Off-nominal Trajectory Computation Applied to Unmanned Aircraft System Traffic Management

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***Abstract*—An Unmanned Aircraft System (UAS) Traffic Management System (UTM) relies significantly on automation, introducing the need for efficient and accurate trajectory computation to enable coordination and safety. The main objective of this paper is to present and to organize prior work and relevant concepts with the goal of developing a framework for UAS trajectory prediction in the presence of anomalous events. Literature documenting UAS safety and risk assessment has provided multiple pointers for identification and characterization of system failures that cause trajectory deviations or changes to its associated qualities. A UAS trajectory modeling framework considering endogenous and exogenous factors affecting the trajectory is introduced and used in this exposition. In addition, a general formulation of the trajectory computation challenge is presented along with key requirements for potential solution approaches.**

I. INTRODUCTION

An Unmanned Aircraft System (UAS) Traffic Management System (UTM) relies significantly on automation, introducing the need for efficient and accurate trajectory computation to enable coordination and safety. Computation of UAS trajectories under nominal conditions presents numerous technical challenges, particularly for small UASs (sUAS). Their relatively small size and weight, relatively slow operating speeds, operations at relatively low altitudes, where wind patterns are harder to characterize, are a few of the factors that contribute to the complexity of the problem. Further, commercial proliferation and a variety of applications has engendered a large diversity of airframes, diversity of control and guidance logic, diversity of schemes for managing contingencies, etc. However, a more peculiar aspect of the research agenda for trajectory prediction of UAS is the consideration of anomalous events triggering off-nominal conditions. UAS represent a relatively new category of National Airspace System (NAS) users, and in contrast to Conventionally Piloted Aircraft (CPA), these systems are still evolving towards maturity in terms of reliability and safety, as attested by multiple recent safety-related studies and reports. Hence, the potential trajectories followed by unmanned aircraft subject to failures is important not only to manufacturers and operators, who are preoccupied with design and operational safety, but also to other airspace users, and to stakeholders on the ground.

The paper starts by introducing a UAS trajectory modeling framework being developed for NASA's UTM project. Next, failure events are studied in the context of current safety-related reports and hazards analysis. Finally, the actual trajectory computation problem is presented and formulated in a general manner. In the last section, a brief review of current and past relevant work is also included for interested readership.

II. UAS TRAJECTORY MODELING FRAMEWORK

Figure 1 (extracted from [1]) presents a general approach to UAS trajectory modeling, which is used as the foundation for the analysis presented in this paper. It shows at a high level all the factors that affect the UAS trajectory. Factors inherent to the system ("endogenous") are shown in the inner and bottom blocks, including items such as: flight dynamics, controls, navigation and guidance, mission planning, etc. These elements represent models of UAS functions. Exogenous factors are shown on the lateral sides. They include: system failure hazards—the main concern for this paper, highlighted in red, and environmental conditions.

Although failure mode analysis literature is focused on aspects such as safety, regulations, integration of UAS into the civil airspace, etc., in the context of this research, it is used to identify key failure modes that affect the trajectory. The safety perspective is also useful for prioritization in terms of risk. Hence, modeling of more common failures would need to be tackled first.

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System failures may be characterized using the reference model stack introduced in Figure 1. This stack represents a functional decomposition of the system model that is suited to trajectory modeling and prediction.

Although environmental factors may cause the execution of trajectories in off-nominal modes (e.g.: due to violation of aircraft performance limitations), those cases are treated separately and are not considered in this document, whose focus is system failures.

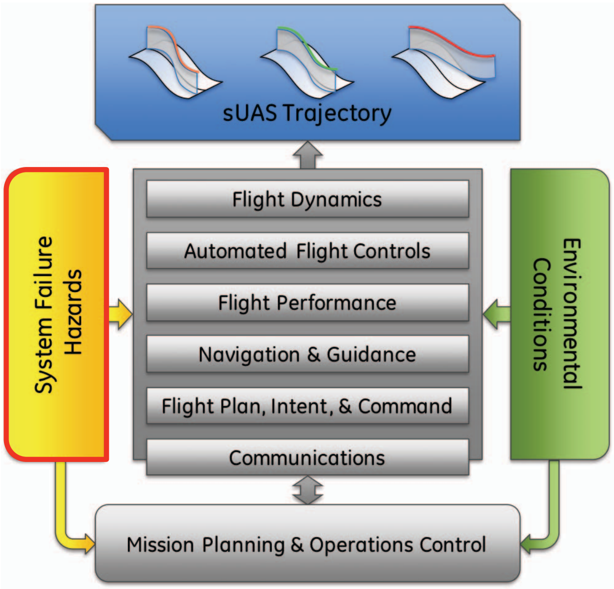

Fig. 1. Stack of factors influencing sUAS trajectory

As shown notionally in the upper portion of Figure 1, trajectories produced by this process may be classified into three broad categories depending on their deviation from a nominal trajectory:

- Trajectories following the nominal trajectory within prescribed bounds.
- Trajectories associated with degraded performance, i.e.: trajectory containment bounds may not be guaranteed.
- Trajectories presenting substantial deviations from the nominal trajectory.

Since UAS present much more operational flexibility than CPA, Ren et al. introduce the concept of a *pattern-based trajectory definition model* with two salient features:

- Two 4D (4DT) trajectory constructs: A conventional one—defined by a 3D/4D path using explicit trajectory objects, and an unconventional one that incorporates abstract trajectory objects, including spatial objects (e.g.: 3D volumes) that characterize possible locations where UAS(s) may operate at a given time.
- A series of "provisions" that accommodate supplementary information for mission-specific requirements. These information items contain parameters and constraints that pertain to specific types of missions and operations, for instance: ratio of vehicles per volume (for multi-UAS a.k.a "swarm" operations), population density for urban operations, geometry and location of assets being surveilled by industrial inspection drones, airspace restrictions, etc. Flexibility conferred by the use of provisions may be also utilized to embed information and models necessary for trajectory prediction under off-nominal conditions. Moreover, these provisions, make the proposed framework extensible. For instance, it allows for more accurate computations when higher fidelity models are available, or conversely, for simplified approximations when UAS-specific data are limited.

The pattern-based trajectory definition model is shown notionally in Figure 2

III. UAS FAILURE EVENTS

A. *Safety-related Reports*

Recent literature was studied to obtain a list of common UAS system failures that impact UAS trajectories. In [2], the authors analyze statistical properties of UAS accidents and incidents and also a categorization by occurrence type, phase of flight, and safety issue. Characterization is performed in a comparative manner, using CPA statistics as a baseline. Although the number of cases analyzed is relatively small (152 cases in the period between 2006 and 2015), the authors identify 20 relevant international databases that may be used as data sources in future studies. In the absence of occurrence classifications specific to UAS, those used by the Global Safety Information Exchange (GSIE–detailed by IATA) for CPA were applied, namely:

- Controlled Flight Into Terrain (CFIT)
- Loss Of Control–In-flight (LOC–I)
- Runway Safety (RS)
- Ground Safety (GS)
- Operational Damage (OD)
- Medical (MED)
- Unknown (UNK)

UAS flight was assumed to comprise four phases: takeoff (including climb out), en-route (cruise), approach (including descent), and landing. Four safety issues were

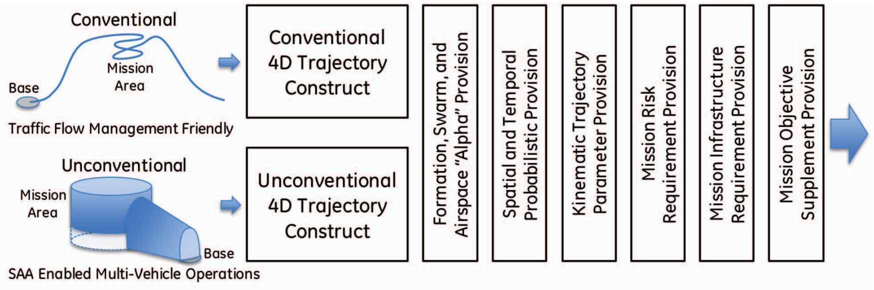

Fig. 2. Pattern-based trajectory definition model

considered: human factors (HF), organizational issues (OI), equipment problems (EP), and environmental issues. Results show that the ratio of incidents to accidents, 74% to 26% is analogous to the ratio known for CAT events. The relative ratios of incidents vs. accidents by phase of flight is also similar. Discrepancies between UASs and CPA arise in types of occurrences, where OD and CFIT are twice as likely, and LOC-I more than three times as likely. In terms of safety issues, equipment problems (EP) accounts for 64% of all UAS events, in contrast to CPA, where HF is the most relevant issue (75%).

The work by Sharma et al. [3] also examined UAS-related incidents leveraging multiple databases, albeit restricted to US territory. Sharma's report focuses on two types of incidents: Near Mid-Air Collisions (NMAC), and National Airspace System (NAS) violations, both hazards resulting from potential trajectory deviations. Relevant information regarding time distribution and location of the "sightings" may be obtained from this report, for example:

- Most UAS-related events occurred between 2015 and 2016, which is consistent with the steady expansion of the UAS market.
- The highest number of reported events took place in California, followed by New York. This may coincide with the presence of metroplex areas in those regions.
- Most events occurred in Class B airspace. The majority of those incidents happened in the altitude corridor between 1000 and 2000 feet AGL.

Although the example studies presented previously provide information about potential consequences of failures and risks, they do not provide much information related to the source or cause of the observed deviations. Hence, it was not possible to derive prioritization to guide the development of specific models or approaches.

There is also work related to specific UAS vulnerabilities, as presented in [4], where the authors study GPS-unavailability. Similarly, the work by Foster and Hartman [5] studies the vortex ring state (VRS) and loss of power in rotorcraft, as representative system failures.

B. *Hazard Analysis*

To understand how faults may affect specific UAS functions, and ultimately their trajectory, we examined reports that included safety analyses that connect, deviations with: potential causes, UAS functions, potential consequences, and risk. Two examples of work that document the functional decomposition methodology to UAS are [6] and [7]. In [6], functional decomposition is utilized in the context of a Functional Hazard Assessment following the Safety Assessment process outlined in SAE ARP 4761 [8]. A similar methodology was applied to the stack presented in Figure 1. A summarized view of this analysis with examples is presented in Table I. Other sources that provided additional information for the elaboration of a more comprehensive version of Table I are: [9], [10], and [5].

The goal of this analysis is not to assess safety per se, but rather, to identify failures causing trajectory deviations, to categorize them and to categorize the types of resulting deviations. Thus, categories of faults correspond to elements of the trajectory prediction stack. Resulting trajectory deviations may be grouped into three categories:

A. Degraded execution. The mission can proceed with some detriment to the ability of the aircraft to follow the nominal trajectory. This may be interpreted as a loss of quality or performance, which usually manifests itself as a loss of accuracy (e.g.: Required Navigation Performance (RNP)).

B. Fail-safe maneuvering. Detected failures trigger mitigation measures such as deploying parachutes, autorotation, emergency landing, etc. Management of contingencies may be performed by automated contingency management onboard the aircraft or by an operator.

C. Uncontrolled flight. Failures cannot be mitigated directly causing the aircraft to follow an unintended path.

Downstream risk-related consequences of these deviations, such as violation of geofencing constraints, which may ultimately lead to violation of airspace restrictions, etc. are considered outside of the scope of this work.

IV. TRAJECTORY COMPUTATION

As mentioned earlier, computation of UAS trajectories in off-nominal conditions has important uses, and it may be deemed as a key capability for the safe operation of UTM, and ultimately, for the safe integration of UAS into the NAS. The diversity of aircraft types, the innumerable system failure sources, and failure modes, make this problem particularly challenging. A preliminary list of requirements for trajectory computation is shown below:

- Generality. A wide variety of vehicle configurations, control characteristics, instrumentation, propulsion, etc. should be accommodated.
- Scalability. Since trajectory computations may feed real-time decision support tools, they should be generated rapidly.
- Extensibility. As more experimental validation data and more sophisticated models become available, it should be possible to increase the level of model fidelity, and thus the accuracy of the trajectory computation tools.
- Flexibility. Computations should allow for adaptation to multiple uses. Predictions may be used in conjunction with tools to determine potential ground risks, to protect other airspace users, or to take risk mitigation measures.

A. Sample Prior Work

Prior work in this specific area is rather scarce. One of the exceptions is the work published by Foster and Hartman ([5]). In [5], the authors take an experimental approach to characterizing a quadrotor and its components (airframe, propellers) in a wind tunnel. Of particular interest is the onset of an off-nominal condition applicable to rotorcraft known as vortex ring state (VRS). Under VRS conditions, a rotorcraft loses thrust (increased descent rate) and enters an oscillatory state, in which control inputs do not affect aircraft motion. The integrated equations of motion in conjunction with the propulsion and aerodynamic models are used to predict the trajectory of a quadrotor in three cases: abrupt failure of all motors, stuck throttle on all four motors, and VRS encounter during descent.

The paper by Ancel et al. ([11]) studies the problem in a qualitative manner. After including a list of prior work, it concludes that the prevalent approach to solving the problem is the use of point-mass or ballistics methods applicable primarily to complete power or control failures.

Other examples of prior work addressing off-nominal conditions for UAS as well as for CAT may be found in [12], [13], [14], and [15].

B. System Failure Impact on Trajectory

From the perspective of the aircraft trajectory, system failures may be modeled using four different mechanisms, as illustrated by the examples shown in Table II.

In practice, mode switch and events may be viewed as two forms of the same type of mechanism. Figure 3 presents a more detailed view of a section of Figure 1, illustrating the application of the aforementioned mechanisms.

C. Aircraft Classifications

Aircraft classifications can be used as a computational artifact for the trajectory prediction process to meet some of the requirements presented above. In [16], Ren et al. present a comprehensive categorization effort used in conjunction with the framework presented in Section II for nominal conditions. However, for off-nominal conditions, given other sources of complexity, such as non-linearities, uncertain parameters, hybrid dynamics, noise, etc., a simplified categorization as proposed in [10] may suffice. One advantage of this approach is that it could allow for the trajectory prediction process to be used in conjunction with risk analyses. In this simplified categorization scheme, three configurations and three weight categories are considered, respectively: fixed wing (FW), multirotor (MR), and unmanned helicopter (UH), and Micro UAS ($W \leq 4.4$lbs), Mini UAS ($4.4 < W \leq 20$ lbs) and Small UAS ($20 < W \leq 55$ lbs). The application of simplified categorization schemes while retaining flexibility and accuracy when needed is an aspect that requires further investigation.

D. Trajectory Computation Problem Formulation

As shown in Section IV-B, computation of trajectories under the effect of system failures present multiple sources of uncertainty, which may be captured by: noise, uncertain parameters, and mode switches. The result of interest is in most cases not a trajectory capturing a single random probable behavior, but rather:

TABLE I
EXAMPLE OF HAZARD ANALYSIS APPLIED TO UAS TP FUNCTIONS

ID	Function	Deviations (examples)	Potential causes (examples)
FH-1	Flight Dynamics	Aircraft loss of control	Power plant failure Vortex ring state (VRS) Foreign object damage (hail, bird strike, undetected obstacle, etc.)
FH-2	Automated Flight Controls	Lost/degraded controllability/ observability	Inertial sensor errors Actuator damage Autopilot error
FH-3	Flight Performance	Lost/degraded controllability	Energy depletion Flight envelope violation Erroneous determination of energy reserves
FH-4	Navigation and Guidance	Lost/degraded navigation capability	GPS failures / errors Erroneous waypoints/routes/maneuvers Erroneous position determination
FH-5	Flight Plan, Intent, & Command	Inability to keep separation from restricted operating zones	Erroneous navigation database entries Incorrect sequencing of aircraft intent commands Malformed commands
FH-6	Communications	Lost/degraded control/telemetry link	Electromagnetic interference (EMI) Malicious jamming Violation of comm. range limitations
FH-7	Mission Planning & Operations Control	Lost/degraded mission planning and execution capability	Operator errors Mission planning software errors Erroneous determination of meteorological conditions

TABLE II
MECHANISMS FOR INCORPORATION OF SYSTEM FAILURES

Mechanism	System faults (examples)
Disturbances	Erroneous operator commands, inertial sensor errors
Uncertain Parameters	Actuator damage, center of gravity shift
Mode Switch	Vortex ring state, parachute release, power plant failure
Events	Loss of link, energy depletion, single event upset phenomena, GCS failure

- the set of all possible behaviors,
- a set of representative behaviors,
- a set of "bad" behaviors,
- the "worst-case" behavior,

captured by the evolution of the aircraft state over time. The aircraft state vector may include continuous as well as discrete states. Further, the concept of state may be generalized to account for a probabilistic representation ("belief") described by a probability density function over the space of state variables.

When the state vector is viewed as a combined state of continuous and discrete variables, the system is called *hybrid*. Further, when signals are corrupted by noise, the system may be viewed as *stochastic*. In addition, when a set of possible behaviors needs to be computed, the problem is known as *reachability analysis*. Hence, in its most general form, the trajectory computation problem for off-nominal conditions may be cast as a reachability analysis of a stochastic hybrid system [17], [18].

To illustrate this formulation, we present a deterministic hybrid system model as presented in [19]. The aircraft dynamics may be modeled by a hybrid automaton H with discrete state $\{q_1, q_2, \ldots, q_k\}$, for instance, representing nominal, off-nominal conditions, or

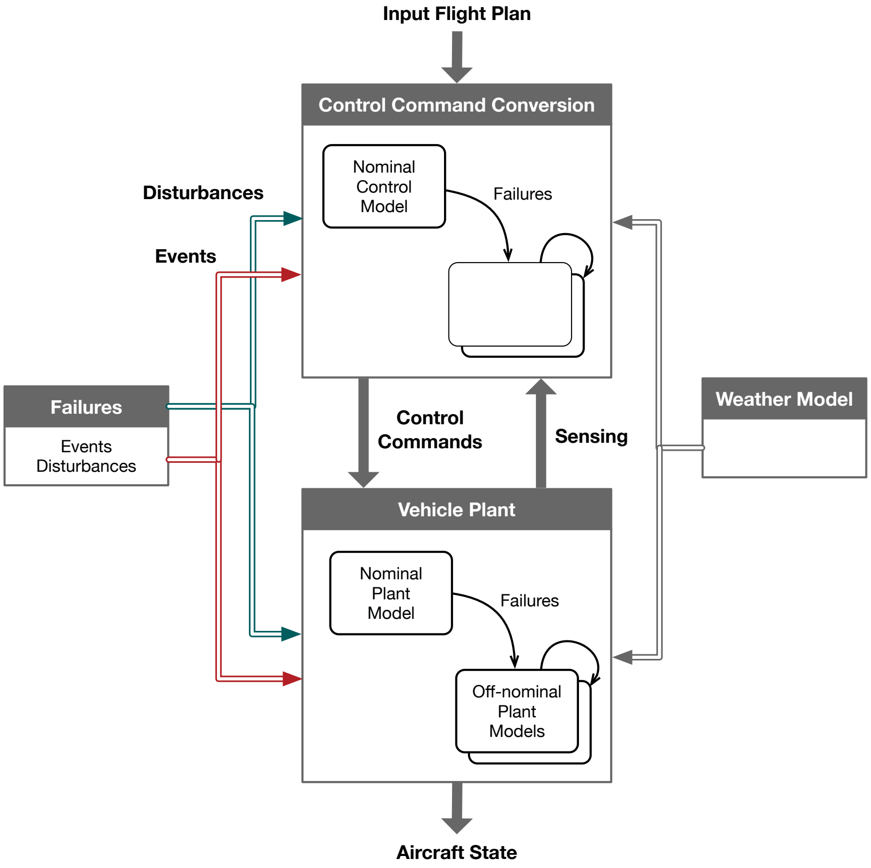

Fig. 3. Simplified off-nominal trajectory computation scheme

different flight modes. Each discrete state has associated continuous dynamics $\dot{x} = f(q_i, x, v)$, with $x \in \Re^n$, and continuous inputs u and disturbances d, such that $v = (u, d)$. Then H is a collection

$$H = (Q, X, \Sigma, V, \mathrm{Init}, f, \mathrm{Inv}, R)$$

where

- $Q \cup X$ is the set of state variables;
- $\Sigma = \Sigma_1 \cup \Sigma_2$ is a finite collection of discrete input variables, where Σ_1 are control inputs, and Σ_2 disturbances, for example, faults;
- $V = U \cup D$ are the continuous input variables, where U are control inputs and D are disturbances;
- $\mathrm{Init} \subseteq Q \times X$ is a set of initial states
- $f : Q \times X \times V \to TX$ is a vector field describing the evolution of x for each $q \in Q$;
- $\mathrm{Inv} \subseteq Q \times X \times \Sigma \times V$ is an *invariant* that defines combinations of states and inputs for which continuous evolution is allowed;
- $R : Q \times X \times \Sigma \times V \to 2^{Q \times X}$ is a "reset" relation that encodes discrete transitions.

The reach set $\mathcal{R}(q, x, t)$ is the set of all states in $Q \times \Re^n$ that can be reached at time t starting in state Init at time 0. This set may be projected into the 3D space ahead of the vehicle or into the 2D space on the terrain, thus allowing for computation of all possible impact points, as illustrated in Figure 4.

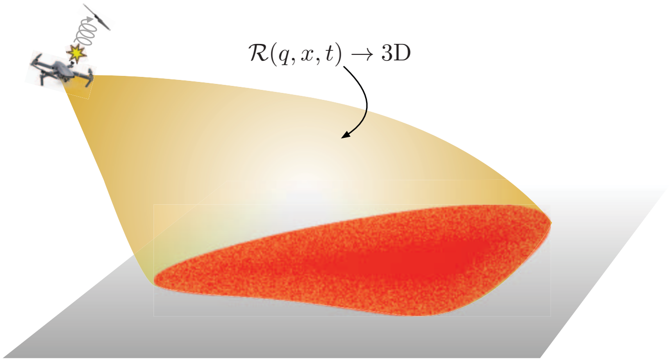

Fig. 4. Off-nominal trajectory prediction as reach set computation

This problem becomes intractable when treated naively on realistic systems. However, in recent years, a rich set of computation techniques has been developed [20]. Significant progress has been made by the formal methods and verification communities, for whom, one of their concerns is to obtain formal guarantees that a system never enters an unsafe region in the system's state space. These techniques have been applied to several domains, such as automotive [21], robotics [22], and power control [23], to name a few. Now, the key task that lies ahead is to formulate useful approximations and heuristics that provide a good balance between the four requirements presented at the beginning of this section for a relevant class of faults, for instance, the ones listed on Table I.

E. Prototype Implementation Efforts

A prototype trajectory prediction tool is being developed to integrate modeling methods and templates developed under this research effort so that their functions may be simulated and verified, and their effectiveness validated. The prototype tool will decompose a given sUAS flight mission scenario into trajectory objects. These objects serve as the basis for generating predicted trajectories, which are represented using the trajectory definition model. Diverse types of sUAS airframe configurations are modeled, including implementation for three sUAS models representing dynamics, performance, and automatic flight control logic of a conventional takeoff and landing (CTOL) fixed wing vehicle, a vertical flight (powered lift) aircraft, and a VTOL airframe configuration, respectively. Models of atmospheric boundary layer (ABL) weather dynamics for winds, temperature, density, and (potentially) precipitation for the mission to be simulated are also under development. The prototype tool is expected to model and predict sUAS trajectories under both nominal and off-nominal (system failures and hazardous environmental) conditions.

To incorporate the required features, an open system architecture is being used, where system components can either be swapped, reconfigured, or re-initialized with different parameters. This is particularly important to give UTM ecosystem partners the ability to incorporate custom models. System components include those that support the conventional trajectory segment construction process, unconventional trajectory segment construction process, and those that provide common capabilities and system integration functionality.

V. CONCLUSIONS

The problem of computing UAS trajectories in off-nominal conditions is still new as these systems are in the process of being deployed and relevant data are being collected. Past work focused on commercial aircraft provides useful pointers, but the scope of the problem was limited, and hence its applicability to the UAS domain.

A very general formulation such as reachability analysis of stochastic hybrid systems may make the problem intractable. However, hybrid systems and formal methods communities have been making rapid progress and developing tools and scalable techniques that may be drawn upon. Further, data-driven techniques enhanced by machine learning, used in combination with the aforementioned tools hold the promise of providing practical solutions addressing the requirements listed in this work.

The pattern-based trajectory definition model framework introduced in this paper furnishes enough structure and flexibility to serve as a basis for future work in UAS trajectory prediction for nominal as well as for off-nominal conditions. A current implementation of this framework in the form of a tool is being undertaken by the authors and used as a basis for refinement.

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