# HabEx Telescope WFE stability specification derived from coronagraph starlight leakage

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# Executive Summary #1

- Exoplanet Science is hard.  It requires that the telescope and coronagraph be designed as an integrated system.

- We describe a rigorous systems engineering methodology for deriving telescope performance specifications from coronagraph performance based on a raw contrast stability error budget.

- To illustrate the methodology, we apply it to four different architectures:
    1. 4-m Off-Axis Unobscured Monolithic Circular Aperture with VVC-4 Coronagraph
    2. 4-m Off-Axis Unobscured Monolithic Circular Aperture with VVC-6 Coronagraph
    3. 4-m Off-Axis Unobscured Monolithic Circular Aperture with HLC Coronagraph
    4. 6-m On-Axis Obscured Hex Segmented Aperture with APLC Coronagraph

- HabEx Baseline (4-m Monolith - VVC-6) has the best performance.

- Architecture 4 (6-m Segmented – APLC) has the worst total performance.

# Telescope Wavefront Stability Tolerances for 4 Coronagraphs:

**VVC 6**

| (C0 = 100 ppt) | tiptilt | defocus | astigmatism | coma | trefoil | spherical | secTrefoil |
|---|---|---|---|---|---|---|---|
| Sensitivities (ppt/pm) | 0.00024 | 0.00029 | 0.00017 | 0.00017 | 0.90 | 0.00029 | 1.04 |
| Allocations (ppt) | 1.52 | 1.52 | 1.52 | 1.52 | 27.5 | 1.52 | 28.9 |
| Tolerances (pm) | 6361 | 5170 | 9095 | 9026 | 30 | 5196 | 28 |

**VVC 4**

| (C0 = 100 ppt) | tiptilt | defocus | astigmatism | coma | trefoil | spherical | secTrefoil |
|---|---|---|---|---|---|---|---|
| Sensitivities (ppt/pm) | 0.00020 | 0.00027 | 0.77 | 0.82 | 0.64 | 1.35 | 0.86 |
| Allocations (ppt) | 1.10 | 1.10 | 16.9 | 18.0 | 14.0 | 20.8 | 18.8 |
| Tolerances (pm) | 5427 | 3996 | 22 | 22 | 22 | 15 | 22 |

**HLC**

| (C0 = 300 ppt) | tiptilt | defocus | astigmatism | coma | trefoil | spherical | secTrefoil |
|---|---|---|---|---|---|---|---|
| Sensitivities (ppt/pm) | 0.0095 | 0.305 | 0.037 | 0.990 | 0.073 | 1.738 | 0.042 |
| Allocations (ppt) | 1.4 | 8.8 | 1.4 | 27.5 | 2.1 | 27.5 | 1.4 |
| Tolerances (pm) | 153 | 29 | 39 | 28 | 29 | 16 | 35 |

peak to valley

**Segm**

| (C0 = 100 ppt) | g_bend | g_powerS | g_spherS | g_comaS | g_comaZ | g_trefZ | g_hexfZ | s_piston | s_tiptilt | s_powerS | s_astigZ | s_trefZ | s_hexfZ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Sensitivities (ppt/pm) | 0.15 | 0.21 | 0.090 | 0.59 | 1.32 | 0.89 | 0.12 | 5.53 | 3.26 | 1.44 | 1.71 | 1.14 | 0.15 |
| Allocations (ppt) | 2.3 | 3.3 | 1.4 | 9.1 | 14.7 | 13.8 | 1.9 | 14.7 | 14.7 | 14.7 | 14.7 | 14.7 | 2.3 |
| Tolerances (pm) | 15.5 | 15.5 | 15.5 | 15.5 | 11.2 | 15.5 | 15.5 | 2.7 | 4.5 | 10.2 | 8.6 | 12.9 | 15.5 |

peak to valley          standard deviation

# What does it take to see an exo-Earth?

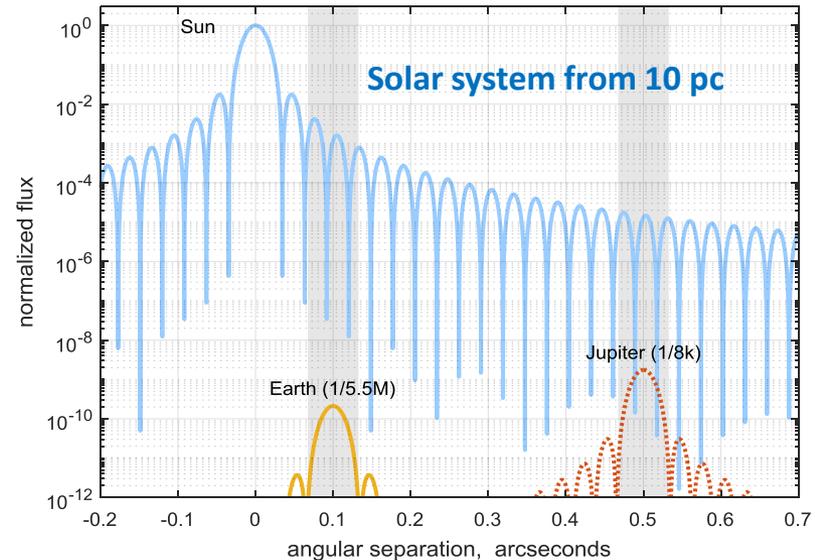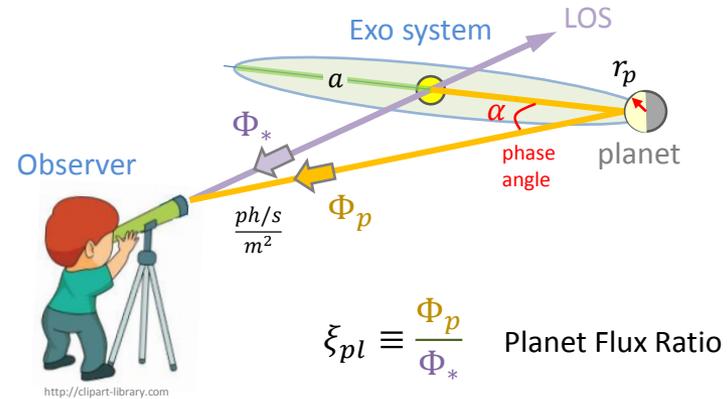The flux ratio of the earth, relative to the sun, is $\sim 2.1 \times 10^{-10}$

$$(210\ ppt)$$

If we could look at our solar system
- in the visible band
- from 10 pc away
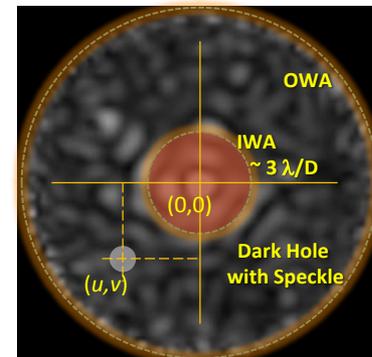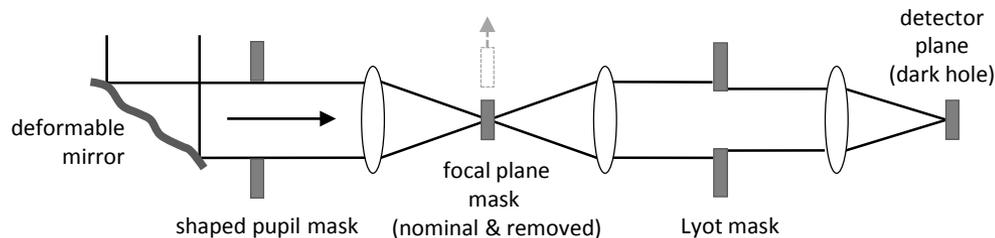- using a 4-meter telescope
- without coronagraph,

the Earth would be buried under the third airy ring of the sun,
$$\text{by a factor of} > 5 \text{ million}$$

- Need to divert diffracted starlight
- We do this with a coronagraph

$$\xi_{pl} \equiv \frac{\Phi_p}{\Phi_*} \quad \text{Planet Flux Ratio}$$

# Coronagraph Elements

- Coronagraph suppresses starlight to allow detection of planet.
- Control diffraction by manipulating the phase and amplitude at a number of planes. Typically via 3 masks and 1-2 deformable mirrors.
- Result is a Dark Hole within which starlight is suppressed strongly relative to planet light.
- Inner and outer working angles are radial limits of a dark hole:
  - IWA is the angle below which the planet light throughput drops to < 0.5 of its peak value within the dark hole.
  - OWA the maximum angle where starlight suppression occurs, limited by the number of deformable mirror actuators.

- Most important measure of coronagraph performance is contrast

- **Contrast** is defined as the fraction of star's light that leaks into the planet location (u,v) relative to the light arriving at (u,v) if the star were at the planet's location (u,v).

| evaluate | source |
|----------|--------|

$$C_{CG}(u,v) \equiv \frac{I_{star}(u,v;0,0)}{I_{star}(u,v;u,v)}$$



(0,0)

(u,v)
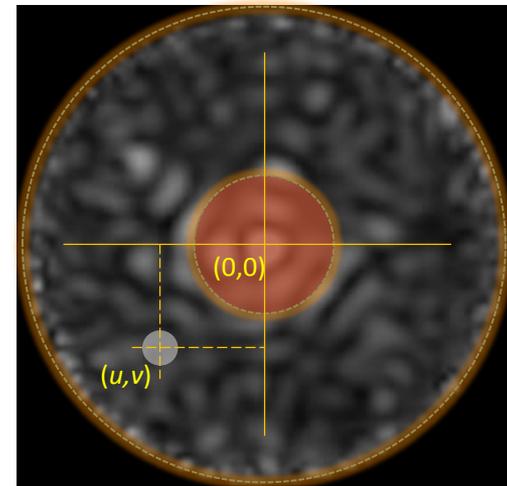
- Another important attribute is throughput.

- **Core throughput** is the fraction of the entering light from a planet that ends up in the planet's point spread function (PSF) "core"

# Defining Core Throughput & Inner Working Angle

- Core throughput is a normalized encircled energy as a function of off-axis angle. It is the fraction of photons incident on the collecting aperture that end up within the half-max contour of the image plane PSF as a function of off-axis 'working' angle.

- Inner working angle is where throughput drops to ½ of its max value

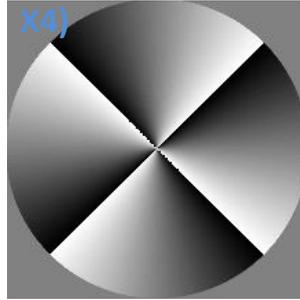- Throughput drops because of Coronagraph vignetting near IWA.
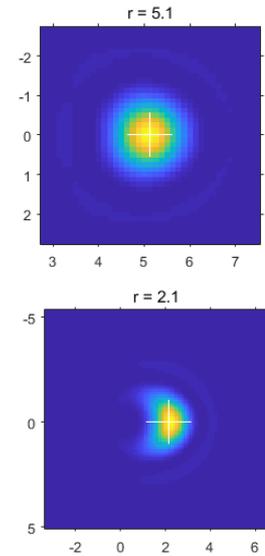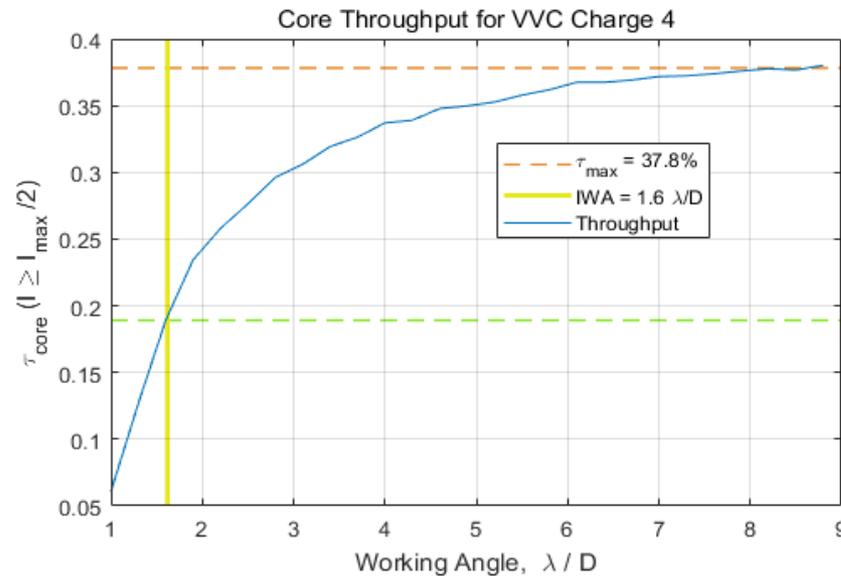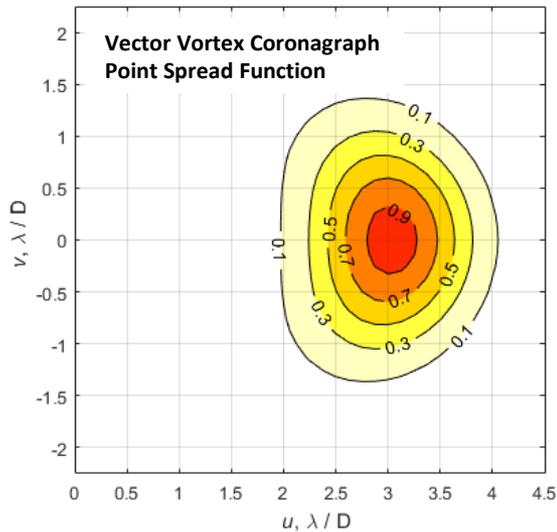
# Architectures Studied

Performance of 4 telescope/coronagraph architectures studied.
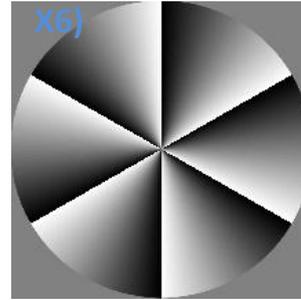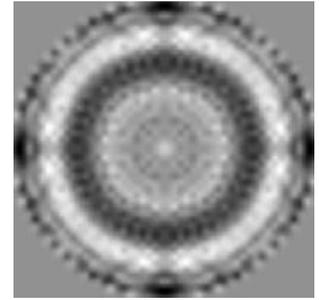
**Unobscured Monolithic (4m)**
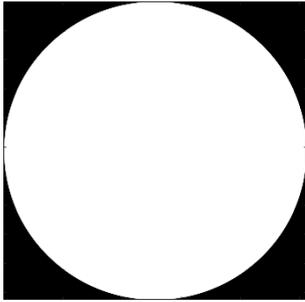
**Vector Vortex Coronagraph, Charge 4 (VVC-X4)**

**Vector Vortex Coronagraph, Charge 6 (VVC-X6)**

**Hybrid Lyot Coronagraph**

**Obscured Segmented (6m)**
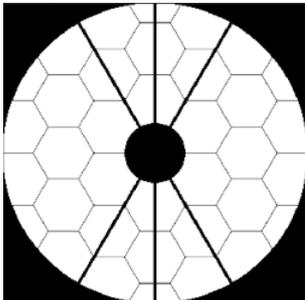
**Apodized Pupil Lyot Coronagraph (APLC)**
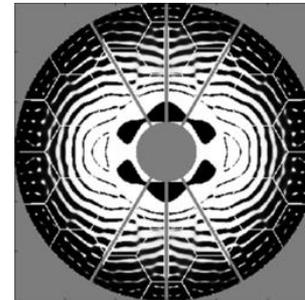
# Core Throughput Comparison

- Comparison of core throughput versus separation angle for the four cases considered in this study.

- The separation for an Exo-Earth at 10 pc (100 mas) is indicated with the vertical line.



Note:  6m aperture has aprox 2X more collecting area than 4m aperture. Thus 'comparative' throughput @ 100 mas would be ~10%.

# Understanding the categories of error

Imaging requires that planet signal has an adequate SNR ($R$).

There are two measurement error categories
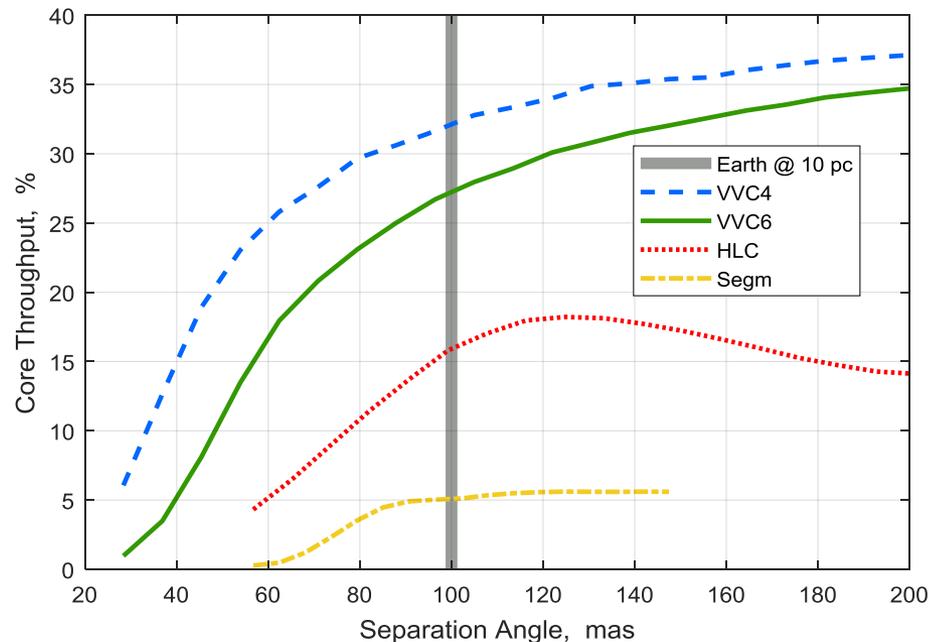
### Random Errors ($\sigma_r$)

Shot noise from signal and background sources (e.g. Zodi)
detector noise

### Systematic Errors ($\sigma_s$)

Optical System Fixed Errors
Optical System Drift – Mechanical & Thermal

residual speckle with a $\lambda/D$
spaced grid superimposed on top



$\sigma_s$ is related to the sdev of the residual intensity variations after averaging within each $\lambda/D$ sized pixel

planet signal

meas. noise

$$S / N = R$$

$$S = r_{pl}t$$

$$N = \sqrt{\sigma_r^2 + \sigma_s^2}$$

$\sigma_r$: random error
$\sigma_s$: systematic error
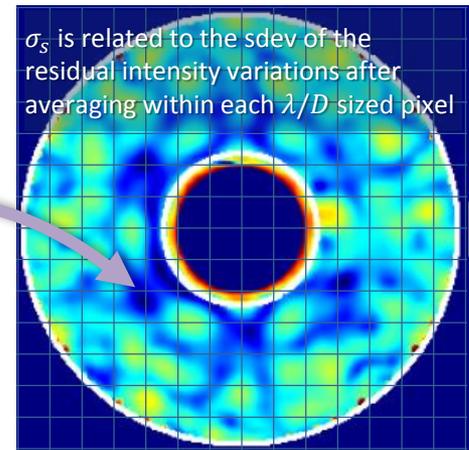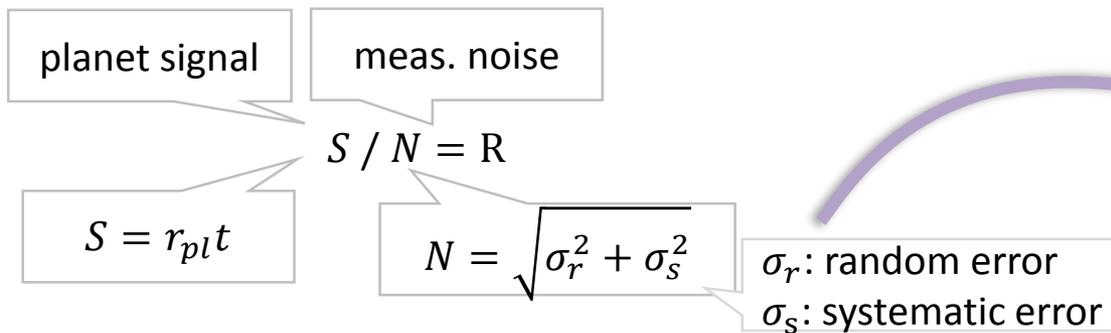
In this image there is no planet

# Correcting Optical System Fixed Errors

- Coronagraph requires 'flat' wavefront, but real systems have errors.
- Deformable Mirror can correct errors over a given spatial frequency range (base on its actuators) to create dark-hole.
- Imperfect Correction results in residual contrast systematic noise.



| | | |
|---|---|---|
| 1 | flat | the WFE or DM surface shape assumed by the design |
| 2 | nm's rms | the practical limit of DM surface fidelity to ideal |
| 3 | | the high-contrast solution near this part of the DM phase space |
| 4 | pm's rms | equivalent relative error with same dark hole E field |
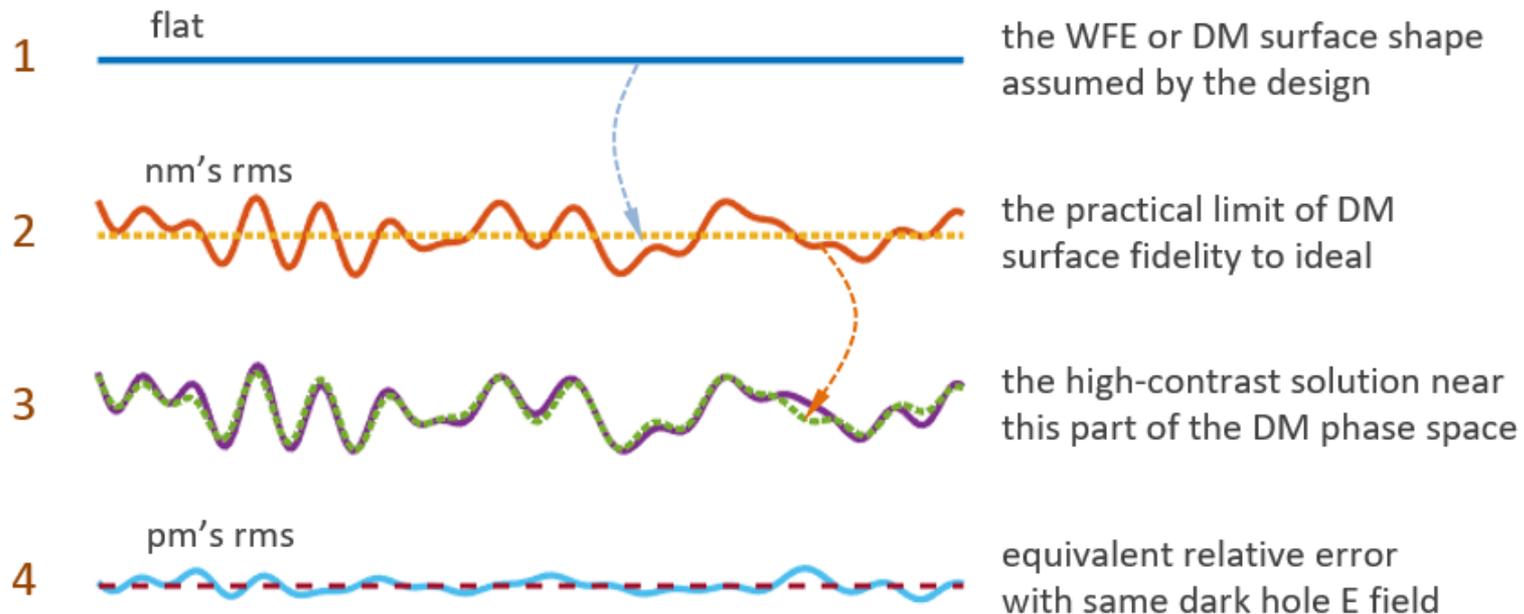
# Speckle Subtraction and Stability

- One way to reduce residual speck noise is Speckle Subtraction via Reference Differential Imaging (RDI)

  - Calibrate Dark-Hole Speckles on Reference Star

  - Subtract Speckles from Target Star signal.

  - Requires that Telescope is Stable over Slew
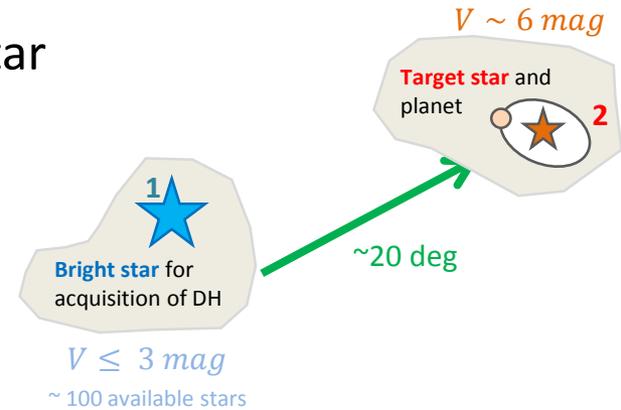


- Any Telescope Perturbation caused by slew appears in cross term.

$$C \propto |E + \Delta E|^2 = |E|^2 + |\Delta E|^2 + 2\,\mathcal{R}\{E^*\Delta E\}$$

existing field    perturbation     Cross term
(static field)    (instability)

- Instability is amplified by the existing E field in the cross term

# Time to SNR and Contributions to Error

- To detect Exo-Earth with SNR = 7, Noise must be < 30 ppt.

- Time to Detect depends on Noise.
  - Random Noise is reduced by longer integration time
  - But systematic noise (e.g. residual speckle) increases with time

- Science Integration time depends on initial residual speckle noise (WFE) and WFE growth with time.

- If Speckle Noise increases too fast, then need to recalibrate dark-hole.



Imaging, 550 nm, 20% BW,  Vortex_X6

Legend: Alloc for SNR = 7, res spec, All Random, zodi shot, planet shot, spec shot, detector



Sun Earth 3xZo 10 pc  Vortex_X6

time to SNR = 7    SNR at t = 25 hrs

residual speckle    $\sigma_s$  $(= f_{pp}\sigma_{\Delta C})$
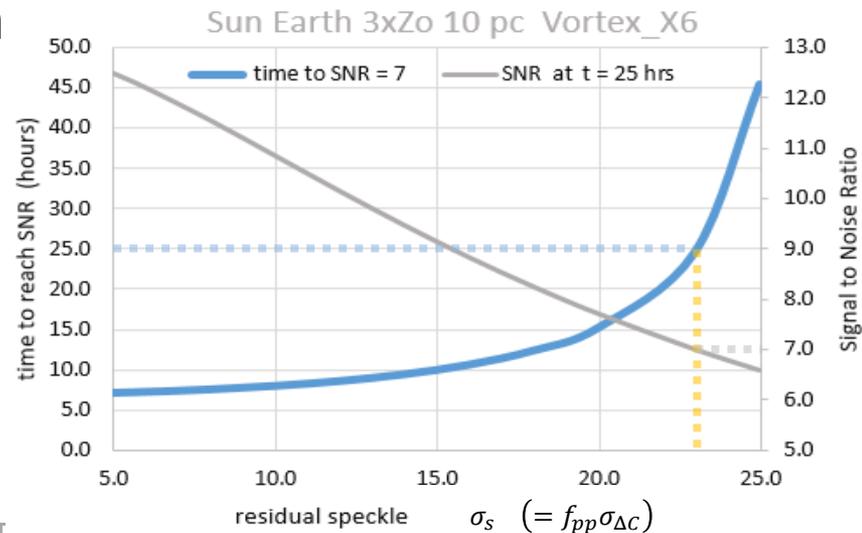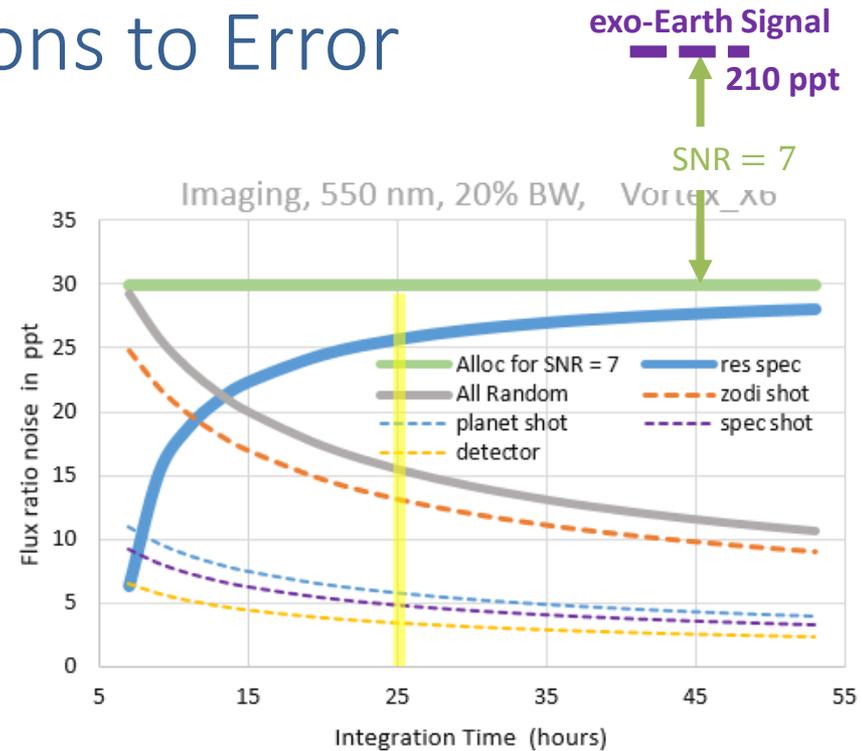
# Contrast Instability Error Budget

For a given desired integration time create an Error Budget for the Telescope Contrast Instability.

- Start with Planet Flux Ratio and desired SNR = 7 to get Flux Ratio Noise

- Allocate noise between Random and Systematic

- Apply Post Processing Factors

- Contrast Instability is what must be achieved by the optical system.



sub-allocate this error

# Allocating Contrast Residual and Instability

- How much star light gets scattered into a given speckle angular separation depends upon the amplitude of the wavefront error of a given spatial frequency (i.e. per the grating equation)

- Because WFE typically decreases with spatial frequency (PSD), residual speckle error sensitivity decreases with angular separation from star (requires higher spatial frequency error to scatter light)

- Contrast Instability must be allocated by Spatial Error Tolerance

allocation · tolerance

$$\epsilon_i = \left(\frac{\partial \epsilon}{\partial x_i}\right) \cdot \delta x_i$$

sensitivity

- A convenient tolerance allocation is Zernike polynomials.

- Each Zernike polynomial WFE has a different Contrast Sensitivity.

# Residual speckle dependence on WFE Trefoil

- Assume that 10 ppt Contrast Instability is allocated to Trefoil
- Left shows residual speckle for 10 pm (PV) of trefoil WFE added between the reference and target star observations (VVC-6 case).
- Right shows Contrast Instability as a function of Trefoil WFE amplitude versus radial distance from star of integrated annular region
- Pink-shaded region shows radial distance where Contrast Instability allocation of 10 ppt is exceeded for given Trefoil error.
- To see an Exo-Earth at 3.5 λ/D, Trefoil cannot exceed ~12.5 pm PV.
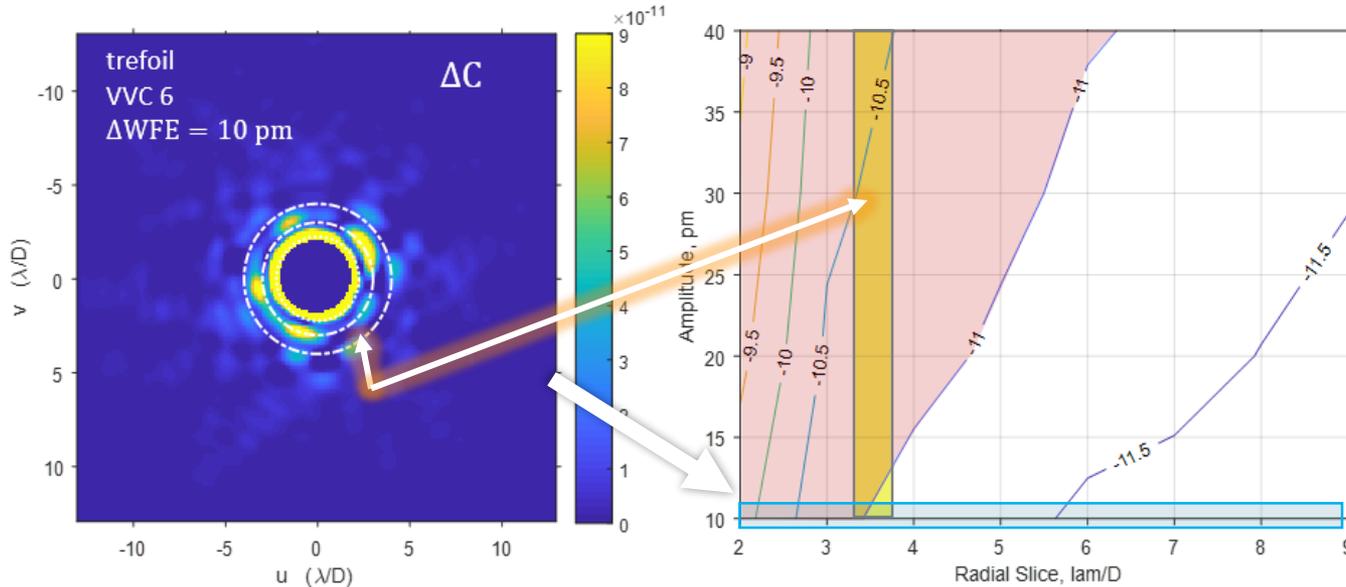
# Contrast Sensitivity for 10 pm P-V error

Plots show Contrast Sensitivity (for each case studied) to various Systematic WFE Changes at radial slice separation from Star for an Exo-Earth at 10 pc observed at 550 nm (center).

- Vertical Line is Instrument $10^{-10}$ Raw Contrast needed to see Exoplanet

- Horizontal $10^{-11}$ Delta-Contrast Line is typical allocation per Zernike

- VVC X4 and X6 are insensitive to some low-order errors.

- Obscured segmented system is extremely sensitive to errors.
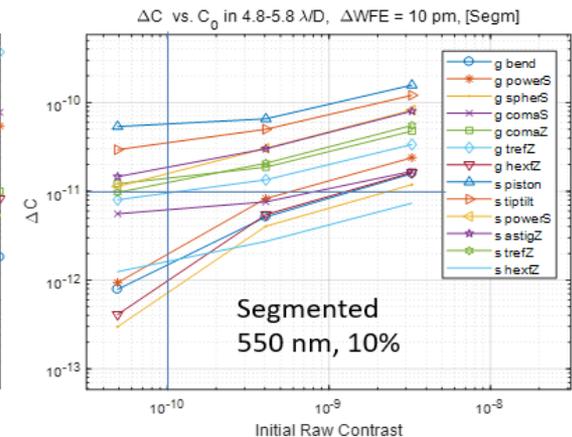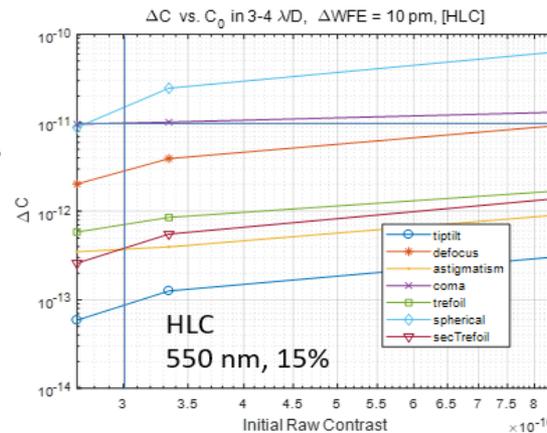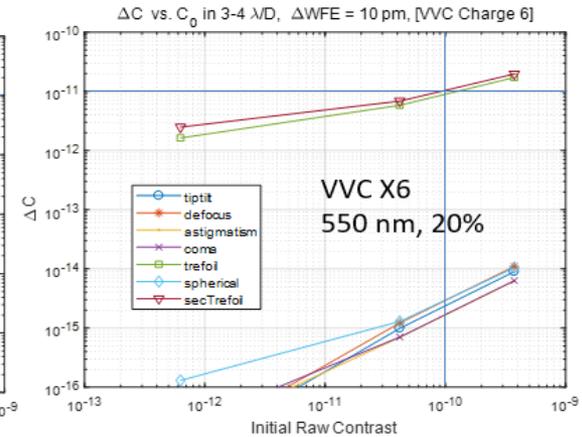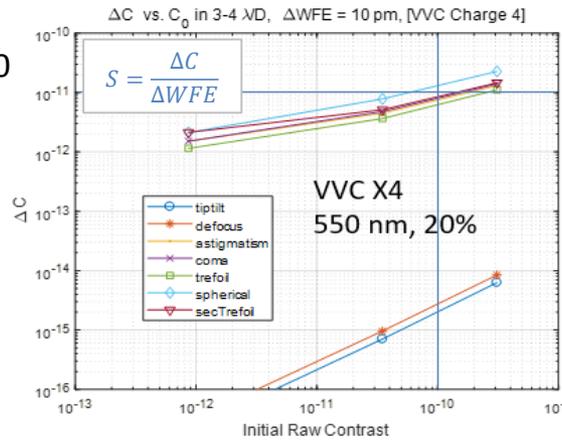
# Wavefront Error Tolerances for Cases Studied

Invert the Sensitivity Plots to determine WFE Allocations per Error

(Note: these are consistent with our previously published numerical simulation results)

**VVC 6**

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**VVC 4**

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peak to valley

**Segm**

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peak to valley                    standard deviation

# Summary and Conclusions

- Exoplanet Science is hard. It requires that the telescope and coronagraph be designed as an integrated system.

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