Power Spectral Density Uncertainty Estimates – Fully-Instrumented Junction Flow Model in the 14X22-ft Wind Tunnel

The uncertainty estimates in the power spectral density, or autospectra, of the unsteady pressures consist of two parts. The first is the systematic uncertainty derived from a Monte Carlo (MC) analysis conducted by perturbing the unsteady pressure sensor sensitivity. The uncertainty in sensitivity that was used in this MC analysis was 0.31% of value. This uncertainty is based on the amplitude accuracy of the A/D converter used for the voltage measurements, and the total uncertainty of the pressure controller used for the sensor calibrations. The perturbed sensitivities were then used to calculate unsteady pressure time series from the acquired voltage time series. Subsequently, the power spectra were calculated. From the integration of the power spectra, the mean square values were calculated for each iteration of the MC analysis. After the desired number of MC cycles were obtained, the standard deviation of the mean square ensemble was calculated. Twice the standard deviation was used to determine the uncertainty, representing a 95% level of confidence. It was normalized using the mean of the mean square values calculated in each MC cycle. Convergence of the uncertainties was determined to decide the minimum number of MC cycles necessary. It was found that N_{mc} =1000 cycles provided adequate convergence, after examining six different iteration totals from 100 to 5000.

Side note: The mean square was used as the defining parameter for the systematic uncertainty in the power spectra (which represents the rate of change of mean square value with frequency). It is simply the summation of the product of the individual power spectra (mean square divided by the frequency band) points multiplied by the resolution bandwidth. This was much easier to handle than retaining N_{mc} spectra and making individual calculations for each point in the spectrum. In addition, the usual definition of the statistical mean square implies a non-biased estimator; however, the mean square as calculated here is based on the integration of the power spectrum. The power spectrum, being a biased estimator, has a larger uncertainty than the statistical mean square, although still very small.

The second part of the uncertainty analysis for power spectral density was performed using the techniques of Bendat and Piersol (Random Data, 2010, pp. 274-279) to calculate the normalized rms error. Twice this error represents the uncertainty at a 95% confidence level. Their analysis calculates a normalized systematic (primarily resolution bandwidth bias) and a normalized random error. The quantities that go into this calculation are the resolution bandwidth, the total acquisition time, the power spectrum, and the second derivative of the spectrum. Since this second derivative can be quite large in some cases, thereby considerably overestimating and exaggerating the error, we used the average of the second derivative, which for a majority of the values is well-behaved and a reasonable estimate.

The total uncertainty in the power spectral density was calculated as the root-sum-square of the result from the first and second parts in the analysis. The final value was determined by examining the results for a number of sensors in the 54-sensor set at several angles of attack. Overall, the average uncertainty in power spectral density was 14.2%, taken over 13 sensors, three angles of attack, for two tests, spaced four months apart.