

## Improvement of RANS models by machine learning for a bump configuration

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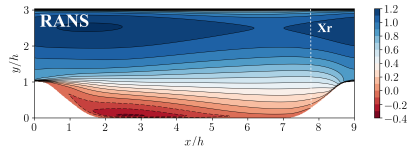
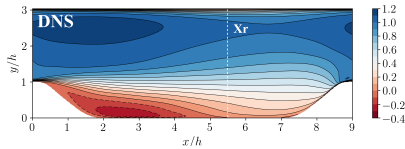


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## Motivation

RANS simulations are widely used in engineering but they lack accuracy.

- flows with adverse pressure gradient, large separations, curvature, swirl, ...



Periodic hill configuration at  $Re=2800$  : mean streamwise velocity field.

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Improve predicted capabilities of classic turbulence models

- 2021 : Data assimilation + Machine learning (Boussinesq correction)
- 2022 : Machine learning (Eddy-viscosity correction)

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## Requirements

- High fidelity test cases, reflecting critical physics

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Solve the exact equations :  $\mathcal{R}_e(u_e(\mathbf{x}, t)) = 0$

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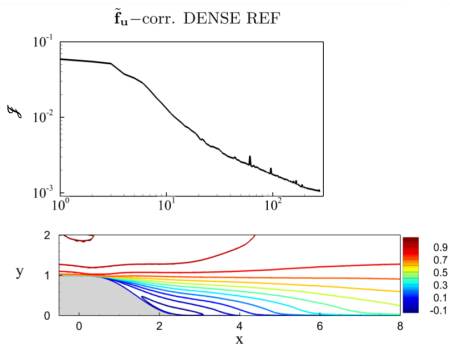
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Computations are performed using the NN augmented turbulence model

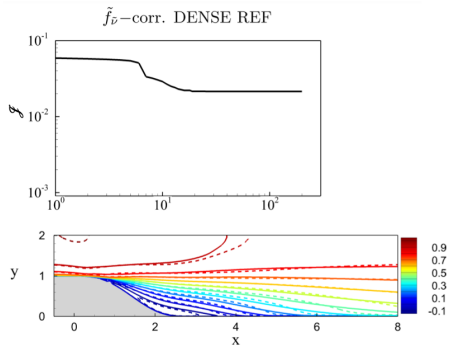
## ■ Boussinesq correction :

$$\frac{\partial u_i u_j}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial(2\nu S_{ij})}{\partial x_j} + \frac{\partial \tau_{ij}^{SA}}{\partial x_j} + \tilde{f}_{u_i}$$

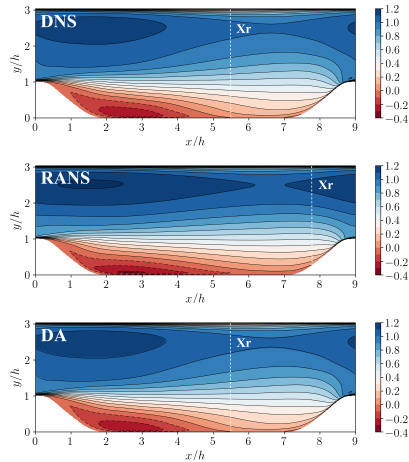


## ■ Eddy-viscosity correction :

$$u_j \frac{\partial \tilde{\nu}}{\partial x_j} = P - D + T + \tilde{f}_{\tilde{\nu}}$$



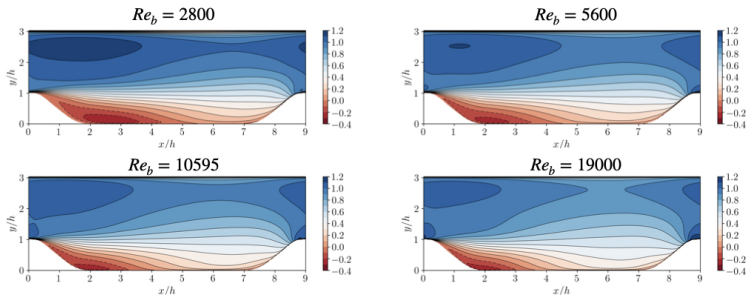
## Mean streamwise velocity field



# Machine learning strategy - Volpiani et al. (2021, PRF)

Database of training flows to predict flow past periodic hills

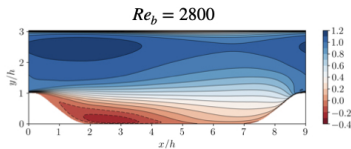
Training flow scenario	Selected cases	Fit on training data only	Fit on entire data without rotation
Scenario I	PH-2800, PH-10595, PH-19000	96.5 %	95.6 %
Scenario II	PH-2800, PH-5600, PH-10595	96.2 %	93.5 %



# Machine learning strategy

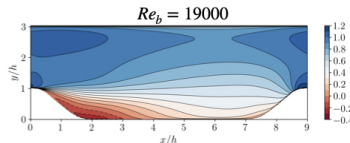
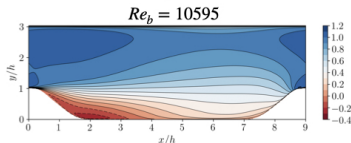
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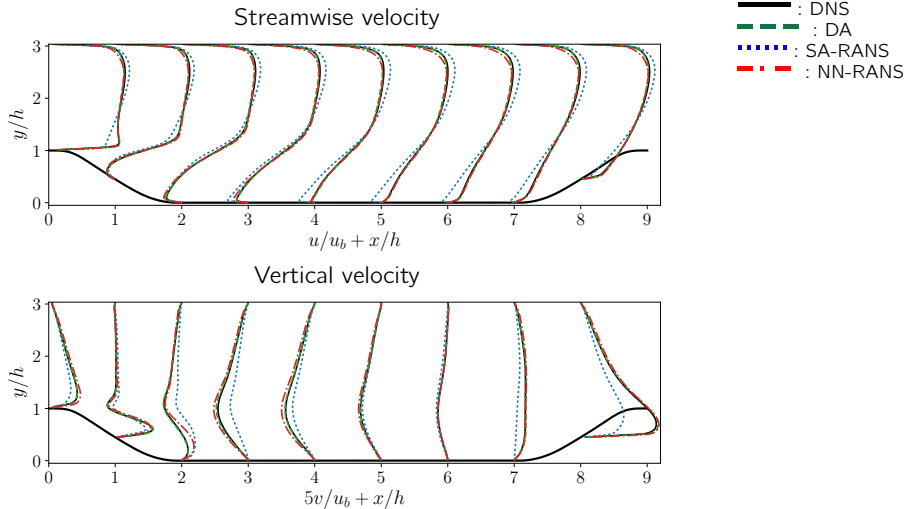
$Re_b = 5600$

?

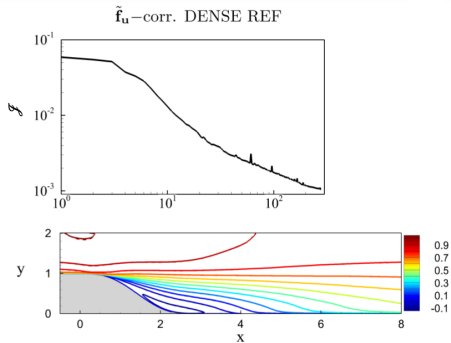


# RANS using a neural network-based model

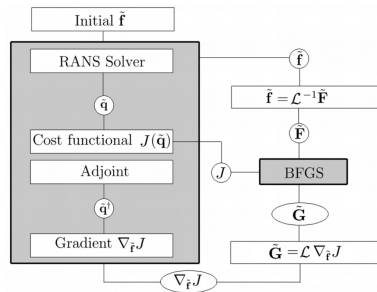
Interpolation in Reynolds number : scenario I is used to predict flow at  $Re_b = 5600$



# Data assimilation following Franceschini et al. (2020, PRF)



Assimilation of velocity measurements based on the volume force  $\mathbf{f}_{u_i}$

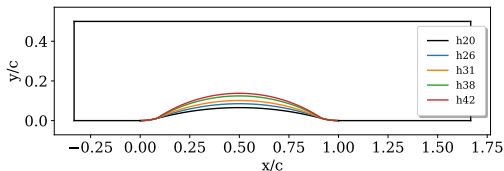


Coupling between the optimization algorithm (BFGS) and the fluid solver.

## RANS equations :

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0, \quad \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_j} [2(\nu + \nu_t^{SA} + \Delta \nu_t) S_{ij}]$$

- $\nu_t^{SA}$  is determined by solving the one equation Spalart-Allmaras (SA) turbulence model
- $\Delta \nu_t$  is determined using the LES statistical data :  $\nu_t^{LES} = \frac{-\overline{u_i' u_j'} \partial_j \bar{u}_i}{2S_{ij} S_{ij}} \approx \frac{\max(0, -\overline{u_i' u_j'} \partial_j \bar{u}_i)}{\max(0, 2S_{ij} S_{ij}) + \epsilon}$

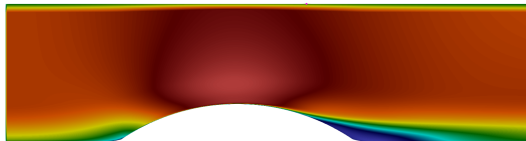
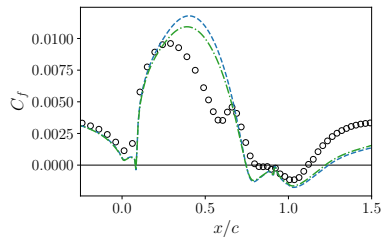
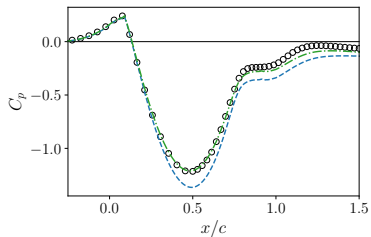


Family of bumps from Matai & Durbin (JFM, 2019).



# Baseline RANS-SA simulation for the flows over a family of bumps

Discussion on boundary conditions : slip wall, as LES (shorter domain), LES bump h42.



Streamwise velocity for bump h42 from Matai & Durbin (JFM, 2019).

RANS equations :

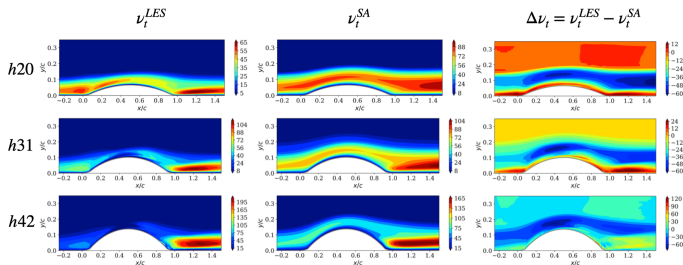
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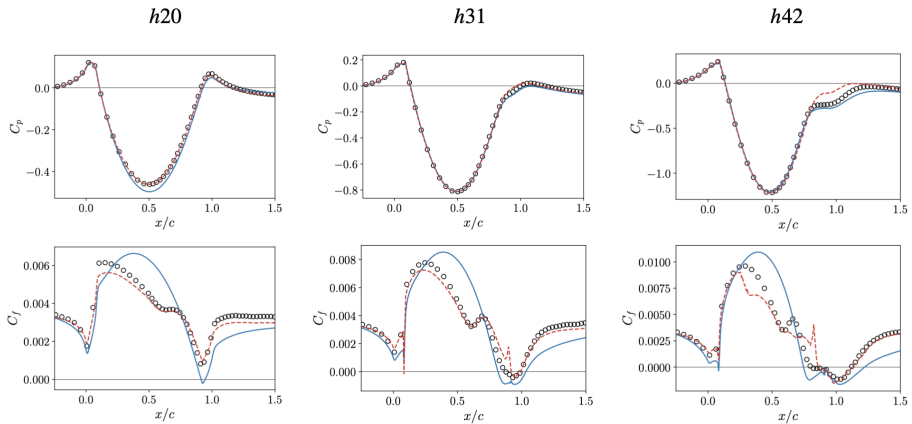
RANS equations :

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# Eddy-viscosity correction



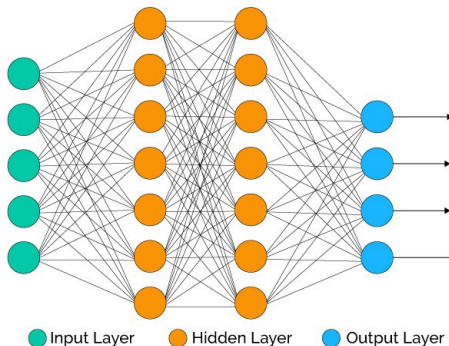
Corrected  $C_p$  and  $C_f$  profiles : RANS-SA, RANS- $\nu_t^{LES}$ .

## Input layer :

Feature	Description	Formula
$q_1$	Q-criterion	$\frac{\ \mathbf{\Omega}\ ^2 - \ \mathbf{S}\ ^2}{\ \mathbf{\Omega}\ ^2 + \ \mathbf{S}\ ^2}$
$q_2$	Ratio of pressure normal stresses to shear stresses	$\frac{\sqrt{\frac{\partial p}{\partial x_k} \frac{\partial p}{\partial x_k}}}{\sqrt{\frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} + \frac{1}{2} \frac{\partial u_i^2}{\partial x_k}}}$
$q_3$	Gorle et al. marker	$\frac{\ \bar{u}_i \bar{u}_j \frac{\partial u_i}{\partial x_j}\ }{\ \bar{u}_i \bar{u}_j \frac{\partial u_i}{\partial x_j}\  + \sqrt{\bar{u}_i \bar{u}_j \frac{\partial u_i}{\partial x_j} \bar{u}_k \frac{\partial u_k}{\partial x_j}}}$
$q_4$	Streamline pressure gradient	$\frac{\bar{u}_k \frac{\partial p}{\partial x_k}}{\ \bar{u}_k \frac{\partial p}{\partial x_k}\  + \sqrt{\frac{\partial p}{\partial x_k} \frac{\partial p}{\partial x_k} \bar{u}_i \bar{u}_i}}$
$q_5$	Viscosity ratio	$\frac{\nu_t + 100\nu}{c_{b1} \tilde{S} \bar{\nu}}$
$q_6$	SA ratio of production to destruction	$\frac{ c_{b1} \tilde{S} \bar{\nu}  + c_{w1} f_w \left(\frac{\bar{\nu}}{a}\right)^2}{c_{b1} \tilde{S} \bar{\nu}}$
$q_7$	SA ratio of production to diffusion	$\frac{ c_{b1} \tilde{S} \bar{\nu}  + \frac{c_{w2}}{\sigma} \frac{\partial \bar{\nu}}{\partial x_k} \frac{\partial \bar{\nu}}{\partial x_k}}{c_{b1} \tilde{S} \bar{\nu}}$
$q_8$	Turbulence intensity	$\frac{k_{qcr}}{k_{qcr} + \frac{1}{2} \bar{u}_i^2}$

Hidden layer :  $4 \times 80$ neurons

Output layer :  $\Delta \nu_t$  or  $\nu_t^{LES}$



● Input Layer    ● Hidden Layer    ● Output Layer

# Predicting the viscosity discrepancy $\Delta\nu_t$ using NN1

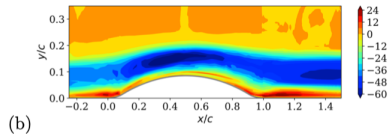
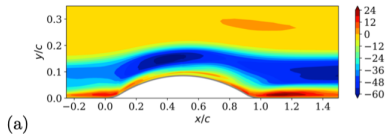
Training cases : h20, h31, h42

Validation cases : h26, h38

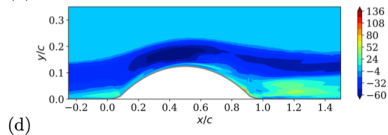
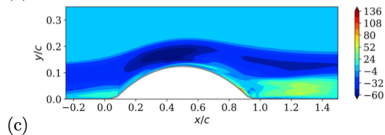
$\Delta\nu_t$

$\Delta\nu_t^{NN}$

h26



h38



# Predicting the viscosity discrepancy $\nu_t^{LES}$ using NN2

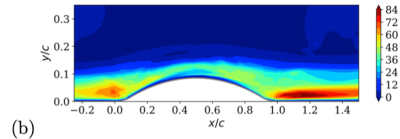
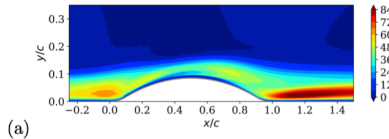
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Validation cases : h26, h38

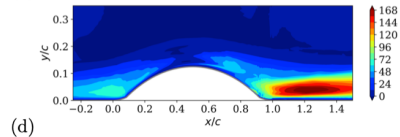
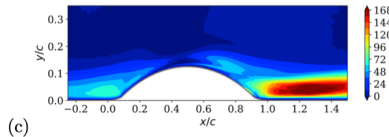
$\nu_t^{LES}$

$\nu_t^{NN}$

h26

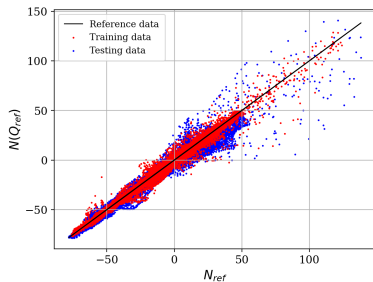


h38

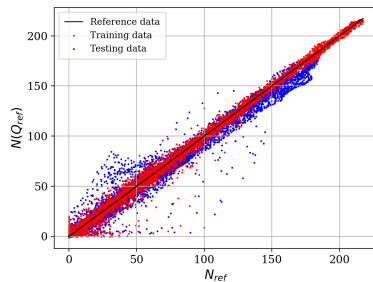


# Comparison between NN1 vs NN2

NN1 -  $\Delta \nu_t^{NN}$



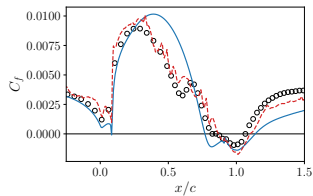
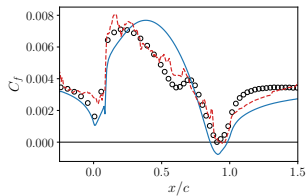
NN2 -  $\nu_t^{NN}$





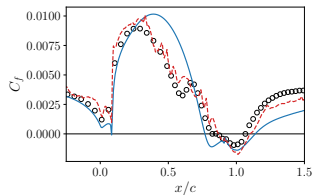
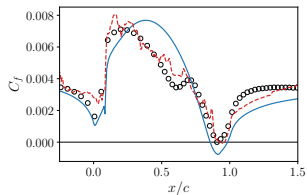
# A posteriori results using NN1 and NN2

$$\text{NN1} - \Delta \nu_t^{NN}$$

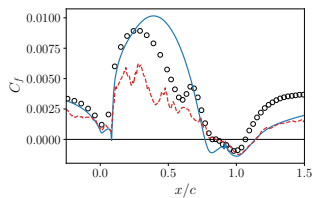
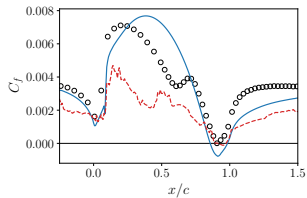


# A posteriori results using NN1 and NN2

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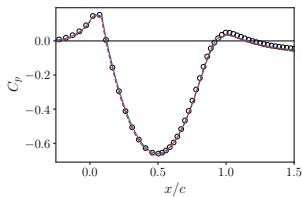


NN1 -  $\Delta\nu_t^{NN}$  no treatment

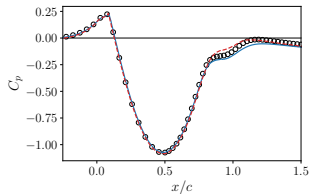
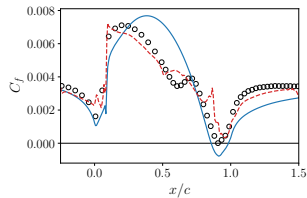


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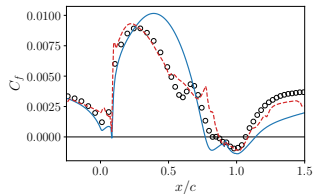
NN2 -  $\nu_t^{NN}$



h26



h38



## Conclusions

- In the past, we employed DA+ML to improve RANS models
- This strategy gives accurate results but it is quite complex and time consuming
- More recently, we employed ML to correct directly the unknown eddy-viscosity term in the RANS equations
- Advantages : simple, fast, no need to transport any turbulent variables
- Although not perfect, the NN-based model improved  $C_p$  and  $C_f$  predictions
- **Machine learning-based turbulence models can be used to improve CFD capabilities**

## Publication

- Volpiani et al. (2021). Machine learning-augmented turbulence modelling for RANS simulations of massively separated flows. *Physical Review Fluids*.
- Volpiani, Bernardini & Francesquini (2022). Neural network-based eddy-viscosity correction for RANS simulations of flows over bi-dimensional bumps. *International Journal of Heat and Fluid Flow*.