NAVIGATE SPACE

A SPACE COMMUNICATIONS AND NAVIGATION WORKBOOK

PRACTICE PROBLEM ANSWER KEY

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NASA's Goddard Space Flight Center

Space Communications and Navigation (SCaN) program office
National Aeronautics and Space Administration

www.nasa.gov

Orbit Determination

Practice Problem (pg. 13)

We calculate the velocity necessary to maintain orbits 2,000,000,000 meters away from each of the planets using this equation:

$$\left(\frac{Gm_1}{r}\right)^{\frac{1}{2}} = v$$

The variable G is the gravitational constant below:

$$G = 6.67 \times 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$$

MERCURY Mass: 0.330×10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 0.330 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 104.907\ \mathrm{m/s}$$

VENUS Mass: 4.87×10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 4.87 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 403.006\ \mathrm{m/s}$$

EARTH Mass: 5.97×10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 446.205 \text{ m/s}$$

MARS Mass: 0.642 x 10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 0.642 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 146.324 \text{ m/s}$$

JUPITER Mass: 1898 x 10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 1898 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 7956.023 \text{ m/s}$$

SATURN Mass: 568 x 10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 568 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 4352.333 \text{ m/s}$$

URANUS Mass: 86.8 x 10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 86.8 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 1701.405 \text{ m/s}$$

NEPTUNE Mass: 102 x 10^{24} kg

$$\left(\frac{6.67 \times 10^{-11} \times 102 \times 10^{24}}{2{,}000{,}000{,}000}\right)^{\frac{1}{2}} = v$$

$$v = 1844.370 \text{ m/s}$$

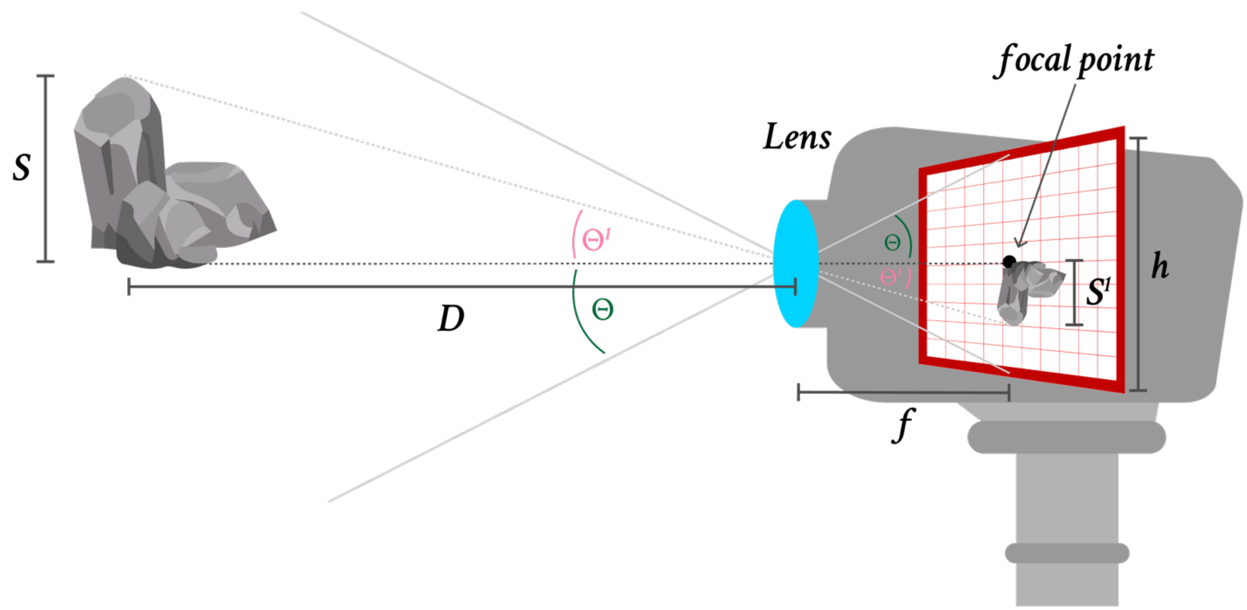
Joshua Lyzhoft

Practice Problem (pg. 17)

Imagine that NASA launches a rover to the Moon to prospect for resources like water. Missions like this are becoming more common as NASA expands human and robotic presence on the lunar surface with the Artemis missions.

The rover needs to navigate from its original location to a deep crater. Scientists believe that water ice might exists in the extremely cold, permanently shadowed regions at the bottoms of craters like the one our rover is exploring.

A large boulder in our rover's field of view marks the spot right at the edge of the crater where our rover must park. Mission controllers on Earth need to command the rover to move the exact distance to place it on the spot. Move too far and the rover may fall into the crater. Move too little and scientists will lose valuable science data.



We know the focal length of the rover's camera is .5 m. Based on observations from previous missions, we know the boulder is 10 m tall. The boulder appears to be .01 m in the sensor array. Each pixel is 1x10-5 m tall.

a) *What is the distance from the boulder to the rover's lens in meters?*

To determine the distance from the boulder to the rover's lens, we can use the tangent function, just as we learned on page 16 of the workbook. Since the angle (θ) is on both sides of the lens, we know that we can form two similar triangles, one for the left of the lens and one for the right of the lens.

$$\tan(\theta') = \frac{S'}{f}$$

$$\tan(\theta') = \frac{S}{D}$$

By substituting $\tan(\theta')$ is in both equations, we get:

$$\frac{S}{D} = \frac{S'}{f}$$

Replacing them with the values from the problem,

$$\frac{10m}{D} = \frac{.01\ m}{.5\ m}$$

To solve for D, we cross-multiply:

$$D = (.5m/.01m) \times 10m = 50 \times 10m = 500m$$

The rover is 500 meters away from the boulder.

b) *If the rover drives at a speed of .08 m/s, how long should mission control instruct the rover to move in order to reach the boulder?*

Since we have the distance in meters, we can use d=vt to calculate how much time our rover needs to drive:

d = 500 meters
v = .08 meters per second

$$500 = .08t$$

t = 6,250 seconds

So, mission control must instruct the rover to drive for 6,250 seconds, or just over 104 minutes.

c) ***Given that the height of the sensor array (h) is 2000 pixels, what is the field of view (Θ) in degrees?***

Given that we have the pixel size of .1 m and the number of pixels to be 2000, we can find the overall height of the sensor array. This is simply the pixel size times the number of pixels:

$$S'' = \frac{1x10^{-5}\ m}{pix} \times 2000pix \times \frac{.01\ m}{1\ x\ 10^{-2}m} = .02\ m$$

Then, using the tangent function with a right triangle:

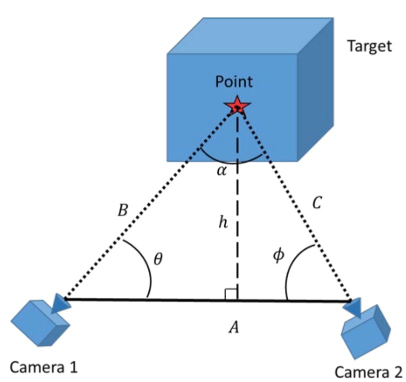
$$\tan(\theta) = \frac{S''}{f} = \frac{.02\ m}{.5\ m} = 0.04$$

Thereafter, we can use an invert tangent function to convert that number to degrees:

$$\theta = \arctan(0.04) \times \left(\frac{180deg}{\pi rad}\right) = 2.29061004\ degrees$$

d) ***If we do not have the size of the object, how might an individual determine the distance? (Hint: There might need to be more than one camera.)***

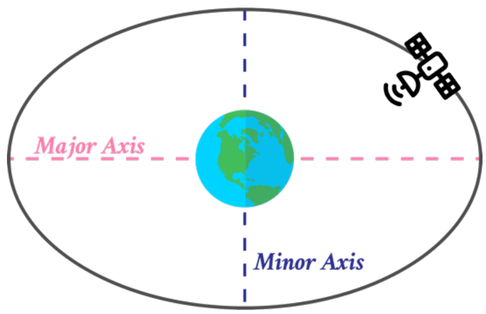
This is a complicated scenario that requires thinking in three dimensions. If an individual had two cameras, they could pin-point the same spot on the target and measure where it is in both cameras' sensors. To finish calculating the distance, they must also have the location of each camera with respect to each other. Then, some simple geometry can give you an estimate of the distance to the body (see image below).



Jenny Donaldson

Practice Problem (pg. 18)

Orbital mechanics is the study of the motion of spacecraft moving under the influence of forces such as gravity. Most orbits around a large central gravitational body, such as Earth, are elliptical, which means they follow a repeating path in the shape of an ellipse, as seen in the diagram below. A special case of the ellipse, when the major and minor axes are the same, is called a circle.



In the 17th century, Johannes Kepler described elliptical orbits when studying the patterned motion of planets orbiting the Sun. Kepler's third law of planetary motion states that the square of the orbital period of a satellite is directly proportional to the cube of the semi-major axis of its orbit. In other words, how long it takes to complete one revolution of an orbit (period, T), is directly related to size of the orbit (semi-major axis, a) and can be calculated with the following formula:

$$T = 2\pi\sqrt{\frac{a^3}{Gm}}$$

G is the universal gravitational constant, and m is the mass of the object the satellite is orbiting.

Earth's elliptical orbit around the Sun has a semi-major axis of 1.496×10^{11} meters and the mass of the Sun is 1.989×10^{30} kilograms.

a) Calculate the period of Earth's orbit around the Sun in seconds.

Plug the numbers into the equation supplied, using 3.14159 as the value for pi (*л*).

$$T = 2\pi\sqrt{\frac{a^3}{Gm}}$$

$$T = 2 \times 3.14159 \times \sqrt{\frac{(1.496 \times 10^{11})^3}{(6.67 \times 10^{-11} \times 1.989 \times 10^{30}}}$$

$$T = 6.28318 \times \sqrt{\frac{3.348071963 \times 10^{33}}{13.26663 \times 10^{19}}}$$

$$T = 6.28318 \times \sqrt{0.25236792885 \times 10^{14}}$$

$$T = 6.28318 \times 5023623.48167$$

$$T = 31564330.5876$$

Rounding that and converting that to exponential notation:

T = 3.156433 x 10^7 seconds

The period of Earth's orbit is 3.156433 x 10^7 seconds.

b) ***Calculate the period of Earth's orbit around the Sun in days.***

First, we must calculate how many seconds there are in a day.

There are 24 hours in a day.

There are 60 minutes in each hour.

There are 60 seconds in each minute.

Therefore: 24 x 60 x 60 = 86,400 seconds per day.

Dividing that by the number of seconds we calculated in the previous question, we get the correct answer:

$$\frac{3.156433 \times 10^7}{86,400} = 365.327893519 \text{ days}$$

So, there are just over 365 days in a year! The small amount of time over the 365 is the reason there is a leap day every four years.

The orbit of the International Space Station around Earth has a semi-major axis of 6.7981 x 10^6 meters. Earth's mass is 5.97 x 10^{24} kg.

c) ***Calculate the orbital period of the space station in minutes.***

Similarly to the first example, we plug the values into the equation provided.

$$T = 2 \times 3.14159 \times \sqrt{\frac{(6.7981 \times 10^6)^3}{6.67 \times 10^{-11} \times 5.97 \times 10^{24}}}$$

$$T = 6.28318 \times \sqrt{\frac{314.168505637 \times 10^{18}}{39.8199 \times 10^{13}}}$$

$$T = 6.28318 \times \sqrt{7.88973617807 \times 10^5}$$

$$T = 6.28318 \times 888.241868979$$

T = 5580.98354633 seconds

Since there are 60 seconds per minute, we divide that number by 60 to calculate the orbital period in minutes.

$$T = \frac{5580.98354633}{60}$$

T = 93.0163924389 minutes

Rounding to the thousandths:

T = 93.0164 minutes

So, the space station orbits the Earth once every 93 minutes. This means that astronauts experience a sunrise every hour and a half!

d) How many times does the space station orbit Earth in one day?

First, we need to know the number of minutes in a day.

24 hours times 60 minutes per hour is 1440 minutes per day.

Then, we divide that by the orbital period we calculated in the previous question:

$$\frac{1440}{93.0164} = 15.4811409601$$

Rounding to the thousandths, we get:

15.4811 orbits per day

So, the space station completes 15 orbits around the Earth per day. That's 15 sunrises!

Joel Parker

Practice Problem (pg. 19)

How do we know if a navigation signal is strong enough to be received by our spacecraft?

Link budget calculations use known features of a signal—like transmit power and the distance to the receiver—to determine unknown features, such as how large an antenna is needed to receive a signal. The largest source of loss in signal strength is path loss, which happens as signals travel vast distances, becoming weaker.

To perform a link budget calculation, we use units of decibels (dB), which express common numbers in exponential form. This is useful for expressing very large or very small numbers. To convert watts (P_W) to decibel-watts (P_{dBW}), use a logarithm, like so:

$$P_{dBW} = 10\log_{10}\left(\frac{P_w}{1\ \text{W}}\right)$$

Given the .001 watts, we get:

$$P_{dBW} = 10\log_{10}\left(\frac{0.001\ \text{W}}{1\ \text{W}}\right)$$

Here we express the quantity 0.001W in terms of decibels relative to 1 Watt. The benefit of working in decibels is that complex equations can be expressed using simple additions and subtractions. The equations for the signal power at the receiver (P_R) and the path loss (A_d) — found on page nine of this workbook — take advantage of this.

a) ***GPS satellites orbit about 20,200 km above Earth. The International Space Station flies at about 400 km. What is the path loss in dBW as the signal travels between the two, assuming the GPS signal has a frequency of 1575.42 MHz?***

   First, we need the equation for free space path loss, (A_d), found on page nine of the workbook:

$$A_d = 10\log_{10}\left[\left(\frac{4лdf}{c}\right)^2\right]$$

We’ve been supplied the frequency (f) so we need to gather the distance (d) by subtracting the altitude of the space station from the altitude of the GPS satellite:

$$22{,}200 - 400 = 19800 \text{ km}$$

Converting to meters, we have: 19,800,000 meters. We plug that in, along with the frequency (converted from 1575.42 MHz to 1,575,420,000 Hz) and the values 3.14159 for pi (*л*) and 300,000,000 for the speed of light (c) into the equation:

$$A_d = 10\log_{10}\left[\left(\frac{4 \times 3.14159 \times 19{,}800{,}000 \times 1{,}575{,}420{,}000}{300{,}000{,}000}\right)^2\right]$$

A_d = 182.322995746 dBW

So, the signal lost about 182 dBW of strength travelling to the space station.

b) *What is this path loss in Watts? (Hint: Reverse the decibel equation.)*

Because we know the path loss in dBW, we can write the decibel equation as follows:

$$182.322995746 = 10\log_{10}\left(\frac{P_W}{1}\right)$$
$$18.2322995746 = \log_{10}\left(\frac{P_W}{1}\right)$$

Reversing the logarithm, we can write this as:

$10^{18.2322995746} = P_w$

$P_w = 1.7072596 \times 10^{18}$ Watts

So, the GPS signal lost about 1.71×10^{18} Watts of strength travelling to the space station.

c) *GPS signals are transmitted at about 13 dBW, using an antenna with a gain of about 15 dBW. Assuming the space station has a receive antenna with a gain of 10 dBW, what is the signal power it receives?*

We know the link budget equation from page 9 of the workbook, which we can use to solve this problem:

$$P_R = P_T + G_T - A_d + G_R$$

Plugging in the values supplied and our answer from the first part of the question, we see that:

$$P_R = 13 + 15 - 182.32 + 10$$

$$P_R = -144.32 \text{ dBW}$$

So, the power received by the space station is -144.32 dBW.

d) Now consider that receiver and antenna are mounted on a spacecraft going to the Moon. The Moon is about 400,000 km from the center of the Earth, which has a radius of 6,378 km. What is the path loss in dBW?

Looking back at page 11 of the workbook, we remember that a spacecraft headed to the Moon would receive GPS signals from a GPS spacecraft on the opposite side of the Earth. So, to get the total distance, we need to add the altitude of the GPS spacecraft to the radius of Earth and the distance from the center of Earth. (Note that usually these signals would be received at an angle off of Earth, so these calculations would ordinarily involve a bit of geometry, but we've simplified here.)

$$d = 400{,}000 + 6{,}378 + 20{,}200$$

$$d = 426{,}578 \text{ km}$$

Converted into meters, that's 426,578,000 m. We can plug that into our equation for free space path loss:

$$A_d = 10 \log_{10} \left[\left(\frac{4\pi d f}{c} \right)^2 \right]$$

$$A_d = 10 \log_{10} \left[\left(\frac{4 \times 3.14159 \times 426{,}578{,}000 \times 1{,}575{,}420{,}000}{300{,}000{,}000} \right)^2 \right]$$

$$A_d = 208.989661015 \text{ dBW}$$

So, the signal lost about 209 dBW of strength travelling to the Moon.

e) What is the signal power received by this lunar spacecraft in dBW?

Returning to the link budget equation:

$$P_R = P_T + G_T - A_d + G_R$$

$$P_R = 13 + 15 - 208.989661015 + 10$$

$$P_R = -170.989661015 \text{ dBW}$$

So, the signal power received by our lunar GPS receiver is about -171 dBW.

f) What is this received signal power in Watts?

Using the logarithmic equation to turn dBW to W:

$$P_{dBW} = 10 \log_{10}\left(\frac{P_w}{1 \text{ W}}\right)$$

$$-170.989661015 = 10 \log_{10}\left(\frac{P_w}{1 \text{ W}}\right)$$

$$-17.0989661015 = 10 \log_{10}\left(\frac{P_w}{1 \text{ W}}\right)$$

$$10^{-17.0989661015} = P_w$$

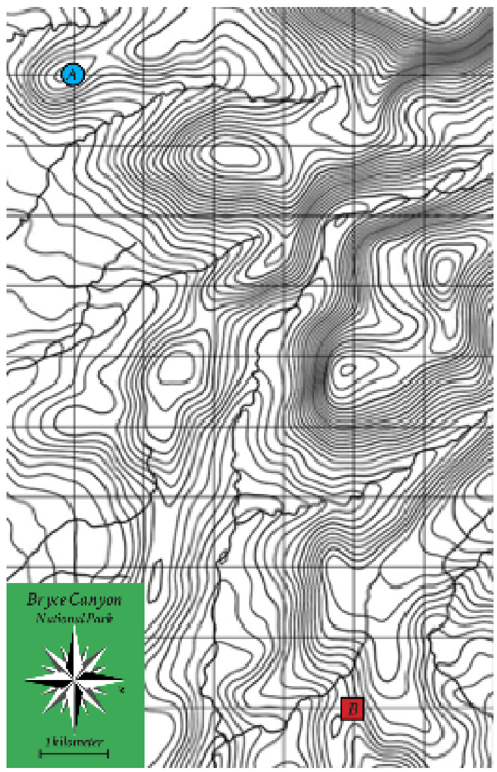
$$P_w = 7.962215 \times 10^{-18} \text{ W}$$

The received signal power is about 8 x 10^{-18} Watts.

Ben Ashman

Practice Problem (pg. 20)

In the Bryce Canyon National Park, wildlife biologists track bears using GPS collars. For this problem, imagine a bear's location is tracked using distance measurements from terrestrial navigation beacons rather than GPS satellites. Two beacons are indicated above: beacon A, a blue circle, and B, a red square. The distance between grid lines is 1 kilometer (km).



a) ***If the reading from the bear's collar indicates a distance of 5 km from beacon A and 7 km from beacon B, write the equations showing the relationship between these distances and the bear's location, [x, y]. Assume altitude variation is negligible and write the beacon coordinates in km relative to an origin at the bottom left corner. (The math for this is complicated, so just write out the equations —don't try and solve them.)***

   Looking back at page 7 of the workbook, we know we would write the equations as:

$$5 = \sqrt{(1-x)^2 + (1-y)^2}$$

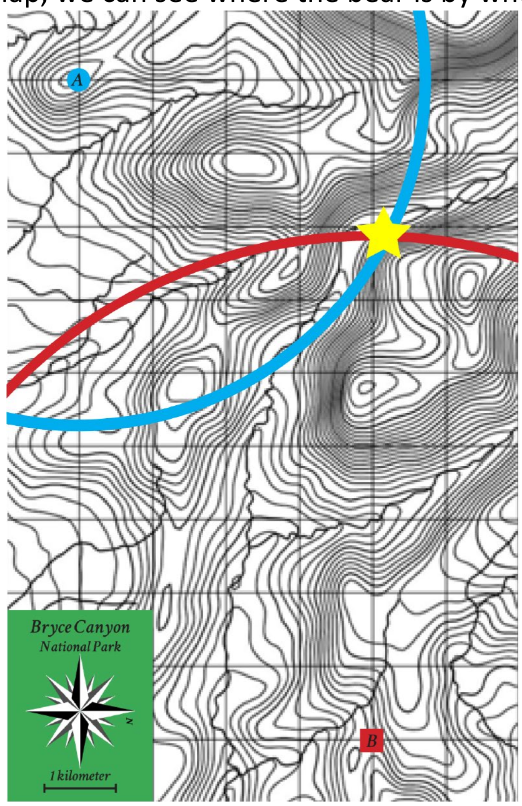
$$7 = \sqrt{(10-x)^2 + (5-y)^2}$$

b) How many possible solutions are there to the equations in part a?

Because we're working in two dimensions and there are two variables, we know there are two possible solutions to these equations. Put another way, circles drawn around A and B with 5 km and 7 km radii, respectively, intersect at two points.

c) Where is the bear? Assume that the bear's location falls solely within the map's presented area. The equations are difficult to solve analytically or numerically, so draw circles on the map to pinpoint the bear's location.

Drawing circles on the map, we can see where the bear is by where the lines intersect:



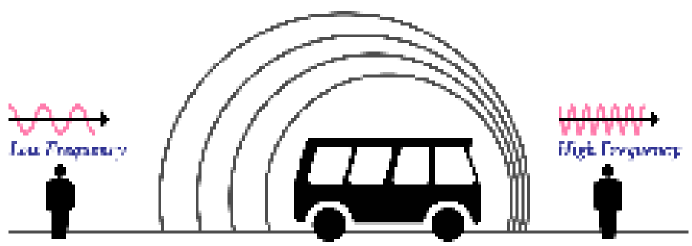
$\begin{bmatrix} x \\ y \end{bmatrix} \approx \begin{bmatrix} 5 \\ 8 \end{bmatrix}$ or at 5, 8 on the grid.

Cheryl Gramling

Practice Problem (pg. 21)

Have you ever heard a car pass you while blowing its horn? The sound of the horn changes as the car comes closer and moves away from you.

That change in sound comes from a shift in the frequency of the sound wave the horn makes. This Doppler Effect — named after Christian Doppler who described the phenomenon in 1842 — is a change in the frequency and wavelength of a wave, in this case a sound wave. This is caused by the change in distance between the creator of the wave (the car horn) and the observer that hears the wave.



What can you learn about a spacecraft using the Doppler Effect? When a spacecraft passes over a ground station, it communicates with the station using a radio signal at a certain frequency. This is like the sound wave produced by the car horn.

As the spacecraft comes closer to the ground station, the speed of the spacecraft appears to change at the ground station, and the frequency of that signal increases because of the Doppler effect. The spacecraft's speed and direction, together known as velocity, can be computed from the Doppler shift of the signal. You can find the equations for this on page nine.

a) ***A ground station in Hawaii sends a signal to a spacecraft with a frequency of 2x10^9 Hertz. After traveling at the speed of light, the signal that the spacecraft receives has been Doppler shifted up by 600 Hz. What is the observed change in velocity of the spacecraft?***

Looking at the doppler shift equations on page nine, we find that:

$$f_{obs} = f_o\left(\frac{c}{c-v}\right)$$

The observed frequency is the initial frequency plus 600 Hz and we have a value for the speed of light (c) so:

$$2\times10^9 + 600 = (2\times10^9)\left(\frac{300{,}000{,}000}{300{,}000{,}000 - v}\right)$$

$$2 \times 10^9 + 600 = (2 \times 10^9)\left(\frac{300{,}000{,}000}{300{,}000{,}000 - v}\right)$$

$$1.0000003 = \left(\frac{300{,}000{,}000}{300{,}000{,}000 - v}\right)$$

$$1.0000003(300{,}000{,}000 - v) = 300{,}000{,}000$$

$$300{,}000{,}090 - 1.0000003v = 300{,}000{,}000$$

$$-1.0000003v = -90$$

$$v = 89.999973 \text{ m/s}$$

The spacecraft is travelling at about 90 m/s.

b) *When a spacecraft is almost done sending its climate change data to a ground station in Svalbard, Norway, it has passed over the ground station and is nearing the horizon. The spacecraft transmits to Svalbard at a frequency of 15x10^9 Hz, and Svalbard receives that signal at 14.9998x10^9 Hz. What is the spacecraft's velocity as observed by Svalbard?*

Looking at the doppler shift equations on page nine, we find that:

$$f_{obs} = f_o\left(\frac{c}{c - v}\right)$$

Plugging in our variables:

$$14.9998 \times 10^9 = (15 \times 10^9)\left(\frac{300{,}000{,}000}{300{,}000{,}000 - v}\right)$$

$$0.99998666666 = \left(\frac{300{,}000{,}000}{300{,}000{,}000 - v}\right)$$

$$0.99998666666 \times (300{,}000{,}000 - v) = 300{,}000{,}000$$

$$299995999.998 - 0.99998666666v = 300{,}000{,}000$$

$$-0.99998666666v = 4000.00199997$$

$$v = -4000.05533407 \text{ m/s}$$

The spacecraft is travelling away from the ground station at about 4,000 m/s.