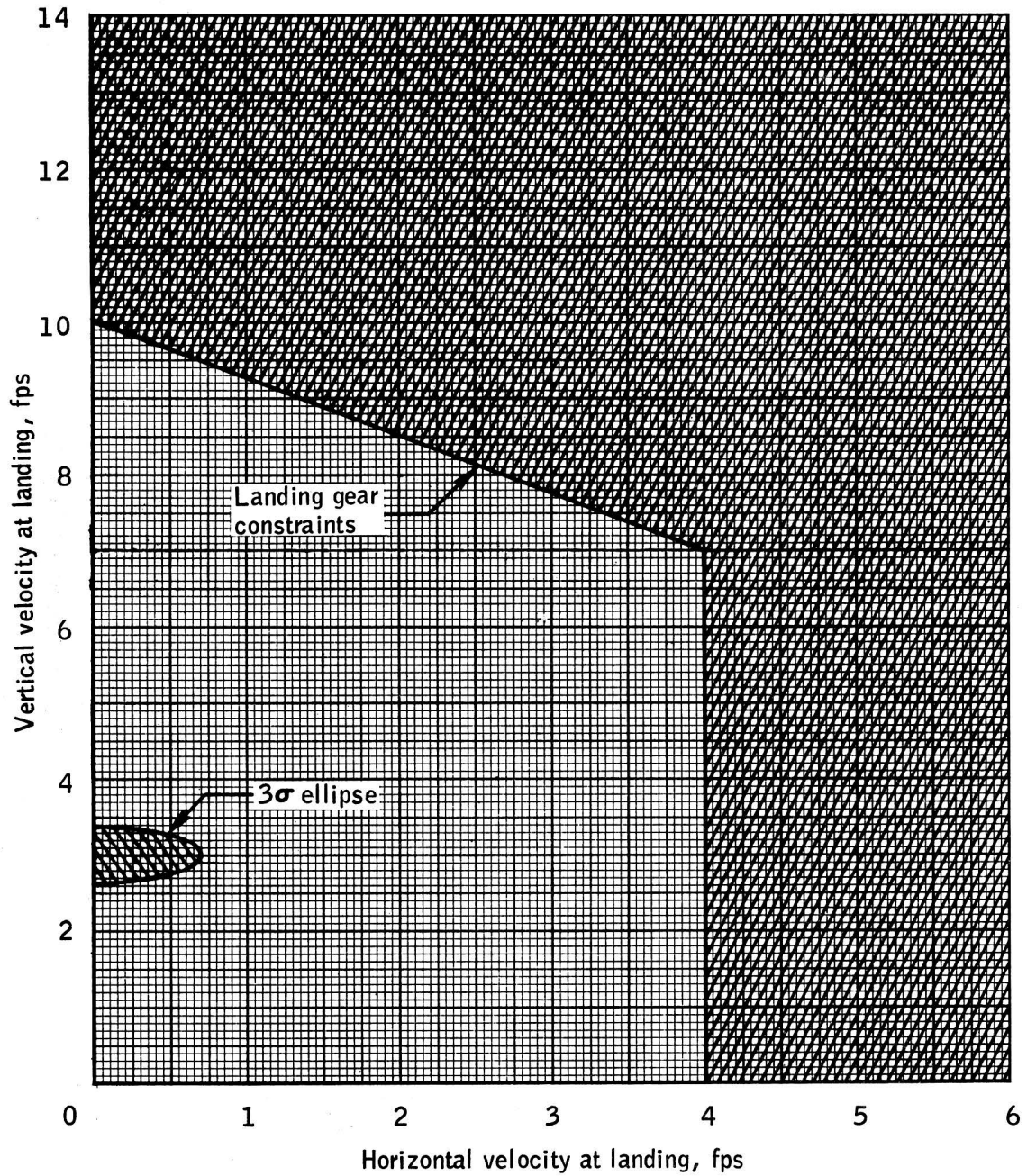
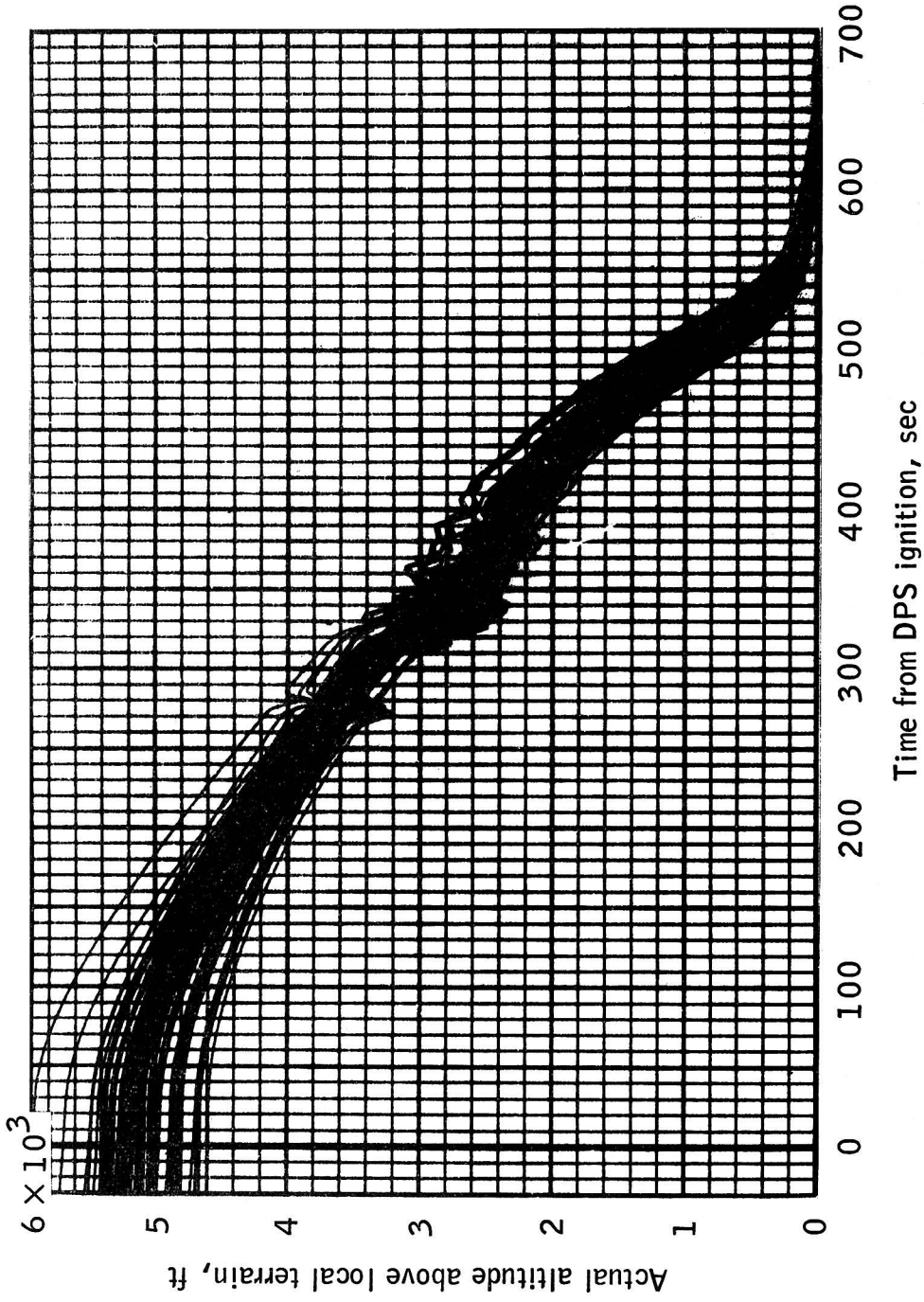


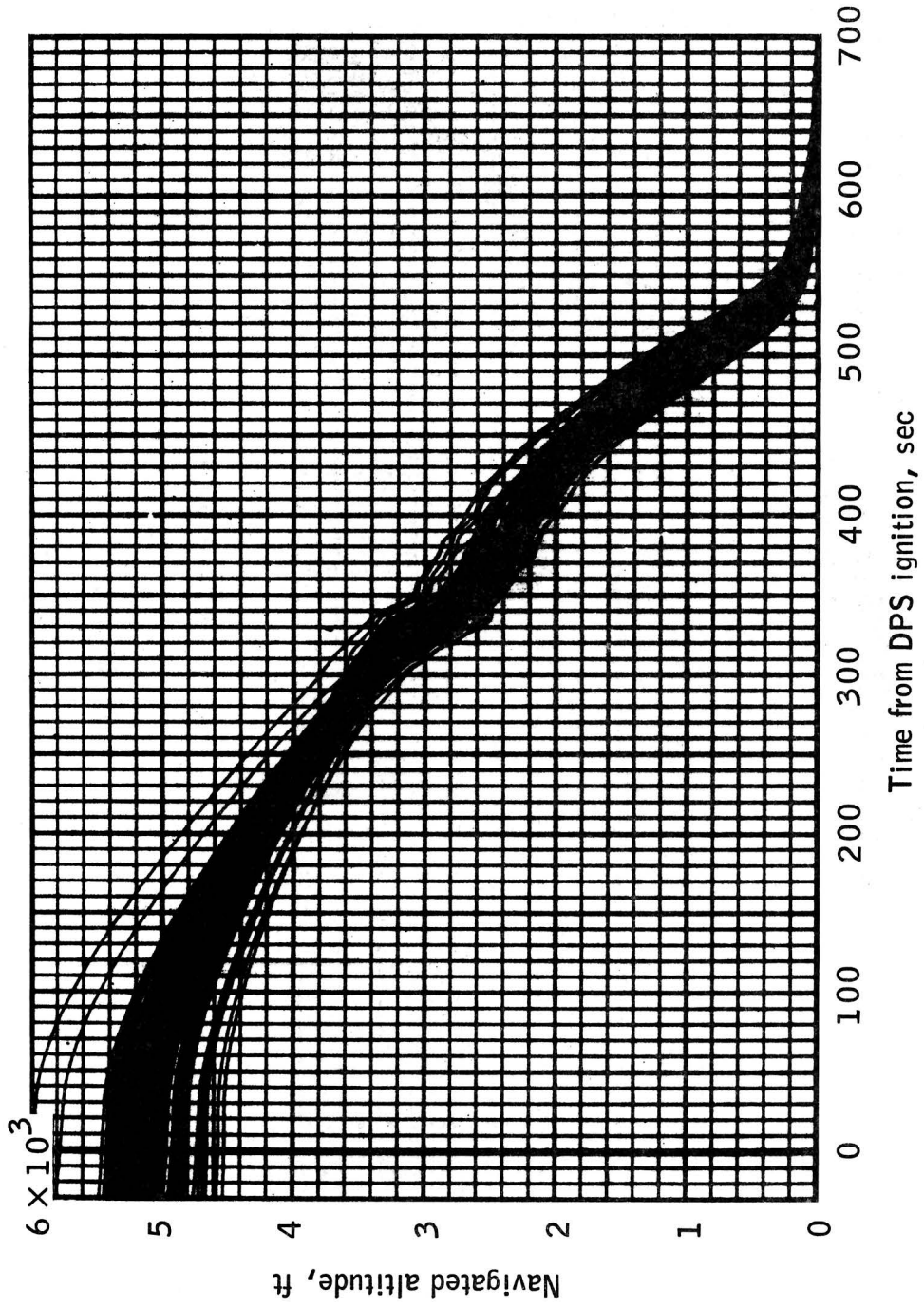
Figure 3.- Landing gear constraints at landing and three-sigma velocity error ellipse at probe contact.



(a) Actual altitude.

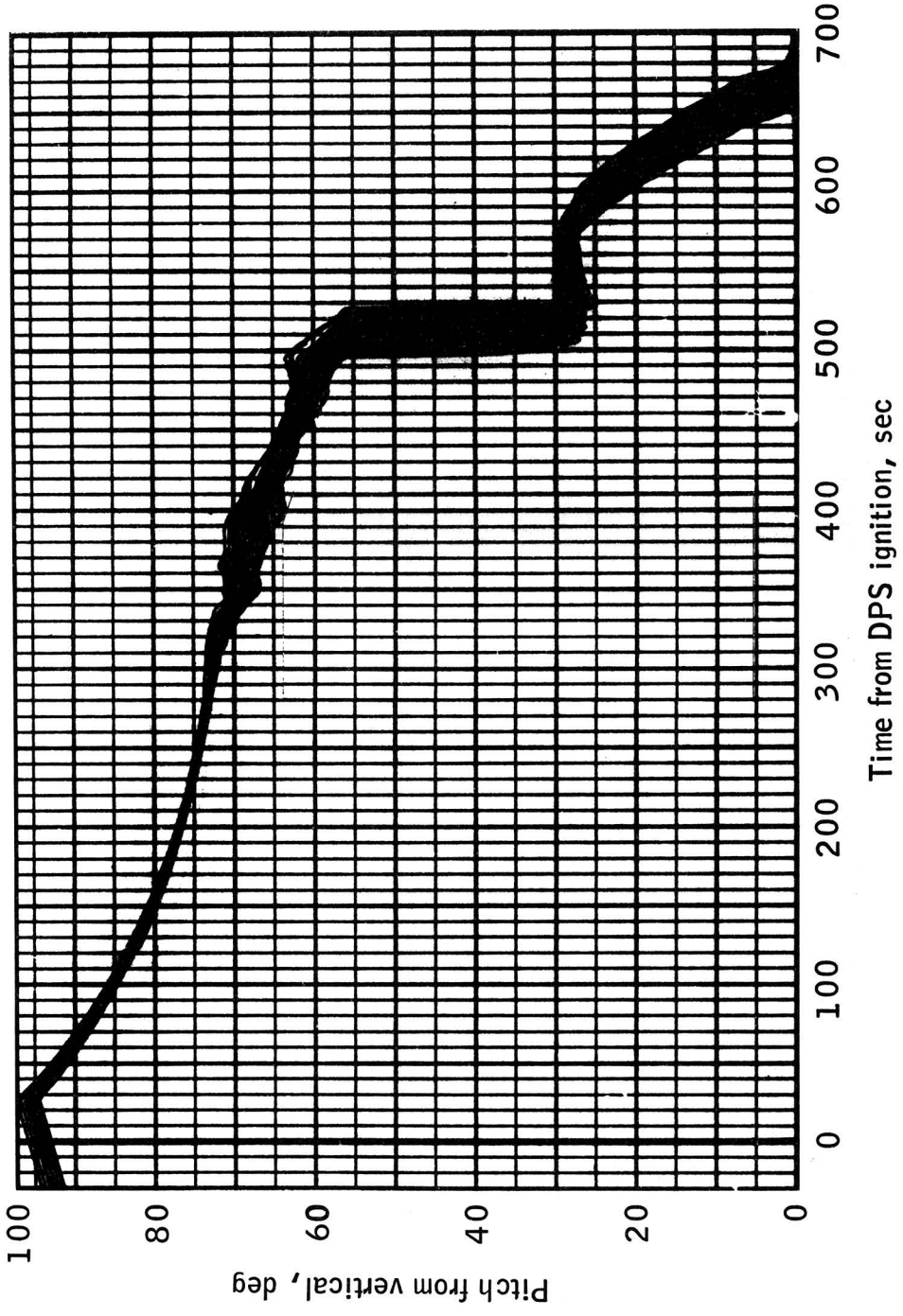
Figure 4.- Time histories of selected parameters.





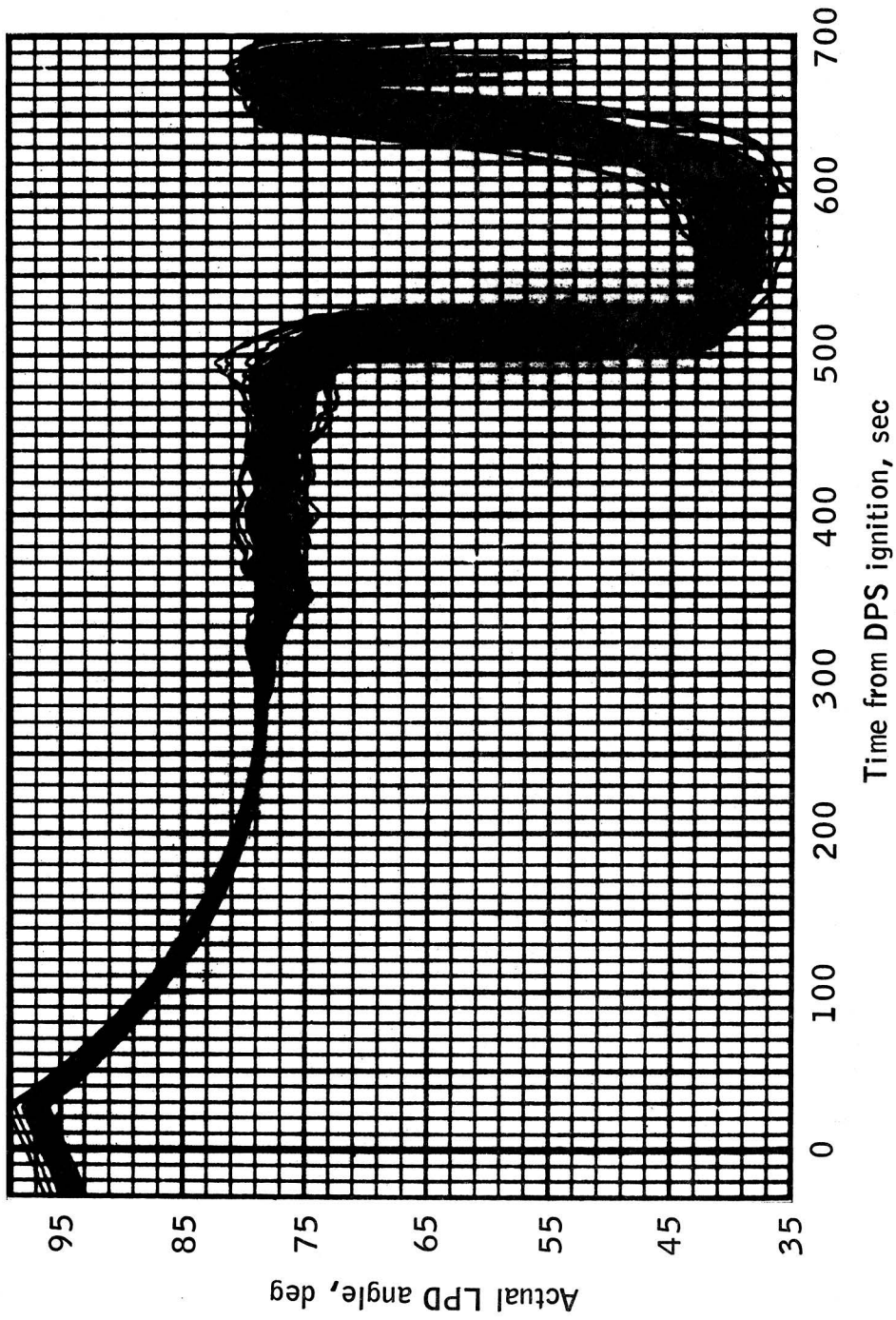
(b) Navigated altitude.

Figure 4.- Continued.



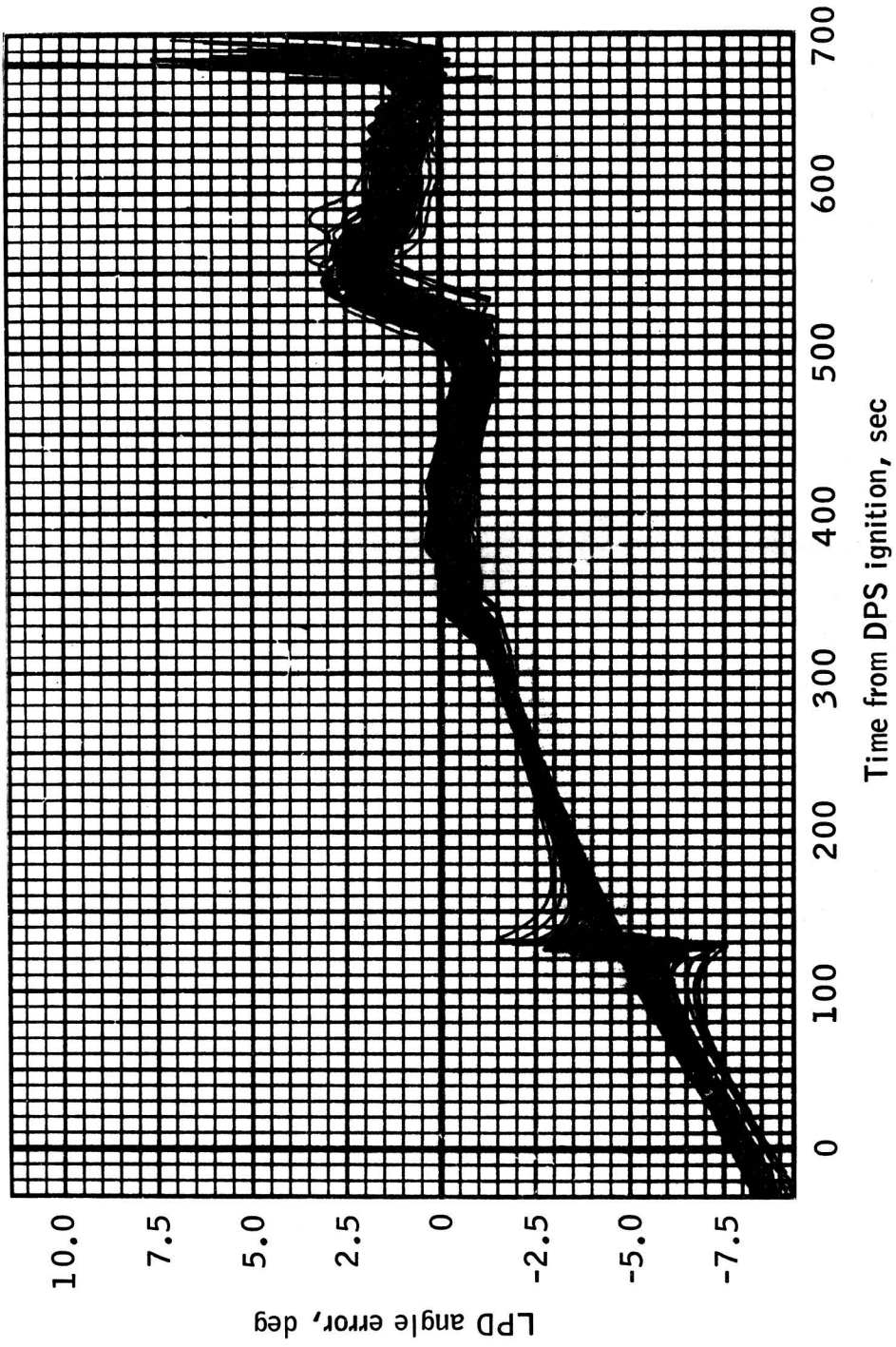
(c) Pitch from vertical.

Figure 4. - Continued.



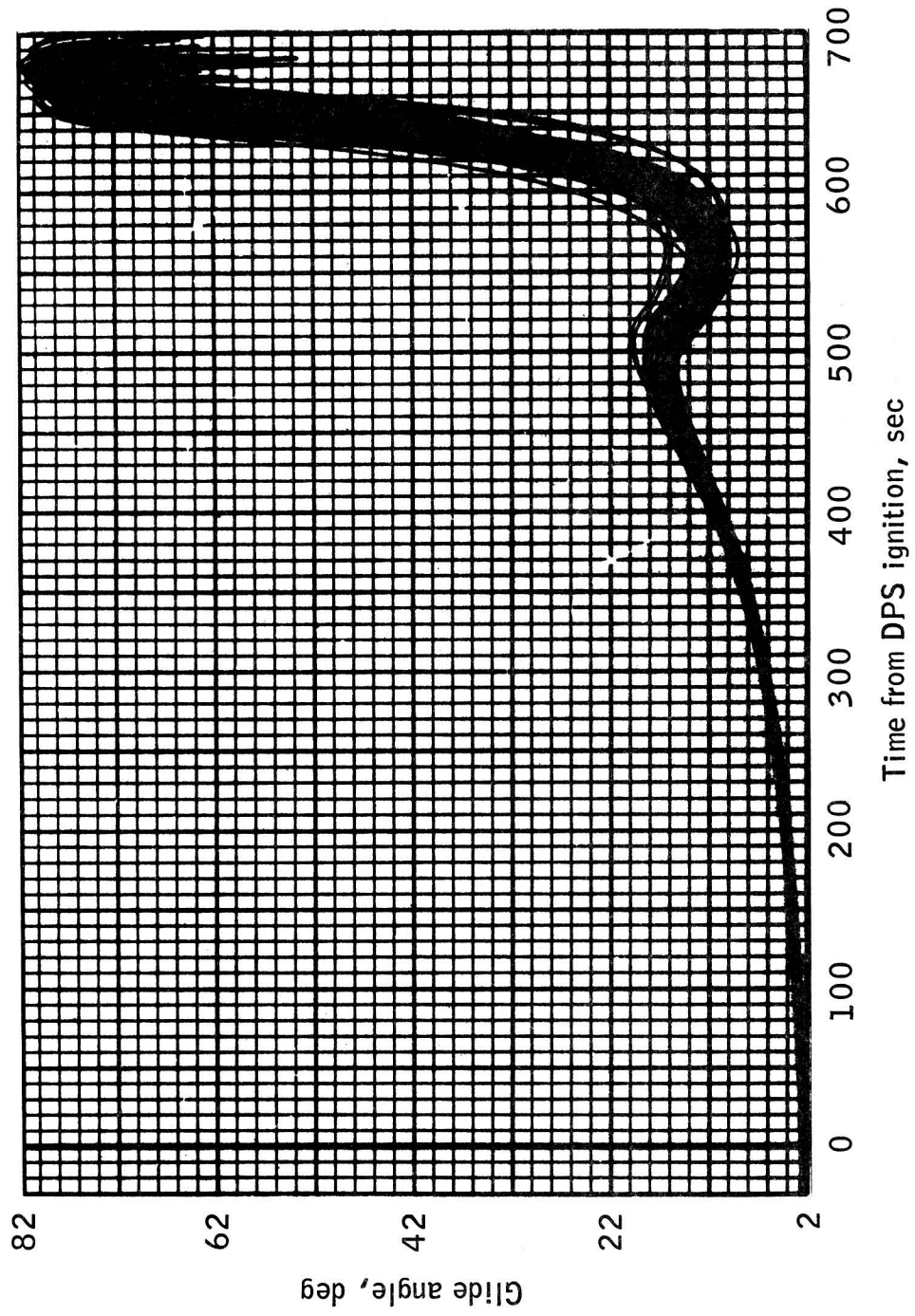
(d) Actual LPD angle.

Figure 4.- Continued.



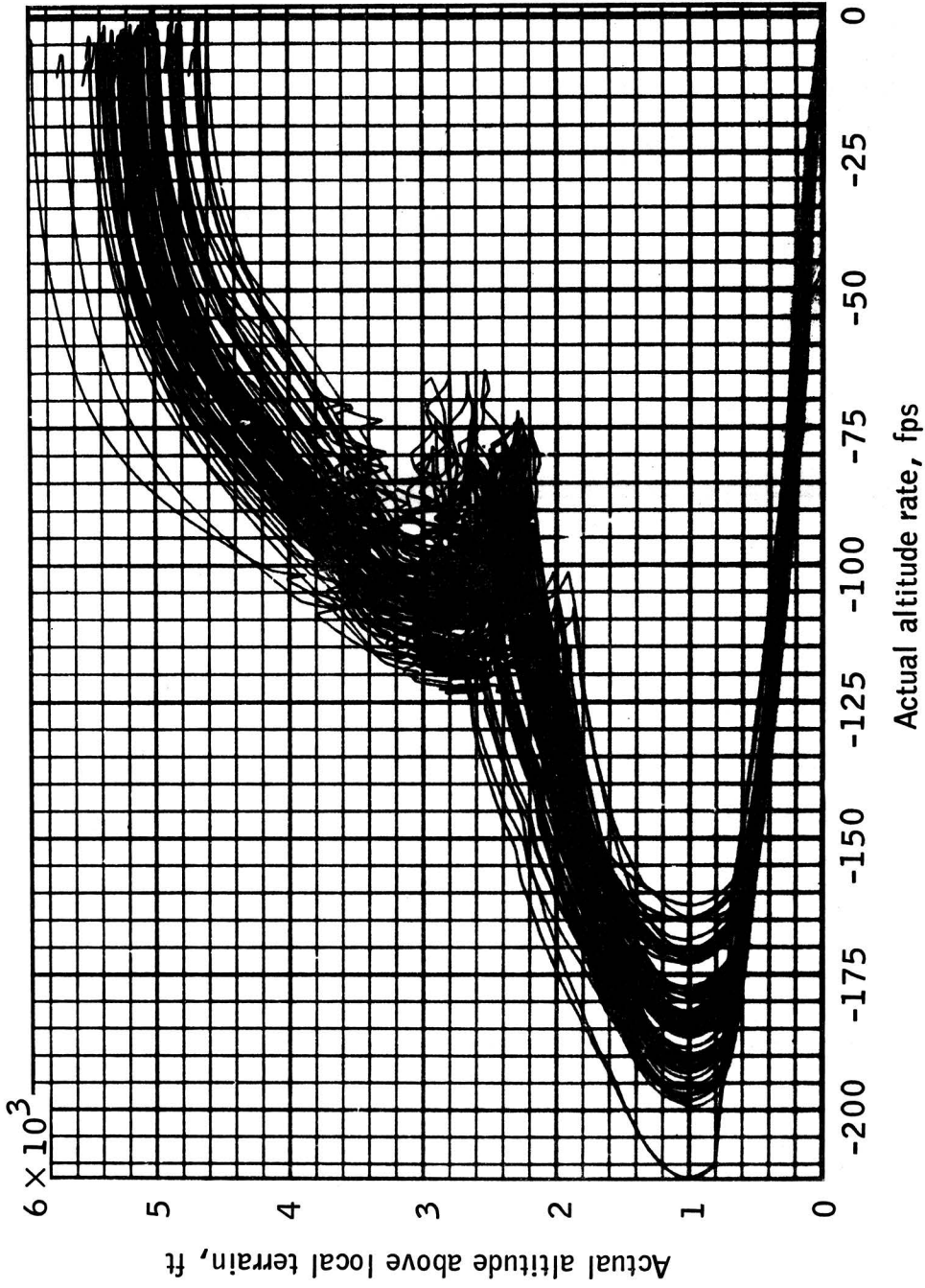
(e) LPD angle error.

Figure 4.- Continued.



(f) Glide angle.

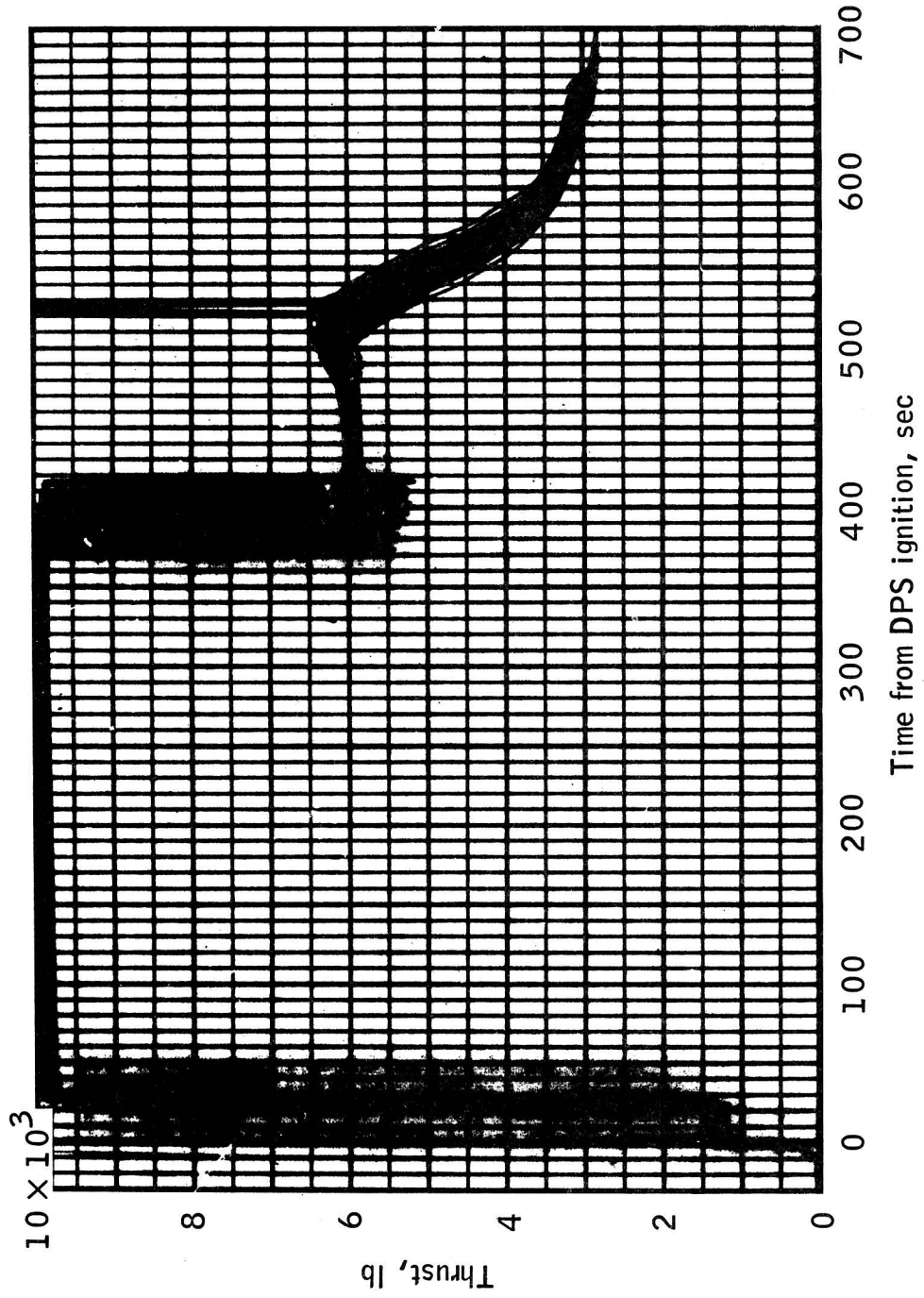
Figure 4.- Continued.



(g) Actual altitude.

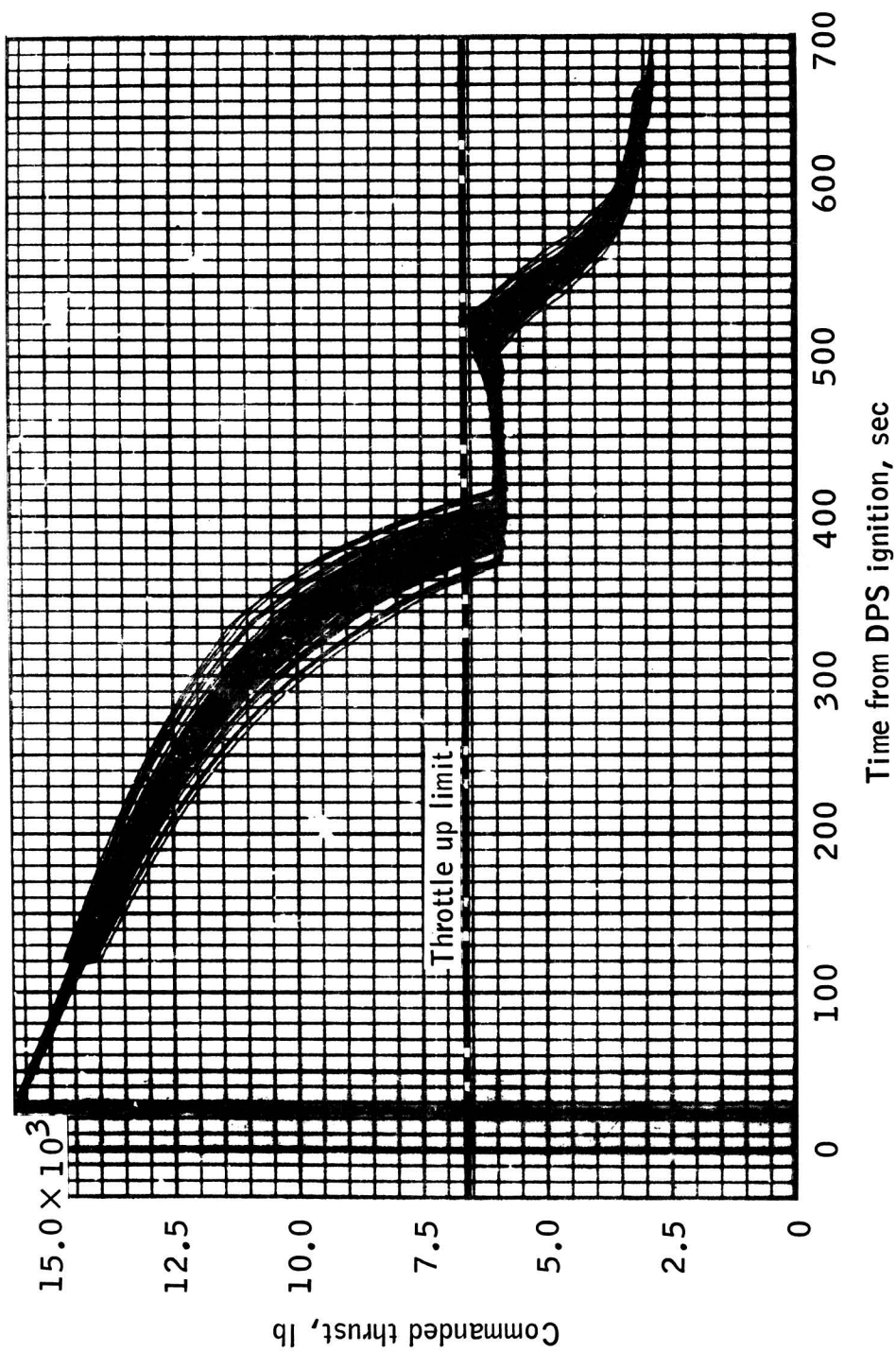
Figure 4.- Continued.





(h) Thrust.

Figure 4.- Continued.



(i) Commanded thrust.

Figure 4.- Concluded.



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## PART 2 - LUNAR ASCENT

A 100-case Monte Carlo dispersion analysis of the Apollo 13 LM powered ascent from the lunar surface indicates no significant changes from the dispersions for Apollo 12 (ref. 1). The dispersions, computed by this analysis, are summarized in the following table.

TABLE I.- ASCENT MANEUVER SUMMARY

Parameter	Nominal	Mean	$3\sigma$ deviation
Actual altitude, ft . . . . .	59 939	59 833	2141
Actual out-of-plane <sup>a</sup> distance, ft . . . . .	0.0	0.0	13 734
Actual radial velocity, fps . . . . .	36.3	35.5	11.7
Actual out-of-plane <sup>a</sup> velocity, fps . . . . .	0.0	0.0	30.3
Actual local horizontal velocity, fps . . . . .	5533.4	5533.8	9.0
Magnitude of total velocity, fps . . . . .	5533.5	5533.9	10.5
Magnitude of PGNC total velocity, fps . . . . .	5533.8	5533.8	8.5
Desired pitch from local horizontal, deg . . . . .	-103.0	-103.0	1.0
Perilune altitude, ft . . . . .	53 134.7	53 177.5	2508.0
Apolune altitude, ft . . . . .	266 533	267 880	17 121
True anomaly, deg . . . . .	20.9	20.5	7.8
Burn time, sec . . . . .	428.1	427.6	14.4
Insertion weight, lb . . . . .	5808	5807	39
Propellant used, lb . . . . .	4861	4861	45
Usable propellant remaining, lb . . . . .	318	313	51
Total $\Delta V$ , fps . . . . .	6044.8	6045.8	22.2

<sup>a</sup>Includes CSM uncertainties.

## REFERENCE

1. West, J. V.: Apollo 12 (Mission H-1) Dispersion Analysis, Vol. IV - Descent and Ascent Dispersion Analysis, Part 2 - Lunar Ascent. MSC IN 69-FM-280, Nov. 4, 1969.



