

Lidar-derived Navigational Geofences for Low Altitude Flight Operations

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Safe Unmanned Aerial Vehicle (UAV) operations near the ground require navigation methods that avoid fixed obstacles such as buildings, power lines and trees. Aerial lidar surveys of ground structures are available with the precision and accuracy to geolocate obstacles, but the high volume of raw survey data can exceed the compute power of onboard processors and the rendering ability of ground-based flight planning maps. Representing ground structures with bounding polyhedra instead of point clouds greatly reduces the data size and can enable effective obstacle avoidance, as long as the bounding geometry envelopes the structures with high spatial fidelity. This report describes in detail four methods to compute bounding geometries of ground obstacles from lidar point clouds. The four methods are: 1) 2.5D Maximum Elevation Box, 2) 2.5D Ground Map Extrusion, 3) 3D Bounding Cylinder, and 4) 3D Bounding Box. The methods are applied to five point cloud datasets from lidar surveys of UAV flight research sites in Georgia and Virginia with an average point spacing that ranges from 0.1m to 0.6m. The methods are assessed using survey areas with geometrically heterogeneous ground structures: buildings, vegetation, power lines, and sub-meter structures such as road signs and guy wires. The 2.5D Maximum Elevation Box method is useful for simple structures. The 2.5D Ground Map Extrusion method efficiently encloses vegetation, but requires hand-drawn ground footprints. The 3D Bounding Cylinder method excels at enclosing linear structures such as power lines and fences. The 3D Bounding Box method excels at enclosing planar structures such as buildings. The methods are compared on the basis of data compression and boundary fidelity on selected areas. The 2.5D methods yield the highest data compression but the polyhedra produced by them enclose significant amounts of empty space. Boundary fidelity is superior for the 3D methods, though this fidelity comes at the cost of a roughly thirtyfold lower data compression ratio than the 2.5D Maximum Elevation Box method. A mix of these output geometries is proposed for autonomous UAV navigation with limited on-board computing. Both the accuracy and spatial detail of emerging satellite-based survey technology lower than that of aerial lidar scanning survey technology. Sub-meter structures and thin linear structures are not reliably mapped at present by satellite-based surveys.

I. Nomenclature

<i>2.5D</i>	=	solid with full two dimensional detail parallel to ground, extruded in altitude
<i>3D</i>	=	three dimensional
<i>ECEF</i>	=	earth-centered-earth-fixed coordinate system standard
<i>Eigenaxis</i>	=	component of coordinate system comprised of orthogonal eigenvectors
<i>GIS</i>	=	geographic information system
<i>ICAROUS</i>	=	independent configurable architecture for reliable operations of unmanned systems
<i>kV</i>	=	thousands of volts
<i>KML</i>	=	Keyhole Markup Language

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<i>LAS</i>	=	file format for lidar point cloud
<i>PCL</i>	=	Point Cloud Library
<i>UAV</i>	=	unmanned aerial vehicle
<i>WGS84</i>	=	World Geodetic System coordinate standard, 1984 revision
<i>XML</i>	=	Extensible Markup Language

II. Introduction

Buildings, power lines and trees are collision hazards in Unmanned Aerial Vehicle (UAV) operations at altitudes just above ground level. Techniques to avoid these obstacles are needed for safe low altitude navigation. Assuming GPS and other onboard sensors provide UAV geolocation on the order of meters, avoidance can be accomplished while planning the flight (proactively) and during the flight (reactively) if the GPS coordinates of the boundaries of obstacles are known. Before a flight, a UAV operator can choose flight waypoints that are a safe distance from obstacles if the obstacle boundaries are displayed in a flight planning map. During flight, onboard autonomy can compare the vehicle coordinates to the boundary coordinates and adjust the flight path to avoid collision in the event that the vehicle strays off-course due to off-nominal conditions such as high winds.

There are two prerequisites to accomplishing these operational practices. First, accurate surveys of obstacle boundaries are required, and second, the obstacle boundaries must be represented compactly. In modern practice, surveys of ground structures combine traditional tripod-based human measurements (to establish known reference control points) with lidar point clouds from aerial [1] or ground [2] scans which are fitted to the control points [3]. Richly detailed surveys with decimeter level accuracy are commonplace with this method. Compact representation of obstacle boundaries is needed, because detailed three dimensional (3D) lidar surveys of a site generate enormous data sets. The raw data size will overwhelm both the visualization software used to plan flights and the onboard computing [4] used for inflight obstacle avoidance. Naïve data reduction such as downsampling prior to boundary determination risks the removal of critical obstacle features.

This report first describes in detail four methods to compute a compact representation: 1) 2.5D Maximum Elevation Box, 2) 2.5D Ground Map Extrusion, 3) 3D Bounding Cylinder, and 4) 3D Bounding Box. The methods are applied to five point cloud datasets from lidar surveys of flight research sites in Georgia and Virginia. Research flights were conducted at these sites to test technologies for UAV inspection of high voltage electrical infrastructure [4]-[9] and for safe UAV urban operations [10]. The spatial sampling (average point spacing) of the datasets ranges from 0.1m to 0.6m.

A. Brief summary of literature

Simplified boundary representations of ground structures have been available for some years. Commonly referred to as digital elevation maps, these representations are used, for example, to map tree canopies in environmental and geological studies [11], buildings [12], and piping [13]. In early studies (see [14] for a review) digital elevation maps represented x and y dimensions with much more fidelity than the z dimension, and were also called 2.5D models [15].

As aerial lidar scan and computing technologies advanced, full 3D models were developed to classify ground obstacles. While the variety of ground structures under study and its partitioning into geometric classes differs from study to study (e.g., [16] [17]), the majority of researchers considered at least three distinct geometric groups: planar structures, linear structures, and vegetation. Man-made structures such as buildings and bridges are predominantly collections of planar facets, while wires and poles are linear, and trees and other vegetation are neither (or quasi-fractal [18]). Planar models are almost universally employed for building facets, while linear structures are modeled either as cylinders [12][13][19], or, in the case of suspended lines such as cables and electrical conductors, as catenary curves [20][21][22]. Because the geometry of vegetation is so complex, diverse, and difficult to model [18][23][24], a classification pipeline is commonly used: first planar and linear structures are identified and enclosed with 3D bounding geometries, and then the enclosed points are removed from the point cloud, leaving only lidar points from vegetation [25]. The remainder is then modeled with a battery of techniques [25], fitted with a convex hull [26], or (analogous to draping it with an elastic sheet) simply enclosed with a 2.5D surface spline [27].

Surveys derived from satellite-based [28] and UAV-based [29] photogrammetric stereo imaging are rapidly progressing. While inadequate for low altitude UAV navigation (see Discussion), survey data from these technologies

can be processed with the methods described in this report and used for low resolution applications such as topographical classification and forestry management.

Accurate geolocation of a UAV is not assured, particularly near ground structures [5]. GPS position estimates can be degraded and even lost due to reflection and obstruction of the signals from orbital satellites, and nearby ferrous material can severely distort onboard magnetometers readings. If the bounds of geolocation error are characterized along a flight path, safety buffers around obstacle boundaries can compensate for positioning error [4][5].

B. Outline of the report

The performance of each method is described in Section III and the methods are compared in Section IV using illustrative subsets of the five lidar surveys. The survey data includes ground structures with heterogeneous geometries: buildings, vegetation, power lines, and small structures such as road signs and guy wires. Both the boundary fidelity and the degree of data compression afforded by the method form the basis for comparison. In Section V, the suitability for autonomous UAV navigation of the output geometries from the methods is discussed in the context of computing requirements and spatial fidelity, and the accuracy and spatial detail of emerging satellite-based survey technology is critically assessed.

III. Lidar Data and Bounding Methods

A. Lidar Data

Five sets of aerial lidar survey data were used to assess and compare the methods. All data was received in the LAS (LASer) point cloud file format. When converting from the state plane coordinate system of the input survey to the WGS84 (World Geodetic System 1984) latitude/longitude/altitude coordinate system, a grid unit spacing of at least six decimal places of decimal latitude and longitude was used, which corresponds to approximately 0.1 meter spatial resolution at middle latitudes. The datasets are:

- A. A survey of a 1.0 square mile area centered on the Southern Company Klondike Training Facility in Lithonia, Georgia collected in May, 2015. At an average point spacing of 0.3m, the full 282M point dataset size is 7.3GB.
- B. A survey of a 0.3 mile by 0.2 mile area centered on the Dominion Energy Training Facility in Chester, Virginia collected in November, 2016. At an average point spacing of 0.33m, the full 8M point dataset size is 270MB.
- C. A survey of a 29 mile transmission line corridor originating at a Dominion Energy facility in central Virginia collected in October, 2015. At an average point spacing of 0.6m, the full 117M point dataset size is 3.4GB.
- D. A survey of a 1.5 square mile area centered on the NASA Langley Research Center in Hampton, Virginia collected in April, 2015. At an average point spacing of 0.2m, the full 117M point dataset size is 3.2GB.
- E. A survey of a 1.0 square mile area centered on the NASA Langley Research Center in Hampton, Virginia collected in February, 2018. At an average point spacing of 0.1m, the full 195M point scan dataset size is 5.2GB.

B. Bounding Methods

Four methods for creating bounding geometries that enclose lidar-surveyed ground obstacles are described in this section:

1. 2.5D Maximum Elevation Box,
2. 2.5D Ground Map Extrusion,
3. 3D Bounding Cylinder, and
4. 3D Bounding Box.

Since the two 3D methods have similar processing steps, they are described together. Figure 1 schematically illustrates the four methods.

1. 2.5D Maximum Elevation Box

The 2.5D Maximum Elevation Box method, written in C++, constructs bounding rectangular polyhedra aligned with a latitude/longitude grid which extend perpendicularly from the ground plane to the maximum enclosed altitude. This method was applied to lidar dataset A, provided by Southern Company. The structures in this survey are largely aligned to lines of latitude and longitude; with suitable rotation in the ground plane, the method can be applied to other structure surveys as well. After ground point removal the lidar points are converted from the local state projection to the WGS84 (x,y,z: latitude, longitude, altitude) coordinate system.

After sorting based upon x and y values, the set of lidar points cloud is traversed, first along lines of latitude and then along lines of longitude, finding clusters of points within a radius set to twice the minimum point cloud spacing. A provisional polyhedron is constructed with a height equal to the maximum altitude within each cluster. As points are traversed and examined, they are marked as processed to avoid overlaps and double counting. After all points are traversed, provisional polyhedra are merged along lines of latitude such that grid locations with lidar points above ground level are set to the maximum value along the line. This simplification results in a striped morphology that was appropriate for the elongate high voltage structures in the flight test range (Fig. 1, left).

2. 2.5D Ground Map Extrusion

The 2.5D Ground Map Extrusion method (Fig. 1, middle) merges two representations of ground structures: hand drawn footprints and aerial lidar survey data. A high fidelity survey database of the NASA Langley Research Center has been built and maintained using a variety of techniques including traditional tripod-based measurements, aerial and ground lidar scans, and airborne photogrammetry via survey-grade UAVs. Ground structure footprints are periodically entered and revised as graphical elements (circles, ellipses and Bezier curves) and stored with corresponding structure elevation estimates using the ArcGIS geographical information systems toolset.

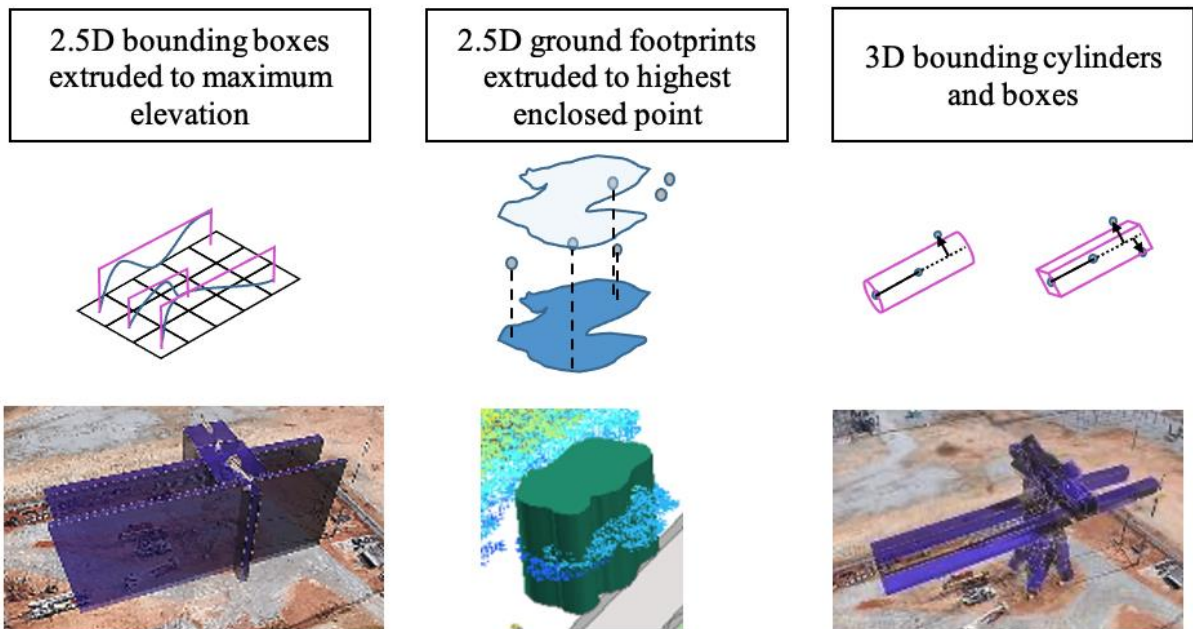


Figure 1. Polyhedra construction methods. Left to right: 2.5D Maximum Elevation Box, 2.5D Ground Map Extrusion, 3D Bounding Cylinder, and 3D Bounding Box. The 2.5D Maximum Elevation Box method is simplest and produces the greatest data compression at the cost of feature fidelity and ‘dead space’. The 2.5D Ground Map Extrusion method is less compact and follows the lidar point cloud more closely, but requires hand-drawn ground footprints and fails to enclose lidar points that do not overly the footprints. Feature fidelity is highest for the 3D Bounding Cylinder and 3D Bounding Box methods, but these methods yield the lowest data compression. © Map data: Google & DigitalGlobe

This method attempts a compromise between the inherently simplified graphical footprints and the exact but enormous raw lidar survey data by combining them to construct 2.5D right prisms with complex, multi-vertex base polygons. Aerial scan lidar is converted from its initial LAS format to an ArcGIS ‘multipoint’ format in the same local state

plane coordinate system as the footprints. Python code running in the ArcGIS tool then performs an ArcGIS ‘spatial join’ operation, which finds all (x,y,z) triplets of the point cloud which are enclosed by the footprint polygons, and an ‘extrude’ operation which finds the maximum altitude of the enclosed points. Finally, custom Python code exports each footprint curve from ArcGIS, discretizes it to a polygon, constructs a 2.5D prism with the height found in the extrude operation, and converts it to the WGS84 (latitude/longitude/altitude) coordinate system. Footprint curve discretization is conducted such that there is no greater than a two centimeter deviation from the original hand-drawn curve.

The 2.5D geometries (Fig. 2, left) are geospatially verified by visual inspection after conversion to KML (Keyhole Markup Language) and importing to Google Earth (Fig. 2, top right) and also after conversion to XML (Extensible Markup Language) and importing to the MavProxy [30] ground station platform (Fig. 2, bottom right). Additionally, the XML is imported to the NASA Polycarp [31] geometry engine to verify that the polyhedra are free of problematic geometries such as repeated vertices and nonoriented (“bow-tie”) topologies. This geometry engine is used by the Safeguard and ICAROUS safety technologies (see Discussion).

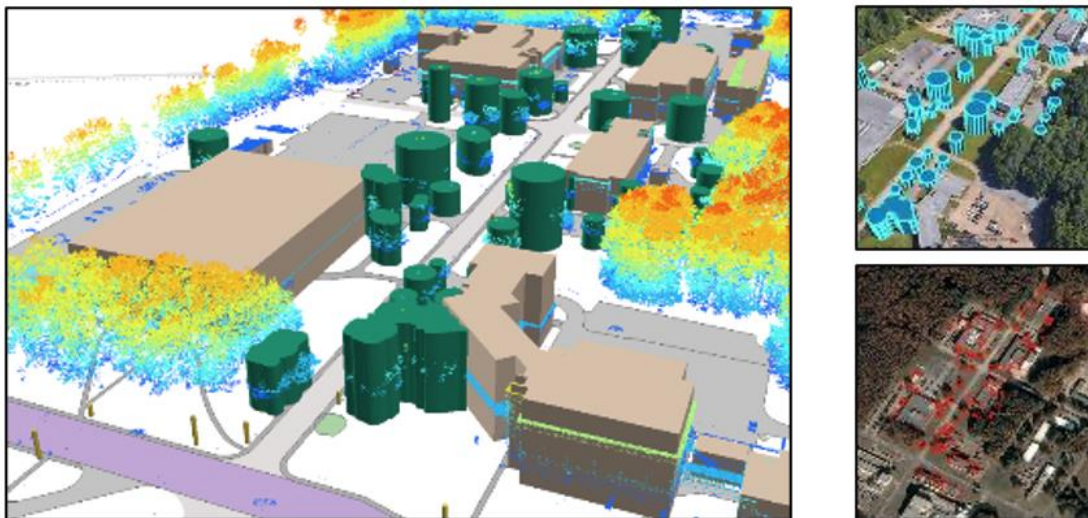


Figure 2. 2.5D Ground Map Extrusion method. Left: Hand-drawn footprint curves for vegetation are digitized and extruded to the highest elevation of the lidar points enclosed. Building footprints are extruded to their surveyed height. Right: The constructed polyhedra for vegetation are verified as KML in Google Earth (top right) and, along with the building footprints, as XML in the MavProxy [30] ground station map (bottom right). © Map data: Google & DigitalGlobe

3. 3D Bounding Cylinder and Box Methods

The 3D bounding methods (Fig. 1, right) cluster an input lidar point cloud via the steps described in the following paragraphs. Spatial constants (e.g., d_{min}) used in the steps were optimized for datasets with an average point spacing of 0.3 m or less.

Overview. After an initial grouping of lidar points into clusters based on proximity, each lidar point may be a member of multiple, spatially overlapping neighborhoods. The 3D methods are designed to find a set of neighborhoods just sufficient to enclose the point cloud. This is accomplished by repeatedly growing existing neighborhoods and eliminating neighborhoods made redundant by that growth. Initially the neighborhood count is on the order of the lidar point count, and as the growth/elimination iterations proceed, the neighborhood count drops rapidly.

The growth/elimination sequence and the metrics used to determine which neighborhoods to grow/eliminate are described in the remainder of this Section.

Create base neighborhoods. Aerial lidar survey data is preprocessed by removing points in the ground plane and converting from LAS format in a survey coordinate system (e.g., State Plane Georgia) to a point array $P_{1...n}$ of (x,y,z) triplets in the earth-centered-earth-fixed (ECEF) coordinate system. For each point P_n a local neighborhood N_n is

found which contains all points no more than $d_{min}=1m$ distant from P_n . For each neighborhood, an eigenaxis-aligned oriented cylinder or oriented bounding box is found using the Point Cloud Library (PCL) moment of inertia estimator, which computes the major, middle and minor eigenvalues and eigenvectors. The eigenvectors form an orthogonal basis $(\hat{a}, \hat{b}, \hat{c})$, and the oriented cylinder or box is aligned with a major axis in the \hat{a} direction and dimensions sufficient to contain all of the points in the neighborhood.

Remove outliers. The distribution of a neighborhood’s point distances from the neighborhood centroid is computed, and those points with a distance greater than 1.5 times the interquartile range are considered as outlier points and removed from the neighborhood.

Sort neighborhoods (cylindrical method). Neighborhoods are sorted from largest to smallest according to the number of points enclosed in the neighborhood. If two or more neighborhoods enclose the same number of points, they are sorted according to a measure of ideal cylindricality (the ratio of major eigenvalue to middle eigenvalue).

Sort neighborhoods (box method). A tally *neighborhood_agreement* is initialized to zero for each base neighborhood. Duplicate neighborhoods are eliminated by finding neighborhoods which have the exact same points and removing all except one from further consideration. The *neighborhood_agreement* tally is incremented for each duplicate of a particular neighborhood that is found and eliminated. The primary, secondary, and tertiary eigenaxes $(\hat{a}, \hat{b}, \hat{c})$ of each remaining oriented bounding box is recalculated and all points that fall within its volume are added to its neighborhood grouping.

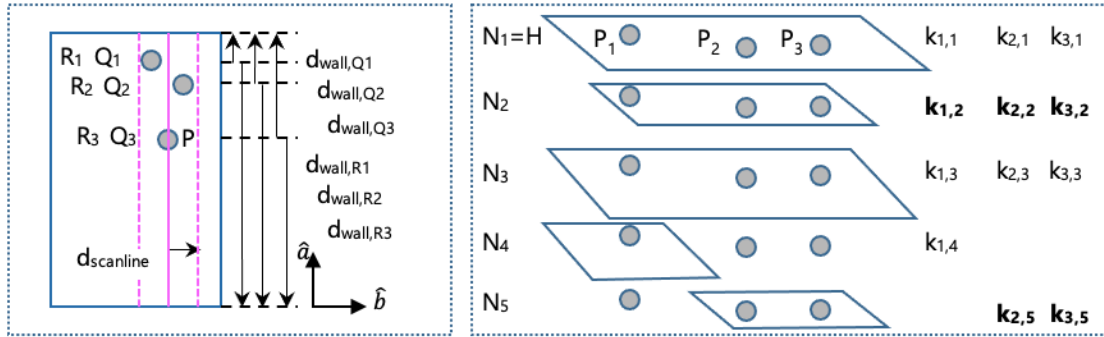


Figure 3. Spatial and membership metrics used to sort and eliminate neighborhoods in the 3D Bounding Box method. The spatial fitness scanline construction and wall distance measurements are shown at left. At right is a representation of the membership decisions that arise from considering points P_1, P_2 , and P_3 in a seed neighborhood H which has points that are contained in four other, overlapping neighborhoods (rendered in two dimensions and exploded in the vertical dimension to remove overlap). The neighborhood designations N_1 to N_5 are shown to the left of the diagram and the corresponding fitness metrics $k_{point, neighborhood}$ are enumerated at the right of the diagram. In this example, neighborhoods N_2 and N_5 fit their constituent points most tightly (boldface k values). The remaining three neighborhoods will be removed in this or later rounds of fitness ranking.

Next, the fitness of the bounding box is calculated using a spatial metric which attempts to quantify how well the points fill the bounding box, by finding the distance of the points to each wall of the bounding box for all six possible permutations of axes $(\hat{a}', \hat{b}', \hat{c}') \in \{(\hat{a}, \hat{b}, \hat{c}), (\hat{a}, \hat{c}, \hat{b}), (\hat{b}, \hat{a}, \hat{c}), \dots\}$. This calculation is depicted at left in Fig. 3. First, the coordinates of each point in the neighborhood are transformed from the input axes to the current local eigenaxes $(\hat{a}', \hat{b}', \hat{c}')$. Then in the plane described by primary and secondary eigenaxes (\hat{a}', \hat{b}') , each point P enclosed in the box is selected in turn, and points within a fixed buffer distance $d_{scanline}$ from P along the secondary axis \hat{b}' are considered. The point Q with the lowest distance d_{wall} to the bounding box wall intersected by the primary axis in the positive direction is found, and the point R with the lowest distance to wall in the opposite (negative) direction is found. The identities and distances of Q and R are ‘tagged’ to avoid double counting and, the distances $d_{wall,Q,min}$ and $d_{wall,R,min}$ are added to a running sum for (\hat{a}', \hat{b}') . The next point in the box is chosen as P , a new scan line is re-centered on it, new Q and R points within the scan line are selected, and $d_{wall,min}$ values are found and tallied, etc. After the scanline is swept across the box along the \hat{b}' axis, the next axis combination $(\hat{a}', \hat{b}', \hat{c}')$ is considered, and these summations are calculated, and so on until sums for all axis combinations are found.

The sums are combined for pairs of the six axis permutations with the same primary axis (e.g., for $\{\hat{a}, \hat{b}, \hat{c}\}$ and $\{\hat{a}, \hat{c}, \hat{b}\}$), resulting in three unnormalized eigenaxis fitness values and three counts of how many points were used for each eigenaxis fitness value. The three counts of points used are summed to find a total count of points used, and eigenaxis fitness values are normalized by dividing the three fitness values by this total point count. This yields the fitness value for each possible primary eigenaxis for the bounding box.

At this point in the computation, individual lidar points may be members of more than one box group (neighborhood). The eigenaxis fitness values are used to sort the set of boxes and remove boxes with poor fit and low point membership as depicted at the right in Fig. 3. An arbitrary ‘seed’ neighborhood H is chosen, and each point $P_i = P_1, P_2, \dots, P_h$ inside H is considered in turn. P_i commonly belongs to $k_i > 1$ neighborhoods; the k_i neighborhoods which contain P_i are sorted from best to worst fitness, where the best has the lowest sum of the three normalized eigenaxis fitness values. There is great spatial overlap amongst the resultant set of neighborhoods $k_1 \dots k_h$. For example, many neighborhoods containing P_1 will also contain P_2 , and therefore sets k_1 and k_2 will generally have common members. The neighborhoods contained in $k_1 \dots k_h$ are ranked according to the number of times (from 0 to a maximum possible count of h) that they have the best fitness. All neighborhoods with a rank of 0 are eliminated.

Grow/eliminate neighborhoods (cylindrical method). Unenclosed points are added to existing neighborhoods in order of the previous sorting, i.e., from largest to smallest fitness. For each point in a neighborhood N , any point $P_{candidate}$ within a 0.75m radius of the major axis is found and added to N if it meets the following criteria:

- a) The point falls within an infinite cylinder with 0.4m radius, aligned to the major eigenvector of N .
- b) The point is ‘sufficiently parallel’. The major eigenvector of base neighborhood $M_{candidate}$ (which was formed in the first step to contain $P_{candidate}$ and its nearest neighbor) is compared to the major eigenvector of N . If the angular difference between the major eigenvectors of N and $M_{candidate}$ is less than ~ 26 degrees, the point is added to N . (The angular difference value is heuristically chosen as one which does not cause merging of neighboring wires in parallel runs of power line conductors, but which does merge points from a single conductor in the run. This heuristic is necessary because of sampling imperfections in the lidar that record a wire as a cylindrical cloud of points which are not colinear. Examples of the ‘jitter’ caused by sampling can be seen in Figs. 6 and 7.)

The eigenvalues and eigenvectors are recalculated using PCL’s moment of inertia estimator and then neighborhoods are ‘greedily’ regrown as follows. For each neighborhood, starting with its origin point the neighborhood is grown (more points added) using the criteria a) and b), but using the recalculated eigenvalues and eigenvectors. Once a point is added to a neighborhood in this step, it is not added to any other neighborhood.

If a resulting neighborhood has less than four points, the neighborhood is eliminated. The points which were contained in an eliminated neighborhood are now unenclosed ‘orphans’ which will be added to surviving neighborhoods in subsequent growth steps.

Grow/eliminate neighborhoods (box method). Unenclosed (orphan) points are added to surviving neighborhoods in a way that approximates incremental, greedy Manhattan box growth on a 3D grid defined by a neighborhood’s eigenaxes. (See [45] for a description of ‘greedy’ sorting methods, which choose subsets by a problem-specific weighting rather than by exhaustive consideration of all possible subsets. Rectilinear, Manhattan approximations of smooth contours on a spatial grid were first employed in circuit board routing [46].) For each neighborhood, ‘orphan’ points are tentatively added if they are within a distance $d_{orphan} = 1\text{m}$ of points in the neighborhood, and fall within the bounding box if it were projected infinitely along its primary eigenaxis. Second, the bounding box and corresponding fitness metrics are recalculated, and if the fitness values for each eigenaxis of the new bounding box are less than a fixed value $F_{min} = 0.3$, the added ‘orphan’ points in the neighborhood are retained and removed from the pool of ‘orphan’ points. Third, this process is repeated until no new points are added to the neighborhood in the direction of its primary eigenaxis. If any of the fitness values exceed F_{min} , then the temporarily added points are removed.

These three steps are repeated for the secondary and tertiary eigenaxes, including or rejecting ‘orphan’ points depending on their fitness. If at least one point is added to the neighborhood along any axis, the three steps are restarted, beginning again with the primary axis. After this axis-wise Manhattan growth, the neighborhood’s oriented bounding box is recalculated and all points that fall within its (likely expanded) 3D box are added to the neighborhood, except for Tukey outliers as determined by a 1.5 interquartile range threshold along each eigenaxis.

After this growth step, neighborhoods are identified which are duplicates, i.e., contain the exact same lidar points, and all but one duplicate is eliminated.

Merge neighborhoods and create bounding polyhedra. At this writing, merging the final set of boxes is not implemented for the 3D Bounding Box method, so that unmerged box neighborhoods, converted to WGS84 coordinates, serve as the bounding polyhedra.

For the 3D Bounding Cylinder method, as a final data reduction step, surviving cylindrical neighborhoods are merged iteratively based on adjacency and overlap. Neighborhood N and neighborhood M are considered adjacent if there exists a pair of points, one from N and one from M , which are within a distance $d_{adjacent}=1\text{m}$ of each other. This value is set to prevent merging of well-separated parallel powerline conductors while enforcing merging of long lines of lidar points from single conductors which may be incidentally separated by sampling offsets. To handle the special case of conductor pairs (see Fig. 6 for an example), two candidate neighborhoods N and M with points closer than $d_{adjacent}=1\text{m}$ are considered “super-adjacent” and merged only if for every point in N there exists a point in M that is within $d_{adjacent}$ of it. Merging proceeds for all neighborhoods and is then repeated until no merging occurs.

Finally, using PCL’s Moment of Inertia Estimator, an oriented bounding box is computed for each cylindrical neighborhood. The set of oriented bounding boxes found in this way is converted to the WGS latitude/longitude/altitude coordinate space and serve as the bounding polyhedra for linearly oriented lidar points such as powerline conductors.

IV. Results

A. Enclosure Style and Fidelity for 2.5D Maximum Elevation Box and 3D Bounding Cylinder Methods

Two areas from dataset A were chosen to compare the 2.5D Maximum Elevation Box method and 3D Bounding Cylinder method (top of Fig. 4). At left is a section of a 500kV transmission line with a steel truss tower and two conductors (referred to herein as the 500kV Section). About 8000 lidar points remain after removal of ground points and the termination structures and associated guy wires at either end of the structure. At right is a nearby heterogeneous area in the same facility (referred to herein as the North Quarter) with a mixture of trees, buildings, poles, towers, electrical overhead conductors, and ground bus bar structures. About 2.3M lidar points remain after removal of ground points from survey data of this area.

The 2.5D Maximum Elevation Box method is adequate for the lidar point cloud of the 500kV Section (middle left of Fig. 4). Though the method produces bounding polyhedra that are overly conservative in the z direction, bounding all of the flight volume below the conductors, it is efficient in the lateral plane. The bounding polyhedra created by the 3D Bounding Cylinder method (bottom left of Fig. 4) tightly envelope the conductors, though the steel truss tower is inefficiently represented.

The more diverse North Quarter obstacle field allows a much richer basis for comparison between the 2.5D Maximum Elevation Box method and 3D Bounding Cylinder method. The bounding polyhedra created by the 2.5D Maximum Elevation Box method (middle right of Fig. 4) delineate the bus bars in the foreground efficiently, but the trees and powerlines in the background are grouped together into loosely bounding polyhedra. In contrast, the bounding polyhedra created by the 3D Bounding Cylinder method (bottom left of Fig. 4) tightly envelope the bus bars and power lines. The computed boundary of the irregular foliage geometry, while tightly enveloping, is a jumble of polyhedra.

These visual metrics are reflected in the relative data compression of the two methods (Table 1). The 2.5D Maximum Elevation Box method reduces the data size by three to four orders of magnitude for the areas tested [4]. That is, the 2.5D Maximum Elevation Box method polyhedral vertex count is 1000-5000 times lower than the number of raw lidar points (after ground removal) as tested with these two data sets. This reduction is largely due to two geometric simplifications:

- a) Clustering by z value encloses space underneath an object even if there are no underlying lidar points.
- b) Elevation along a line of latitude is simplified to a single (max) elevation for long runs in longitude, so that open air over an object may be enclosed.

In contrast, the 3D Bounding Cylinder method reduces the data volume approximately by 1.5 to 2 orders of magnitude (a ratio of 50-100) with the same data set. While the polyhedral fit is superb, compression is poor for foliage and complex structures.

A close-up view of the North Quarter results for the two methods is shown in Figure 5. Large swaths of empty area within the array of bus bar structures at right are enclosed by the 2.5 Maximum Elevation Box method, while the 3D Bounding Cylinder method bounds them tightly. At left, the mixture of foliage and powerlines is clumped together by the 2.5 Maximum Elevation Box method. The 3D Bounding Cylinder method handily segregates power lines from foliage, but the foliage is enclosed by many polyhedra with diverse orientation.

As expected based on their relative complexity, the run time of the 2.5 Maximum Elevation Box method is much shorter than the run time for the 3D Bounding Cylinder method. To compute bounding geometry for the North Quarter point cloud, using a machine with 32 3.1 GHz Xeon CPUs and 128GB of RAM, the 2.5D Maximum Elevation Box method requires about two minutes of run time using a single thread, while the 3D cylindrical method requires 215 minutes (3.5 hours) of run time using 24 threads. Assuming linear acceleration as threads are added, the 3D method's run time is about 2400 times longer than the 2.5D method's run time.

Table 1. Data reduction of two areas using 2.5D Maximum Elevation and 3D Bounding Cylinder Methods

	Input lidar	2.5D Maximum Elevation		3D Bounding Cylinder	
		Polyhedra count	Ratio	Polyhedra count	Ratio
500 kV Section	7,965	6	1328	146	55
North Quarter	2,388,823	406	5883	22,380	107

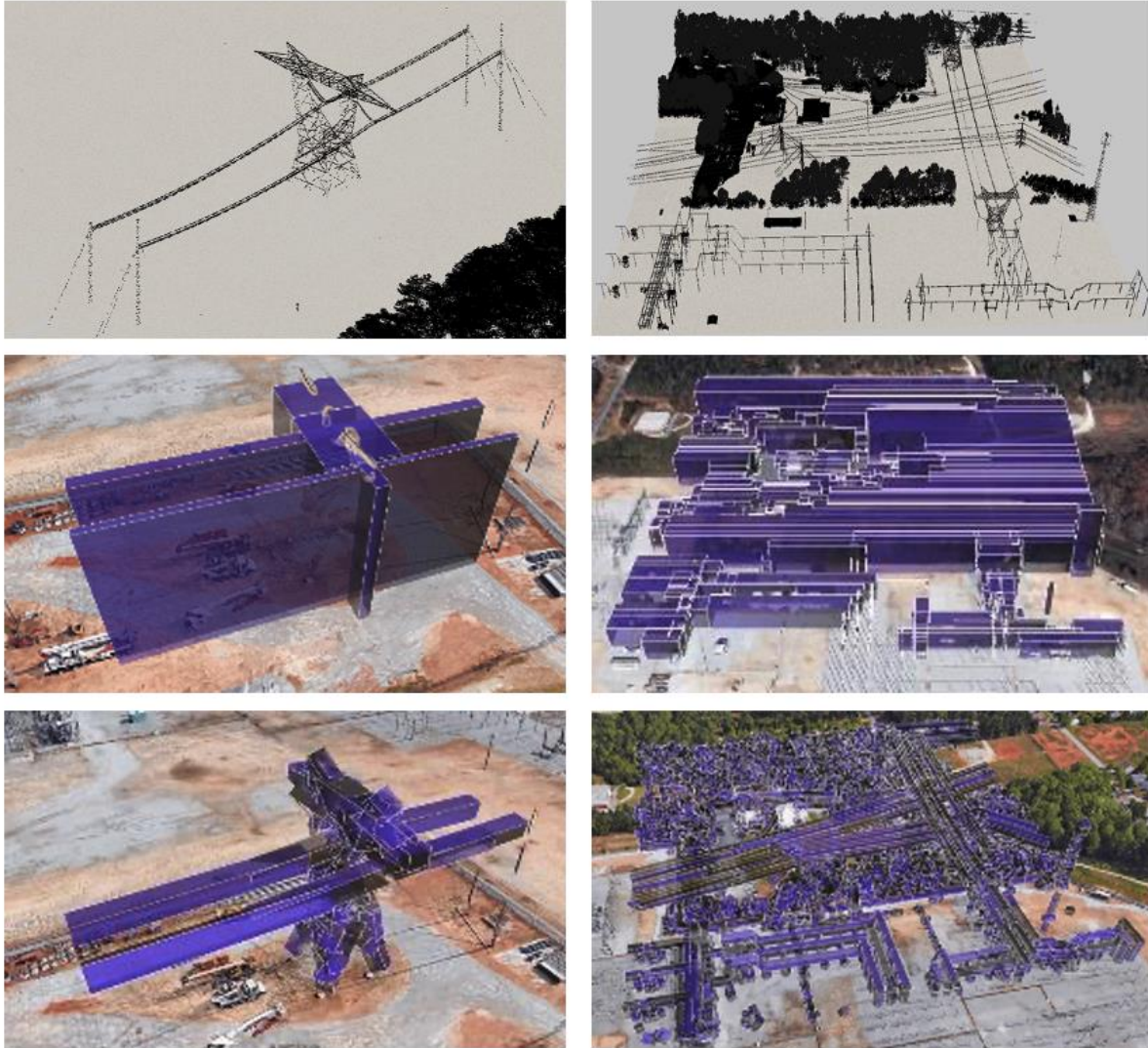


Figure 4. Comparison of 2.5D Maximum Elevation Box and 3D Bounding Cylinder methods on two areas from dataset A. Left: 500 kV Section. Right: North Quarter. Top row: input lidar after ground point removal. Middle row: 2.5D Maximum Elevation Box representation. Bottom row: 3D Bounding Cylinder representation. © Lidar data: Southern Company; Map data: Google & DigitalGlobe

The internal representation of the 3D Bounding Cylinder method is illustrated in Figure 6 for a section of a 360kV Dominion Energy transmission line structure from dataset B. Three conductor pairs are suspended by two steel truss towers in this structure. At top, the individual lidar points are colored according to the membership in a neighborhood as described in Section II.B.3. The bounding polyhedra constructed with a one meter safety buffer are shown at bottom. The constraints employed to enforce cylindrical clustering (0.4m radial proximity) cause the catenary curve of the suspended cables to be segmented. The data compression afforded by the method is considerable ($\sim 100:1$) for the linear conductors. Most of the 370 polyhedra that enclose the approximately 15000 lidar points are located on the truss elements of the steel towers.

While the 3D Bounding Cylinder method is adequate for most linear structures (poles, conductors, bus bars), very thin objects such as guy wires are not possible to enclose completely due to lidar sampling errors. Figure 7 shows an attempt to bound a lidar scan of a switching tower at the Dominion Energy Chester facility. The beams and poles of the tower resolve well in the aerial scan and were bounded adequately by the method, but there are dropouts in the aerial lidar survey scan data for the thin ($\sim 3/8$ in) guy wires which mechanically support the tower. The one meter maximum gap d_{min} rule used by the method to avoid bridging across spatially separated objects breaks the bounding geometry of the guy wires into multiple polyhedra (Figure 7, right).

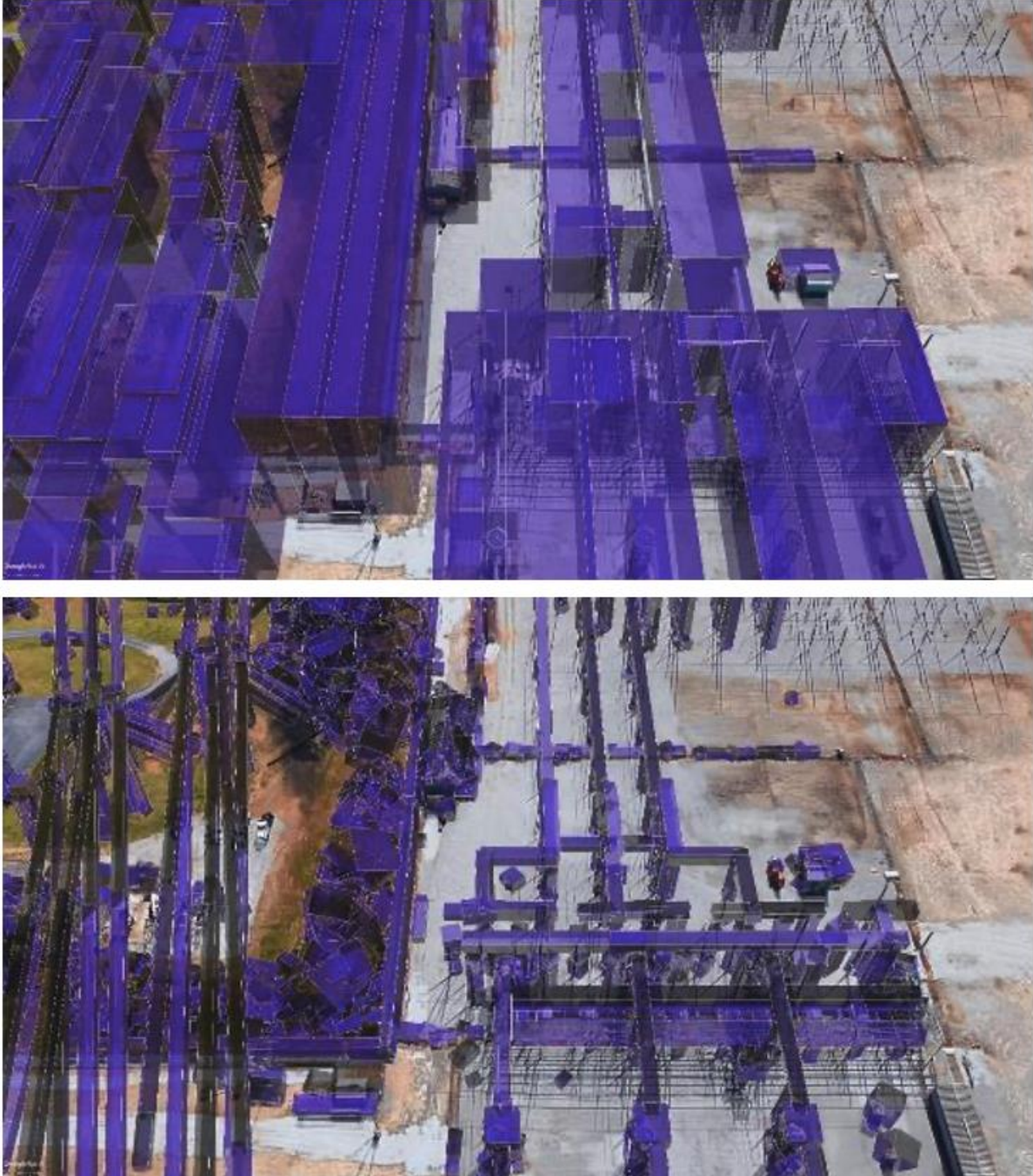


Figure 5. Close up of bounding polyhedra for the heterogeneous North Quarter section of the Southern Company Klondike test facility. Top: 2.5D Maximum Elevation Box method; bottom: 3D Bounding Cylinder method. © Lidar data: Southern Company; Map data: Google & DigitalGlobe

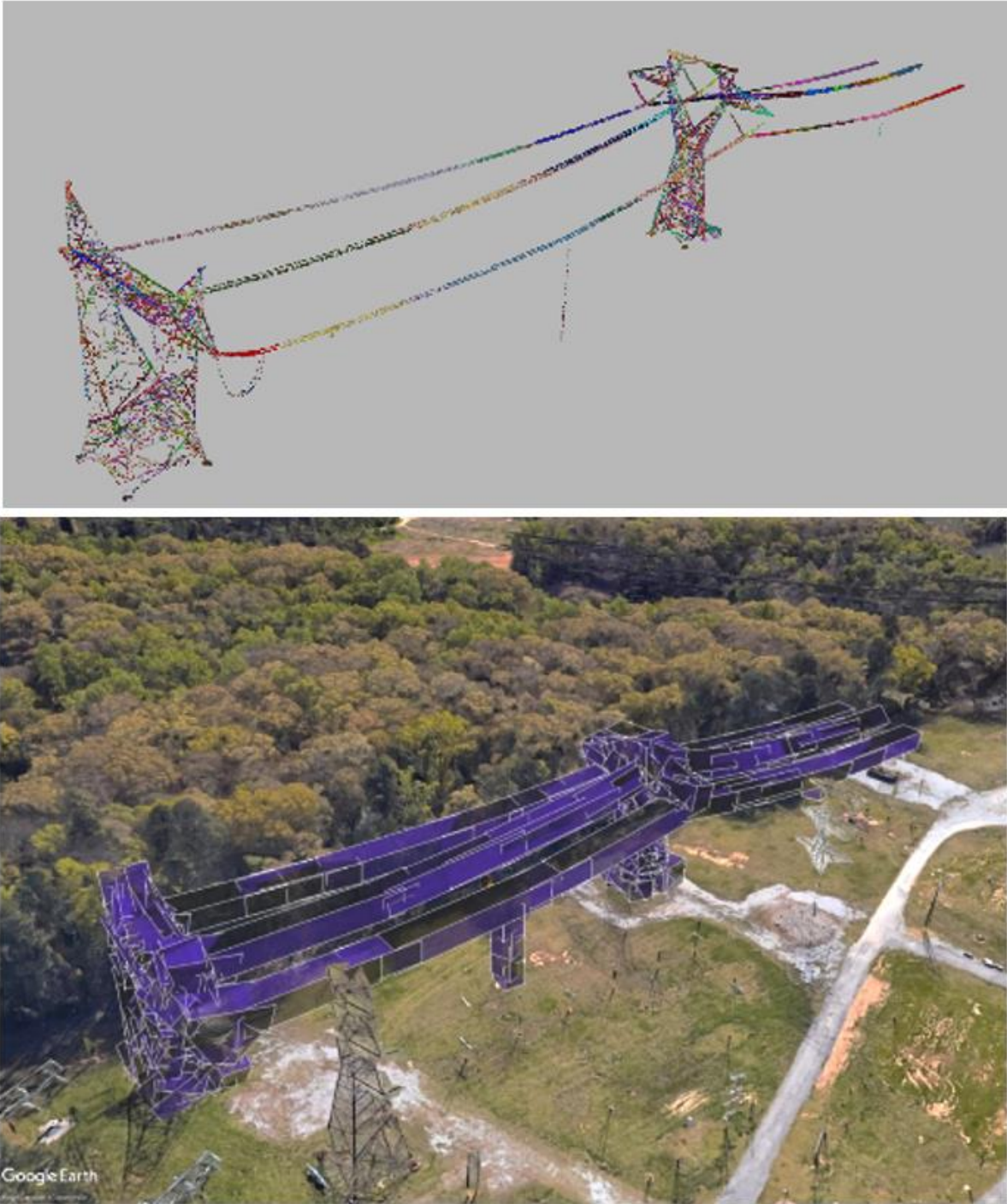


Figure 6. The internal representation of the 3D Bounding Cylinder method (top) to enforce collinearity results in segmenting of the catenary arc of conductors at bottom. The approximately 15000 lidar points from dataset B are enclosed by 370 bounding polyhedra. © Lidar data: Dominion Energy; Map data: Google & DigitalGlobe



Figure 7. Aerial lidar scans fail to adequately sample fine structures such as guy wires. The 3D Bounding Cylinder method attempts to bridge gaps in sampling by projecting 1 meter along a given cylindrical eigenaxis to search for points in neighborhood construction. If there are no collinear points within a meter, a guy wire with drops in scan sampling is represented as more than one neighborhood (left), yielding multiple bounding polyhedra for the span (right). © Lidar data: Dominion Energy; Map data: Google & DigitalGlobe

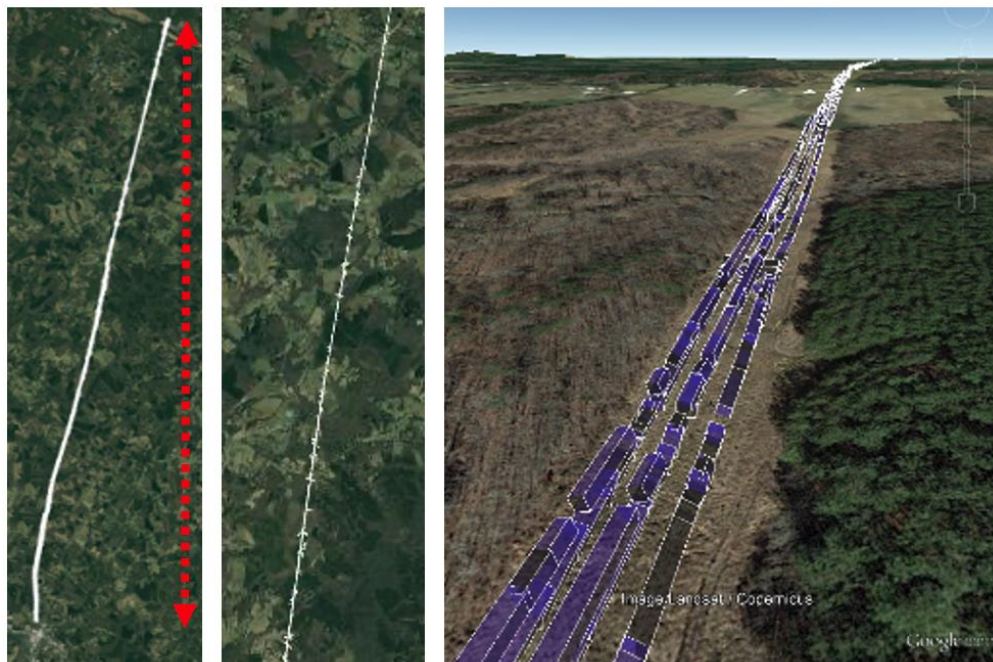


Figure 8. Enclosure of a 29 mile transmission line corridor from dataset C with the 3D Bounding Cylinder method. Left: entire set of polyhedra; red arrow is 27 miles long. Middle: detail of 11 mile flight region. Right: low altitude view. The cylindrical model parameters used to created bounding polyhedra in this report were optimized for datasets with finer sampling (~0.3m). At the relatively low point spacing (0.6m average) of this dataset, lidar sampling dropouts result in gaps in the enclosure geometry, but fidelity is otherwise excellent. 2.2M lidar points are compressed to 71K polyhedra in this example. © Lidar data: Dominion Energy; Map data: Google, Landsat/Copernicus & DigitalGlobe

Dataset C has a sampling (point spacing) twice as coarse as the other transmission line survey datasets used in this study, but spans the entire length of a 29 mile transmission line. Extensive manual inspection and data preparation are needed to produce survey sets such as this one, which are used to monitor foliage [32], since lidar points along the transmission corridor must be relabeled by object type (i.e., foliage, conductor, ground). An eleven mile section of

this corridor selected for a pilot UAV inspection overflight [33] was used to test the data capacity and enclosure fidelity of the 3D Bounding Cylinder method. The results (Figure 8) exhibit the features of the method described above: high data compression (2.2M lidar points enclosed by 71K polyhedra), efficient conductor rendering, and scattered dropouts from lidar sampling errors.

B. Enclosure Style and Fidelity for 2.5D Ground Extrusion Method

The 2.5D Ground Map Extrusion method takes advantage of existing geographic information system (GIS) data to simplify vegetation in lidar survey scans. A dedicated GIS group at NASA Langley Research Center maintains a database of key geographical features at the Center, and continually updates its breadth, accuracy, and resolution as new survey technologies become available. Major permanent structures such as buildings and roads are represented with sub-meter accuracy sets of ground boundary (footprint) coordinates and associated elevations in commercial GIS software. Trees, signage and other minor structures are entered on an *ad hoc* basis as graphical footprint drawings and a single elevation.

To build an obstacle map of a flight range on North Dryden Boulevard, Hampton, Virginia, building footprints and elevations were combined with polyhedra generated by the 2.5D Ground Map Extrusion method on foliage in two sets of aerial lidar scans. Results from the dataset D (with 0.2m average point spacing) are shown at left in Figure 9, with buildings colored blue, light poles colored yellow, and foliage (bounded with a best-fit rectangle) colored green. After software to digitize the ground footprints was developed, the method was applied to a second, newer lidar survey (dataset E) with 0.1m average point spacing. As shown at the right in Figure 9, the method bounds most of lidar points at locations where footprints exist.

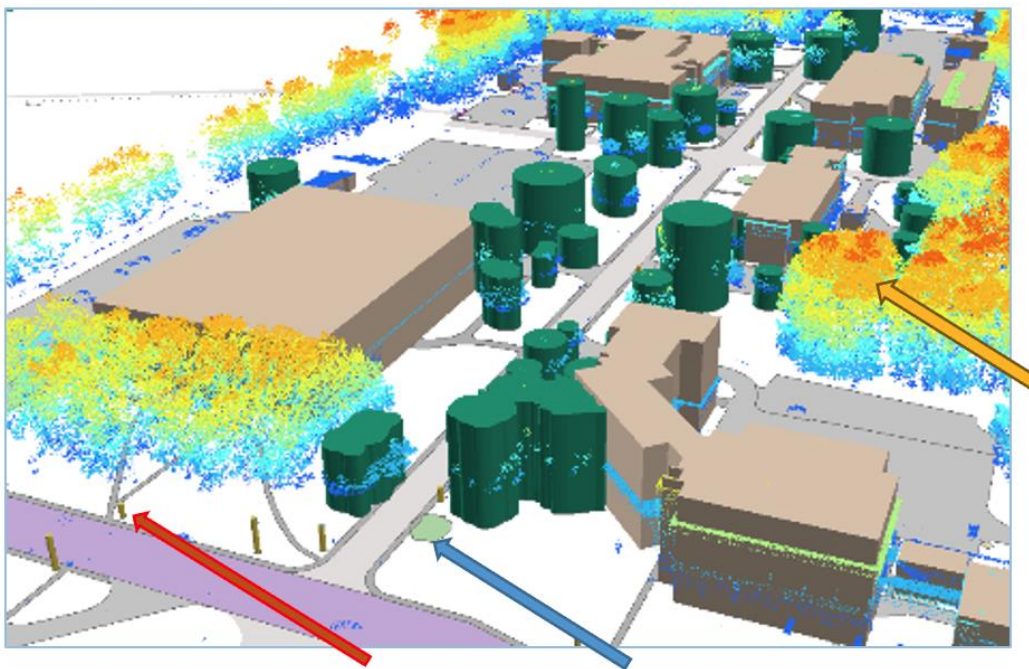


Figure 9. Obstacle field created with the 2.5D Ground Map Extrusion method maps for a flight corridor at NASA Langley Research Center. Building footprints with sub-meter accuracy are taken from the LaRC GIS database and extruded to the corresponding elevation. Extruded buildings (brown) are combined with computed boundary results from lidar dataset E. Raw lidar, after ground point removal, is rendered with a color gradient according to elevation (blue-to-red dots). The bounding polyhedra for trees (green) and light poles (yellow) are computed with the 2.5D Ground Map Extrusion method. Arrows point to exemplars of the method’s limitations: footprints are not drawn for some groups of trees (yellow arrow at right), some trees have been cut down since the footprints were created (blue arrow at bottom), and lidar sampling of thin light pole structures produces height errors (red arrow at bottom).

Limitations of this method are also evident in Figure 9. The foliage footprint outlines are quite close to the actual footprint, but since they are created manually, and since structures (especially trees) change over time, they are not exact. Trees without a footprint entry in the GIS database are not captured at all (yellow arrow). Thin, vertically

oriented structures such as street signs and light posts are poorly sampled by an aerial scan, so that the computed polyhedron height is often inaccurate (red arrow). The data compression produced by the 2.5D Ground Map Extrusion method cannot be computed readily, as it incorporates a mixture of building geometry which is already simplified and misses the foliage areas that have no drawn ground footprint. Qualitatively, compression appears to be much higher than the 3D Bounding Cylinder method, due to the reduction of foliage data volume, and somewhat lower than the 2.5D Maximum Elevation Box method, since its base polyhedra have many more vertices than the rectangular bases of the 2.5D Maximum Elevation Box method.

C. Preliminary Results for 3D Bounding Box Method

The 3D Bounding Box Filter method was developed last in this research project. Merging of box neighborhoods is not yet implemented, so compression statistics are not available; the results for this method are discussed qualitatively in this section.

A portion of the heterogeneous North Quarter section of the Southern Company Klondike test facility from dataset A with trees, poles and buildings is shown in Figure 10 to compare the 3D Bounding Cylinder method and 3D Bounding Box method. At top left is the lidar point cloud with ground points removed; at top right, the lidar is shown again with annotated regions of interest; at bottom left, the internal representation of the 3D Bounding Cylinder neighborhoods in the Point Cloud Library tool is shown; and at bottom right, the internal representation of the 3D Bounding Box neighborhoods in the Point Cloud Library tool is shown.

The 3D Bounding Cylinder method excels at clustering lidar points of power lines and poles. This can be seen at the upper right of the two images at the bottom of Figure 10. Most of the conductor spans are rendered with a single color in the 3D Bounding Cylinder method results, representing a single neighborhood cluster. In contrast, the spans are comprised of multiple neighborhood clusters in the 3D Bounding Box method results.

The box filter method excels at clustering planar lidar points of rooftops. This can be seen in the foreground of the two images at the bottom of Figure 10. The 3D Bounding Cylinder method ‘paints stripes’ across the rooftops, while the 3D Bounding Box method tiles the roofs with large rectangles. With the addition of a merge step, this method appears capable of representing each roof face with a single neighborhood.

Neither method copes well with the complex geometry of foliage [23][24]. A small area of the same flight range shown in Fig. 9 is used to illustrate this point. As shown in Figure 11, the internal neighborhood representation for foliage for both methods is characterized by topological incoherence and spatial inhomogeneity. Foliage aside, this example again illustrates the comparative strength of the 3D Bounding Box method for computing compact 3D rooftop boundaries.

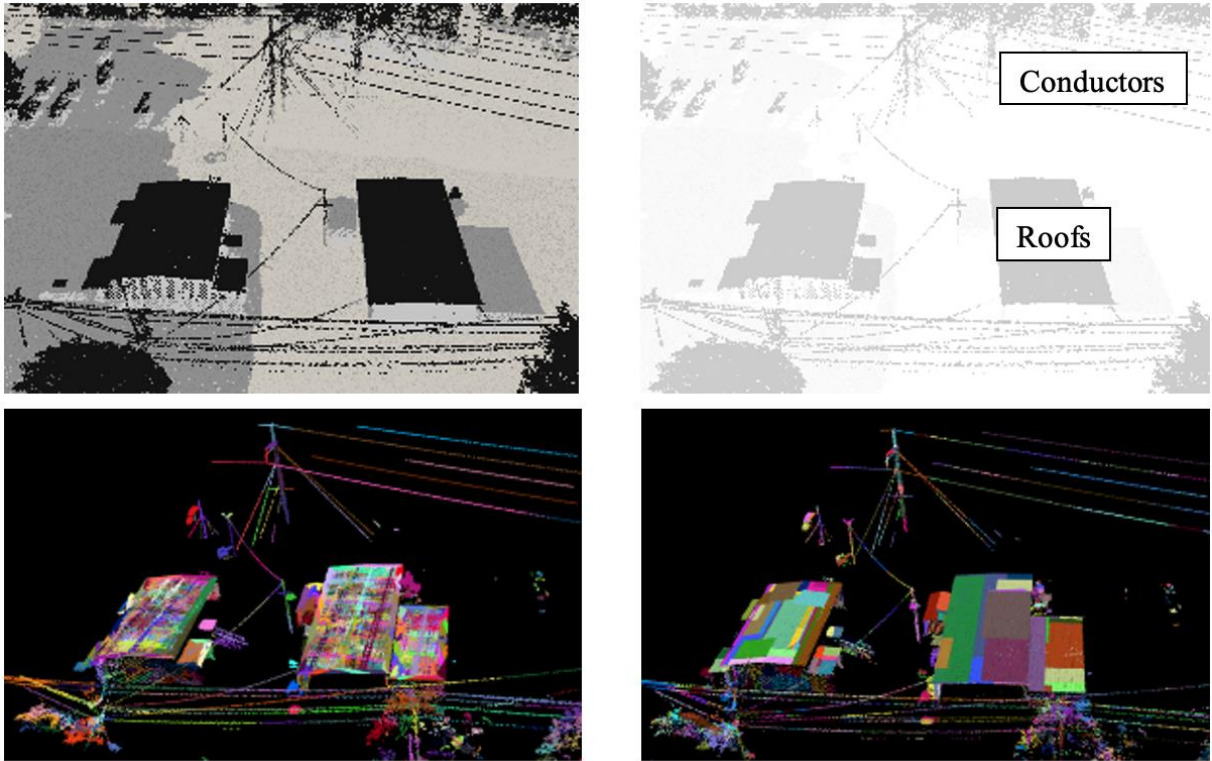


Figure 10. A comparison of the 3D polyhedron bounding methods in an area from dataset A with trees, poles and buildings. Top: input and annotated lidar. Bottom: internal neighborhood representation in PCL for the 3D Bounding Cylinder method (left) and 3D Bounding Box method (right). The 3D Bounding Cylinder method is more effective with linear objects, while the 3D Bounding Box method is more effective with planar objects. © Lidar data: Southern Company; Map data: Google & DigitalGlobe

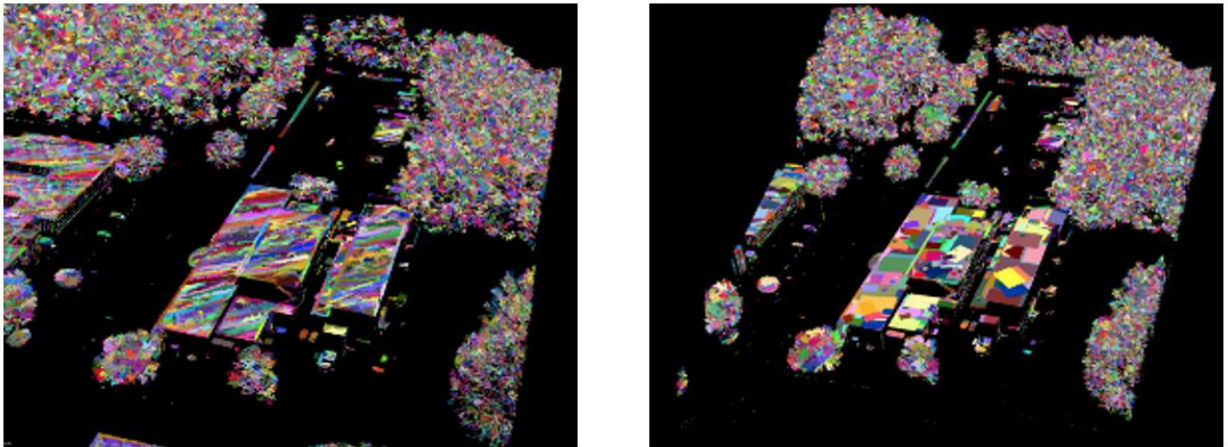


Figure 11. A comparison of the 3D polyhedron bounding methods for a NASA Langley Research Center flight range using the lidar survey from dataset D. The poor performance of both methods for foliage is evident.

V. Discussion

Summary of results. The results for the four methods primarily show the inverse relationship between spatial fidelity and data compression. The 2.5D methods can construct ground structure boundaries that are compact but spatially

approximate, while the boundary geometry constructed by the 3D methods is rather less compact (Table 1) but more tightly encloses the actual boundaries.

Of the 2.5D methods, the Ground Map Extrusion technique strikes the better balance between compactness and fidelity, due to the human judgment applied in drawing the ground footprints used for extrusion. The 3D methods are each superb for one geometric class (linear structures for the 3D Bounding Cylinder method and planar structures for the 3D Bounding Box method). Foliage and complex structures such as lattice towers are handled poorly by both 3D methods.

Figures 4 and 5 summarize well the tradeoff between obstacle boundary fidelity and compactness. The 2.5D Maximum Elevation Box method (middle row of Fig. 4, top of Fig. 5) encloses a great deal of empty space, particularly below elevated structures. The 3D Bounding Cylinder method (bottom row of Fig. 4, bottom of Fig. 5) encloses almost no empty space, at the cost of poor compactness in complex areas such as the trees and the trusswork of the steel lattice towers. Table 1 quantifies this cost for two sections of a survey with 0.3 meter average point spacing: while the 2.5D Maximum Elevation Box method compresses the lidar points by 3-4 orders of magnitude, the 3D Bounding Cylinder method compresses them by 1.5-2 orders of magnitude. The 2.5D Ground Map Extrusion method compression is slightly lower compared to the 2.5D Maximum Elevation Box method, because its base polyhedra generally have many more vertices than the rectangular bases of the 2.5D Maximum Elevation Box method. The 3D Bounding Box method (once a final merge step is implemented) is expected to yield data compression ratios similar to the 3D Bounding Cylinder method, with better compression on planar structures than linear structures.

Optimizing compactness. There is no single bounding method which optimizes the exactness/compactness tradeoff. As mentioned in the Introduction, in some studies an ensemble of methods is applied in series. For example, a pipeline

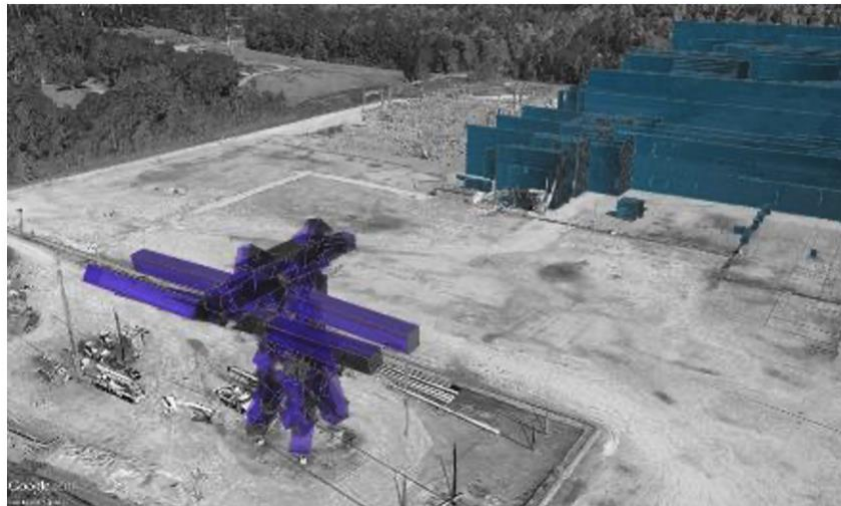


Figure 12. Mixed polyhedra representation for UAV-based inspection of a 500kV transmission line tower and conductors. The 500 kV Section is rendered with the 3D Bounding Cylinder method (foreground, purple) while the North Quarter, of less immediate concern as an obstacle, is rendered with the 2.5D Maximum Elevation Box method (background, blue). © Lidar data: Southern Company; Map data: Google & DigitalGlobe

could be constructed using a method which finds and bounds 3D planar areas (e.g., buildings) and then removes the bounded planar lidar points first, finds and bounds 3D linear structures (e.g., power lines) and then removes the bounded linear lidar points second, and encloses the remaining lidar points (presumably foliage) in a convex hull or surface spline. Another approach is suggested in a study which addressed compactness for compute-bound onboard aerial navigation (Figure 12, from [4]). The capacity and throughput of onboard processors, while improving, is lower than desktop or laptop processors by at least an order of magnitude. The two portions of dataset A compared in Table 1 were combined to produce a reference obstacle field in a detailed compute capacity analysis for a version of the ICAROUS (Independent Configurable Architecture for Reliable Operations of Unmanned Systems) flight path conformance software [31][35][36] running on a single-threaded processor [4]. That analysis found that for conservative UAV velocities, the obstacle field geometry should not exceed about 200 polyhedra. While more powerful onboard processors and more optimized software have become available since that study, it is safe to say

that the volume of raw lidar data (at over 2.3 million points for this reference obstacle field) still exceeds the capability of onboard obstacle avoidance autonomy. At the time of the analysis, only the 2.5D Maximum Elevation Box method reduced the data volume sufficiently to avoid overwhelming the onboard processor. A hybrid geometric representation was proposed (Figure 12) to manage the data volume which attempts to limit loss of fidelity only in the immediate, most safety-critical area. In this hybrid approach, 3D bounding polyhedra are computed for structures near the current UAV position (foreground, purple) and 2.5D bounding polyhedra for more distant structures (background, blue).

Applications to UAV autonomy. Widely available geo-conformance monitors, such as the one embedded in the ArduCopter [37] controller for the Pixhawk [38] autopilot, and more sophisticated monitors such as Safeguard [39][40] enforce UAV containment within a prescribed flight corridor that is specified as a 2.5D geofence. (See [10] for research flights that used both of these monitors.) The bounding polyhedra produced by the 2.5D Maximum Elevation Box method can serve as the geofence source for both ArduCopter and Safeguard. Geofence geometries created by the 2.5D Ground Map Extrusion method such as those shown in Figure 2 will exceed ArduCopter’s vertex capacity, however, while the high capacity geometry processing in the current Safeguard technology can easily cope with its higher vertex count.

The 3D bounding methods are particularly useful for onboard autonomy such as ICAROUS, which continually computes flight paths between planned waypoints that avoid obstacles, such as geofences and ‘safety bubbles’ around other vehicles in the airspace. Figure 13 shows an example of the internal representation that ICAROUS computes (adopted from [36]) and a typical flight path [10] with ICAROUS constrained to plan 2D avoidance maneuvers. With this or similar autonomy onboard a UAV, assuming reliable GPS reception [5] and precomputed bounding geometry as described in this report, inspection flights underneath structures such as bridges and powerlines could be conducted with added safety.

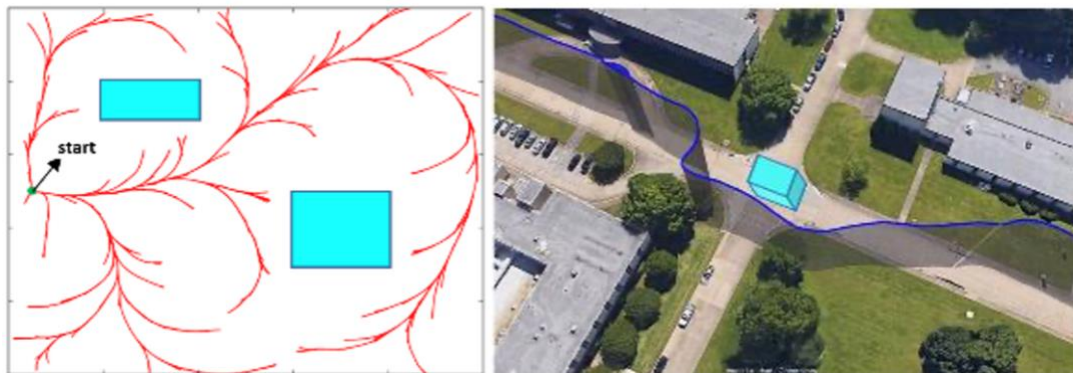


Figure 13. Internal representation (left) and flight results (right) of ICAROUS path conformance autonomy, adopted from [36] and [10], respectively. Geofences are colored cyan in both images. Potential flight paths are shown in red at left. Actual flight path is shown in blue at right. This example illustrates contingency flight paths computed for 2.5D geofences, though the technology can compute and execute 3D paths as well. © Map data: Google & Landsat/Copernicus

Emerging satellite survey technology. This report details methods to create bounding polyhedra for survey data gathered using aerial lidar scans. At the time that this research began, surveys using satellite photogrammetry [41] were not widely available with precision (average point spacing) and accuracy [42] comparable to surveys using lidar. Survey data from satellite photogrammetry is now readily obtainable at low cost, in formats that the methods described herein can process. The common figure of merit for satellite photogrammetry, LE90, is a statistical estimate of errors in accuracy at the ninety percent level [43]; for commodity satellite-based surveys the LE90 distance is 3m or more [44], about ten times higher than current lidar surveys. The spatial sampling, a measure of precision, is coarser than that of lidar (about 50cm for satellite photogrammetry vs. 10cm for high fidelity lidar data such as in dataset E), so that gaps in bounding polyhedra (Figure 7, Figure 8) that arise from data dropouts will be more severe than for lidar.

The research described in this report demonstrated, on a diverse set of lidar survey data, that both sub-meter sampling and sub-meter accuracy are required to resolve and map power lines, guy wires and small structures such as road signs. The current precision and accuracy of satellite photogrammetry are therefore suitable only for large obstacles such as buildings and trees. To ensure low-altitude UAV collision avoidance, satellite-based survey data must be supplemented with lidar survey data to safely map all collision hazards.

VI. Conclusion

This report described in detail four methods to compute bounding geometries of ground obstacles from lidar point clouds. The four methods are: 1) 2.5D Maximum Elevation Box, 2) 2.5D Ground Map Extrusion, 3) 3D Bounding Cylinder, and 4) 3D Bounding Box. Custom software was used for the 2.5D methods. For the 3D methods, the open source Point Cloud Library software was used to find clusters of point cloud data, and custom software tested clusters according to tunable geometric criteria, purged outliers, merged clusters, and constructed polyhedra from the resultant point clouds.

The methods were evaluated with heterogeneous survey data including buildings, vegetation, power lines, and small structures such as road signs and guy wires. The level of detail of the test data ranged from 0.1m to 0.6m.

The performance of each method was described using areas from the lidar survey that highlight strengths and weakness of each, and the methods were compared on the bases of boundary fidelity and data compression. The 2.5D Maximum Elevation Box method reduces the data volume the most, as measured by the ratio of the input lidar point count and the output polyhedral vertex count. However, the bounding polyhedra it produces enclose a relatively high volume of empty space. The 2.5D Ground Map Extrusion method bounding fidelity is higher and its compression is slightly lower, because the 2D polygons which are extruded generally have more vertices than the four rectangle corners characteristic of the 2.5D Maximum Elevation Box method. The extruded geometry is human drawn, and so this is not a fully automated method.

Both 3D methods have high spatial fidelity and enclose very little empty space. Each 3D method is most effective on a distinct geometry type. The 3D Bounding Cylinder method excels at enclosing suspended cables such as power lines and linear structures such as bus bars and fences. The 3D Bounding Box method excels at enclosing planar structures such as buildings. The high fidelity comes at the cost of a compression ratio that is ~30 times lower than the 2.5D Maximum Elevation Box method. Another consequence of their geometric selectivity is that the geometry count is high on vegetation and structures with a high degree of angular variation such as lattice towers. The 3D Bounding Cylinder method is particularly susceptible to sampling dropouts for structures much thinner than the average point spacing, which produce gaps in the output polyhedra.

Both the 2.5D and 3D methods are useful for low altitude UAV navigation. Geo-containment autonomy is typically based on 2.5D keep-in geofences. Flight path conformance autonomy is capable of planning avoidance paths both in 2D and 3D. The compute power of onboard processors is relatively limited, however, and so a mix of 2.5D and 3D obstacle representation may be needed to stay within the limits of airborne processor throughput and capacity.

Satellite-based photogrammetry data was not processed in this report, but its spatial accuracy and resolution was evaluated in the light of these results. While suitable for low resolution applications such as forestry surveys and terrain mapping, it is not currently capable of resolving all of the obstacles needed for safe UAV navigation at low altitudes.

VII. Acknowledgments

We are grateful to the many NASA Langley colleagues for discussions and assistance in the course of this multi-year development effort: Mike Logan, Evan Dill, and Steve Young (flight range design); George Hagen, Swee Balachandran, and Cesar Munoz (ICAROUS capacity testing); Robert Gage and Berch Smithson (coordinate systems); and, Sharon Graves, Maria Consiglio and John Koelling (program support). J. Smith acknowledges the University Space Research Association for internship logistics. This work was performed with support from the NASA Safe Autonomous Systems Operations program, Unmanned Traffic Management program, and System Wide Safety program. Paul Schneider and Dexter Lewis of Southern Company provided lidar of the Klondike Training site. Steve Eisenrauch and Amanda Palmore of Dominion Energy provided lidar of the Chester training site and the 29 mile central Virginia line. Berch Smithson of the Midland GSS Joint Venture provided lidar surveys of the NASA Langley Research Center.

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