



Uncertainty Estimates for LDV Probe 2 via a Statistical Analysis of Residuals

Michael A. Kegerise
NASA Langley Research Center

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Factors that Contribute to Total Uncertainty

- Sampling variance for a given statistic
- Uncertainty in LDV beam unit vector measurements (impacts the beam angles, fringe spacings, velocity transformation matrix, and ultimately the 3 components of velocity)
- Uncertainty in Doppler frequency measurement

Previous UQ only accounted for these factors – see NASA TM 2019-220286

- Other factors
 - Repeatability of model location in the test section
 - Repeatability of the model attitude settings (pitch, roll, and yaw)
 - Run-to-run variation in the flow conditions (chord Reynolds number set point)
 - Repeatability due to a rebuild of the model (including a rebuild & repositioning of the LDV system)
 - Repeatability in the wing-root/fuselage gasket
 - Repeatability of the measurement-volume location in the flow field
 - Effect of screen blockage on freestream flow characteristics; e.g., flow nonuniformity introduced between T640 and T653
- Uncertainties associated with inverting the model
 - Freestream flow nonuniformity
 - Freestream flow angularity
 - Model sting & mast blockage effects
 - Gravity effects on wing deflection



Calculation of Residuals

- Following the method outlined by Aeschliman and Oberkampf (AIAA J. Vol. 36, No. 5, May 1998) and later by Rhode and Oberkampf (J. Spacecraft & Rockets Vol. 54, No. 1, Jan-Feb 2017)
- Uncertainty is estimated from a statistical analysis of residuals determined through comparison of measurements from certain pairings of runs and measurement points in space
 - Comparison of replicate runs at the same Reynolds number, model attitude, test-section location, and flow-field location yields information about the random component of uncertainty
 - Comparison of replicate runs with the model upright or inverted, but otherwise at the same Reynolds number, model attitude, and flow-field location yields some information about the component of uncertainty due to flow nonuniformity and flow angularity (but also brings in uncertainty related to sting/mast blockage effects)

- Given a pair of runs, denoted by run numbers r and s , calculate the average value of a statistic at a given point in the flow field:

$$\bar{\theta}_i^{r,s} = \frac{1}{2} (\theta_i^r + \theta_i^s)$$

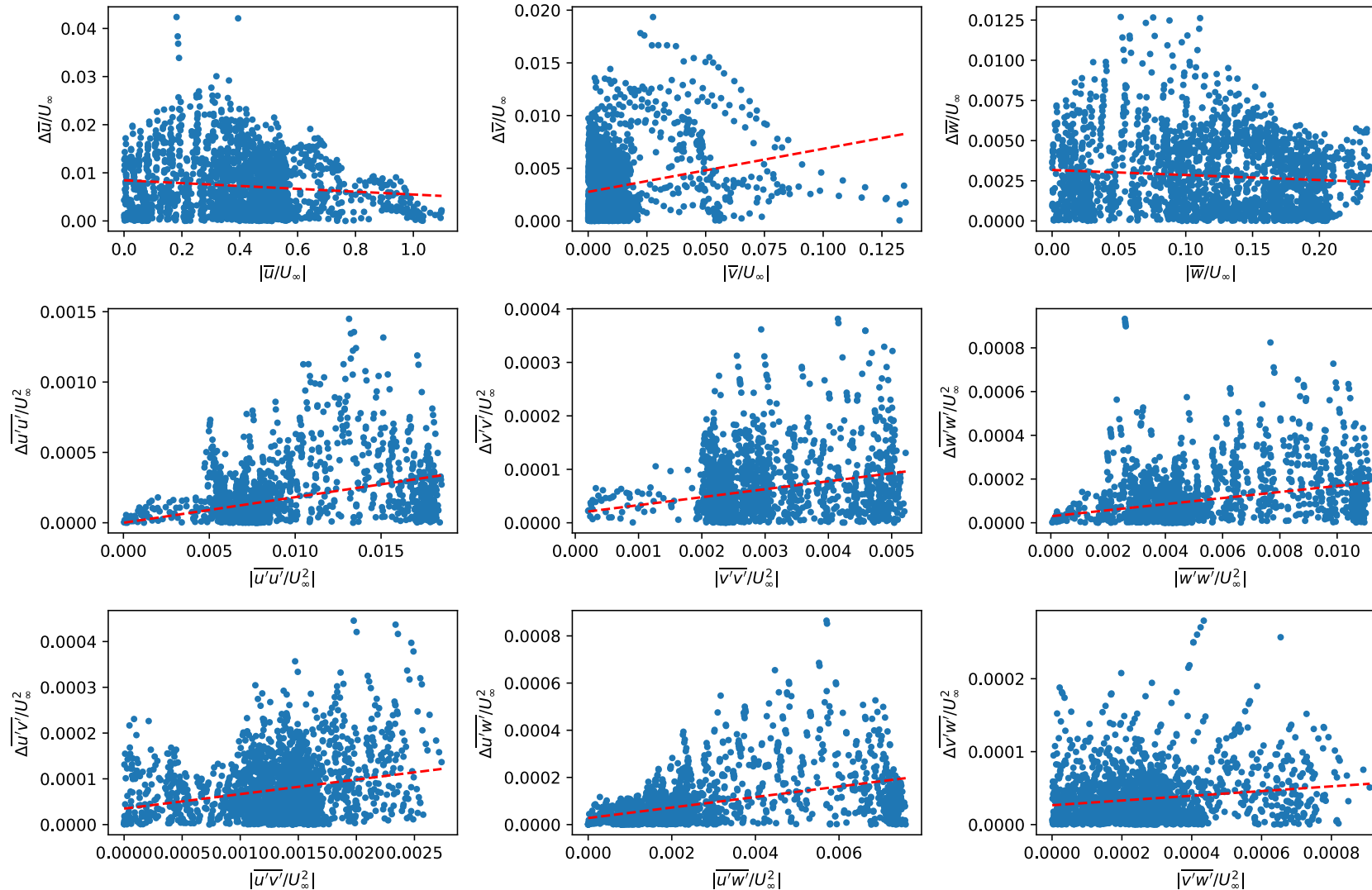
- Define the residual as the absolute value of the difference between the statistic from one of the runs and the average value:

$$\Delta\theta_i^{r,s} = \left| \theta_i^r - \bar{\theta}_i^{r,s} \right|$$

- For each component of uncertainty, calculate the sample variance from the residuals:

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N [\Delta\theta_i]^2$$

Example: All Residuals with Trend Line, $\alpha = 5$ deg



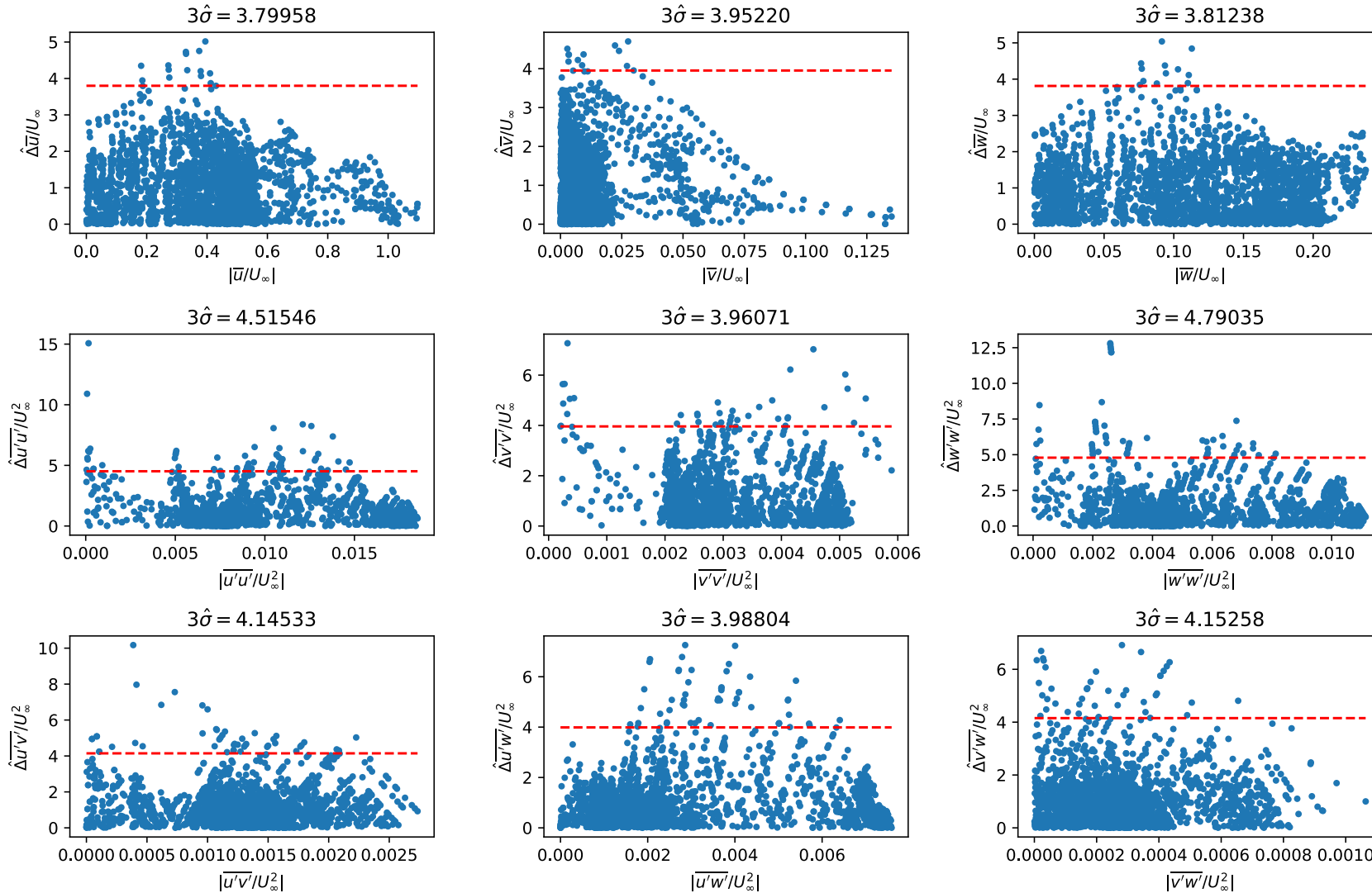
- All residuals plotted against the absolute value of the corresponding mean for a pair of replicates
- Trendlines are constrained linear regressions to the residuals:

$$\Delta\theta_{ls} = p_0 |\theta| + p_1$$

$$p_0 : (-\infty, \infty)$$

$$p_1 : [0, \infty)$$

Example: Normalized Residuals, $\alpha = 5$ deg



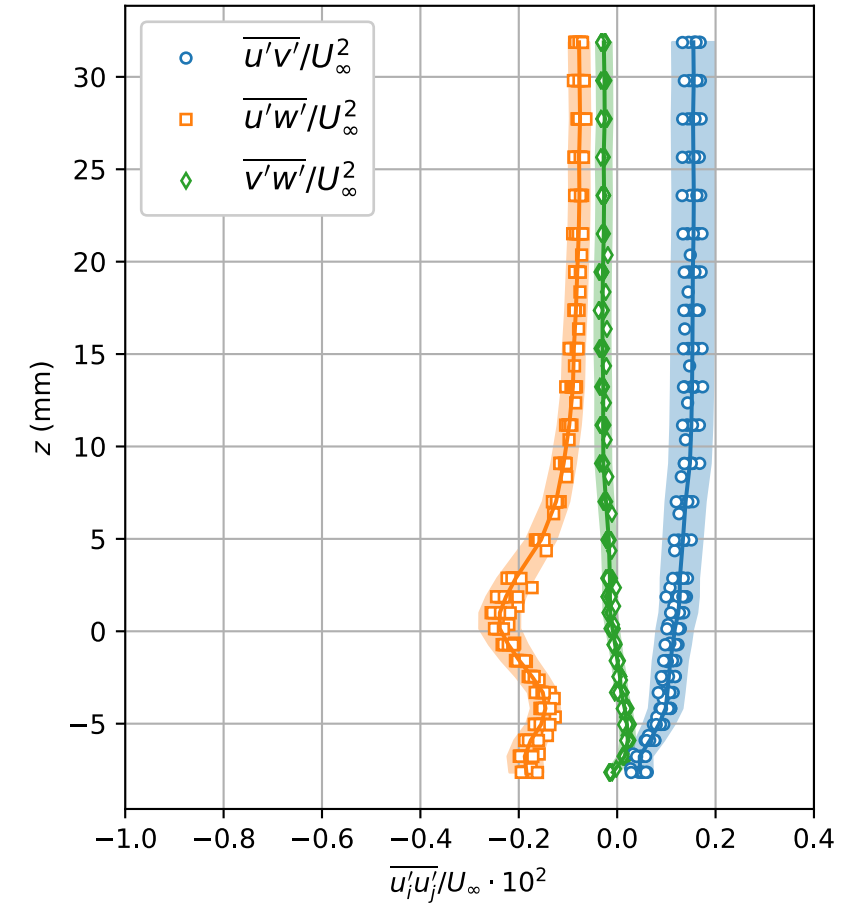
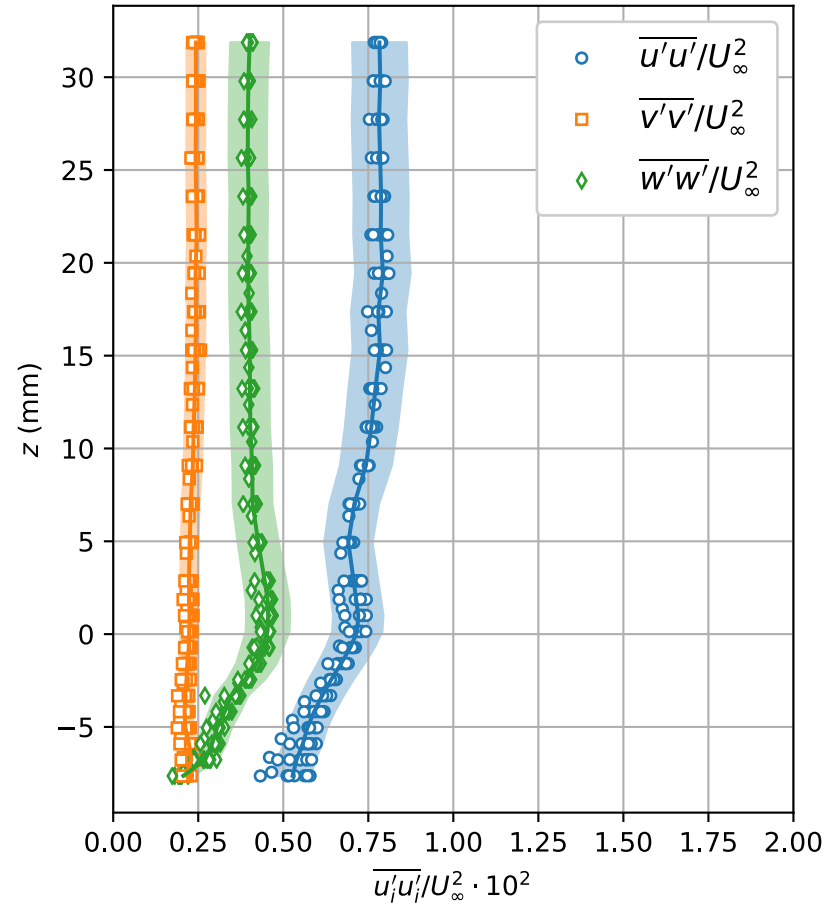
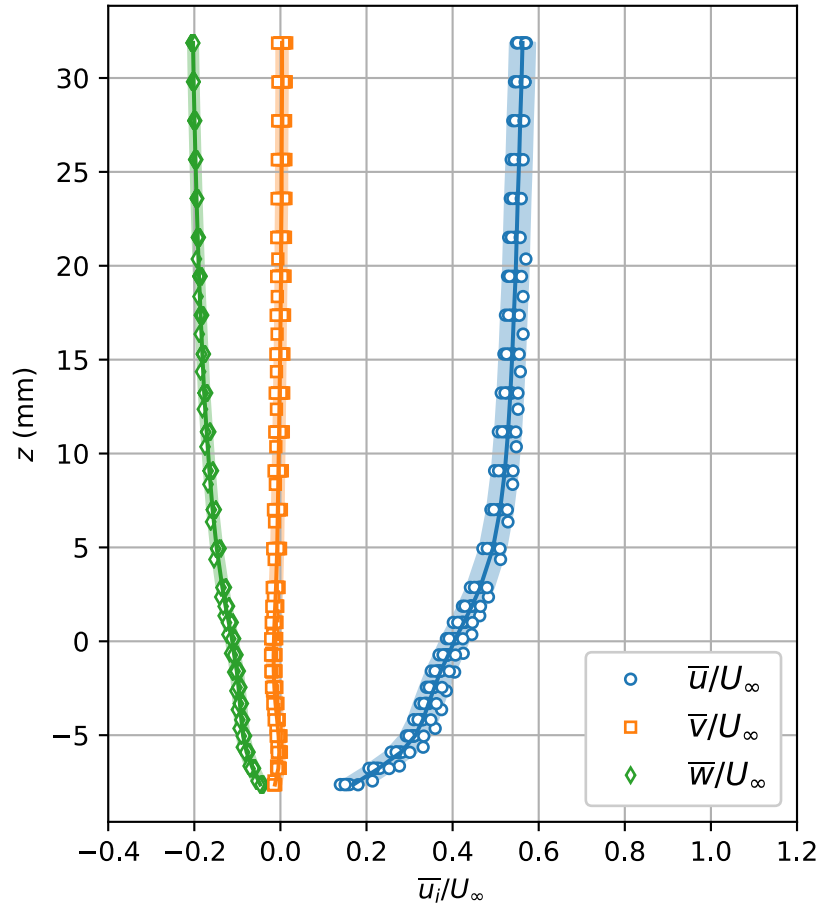
- Residuals for each statistic are normalized by their respective linear regression
- Variance of normalized residuals was calculated

$$\hat{\sigma}^2 = \frac{1}{N} \sum_{i=1}^N \left[\frac{\Delta \theta}{\Delta \theta_{ls}} \right]^2$$

- Total uncertainty is given by:

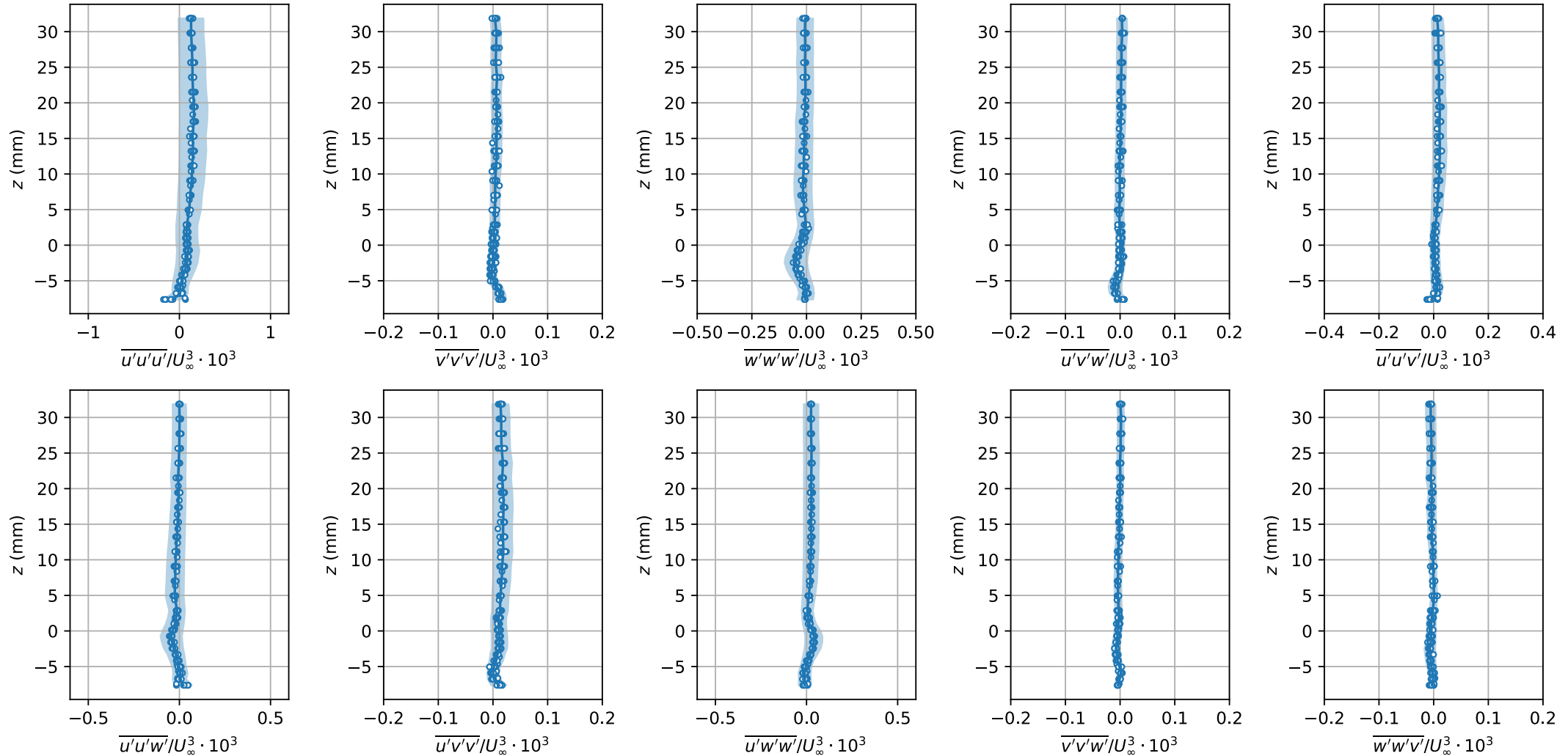
$$U_\theta = \pm 3\hat{\sigma}_\theta \Delta \theta_{ls}$$

Example: $x = 2852.6$ mm, $y = -239.1$ mm, $\alpha = 5$ deg

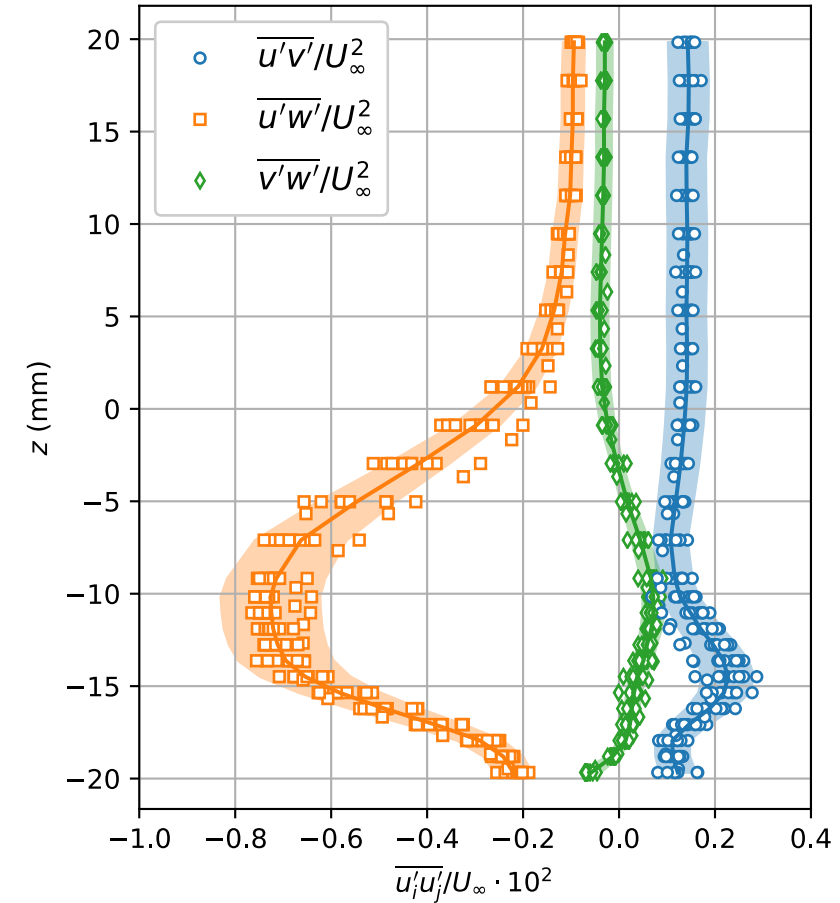
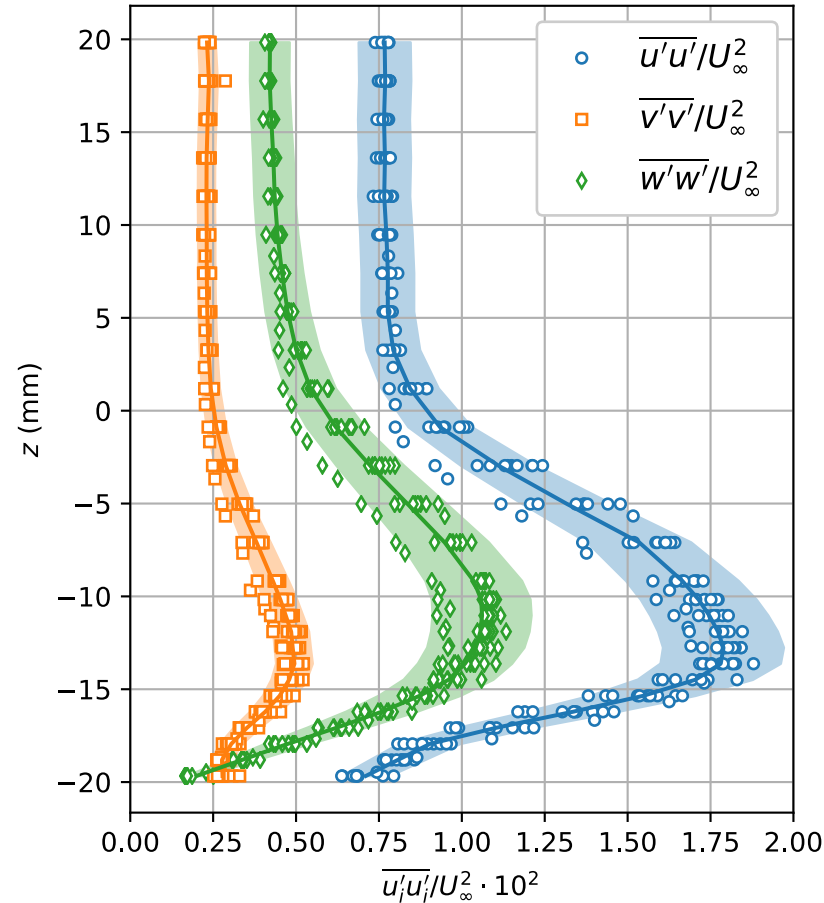
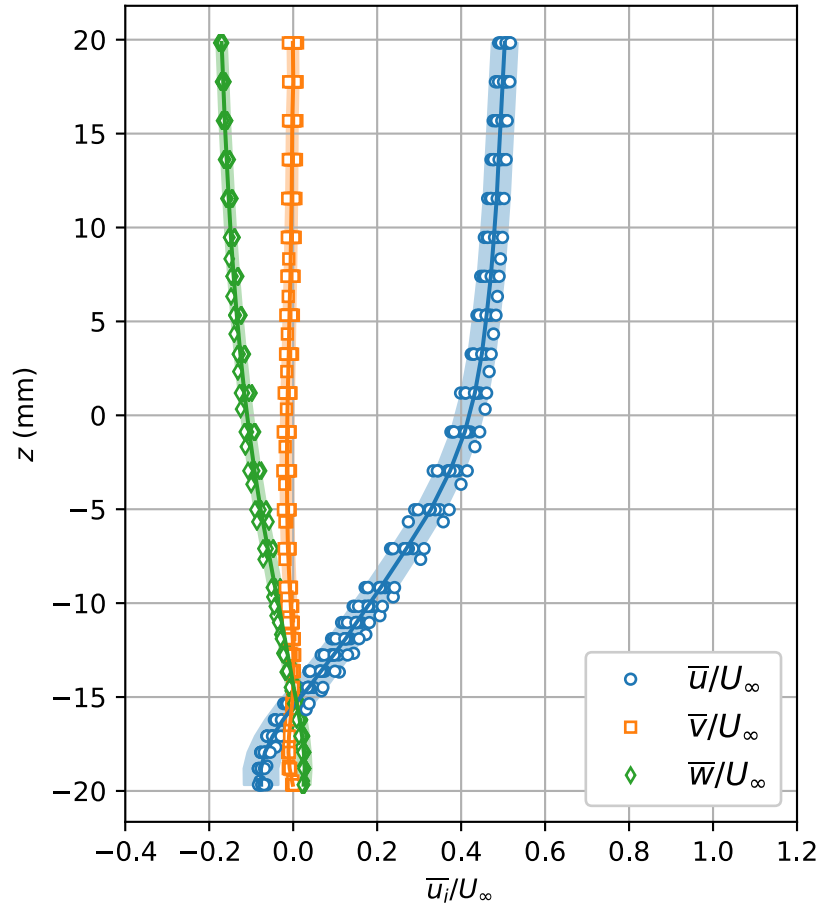


- Solid lines denote mean of all data sets, open symbols denote the experimental data
- Shaded uncertainty bands are about the mean line

Example: $x = 2852.6$ mm, $y = -239.1$ mm, $\alpha = 5$ deg

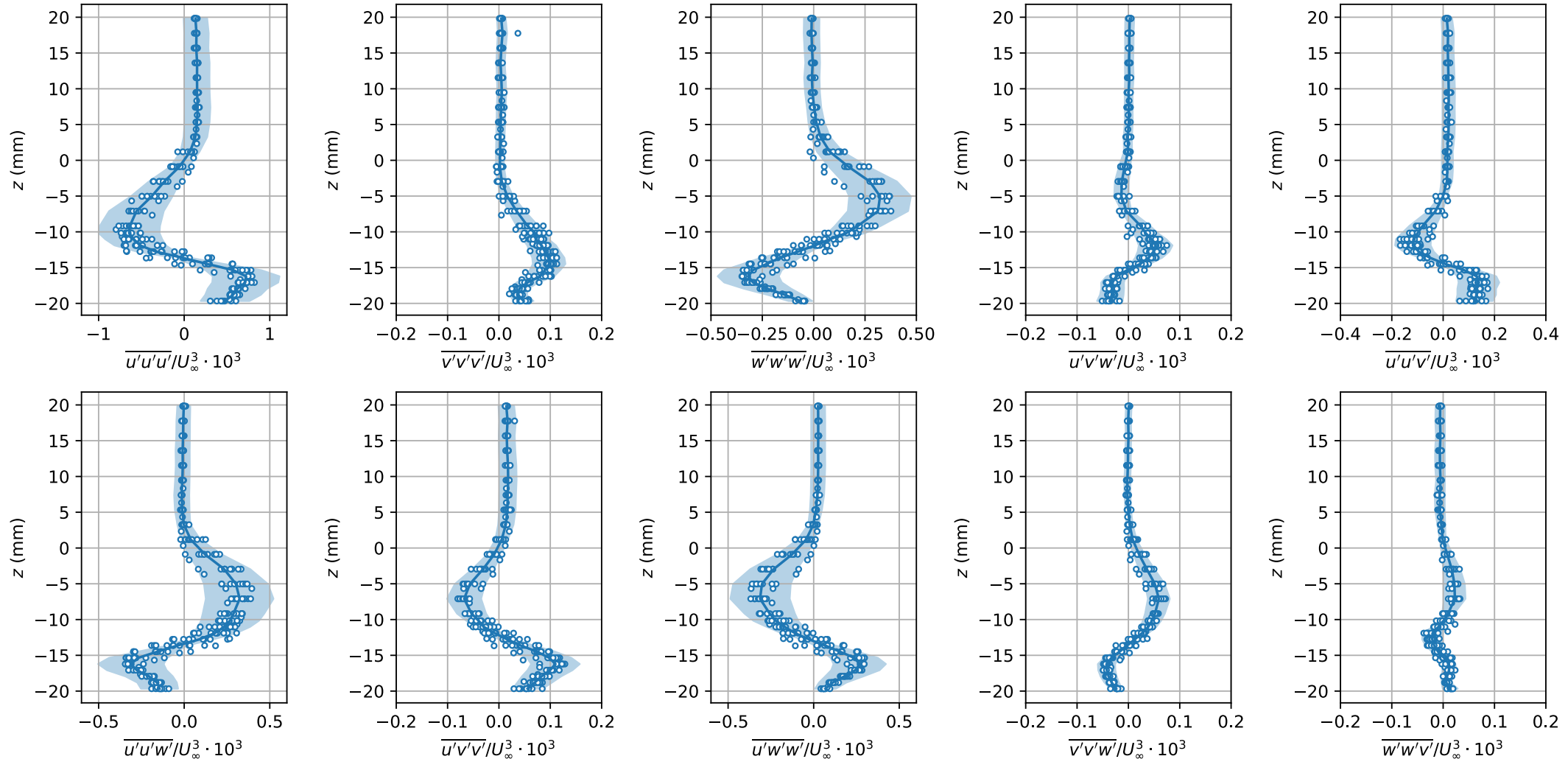


Example: $x = 2892.6$ mm, $y = -239.1$ mm, $\alpha = 5$ deg

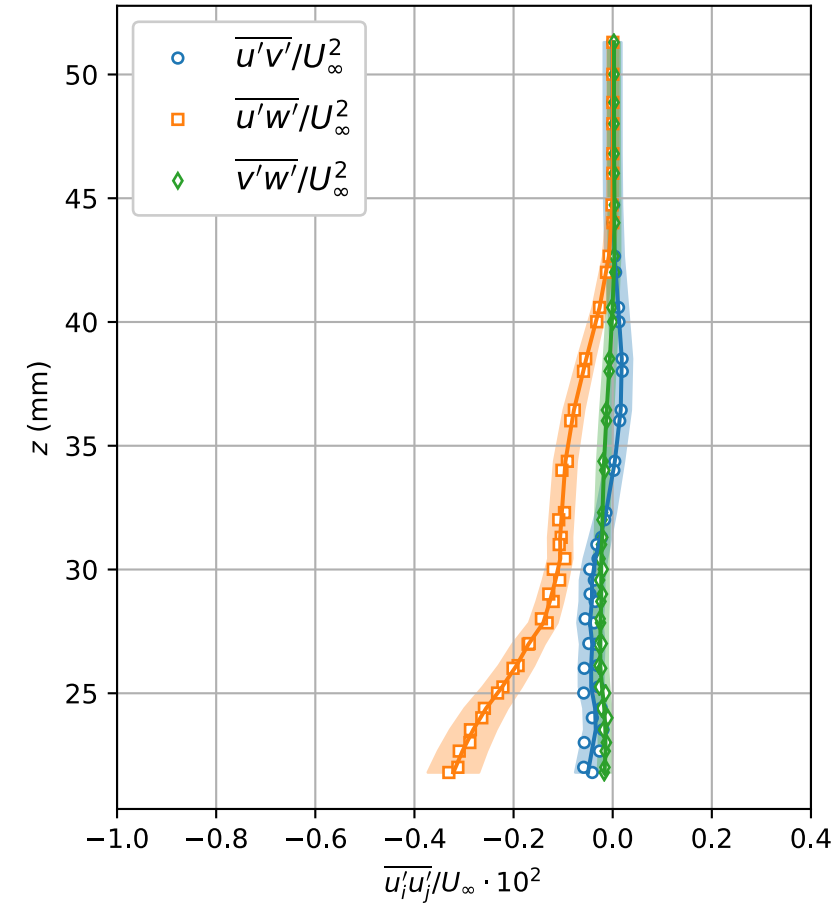
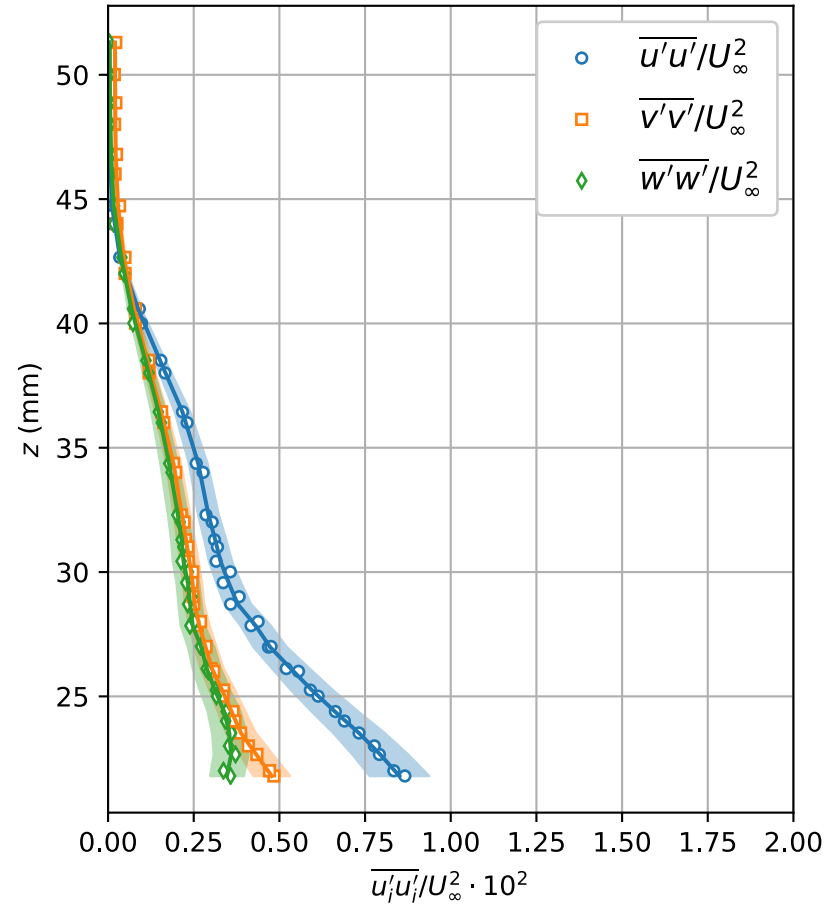
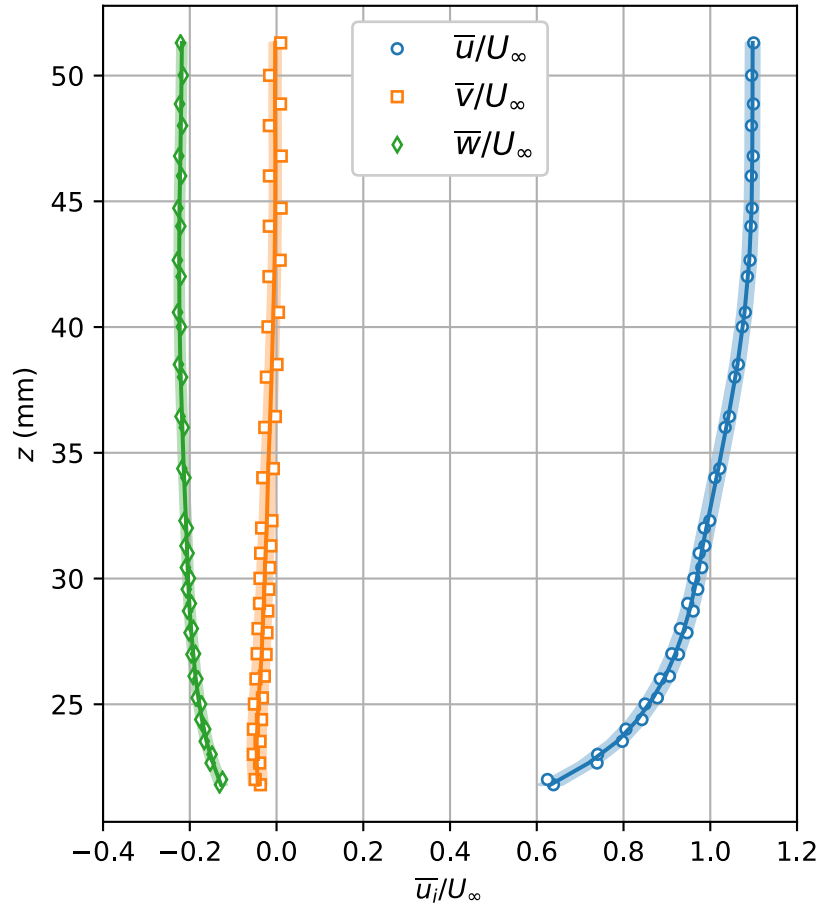


- Solid lines denote the mean value for data, open symbols denote the experimental data
- Shaded uncertainty bands are about the mean line

Example: $x = 2892.6$ mm, $y = -239.1$ mm, $\alpha = 5$ deg



Example: $x = 2747.6$ mm, $y = -266.1$ mm, $\alpha = 5$ deg



- Solid lines denote the mean value for data, open symbols denote the experimental data
- Shaded uncertainty bands are about the mean line



Example: $x = 2747.6$ mm, $y = -266.1$ mm, $\alpha = 5$ deg

