National Aeronautics and Space Administration

INSTRUCTOR'S GUIDE

NASA Space Communications and Navigation

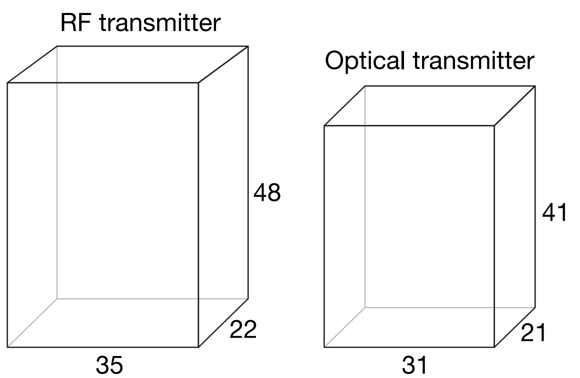
Geometry of Circles

GRADES 9–10

Answer Key

1. Sketch a diagram of each box-shaped payload.

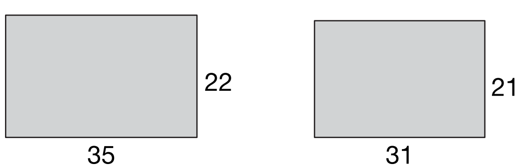
Using box-shaped payloads might not fill the volume efficiently, but the shape has been picked for simplicity of the exercise.



2. When placing each payload inside the rocket fairing, which face should be oriented toward the base of the rocket so that it fits in the smallest space?

Select the smallest face (just pick the two smallest dimensions from among the supplied length/width/heights).

RF: **22** < **35** < 48   Optical: **21** < **31** < 41



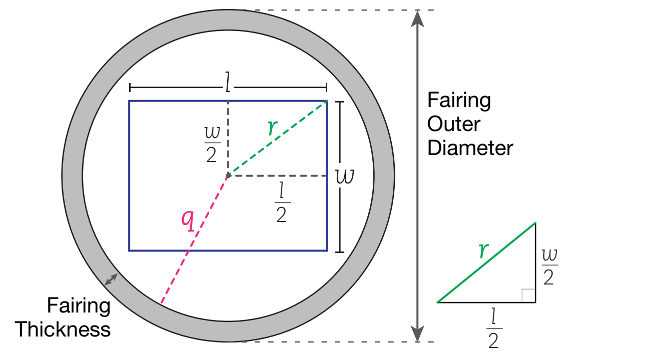
To see an example of how NASA folds spacecraft to fit inside rocket fairings, animations of the Tracking and Data Relay Satellite, known as TDRS-K, can be seen here: ***http://go.nasa.gov/TDRS-K_videos.*** For an animation of the James Webb Space Telescope (JWST) deployment, visit: ***http://go.nasa.gov/JWST_deployment***

Additional JWST educational activities and resources can be found at ***http://go.nasa.gov/JWST_educators_formal*** and ***http://go.nasa.gov/JWST_activities.***

3. What is the smallest fairing each of our payloads can use?

a. Assuming that we center the payload in the fairing, determine the largest radius of the payload's footprint.

The largest radius of the payload is the hypotenuse *r*.



$$r^2 = \left(\frac{w}{2}\right)^2 + \left(\frac{l}{2}\right)^2$$

$$r = \sqrt{\left(\frac{w}{2}\right)^2 + \left(\frac{l}{2}\right)^2}$$

$$q = \frac{\left(\text{Fairing Outer Diameter}\right) - 2 \times \left(\text{Fairing Thickness}\right)}{2}$$

Evaluate: Is $r < q$?

Optical Transmitter:

$$r_{Opt} = \sqrt{\left(\frac{21\text{ cm}}{2}\right)^2 + \left(\frac{31\text{ cm}}{2}\right)^2}$$

$$r_{Opt} = \sqrt{(10.5\text{ cm})^2 + (15.5\text{ cm})^2}$$

$$r_{Opt} = \sqrt{110.25\text{ cm}^2 + 240.25\text{ cm}^2}$$

$$r_{Opt} = \sqrt{350.5\text{ cm}^2}$$

$$r_{Opt} = \mathbf{18.7\text{ cm}}$$

RF Transmitter:

$$r_{RF} = \sqrt{\left(\frac{22\text{ cm}}{2}\right)^2 + \left(\frac{35\text{ cm}}{2}\right)^2}$$

$$r_{RF} = \sqrt{(11\text{ cm})^2 + (17.5\text{ cm})^2}$$

$$r_{RF} = \sqrt{121\text{ cm}^2 + 306.25\text{ cm}^2}$$

$$r_{RF} = \sqrt{427.25\text{ cm}^2}$$

$$r_{RF} = \mathbf{20.7\text{ cm}}$$

(continued)

b. Determine the largest radius that would fit inside each of our four rocket fairing options.

$$\text{Max. diameter } (p) = \text{Fairing outer diameter} - \left(2 \times \text{Fairing thickness}\right)$$

$p_A = 0.26\text{ m} - 2(0.02\text{ m}) = 0.22\text{ m}$

$p_B = 0.44\text{ m} - 2(0.03\text{ m}) = 0.38\text{ m}$

$p_C = 0.50\text{ m} - 2(0.04\text{ m}) = 0.42\text{ m}$

$p_D = 0.76\text{ m} - 2(0.08\text{ m}) = 0.60\text{ m}$

$\text{Max. radius } (q) = \frac{p}{2}$

$q_A = 0.11\text{ m}$ $q_C = 0.21\text{ m}$

$q_B = 0.19\text{ m}$ $q_D = 0.30\text{ m}$

c. Convert your payload radii and rocket fairing radii to common units.

We're about ready to make the crucial comparisons here, but our payload radii and our fairing radii are in different units (m and cm). Get both measurements in the same unit. We'll use cm here, but either is valid:

$q_A = 0.11\text{ m} \times \left(\frac{100\text{ cm}}{\text{m}}\right) = 11.0\text{ cm}$ $q_C = 21.0\text{ cm}$

$q_B = 19.0\text{ cm}$ $q_D = 30.0\text{ cm}$

d. Compare the rocket fairing radii to the payload radii. Which payloads can fit in which rockets? What's the least expensive (smallest) rocket we can use for each of our two payloads?

Which fairings have a largest possible radius (q) that's larger than the largest radius (r) of the payload?

<u>Optical Transmitter:</u>

	q vs. r	
Rocket A:	Is 11.0 cm ≥ 18.7 cm?	No. It doesn't fit.
Rocket B:	Is 19.0 cm ≥ 18.7 cm?	Yes. It fits.
Rocket C:	Is 21.0 cm ≥ 18.7 cm?	Yes. It fits.
Rocket D:	Is 30.0 cm ≥ 18.7 cm?	Yes. It fits.

<u>RF Transmitter:</u>

	q vs. r	
Rocket A:	Is 11.0 cm ≥ 20.7 cm?	No. It doesn't fit.
Rocket B:	Is 19.0 cm ≥ 20.7 cm?	No. It doesn't fit.
Rocket C:	Is 21.0 cm ≥ 20.7 cm?	Yes. It fits.
Rocket D:	Is 30.0 cm ≥ 20.7 cm?	Yes. It fits.

While laser communications can provide more information at once to scientists, clouds can disrupt laser signals as they enter Earth's atmosphere. LCRD sends its information to two ground stations located on Table Mountain in California, and on Haleakalā Volcano on the island of Maui, Hawaii. These remote and high-altitude locations were chosen for their clear weather conditions.

NOTE: The rocket dimensions selected for this exercise are based on real sounding rockets used by NASA. Sounding rockets are smaller vehicles and make suborbital flights: they do not have the power to make it into orbit. These rockets fall back to Earth on parachutes after their fuel is spent. Learn more about sounding rockets at *http://go.nasa.gov/WFF_soundingrockets*.

The payload and fairing volumes for this exercise are loosely based on real transmitters but were otherwise selected for mathematical convenience.