


CURRENT STATUS OF PDE-BASED TRANSITION MODELING FOR AERODYNAMICS APPLICATIONS



JIM CODER

Associate Professor, Aerospace Engineering
Penn State University

Presented at the 2022 Symposium on Turbulence Modeling:
Roadblocks, and the Potential for Machine Learning

29 July 2022

LESSONS FOR A YOUNG ENGINEER

LESSON #1

Know who the important people are *before* you give your presentation

LESSON #2

If you're going to argue with Philippe, you need to *really* know what you're talking about

IMPACT OF TRANSITION ON AIRCRAFT



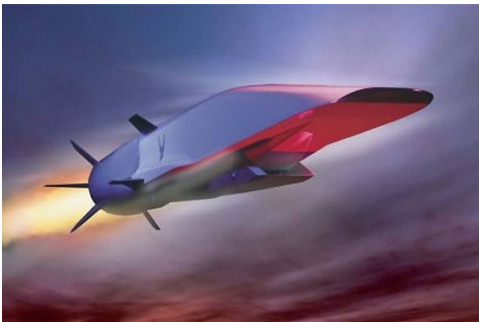
Ultra Efficient Aircraft

- NLF wings enable revolutionary leaps in performance
- Crucial for reducing carbon footprint as air travel continues to grow



Rotorcraft

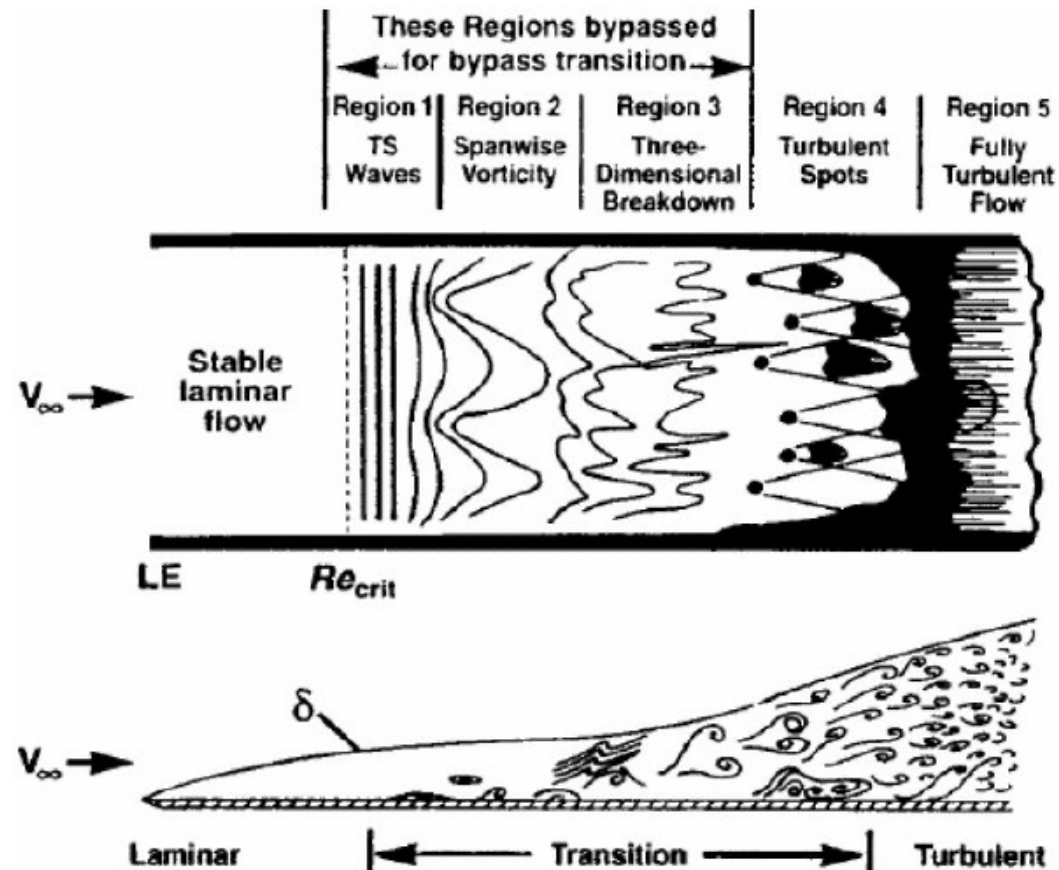
- Hover lift capabilities directly impacted by boundary-layer transition
- Laminar flow enables expansion of operating envelope in new designs
 - Small-scale UAS anticipated to operate in transitional flow regime
 - Trailing-edge noise directly impacted by boundary-layer state



Hypersonic Systems

- Transition has leading-order impact on surface heating, driving vehicle design
 - Transitional SBLI phenomena can introduce large unsteady loads and influence control effectiveness

LAMINAR-TURBULENT TRANSITION



Schlichting (1979)

Transition in flight is (assumed) dominated by linear mechanisms

Linear instability growth is solution of a non-local eigenvalue problem

$$-i\omega \mathbf{I} \hat{q} = \frac{1}{Re} (\mathbf{D}_{yy} - \alpha^2 \mathbf{I}) \hat{q} - i\alpha \mathbf{A} \hat{q} - \mathbf{B} \mathbf{D}_y \hat{q}$$

$$q' = \hat{q}(y) e^{i(\alpha x + \beta z - \omega t)}$$

Physics are incompatible with Reynolds averaging

QUALITIES OF A CFD TRANSITION MODEL?

What **do** we want from the transition model?

- Accurate prediction of transition onset location from a variety of mechanisms
- General 3D formulation, applicable to arbitrarily complex geometries
- Amenability to massive parallelization (i.e. don't be a bottleneck)
- Care-free application with minimal additional user intervention
- Robust convergence

What **don't** we want from the transition model?

- Non-local searches or integral operations
- Excessive increases in computational cost
- Alter post-transition turbulence model behavior

PDE-BASED TRANSITION MODELING

$$\frac{\partial \rho \phi}{\partial t} + \frac{\partial \rho u_j \phi}{\partial x_j} = P_\phi - E_\phi + \frac{\partial}{\partial x_j} \left[\sigma (\mu + \mu_t) \frac{\partial \phi}{\partial x_j} \right]$$

Advection-diffusion-type PDEs with **single-point closure**

- Fully compatible with Navier-Stokes solution algorithms
- Generalized, Galilean-invariant, 3D formulation

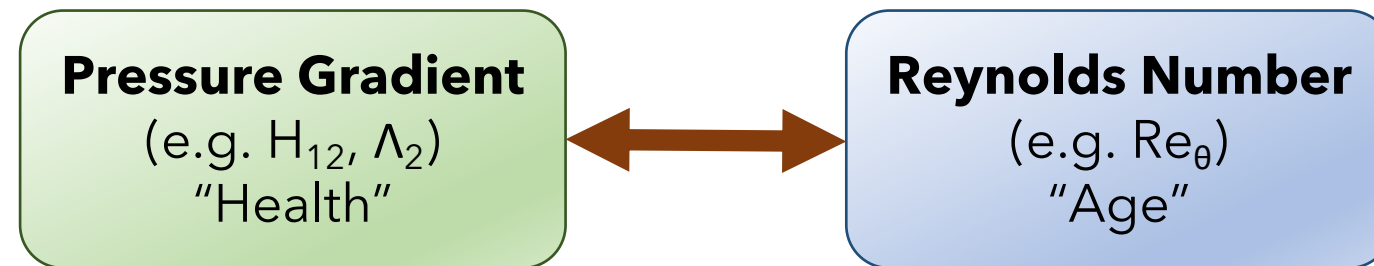
Two broad categories of PDE-based models

- Physics-based - Direct modeling of underlying physics
- Phenomenological - Modeling of surrogate indicators

PHENOMENOLOGICAL MODELS

Directly modeling linear mechanisms is paradoxical in a RANS setting, so we look for surrogate indicators

- For Tollmien-Schlichting instabilities,



In current practice, this is a two-step process

- Integral boundary-layer properties already established as modeling surrogates
- *Local* surrogates are then needed to estimate the *non-local* surrogates

PROMINENT MODEL TYPES

Modeling Variables

Local Correlation Transition Models (“Menter-type”)

- Streamwise transition prediction using local pressure gradient parameter and estimated transitional Re_θ
- Varying numbers of transport equations

$$\gamma, \lambda_\theta, \widetilde{Re}_{\theta t}$$

Stability-Based Transport Models

- Evolve the margin to transition along streamlines
- Amplification Factor Transport models of Coder et al.
- AHD-based models of Pascal et al. and Stroer et al.

$$\gamma, H_{12}, \tilde{n} \\ \gamma, \bar{\lambda}_\theta, \bar{l}_c, Re_{\theta c}$$

Algebraic Models

- Based solely on local Reynolds number
- Bas-Cakmakcioglu (SA-BCM model) seeing increased popularity

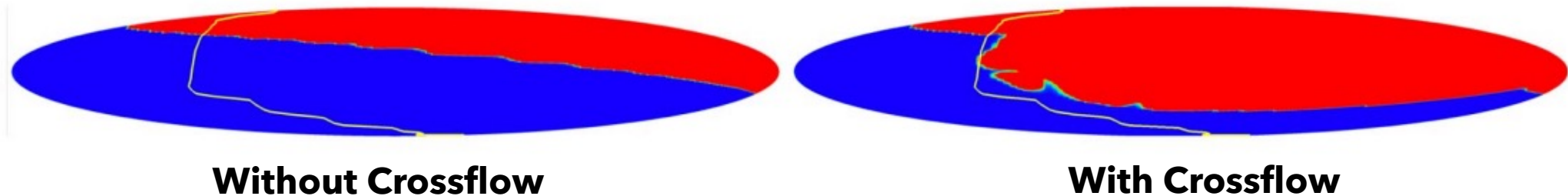
$$\gamma_{BC}, Re_{\theta c}$$

CROSSFLOW TRANSITION MODELING

Dominant crossflow models are based helicity

$$H_{crossflow} = \frac{y |(\vec{U} - \vec{U}_{grid}) \cdot \vec{\Omega}|}{|\vec{U} - \vec{U}_{grid}|}$$

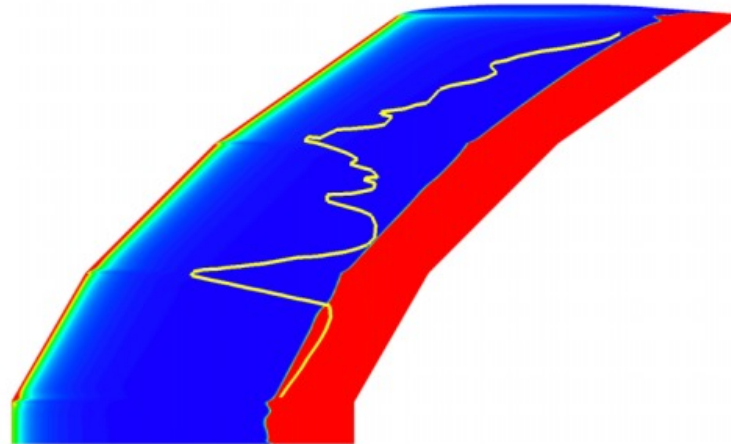
- Not strictly Galilean invariant, but “hacks” are frequently used



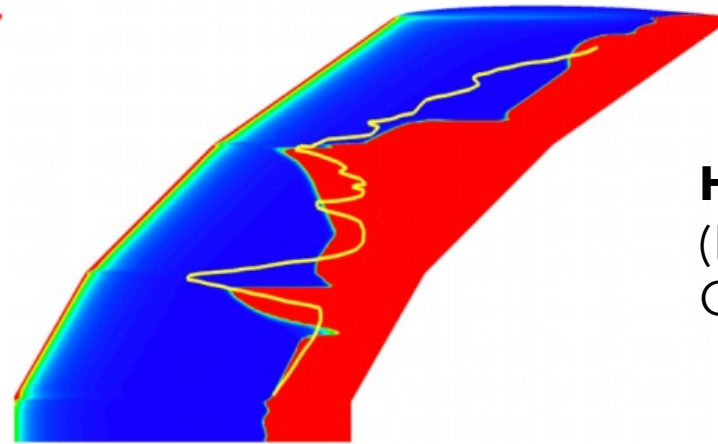
Menter’s Galilean-invariant crossflow model buried in ANSYS documentation, and implemented by Nichols

- Involves higher derivatives, not as consistently accurate

CROSSFLOW TRANSITION MODELING



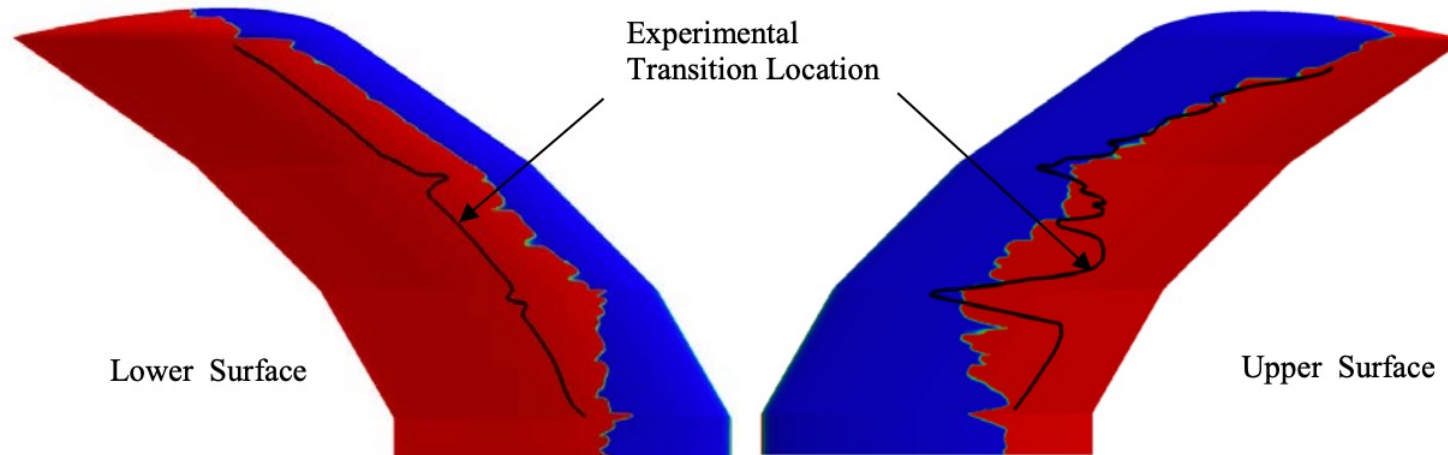
a) Upper surface, AFT2019b



b) Upper surface, AFT2019b+CF

Helicity-Based Model

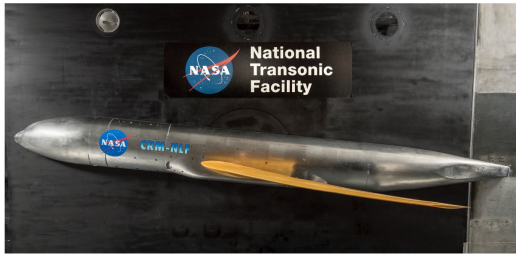
(Langtry et al., 2015;
Carnes and Coder, 2021)



Galilean Invariant Model

(Menter; Nichols, 2019)

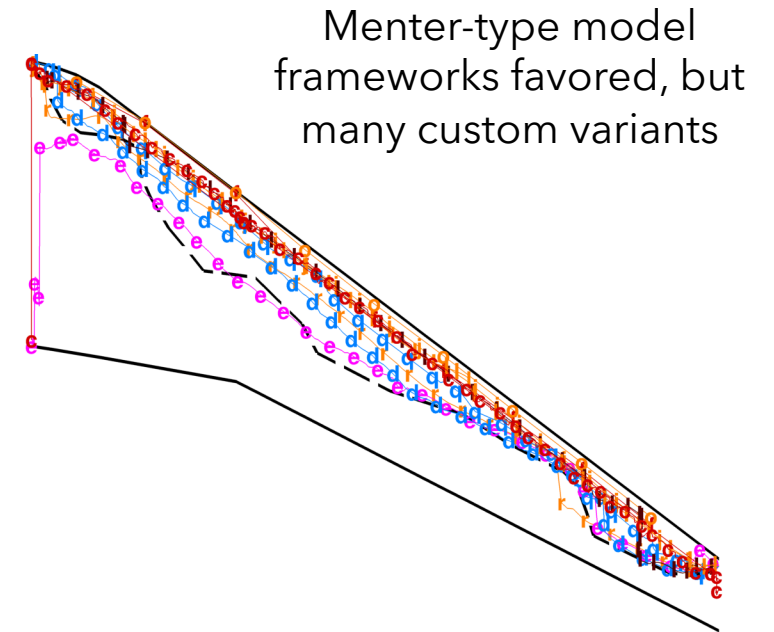
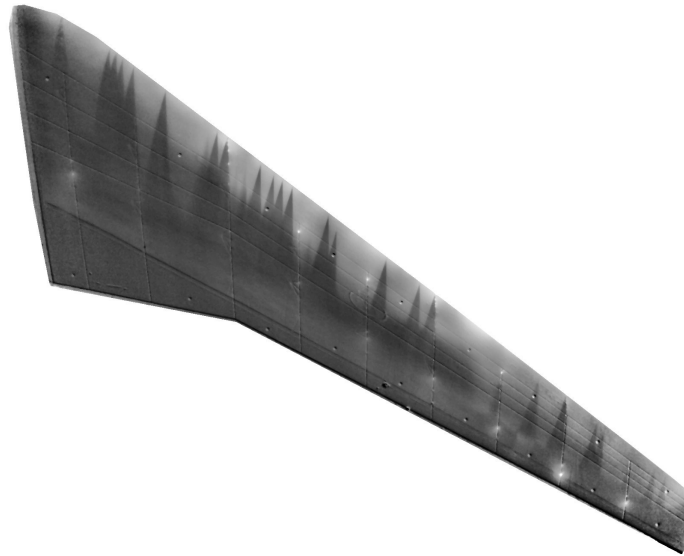
FIRST AIAA CFD TRANSITION MODELING AND PREDICTION WORKSHOP



Objectives

- Assess current state of the art in transition prediction for industrial CFD Determine and document best practices for transitional flow simulations
- Verify transition/turbulence model implementations
- Encourage risk taking and promote improvements to CFD prediction capabilities

**18 participant teams representing
13 countries with strong mix of
government, industry, and academia**



Observations and Conclusions

- Transition model verification is still an open question, and subtle differences in model variants can have strong impact on results
- RANS-based transition modeling is reduced-order by nature and lacks well-stated PDEs
- Verification and validation of transition models is challenging
- We have incomplete characterization of test conditions

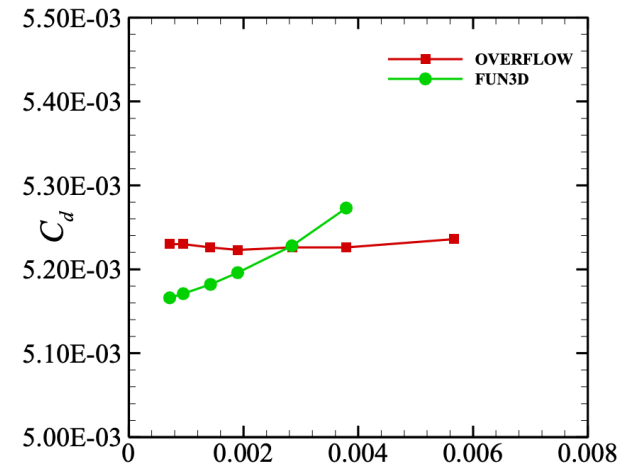
BENCHMARKING AND VERIFICATION

Follow-on Special Session held last month at AIAA Aviation to explore numerical aspects of models

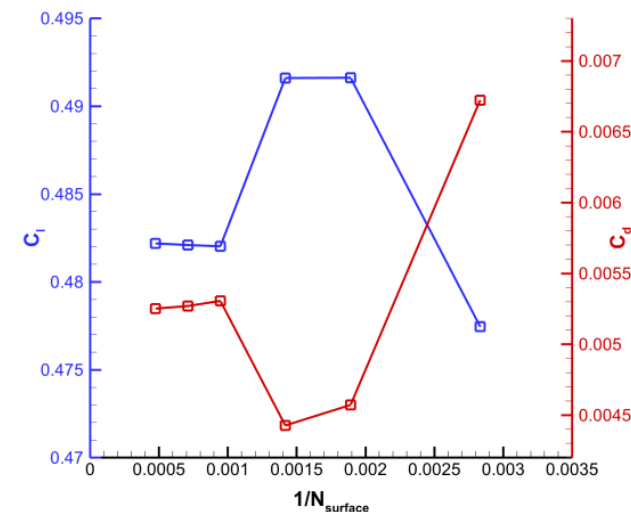
Two groups presented results related to verification of the Langtry-Menter model

- Code-to-code comparisons of 2D cases
- Benchmark solutions for Workshop test cases, including 3D cases
- Steps towards codifying best practices

There is still much work to be done!



T3A- ZPG Flat Plate
(Venkatachari et al.)



NLF(1)-0416 Airfoil
(Carnes and Coder)

COUPLING WITH TURBULENCE MODELS

Menter-type local-correlation transition models lack universal post-transition solutions and introduce singular behaviors

- TKE profile dependent on transitional Re

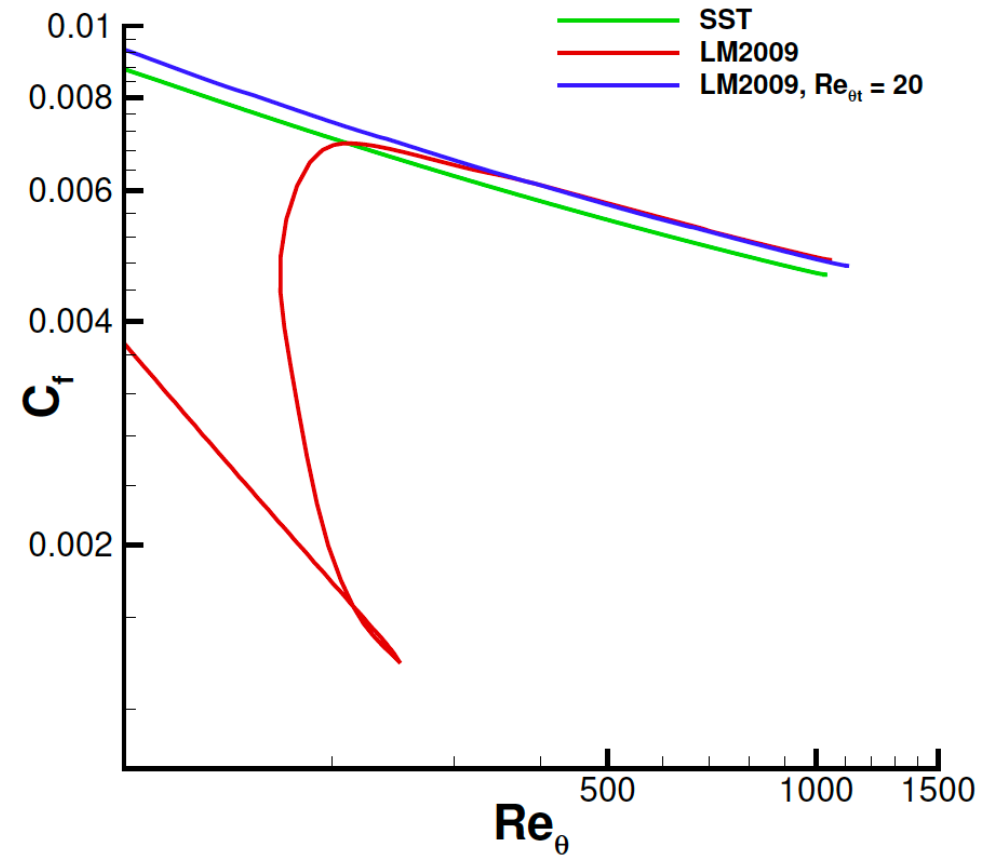
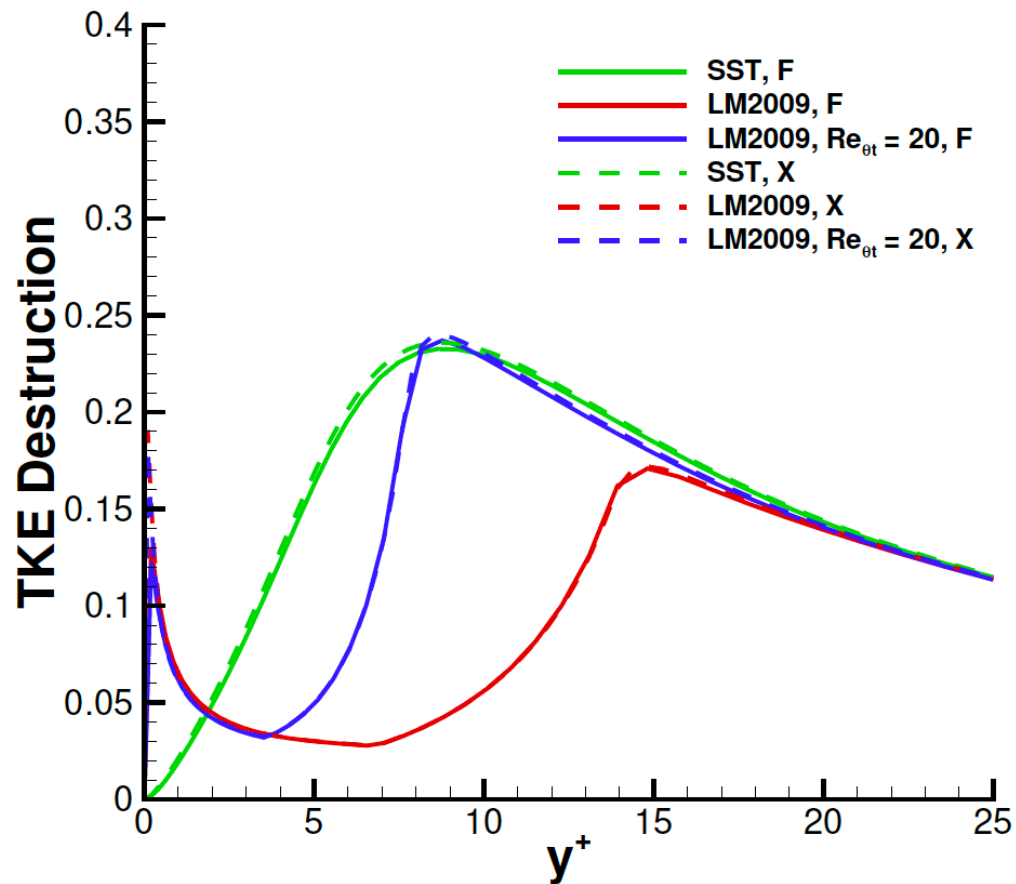
$$\rho \frac{Dk}{Dt} = \gamma_e P_k - \max(\gamma_e, 0.1) D_k + \text{Diffusion}$$

$$\gamma_e \frac{k^+}{\omega^+} \left(\frac{du^+}{dy^+} \right)^2 - \max(\gamma_e, 0.1) \beta^* k^+ \omega^+ + \frac{d}{dy^+} \left[\left(1 + \sigma_k \frac{k^+}{\omega^+} \right) \frac{dk^+}{dy^+} \right]$$

- Asymptotic behaviors change near the wall

$$\begin{array}{ll} \mathbf{SST:} & k \sim y^{3.2295} \\ \mathbf{LCTM:} & k \sim y^{1.4849} \end{array} \quad \omega \sim \frac{1}{y^2}$$

COUPLING WITH TURBULENCE MODELS



COUPLING WITH TURBULENCE MODELS

Many SA-based coupling approaches employ a naïve analogy with “noft2” variant

$$\frac{D\tilde{\nu}}{Dt} = \gamma c_{b1} \tilde{S} \tilde{\nu} - \gamma_{lim} c_{w1} f_w \frac{\tilde{\nu}^2}{d^2} + \frac{1}{\sigma} \left[\frac{\partial}{\partial x_j} \left((\nu + \tilde{\nu}) \frac{\partial \tilde{\nu}}{\partial x_j} \right) + c_{b2} \frac{\partial \tilde{\nu}}{\partial x_j} \frac{\partial \tilde{\nu}}{\partial x_j} \right]$$

- Truly universal behavior is lost after transition, but the model is very resilient
- Laminar flow is not a stable solution, creating strong dependency on free-stream BC
- Model developers should consider these behaviors

SA-AFT model couples via the f_{t2} term, but is overly dependent on the non-linear diffusion propagating eddy viscosity upstream

FREE-STREAM TURBULENCE

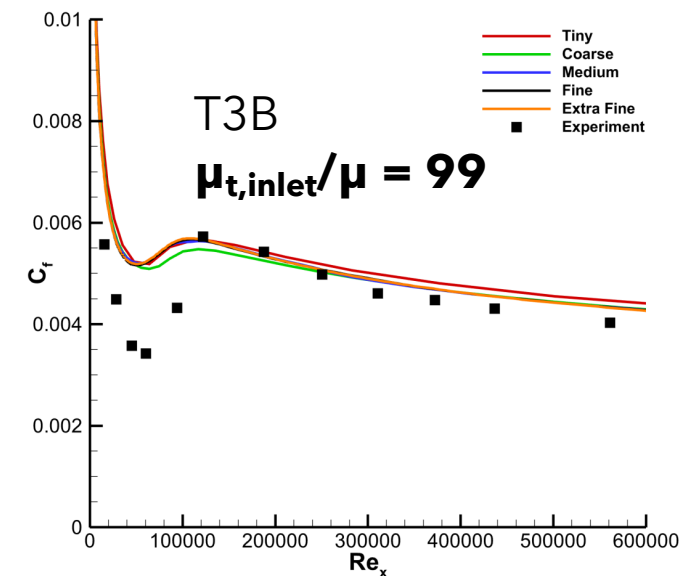
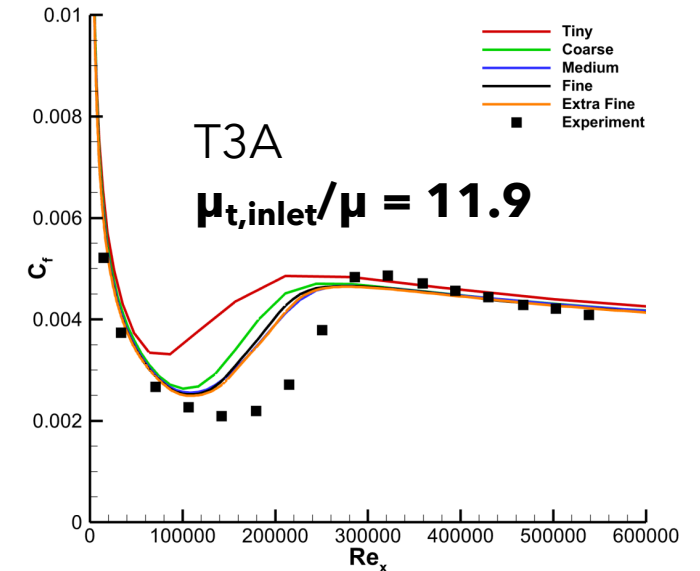
Transition models are sensitive to turbulence quantities outside the boundary layer

- TKE in free stream drives the transition criteria
- Decay of TKE is very rapid for external flows
- Large eddy viscosity values can overwhelm the laminarization

Form of TKE production term can have leading-order influence

- Stagnation-Point Anomaly
- Strain vs. Vorticity vs. Kato-Launder

Turbulence sustaining terms still an open question for transition-sensitized equations



OUTLOOK AND OPPORTUNITIES

Prominent PDE-based transition models are artisanal and anchored to relations constructed from integral boundary-layer properties

- Use of local variables is a ROM of a ROM

Some flow regimes defy integral modeling (i.e. hypersonics)

- Creativity required to construct localized shape factors and Reynolds numbers
- Data-driven methods may help downselect candidate model surrogates

Equations remain difficult to converge, and grid resolutions are finer than for fully turbulent

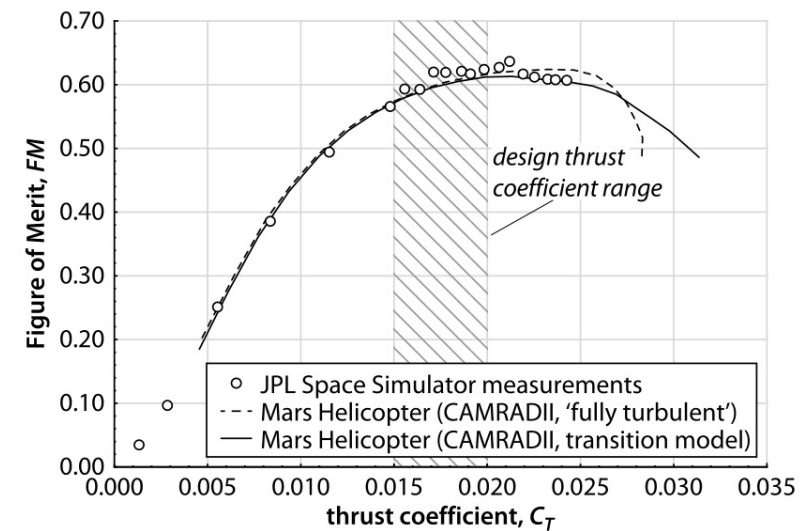
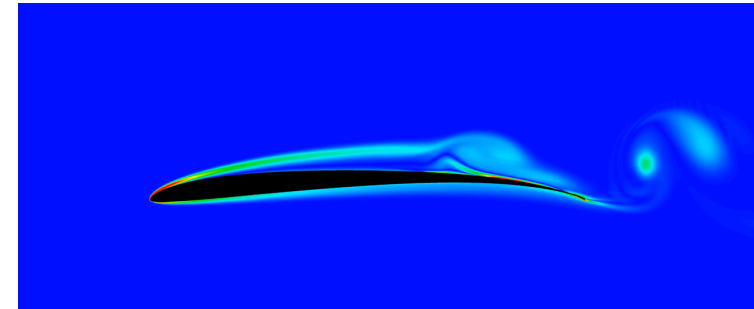
Nevertheless, these transition models are making headway for use in aerodynamic design

INTERPLANETARY SUCCESS STORY



NASA Ingenuity Mars Helicopter

PDE-based transition modeling used
for rotor airfoil aerodynamics model



Koning, Johnson, and Grip (*AIAA Journal*, 2019)

ACKNOWLEDGMENTS

The work presented here is an accumulation of research partially supported by the U.S. Army (Agreement No. W911 W6-17-2-0003), DoD HPCMP (NDSEG program), NASA (Award NNX17AJ95A) and through interactions facilitated by the AIAA Applied Aerodynamics Technical Committee's CFD Transition Modeling Discussion Group

Special thanks to Philippe Spalart for his immeasurable impact on computational fluid dynamics and aerodynamics, and for the opportunity to know him

QUESTIONS?