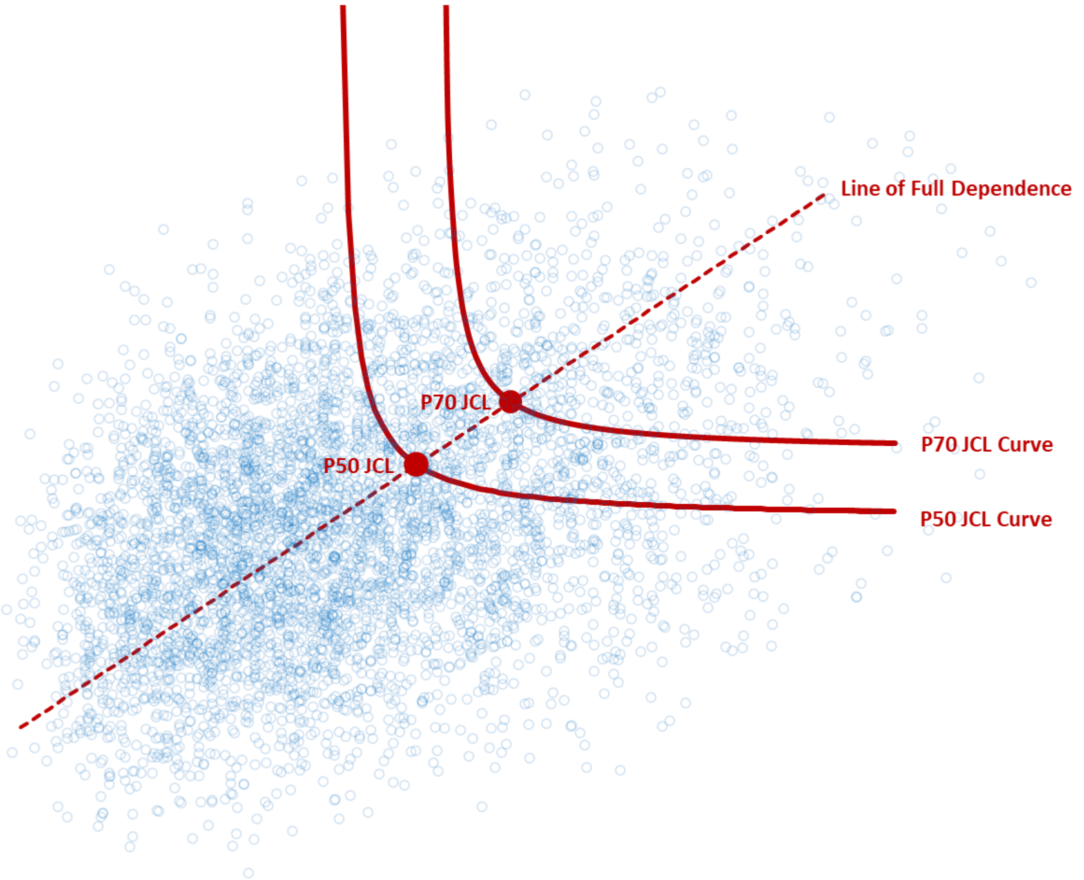
Standardization of JCL Value Selection

Louis Fussell

Programmatic Risk Analyst
Strategic Business Integration Office
Johnson Space Center
E-mail: louis.r.fussell@nasa.gov
Mobile: 832.392.2601

Objective

- Explain the need for a standard JCL selection process
- Describe relevant research
- Describe characteristics of a JCL Curve
- Define equations for the JCL Curve
  - Rational and logarithmic equations
  - Prove accuracy of equations
- Provide Excel functions for finding the *perfect* JCL Point



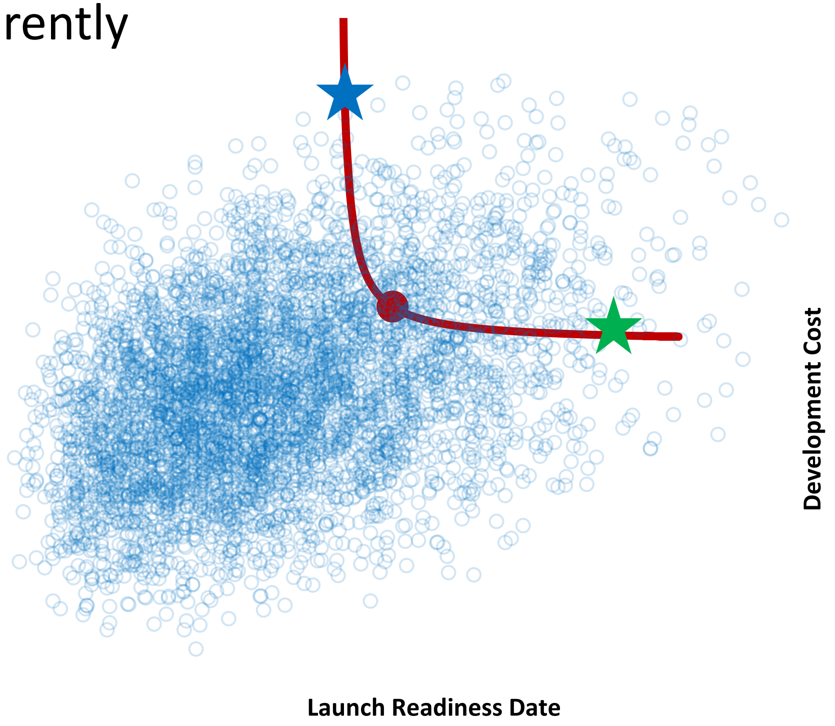
Why We Need a Standard Process

- At Key Decision Points, decision makers may see results from several JCL models
  - Project, Center, Program, Headquarters, and Standing Review Board
  - Need to standardize implementation of the JCL policy

- JCL analysts may agree on the model inputs, but analyze results differently
  - Select a point under the cost cap ★
  - Select a point within the launch window ★
  - Creates tension between administrative and technical/scientific realities

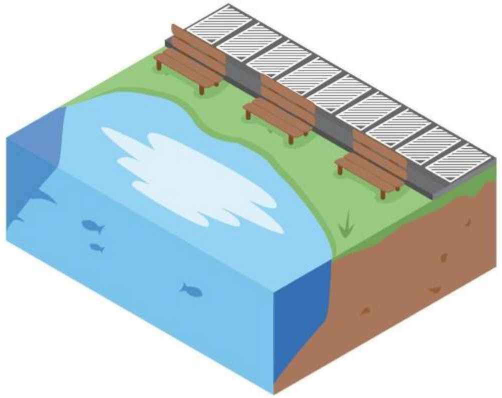
- Eliminate tension by standardizing JCL Point selection
  - Report the "most likely" JCL Point

- Focus decision makers on the JCL model inputs
  - What is driving the JCL result and what can be done about it?

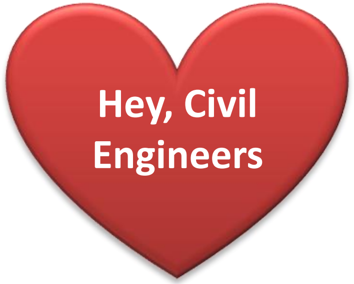


Inspiration from the Study of Hydrology

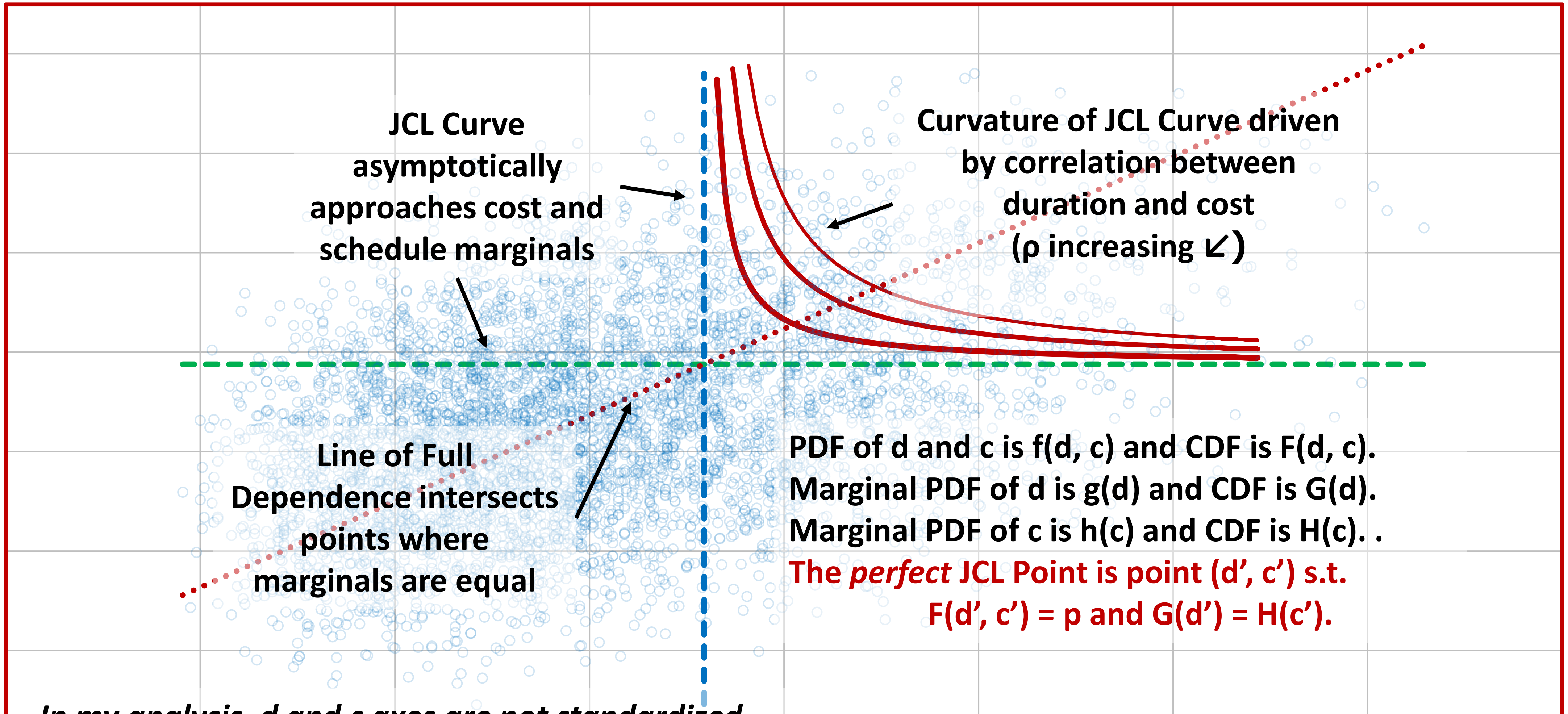
- JCL-like measures used to study flood frequency analysis since 1980’s
  - Bivariate distributions of flood peak and flood volume
  - P-level (similar to JCL) based on flood frequency
  - Variable dependencies derived from historical data
  - Copulas used to model bivariate distributions



- Focus on designing flood mitigation based on most likely scenario
  - Salvadori et al. (2011) recognized some points on p-level curve are more likely to be simulated
    - Apparent from looking at cloud of points
  - Chebana and Ourada (2011) divided the p-level curve into “naïve” and “proper” parts
    - Naïve part is the extremes of the curve
  - Volpi and Fiori (2012) derived conditional distribution for points along the p-level curve
    - Used conditional distribution to define critical events on p-level curve
    - Identified most likely point as being on the “Line of Full Dependence”



Anatomy of a JCL Curve

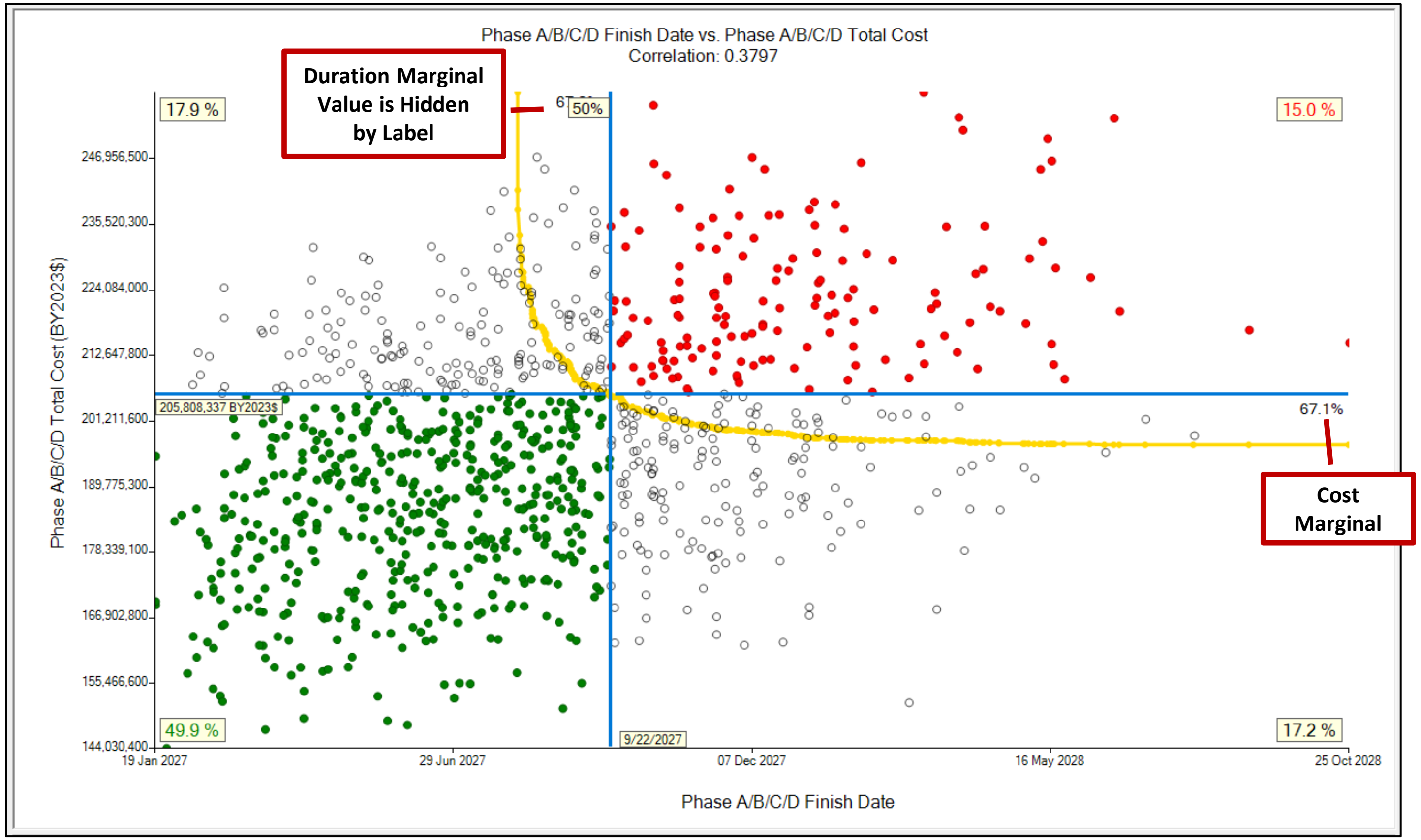


PDF of d and c is f(d, c) and CDF is F(d, c).
Marginal PDF of d is g(d) and CDF is G(d).
Marginal PDF of c is h(c) and CDF is H(c). .
The *perfect* JCL Point is point (d’, c’) s.t.
F(d’, c’) = p and G(d’) = H(c’).

In my analysis, d and c axes are not standardized.

One Can Manually Find the *Perfect* JCL Point in JACS and PRA

- Display the Frontier Line for the desired p-level and show the Crosshairs
- Lock the Crosshairs to the Frontier Line
- Move the Crosshairs around until the marginals are close to equal



It would be nice if the *perfect* JCL Point were the default!

JCL Curve Equations

- Defined by horizontal and vertical asymptotes and single point on curve
  - Md is the duration marginal, where G(d) = p <- vertical asymptote
  - Mc is the cost marginal , where H(c) = p <- horizontal asymptote
  - (Jd, Jc) is a point on the p-level JCL curve, where F(d', c') = p
    - Choosing (Jd, Jc) near the Line of Full Dependence increases accuracy of results
- Note: Equations for curvature factors, Kr and Kl, are not the same

- **Rational Equation**
  - Lesson from 9th Grade algebra
  - Rational Equation for JCL Curve

$$C = \frac{D * Mc + Kr}{D - Md};\ for\ D > Md$$

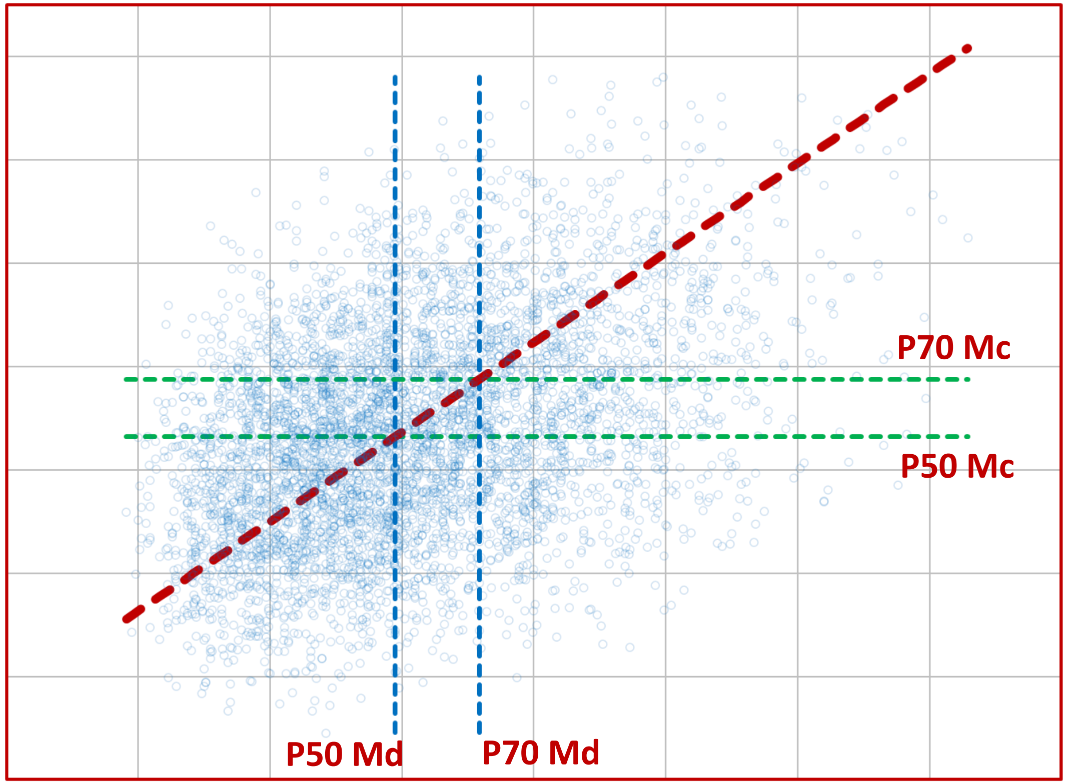
$$Kr = (Jd)(Jc) - (Jd)(Mc) - (Jc)(Md)$$

- **Logarithmic Equation**
  - Adapted from Viopi and Fiori (2014)
  - Logarithmic Equation for a JCL Curve

$$C = -Mc * \frac{D + Kl}{Md + Kl} * \ln\left(1 - \frac{Md + Kl}{D + Kl}\right); for\ D > Md$$

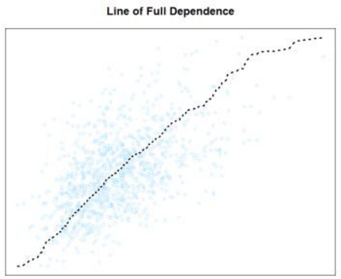
$$Kl\ s.t.\ Jc = -Mc * \frac{Jd+Kl}{Md+Kl} * \ln\left(1 - \frac{Md+Kl}{Jd+Kl}\right)$$

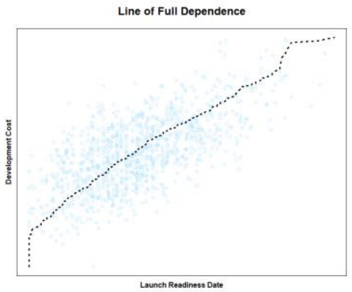
Line of Full Dependence Equation



- Plot of Duration quantiles vs Cost quantiles
  - Sort duration values and sort cost values and plot result
- Line of Full Dependence may stray at extremes
  - Indicator of tail dependence
- Line of Full Dependence is linear'ish through area of interest
  - See examples below
- Equation is the slope-point form of a linear equation

$$C = \left(\frac{P70Mc - P50Mc}{P70Md - P50Md}\right) * (D - P50Md) + P50Mc$$





Putting it All Together

- Find the intercept of the JCL Curve equation and the Line of Full Dependence Equation

- Example: Rational Equation for P50 JCL Point

1. $C = \frac{D*P50Mc+Kr}{D - \mathrm{P50}Md}$ (Rational Equation)

2. $C = \left(\frac{P70Mc - P50Mc}{P70Md - P50Md}\right) * (D - P50Md) + P50Mc$ (Line of Full Dependence Equation)

3. $\frac{D*P50Mc+Kr}{D - P50Md} = \left(\frac{P70Mc - P50Mc}{P70Md - P50Md}\right) * (D - P50Md) + P50Mc$

4. $Let\ m = \frac{P70Mc - P50Mc}{P70Md - P50Md}$

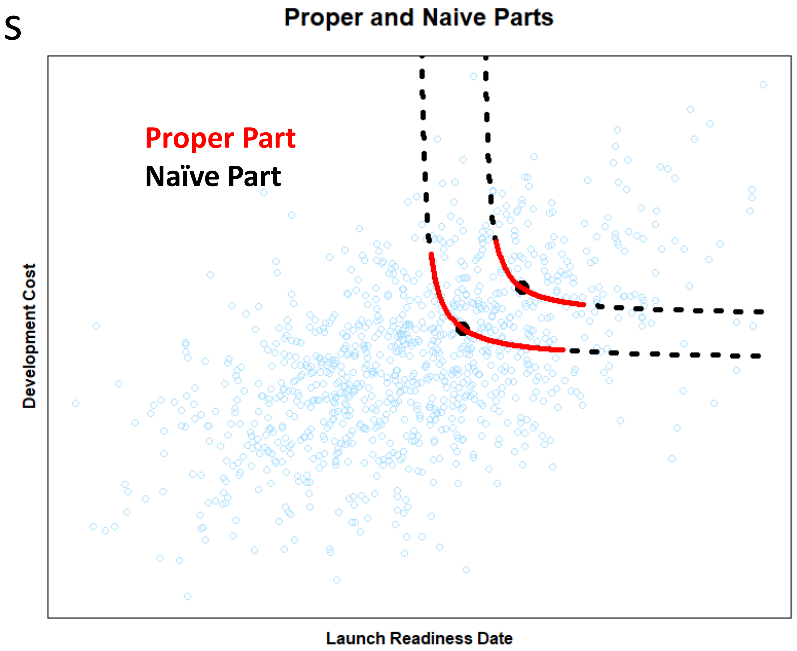
5. $D = P50Md \pm \sqrt{\frac{Kr - (P50Md*P50Mc)}{m}}$

6. *Substitute D into Equation 1 to find C*

Logarithmic Equation requires numerical analysis to solve.

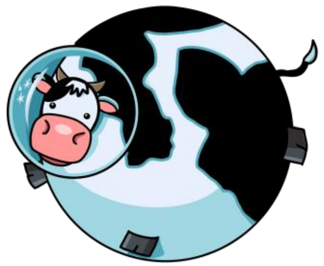
Check for Accuracy – Spherical Cow Cases

- Spherical cow cases
  - Define f(d, c) using a known copula with known marginal distributions
  - Rely on algorithms in R to identify the “true” *perfect* JCL Points
    - p = 0.5 and p = 0.7
  - Use the “true” *perfect* JCL Point to define JCL Curve equations



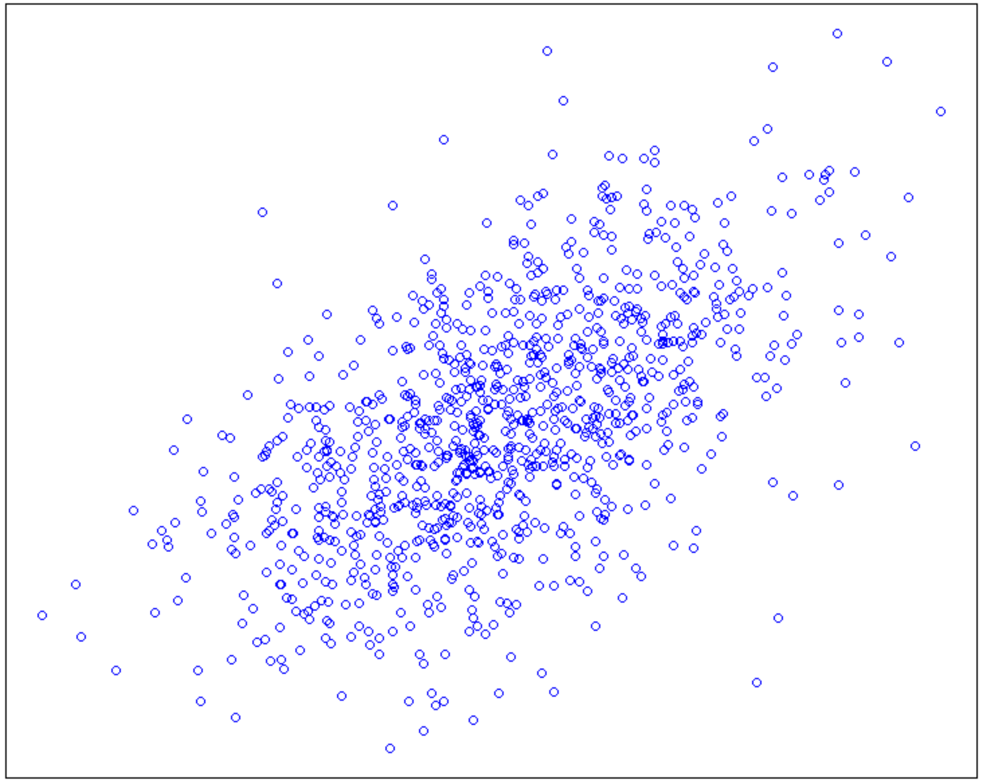
- Two spherical cow test cases
  - Normal copula with normal marginals
  - Gumbel copula with lognormal marginals

- Accuracy test
  - Visually compare equation plots to contour lines generated by R package “copula”
  - Define proper part of JCL Curve as those points within a percentage of the JCL Point
    - Determine JCL values of points calculated in the proper region using R package “copula”
    - Calculate MAE between JCL values and the desired p-level
  - Calculate AE between duration and cost marginals, Md and Mc, using R package “copula”

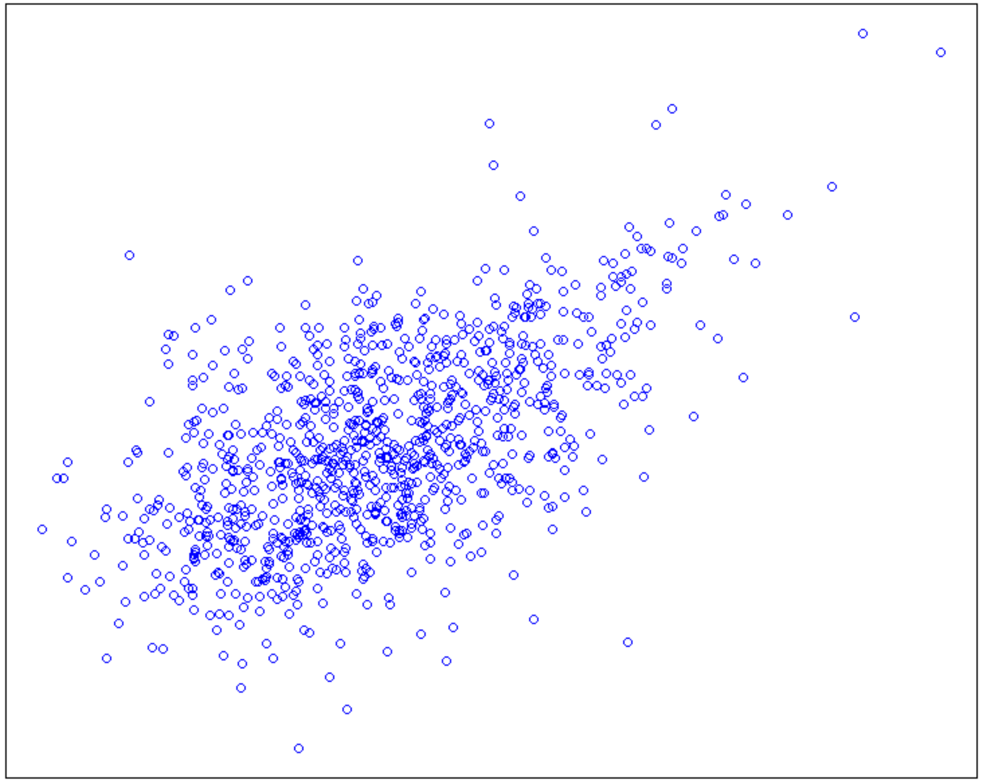


Two Test Cases Have Different Tail Dependence

Normal Copula with Normal Marginals



Gumbel Copula with Lognormal Marginals

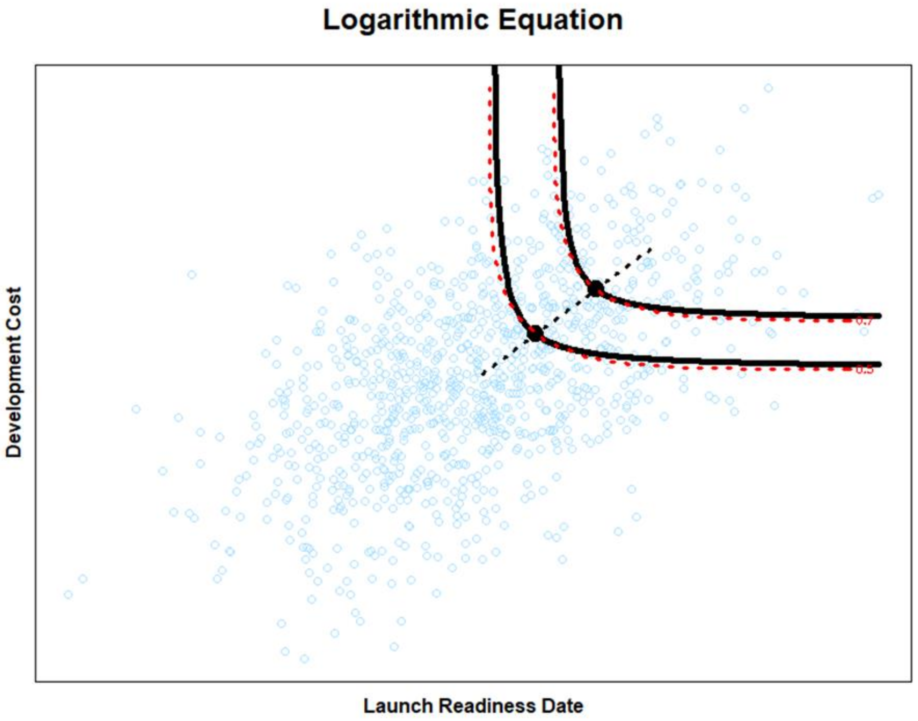


Copula is sampled to give visual of distribution. In perfect cases, samples are not used to derive equations.

Spherical Cow Case with Normal Copula and Normal Margins

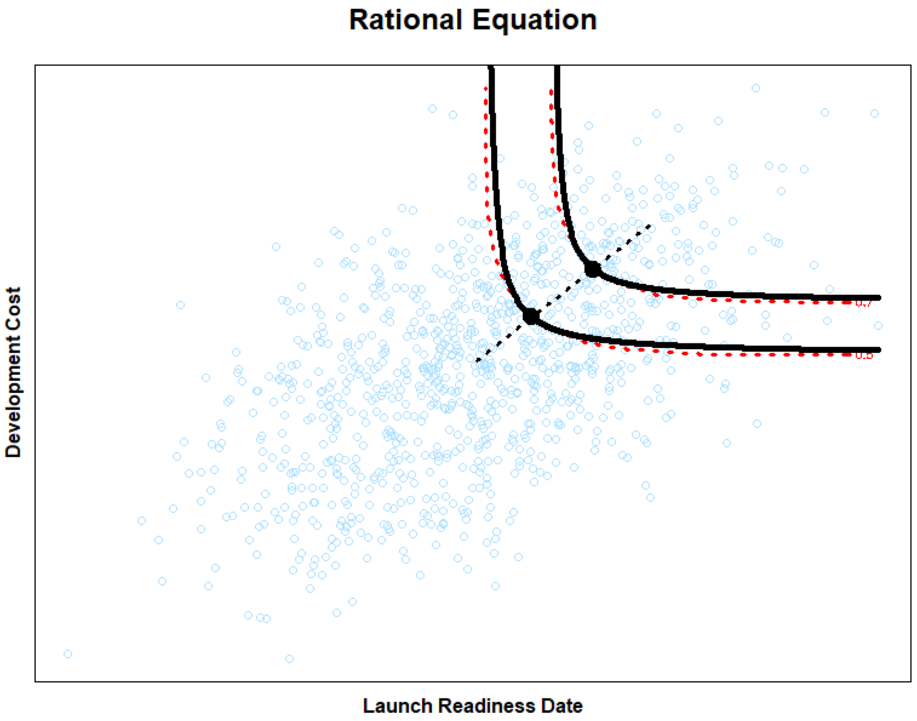
P50JCL: MAE of JCL Values = 0.013, AE of Marginals = 0.001

P70JCL: MAE of JCL Values = 0.007, AE of Marginals = 0.002



P50JCL: MAE of JCL Values = 0.013, AE of Marginals = 0.001

P70JCL: MAE of JCL Values = 0.011, AE of Marginals = 0.002

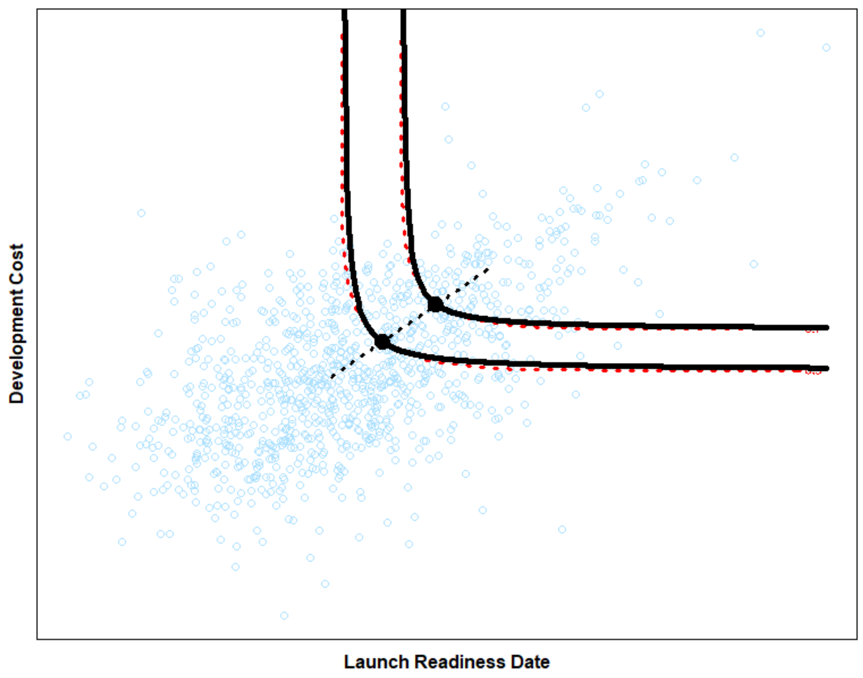


Spherical Cow Case with Gumbel Copula and Lognormal Margins

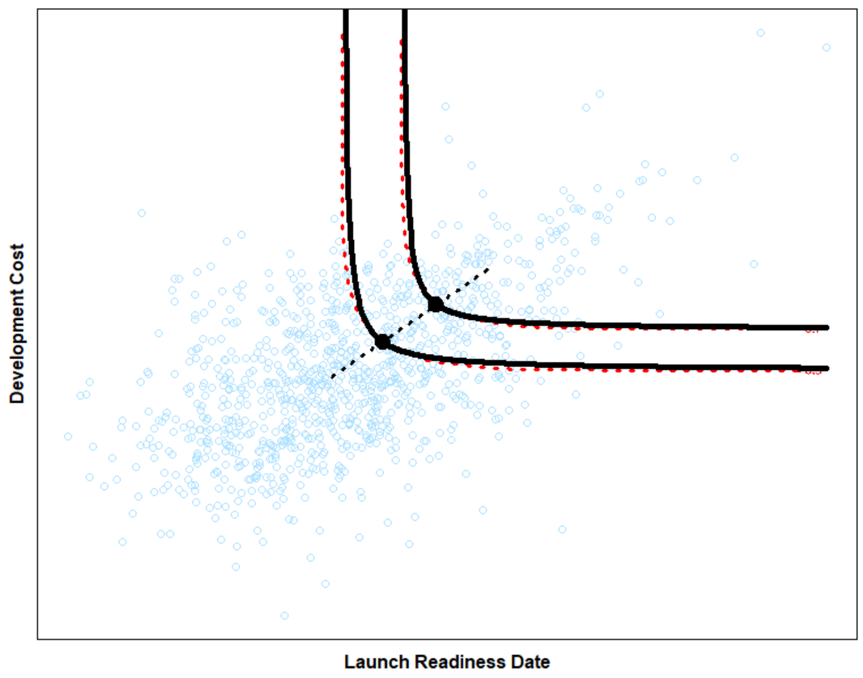
P50JCL: MAE of JCL Values = 0.009, AE of Marginals = 0.0
P70JCL: MAE of JCL Values = 0.004, AE of Marginals = 0.0

P50JCL: MAE of JCL Values = 0.009, AE of Marginals = 0.0
P70JCL: MAE of JCL Values = 0.007, AE of Marginals = 0.0

Logarithmic Equation



Rational Equation



Check for Accuracy – Control Case

- Control Case
  - Define f(d, c) using a known copula with known marginal distributions
  - Draw random samples from the copula distribution
  - Rely on algorithms in R to identify the *sample* p-level JCL points
  - Use the *sample* p-level JCL point to define JCL curve equations

- Two control test cases
  - Normal copula with normal marginals
  - Gumbel copula with lognormal marginals

- Accuracy test
  - Visually compare equation plots to contour lines generated by R package “copula”
  - Define proper part of JCL curve as those points within a percentage of the JCL point
    - Determine JCL values of points calculated in the proper region using R package “copula”
    - Calculate MAE between JCL values and the desired p-level
  - Calculate AE between duration and cost marginals, Md and Mc, using R package “copula”
  - Calculate MAPE between computed JCL point and JCL point obtained using R package “copula”

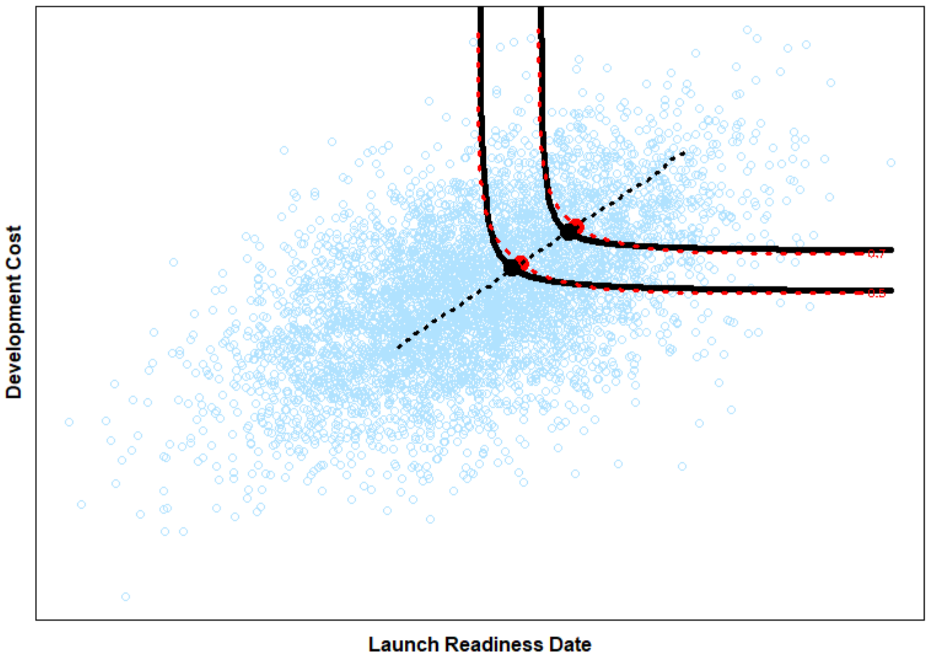
Sample data generated from copula are used to derive equations.

Control Case with Normal Copula and Normal Margins

P50JCL: MAE of JCL Values = 0.013, AE of Marginals = 0.01

P70JCL: MAE of JCL Values = 0.014, AE of Marginals = 0.00

Logarithmic Equation



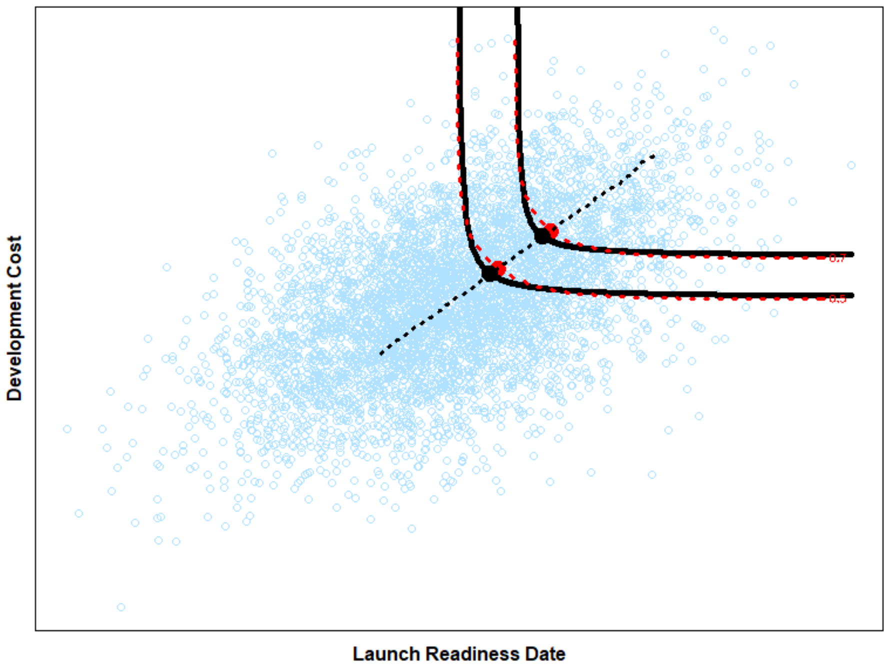
P50JCL: MAPE of JCL Point = 0.35%

P70JCL: MAPE of JCL Point = 0.37%

P50JCL: MAE of JCL Values = 0.013, AE of Marginals = 0.007

P70JCL: MAE of JCL Values = 0.015, AE of Marginals = 0.004

Rational Equation

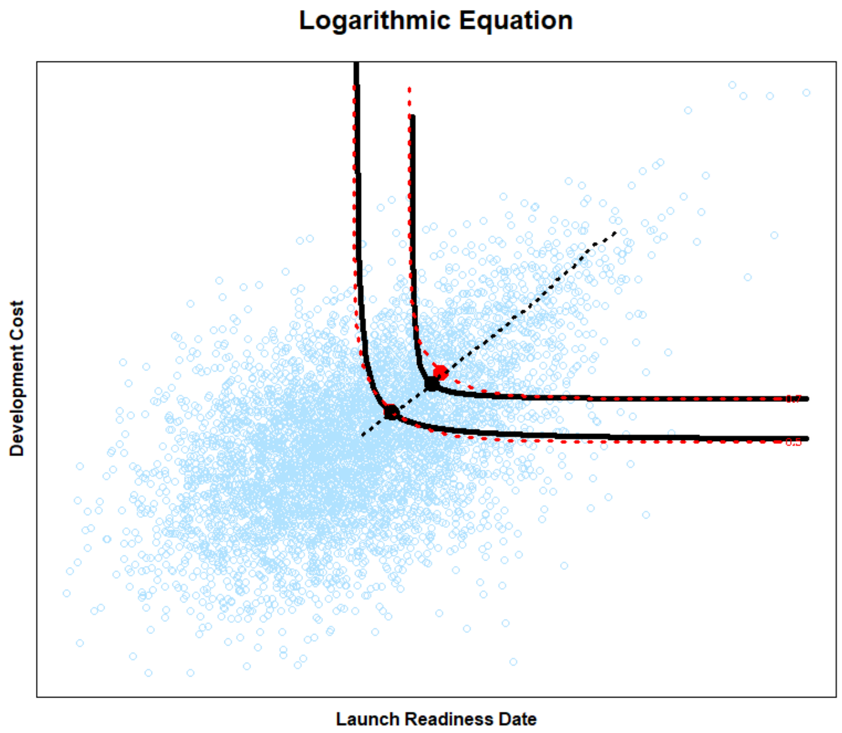


P50JCL: MAPE of JCL Point = 0.37%

P70JCL: MAPE of JCL Point = 0.39%

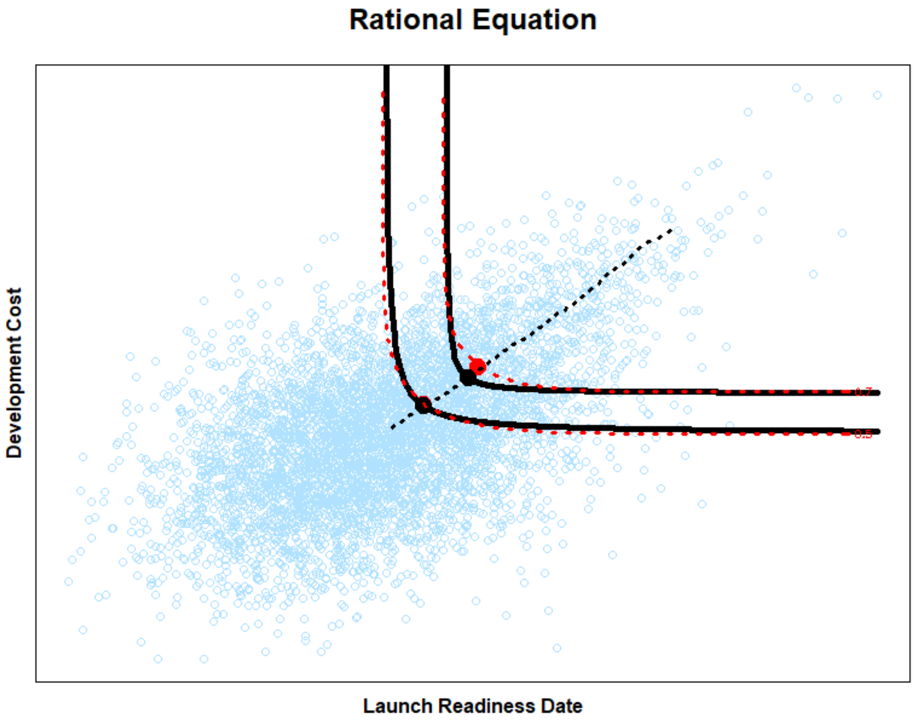
Control Case with Gumbel Copula and Lognormal Margins

P50JCL: MAE of JCL Values = 0.008, AE of Marginals = 0.006
P70JCL: MAE of JCL Values = 0.035, AE of Marginals = 0.013



P50JCL: MAPE of JCL Point = 0.07%
P70JCL: MAPE of JCL Point = 0.97%

P50JCL: MAE of JCL Values = 0.008, AE of Marginals = 0.000
P70JCL: MAE of JCL Values = 0.035, AE of Marginals = 0.012



P50JCL: MAPE of JCL Point = 0.04%
P70JCL: MAPE of JCL Point = 0.96%

Comparison of Logarithmic and Rational Equations

- Logarithmic Equation and Rational Equation produce similar results
  - In the control case, deriving equations from sample data produced acceptable results
  - *Calculated* JCL Point was with 1% MAPE of *true* JCL Point

- Rational equation should be considered a good approximation of the logarithmic equation
  - Let the point O be the JCL point F(d', c') that we calculated
  - Let R(O) and L(O) be the rational equation and logarithmic equation evaluated at O, respectively
  - In the perfect case, R(O) ≈ L(O), R'(O) ≈ L'(O), and R''(O) ≈ L''(O)
  - Believe the Rational equation is a *Padé approximant* of the Logarithmic equation
    - The "best" approximation of a function near a specific point by a rational function of given order

- Rational equations are "easier" to solve
  - Rational equation can be solved analytically
  - Logarithmic equation requires numerical analysis

Recommend Rational Equation

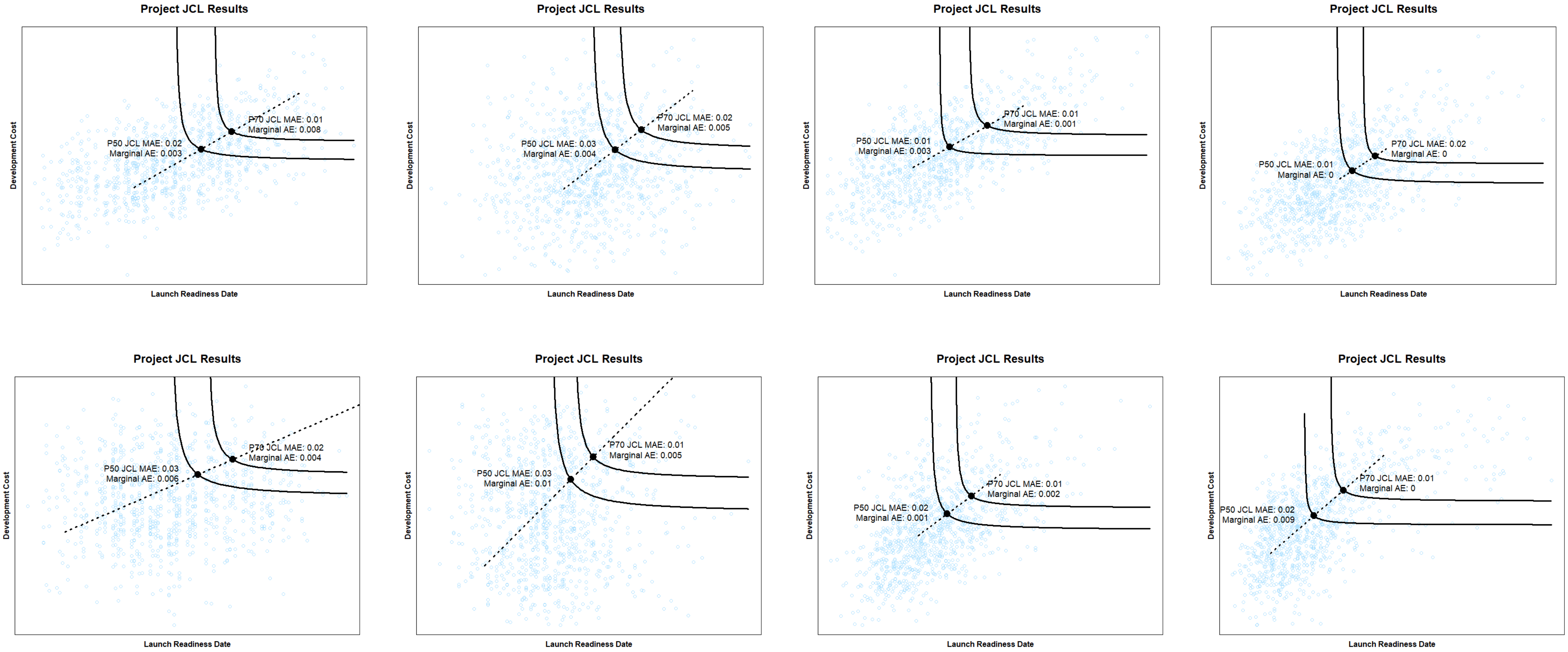
Check for Accuracy – Practical Case

- Practical Case
  - Obtain random samples from a Monte Carlo simulation
    - Risk-adjusted, cost-loaded, integrated schedule model
  - Use the *sample* p-level JCL point to define JCL curve equations

- Recall goal is to find the *perfect* JCL Point where F(d’, c’) = p and G(d’) = H(c’)

- Accuracy test
  - Visually compare equation plots to contour lines generated by R package “copula”
  - Define proper part of JCL curve as those points within a percentage of the JCL point
    - Determine the empirical JCL values of points given the random samples
    - Calculate MAE between JCL values and the desired p-level
  - Calculate AE between the empirical duration and cost marginals, Md and Mc

Practical Case – Rational Equation Applied to JCL Model Data



- ✓ JCL Values in the proper region are close to desired p-level
- ✓ Duration and cost marginals are close to equal.

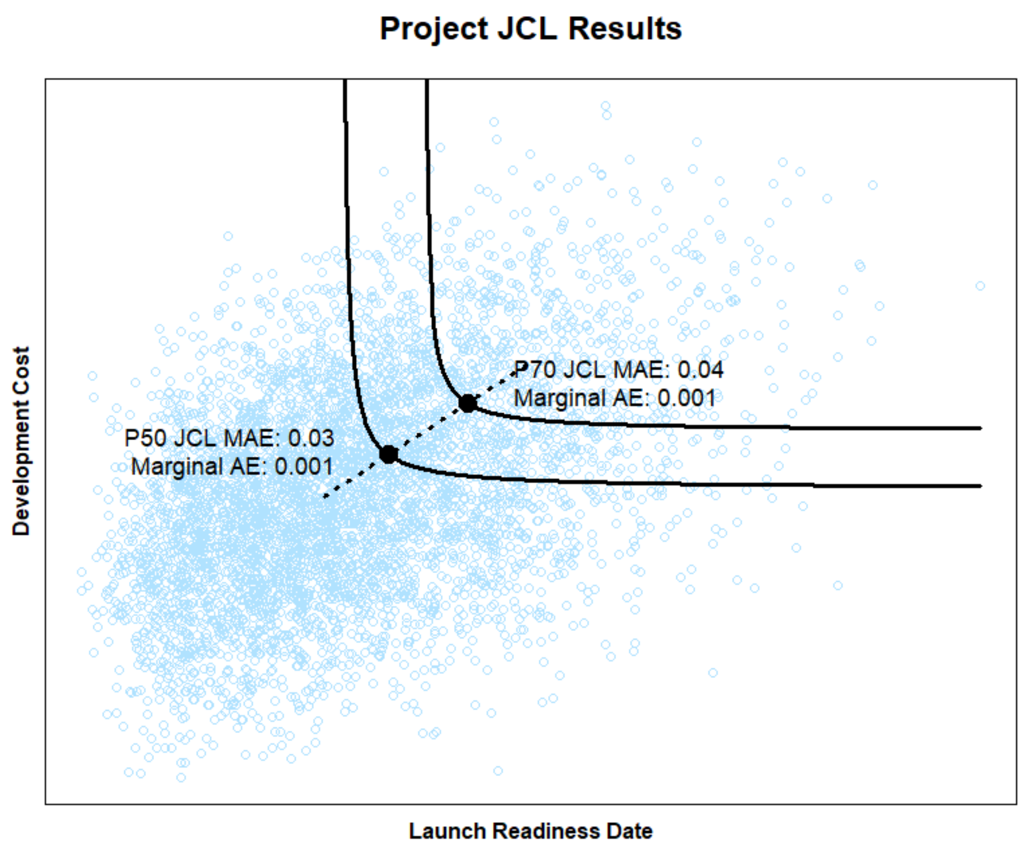
Excel Formulas for Rational Equation

	A	B
1	Variable Name	Sample Data
2	Duration P50	7/27/2027
3	Cost P50	$196,483
4	Duration P70	9/29/2027
5	Cost P70	$207,575
6	D coordinate of point on P50 JCL curve	9/15/2027
7	C coordinate of point on P50 JCL curve	$205,644
8	D coordinate of point on P70 JCL curve	10/28/2027
9	C coordinate of point on P70 JCL curve	$220,379

Data Obtained from JCL Model

MS Excel Formulas to Calculate JCL Point

10	Variable Name	Intermediate Calcs
11	M - Slope of Line of Full Dependence	=(B5-B3)/(B4-B2)
12	K - Curvature Factor	=B6*B7-B2*B7-B3*B6
13	Variable Name	Final Calcs
14	P50 JCL Value D	=B2+SQRT((B12+B2*B3)/B11)
15	P50 JCL Value C	=(B$3*B14+B12)/(B14-B2)



Future Research

- Convince Tecolote that this approach should be built into JACS
- Understand how the correlation coefficient factors into curvature factors (K)
  – Which correlation coefficient?
- Develop an algorithm for calculating confidence intervals around the *perfect* JCL Point
- Prove that the Rational Equation is a Padé approximant of the Logarithmic Equation
- Understand how R “copula” algorithms draw the contour curves
- Please send me your JCL models!