

Complex Modes in Lightly Damped Structural Systems: A Two-Mode Interaction Perspective

Complex mode shapes can emerge in a linear structure even when overall damping is low. This behavior can occur when two structural modes are closely spaced in frequency and are coupled through nonproportional damping. As the damping interaction increases, the two modal frequencies move toward one another while the mode shapes become increasingly complex. At a critical interaction level, the two frequencies coalesce. Beyond this point, the frequencies remain approximately equal while the modal decay rates separate. At very strong interaction, the mode shapes become approximately real again, but as different combinations of the original uncoupled modes. This framework provides a simple physical interpretation of mode veering, complex mode formation, eigenvalue coalescence, and decay-rate splitting in lightly damped structural systems.

Why Complex Modes Can Appear at Low Damping

Classical modal analysis often treats lightly damped structures as nearly proportional systems with real mode shapes. That interpretation can become misleading when two modes are closely spaced. In modal coordinates, nonproportional damping introduces an off-diagonal coupling term ϵ . The relevant question is therefore not simply whether damping is small, but whether the coupling is large relative to the baseline half-frequency separation $\delta = (\omega_2 - \omega_1)/2$.

Reduced Perturbation Framework

For a closely spaced pair, define the central eigenvalue $\lambda_0 \approx -\zeta\omega + i\omega$ and write the coupled eigenvalues as $\lambda_{1,2} = \lambda_0 \pm s$. Under light-damping and close-mode asymptotic assumptions, the local quartic eigenvalue problem reduces to $s^2 = \epsilon^2/4 - \delta^2$. This single relation determines the direction of eigenvalue separation in the complex plane, as summarized in Figure 1. For $0 < \epsilon < 2\delta$, $s = \pm i\alpha$ and the split is predominantly vertical (frequency). At $\epsilon = 2\delta$, $s = 0$ and the pair coalesces. For $\epsilon > 2\delta$, $s = \pm\beta$ and the split is predominantly horizontal (decay rate). As ϵ increases further, β increases and the two decay rates separate more strongly while their frequencies remain approximately equal.

Engineering Interpretation and Recommended Practice

Complexity in an identified mode shape should not automatically be interpreted as evidence of heavy damping, nonlinearity, or poor test data. For a close modal pair, examine the complex poles together with mode-shape phase. Frequency convergence accompanied by near-quadrature modal components is consistent with the imaginary- s regime; nearly equal frequencies accompanied by decay-rate separation is consistent with the real- s regime. The framework also provides a practical test-analysis bridge: $s = (\lambda_1 - \lambda_2)/2$ can be obtained from identified poles, while δ can be estimated from the uncoupled or pretest analytical modal spacing. These quantities can then be used to estimate the effective interaction strength and determine proximity to the transition boundary.

References

1. Majed, A., Henkel, E. E., and Sills, J. W., Emergence of Complex Modes in Lightly Damped Structural Systems: A Reduced Perturbation Framework, NASA/TM-20260007553, August 2026.

Two-Mode Interaction in a Lightly Damped System

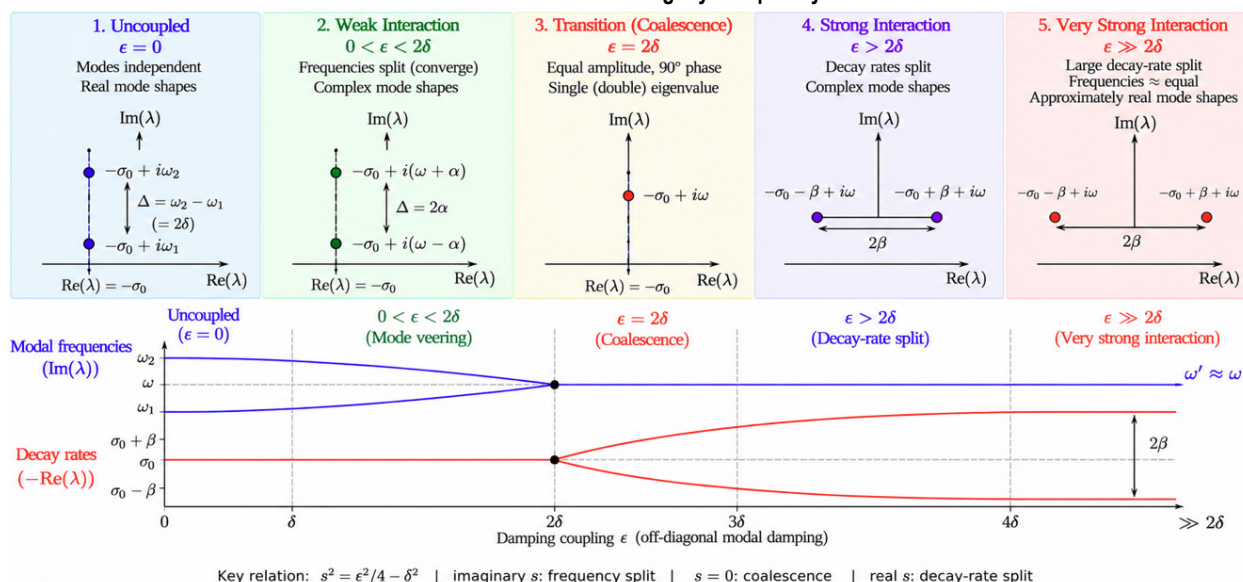


Figure 1. Simplified two-mode interaction diagram. Increasing off-diagonal modal damping coupling moves the system from frequency splitting, through coalescence, to decay-rate splitting.

