National Aeronautics and Space Administration

NASA

NAVIGATE SPACE

SPACE COMMUNICATIONS AND NAVIGATION (SCaN) WORKBOOK

Educational Product	
Students	Grades 8-12

TEACHERS

This workbook can be used in parallel with courses in algebra, geometry or trigonometry to apply abstract mathematical skills to the practical engineering challenges presented by space navigation. This text includes basic space navigation concepts alongside practice problems developed by NASA navigation engineers. The complex mathematics of space navigation have been simplified for use as a teaching tool.

Please note that, per NASA guidelines, we have used imperial units in the text to describe distances from Earth. However, we have elected to use the metric system when teaching mathematical concepts, as most NASA engineers use the metric system when doing calculations.

EXPLORATION AND SPACE COMMUNICATIONS PROJECTS DIVISION
Goddard Space Flight Center

SPACE COMMUNICATIONS AND NAVIGATION (SCaN) PROGRAM OFFICE
National Aeronautics and Space Administration

INTRODUCTION

NASA

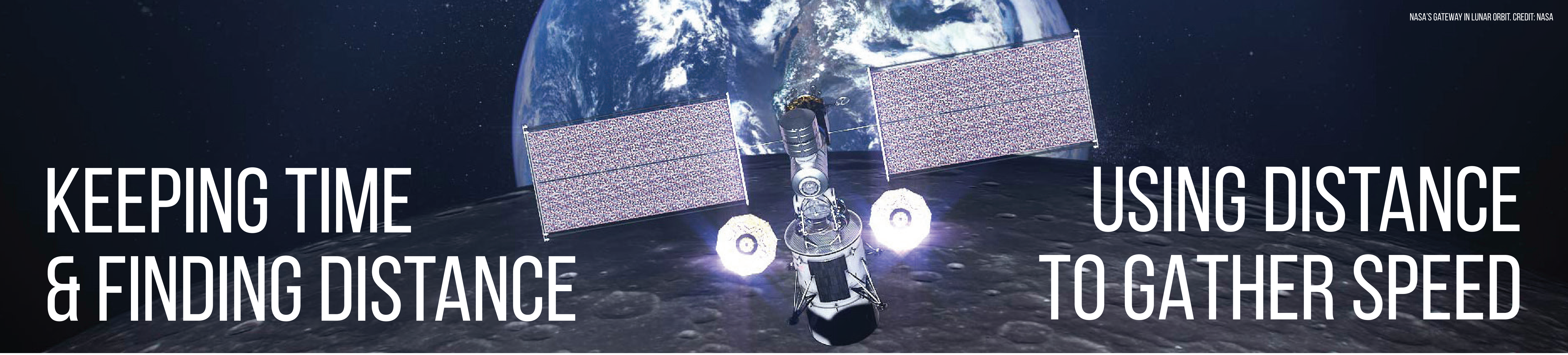
navigates throughout the solar system and beyond, revealing the mysteries of the universe. In this workbook, you'll use basic mathematic concepts to explore space navigation. We'll use simplified, real-world examples supplied by navigation engineers to delve deep into the exciting world of space exploration. As NASA goes forward to the Moon and journeys on to Mars, maybe you could be the one to plan our next big mission!

NASA's **Space Communications and Navigation** (SCaN) program office oversees much of NASA's work in navigation, particularly in the areas of technology development and navigation policy. Additionally, SCaN provides strategic oversight to NASA communications infrastructure and innovation. NASA's two major networks are the Near Space Network and the Deep Space Network.

NASA's **Near Space Network** provides communications and navigation services to user missions from the launch pad to two million kilometers away through a combination of commercial and government space communications infrastructure worldwide and in orbit. As a single, end-to-end network, they serve missions throughout their entire lifecycle. Users confidently rely on the expertise of NASA's Goddard Space Flight Center, which has a legacy of excellence in managing NASA communications services.

NASA's **Deep Space Network** uses antennas up to 230 feet in diameter to communicate with missions as far away as Voyager 1, over 13 billion miles from Earth. The Jet Propulsion Laboratory manages the network and its ground stations in California, Spain, and Australia.

As you make your way through the workbook, visit https://go.nasa.gov/30ke8KB for further explanation of topics and answers to practice problems!


NASA'S GATEWAY IN LUNAR ORBIT. CREDIT: NASA

KEEPING TIME & FINDING DISTANCE

Distance equals velocity multiplied by time.

$$d = vt$$

This equation is the foundation of space navigation.

Waves of energy, or electromagnetic radiation, can communicate data from spacecraft. They propagate through the universe at a constant speed regardless of frequency. This speed, the speed of light, is 299,272,458 meters per second, (to simplify, we'll use the value 300,000,000 meters per second in this workbook). The speed of light is represented by the letter "c" in mathematical formulas.

$$c = 300{,}000{,}000 \text{ m/s}$$

Since the velocity of electromagnetic radiation is always the speed of light:

$$d = vt \text{ becomes } d = ct$$

Say a spacecraft communicates through a Near Space Network ground station in McMurdo, Antarctica. A ground antenna sends a signal to the spacecraft and the spacecraft returns the signal. That time, divided by two, is the time it takes for electromagnetic radiation to travel from the spacecraft to the antenna.

Let's say the time elapsed was 10/6 of a second. Half of that is 5/6 of a second.

$$d = c \times \frac{5}{6} \longrightarrow d = 300{,}000{,}000 \times \frac{5}{6}$$

$$d = 250{,}000{,}000 \text{ m}$$

The spacecraft is 250,000,000 meters from the antenna!

Using this basic equation, NASA can find out a lot about our spacecraft.

ELECTROMAGNETIC RADIATION USED IN COMMUNICATIONS

RADIO

Radio, the tried and true method of space communications, has supported NASA missions since the inception of the agency. To this day, many spacecraft rely on radio antennas for communications services.

MICROWAVE

Microwave communications is very similar to radio, but occurs at a higher frequency on the electromagnetic spectrum. A higher frequency waveform allows communications equipment to transmit more data per second.

OPTICAL

Optical communications uses infrared lasers, which can offer even higher data rates than radio or microwave. Additionally, optical telescopes can have lower size, weight and power requirements than radio antennas.

X-RAYS

NASA is also experimenting with X-ray communications, which will offer even higher data rates than infrared lasers! Because of their high-energy nature, X-ray communications can only be used between two assets in space, and not in communications with the ground.

USING DISTANCE TO GATHER SPEED

Let's say we calculate two distances for another spacecraft, one after another, 5 minutes apart.

For the first distance, we get 1,500,000 meters.

For the second distance, we get 1,800,000 meters.

That means the spacecraft moved 300,000 meters in 5 minutes.

Since 5 minutes is 300 seconds, we know our spacecraft is moving at 1,000 meters per second away from Earth. Using only the speed of light and the time it takes for spacecraft communications to reach earth, we've managed to calculate the velocity of our spacecraft relative to Earth!

Let's say we calculate another distance, another five minutes later. We get 2,400,000 meters.

$$2{,}400{,}000 - 1{,}800{,}000 = 600{,}000 \text{ m}$$

$$600{,}000 \text{ m}/300 \text{ s} = 2{,}000 \text{ m/s}$$

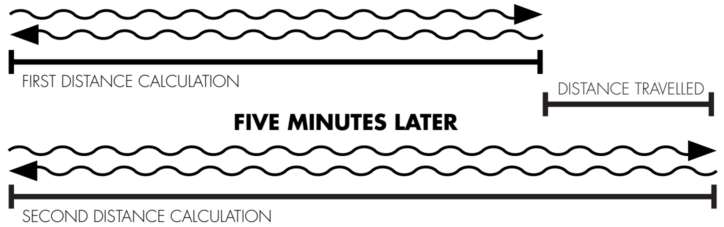
Since acceleration is the change in velocity over time:

$$a = \frac{\Delta v}{t} \longrightarrow a = \frac{600{,}000 - 300{,}000}{300}$$

$$a = 1{,}000 \text{ m/s}^2$$

The more distances we calculate, the more we know about the spacecraft's position and movement. If we gather distances from multiple sources and apply a little geometry, we can accurately determine the spacecraft's location.

In the next lesson, we'll learn about one way to determine location, a process called "trilateration."



IT'S ALL IN THE TIMING

When calculating distances using the speed of light, accurate timekeeping is enormously important. A small inaccuracy in time can lead to a huge inaccuracy in the distance calculated. To ensure correct calculations, we can derive time from atomic clocks, which use the ultra-regular oscillations of atoms and atomic particles to tick away the seconds.

These atomic clocks can be located on the ground or in space, like the ones on GPS satellites. Space-bound atomic clocks can be prohibitively large (as large as a refrigerator), but NASA is developing smaller ones like the Deep Space Atomic Clock (DSAC), which is about the size of a four-slice toaster.

Additionally, spacecraft can look to the stars for accurate time measurements. Pulsars, rotating neutron stars, emit X-ray radiation at intervals regular enough to provide timekeeping with accuracy similar to atomic clocks. Instruments like NASA's Neutron star Interior Composition Explorer (NICER) are testing the use of pulsars for navigation purposes with investigations like the Station Explorer for X-ray Timing and Navigation (SEXTANT), which uses NICER instruments.

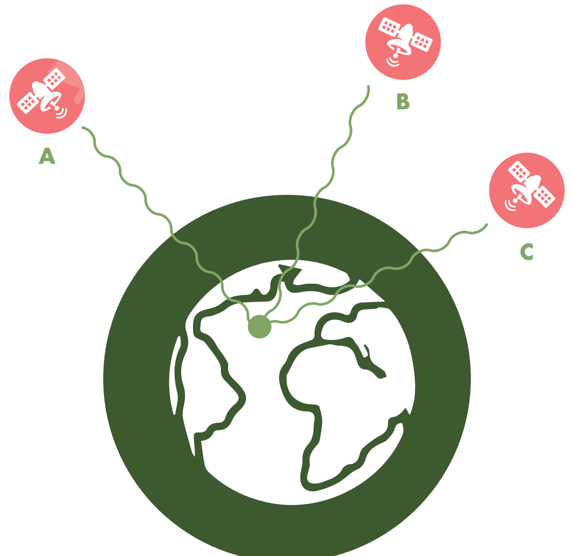

A PLANE CRASH IN ALASKA. CREDIT: U.S. NATIONAL TRANSPORTATION SAFETY BOARD

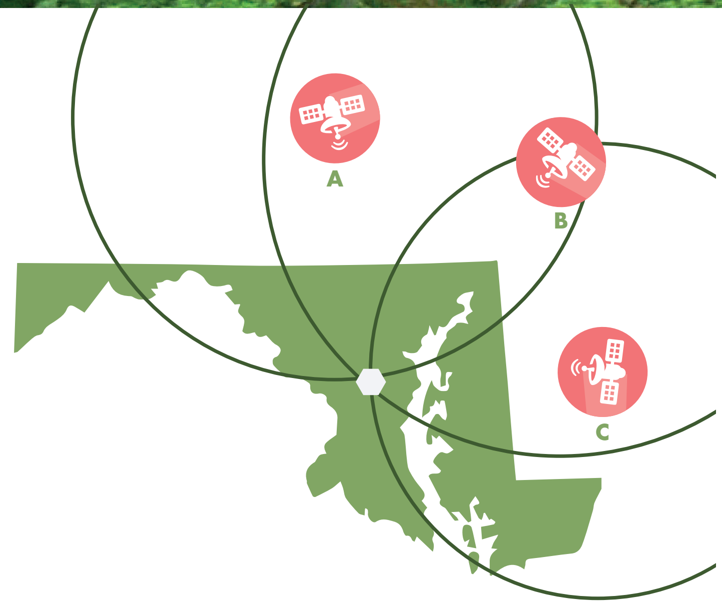
FINDING YOURSELF IN THE GREAT UNKNOWN

TERRESTRIAL TRILATERATION

When we calculate distances using the time it takes signals to travel between objects, we have only determined an area where our spacecraft could be. The spacecraft could be on a sphere with a radius from the antenna equal to the distance we measured. We don't yet know where the spacecraft is specifically. If we gather multiple spheres from different signal sources, we can narrow potential positions down to a solitary point.

The math you need to get to that point is called trilateration.





In the example to the right, we use GPS satellites to trilaterate our location on the ground. Since we know the exact position of GPS satellites in their orbits, we can draw imaginary spheres extending from each that indicate the satellites distance from us. The spheres only intersect at one spot - Goddard Space Flight Center, home of NASA's Search and Rescue office!

DID YOU KNOW?

NASA's Search and Rescue (SAR) office has served as the technology development arm of the international satellite-aided search and rescue program since its inception in the 1970s. The SAR office has developed second-generation emergency beacons that take full advantage of a new constellation of satellite-based search and rescue instruments. These beacons will offer greatly improved location accuracy and detection times for users worldwide. For more information about their work, visit esc.gsfc.nasa.gov/sar.

On this page, we'll walk you through how to do some of the math related to trilateration. Performing the math can be exceedingly complex, so just practice setting up the equations for now.

Imagine you are at some unknown position on Earth. Your position is defined by coordinates u_x, u_y and u_z in a coordinate system with three dimensions: x, y and z.

$$\vec{u} = \begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix}$$

If you're not familiar with this format, look up matrix and vector notation.

If you know the position of GPS satellite a,

$$\vec{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Then your measurement of distance can be written:

$$d_A = |\vec{a} - \vec{u}| = \sqrt{(a_x - u_x)^2 + (a_y - u_y)^2 + (a_z - u_z)^2}$$

Now, imagine you have measured the distance to three GPS satellites:

$$d_a = \sqrt{(a_x - u_x)^2 + (a_y - u_y)^2 + (a_z - u_z)^2}$$

$$d_b = \sqrt{(b_x - u_x)^2 + (b_y - u_y)^2 + (b_z - u_z)^2}$$

$$d_c = \sqrt{(c_x - u_x)^2 + (c_y - u_y)^2 + (c_z - u_z)^2}$$

You must find the coordinates of $\vec{u}$ that make these equations true. Remember: you know the GPS satellite positions $\vec{a}$, $\vec{b}$ and $\vec{c}$ as well as the distance measurements from those satellites: d_a, d_b and d_c. If the GPS satellite positions in kilometers are,

$$\vec{a} = \begin{bmatrix} 26,200 \\ -400 \\ 4,400 \end{bmatrix} \quad \vec{b} = \begin{bmatrix} -17,500 \\ -19,800 \\ 2,500 \end{bmatrix} \quad \vec{c} = \begin{bmatrix} 4,500 \\ -17,000 \\ 19,900 \end{bmatrix}$$

and you measure distances (rounded to the nearest 1000 km),

$$d_a = 25,000 \quad d_b = 24,000 \quad d_c = 20,000$$

then which city are you in? Try plugging the coordinates below into the equations to find out!

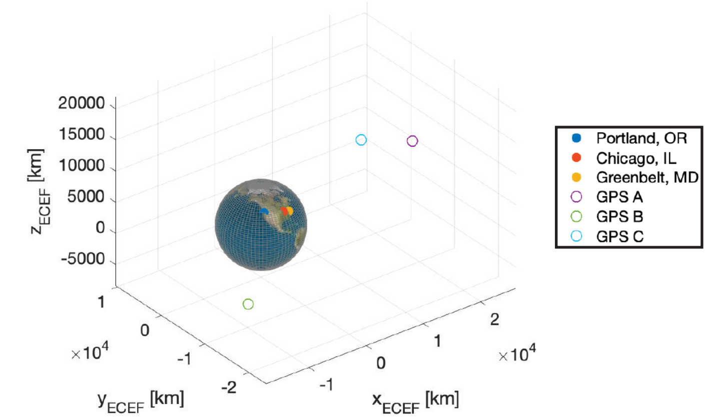
Portland, Oregon: $\vec{u} = \begin{bmatrix} -2,420 \\ -3,770 \\ 4,530 \end{bmatrix}$

Greenbelt, Maryland: $\vec{u} = \begin{bmatrix} 1,130 \\ -4,830 \\ 3,990 \end{bmatrix}$

Chicago, Illinois: $\vec{u} = \begin{bmatrix} 200 \\ -4,750 \\ 4,240 \end{bmatrix}$

Turns out you're in Greenbelt, Maryland!

The puzzle formed by these equations can be treated as a non-linear least squares problem by a GPS reciever. These receivers use a variety of mathematical methods to solve this problem, finding the location that best agrees with the distance measurements.



EARTH FROM SPACE. CREDIT: NASA

A FEW COMPLICATIONS

Of course, GPS trilateration is much more complicated than we make it seem here. Accurately locating something using GPS signals requires advanced, college-level math. There are a number of challenges that engineers must overcome when using GPS signals:

EARTH IS NOT FLAT
Rather than finding a location on a flat plane, you must find a location on an irregular spheroid with hills, valleys, mountains and trenches.

REFRACTION
As GPS signals move through parts of the atmosphere with different densities, the signals refract, changing direction slightly. If you don't adjust for these subtle changes in direction, your solutions can be inaccurate.

RELATIVITY
Einstein's theories describe how time flows slightly faster where there is less gravity. Therefore, navigation engineers must adjust for the subtle change in the speed of time between Earth and the satellites' orbits.

DID YOU KNOW?

Gladys West, a "hidden figure" of the space program, developed much of the math behind the Global Positioning System in the 1970s and 80s. West used her expertise in satellite geodesy — the measurement of Earth's form and dimensions from space — to improve the accuracy of location calculations. Since then, many mathematicians and physicists have lent their knowledge and expertise to further refine GPS calculations and overcome the challenges mentioned on this page!

GLADYS WEST. CREDIT: U.S. AIR FORCE

VENUS. CREDIT: NASA

ADVANCED INVESTIGATIONS

DOPPLER SHIFT

Navigation engineers can also determine velocity using a phenomenon called the Doppler effect *(further explanation on pg 21)*. When the distance between a ground station and a satellite is decreasing, the wavelengths of a signal received from the satellite compress, increasing the frequency. When the distance between a ground station and a satellite is increasing, the wavelengths of the signal received from a satellite stretch, decreasing the frequency. By analyzing the change in frequency between transmission and reception, navigation engineers can determine a satellite's velocity.

The frequency and wavelength of a wave are related by the speed of light, where frequency equals the speed of light divided by the wavelength, λ.

$$f = \frac{c}{\lambda}$$

The frequency, measured in Hertz (Hz), observed by the ground station is equal to the initial frequency times the speed of light, divided by the speed of light minus the velocity of the satellite.

$$f_{obs} = f_o \times \left(\frac{c}{c - v}\right)$$

Let's say a satellite sends us a signal of 100 MHz, but we receive that signal at 100.000333 MHz. Since the frequency increased, we know the satellite is moving towards us. Now, let's calculate how fast:

$$100 = 100.000333 \times \left(\frac{c}{c - v}\right)$$

$$\frac{100}{100.000333} = \frac{300{,}000{,}000}{300{,}000{,}000 - v}$$

$$.999967 \times (300{,}000{,}000 - v) = 300{,}000{,}000$$

$$300{,}000{,}000 - v = 300{,}009{,}900$$

$$-v = 9{,}900 \text{ becomes } v = -9{,}900 \text{ m/s}$$

Therefore, the spacecraft is travelling towards the ground station at a velocity of 9,900 meters per second.

LINK BUDGETS

A link budget is a formula for calculating the power received from a transmitter such as a GPS satellite. The power of the signal at the receiver depends on a number of factors, including the transmitted power, the distance between the transmitter and receiver, and the hardware used by both. Here is a simple link budget, where P_R is total power recieved.

$$P_R = P_T + G_T - A_d + G_R$$

The signal is transmitted with a power P_T. This power is typically focused in a desired direction by an antenna (such as toward the Earth, in the case of GPS satellites). This focusing has the effect of increasing the power the receiver sees by G_T, a quantity known as transmit antenna gain. The signal must then travel across the space between the transmitter and receiver. Signals weaken with distance traveled, decreasing according to the free space path loss, A_d, defined as:

$$A_d = 10\log10\left[\left(\frac{4\pi df}{c}\right)^2\right]$$

where f is the frequency of the signal and d is the distance it travels. This free space path loss is proportional to the inverse square of the distance between the transmitter and receiver. Finally, the signal is received by an antenna whose design adds some receive antenna gain, G_R.

The exact values for these parameters vary, but the signal leaving a GPS satellite (including transmit power and antenna gain) may be approximately 14 decibel watts, dBW *(further explanation of decibal watts may be found on pg 19)*. This loses 125 dB as it travels to the surface of Earth from the satellite. With a 7 dB receive antenna gain, the GPS signal is only -104 dBW — weaker than the power you would see from a car tail light 3,000 kilometers away!

GPS ACE

Members of NASA's navigation team used link budget analysis to reverse-engineer characteristics of GPS satellite antennas in a project called the GPS Antenna Characterization Experiment (GPS ACE). They documented how well these satellites could support spacecraft that fly well above the GPS satellites themselves. In fact, the team determined that GPS signals are strong enough to be used in lunar orbit!

NASA'S GATEWAY IN LUNAR ORBIT. CREDIT: NASA

THE SPACE SERVICE VOLUME

Many spacecraft close to Earth rely on the global navigation satellite system (GNSS), the collection of six international GPS constellations, for navigation. GNSS signals provide these spacecraft with timing data crucial to determining their orbits and trajectories, just as they would a user on Earth.

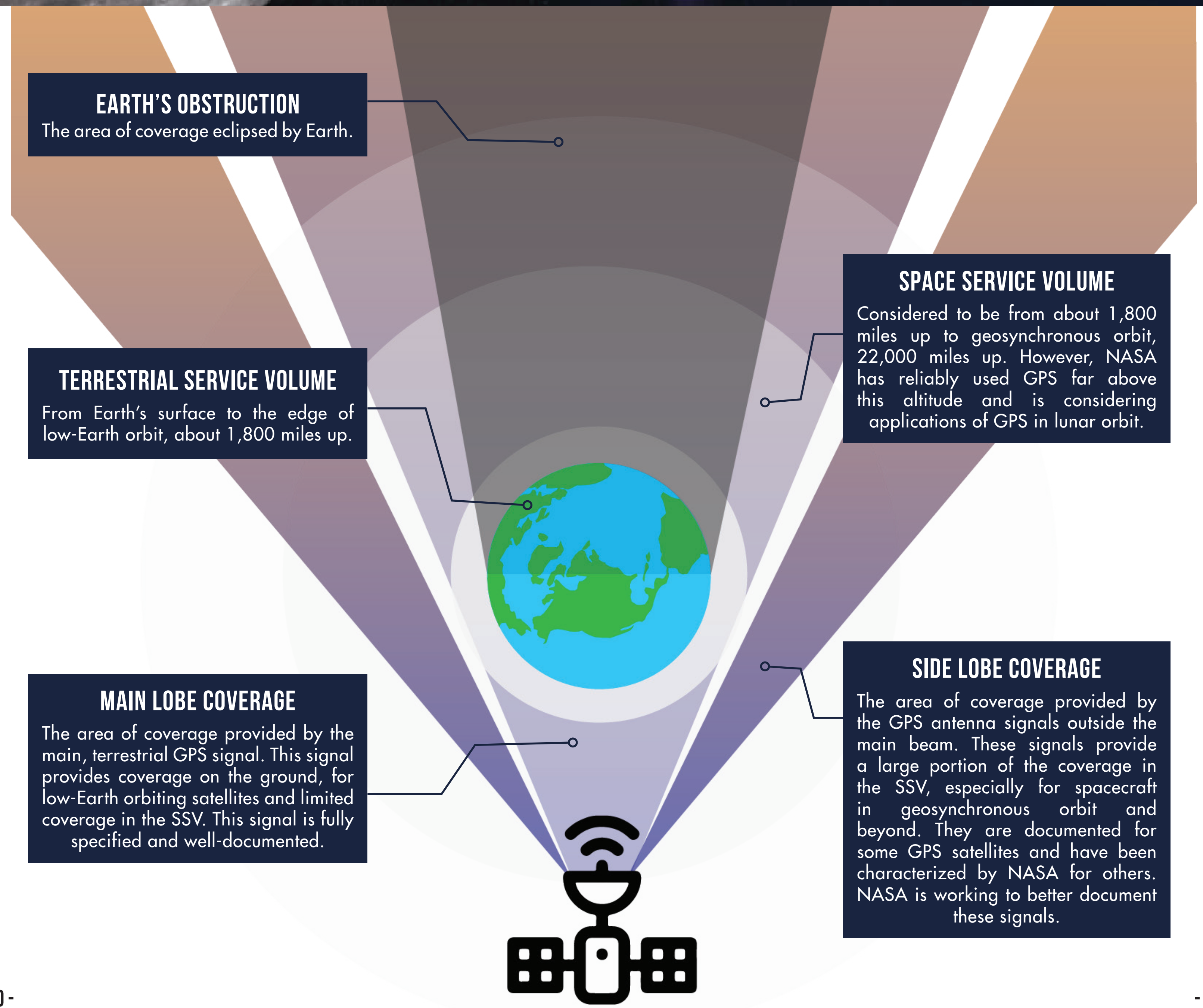
The area of space supported by GNSS is called the Space Service Volume (SSV). The formal outer limit of the SSV is an altitude of about 22,000 miles, or geosynchronous orbit.

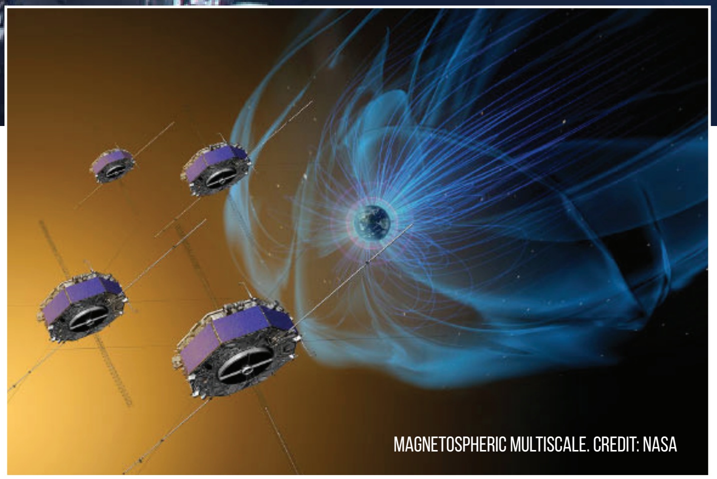
Spacecraft with higher altitudes can also use GNSS signals, but they do not use the coverage traditionally documented by GNSS service providers. Rather, they rely on signals from GNSS antenna side lobes.

Communications engineers usually consider these side lobes wasted energy, signals sent out in unwanted directions. However, because GNSS side lobe signals extend past Earth, satellites at higher altitudes on the opposite side of the globe can utilize their signals.

Documenting these side lobes can help extend GNSS services to spacecraft outside the current SSV. NASA simulations show GNSS services could even be used at lunar orbit. Engineers are considering applications of GNSS in developing Gateway, a lunar-orbiting platform that will enable sustained surface exploration of the Moon.

Currently, NASA navigation experts and international service providers are advocating for better GNSS documentation to make innovations like lunar GPS a possibility.





MAGNETOSPHERIC MULTISCALE. CREDIT: NASA

DID YOU KNOW?

After navigation maneuvers conducted in February 2019, the four Magnetospheric Multiscale (MMS) mission spacecraft now reach over 116,300 miles from Earth at their highest point. That's about halfway to the Moon! At this altitude, MMS continued to receive strong enough GPS signals to determine its position, shattering previous records it set in October 2016 in February 2017. This demonstrates that GPS signals extend farther than expected and that future missions can reliably use GPS at extreme altitudes.

MMS studies a fundamental process that occurs throughout the universe, called magnetic reconnection. This is when magnetic fields collide and explosively release particles in all directions. Near Earth, reconnection is a key driver of space weather, the dynamic system of energy, particles and magnetic fields around Earth. They can adversely impact communications networks, electrical grids and GPS navigation. Physicists predicted magnetic reconnection for years, but never directly observed it until the MMS mission.

PLANET COLLAGE. CREDIT: NASA

ORBIT TYPES

Different orbits provide different views of Earth and deep space. Navigation experts at NASA must determine which type of orbit a spacecraft needs to complete its science mission.

The planets of our solar system orbit around the Sun in various elliptical orbits. Elliptical orbits resemble a long oval with an extreme high point called the apogee and low point called the perigee.

The United States launched its first satellite, Explorer-1, on January 31, 1958, as a part of the International Geophysical Year. This was a period of intense technological invention during the Cold War. Explorer-1 used an elliptical orbit to detect charged particle radiation trapped by Earth's magnetic field. The radiation belts discovered around Earth are known as the Van Allen Belts, named after James Van Allen, the engineer behind the Explorer series instruments. Today, NASA studies these regions with more advanced probes, but still uses elliptical orbits.

You might think that a circular orbit would be more common than an elliptical one, but that couldn't be further from the truth! Circular orbits at a fixed distance with a stable speed and no apogees or perigees are one of the rarest kind of orbits. Earth's orbit around the Sun, compared to other planets in our solar system, is almost circular, which is a factor that makes it habitable.

Though completely circular orbits rarely occur naturally, NASA places satellites in circular orbits all the time!

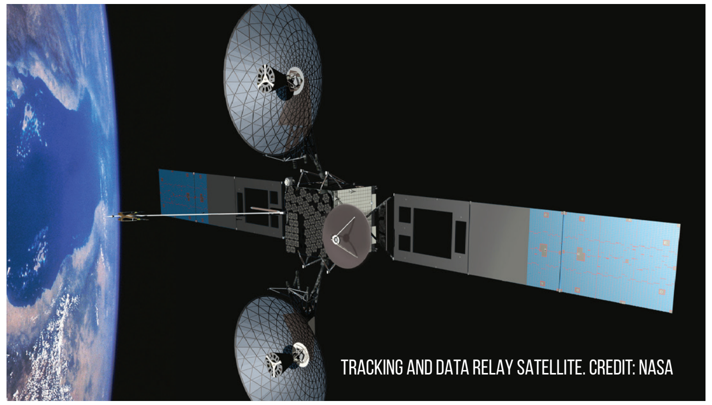
Many of NASA's weather and communications satellites are in a circular orbit called geosynchronous orbit – about 22,000 miles above Earth's surface. Geosynchronous orbits match Earth's rotation on its axis, so it seems as though they remain in the same place relative to a fixed point on the ground at all times.

NASA's fleet of Tracking and Data Relay Satellites (TDRS) are in geosynchronous orbit, using radio frequency to transmit data to Earth from spacecraft located in low-Earth orbit, including the International Space Station.

Beyond elliptical and circular, there are other ways to describe an orbit: equatorial, polar and more. For example, many of NASA's Earth science spacecraft orbit around the poles, earning the name "polar orbiting satellites."

LAGRANGE POINTS

Lagrange points, or libration points, are points in space near two large bodies where a small object can be "parked" stationary relative to the large bodies. There, the combined gravitational forces equal the centripetal force felt by a smaller body. Sun-Earth Lagrange points are useful for NASA science spacecraft!



TRACKING AND DATA RELAY SATELLITE. CREDIT: NASA

ORBIT DETERMINATION

Calculating the velocity a spacecraft needs to maintain a stable, circular orbit requires the force of gravity to equal the centripetal force of the spacecraft orbiting the planet. Centripetal force is a force that keeps something moving along a curved path.

Below are the equations to calculate both the force of gravity (F_g) and the centripetal force (F_c).

F_g = gravitational force of attraction (Newtons)

$$F_g = \frac{Gm_1m_2}{r^2}$$

m_1 = mass of planet (kilograms)

m_2 = mass of orbiting spacecraft (kilograms)

r = distance between the spacecraft and the center of the planet (meters)

G = universal gravitational constant

$$G = 6.67 \times 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$$

F_c = centripetal force

$$F_c = m_2\left(\frac{v^2}{r}\right)$$

m_2 = mass of orbiting spacecraft (kilograms)

v = spacecraft velocity tangent to its orbit

When $F_c = F_g$,

$$\frac{Gm_1m_2}{r^2} = m_2\left(\frac{v^2}{r}\right)$$

This equation can be reduced to something much simpler. Multiply both sides by r/m_2, then take the square root of both sides. The equation that remains is:

$$\left(\frac{Gm_1}{r}\right)^{\frac{1}{2}} = v$$

It turns out that the velocity needed to maintain a stable, circular orbit around a planet is the square root of the gravitational constant times the mass of the planet divided by the distance between the spacecraft and the center of the planet.

Below, calculate the velocity necessary to maintain orbits 2,000,000,000 meters away from each of the planets.

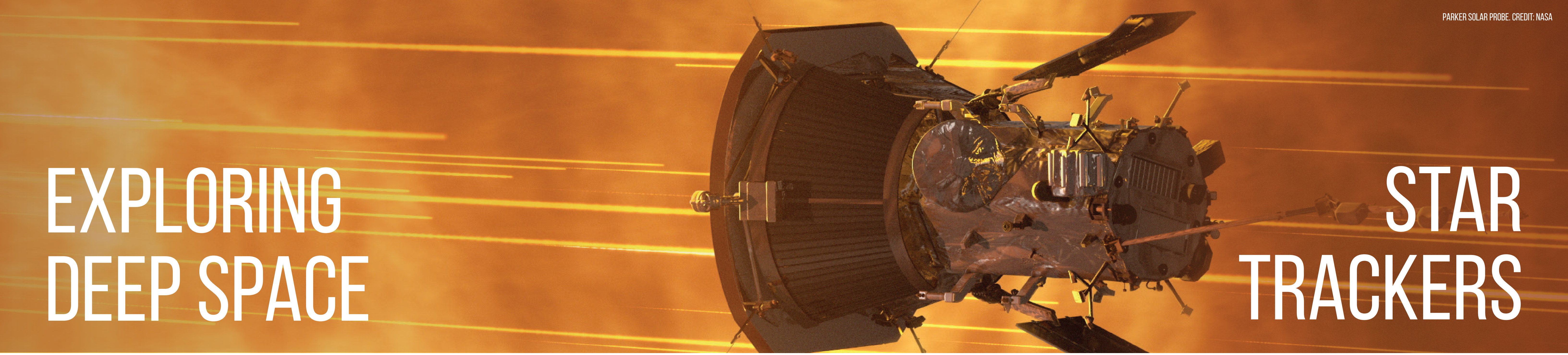
MERCURY	Mass: 0.330×10^{24} kg
VENUS	Mass: 4.87×10^{24} kg
EARTH	Mass: 5.97×10^{24} kg
MARS	Mass: 0.642×10^{24} kg
JUPITER	Mass: 1898×10^{24} kg
SATURN	Mass: 568×10^{24} kg
URANUS	Mass: 86.8×10^{24} kg
NEPTUNE	Mass: 102×10^{24} kg

DELVING FURTHER

There are a number of factors that can make orbit determination challenging:

- In an elliptical orbit, the velocity of the orbiting body changes, becoming faster as it approaches the planet and slower as it moves away. This is because as the distance between two objects changes, the force of gravity between them does as well.
- When two large bodies orbit one another, the math becomes a little more complicated. In a binary system, the center of the orbit lies in between the two bodies, rather than at the center of one.

We won't address these calculations in this workbook, but you can find further information on orbit calculation from a teacher or online.



PARKER SOLAR PROBE. CREDIT: NASA

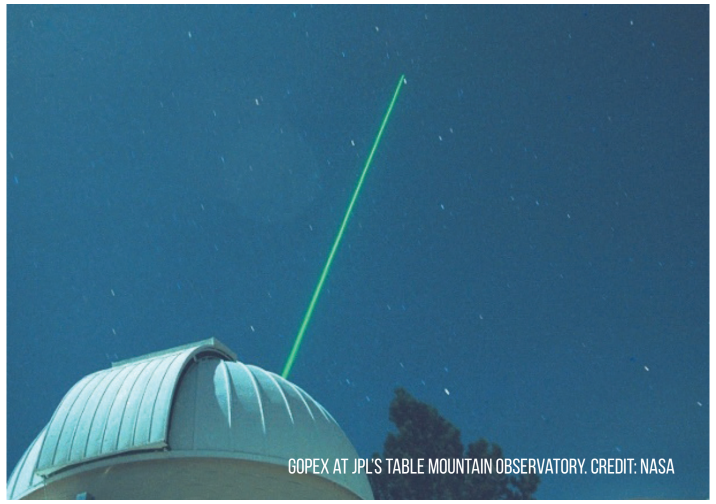
EXPLORING DEEP SPACE

Some NASA missions venturing deep into space use the gravitational pull of other planets to speed up, slow down or change direction. These gravitational assists are often planned years in advance and require a specific launch window.

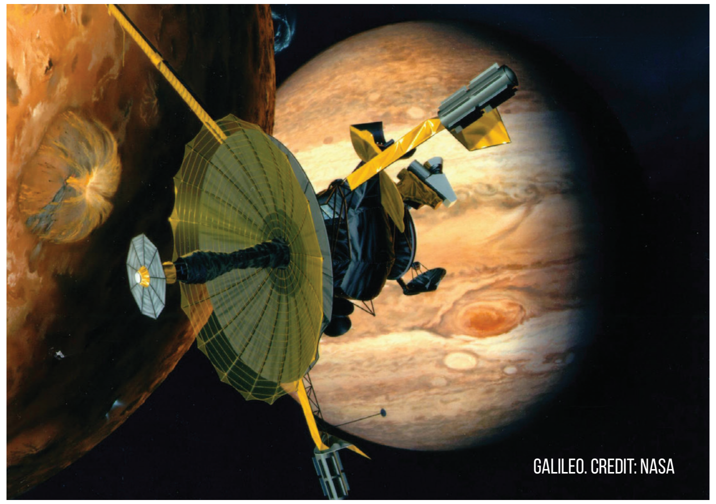
In the 1970s, the twin Voyager missions took advantage of a rare alignment of Jupiter, Saturn, Uranus and Neptune that takes place only once every 175 years. This arrangement allowed the two spacecraft to swing from one planet to the next without the need for large, onboard propulsion systems. Each flyby bent the spacecraft's path and increased their velocity enough to deliver them to the next planet. Using these gravitational assists and the unique four-planet arrangement, the flight to Neptune was reduced from 30 years to 12. Now, both Voyager spacecraft are in interstellar space, over 11 billion miles from Earth, outside the influence of our Sun.

Another flagship mission that utilized gravitational assists was NASA's Galileo mission, which launched in 1989. The Galileo spacecraft, bound for Jupiter, was named after the Italian Renaissance scientist Galileo Galilei. Galileo discovered Jupiter's major moons in 1610 with the first astronomical telescope. Three gravity assists were needed for the Galileo spacecraft to reach Jupiter. In February 1990, the spacecraft used Venus' gravitational pull to increase the spacecraft's speed, followed by two additional gravity assist flybys of Earth. The spacecraft reached Jupiter in 1995. The Galileo mission also played an important role in early demonstrations of optical communications technology with the Galileo Optical Experiment (GOPEX), which demonstrated a laser "uplink" from Earth to a spacecraft.

Since NASA's inception, the study of our Sun has been a top priority to the agency. 60 years of technology development led to the Parker Solar Probe, which launched toward the Sun in 2018. Parker will use seven Venus flybys to gradually shrink its orbit, coming as close as 3.83 million miles from the Sun – seven times closer than any previous mission.



GOPEX AT JPL'S TABLE MOUNTAIN OBSERVATORY. CREDIT: NASA



GALILEO. CREDIT: NASA

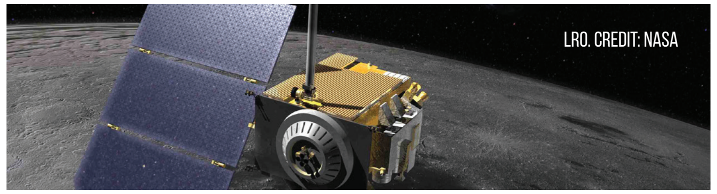
STAR TRACKERS

Most navigation experts consider the near-Earth region to be within two million kilometers of Earth. Deep space navigation does not typically benefit from near-Earth systems like GNSS, but over the years, NASA has developed a number of ways to navigate the solar system safely — techniques that go beyond ranging the satellites with ground stations to judge their distances.

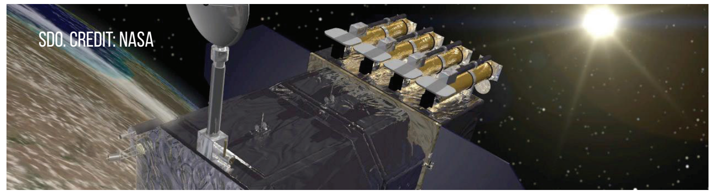
The concept of a star tracker can be traced back to mariners navigating the oceans in a time before GPS. They used bright and commonly known star patterns to determine their position and the direction they faced.

Today's star trackers are vastly more complex but use the same concept, searching for familiar star patterns to determine the orientation and attitude of a spacecraft. A star tracker is an optical device that acts as the eyes of the spacecraft. The device captures star field images, measures their position in relation to the spacecraft and identifies particular stars so that they can be logged and entered into a star catalog.

Typically, a star tracker will include a processor to compare the star field pattern it observes to previously logged star field patterns. NASA's Lunar Reconnaissance Orbiter (LRO) and Solar Dynamics Observatory (SDO) host two star trackers each. Having two star trackers increases position accuracy and provides redundancy – if one fails, the second one can take over.



LRO. CREDIT: NASA



SDO. CREDIT: NASA

ADVANCED INVESTIGATION

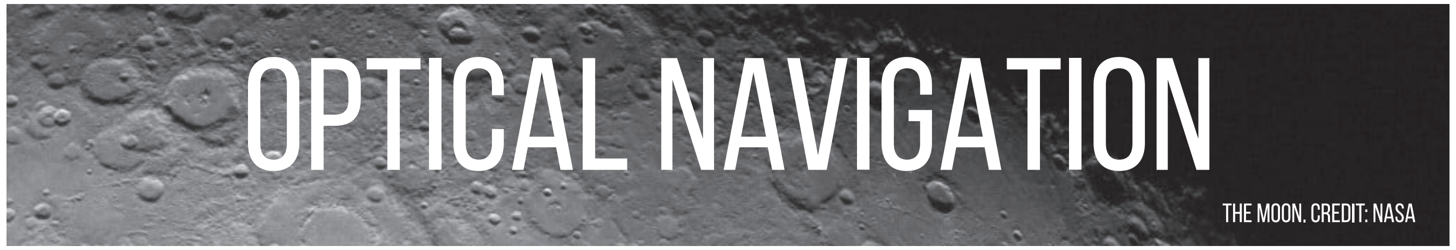
Delta-Differential One-Way Ranging (DDOR) is a navigation technique that relies on two widely separated ground stations receiving signals from the same spacecraft. The difference in time between the arrival of the signal at the ground stations can be used to calculate a satellite's position.

These times are affected by several factors. Particularly, when radio waves travel through the atmosphere, the different densities of air cause disturbances that must be accounted for. NASA has ways of adjusting for this.

DDOR corrects the error by looking for a quasar – a galactic nucleus – in a direction close to the spacecraft. Since the quasar's direction is known, the delay time of the quasar can be subtracted from the delay time of the spacecraft to account for the atmosphere.

DDOR, alongside other navigation techniques, offers increased spacecraft targeting accuracy, improved mission reliability, reduced tracking time and reduced cost.

OPTICAL NAVIGATION



THE MOON. CREDIT: NASA

NASA can also navigate spacecraft relative to a target using images taken from the spacecraft. In optical navigation, a camera takes images of a target, producing measurements that can be used to estimate distance and trajectory. Optical navigation works similarly to human eyes, collecting light from a target and processing the information in the spacecraft's 'brain.'

Cameras, like eyes, have lenses. Generally, these lenses take the subject in view and magnify it, like a telescope. Complex lenses can distort images, like a fisheye lens.

A camera collects the light at its lens and focuses the light on the sensor array creating a picture out of individual pixels on a grid. The size of the image we see is directly related to the size of the target, the distance to the object, focal length of the camera and the pixel size, otherwise known as the pixel pitch.

In optical navigation, we use geometry and trigonometry alongside known characteristics of a camera's lens and sensor to calculate distances. Below is a diagram of a simple pinhole camera looking at a tree.

S = height of the target.

D = the distance from the Target to the camera lens.

f = the focal length, the distance between the sensor array and the camera lens.

S' = the height of the target on the sensor array.

Θ = the field of view for the camera, the angle between the focal point and the edge of the sensor array.

Θ' = the angular size of the target, the angle between the focal point and the height of the target.

The focal length (F) and field of view (Θ) are generally known variables for a camera — they are designed with those specifications in mind. Once the camera takes a picture, we can easily measure the height of the target on the sensor array, the number of pixels on the y-axis of the sensor array times the pixel height.

Drawing a right triangle using the height of the target on the sensor array (S'), and a perpendicular line from the focal point to the lens with the known focal length (f), we can use the tangent function to determine the angular size of the target.

$$\tan \Theta_1 = \frac{S_1}{f}$$

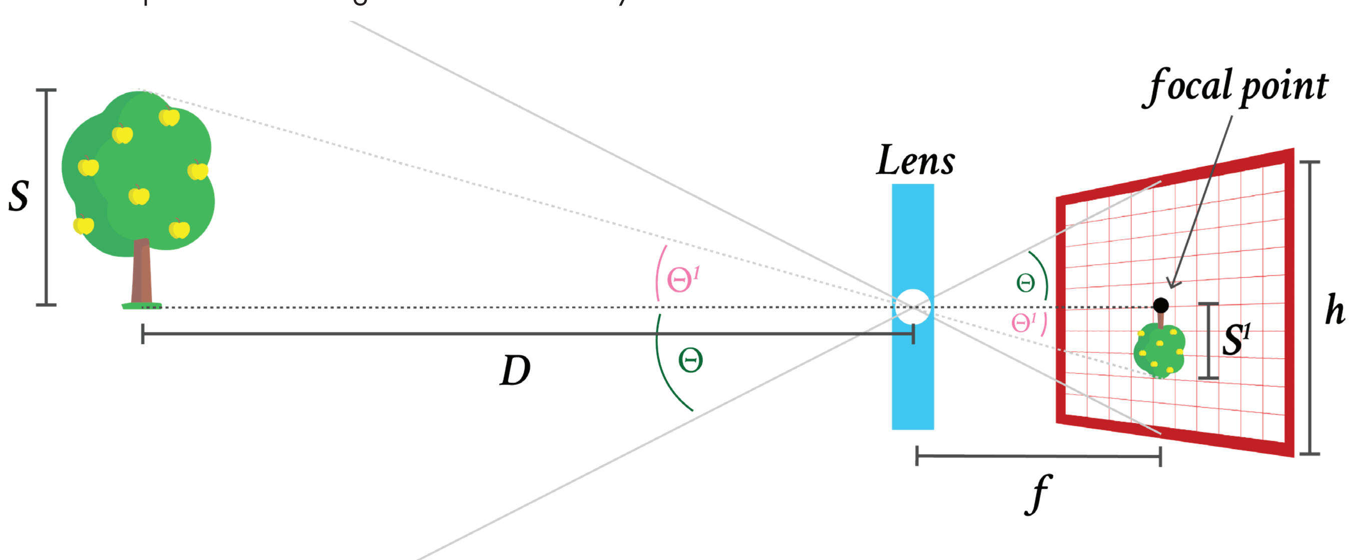
Drawing a right triangle using the distance from the target to the camera lens (D) and the height of the target (S), we can use the tangent function again.

$$\tan \Theta_1 = \frac{S}{D}$$

Combining these equations, we see that:

$$\frac{S_1}{f} = \frac{S}{D}$$

Because S' and f are known variables, if we have a value for either S or D, we can easily calculate the other. Optical navigation calculations like this can be used for a wide variety of applications in space.



JOSH LYZHOFT

A MINNESOTAN BATTLING ASTEROIDS

In 2016, Joshua Lyzhoft drove over 17 hours from Minnesota, his home state, to Maryland to begin his career at Goddard Space Flight Center with a Pathways Program internship. Josh came to NASA with a doctorate in aerospace engineering focused on planetary defense, a rapidly advancing field that examines the risks of near-Earth asteroids and develops technologies to alleviate their danger.

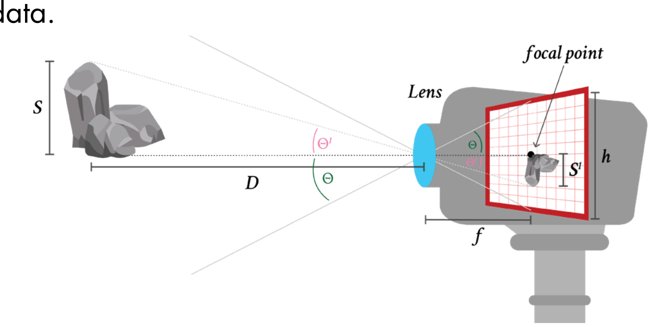
At NASA, he works on the Double Asteroid Redirection Test (DART) spacecraft, which will slam into the smaller of two asteroids in a binary system. In the aftermath, NASA will study how the spacecraft changed the relationship between the two asteroids. The mission will serve as a proof of concept for future planetary defense efforts. In addition to DART, Josh lends his talents in optical navigation and orbit determination to other missions.

When not hunting down asteroids, Josh enjoys hunting for mushrooms. A true Midwesterner, he loves the outdoors: fishing, foraging and chopping wood.

Imagine that NASA launches a rover to the Moon to prospect for resources like water. Missions like this are becoming more common as NASA expands human and robotic presence on the lunar surface with the Artemis missions.

The rover needs to navigate from its original location to a deep crater. Scientists believe that water ice might exist in the extremely cold, permanently shadowed regions at the bottoms of craters like the one our rover is exploring.

A large boulder in our rover's field of view marks the spot right at the edge of the crater where our rover must park. Mission controllers on Earth need to command the rover to move the exact distance to place it on the spot. Move too far and the rover may fall into the crater. Move too little and scientists will lose valuable science data.



We know the focal length of the rover's camera is .5 m. Based on observations from previous missions, we know the boulder is 10 m tall. The boulder appears to be .01 m in the sensor array. Each pixel is 1×10^{-5} m tall.

PROBLEMS

Commanding the Rover

a. What is the distance from the boulder to the rover's lens in meters?

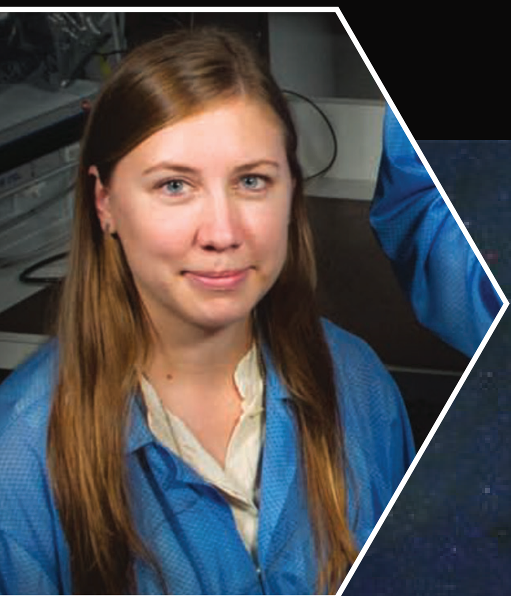
b. If the rover drives at a speed of .08 m/s, how long should mission control instruct the rover to move in order to reach the boulder?

Learning More About the Rover

c. Given that the height of the sensor array (h) is 2000 pixels, what is the field of view (Θ) in degrees?

Advanced Investigation: Thinking Critically

d. If we do not have the size of the object, how might an individual determine the distance? (Hint: There might need to be more than one camera.)



JENNY DONALDSON

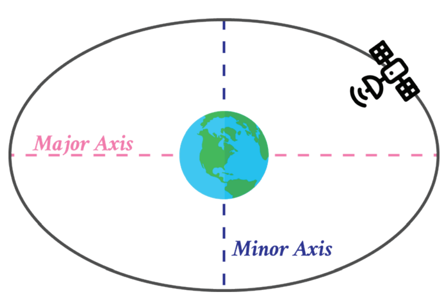
DANCING AMONG THE STARS.

In 2016, two teams of engineers combined two technologies to create a powerful navigation tool, NavCube, that could demonstrate advanced X-ray communications technologies in space. For their profound creativity, the teams won a NASA Innovators of the Year award.

Jenny Donaldson led the development of NavCube. Previously, she worked on the team that developed one of the contributing technologies, the Navigator GPS, an instrument uniquely suited to "hearing" GPS signals at ultra-high altitudes. In addition to helping adapt the Navigator into NavCube, Jenny used the Navigator to perform the GPS Antenna Characterization Experiments (ACE), which helped define the characteristics of GPS antennas.

As a child and young adult, Jenny pursued a career as a ballerina. In fact, she continues to dance in her spare time! She didn't become interested in engineering until she read a piece about Mars rovers, falling in love with space exploration. She then pursued a degree in engineering, eventually coming to NASA as an intern and then a full-fledged employee.

Orbital mechanics is the study of the motion of spacecraft moving under the influence of forces such as gravity. Most orbits around a large central gravitational body, such as Earth, are elliptical, which means they follow a repeating path in the shape of an ellipse, as seen in the diagram below. A special case of the ellipse, when the major and minor axes are the same, is called a circle.



In the 17th century, Johannes Kepler described elliptical orbits when studying the patterned motion of planets orbiting the Sun. Kepler's third law of planetary motion states that the square of the orbital period of a satellite is directly proportional to the cube of the semi-major axis of its orbit. In other words, how long it takes to complete one revolution of an orbit (period, T), is directly related to size of the orbit (semi-major axis, a) and can be calculated with the following formula:

$$T = 2\pi\sqrt{\frac{a^3}{Gm}}$$

G is the universal gravitational constant, and m is the mass of the object the satellite is orbiting.

PROBLEMS

Earth's elliptical orbit around the Sun has a semi-major axis of 1.496×10^{11} meters and the mass of the Sun is 1.989×10^{30} kilograms.

a. Calculate the period of Earth's orbit around the Sun in seconds.

b. Calculate the period of Earth's orbit around the Sun in days.

The orbit of the International Space Station around Earth has a semi-major axis of 6.7981×10^6 meters. Earth's mass is 5.97×10^{24} kg.

a. Calculate the orbital period of the space station in minutes.

b. How many times does the space station orbit Earth in one day?

JOEL PARKER

PRAGUE. MUNICH. SOCHI. BREMEN. VIENNA. KYOTO.

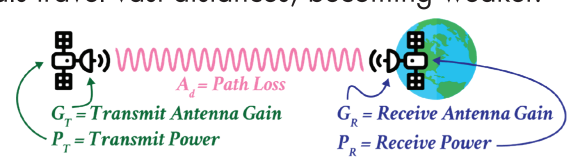
Maps neatly frame Joel Parker's desk. Each creased leaflet marks a meeting conducted - a milestone crossed. He collects one for every city he visits while representing NASA as a navigation expert.

Joel's work occupies the intersection of engineering and policy. He leads the effort to extend the Space Service Volume, working with domestic and international GNSS providers. Joel's goal is to ensure that GNSS navigation signals are a robust option for high-altitude NASA missions. He also serves as the Flight Dynamics Lead for the Plankton, Aerosol, Cloud, ocean Ecosystem (PACE) mission, which will launch in 2022 into polar orbit. PACE will advance the assessment of ocean health by measuring the distribution of phytoplankton, tiny plants that sustain the marine food chain.

When not at work, Joel finds creative outlets by cooking for his family and working with his wife to restore their 1930s, Craftsman-style home in Hyattsville, Maryland. Raising a young daughter, Madeline, and taking care of two dogs takes up the rest of his time.

How do we know if a navigation signal is strong enough to be received by our spacecraft?

Link budget calculations use known features of a signal—like transmit power and the distance to the receiver—to determine unknown features, such as how large an antenna is needed to receive a signal. The largest source of loss in signal strength is path loss, which happens as signals travel vast distances, becoming weaker.



To perform a link budget calculation, we use units of decibels (dB), which express common numbers in exponential form. This is useful for expressing very large or very small numbers. To convert watts (P_W) to decibel-watts (P_{dBW}), use a logarithm, like so:

$$P_{dBW} = 10\log_{10}\left(\frac{P_W}{1W}\right)$$

Given the .001 watts, we get:

$$P_{dBW} = 10\log_{10}\left(\frac{.001W}{1W}\right)$$

Here we express the quantity 0.001W in terms of decibels relative to 1 Watt. The benefit of working in decibels is that complex equations can be expressed using simple additions and subtractions. The equations for the signal power at the receiver (P_R) and the path loss (A_d) — found on page nine of this workbook — take advantage of this.

PROBLEMS

a. GPS satellites orbit about 20,200 km above Earth. The International Space Station flies at about 400 km. What is the path loss in dBW as the signal travels between the two, assuming the GPS signal has a frequency of 1575.42 MHz?

b. What is this path loss in Watts?

(Hint: Reverse the decibel equation.)

c. GPS signals are transmitted at about 13 dBW, using an antenna with a gain of about 15 dBW. Assuming the space station has a receive antenna with a gain of 10 dBW, what is the signal power it receives?

d. Now consider that receiver and antenna are mounted on a spacecraft going to the Moon. The Moon is about 400,000 km from the center of the Earth, which has a radius of 6,378 km. What is the path loss in dBW?

e. What is the signal power received by this lunar spacecraft in dBW?

f. What is this received signal power in Watts?

BEN ASHMAN

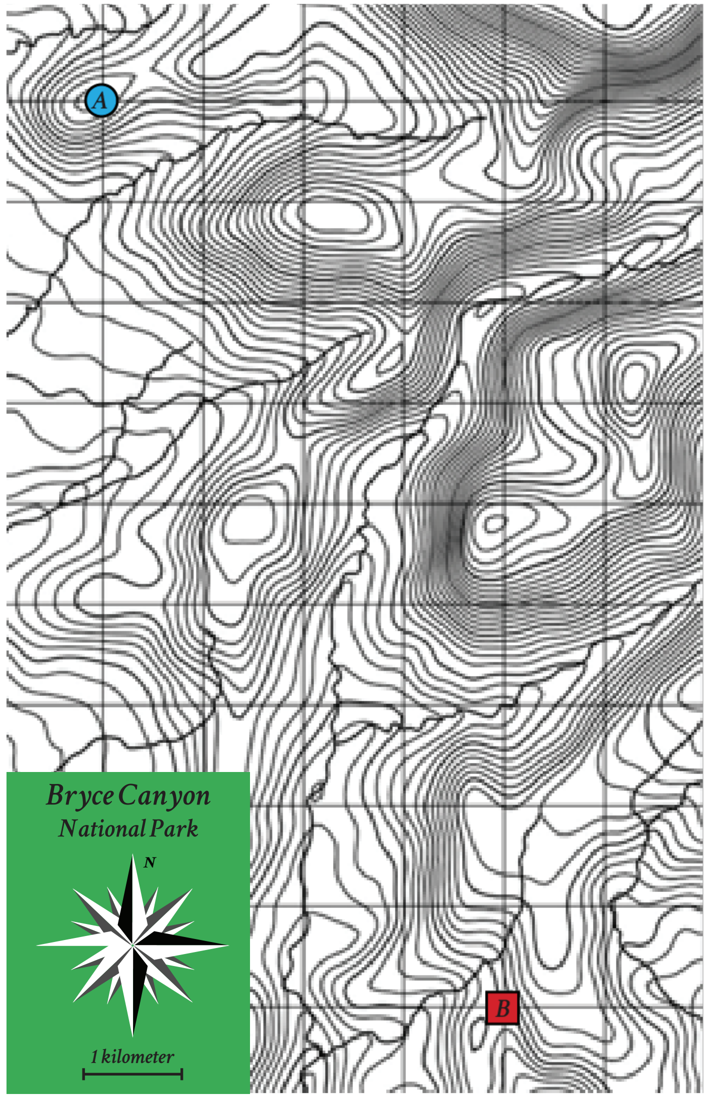
PINBALL WIZARD. GPS PIONEER.

One Sunday, Ben Ashman drove a cherry-red moped from his home in the nation's capital to the Visitor Center at Goddard Space Flight Center. He spoke with a group of children and their parents about an issue that plagues most on a daily basis:

How do we get where we need to go?

Today, mobile devices with continuous access to GPS technology make navigation easy. However, Ben isn't concerned with GPS on Earth; he runs simulations to show that GPS signals could be used in lunar orbit, guiding astronauts around the Moon!

In addition to this work, Ben serves on the navigation team for the Origins Spectral Interpretation Resource Identification Security – Regolith Explorer (OSIRIS-REx), a spacecraft designed to return a sample from Bennu, a distant asteroid. Outside work, Ben plays keys in local bands and enjoys pinball.



In the Bryce Canyon National Park, wildlife biologists track bears using GPS collars. For this problem, imagine a bear's location is tracked using distance measurements from terrestrial navigation beacons rather than GPS satellites. Two beacons are indicated above: beacon A, a blue circle, and B, a red square. The distance between grid lines is 1 kilometer (km).

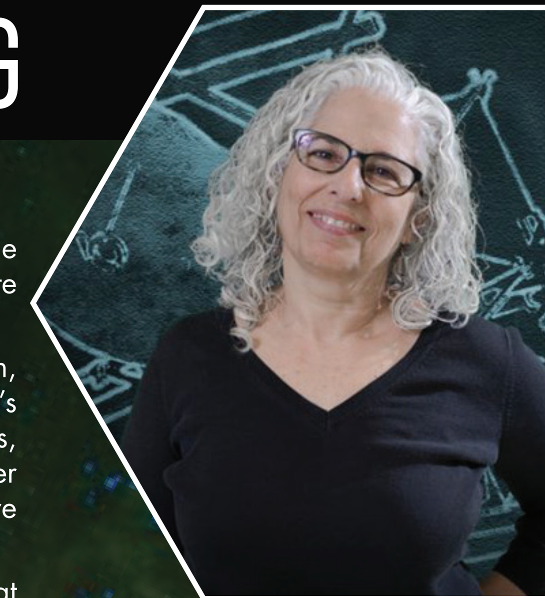
PROBLEMS

a. If the reading from the bear's collar indicates a distance of 5 km from beacon A and 7 km from beacon B, write the equations showing the relationship between these distances and the bear's location, [x, y]. Assume altitude variation is negligible and write the beacon coordinates in km relative to an origin at the bottom left corner. (The math for this is complicated, so just write out the equations —don't try and solve them.)

b. How many possible solutions are there to the equations in part a?

c. Where is the bear? Assume that the bear's location falls solely within the map's presented area. The equations are difficult to solve analytically or numerically, so draw circles on the map to pinpoint the bear's location.

CHERYL GRAMLING



EARTH? MOON? JUPITER? SWING ON BY!

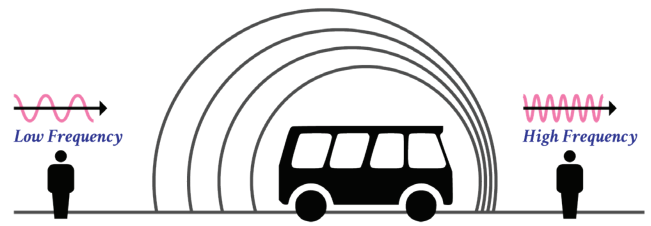
Cheryl Gramling is a practical person, so her innovative spirit comes from the need for practical, economical solutions to problems. This spirit led her to create robust navigation tools and adapt those tools to the needs of new missions.

Cheryl creates complex trajectory designs for missions like the Wind mission, which launched in 1994 and studied plasma in the solar wind and the Earth's magnetosphere. The spacecraft flew a trajectory of double-lunar swingbys, where engineers used the Moon's gravity to change a satellite's orbit. In order to visualize the myriad trajectories of Wind, she developed Swingby, a software tool that helps flight dynamics engineers plan their missions.

Cheryl currently serves as head of the Navigation and Mission Design Branch at Goddard Space Flight Center. A key thrust of Cheryl's career and the branch's work is the development of autonomous navigation, guidance and control systems. These allow spacecraft — especially those at libration points or further out in the solar system — to perform time-critical maneuvers without the time delay involved in receiving commands from Earth.

Have you ever heard a car pass you while blowing its horn? The sound of the horn changes as the car comes closer and moves away from you.

That change in sound comes from a shift in the frequency of the sound wave the horn makes. This Doppler Effect — named after Christian Doppler who described the phenomenon in 1842 — is a change in the frequency and wavelength of a wave, in this case a sound wave. This is caused by the change in distance between the creator of the wave (the car horn) and the observer that hears the wave.



What can you learn about a spacecraft using the Doppler Effect? When a spacecraft passes over a ground station, it communicates with the station using a radio signal at a certain frequency. This is like the sound wave produced by the car horn.

As the spacecraft comes closer to the ground station, the speed of the spacecraft appears to change at the ground station, and the frequency of that signal increases because of the Doppler effect. The spacecraft's speed and direction, together known as velocity, can be computed from the Doppler shift of the signal. You can find the equations for this on page nine.

PROBLEMS

a. A ground station in Hawaii sends a signal to a spacecraft with a frequency of $2\text{x}10^9$ Hertz. After traveling at the speed of light, the signal that the spacecraft receives has been Doppler shifted up by 600 Hz. What is the velocity of the spacecraft?

b. When a spacecraft is almost done sending its climate change data to a ground station in Svalbard, Norway, it has passed over the ground station and is nearing the horizon. The spacecraft transmits to Svalbard at a frequency of $15\ \text{x}10^9$ Hz, and Svalbard receives that signal at $14.9998\ \text{x}10^9$ Hz. What is the spacecraft's velocity as observed by Svalbard?

SIP INTERNS SUMMER 2019. CREDIT: NASA

NAVIGATE TO NASA

NASA is expanding our reach among the stars, establishing a sustained presence at the Moon, journeying to Mars and studying the cosmos. The Artemis missions to the Moon, bold inquiry into the origins of the universe and continued examination of our changing planet require the hard work and expertise of humanity's brightest. NASA looks to you, the Artemis generation, for the innovations and breakthroughs we need.

You've completed this workbook. Presumably, you're a student interested in science, technology, engineering or mathematics.

How do you navigate your way to NASA?

BECOME A CITIZEN SCIENTIST

There are many ways to get involved. You can become a citizen scientist, joining NASA investigations into the world around us and the stars above. This doesn't always require fancy equipment. Many citizen science opportunities can be done by anyone, anywhere, with just a cellphone or laptop.

To learn more about citizen science, visit: ***science.nasa.gov/citizenscience.***

EXPLORE ON YOUR OWN

Interested in communications technologies? Study for your ham radio license! Ham, or amateur, radio hobbyists build and operate their own radios and communicate with other ham operators recreationally.

Did you know that the International Space Station has a ham radio? As the station orbits overhead, you can send messages to it! Amateur Radio on the International Space Station (ARISS) is a program that connects students worldwide with astronauts on the orbiting laboratory over ham frequencies.

To learn more about ARISS, visit: ***ariss.org.***

APPLY FOR AN INTERNSHIP

NASA offers internship opportunities to students from high school through doctoral research. These internships are paid opportunities that give you real world experience in aerospace.

To learn more about internships, visit ***intern.nasa.gov***.

If you're specifically interested in navigation, the Space Communications and Navigation (SCaN) Internship Project (SIP) might be a great option for you. SCaN offers internships to prospective students through Goddard Space Flight Center in Greenbelt, Maryland, Glenn Research Center in Cleveland, Ohio, and NASA Headquarters in Washington, D.C.

To learn more about SIP, visit: ***go.nasa.gov/37PQpD4.***

PURSUE A DEGREE IN STEM

A college degree in science, technology, engineering or mathematics is a great way to advance your understanding of key aerospace disciplines. A STEM degree can prepare you to work with NASA or anywhere else your interest in aerospace takes you!

CONGRATULATIONS

You've completed the workbook!

For more information about the work of NASA's Space Communications and Navigation (SCaN) program office, visit:

nasa.gov/SCaN

For information the role Goddard Space Flight Center plays in communications and navigation, visit:

esc.gsfc.nasa.gov

Check your answers to the practice problems at **https://go.nasa.gov/30ke8KB**

EXPLORATION AND SPACE COMMUNICATIONS PROJECTS DIVISION
Goddard Space Flight Center

SPACE COMMUNICATIONS AND NAVIGATION (SCaN) PROGRAM OFFICE
National Aeronautics and Space Administration

National Aeronautics and Space Administration
Goddard Space Flight Center
8800 Greenbelt Road
Greenbelt, MD 20771
esc.gsfc.nasa.gov

www.nasa.gov

NP-2020-9-586-GSFC