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**REPORT 1135**

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**EQUATIONS, TABLES, AND CHARTS FOR  
COMPRESSIBLE FLOW**

**By AMES RESEARCH STAFF**

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## EQUATIONS, TABLES, AND CHARTS FOR COMPRESSIBLE FLOW <sup>1</sup>

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### SUMMARY

This report, which is a revision and extension of NACA TN 1428, presents a compilation of equations, tables, and charts useful in the analysis of high-speed flow of a compressible fluid. The equations provide relations for continuous one-dimensional flow, normal and oblique shock waves, and Prandtl-Meyer expansions for both perfect and imperfect gases. The tables present useful dimensionless ratios for continuous one-dimensional flow and for normal shock waves as functions of Mach number for air considered as a perfect gas. One series of charts presents the characteristics of the flow of air (considered a perfect gas) for oblique shock waves and for cones in a supersonic air stream. A second series shows the effects of caloric imperfections on continuous one-dimensional flow and on the flow through normal and oblique shock waves.

### INTRODUCTION

The practical analysis of compressible flow involves frequent application of a few basic results. A convenient compilation of equations, tables, and charts embodying these results is therefore of great assistance in both research and design. The present report makes one of the first such compilations (ref. 1) more readily available in a revised and extended form. The revisions include a complete rewriting of the lists of equations, as well as the correction of certain typographical errors which appeared in the earlier work. The extensions are primarily in the directions dictated by increasing flight speeds, that is, to higher Mach numbers and to higher temperatures with the accompanying gaseous imperfections.

Compilations similar to those of reference 1 have been given in other publications, as, for example, references 2 through 6. These references have been utilized in extending the tables and charts to higher values of the Mach number. The extension to imperfect gases is based on the relations presented in references 7 and 8.

### SYMBOLS AND NOTATION

#### PRIMARY SYMBOLS

$a$  speed of sound  
 $A$  cross-sectional area of stream tube or channel

$C_N$  normal-force coefficient for cones,  $\frac{\text{normal force}}{q_\infty S_b}$   
 $c_p$  specific heat at constant pressure  
 $c_v$  specific heat at constant volume  
 $h$  enthalpy per unit mass,  $u + pv$   
 $l$  characteristic reference length  
 $M$  Mach number,  $\frac{V}{a}$   
 $p$  pressure <sup>2</sup>  
 $q$  dynamic pressure,  $\rho V^2/2$   
 $q$  heat added per unit mass  
 $R$  gas constant  
 $R$  Reynolds number,  $\frac{\rho V l}{\mu}$   
 $S_b$  base area of cone  
 $s$  entropy per unit mass  
 $T$  absolute temperature <sup>2</sup>  
 $u$  internal energy per unit mass  
 $v$  specific volume,  $\frac{1}{\rho}$   
 $u, v$  velocity components parallel and perpendicular respectively, to free-stream flow direction  
 $\tilde{u}, \tilde{v}$  velocity components normal and tangential, respectively, to oblique shock wave  
 $V$  speed of flow  
 $V_m$  maximum speed obtainable by expanding to zero absolute temperature  
 $w$  external work performed per unit mass  
 $\alpha$  angle of attack  
 $\beta$   $\sqrt{|M^2 - 1|}$   
 $\gamma$  ratio of specific heats,  $\frac{c_p}{c_v}$   
 $\delta$  angle of flow deflection across an oblique shock wave  
 $\theta$  shock-wave angle measured from upstream flow direction  
 $\Theta$  molecular vibrational-energy constant  
 $\mu$  Mach angle,  $\sin^{-1} \frac{1}{M}$   
 $\mu$  absolute viscosity  
 $\nu$  Prandtl-Meyer angle (angle through which a supersonic stream is turned to expand from  $M=1$  to  $M>1$ )

<sup>1</sup> Supersedes NACA TN 1428, "Notes and Tables for Use in the Analysis of Supersonic Flow" by the Staff of the Ames 1- by 3-foot Supersonic Wind-Tunnel Section, 1947.  
<sup>2</sup> When used without subscripts,  $p$ ,  $\rho$ , and  $T$  denote static pressure, static density, and static temperature, respectively.

$\xi$	pressure ratio across a shock wave, $\frac{p_2}{p_1}$
$\rho$	mass density <sup>2</sup>
$\sigma$	semivertex angle of cone

## SUBSCRIPTS

$\infty$	free-stream conditions
1	conditions just upstream of a shock wave
2	conditions just downstream of a shock wave
$t$	total conditions (i. e., conditions that would exist if the gas were brought to rest isentropically)
*	critical conditions (i. e., conditions where the local speed is equal to the local speed of sound)
$c$	conditions on the surface of a cone
$r$	reference (or datum) values
perf	quantity evaluated for a gas which is both thermally and calorically perfect
therm perf	quantity evaluated for a gas which is thermally perfect but calorically imperfect
$( )_p$	derivative evaluated at constant pressure
$( )_s$	derivative evaluated at constant entropy
$( )_T$	derivative evaluated at constant temperature
$( )_v$	derivative evaluated at constant specific volume
$( )_{rev}$	quantity evaluated over a reversible path

## NOTATION

The notation in brackets [ ] after many of the equations signifies that the equation is valid only within certain limitations. For example:

[perf]	means that the equation is restricted to a gas which is both thermally and calorically perfect. (By "thermally perfect" it is meant that the gas obeys the thermal equation of state $p = \rho RT$ . By "calorically perfect" it is meant that the specific heats $c_p$ and $c_v$ are constant.)
[therm perf]	means that the only restriction on the gas is that it must be thermally perfect. Equations so marked may be used for calorically imperfect gases. (They are, of course, also valid for completely perfect gases.)
[isen]	means that the flow process must take place isentropically. Equations so marked may not be applied to the flow across a shock wave.
[adiab]	means that the only restriction on the flow process is that it must take place adiabatically—that is, without heat transfer. (Such a flow process may or may not be isentropic depending on whether it is or is not reversible.) Equations so marked may be applied to the flow across a shock wave.

An equation without notation has no restrictions beyond those basic to the study of thermodynamics and/or inviscid compressible flow.

## FUNDAMENTAL RELATIONS

## THERMODYNAMICS

## THERMAL EQUATIONS OF STATE

A thermal equation of state is an equation of the form

$$p = p(v, T) \quad (1)$$

Several of the more commonly used thermal equations of state are the following:

Equation for thermally perfect gas

$$p = \frac{RT}{v} = \rho RT \text{ [therm perf]} \quad (2)$$

or

$$\frac{dp}{p} - \frac{d\rho}{\rho} - \frac{dT}{T} = 0 \text{ [therm perf]} \quad (3)$$

Equations for thermally imperfect gas

Van der Waals' equation (ref. 9)

$$p = \frac{RT}{v-b} - \frac{a}{v^2} \quad (4)$$

where  $a$  is the intermolecular-force constant and  $b$  is the molecular-size constant (see ref. 9, pp. 390 et seq. for numerical values).

Berthelot's equation (ref. 7)

$$p = \frac{RT}{v-b} - \frac{c}{v^2T} \quad (5)$$

where  $b$  is the molecular-size constant and  $c$  is the intermolecular-force constant (see ref. 7 for numerical values).

Beattie-Bridgeman equation (ref. 10)

$$p = \frac{RT}{v^2} \left( 1 - \frac{c}{vT^3} \right) \left[ v + B_0 \left( 1 - \frac{b}{v} \right) \right] - \frac{A_0}{v^2} \left( 1 - \frac{a}{v} \right) \quad (6)$$

where  $a$ ,  $A_0$ ,  $b$ ,  $B_0$ , and  $c$  are constants for a given gas (see ref. 10, p. 270 for numerical values).

## CALORIC EQUATION OF STATE

A caloric equation of state is an equation of the form

$$u = u(v, T) \quad (7)$$

It can be shown that

$$du = c_v dT + \left[ T \left( \frac{\partial p}{\partial T} \right)_v - p \right] dv \quad (8a)$$

$$du = c_v dT \text{ [therm perf]} \quad (8b)$$

If the gas is calorically perfect—that is, the specific heats are constant—equation (8b) can be integrated to obtain

$$u = c_v T + u_r \text{ [perf]} \quad (9)$$

<sup>2</sup> When used without subscripts,  $p$ ,  $\rho$ , and  $T$  denote static pressure, static density, and static temperature, respectively.

ENERGY RELATIONS

The law of conservation of energy gives

$$\left. \begin{aligned} dq &= du + dw \quad (\text{first law of thermodynamics}) \\ &= du + p \, dv = dh - v \, dp \end{aligned} \right\} \quad (10a)$$

$$\left. \begin{aligned} dq &= c_p \, dT + p \, dv \\ &= c_p \, dT - v \, dp \end{aligned} \right\} \quad [\text{therm perf}] \quad (10b)$$

SPECIFIC HEATS

The specific heats at constant pressure and constant volume are defined by

$$c_p \equiv \left( \frac{\partial q}{\partial T} \right)_p = \left( \frac{\partial h}{\partial T} \right)_p \quad (11)$$

$$c_v \equiv \left( \frac{\partial q}{\partial T} \right)_v = \left( \frac{\partial u}{\partial T} \right)_v \quad (12)$$

It can be shown that

$$c_p - c_v = \left[ \left( \frac{\partial u}{\partial v} \right)_T + p \right] \left( \frac{\partial v}{\partial T} \right)_p = -T \frac{\left( \frac{\partial p}{\partial T} \right)_v^2}{\left( \frac{\partial p}{\partial v} \right)_T} \quad (13a)$$

$$c_p - c_v = R \quad [\text{therm perf}] \quad (13b)$$

The ratio of specific heats is defined as

$$\gamma \equiv \frac{c_p}{c_v} \quad (14)$$

According to the kinetic theory of gases, for many gases over a moderate range of temperature,

$$\gamma = \frac{n+2}{n} \quad (15)$$

where  $n$  is the number of effective degrees of freedom of the gas molecule. Useful relations for thermally perfect gases are

$$c_p = \frac{dh}{dT} = c_v + R = \frac{\gamma R}{\gamma - 1} \quad [\text{therm perf}] \quad (16)$$

$$c_v = \frac{du}{dT} = c_p - R = \frac{R}{\gamma - 1} \quad [\text{therm perf}] \quad (17)$$

ENTHALPY

The enthalpy of a gas is defined by

$$h \equiv u + pv \quad (18)$$

It follows that

$$\begin{aligned} dh &= du + p \, dv + v \, dp = dq + v \, dp \\ &= \left[ c_p + v \left( \frac{\partial p}{\partial T} \right)_v \right] dT + \left[ v \left( \frac{\partial p}{\partial v} \right)_T + T \left( \frac{\partial p}{\partial T} \right)_v \right] dv \end{aligned} \quad (19a)$$

$$dh = (c_p + R) dT = c_p dT \quad [\text{therm perf}] \quad (19b)$$

$$h = (c_p + R) T + u_v = c_p T + u_v \quad [\text{perf}] \quad (20)$$

ENTROPY

The entropy is defined by

$$ds \equiv \left( \frac{dq}{T} \right)_{\text{rev}} \quad (21)$$

It follows that

$$ds = \left( \frac{du + dw}{T} \right)_{\text{rev}} = \left( \frac{du + p \, dv}{T} \right)_{\text{rev}} = c_p \frac{dT}{T} + \left( \frac{\partial p}{\partial T} \right)_v dv \quad (22a)$$

$$\left. \begin{aligned} ds &= c_p \frac{dT}{T} + R \frac{dv}{v} \\ &= c_p \frac{dT}{T} - R \frac{d\rho}{\rho} \\ &= c_p \frac{dT}{T} - R \frac{dp}{p} \\ &= c_p \frac{dp}{p} - c_p \frac{d\rho}{\rho} \end{aligned} \right\} \quad [\text{therm perf}] \quad (22b)$$

$$\left. \begin{aligned} s - s_r &= c_p \ln \frac{T}{T_r} - R \ln \frac{\rho}{\rho_r} \\ &= c_p \ln \frac{T}{T_r} - R \ln \frac{p}{p_r} \\ &= c_p \ln \frac{p}{p_r} - c_p \ln \frac{\rho}{\rho_r} \end{aligned} \right\} \quad [\text{perf}] \quad (23a)$$

$$\left. \begin{aligned} s - s_r &= c_p \ln \frac{T/T_r}{(\rho/\rho_r)^{\gamma-1}} \\ &= c_p \ln \frac{T/T_r}{(p/p_r)^{(\gamma-1)/\gamma}} \\ &= c_p \ln \frac{p/p_r}{(\rho/\rho_r)^\gamma} \end{aligned} \right\} \quad [\text{perf}] \quad (23b)$$

$$\frac{p}{\rho^\gamma} = \frac{p_r}{\rho_r^\gamma} e^{(s-s_r)/c_p} \quad [\text{perf}] \quad (24)$$

The second law of thermodynamics requires that

$$s - s_r \geq 0 \quad [\text{adiab}] \quad (25)$$

CONTINUOUS ONE-DIMENSIONAL FLOW

BASIC EQUATIONS AND DEFINITIONS

The basic equations for the continuous flow of an inviscid non-heat-conducting gas along a streamline are as follows:

Thermal equation of state

$$\frac{p}{\rho} = RT \quad [\text{therm perf}] \quad (26)$$

Dynamic equation

$$\frac{1}{\rho} dp + V \, dV = 0 \quad (27)$$

Energy equation

$$\left. \begin{aligned} du + d\left(\frac{p}{\rho}\right) + VdV = 0 \\ dh + VdV = 0 \end{aligned} \right\} \text{[adiab]} \quad (28a)$$

$$\left. \begin{aligned} c_p dT + VdV = 0 \\ \frac{\gamma}{\gamma-1} d\left(\frac{p}{\rho}\right) + VdV = 0 \end{aligned} \right\} \text{[adiab, therm perf]} \quad (28b)$$

Additional useful variables are defined as follows:

Speed of sound

$$a = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_s} = \sqrt{\gamma \left(\frac{\partial p}{\partial \rho}\right)_T} \quad (29a)$$

$$= \sqrt{\gamma \frac{p}{\rho}} = \sqrt{\gamma RT} \quad \text{[therm perf]} \quad (29b)$$

$$\begin{aligned} &\cong 49.0 \sqrt{T} \text{ ft/sec for air} \\ &\text{if } T \text{ is in degrees Rankine} \\ &(\text{=degrees Fahrenheit} + 459.6) \end{aligned} \quad (29c)$$

Mach number

$$M = \frac{V}{a} \quad (30)$$

Dynamic pressure

$$q = \frac{1}{2} \rho V^2 \quad (31a)$$

$$= \frac{\gamma}{2} p M^2 \quad \text{[therm perf]} \quad (31b)$$

#### INTEGRATED FORMS OF ENERGY EQUATION

The energy equation (28) can be integrated at once to obtain

$$h + \frac{V^2}{2} = \text{constant} = h_t \quad \text{[adiab]} \quad (32a)$$

$$\left. \begin{aligned} c_p T + \frac{V^2}{2} &= c_p T_t \\ \frac{\gamma}{\gamma-1} \left(\frac{p}{\rho}\right) + \frac{V^2}{2} &= \frac{\gamma}{\gamma-1} \left(\frac{p_t}{\rho_t}\right) \\ \frac{a^2}{\gamma-1} + \frac{V^2}{2} &= \frac{a_t^2}{\gamma-1} \\ \frac{a^2}{\gamma-1} + \frac{V^2}{2} &= \frac{1}{2} \left(\frac{\gamma+1}{\gamma-1}\right) a_*^2 \\ \frac{a^2}{\gamma-1} + \frac{V^2}{2} &= \frac{V_m^2}{2} \end{aligned} \right\} \text{[adiab, perf]} \quad (32b)$$

The three reference speeds  $a_t$ ,  $a_*$ , and  $V_m$  are related by

$$\left. \begin{aligned} \left(\frac{a_t}{a_*}\right)^2 &= \frac{\gamma+1}{2} \\ \left(\frac{V_m}{a_*}\right)^2 &= \frac{\gamma+1}{\gamma-1} \\ \left(\frac{V_m}{a_t}\right)^2 &= \frac{2}{\gamma-1} \end{aligned} \right\} \text{[adiab, perf]} \quad (33)$$

#### PRESSURE-DENSITY RELATION

From equations (27) and (28b) it follows that

$$\frac{p}{\rho^\gamma} = \text{constant} = \frac{p_t}{\rho_t^\gamma} \quad \text{[isen, perf]} \quad (34)$$

from which

$$\frac{p}{p_t} = \left(\frac{\rho}{\rho_t}\right)^\gamma = \left(\frac{T}{T_t}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{a}{a_t}\right)^{\frac{2\gamma}{\gamma-1}} \quad \text{[isen, perf]} \quad (35)$$

#### BERNOULLI'S EQUATION

Combination of equations (32b) and (35) gives Bernoulli's equation for compressible flow in the form

$$\frac{\gamma}{\gamma-1} \left(\frac{p_t}{\rho_t}\right) \left(\frac{p}{p_t}\right)^{\frac{\gamma-1}{\gamma}} + \frac{V^2}{2} = \frac{\gamma}{\gamma-1} \left(\frac{p_t}{\rho_t}\right) \quad \text{[isen, perf]} \quad (36)$$

#### RELATIONS BETWEEN LOCAL AND FREE-STREAM CONDITIONS

With the aid of the foregoing equations it can be shown that

$$\frac{T}{T_\infty} = 1 - \frac{\gamma-1}{2} M_\infty^2 \left[ \left(\frac{V}{V_\infty}\right)^2 - 1 \right] \quad \text{[adiab, perf]} \quad (37)$$

$$\frac{p}{p_\infty} = \left\{ 1 - \frac{\gamma-1}{2} M_\infty^2 \left[ \left(\frac{V}{V_\infty}\right)^2 - 1 \right] \right\}^{\frac{\gamma}{\gamma-1}} \quad \text{[isen, perf]} \quad (38)$$

$$\frac{\rho}{\rho_\infty} = \left\{ 1 - \frac{\gamma-1}{2} M_\infty^2 \left[ \left(\frac{V}{V_\infty}\right)^2 - 1 \right] \right\}^{\frac{1}{\gamma-1}} \quad \text{[isen, perf]} \quad (39)$$

In small-disturbance theory, where it is assumed that  $(V - V_\infty) \ll V_\infty$ , these equations take on the simplified form

$$\frac{T}{T_\infty} \cong 1 - (\gamma-1) M_\infty^2 \frac{V - V_\infty}{V_\infty} \quad \text{[adiab, perf]} \quad (40)$$

$$\frac{p}{p_\infty} \cong 1 - \gamma M_\infty^2 \frac{V - V_\infty}{V_\infty} \quad \text{[isen, perf]} \quad (41)$$

$$\frac{\rho}{\rho_\infty} \cong 1 - M_\infty^2 \frac{V - V_\infty}{V_\infty} \quad \text{[isen, perf]} \quad (42)$$

#### USEFUL RATIOS

On the basis of the above results, useful relations can be derived expressing various dimensionless ratios as functions of a single parameter. These relations are given below, grouped according to which of the various parameters ( $M$ ,  $V/a_*$ ,  $V/a_t$ , or  $V/V_m$ ) is used as the independent variable.

In each case the second form of the equation applies for  $\gamma = \frac{7}{5}$ .

Parameter  $M$ .—

$$\frac{T}{T_t} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} = \left(1 + \frac{M^2}{5}\right)^{-1} \quad \text{[adiab, perf]} \quad (43)$$

$$\frac{p}{p_t} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} = \left(1 + \frac{M^2}{5}\right)^{-\frac{7}{2}} \quad \text{[isen, perf]} \quad (44)$$

$$\frac{\rho}{\rho_t} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{\gamma-1}} = \left(1 + \frac{M^2}{5}\right)^{-\frac{5}{2}} \quad \text{[isen, perf]} \quad (45)$$

$$\frac{a}{a_t} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{1}{2}} = \left(1 + \frac{M^2}{5}\right)^{-\frac{1}{2}} \quad \text{[adiab, perf]} \quad (46)$$

$$\frac{q}{p} = \frac{\gamma}{2} M^2 = \frac{7}{10} M^2 \quad [\text{therm perf}] \quad (47)$$

$$\begin{aligned} \frac{q}{p_t} &= \frac{\gamma}{2} M^2 \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma}{\gamma-1}} \\ &= \frac{7}{10} M^2 \left(1 + \frac{M^2}{5}\right)^{-\frac{7}{2}} \quad [\text{isen, perf}] \quad (48) \end{aligned}$$

$$\begin{aligned} \left(\frac{V}{a_t}\right)^2 &= M^2 \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \\ &= M^2 \left(1 + \frac{M^2}{5}\right)^{-1} \quad [\text{adiab, perf}] \quad (49) \end{aligned}$$

$$\begin{aligned} \left(\frac{V}{a_*}\right)^2 &= \frac{\gamma+1}{2} M^2 \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \\ &= \frac{6M^2}{5} \left(1 + \frac{M^2}{5}\right)^{-1} \quad [\text{adiab, perf}] \quad (50) \end{aligned}$$

$$\begin{aligned} \left(\frac{V}{V_m}\right)^2 &= \frac{\gamma-1}{2} M^2 \left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} \\ &= \frac{M^2}{5} \left(1 + \frac{M^2}{5}\right)^{-1} \quad [\text{adiab, perf}] \quad (51) \end{aligned}$$

Parameter  $\frac{V}{a_*}$ —

$$\frac{T}{T_t} = 1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2 = 1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2 \quad [\text{adiab, perf}] \quad (52)$$

$$\begin{aligned} \frac{p}{p_t} &= \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2\right]^{\frac{\gamma}{\gamma-1}} \\ &= \left[1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2\right]^{\frac{7}{2}} \quad [\text{isen, perf}] \quad (53) \end{aligned}$$

$$\begin{aligned} \frac{\rho}{\rho_t} &= \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2\right]^{\frac{1}{\gamma-1}} \\ &= \left[1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2\right]^{\frac{5}{2}} \quad [\text{isen, perf}] \quad (54) \end{aligned}$$

$$\begin{aligned} \frac{a}{a_t} &= \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2\right]^{\frac{1}{2}} \\ &= \left[1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2\right]^{\frac{1}{2}} \quad [\text{adiab, perf}] \quad (55) \end{aligned}$$

$$\begin{aligned} \frac{q}{p} &= \frac{\gamma}{\gamma+1} \left(\frac{V}{a_*}\right)^2 \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2\right]^{-1} \\ &= \frac{7}{12} \left(\frac{V}{a_*}\right)^2 \left[1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2\right]^{-1} \quad [\text{adiab, perf}] \quad (56) \end{aligned}$$

$$\begin{aligned} \frac{q}{p_t} &= \frac{\gamma}{\gamma+1} \left(\frac{V}{a_*}\right)^2 \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2\right]^{-\frac{1}{\gamma-1}} \\ &= \frac{7}{12} \left(\frac{V}{a_*}\right)^2 \left[1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2\right]^{-\frac{5}{2}} \quad [\text{isen, perf}] \quad (57) \end{aligned}$$

$$\begin{aligned} M^2 &= \frac{2}{\gamma+1} \left(\frac{V}{a_*}\right)^2 \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2\right]^{-1} \\ &= \frac{5}{6} \left(\frac{V}{a_*}\right)^2 \left[1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2\right]^{-1} \quad [\text{adiab, perf}] \quad (58) \end{aligned}$$

$$\left(\frac{V}{a_t}\right)^2 = \frac{2}{\gamma+1} \left(\frac{V}{a_*}\right)^2 = \frac{5}{6} \left(\frac{V}{a_*}\right)^2 \quad [\text{adiab, perf}] \quad (59)$$

$$\left(\frac{V}{V_m}\right)^2 = \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2 = \frac{1}{6} \left(\frac{V}{a_*}\right)^2 \quad [\text{adiab, perf}] \quad (60)$$

Parameter  $\frac{V}{a_t}$ —

$$\frac{T}{T_t} = 1 - \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2 = 1 - \frac{1}{5} \left(\frac{V}{a_t}\right)^2 \quad [\text{adiab, perf}] \quad (61)$$

$$\begin{aligned} \frac{p}{p_t} &= \left[1 - \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2\right]^{\frac{\gamma}{\gamma-1}} \\ &= \left[1 - \frac{1}{5} \left(\frac{V}{a_t}\right)^2\right]^{\frac{7}{2}} \quad [\text{isen, perf}] \quad (62) \end{aligned}$$

$$\begin{aligned} \frac{\rho}{\rho_t} &= \left[1 - \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2\right]^{\frac{1}{\gamma-1}} \\ &= \left[1 - \frac{1}{5} \left(\frac{V}{a_t}\right)^2\right]^{\frac{5}{2}} \quad [\text{isen, perf}] \quad (63) \end{aligned}$$

$$\begin{aligned} \frac{a}{a_t} &= \left[1 - \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2\right]^{\frac{1}{2}} \\ &= \left[1 - \frac{1}{5} \left(\frac{V}{a_t}\right)^2\right]^{\frac{1}{2}} \quad [\text{adiab, perf}] \quad (64) \end{aligned}$$

$$\begin{aligned} \frac{q}{p} &= \frac{\gamma}{2} \left(\frac{V}{a_t}\right)^2 \left[1 - \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2\right]^{-1} \\ &= \frac{7}{10} \left(\frac{V}{a_t}\right)^2 \left[1 - \frac{1}{5} \left(\frac{V}{a_t}\right)^2\right]^{-1} \quad [\text{adiab, perf}] \quad (65) \end{aligned}$$

$$\begin{aligned} \frac{q}{p_t} &= \frac{\gamma}{2} \left(\frac{V}{a_t}\right)^2 \left[1 - \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2\right]^{-\frac{1}{\gamma-1}} \\ &= \frac{7}{10} \left(\frac{V}{a_t}\right)^2 \left[1 - \frac{1}{5} \left(\frac{V}{a_t}\right)^2\right]^{-\frac{5}{2}} \quad [\text{isen, perf}] \quad (66) \end{aligned}$$

$$\begin{aligned} M^2 &= \left(\frac{V}{a_t}\right)^2 \left[1 - \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2\right]^{-1} \\ &= \left(\frac{V}{a_t}\right)^2 \left[1 - \frac{1}{5} \left(\frac{V}{a_t}\right)^2\right]^{-1} \quad [\text{adiab, perf}] \quad (67) \end{aligned}$$

$$\left(\frac{V}{a_*}\right)^2 = \frac{\gamma+1}{2} \left(\frac{V}{a_t}\right)^2 = \frac{6}{5} \left(\frac{V}{a_t}\right)^2 \quad [\text{adiab, perf}] \quad (68)$$

$$\left(\frac{V}{V_m}\right)^2 = \frac{\gamma-1}{2} \left(\frac{V}{a_t}\right)^2 = \frac{1}{5} \left(\frac{V}{a_t}\right)^2 \quad [\text{adiab, perf}] \quad (69)$$

Parameter  $\frac{V}{V_m}$

$$\frac{T}{T_i} = 1 - \left(\frac{V}{V_m}\right)^2 \quad [\text{adiab, perf}] \quad (70)$$

$$\frac{p}{p_i} = \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{\gamma}{\gamma-1}} = \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{7}{2}} \quad [\text{isen, perf}] \quad (71)$$

$$\frac{\rho}{\rho_i} = \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{1}{\gamma-1}} = \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{5}{2}} \quad [\text{isen, perf}] \quad (72)$$

$$\frac{a}{a_i} = \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{1}{2}} \quad [\text{adiab, perf}] \quad (73)$$

$$\frac{q}{p} = \frac{\gamma}{\gamma-1} \left(\frac{V}{V_m}\right)^2 \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{-1} \\ = \frac{7}{2} \left(\frac{V}{V_m}\right)^2 \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{-1} \quad [\text{adiab, perf}] \quad (74)$$

$$\frac{q}{p_i} = \frac{\gamma}{\gamma-1} \left(\frac{V}{V_m}\right)^2 \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{1}{\gamma-1}} \\ = \frac{7}{2} \left(\frac{V}{V_m}\right)^2 \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{5}{2}} \quad [\text{isen, perf}] \quad (75)$$

$$M^2 = \frac{2}{\gamma+1} \left(\frac{V}{V_m}\right)^2 \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{-1} \\ = \frac{5}{6} \left(\frac{V}{V_m}\right)^2 \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{-1} \quad [\text{adiab, perf}] \quad (76)$$

$$\left(\frac{V}{a_i}\right)^2 = \frac{2}{\gamma-1} \left(\frac{V}{V_m}\right)^2 = 5 \left(\frac{V}{V_m}\right)^2 \quad [\text{adiab, perf}] \quad (77)$$

$$\left(\frac{V}{a_*}\right)^2 = \frac{\gamma+1}{\gamma-1} \left(\frac{V}{V_m}\right)^2 = 6 \left(\frac{V}{V_m}\right)^2 \quad [\text{adiab, perf}] \quad (78)$$

Tables I and II list numerical values of the following ratios with Mach number  $M$  as the independent variable:

$$\frac{p}{p_i}, \frac{\rho}{\rho_i}, \frac{T}{T_i}, \frac{q}{p_i}, \frac{V}{a_*}$$

#### STREAM-TUBE-AREA RELATIONS

If it is assumed that the density and speed are uniform across any section of a given stream tube, then the equation of continuity is

$$\rho V A = \text{constant} = \rho_* a_* A_* \quad (79)$$

By combining this and certain of the foregoing equations, the area ratio  $A_*/A$  can be expressed as a function of any one of the four parameters used above. The final equations are

$$\frac{A_*}{A} = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} M \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{\gamma+1}{2(\gamma-1)}} \\ = \frac{216}{125} M \left(1 + \frac{M^2}{5}\right)^{-3} \quad [\text{isen, perf}] \quad (80)$$

$$\frac{A_*}{A} = \left(\frac{\gamma+1}{2}\right)^{\frac{1}{\gamma-1}} \left(\frac{V}{a_*}\right) \left[1 - \frac{\gamma-1}{\gamma+1} \left(\frac{V}{a_*}\right)^2\right]^{\frac{1}{\gamma-1}} \\ = \left(\frac{6}{5}\right)^{\frac{5}{2}} \left(\frac{V}{a_*}\right) \left[1 - \frac{1}{6} \left(\frac{V}{a_*}\right)^2\right]^{\frac{5}{2}} \quad [\text{isen, perf}] \quad (81)$$

$$\frac{A_*}{A} = \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{V}{a_i}\right) \left[1 - \frac{\gamma-1}{2} \left(\frac{V}{a_i}\right)^2\right]^{\frac{1}{\gamma-1}} \\ = \frac{216}{125} \left(\frac{V}{a_i}\right) \left[1 - \frac{1}{5} \left(\frac{V}{a_i}\right)^2\right]^{\frac{5}{2}} \quad [\text{isen, perf}] \quad (82)$$

$$\frac{A_*}{A} = \left(\frac{2}{\gamma-1}\right)^{\frac{1}{2}} \left(\frac{\gamma+1}{2}\right)^{\frac{\gamma+1}{2(\gamma-1)}} \left(\frac{V}{V_m}\right) \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{1}{\gamma-1}} \\ = 5^{\frac{1}{2}} \left(\frac{216}{125}\right) \left(\frac{V}{V_m}\right) \left[1 - \left(\frac{V}{V_m}\right)^2\right]^{\frac{5}{2}} \quad [\text{isen, perf}] \quad (83)$$

Numerical values of  $A_*/A$  as a function of  $M$  are given in tables I and II.

Equation (79) combined with equations (26), (29b), (45), and (46) can be employed to obtain the mass-flow rate per unit area  $\rho V$  along a stream tube as a function of Mach number, total temperature, and total pressure. Numerical values can be obtained conveniently from chart 1 where the variation with Mach number of the mass-flow rate per unit cross-sectional area is presented for various total temperatures and a total pressure of 1 pound per square inch absolute.

## SHOCK WAVES

### NORMAL SHOCK WAVES

#### BASIC EQUATIONS

The previous relations for isentropic flow are valid on either side of a shock wave, but not across it, because at the shock wave the flow quantities have discontinuities. Jump

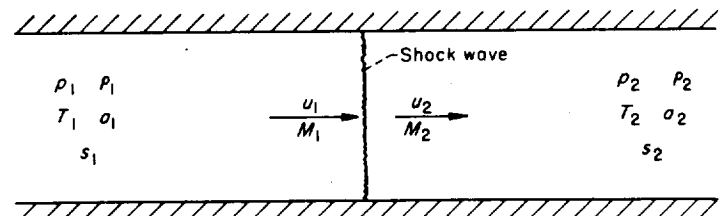


FIGURE 1.—Notation for normal shock wave.

conditions for a steady normal shock wave (fig. 1) result from requiring conservation of

$$\text{mass:} \quad \rho_1 u_1 = \rho_2 u_2 \quad (84)$$

$$\text{momentum:} \quad p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2 \quad (85)$$

$$\text{energy:}^3 \quad \frac{1}{2} u_1^2 + h_1 = \frac{1}{2} u_2^2 + h_2 \quad [\text{adiab}] \quad (86a)$$

<sup>3</sup> The actual relation for conservation of energy is  $\rho_1 u_1 \left(\frac{1}{2} u_1^2 + h_1\right) = \rho_2 u_2 \left(\frac{1}{2} u_2^2 + h_2\right)$ ; it reduces to the above form in view of equation (84).

$$\left. \begin{aligned} \frac{1}{2} u_1^2 + c_p T_1 &= \frac{1}{2} u_2^2 + c_p T_2 \\ \frac{1}{2} u_1^2 + \frac{\gamma}{\gamma-1} \frac{p_1}{\rho_1} &= \frac{1}{2} u_2^2 + \frac{\gamma}{\gamma-1} \frac{p_2}{\rho_2} \\ \frac{1}{2} u_1^2 + \frac{1}{\gamma-1} a_1^2 &= \frac{1}{2} u_2^2 + \frac{1}{\gamma-1} a_2^2 \end{aligned} \right\} \text{[adiab, perf]} \quad (86b)$$

together with the requirement that the entropy does not decrease:

$$\Delta s \equiv s_2 - s_1 \geq 0 \quad (87)$$

It follows immediately from the energy relation (86) that total enthalpy, total temperature, and total speed of sound are constant across the shock and hence (from the previous relations (33) for adiabatic flow) also the critical speed of sound and limiting speed:

$$h_{t_1} = h_{t_2} \quad \text{[adiab]} \quad (88a)$$

$$\left. \begin{aligned} T_{t_1} &= T_{t_2} \\ a_{t_1} &= a_{t_2} \\ a_{*1} &= a_{*2} \\ V_{m_1} &= V_{m_2} \end{aligned} \right\} \text{[adiab, perf]} \quad (88b)$$

Combining equations (84) to (86) leads to Prandtl's relation

$$u_1 u_2 = a_*^2 = \frac{p_2 - p_1}{\rho_2 - \rho_1} \quad \text{[adiab, perf]} \quad (89)$$

which implies that the flow is supersonic ahead of the shock wave and subsonic behind (the reverse possibility is ruled out by the requirement of nondecreasing entropy), and to the Rankine-Hugoniot relations

$$\frac{p_2}{p_1} = \frac{(\gamma+1) \rho_2 - (\gamma-1) \rho_1}{(\gamma+1) \rho_1 - (\gamma-1) \rho_2} \quad \text{[adiab, perf]} \quad (90)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1) p_2 + (\gamma-1) p_1}{(\gamma+1) p_1 + (\gamma-1) p_2} \quad \text{[adiab, perf]} \quad (91)$$

$$\frac{p_2 - p_1}{\rho_2 - \rho_1} = \gamma \frac{p_2 + p_1}{\rho_2 + \rho_1} \quad \text{[adiab, perf]} \quad (92)$$

USEFUL RELATIONS

Many relations for normal shock waves are conveniently expressed in terms of either upstream Mach number  $M_1$  or the static-pressure ratio across the shock  $\xi \equiv p_2/p_1$ . The following relations apply to adiabatic flow of a completely perfect fluid. The last form of each equation holds for  $\gamma=7/5$ .

Parameter  $M_1$ .—

$$\frac{p_2}{p_1} \equiv \xi = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} = \frac{7M_1^2 - 1}{6} \quad (93)$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{u_1^2}{u_2^2} = \frac{a_*^2}{u_2^2} = \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2} = \frac{6M_1^2}{M_1^2 + 5} \quad (94)$$

$$\frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = \frac{[2\gamma M_1^2 - (\gamma-1)] [(\gamma-1) M_1^2 + 2]}{(\gamma+1)^2 M_1^2} = \frac{(7M_1^2 - 1)(M_1^2 + 5)}{36M_1^2} \quad (95)$$

$$M_2^2 = \frac{(\gamma-1) M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)} = \frac{M_1^2 + 5}{7M_1^2 - 1} \quad (96)$$

$$\frac{p_2}{p_1} = \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} \left[ \frac{2}{(\gamma-1) M_1^2 + 2} \right]^{\frac{\gamma}{\gamma-1}} = \frac{7M_1^2 - 1}{6} \left( \frac{5}{M_1^2 + 5} \right)^{\frac{7}{2}} \quad (97)$$

$$\frac{p_2}{p_{t_2}} = \left[ \frac{4\gamma M_1^2 - 2(\gamma-1)}{(\gamma+1)^2 M_1^2} \right]^{\frac{\gamma}{\gamma-1}} = \left[ \frac{5(7M_1^2 - 1)}{36M_1^2} \right]^{\frac{7}{2}} \quad (98)$$

$$\frac{p_{t_2}}{p_{t_1}} = \frac{\rho_{t_2}}{\rho_{t_1}} = e^{-\frac{\Delta s}{R}} = \left[ \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2} \right]^{\frac{\gamma}{\gamma-1}} \left[ \frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)} \right]^{\frac{1}{\gamma-1}} = \left( \frac{6M_1^2}{M_1^2 + 5} \right)^{\frac{7}{2}} \left( \frac{6}{7M_1^2 - 1} \right)^{\frac{5}{2}} \quad (99)$$

$$\frac{p_{t_2}}{p_1} = \left[ \frac{(\gamma+1) M_1^2}{2} \right]^{\frac{\gamma}{\gamma-1}} \left[ \frac{\gamma+1}{2\gamma M_1^2 - (\gamma-1)} \right]^{\frac{1}{\gamma-1}} = \left( \frac{6M_1^2}{5} \right)^{\frac{7}{2}} \left( \frac{6}{7M_1^2 - 1} \right)^{\frac{5}{2}} \quad (100)$$

(Rayleigh pitot formula)

$$\frac{\Delta s}{c_p} = (\gamma-1) \frac{\Delta s}{R} = -(\gamma-1) \ln \left( \frac{p_{t_2}}{p_{t_1}} \right) = \ln \left[ \frac{2\gamma M_1^2 - (\gamma-1)}{\gamma+1} \right] - \gamma \ln \left[ \frac{(\gamma+1) M_1^2}{(\gamma-1) M_1^2 + 2} \right] = \ln \left( \frac{7M_1^2 - 1}{6} \right) - \frac{7}{5} \ln \left( \frac{6M_1^2}{M_1^2 + 5} \right) \quad (101)$$

$$\frac{p_2 - p_1}{q_1} = \frac{4(M_1^2 - 1)}{(\gamma+1) M_1^2} = \frac{5(M_1^2 - 1)}{3M_1^2} \quad (102)$$

Numerical values from equations (93), (94), (95), (96), (99), and (100) (with  $\gamma=7/5$ ) are given in table II.

For weak shock waves ( $M_1$  only slightly greater than unity) the following series are useful:

$$\frac{p_{t_2}}{p_{t_1}} = 1 - \frac{2\gamma}{3(\gamma+1)^2} (M_1^2 - 1)^3 + \frac{2\gamma^2}{(\gamma+1)^3} (M_1^2 - 1)^4 + \dots = 1 - \frac{35}{216} (M_1^2 - 1)^3 + \frac{245}{864} (M_1^2 - 1)^4 + \dots \quad (103)$$

$$\frac{\Delta s}{R} = \frac{1}{\gamma-1} \frac{\Delta s}{c_p} = \frac{2\gamma}{3(\gamma+1)^2} (M_1^2 - 1)^3 - \frac{2\gamma^2}{(\gamma+1)^3} (M_1^2 - 1)^4 + \dots = \frac{35}{216} (M_1^2 - 1)^3 - \frac{245}{864} (M_1^2 - 1)^4 + \dots \quad (104)$$



Parameter  $\xi \equiv p_2/p_1$ .—

$$M_1^2 = \frac{(\gamma+1)\xi + (\gamma-1)}{2\gamma} = \frac{6\xi+1}{7} \tag{105}$$

$$\frac{\rho_2}{\rho_1} = \frac{u_1}{u_2} = \frac{(\gamma+1)\xi + (\gamma-1)}{(\gamma-1)\xi + (\gamma+1)} = \frac{6\xi+1}{\xi+6} \tag{106}$$

$$\frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = \xi \frac{(\gamma-1)\xi + (\gamma+1)}{(\gamma+1)\xi + (\gamma-1)} = \xi \frac{\xi+6}{6\xi+1} \tag{107}$$

$$M_2^2 = \frac{(\gamma-1)\xi + (\gamma+1)}{2\gamma\xi} = \frac{\xi+6}{7\xi} \tag{108}$$

$$\frac{p_2'}{p_1'} = \xi \frac{p_1}{p_1'} = \xi \left\{ \frac{4\gamma}{(\gamma+1)[(\gamma-1)\xi + (\gamma+1)]} \right\}^{\frac{\gamma}{\gamma-1}} = \xi \left[ \frac{35}{6(\xi+6)} \right]^{\frac{7}{2}} \tag{109}$$

$$\frac{p_2}{p_2'} = \xi \frac{p_1}{p_1'} = \left\{ \frac{4\gamma\xi}{(\gamma+1)[(\gamma+1)\xi + (\gamma-1)]} \right\}^{\frac{\gamma}{\gamma-1}} = \left[ \frac{35\xi}{6(6\xi+1)} \right]^{\frac{7}{2}} \tag{110}$$

$$\frac{p_{t_2}}{p_{t_1}} = \frac{\rho_{t_2}}{\rho_{t_1}} = e^{-\frac{\Delta s}{R}} = \xi^{-\frac{1}{\gamma-1}} \left[ \frac{(\gamma+1)\xi + (\gamma-1)}{(\gamma-1)\xi + (\gamma+1)} \right]^{\frac{\gamma}{\gamma-1}} \\ = \left( \frac{1}{\xi} \right)^{\frac{5}{2}} \left( \frac{6\xi+1}{\xi+6} \right)^{\frac{7}{2}} \tag{111}$$

$$\frac{\Delta s}{c_p} = (\gamma-1) \frac{\Delta s}{R} = -(\gamma-1) \ln \left( \frac{p_{t_2}}{p_{t_1}} \right) = \ln \xi - \gamma \ln \left[ \frac{(\gamma+1)\xi + (\gamma-1)}{(\gamma-1)\xi + (\gamma+1)} \right] = \ln \xi - \frac{7}{5} \ln \left( \frac{6\xi+1}{\xi+6} \right) \tag{112}$$

For weak shock waves ( $\xi$  only slightly greater than unity)

$$\frac{p_{t_2}}{p_{t_1}} = 1 - \frac{\gamma+1}{12\gamma^2} (\xi-1)^3 + \frac{\gamma+1}{8\gamma^2} (\xi-1)^4 + \dots \\ = 1 - \frac{5}{49} (\xi-1)^3 + \frac{15}{98} (\xi-1)^4 + \dots \tag{113}$$

$$\frac{\Delta s}{R} = \frac{1}{\gamma-1} \frac{\Delta s}{c_p} = \frac{\gamma+1}{12\gamma^2} (\xi-1)^3 - \frac{\gamma+1}{8\gamma^2} (\xi-1)^4 + \dots \\ = \frac{5}{49} (\xi-1)^3 - \frac{15}{98} (\xi-1)^4 + \dots \tag{114}$$

In unsteady flow a normal shock wave acts at each instant as a steady shock. Hence all the above relations are valid across a moving normal shock wave if instantaneous velocities are measured relative to the shock.

**OBLIQUE SHOCK WAVES**

In general, a three-dimensional shock wave will be curved, and will separate two regions of nonuniform flow. However, the shock transition at each point takes place instantaneously, so that it is sufficient to consider an arbitrarily small neighborhood of the point. In such a neighborhood

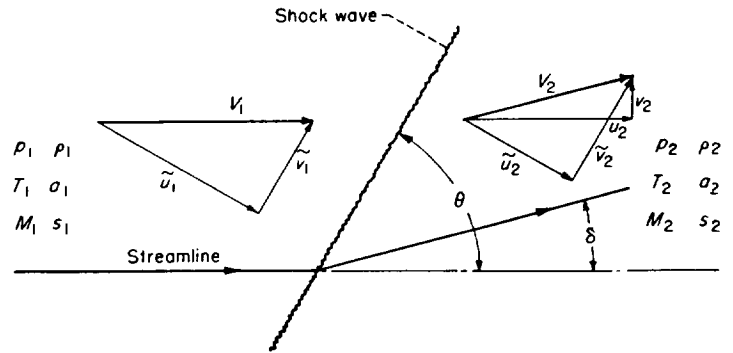


FIGURE 2.—Notation for oblique shock wave.

the shock wave may be regarded as plane to any desired degree of accuracy, and the flows on either side as uniform and parallel. Moreover, with the proper orientation of axes the flow is locally two-dimensional. Hence it is sufficient to consider a straight oblique shock wave in a uniform parallel two-dimensional stream, as shown in figure 2.

**BASIC EQUATIONS**

For a steady oblique shock wave, jump conditions result from requiring conservation of

mass:  $\rho_1 \tilde{u}_1 = \rho_2 \tilde{u}_2$  (115)

normal momentum:  $p_1 + \rho_1 \tilde{u}_1^2 = p_2 + \rho_2 \tilde{u}_2^2$  (116)

tangential momentum:  $\rho_1 \tilde{u}_1 \tilde{v}_1 = \rho_2 \tilde{u}_2 \tilde{v}_2$  (117)

energy<sup>4</sup>:  $\frac{1}{2} (\tilde{u}_1^2 + \tilde{v}_1^2) + h_1 = \frac{1}{2} (\tilde{u}_2^2 + \tilde{v}_2^2) + h_2$  [adiab] (118a)

$$\left. \begin{aligned} \frac{1}{2} (\tilde{u}_1^2 + \tilde{v}_1^2) + c_p T_1 &= \frac{1}{2} (\tilde{u}_2^2 + \tilde{v}_2^2) + c_p T_2 \\ \frac{1}{2} (\tilde{u}_1^2 + \tilde{v}_1^2) + \frac{\gamma}{\gamma-1} \frac{p_1}{\rho_1} &= \frac{1}{2} (\tilde{u}_2^2 + \tilde{v}_2^2) + \frac{\gamma}{\gamma-1} \frac{p_2}{\rho_2} \\ \frac{1}{2} (\tilde{u}_1^2 + \tilde{v}_1^2) + \frac{1}{\gamma-1} a_1^2 &= \frac{1}{2} (\tilde{u}_2^2 + \tilde{v}_2^2) + \frac{1}{\gamma-1} a_2^2 \end{aligned} \right\} \left[ \begin{array}{l} \text{adiab,} \\ \text{perf} \end{array} \right] \tag{118b}$$

together with the requirement that the entropy does not decrease:

$$\Delta s \equiv s_2 - s_1 \geq 0 \tag{119}$$

Again it follows from the energy relation (118) that total enthalpy, total temperature, and total speed of sound are constant across the shock and hence also the critical speed of sound and limiting speed:

$$h_{t_1} = h_{t_2} \quad [\text{adiab}] \tag{120}$$

$$\left. \begin{aligned} T_{t_1} &= T_{t_2} \\ a_{t_1} &= a_{t_2} \\ a_{*1} &= a_{*2} \\ V_{m_1} &= V_{m_2} \end{aligned} \right\} \left[ \begin{array}{l} \text{adiab,} \\ \text{perf} \end{array} \right] \tag{121}$$

<sup>4</sup> Compare remark for normal shock waves, footnote on page 618.

## CONNECTION WITH NORMAL SHOCK

A comparison of equation (115) with (117) shows that the tangential velocity is constant across the shock wave:

$$\tilde{v}_1 = \tilde{v}_2 \quad [\text{adiab}] \quad (122)$$

so that the change in velocity is normal to the shock. It follows that

$$\frac{1}{2} \tilde{v}_1^2 = \frac{1}{2} \tilde{v}_2^2$$

so that the energy equation (118a) reduces to

$$\frac{1}{2} \tilde{u}_1^2 + h_1 = \frac{1}{2} \tilde{u}_2^2 + h_2 \quad [\text{adiab}] \quad (123)$$

Now equations (115), (116), and (123) involve only the component of velocity  $\tilde{u}$  normal to the shock, and are identical with equations (84), (85), and (86) for normal shock waves. Hence an oblique shock wave acts as a normal shock to the component of flow perpendicular to it, while the tangential component is unchanged. This is also clear physically from the "sweepback principle" that the oblique flow is reduced to the normal flow by a uniform translation of axes (Galilean transformation).

Because the speed of sound depends on the tangential velocity, Prandtl's relation differs from that for normal shock waves (see ref. 11, pp. 302-303):

$$\tilde{u}_1 \tilde{u}_2 = a_*^2 - \frac{\gamma-1}{\gamma+1} \tilde{v}^2 \quad [\text{adiab, perf}] \quad (124)$$

where  $a_*$  and  $\tilde{v}$  can be evaluated on either side of the shock.

The Rankine-Hugoniot relations are the same as for normal shock waves:

$$\frac{p_2}{p_1} = \frac{(\gamma+1)\rho_2 - (\gamma-1)\rho_1}{(\gamma+1)\rho_1 - (\gamma-1)\rho_2} \quad [\text{adiab, perf}] \quad (125)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma+1)p_2 + (\gamma-1)p_1}{(\gamma+1)p_1 + (\gamma-1)p_2} \quad [\text{adiab, perf}] \quad (126)$$

$$\frac{p_2 - p_1}{\rho_2 - \rho_1} = \gamma \frac{p_2 + p_1}{\rho_2 + \rho_1} \quad [\text{adiab, perf}] \quad (127)$$

## USEFUL RELATIONS

Because an oblique shock wave acts as a normal shock to the flow perpendicular to it, the previous relations for normal shocks (except those for ratios of static to total pressures) apply to oblique shocks if  $M_1$  and  $M_2$  are replaced by their normal components  $M_1 \sin \theta$  and  $M_2 \sin(\theta - \delta)$ . This gives most of the following relations; the remainder are derived from them by using the kinematic condition that the stream turns through an angle  $\delta$ , together with the previous isentropic-flow relations.

Parameters  $M_1$  and  $\theta$ .—

$$\frac{p_2}{p_1} = \xi = \frac{2\gamma M_1^2 \sin^2 \theta - (\gamma-1)}{\gamma+1} = \frac{7M_1^2 \sin^2 \theta - 1}{6} \quad (128)$$

$$\frac{\rho_2}{\rho_1} = \frac{\tilde{u}_1}{\tilde{u}_2} = \frac{(\gamma+1)M_1^2 \sin^2 \theta}{(\gamma-1)M_1^2 \sin^2 \theta + 2} = \frac{6M_1^2 \sin^2 \theta}{M_1^2 \sin^2 \theta + 5} \quad (129)$$

$$\frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = \frac{[2\gamma M_1^2 \sin^2 \theta - (\gamma-1)][(\gamma-1)M_1^2 \sin^2 \theta + 2]}{(\gamma+1)^2 M_1^2 \sin^2 \theta} \\ = \frac{(7M_1^2 \sin^2 \theta - 1)(M_1^2 \sin^2 \theta + 5)}{36M_1^2} \quad (130)$$

$$M_2^2 \sin^2(\theta - \delta) = \frac{(\gamma-1)M_1^2 \sin^2 \theta + 2}{2\gamma M_1^2 \sin^2 \theta - (\gamma-1)} = \frac{M_1^2 \sin^2 \theta + 5}{7M_1^2 \sin^2 \theta - 1} \quad (131)$$

$$M_2^2 = \frac{(\gamma+1)^2 M_1^4 \sin^2 \theta - 4(M_1^2 \sin^2 \theta - 1)(\gamma M_1^2 \sin^2 \theta + 1)}{[2\gamma M_1^2 \sin^2 \theta - (\gamma-1)][(\gamma-1)M_1^2 \sin^2 \theta + 2]} \\ = \frac{36M_1^4 \sin^2 \theta - 5(M_1^2 \sin^2 \theta - 1)(7M_1^2 \sin^2 \theta + 5)}{(7M_1^2 \sin^2 \theta - 1)(M_1^2 \sin^2 \theta + 5)} \quad (132)$$

$$\frac{\tilde{u}_2}{V_1} = \frac{(\gamma-1)M_1^2 \sin^2 \theta + 2}{(\gamma+1)M_1^2 \sin^2 \theta} \sin \theta = \frac{M_1^2 \sin^2 \theta + 5}{6M_1^2 \sin^2 \theta} \sin \theta \quad (133)$$

$$\frac{\tilde{v}_2}{V_1} = \frac{\tilde{v}_1}{V_1} = \cos \theta \quad (134)$$

$$\frac{u_2}{V_1} = 1 - \frac{2(M_1^2 \sin^2 \theta - 1)}{(\gamma+1)M_1^2} = 1 - \frac{5(M_1^2 \sin^2 \theta - 1)}{6M_1^2} \quad (135)$$

$$\frac{v_2}{V_1} = \frac{2(M_1^2 \sin^2 \theta - 1)}{(\gamma+1)M_1^2} \cot \theta = \frac{5(M_1^2 \sin^2 \theta - 1)}{6M_1^2} \cot \theta \quad (136)$$

$$\frac{V_2^2}{V_1^2} = 1 - 4 \frac{(M_1^2 \sin^2 \theta - 1)(\gamma M_1^2 \sin^2 \theta + 1)}{(\gamma+1)^2 M_1^4 \sin^2 \theta} \\ = 1 - \frac{5}{36} \frac{(M_1^2 \sin^2 \theta - 1)(7M_1^2 \sin^2 \theta + 5)}{M_1^4 \sin^2 \theta} \quad (137)$$

$$\cot \delta = \tan \theta \left[ \frac{(\gamma+1)M_1^2}{2(M_1^2 \sin^2 \theta - 1)} - 1 \right] \\ = \tan \theta \left[ \frac{6M_1^2}{5(M_1^2 \sin^2 \theta - 1)} - 1 \right] \quad (138)$$

$$\tan \delta = \frac{2 \cot \theta (M_1^2 \sin^2 \theta - 1)}{2 + M_1^2 (\gamma + 1 - 2 \sin^2 \theta)} = \frac{5 \cot \theta (M_1^2 \sin^2 \theta - 1)}{5 + M_1^2 (6 - 5 \sin^2 \theta)} \quad (139a)$$

$$= \frac{M_1^2 \sin 2\theta - 2 \cot \theta}{2 + M_1^2 (\gamma + \cos 2\theta)} = 5 \frac{M_1^2 \sin 2\theta - 2 \cot \theta}{10 + M_1^2 (7 + 5 \cos 2\theta)} \quad (139b)$$

$$\frac{p_2}{p_{t_1}} = \frac{2\gamma M_1^2 \sin^2 \theta - (\gamma-1)}{(\gamma+1)} \left[ \frac{2}{(\gamma-1)M_1^2 + 2} \right]^{\frac{\gamma}{\gamma-1}} \\ = \frac{7M_1^2 \sin^2 \theta - 1}{6} \left( \frac{5}{M_1^2 + 5} \right)^{\frac{\gamma}{2}} \quad (140)$$

$$\frac{p_2}{p_{t_2}} = \left\{ 2 \frac{[2\gamma M_1^2 \sin^2 \theta - (\gamma-1)][(\gamma-1)M_1^2 \sin^2 \theta + 2]}{(\gamma+1)^2 M_1^2 \sin^2 \theta [(\gamma-1)M_1^2 + 2]} \right\}^{\frac{\gamma}{\gamma-1}} \\ = \left[ 5 \frac{(7M_1^2 \sin^2 \theta - 1)(M_1^2 \sin^2 \theta + 5)}{36M_1^2 \sin^2 \theta (M_1^2 + 5)} \right]^{\frac{\gamma}{2}} \quad (141)$$

$$\frac{p_{t_2}}{p_{t_1}} = \frac{\rho_{t_2}}{\rho_{t_1}} = e^{-\frac{\Delta s}{R}}$$

$$= \left[ \frac{(\gamma+1)M_1^2 \sin^2 \theta}{(\gamma-1)M_1^2 \sin^2 \theta + 2} \right]^{\frac{\gamma}{\gamma-1}} \left[ \frac{\gamma+1}{2\gamma M_1^2 \sin^2 \theta - (\gamma-1)} \right]^{\frac{1}{\gamma-1}}$$

$$= \left( \frac{6M_1^2 \sin^2 \theta}{M_1^2 \sin^2 \theta + 5} \right)^{7/2} \left( \frac{6}{7M_1^2 \sin^2 \theta - 1} \right)^{5/2} \quad (142)$$

$$\frac{p_{t_2}}{p_1} = \left[ \frac{\gamma+1}{2\gamma M_1^2 \sin^2 \theta - (\gamma-1)} \right]^{\frac{1}{\gamma-1}} \times$$

$$\left\{ \frac{(\gamma+1)M_1^2 \sin^2 \theta [(\gamma-1)M_1^2 + 2]}{2[(\gamma-1)M_1^2 \sin^2 \theta + 2]} \right\}^{\frac{\gamma}{\gamma-1}}$$

$$= \left( \frac{6}{7M_1^2 \sin^2 \theta - 1} \right)^{5/2} \left[ \frac{6M_1^2 \sin^2 \theta (M_1^2 + 5)}{5(M_1^2 \sin^2 \theta + 5)} \right]^{7/2} \quad (143)$$

$$\frac{\Delta s}{c_p} = (\gamma-1) \frac{\Delta s}{R} = -(\gamma-1) \ln \left( \frac{p_{t_2}}{p_{t_1}} \right)$$

$$= \ln \left[ \frac{2\gamma M_1^2 \sin^2 \theta - (\gamma-1)}{\gamma+1} \right] -$$

$$\gamma \ln \left[ \frac{(\gamma+1)M_1^2 \sin^2 \theta}{(\gamma-1)M_1^2 \sin^2 \theta + 2} \right]$$

$$= \ln \left( \frac{7M_1^2 \sin^2 \theta - 1}{6} \right) - \frac{7}{5} \ln \left( \frac{6M_1^2 \sin^2 \theta}{M_1^2 \sin^2 \theta + 5} \right) \quad (144)$$

$$\frac{p_2 - p_1}{q_1} = \frac{4(M_1^2 \sin^2 \theta - 1)}{(\gamma+1)M_1^2} = \frac{5}{3} \frac{M_1^2 \sin^2 \theta - 1}{M_1^2} \quad (145)$$

Values of the following ratios for oblique shock waves can be read from table II, provided  $M_1 \sin \theta$  is used instead of  $M_1$  in the first column:

$$\frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{p_{t_2}}{p_{t_1}}$$

For weak shock waves ( $M_1 \sin \theta$  only slightly greater than unity) the following series are obtained from equations (103) and (104) by replacing  $M_1$  by  $M_1 \sin \theta$ :

$$\frac{p_{t_2}}{p_{t_1}} = 1 - \frac{2\gamma}{3(\gamma+1)^2} (M_1^2 \sin^2 \theta - 1)^3 + \frac{2\gamma^2}{(\gamma+1)^3} (M_1^2 \sin^2 \theta - 1)^4 + \dots$$

$$= 1 - \frac{35}{216} (M_1^2 \sin^2 \theta - 1)^3 + \frac{245}{864} (M_1^2 \sin^2 \theta - 1)^4 + \dots \quad (146)$$

$$\frac{\Delta s}{R} = \frac{1}{\gamma-1} \frac{\Delta s}{c_p} = \frac{2\gamma}{3(\gamma+1)^2} (M_1^2 \sin^2 \theta - 1)^3 -$$

$$\frac{2\gamma^2}{(\gamma+1)^3} (M_1^2 \sin^2 \theta - 1)^4 + \dots$$

$$= \frac{35}{216} (M_1^2 \sin^2 \theta - 1)^3 - \frac{245}{864} (M_1^2 \sin^2 \theta - 1)^4 + \dots \quad (147)$$

Parameters  $\theta$  and  $\delta$ .—

$$\frac{1}{M_1^2} = \sin^2 \theta - \frac{\gamma+1}{2} \frac{\sin \theta \sin \delta}{\cos(\theta-\delta)} = \sin^2 \theta - \frac{\gamma+1}{2} \frac{\tan \delta}{\tan \theta + \cot \theta}$$

$$= \sin^2 \theta - \frac{\gamma+1}{2} \frac{\tan \theta}{\tan \theta + \cot \theta} \quad (148a)$$

$$M_1^2 = \frac{2(\cot \theta + \tan \delta)}{\sin 2\theta - \tan \delta(\gamma + \cos 2\theta)}$$

$$= \frac{10(\cot \theta + \tan \delta)}{5 \sin 2\theta - \tan \delta(7 + 5 \cos 2\theta)} \quad (148b)$$

$$\frac{p_2 - p_1}{q_1} = 2 \frac{\sin \theta \sin \delta}{\cos(\theta-\delta)}$$

$$= 2 \frac{\tan \delta}{\tan \theta + \cot \theta} = 2 \frac{\tan \theta}{\tan \theta + \cot \theta} \quad (149a)$$

$$\frac{\rho_2 - \rho_1}{\rho_2} = \frac{\sin \delta}{\sin \theta \cos(\theta-\delta)} \quad (149b)$$

Parameters  $M_1$  and  $\delta$ .—

No convenient explicit relations exist. However, the value of  $\sin^2 \theta$  can be found by solving the following cubic equation (ref. 12):

$$\sin^6 \theta + b \sin^4 \theta + c \sin^2 \theta + d = 0 \quad (150a)$$

where

$$\left. \begin{aligned} b &= -\frac{M_1^2 + 2}{M_1^2} - \gamma \sin^2 \delta \\ c &= \frac{2M_1^2 + 1}{M_1^4} + \left[ \frac{(\gamma+1)^2}{4} + \frac{\gamma-1}{M_1^2} \right] \sin^2 \delta \\ d &= -\frac{\cos^2 \delta}{M_1^4} \end{aligned} \right\} \quad (150b)$$

The smallest of the three roots corresponds to a decrease in entropy and should be disregarded.

For weak shock waves (small deflections  $\delta$ ) the following series are useful (note that  $\delta$  must be measured in radians):

$$\frac{p_2}{p_1} = 1 + \frac{\gamma M_1^2}{(M_1^2 - 1)^{1/2}} \delta + \gamma M_1^2 \frac{(\gamma+1)M_1^4 - 4(M_1^2 - 1)}{4(M_1^2 - 1)^2} \delta^2 +$$

$$\frac{\gamma M_1^2}{(M_1^2 - 1)^{7/2}} \left[ \frac{(\gamma+1)^2}{32} M_1^8 - \frac{7+12\gamma-3\gamma^2}{24} M_1^6 + \right.$$

$$\left. \frac{3}{4} (\gamma+1)M_1^4 - M_1^2 + \frac{2}{3} \right] \delta^3 + \dots \quad (151)$$

$$\frac{p_2 - p_1}{q_1} = \frac{2}{(M_1^2 - 1)^{1/2}} \delta + \frac{(\gamma+1)M_1^4 - 4(M_1^2 - 1)}{2(M_1^2 - 1)^2} \delta^2 +$$

$$\frac{1}{(M_1^2 - 1)^{7/2}} \left[ \frac{(\gamma+1)^2}{16} M_1^8 - \frac{7+12\gamma-3\gamma^2}{12} M_1^6 + \right.$$

$$\left. \frac{3}{2} (\gamma+1)M_1^4 - 2M_1^2 + \frac{4}{3} \right] \delta^3 + \dots \quad (152)$$

$$\frac{\rho_2}{\rho_1} = 1 + \frac{M_1^2}{(M_1^2 - 1)^{1/2}} \delta + M_1^2 \frac{(3 - \gamma)M_1^2(M_1^2 - 2) + 4}{4(M_1^2 - 1)^2} \delta^2 + \dots \quad (153)$$

$$\frac{T_2}{T_1} = 1 + \frac{(\gamma - 1)M_1^2}{(M_1^2 - 1)^{1/2}} \delta + (\gamma - 1)M_1^2 \frac{(\gamma + 1)M_1^4 - 2(M_1^2 + 2)(M_1^2 - 1)}{4(M_1^2 - 1)^2} \delta^2 + \dots \quad (154)$$

Since flow through weak shock waves is nearly isentropic, compressions through small angles can also be calculated with the aid of table II by regarding them as reversed Prandtl-Meyer expansions (see later section). The resulting numerical accuracy is greater than that obtained by retaining terms up to  $\delta^2$  in the above series, and nearly equal to that obtained by retaining terms up to  $\delta^3$ .

Charts 2, 3, and 4 show the variation of shock-wave angle, pressure coefficient across a shock wave, and downstream Mach number with flow-deflection angle for various upstream Mach numbers.

Parameter  $\xi \equiv p_1/p_2$ .—

$$M_1^2 \sin^2 \theta = \frac{(\gamma + 1)\xi + (\gamma - 1)}{2\gamma} = \frac{6\xi + 1}{7} \quad (155)$$

$$M_2^2 \sin^2(\theta - \delta) = \frac{(\gamma - 1)\xi + (\gamma + 1)}{2\gamma\xi} = \frac{\xi + 6}{7\xi} \quad (156)$$

$$M_2^2 = \frac{M_1^2[(\gamma + 1)\xi + (\gamma - 1)] - 2(\xi^2 - 1)}{\xi[(\gamma - 1)\xi + (\gamma + 1)]} = \frac{M_1^2(6\xi + 1) - 5(\xi^2 - 1)}{\xi(\xi + 6)} \quad (157)$$

$$\frac{\rho_2}{\rho_1} = \frac{(\gamma + 1)\xi + (\gamma - 1)}{(\gamma - 1)\xi + (\gamma + 1)} = \frac{6\xi + 1}{\xi + 6} \quad (158)$$

$$\frac{T_2}{T_1} = \frac{a_2^2}{a_1^2} = \xi \frac{(\gamma - 1)\xi + (\gamma + 1)}{(\gamma + 1)\xi + (\gamma - 1)} = \xi \frac{\xi + 6}{6\xi + 1} \quad (159)$$

$$\tan^2 \delta = \left( \frac{\xi - 1}{\gamma M_1^2 - \xi + 1} \right)^2 \frac{2\gamma M_1^2 - (\gamma - 1) - (\gamma + 1)\xi}{(\gamma + 1)\xi + (\gamma - 1)} = \left[ \frac{5(\xi - 1)}{7M_1^2 - 5(\xi - 1)} \right]^2 \frac{7M_1^2 - (6\xi + 1)}{6\xi + 1} \quad (160)$$

$$\frac{p_{t_2}}{p_{t_1}} = \frac{\rho_{t_2}}{\rho_{t_1}} = e^{-\frac{\Delta s}{R}} = \left[ \frac{(\gamma + 1)\xi + (\gamma - 1)}{(\gamma - 1)\xi + (\gamma + 1)} \right]^{\frac{\gamma}{\gamma - 1}} \xi^{-\frac{1}{\gamma - 1}} = \left( \frac{6\xi + 1}{\xi + 6} \right)^{7/2} \xi^{-5/2} \quad (161)$$

$$\frac{V_2^2}{V_1^2} = 1 - \frac{2(\xi^2 - 1)}{M_1^2[(\gamma + 1)\xi + (\gamma - 1)]} = 1 - \frac{5(\xi^2 - 1)}{M_1^2(6\xi + 1)} \quad (162)$$

For weak shock waves, equations (113) and (114) apply to oblique as well as normal shocks.

SHOCK POLAR

The velocities associated with an oblique shock wave are conveniently represented in the velocity-vector (hodograph) plane. For a given Mach number ahead of the shock wave, all possible velocity vectors behind the shock lie on a single curve.

Only the closed loop represents real shock waves with non-decreasing entropy, and forms Busemann's shock polar (fig. 3). Its equation is

$$v_2^2 = (V_1 - u_2)^2 \frac{u_2 - \frac{a_*^2}{V_1}}{\frac{2}{\gamma + 1} V_1 + \frac{a_*^2}{V_1} - u_2} \quad (163)$$

Other forms of this equation convenient for computation are, given  $V_1$  and  $M_1$ ,

$$\left( \frac{v_2}{V_1} \right)^2 = \left( 1 - \frac{u_2}{V_1} \right)^2 \frac{(M_1^2 - 1) - \frac{\gamma + 1}{2} M_1^2 \left( 1 - \frac{u_2}{V_1} \right)}{1 + \frac{\gamma + 1}{2} M_1^2 \left( 1 - \frac{u_2}{V_1} \right)} = \left( 1 - \frac{u_2}{V_1} \right)^2 \frac{5(M_1^2 - 1) - 6M_1^2 \left( 1 - \frac{u_2}{V_1} \right)}{5 + 6M_1^2 \left( 1 - \frac{u_2}{V_1} \right)} \quad (164a)$$

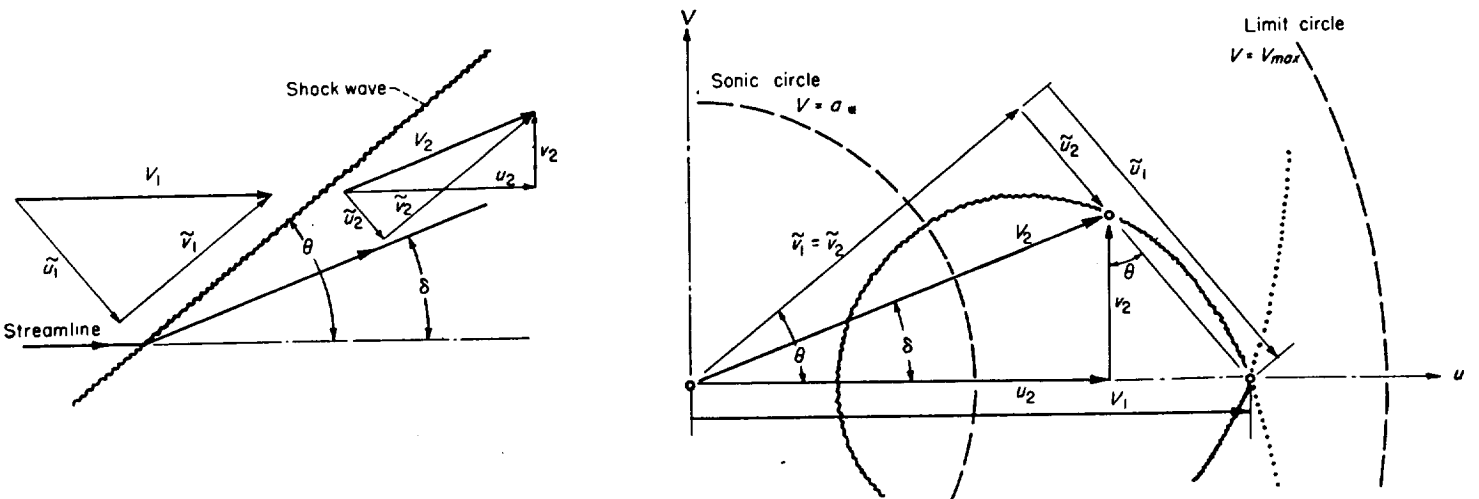


FIGURE 3.—Shock polar.

given  $a_*$  and  $V_1$ ,

$$\left(\frac{V_2}{a_*}\right)^2 = \left(\frac{V_1}{a_*} - \frac{u_2}{a_*}\right)^2 \frac{\frac{V_1}{a_*} \frac{u_2}{a_*} - 1}{1 + \frac{2}{\gamma+1} \left(\frac{V_1}{a_*}\right)^2 - \frac{V_1}{a_*} \frac{u_2}{a_*}}$$

$$\left(\frac{V_1}{a_*} - \frac{u_2}{a_*}\right)^2 \frac{6 \left(\frac{V_1}{a_*} \frac{u_2}{a_*} - 1\right)}{5 \left(\frac{V_1}{a_*}\right)^2 - 6 \left(\frac{V_1}{a_*} \frac{u_2}{a_*} - 1\right)} \quad (164b)$$

and given  $V_1$  and  $V_m$ ,

$$\left(\frac{v_2}{V_m}\right)^2 = \left(\frac{V_1}{V_m} - \frac{u_2}{V_m}\right)^2 \frac{\frac{V_1}{V_m} \frac{u_2}{V_m} - \frac{\gamma-1}{\gamma+1}}{\frac{2}{\gamma+1} \left(\frac{V_1}{V_m}\right)^2 + \frac{\gamma-1}{\gamma+1} - \frac{V_1}{V_m} \frac{u_2}{V_m}}$$

$$= \left(\frac{V_1}{V_m} - \frac{u_2}{V_m}\right)^2 \frac{\left(6 \frac{V_1}{V_m} \frac{u_2}{V_m} - 1\right)}{5 \left(\frac{V_1}{V_m}\right)^2 - \left(6 \frac{V_1}{V_m} \frac{u_2}{V_m} - 1\right)} \quad (164c)$$

The shock-wave angle  $\theta$  and wedge angle  $\delta$  are given in terms of the velocity components by

$$\tan \theta = \frac{V_1 - u_2}{v_2} = \frac{\tilde{u}_1}{\tilde{v}_1} \quad (165)$$

$$\tan \delta = \frac{v_2}{u_2} \quad (166)$$

The shock-wave angle  $\theta_*$  for sonic flow behind the shock is found (by setting  $M_2=1$  in eq. (132)) to be given by

$$\sin^2 \theta_* = \frac{1}{4\gamma M_1^2} \{(\gamma+1)M_1^2 - (3-\gamma) + \sqrt{(\gamma+1)[(\gamma+1)M_1^4 - 2(3-\gamma)M_1^2 + (\gamma+9)]}\}$$

$$= \frac{1}{7M_1^2} [3M_1^2 - 2 + \sqrt{3(3M_1^4 - 4M_1^2 + 13)}] \quad (167)$$

The shock-wave angle  $\theta_{s_{max}}$  for maximum stream deflection behind the shock is given by

$$\sin^2 \theta_{s_{max}} = \frac{1}{4\gamma M_1^2} \{(\gamma+1) M_1^2 - 4 + \sqrt{(\gamma+1)[(\gamma+1) M_1^4 + 8(\gamma-1) M_1^2 + 16]}\}$$

$$= \frac{1}{7M_1^2} [3M_1^2 - 5 + \sqrt{3(3M_1^4 + 4M_1^2 + 20)}] \quad (168)$$

For small deflection angles (hence Mach numbers close to unity), the deflection angle (radians) for sonic flow behind the shock is given approximately in terms of the upstream Mach number by

$$\delta_* = \frac{1}{\sqrt{2}(\gamma+1)} \frac{(M_1^2-1)^{3/2}}{M_1^2} = 0.2946 \frac{(M_1^2-1)^{3/2}}{M_1^2} \quad (169)$$

The maximum stream deflection angle for a specified upstream Mach number is given approximately by

$$\delta_{max} = \frac{4}{3\sqrt{3}(\gamma+1)} \frac{(M_1^2-1)^{3/2}}{M_1^2} = 0.3208 \frac{(M_1^2-1)^{3/2}}{M_1^2} \quad (170)$$

In unsteady flow all the above relations are valid across a moving oblique shock wave if instantaneous velocities are measured relative to the shock.

### SUPERSONIC FLOW PAST WEDGES AND CONES

A shock wave forms ahead of any body in supersonic flight and remains fixed relative to the body if the flight is steady. It stands ahead of blunt shapes, but may be attached to pointed shapes.

Just at the tip of a pointed airfoil or body of revolution the flow is the same as for the initially tangent wedge or cone. The bow wave is attached at sufficiently high Mach numbers for a wedge of semivertex angle  $\delta$  less than  $\sin^{-1}(1/\gamma) = 45.6^\circ$  for  $\gamma=7/5$ , and for a circular cone of semivertex angle  $\sigma$  less than  $57.5^\circ$  for  $\gamma=1.405$ . Below these limits, the wave is attached above a minimum Mach number whose dependence upon nose angle is shown for wedges and cones in figure 4. (These values can be applied to pointed airfoils and bodies of revolution which are not concave.) Also shown in figure 4 are the slightly higher Mach numbers above which the velocity behind the shock wave is supersonic, and for the cone the still higher Mach number above which the flow is supersonic even at the surface. (For wedges these last two coincide.) For thin wedges, these Mach numbers are given approximately by equations (169) and (170).

### FLOW PAST WEDGES

If the bow shock wave is attached to a wedge, it is straight, and the flow behind the shock consists of uniform streams parallel to either face of the wedge. The flow pattern above the upper face (fig. 5) may be regarded as obtained from the straight oblique shock-wave pattern of figure 2 by replacing the streamline behind the shock wave with a solid wall. Flow quantities are determined by the oblique-shock-wave relations, equations (115) to (170). As noted previously, table II can also be applied if  $M_1 \sin \theta$  is used in place of  $M_1$  in the first column.

The flows above and below the wedge are independent, so that inclined wedges can be treated if neither face exceeds the attachment angle shown in figure 4. However, if the angle of attack exceeds the semivertex angle, the flow over the upper (leeward) surface is given by a Prandtl-Meyer expansion (see fig. 4) rather than by the shock relations.

It is clear from the shock polar (fig. 3) that two different shock waves and flow patterns are theoretically possible for a given wedge and Mach number. However, it is believed that only the weaker shock wave (larger  $u_2$  and smaller  $\theta$ ) can occur attached to an isolated convex body.

Charts 2, 3, and 4 show the dependence of shock-wave angle, surface pressure coefficient, and downstream Mach number upon wedge angle for various free-stream Mach numbers.

### FLOW PAST CONES

If the bow shock wave is attached to an uninclined circular cone, the shock wave too has the form of a circular cone. Flow quantities are constant on all concentric conical surfaces lying between the shock wave and the body, and so depend upon only one space variable. The transition across the shock wave is governed by the oblique-shock relations,

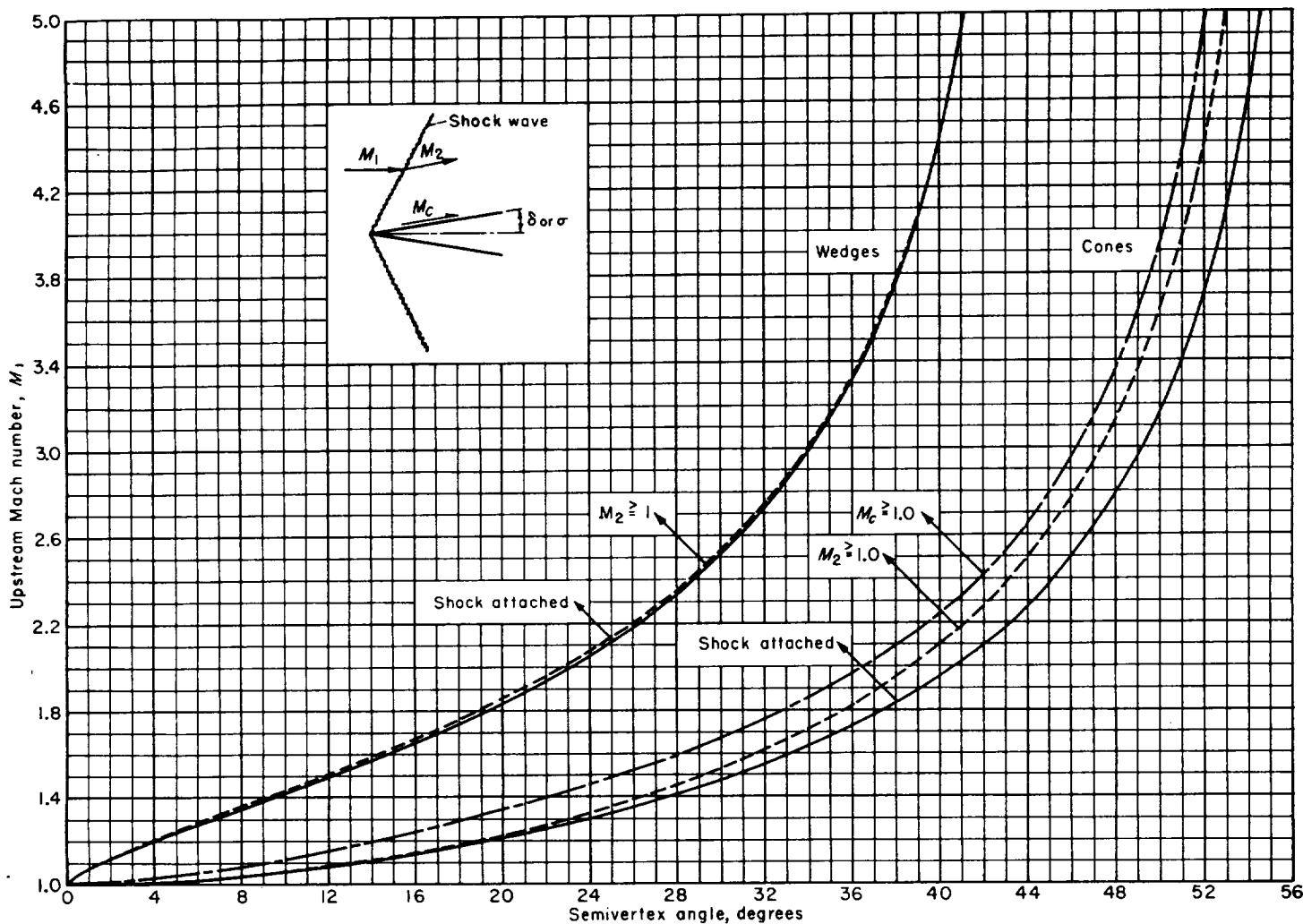


FIGURE 4.—Upstream Mach numbers for shock attachment and for supersonic flow behind shock wave on wedges and cones, and for supersonic flow at surface of cones.

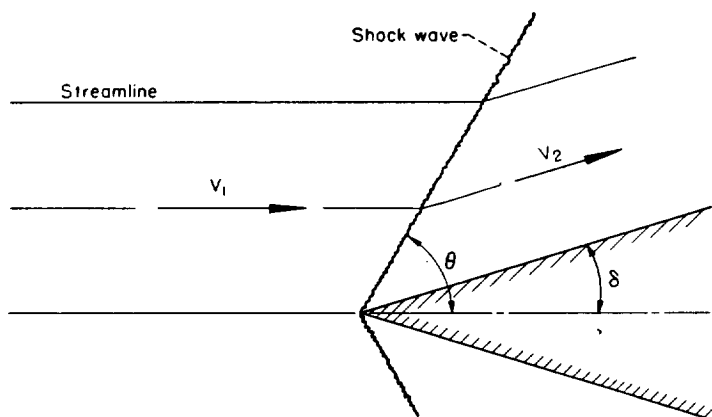


FIGURE 5.—Flow past a wedge.

and is followed by a continuous isentropic compression to surface conditions, as indicated in figure 6. The flow quantities have been extensively tabulated in reference 6 for  $\gamma=1.405$  and for  $\gamma=4/3$ . As in the case of wedges, two solutions exist for each cone and Mach number, but it is believed that only the weaker shock wave can occur on an isolated convex body. Charts 5, 6, and 7 show the dependence of shock-wave angle, surface-pressure coefficient, and

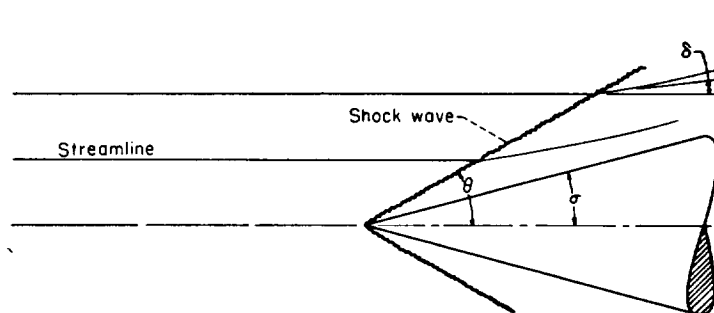


FIGURE 6.—Flow past a cone.

surface Mach number on cone semivertex angle for various free-stream Mach numbers.

The effects of slightly inclining a cone have been considered by Stone (ref. 13) and numerical results are tabulated in reference 14. Chart 8 shows the variation with Mach number of the initial slope of the normal-force curve for various cone angles. Stone has also sought an approximation for larger inclinations by retaining squares as well as first powers of angle of attack (ref. 15), and numerical results have been tabulated (ref. 16); however, these results are not free of error (see refs. 17 and 18).

## PRANDTL-MEYER EXPANSION

A uniform two-dimensional supersonic stream flowing over a convex bend expands isentropically. Convenient relations are found by considering the special case of a stream at Mach number unity flowing around a sharp corner (fig. 7).

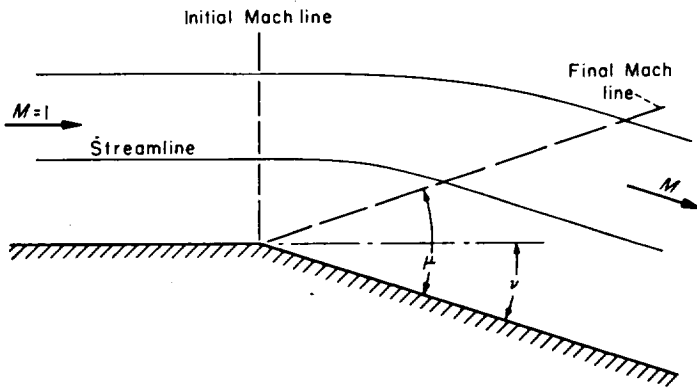


FIGURE 7.—Prandtl-Meyer expansion around a corner.

For a perfect gas, the Prandtl-Meyer angle  $\nu$  through which the stream turns in expanding from  $M=1$  to a supersonic Mach number  $M$  is

$$\nu = \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2-1)} - (90^\circ - \mu) \quad (171a)$$

$$= \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2-1)} - \cos^{-1} \frac{1}{M} \quad (171b)$$

$$= \sqrt{\frac{\gamma+1}{\gamma-1}} \tan^{-1} \sqrt{\frac{\gamma-1}{\gamma+1} (M^2-1)} - \tan^{-1} \sqrt{M^2-1} \quad (171c)$$

(For  $\gamma=7/5$ ,  $\sqrt{\frac{\gamma+1}{\gamma-1}}=2.4495$ , and  $\sqrt{\frac{\gamma-1}{\gamma+1}}=0.40825$ .) The maximum expansion angle, for  $M=\infty$ , is

$$\nu_{\max} = \left( \sqrt{\frac{\gamma+1}{\gamma-1}} - 1 \right) \times 90^\circ = 130.45^\circ \text{ for } \gamma=7/5 \quad (172)$$

The ratio of static to total pressure, corresponding to Mach number  $M$  is given by

$$\left( \frac{p}{p_t} \right)^{\frac{\gamma-1}{\gamma}} = \frac{1}{\gamma+1} \left\{ 1 + \cos \left[ 2 \sqrt{\frac{\gamma-1}{\gamma+1}} (\nu + 90^\circ - \mu) \right] \right\} \quad (173a)$$

$$= \frac{1}{\gamma+1} \left\{ 1 + \cos \left[ 2 \sqrt{\frac{\gamma-1}{\gamma+1}} \left( \nu + \cos^{-1} \frac{1}{M} \right) \right] \right\} \quad (173b)$$

$$= \frac{1}{\gamma+1} \left\{ 1 + \cos \left[ 2 \sqrt{\frac{\gamma-1}{\gamma+1}} \left( \nu + \tan^{-1} \sqrt{M^2-1} \right) \right] \right\} \quad (173c)$$

which falls to zero as  $\nu \rightarrow \nu_{\max}$ . Numerical values of  $\nu$ ,  $\mu$ , and  $p/p_t$  are given in table II as functions of  $M$ .

These relations and the values in table II apply to a uniform stream flowing past any convex surface in the ab-

sence of external disturbances. (They also give a very good approximation at all Mach numbers when, as on an airfoil, external disturbances arise only from interaction with a shock wave, and are disregarded.) If flow quantities are known at one point, the values at any second point can be read from table II by identifying the change in flow angle between the two points with  $\Delta\nu = \nu_2 - \nu_1$ , as indicated in figure 8.

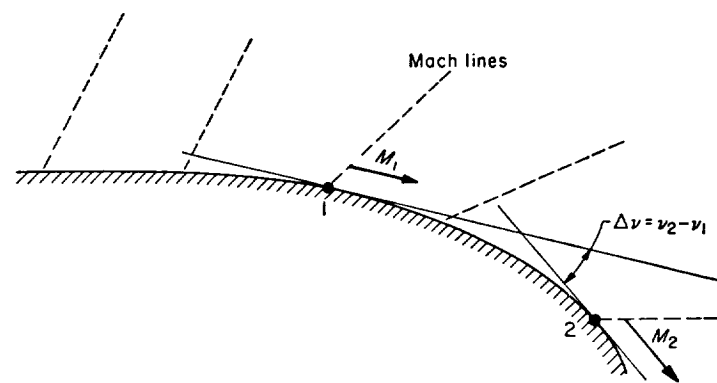


FIGURE 8.—Prandtl-Meyer expansion over a convex surface.

For expansions through small angles  $\Delta\nu$ , the ratio of final to initial static pressures is given by the following series ( $\Delta\nu$  in radians):

$$\frac{p_2}{p_1} = 1 - \frac{\gamma M_1^2}{\sqrt{M_1^2-1}} (\Delta\nu) + \gamma M_1^2 \frac{(\gamma+1) M_1^4 - 4(M_1^2-1)}{4(M_1^2-1)^2} (\Delta\nu)^2 - \frac{\gamma M_1^2}{2(M_1^2-1)^{7/2}} \left[ \frac{\gamma+1}{6} M_1^8 - \frac{5+7\gamma-2\gamma^2}{6} M_1^6 + \frac{5}{3} (\gamma+1) M_1^4 - 2M_1^2 + \frac{4}{3} \right] (\Delta\nu)^3 + \dots \quad (174)$$

Up to and including the term in  $(\Delta\nu)^2$  this series is identical with that for compression through an oblique shock wave (eq. (151) with  $\delta = -\Delta\nu$ ).

## IMPERFECT-GAS EFFECTS

Methods for calculating the flow of a calorically imperfect, thermally imperfect gas and a calorically imperfect, thermally perfect gas at temperatures up to 5000° R are described in this section. The equations presented are in substantially the same form as those given in references 7 and 8. Effects of gaseous imperfections, such as molecular dissociation, which become important at temperatures greater than about 5000° R are not considered.

Atmospheric and wind-tunnel air flows are of primary concern here. In such flows air generally exhibits only caloric imperfections to any appreciable degree. Consequently, numerical results are presented only for the flow of a calorically imperfect, thermally perfect diatomic gas.

## THERMODYNAMICS

## EQUATIONS OF STATE

The thermal equation of state used here for a calorically and thermally imperfect gas is the Berthelot equation

(eq. (5)). The thermal equation of state used for a calorically imperfect, thermally perfect gas is equation (2). The caloric equation of state used for a calorically and thermally imperfect gas is equation (8a). The caloric equation of state used for a calorically imperfect, thermally perfect gas is equation (8b).

## SPECIFIC HEATS

The assumption of a simple harmonic vibrator is used to account for the contribution of the vibrational heat capacity to the specific heats. The equations for the specific heats at constant volume and constant pressure, respectively, are (see ref. 7)

$$c_v = (c_v)_{\text{pert}} \left\{ 1 + (\gamma_{\text{pert}} - 1) \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} + \frac{2c_p}{RT^2} \right] \right\} \quad (175)$$

$$c_v = (c_v)_{\text{pert}} \left\{ 1 + (\gamma_{\text{pert}} - 1) \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} \right] \right\} [\text{therm perf}] \quad (176)$$

$$c_p = (c_p)_{\text{pert}} \left\{ 1 + \frac{\gamma_{\text{pert}} - 1}{\gamma_{\text{pert}}} \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} + \frac{2c_p}{RT^2} \left( 1 + \frac{2 - b_p}{1 - b_p} + \frac{c_p}{2RT^2} \right) \right] \right\} \quad (177)$$

$$c_p = (c_p)_{\text{pert}} \left\{ 1 + \frac{\gamma_{\text{pert}} - 1}{\gamma_{\text{pert}}} \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} \right] \right\} [\text{therm perf}] \quad (178)$$

The ratio of specific heats is then

$$\gamma = \gamma_{\text{pert}} \times$$

$$\left[ \frac{1 + \frac{\gamma_{\text{pert}} - 1}{\gamma_{\text{pert}}} \left\{ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} + \frac{2c_p}{RT^2} \left[ 1 + \frac{2 - b_p}{1 - b_p} + \frac{c_p}{2RT^2} \right] \right\}}{1 + (\gamma_{\text{pert}} - 1) \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} + \frac{2c_p}{RT^2} \right]} \right] \quad (179)$$

or, for a thermally perfect gas,

$$\gamma = 1 + \frac{\gamma_{\text{pert}} - 1}{1 + (\gamma_{\text{pert}} - 1) \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} \right]} [\text{therm perf}] \quad (180)$$

The following values of  $\gamma$  are for temperatures from 400° R to 5000° R, with  $\Theta = 5500^\circ$  R (see ref. 7). For engineering purposes, these are a satisfactory approximation for air.

VARIATION OF RATIO OF SPECIFIC HEATS WITH TEMPERATURE					
T, °R	$\gamma$	T, °R	$\gamma$	T, °R	$\gamma$
500	1.400	1300	1.361	2200	1.322
600	1.399	1400	1.355	2400	1.317
700	1.396	1500	1.349	2600	1.313
800	1.392	1600	1.344	2800	1.309
900	1.387	1700	1.339	3000	1.305
1000	1.381	1800	1.335	3500	1.301
1100	1.375	1900	1.331	4000	1.298
1200	1.368	2000	1.328	4600	1.296
				5000	1.294

## CONTINUOUS ONE-DIMENSIONAL FLOW

## BASIC EQUATIONS AND DEFINITIONS

Basic equations pertinent to this section are equations (26), (27), (28), (29), (30), and (31). The equations for the speed of sound are (see ref. 7)

$$a^2 = RT \left\{ \frac{1}{(1 - b_p)^2} - \frac{2c_p}{RT^2} + \frac{(\gamma_{\text{pert}} - 1) \left( \frac{c_p}{RT^2} + \frac{1}{1 - b_p} \right)^2}{1 + (\gamma_{\text{pert}} - 1) \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} + \frac{2c_p}{RT^2} \right]} \right\} \quad (181)$$

and

$$a^2 = RT \left\{ 1 + \frac{\gamma_{\text{pert}} - 1}{\left[ 1 + (\gamma_{\text{pert}} - 1) \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} \right]} \right\} [\text{therm perf}] \quad (182)$$

## INTEGRATED FORMS OF ENERGY EQUATION

The integrated forms of the energy equation are (see ref. 7)

$$V^2 = 2RT_i \left[ \frac{1 - T_i}{\gamma_{\text{pert}} - 1} + \frac{\Theta}{T_i} \left( \frac{1}{e^{\Theta/T_i} - 1} - \frac{1}{e^{\Theta/T} - 1} \right) + \frac{2c}{RT_i} \left( \frac{\rho}{T} - \frac{\rho_i}{T_i} \right) + \frac{1}{RT_i} \left( \frac{p_i}{\rho_i} - \frac{p}{\rho} \right) \right] [\text{adiab}] \quad (183)$$

and

$$V^2 = 2RT_i \left[ \frac{\gamma_{\text{pert}}}{\gamma_{\text{pert}} - 1} \left( 1 - \frac{T}{T_i} \right) + \frac{\Theta}{T_i} \left( \frac{1}{e^{\Theta/T_i} - 1} - \frac{1}{e^{\Theta/T} - 1} \right) \right] [\text{adiab, therm perf}] \quad (184)$$



In terms of Mach number these equations become, respectively,

$$M^2 = \frac{2T_t \left[ \frac{1 - \frac{T}{T_t}}{\gamma_{\text{perf}} - 1} + \frac{\Theta}{T_t} \left( \frac{1}{e^{\Theta/T_t} - 1} - \frac{1}{e^{\Theta/T} - 1} \right) + \frac{2c}{RT_t} \left( \frac{\rho}{T} - \frac{\rho_t}{T_t} \right) + \frac{1}{RT_t} \left( \frac{p_t}{\rho_t} - \frac{p}{\rho} \right) \right]}{T \left\{ \frac{1}{(1 - b\rho)^2} - \frac{2c\rho}{RT^2} + \frac{(\gamma_{\text{perf}} - 1) \left( \frac{c\rho}{RT^2} + \frac{1}{1 - b\rho} \right)^2}{1 + (\gamma_{\text{perf}} - 1) \left[ \left( \frac{\Theta}{T} \right)^2 \frac{e^{\Theta/T}}{(e^{\Theta/T} - 1)^2} + \frac{2c\rho}{RT^2} \right]} \right\}} \quad [\text{adiab}] \quad (185)$$

and

$$M^2 = \frac{2T_t}{\gamma T} \left[ \frac{\gamma_{\text{perf}}}{\gamma_{\text{perf}} - 1} \left( 1 - \frac{T}{T_t} \right) + \frac{\Theta}{T_t} \left( \frac{1}{e^{\Theta/T_t} - 1} - \frac{1}{e^{\Theta/T} - 1} \right) \right] \quad [\text{adiab, therm perf}] \quad (186)$$

where  $\gamma$  is given by equation (180).

The variations of  $\frac{\left(\frac{V}{a_*}\right)_{\text{therm perf}}}{\left(\frac{V}{a_*}\right)_{\text{perf}}}$  and  $\frac{\left(\frac{T}{T_t}\right)_{\text{therm perf}}}{\left(\frac{T}{T_t}\right)_{\text{perf}}}$  with Mach number for several values of total temperature  $T_t$  are given in charts 9 and 10.

#### PRESSURE AND DENSITY RELATIONS

For isentropic flow, the relations between density and temperature are (see ref. 7)

$$\left(\frac{\rho}{\rho_t}\right) \left(\frac{1 - b\rho_t}{1 - b\rho}\right) = \left(\frac{e^{\Theta/T_t} - 1}{e^{\Theta/T} - 1}\right) \left(\frac{T}{T_t}\right)^{\frac{1}{\gamma_{\text{perf}} - 1}} \exp \left[ \frac{c\rho_t}{RT_t^2} - \frac{c\rho}{RT^2} + \left(\frac{\Theta}{T}\right) \frac{e^{\Theta/T}}{e^{\Theta/T} - 1} - \left(\frac{\Theta}{T_t}\right) \frac{e^{\Theta/T_t}}{e^{\Theta/T_t} - 1} \right] \quad [\text{isen}] \quad (187)$$

and, for a thermally perfect gas,

$$\frac{\rho}{\rho_t} = \left(\frac{e^{\Theta/T_t} - 1}{e^{\Theta/T} - 1}\right) \left(\frac{T}{T_t}\right)^{\frac{1}{\gamma_{\text{perf}} - 1}} \exp \left[ \left(\frac{\Theta}{T}\right) \frac{e^{\Theta/T}}{e^{\Theta/T} - 1} - \left(\frac{\Theta}{T_t}\right) \frac{e^{\Theta/T_t}}{e^{\Theta/T_t} - 1} \right] \quad [\text{isen, therm perf}] \quad (188)$$

The variation of  $\frac{\left(\frac{\rho}{\rho_t}\right)_{\text{therm perf}}}{\left(\frac{\rho}{\rho_t}\right)_{\text{perf}}}$  with Mach number for several total temperatures is presented in chart 11.

For the isentropic flow of a thermally imperfect, calorically imperfect gas, the relation between pressure, density, and temperature can be obtained by a trial-and-error procedure using equations (5) and (187).<sup>5</sup> For the isentropic flow of a thermally perfect gas, the relation between pressure and temperature is

$$\frac{p}{p_t} = \left(\frac{e^{\Theta/T_t} - 1}{e^{\Theta/T} - 1}\right) \left(\frac{T}{T_t}\right)^{\frac{\gamma_{\text{perf}}}{\gamma_{\text{perf}} - 1}} \exp \left[ \left(\frac{\Theta}{T}\right) \frac{e^{\Theta/T}}{e^{\Theta/T} - 1} - \left(\frac{\Theta}{T_t}\right) \frac{e^{\Theta/T_t}}{e^{\Theta/T_t} - 1} \right] \quad [\text{isen, therm perf}] \quad (189)$$

The relation between dynamic and static pressure for a thermally imperfect gas can be obtained by a trial-and-error procedure using equations (5), (31a), (183), and (187). The relation between dynamic and static pressure for a thermally perfect gas can be obtained with equations (31b) and (186), and is

$$\frac{q}{p} = \frac{\gamma_{\text{perf}}}{\gamma_{\text{perf}} - 1} \left(\frac{T_t}{T} - 1\right) + \frac{\Theta}{T} \left(\frac{1}{e^{\Theta/T_t} - 1} - \frac{1}{e^{\Theta/T} - 1}\right) \quad [\text{adiab, therm perf}] \quad (190)$$

The variations of  $\frac{\left(\frac{p}{p_t}\right)_{\text{therm perf}}}{\left(\frac{p}{p_t}\right)_{\text{perf}}}$  and  $\frac{\left(\frac{q}{p_t}\right)_{\text{therm perf}}}{\left(\frac{q}{p_t}\right)_{\text{perf}}}$  with Mach

number for several total temperatures are given in charts 12 and 13.

#### STREAM-TUBE-AREA RELATIONS

The stream-tube-area relation is given by equation (79), or, in more convenient form,

$$\frac{A}{A_*} = \frac{\rho_* a_*}{\rho a M} \quad (191)$$

This ratio can be evaluated for a thermally imperfect gas with the aid of equations (187), (181), (5), and (185), and for a thermally perfect gas with the aid of equations (188),

(182), and (186). The variation of  $\frac{\left(\frac{A}{A_*}\right)_{\text{therm perf}}}{\left(\frac{A}{A_*}\right)_{\text{perf}}}$  with Mach

number for several values of total temperature is presented in chart 14.

<sup>5</sup> In this, as in many of the cases to be presented, no direct solution for flow properties is possible if the gas exhibits both thermal and caloric imperfections. Approximate solutions of this type can be obtained, however, if the degree of imperfection is small (see ref. 7).

**NORMAL SHOCK WAVES**

The requirements for conservation of mass, momentum, and energy across a normal shock wave are given by equations (84), (85), and (86a). The energy relation can be written

$$\frac{u_2^2}{2} - \frac{u_1^2}{2} + \frac{R}{\gamma_{\text{pert}} - 1} (T_2 - T_1) - \left( \frac{2c\rho_2}{T_2} - \frac{2c\rho_1}{T_1} \right) + \left( \frac{p_2}{\rho_2} - \frac{p_1}{\rho_1} \right) + R\theta \left( \frac{1}{e^{\theta/T_2} - 1} - \frac{1}{e^{\theta/T_1} - 1} \right) = 0 \quad [\text{adiab}] \quad (192)$$

or, for a thermally perfect gas,

$$\frac{u_2^2}{2} - \frac{u_1^2}{2} + \left( \frac{\gamma_{\text{pert}}}{\gamma_{\text{pert}} - 1} \right) R(T_2 - T_1) + R\theta \left( \frac{1}{e^{\theta/T_2} - 1} - \frac{1}{e^{\theta/T_1} - 1} \right) = 0 \quad [\text{adiab, therm perf}] \quad (193)$$

No explicit equation has been found to relate the temperature downstream of a normal shock wave in thermally imperfect air to the upstream conditions. A trial-and-error procedure, starting with assumed values of  $\rho_2$  and  $T_2$  and involving equations (5), (84), (85), and (192), can be used to determine the downstream temperature.

For the flow of a thermally perfect gas, the simultaneous solution of equations (84), (85), (193), and (2) yields the following relation from which the temperature behind the shock wave can be found:

$$\left( u_1 + \frac{RT_1}{u_1} \right)^2 - \left( u_1 + \frac{RT_1}{u_1} \right) \sqrt{\left( u_1 + \frac{RT_1}{u_1} \right)^2 - 4RT_2 - 2RT_2 - 2u_1^2 + \left( \frac{\gamma_{\text{pert}}}{\gamma_{\text{pert}} - 1} \right) 4R(T_2 - T_1) + 4R\theta \left( \frac{1}{e^{\theta/T_2} - 1} - \frac{1}{e^{\theta/T_1} - 1} \right)} = 0 \quad [\text{adiab, therm perf}] \quad (194)$$

Since the total temperature  $T_t$  remains constant across a shock wave, other flow parameters behind the shock wave can be found with the aid of previously presented one-dimensional flow relations. The variations of

$$\frac{\left( \frac{T_2}{T_1} \right)_{\text{therm perf}}}{\left( \frac{T_2}{T_1} \right)_{\text{pert}}}, \frac{\left( \frac{\rho_2}{\rho_1} \right)_{\text{therm perf}}}{\left( \frac{\rho_2}{\rho_1} \right)_{\text{pert}}}, \frac{\left( \frac{p_1}{p_{t_2}} \right)_{\text{therm perf}}}{\left( \frac{p_1}{p_{t_2}} \right)_{\text{pert}}}, \frac{\left( \frac{p_2}{p_1} \right)_{\text{therm perf}}}{\left( \frac{p_2}{p_1} \right)_{\text{pert}}}, \frac{M_{2,\text{therm perf}}}{M_{2,\text{pert}}}, \text{ and } \frac{\left( \frac{p_{t_2}}{p_{t_1}} \right)_{\text{therm perf}}}{\left( \frac{p_{t_2}}{p_{t_1}} \right)_{\text{pert}}}$$

with upstream Mach number for several total temperatures are presented in charts 15 through 20, respectively.

**OBLIQUE SHOCK WAVES**

For a thermally imperfect gas, no simple equations can be found to relate the values of the flow parameters across oblique shock waves. In general, trial-and-error procedure, starting with assumed values of  $\rho_2$  and  $T_2$ , and involving the relations for the conservation of mass, momentum, and energy, must be used. (See eqs. (115), (116), (117), and (118a) as well as equations (5) and (183).) For a thermally perfect gas, the Mach number downstream of an oblique shock wave can be found with the aid of the energy equation (see eqs. (118a) and (186)), thus

$$M_2^2 = \frac{2T_1}{\gamma_2 T_2} \left[ \frac{\gamma_1 M_1^2}{2} + \left( \frac{\gamma_{\text{pert}}}{\gamma_{\text{pert}} - 1} \right) \left( 1 - \frac{T_2}{T_1} \right) + \frac{\theta}{T_1} \left( \frac{1}{e^{\theta/T_1} - 1} - \frac{1}{e^{\theta/T_2} - 1} \right) \right] \quad [\text{adiab, therm perf}] \quad (195)$$

where  $\gamma_1$  and  $\gamma_2$  are the functions of  $T_1$  and  $T_2$ , respectively, given by equation (180). The pressure ratio across the shock is given by

$$\frac{p_1}{p_2} = \frac{1}{2} \left\{ (1 + \gamma_2 M_2^2) - \frac{T_1}{T_2} (1 + \gamma_1 M_1^2) + \sqrt{\left[ (1 + \gamma_2 M_2^2) - \frac{T_1}{T_2} (1 + \gamma_1 M_1^2) \right]^2 + 4 \frac{T_1}{T_2}} \right\} \quad [\text{adiab, therm perf}] \quad (196)$$

The density ratio can be determined from the equation of state (eq. (2)) with the aid of the pressure and temperature ratios. The shock-wave and deflection angles are given by (see ref. 8)

$$\sin^2 \theta = \frac{\left( \frac{\gamma_2}{\gamma_1} \right) \left( \frac{T_2}{T_1} \right) \left( \frac{M_2}{M_1} \right)^2 - 1}{\left( \frac{\rho_1}{\rho_2} \right)^2 - 1} \quad [\text{adiab, therm perf}] \quad (197)$$

and

$$\cot \delta = \tan \theta \left( \frac{\gamma_1 M_1^2 - 1}{\frac{p_2}{p_1} - 1} \right) \quad [\text{adiab, therm perf}] \quad (198)$$

respectively.

The variation of  $\theta$  with  $\delta$  for various values of  $M_1$  and  $T_1$  is presented in chart 21. In addition, the variations of

$$\frac{(M_2)_{\text{therm perf}}}{(M_2)_{\text{pert}}} \text{ and } \frac{\left( \frac{p_2 - p_1}{q_1} \right)_{\text{therm perf}}}{\left( \frac{p_2 - p_1}{q_1} \right)_{\text{pert}}}$$

are presented in charts 22 and 23.

Values of the ratios

$$\frac{p_2}{p_1}, \frac{\rho_2}{\rho_1}, \frac{T_2}{T_1}, \frac{p_{t_2}}{p_{t_1}}$$

for the flow of a thermally perfect gas across an oblique shock wave can be determined from the normal-shock relations,

provided that  $M_1 \sin \theta$  is used instead of  $M_1$  and that the static temperature  $T_1$  just upstream of the shock wave is the same for the oblique shock wave as for the normal shock wave.

#### PRANDTL-MEYER EXPANSION

The Prandtl-Meyer angle for the flow of an imperfect gas can be found by graphically integrating the equation (see ref. 8)

$$\nu = - \int_{p_0}^p \frac{dp}{\rho V^2 \tan \mu} \quad [\text{isen}] \quad (199)$$

The relations between  $p$ ,  $\rho$ ,  $V$ , and  $\mu$  can be found with the

aid of equations (5), (187), (183), and (185). For a thermally perfect gas this equation becomes (see, again, ref. 8)

$$\nu = - \int_{p_0}^p \frac{\sin 2\mu}{2\gamma p} dp \quad [\text{isen, therm perf}] \quad (200)$$

The relations between  $\gamma$ ,  $p$ , and  $\mu$  can be found with the aid of equations (180), (189), and (186) using the temperature as a parameter. The graphical integration of equation (200) has been carried out, and the variations of  $\nu_{\text{therm perf}}$  and  $\frac{\nu_{\text{therm perf}}}{\nu_{\text{perf}}}$  with Mach number for various values of total temperature are presented in chart 24.

## APPENDIX A

### VISCOSITY AND THERMODYNAMIC CONSTANTS FOR AIR

#### VISCOSITY

The viscosity of air is nearly independent of pressure; the variation with absolute temperature, between temperatures of about 300° R and 900° R, may be approximated by the formula

$$\frac{\mu}{\mu_r} = \left(\frac{T}{T_r}\right)^{0.76} \quad (A1)$$

For a wider range of temperatures, between about 180° R and 3400° R, Sutherland's formula (see ref. 19) is more accurate:

$$\frac{\mu}{\mu_r} = \frac{T_r + 198.6}{T + 198.6} \left(\frac{T}{T_r}\right)^{3/2} \quad (A2)$$

The viscosity of air, as determined from this relation, may be expressed as

$$\mu = 2.270 \frac{T^{3/2}}{T + 198.6} \times 10^{-3} \frac{\text{lb sec}}{\text{ft}^2} \quad (A3)$$

This latter equation has been employed in the calculations of Reynolds number (chart 25).

#### THERMODYNAMIC CONSTANTS

The value of  $\gamma$  employed for air, when treated as a completely perfect gas, is 7/5. This simple value, which has been employed in table I, table II, charts 1 to 4, and chart 25, is a good approximation to the more precise values obtained from spectroscopic measurements (see ref. 20). Values of  $c_p$ ,  $c_v$ , and  $R$  for air, consistent with the approximation  $\gamma=7/5$ , are

$$c_p = 6006 \text{ ft}^2/\text{sec}^2 \text{ }^\circ\text{R}$$

$$c_v = 4290 \text{ ft}^2/\text{sec}^2 \text{ }^\circ\text{R}$$

$$R = 1716 \text{ ft}^2/\text{sec}^2 \text{ }^\circ\text{R}$$

## APPENDIX B

### REYNOLDS NUMBER

Reynolds number is defined as

$$R = \frac{\rho V l}{\mu} \quad (B1)$$

For sea-level conditions,

$$R \cong 10,000 \text{ (} V \text{ in mph) (} l \text{ in ft)} \quad (B2)$$

In a wind tunnel (subsonic or supersonic), if isentropic expansion is assumed from a total pressure  $p_t$  and equation

(A2) is used for the variation of viscosity with temperature, the Reynolds number per unit reference length is given by

$$\frac{R}{l} = \frac{p_t M}{\mu_t} \sqrt{\frac{\gamma}{(\gamma-1)c_p T_t}} \left(\frac{T_t}{T}\right)^{\frac{\gamma-2}{\gamma-1}} \frac{T_t + \frac{198.6}{T_t}}{1 + \frac{198.6}{T_t}} \quad [\text{perf}] \quad (B3)$$

The Reynolds number per unit length for  $p_t=1$  psia has been plotted in chart 25 as a function of  $M$  for various total temperatures  $T_t$ .

## APPENDIX C

### PRESSURE CONVERSION FACTORS AND CONSTANTS

Multiply by to obtain	lb in. <sup>2</sup>	lb ft <sup>2</sup>	in. H <sub>2</sub> O at 70° F	in. Hg at 70° F	cm. Hg at 70° F	Standard atmos- pheres
lb/in. <sup>2</sup>	1	0.006944	0.03607	0.4892	0.1926	14.70
lb/ft <sup>2</sup>	144	1	5.194	70.45	27.74	2117
in. H <sub>2</sub> O (70° F)	27.73	.1925	1	13.56	5.340	407.6
in. Hg. (70° F)	2.044	.01420	.07373	1	.3937	30.05
cm. Hg. (70° F)	5.192	.03605	.1873	2.540	1	76.33
Standard atmospheres	.06804	.0004725	.002453	.03328	.01310	1

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## TABLES

The tables that follow contain numerical values for certain quantities often required for the solution of problems in compressible flow. The symbols used in these tables are the same as those used in the preceding sections. For convenience, however, the symbols are redefined at the end of table II.

To conserve space, a modified computing-machine notation has been adopted to indicate the position of the decimal point in the tabulated quantities. The location of the decimal point is governed by the following rules:

(a) A group of digits followed by  $-n$  indicates that the decimal point should be  $n$  places to the left of the first digit.

Example:  $.3268_{-3} = .0003268$

(b) A group of digits followed by  $+n$  indicates that the decimal point should be  $n$  places to the right of the last digit.

Example:  $3268_{+3} = 3,268,000$

(c) A group of digits without a suffix indicates that the decimal point is correctly located as printed.

## TABLE I.—SUBSONIC FLOW

The ratios given by equations (43), (44), (45), (48), (50), and (83) are given as functions of Mach number. If, at a point in an isentropic flow, any one of these ratios or the Mach number is known, then all other ratios for that point can be read or interpolated from the table. In addition, the parameter  $\beta = \sqrt{|M^2 - 1|}$ , which is sometimes more convenient to use than the Mach number itself, is also tabulated.

## TABLE II.—SUPERSONIC FLOW

The ratios given in table I for subsonic flow are also given in table II for supersonic flow. The Mach angle  $\mu$  and the Prandtl-Meyer angle  $\nu$  are also given as functions of Mach number. In addition to these point functions for isentropic flow, the normal-shock relations given by equations (93), (94), (95), (96), (99), and (100) are tabulated as functions of the Mach number  $M_1$  ahead of the shock wave. Although these values are for normal shock waves, the values of  $p_2/p_1$ ,  $\rho_2/\rho_1$ ,  $T_2/T_1$ , and  $p_{t_2}/p_{t_1}$  may also be used for oblique shock waves, provided  $M_1 \sin \theta$  is used instead of  $M_1$  in the first column.

EQUATIONS, TABLES, AND CHARTS FOR COMPRESSIBLE FLOW

TABLE I.—SUBSONIC FLOW

γ=7/5

Table I: Subsonic flow data for γ=7/5. Columns include Mach number (M), pressure ratios (p/p1, p/p1), temperature ratios (T/T1), density ratios (ρ/ρ1), area ratios (A/A\*), velocity ratios (V/a\*), and other parameters. The table is organized into two main sections for Mach numbers 0 to 1.00.

TABLE II.—SUPERSONIC FLOW

γ=7/5

Table II: Supersonic flow data for γ=7/5. Columns include Mach number (M or M1), pressure ratios (p/p1, p/p1), temperature ratios (T/T1), density ratios (ρ/ρ1), area ratios (A/A\*), velocity ratios (V/a\*), and other parameters. The table covers Mach numbers from 1.00 to 1.24.

















EQUATIONS, TABLES, AND CHARTS FOR COMPRESSIBLE FLOW

TABLE II.—SUPERSONIC FLOW—Continued

$\gamma=7/5$

Table with 17 columns: M or M1, P/P1, P/P1, T/T1, beta, q/P1, A/A\*, V/a\*, mu, mu, M2, P2/P1, P2/P1, T2/T1, P2/P1, P2/P1. The table contains numerical data for various flow conditions, with values ranging from 7.55 to 8.44 in the first column and corresponding values in the other columns.





TABLE II.—SUPERSONIC FLOW—Continued

γ=7/5

Table with 16 columns: M or M1, p/p1, rho/rho1, T/T1, beta, q/p1, A/A\*, V/a\*, mu, M2, p2/p1, rho2/rho1, T2/T1, p2/p1, p1/p2. It contains a grid of numerical data for Mach numbers ranging from 9.35 to 10.48.





EQUATIONS, TABLES, AND CHARTS FOR COMPRESSIBLE FLOW

TABLE II.—SUPERSONIC FLOW—Continued

γ=7/5

Table with 17 columns: M or M1, p/p1, rho/rho1, T/T1, beta, q/p1, A/A\*, V/a\*, nu, mu, M2, p2/p1, rho2/rho1, T2/T1, p2/p1, p1/p2. Rows contain numerical data for various Mach numbers from 12.30 to 14.08.





TABLE II.-SUPERSONIC FLOW-Continued

γ = 7/5

Table with columns: M or M1, p/p1, rho/rho1, T/T1, beta, q/p1, A/A\*, V/a\*, mu, M2, p2/p1, rho2/rho1, T2/T1, P12/P1, P1/P2. It contains a grid of data points for various Mach numbers and flow parameters.

EQUATIONS, TABLES, AND CHARTS FOR COMPRESSIBLE FLOW

TABLE II.—SUPERSONIC FLOW—Continued

γ=7/5

Table with columns: M or M1, p/pi, rho/prho1, T/T1, beta, q/pi, A/A\*, V/a\*, mu, M2, p2/p1, rho2/rho1, T2/T1, p2/p1, p1/p2. Rows range from M=19.50 to 32.80.



TABLE II.-SUPERSONIC FLOW-Continued

$\gamma=7/5$

Table with columns: M of M1, p/p1, rho/rho1, T/T1, beta, q/p1, A/A\*, V/a\*, mu, mu, M2, p2/p1, rho2/rho1, T2/T1, p2/p1, p1/p2. Rows range from M=33.00 to M=54.00.

TABLE II.—SUPERSONIC FLOW—Concluded

$\gamma=7/5$

$M$ or $M_1$	$\frac{p}{p_1}$	$\frac{\rho}{\rho_1}$	$\frac{T}{T_1}$	$\beta$	$\frac{q}{p_1}$	$\frac{A}{A_*}$	$\frac{V}{a_*}$	$\nu$	$\mu$	$M_2$	$\frac{p_2}{p_1}$	$\frac{\rho_2}{\rho_1}$	$\frac{T_2}{T_1}$	$\frac{p_{t2}}{p_{t1}}$	$\frac{p_1}{p_{t2}}$
55.00	.1826 →	.1106 ←	.1650 →	54.99	.3866 ←	2341 →	2.4474679	125.25	1.042	.3783	3529	5.990	589.1	.7111 ←	.2567 →
56.00	.1609 →	.1011 ←	.1592 →	55.99	.3533 ←	2562 →	2.4473304	125.34	1.023	.3783	3659	5.990	610.7	.6499 ←	.2476 →
57.00	.1422 →	.9256 →	.1537 →	56.99	.3235 ←	2798 →	2.4476071	125.43	1.005	.3783	3790	5.991	632.7	.5950 ←	.2390 →
58.00	.1259 →	.8485 →	.1484 →	57.99	.2965 ←	3052 →	2.4476714	125.52	.9879	.3783	3925	5.991	655.1	.5455 ←	.2308 →
59.00	.1118 →	.7791 →	.1434 →	58.99	.2723 ←	3324 →	2.4477325	125.60	.9712	.3782	4061	5.991	677.8	.5009 ←	.2231 →
60.00	.9937 →	.7165 →	.1387 →	59.99	.2504 ←	3615 →	2.4477905	125.68	.9550	.3782	4200	5.992	700.9	.4606 ←	.2157 →
61.00	.8852 →	.6596 →	.1342 →	60.99	.2306 ←	3926 →	2.4478457	125.76	.9393	.3782	4341	5.992	724.5	.4241 ←	.2087 →
62.00	.7900 →	.6082 →	.1299 →	61.99	.2126 ←	4258 →	2.4478982	125.84	.9241	.3782	4485	5.992	748.4	.3911 ←	.2020 →
63.00	.7065 →	.5615 →	.1258 →	62.99	.1963 ←	4612 →	2.4479483	125.91	.9095	.3782	4630	5.993	772.7	.3611 ←	.1957 →
64.00	.6328 →	.5190 →	.1218 →	63.99	.1814 ←	4990 →	2.4479961	125.98	.8953	.3782	4779	5.993	797.4	.3338 ←	.1896 →
65.00	.5678 →	.4803 →	.1182 →	64.99	.1679 ←	5391 →	2.4480416	126.05	.8815	.3782	4929	5.993	822.5	.3089 ←	.1838 →
66.00	.5103 →	.4451 →	.1147 →	65.99	.1556 ←	5818 →	2.4480857	126.12	.8682	.3782	5082	5.993	847.9	.2863 ←	.1783 →
67.00	.4594 →	.4129 →	.1113 →	66.99	.1444 ←	6271 →	2.4481267	126.18	.8552	.3782	5237	5.993	873.8	.2655 ←	.1730 →
68.00	.4141 →	.3834 →	.1080 →	67.99	.1340 ←	6754 →	2.4481655	126.24	.8426	.3782	5395	5.994	900.1	.2466 ←	.1679 →
69.00	.3740 →	.3565 →	.1049 →	68.99	.1246 ←	7264 →	2.4482045	126.30	.8304	.3782	5554	5.994	926.7	.2293 ←	.1631 →
70.00	.3382 →	.3318 →	.1019 →	69.99	.1160 ←	7804 →	2.4482410	126.36	.8185	.3782	5717	5.994	953.7	.2134 ←	.1585 →
71.00	.3062 →	.3091 →	.9909 →	70.99	.1081 ←	8378 →	2.4482759	126.42	.8070	.3782	5881	5.994	981.1	.1988 ←	.1540 →
72.00	.2777 →	.2882 →	.9636 →	71.99	.1008 ←	8984 →	2.4483093	126.48	.7958	.3782	6048	5.994	1009	.1854 ←	.1498 →
73.00	.2522 →	.2690 →	.9374 →	72.99	.9406 →	9625 →	2.4483414	126.53	.7849	.3781	6217	5.994	1037	.1730 ←	.1457 →
74.00	.2293 →	.2513 →	.9122 →	73.99	.8789 →	1030 →	2.4483722	126.59	.7742	.3781	6389	5.995	1066	.1617 ←	.1418 →
75.00	.2088 →	.2351 →	.8881 →	74.99	.8220 →	1102 →	2.4484018	126.64	.7639	.3781	6562	5.995	1095	.1512 ←	.1381 →
76.00	.1903 →	.2200 →	.8649 →	75.99	.7693 →	1177 →	2.4484302	126.69	.7539	.3781	6739	5.995	1124	.1415 ←	.1345 →
77.00	.1737 →	.2061 →	.8426 →	76.99	.7207 →	1256 →	2.4484576	126.74	.7441	.3781	6917	5.995	1154	.1326 ←	.1310 →
78.00	.1587 →	.1932 →	.8212 →	77.99	.6757 →	1340 →	2.4484838	126.78	.7345	.3781	7098	5.995	1184	.1243 ←	.1276 →
79.00	.1451 →	.1813 →	.8005 →	78.99	.6341 →	1428 →	2.4485091	126.83	.7253	.3781	7281	5.995	1215	.1166 ←	.1244 →
80.00	.1329 →	.1703 →	.7806 →	79.99	.5954 →	1521 →	2.4485335	126.88	.7162	.3781	7467	5.995	1245	.1095 ←	.1214 →
81.00	.1219 →	.1600 →	.7615 →	80.99	.5596 →	1618 →	2.4485569	126.92	.7074	.3781	7654	5.995	1277	.1030 ←	.1184 →
82.00	.1118 →	.1505 →	.7431 →	81.99	.5264 →	1720 →	2.4485795	126.96	.6987	.3781	7845	5.996	1308	.9682 →	.1155 →
83.00	.1027 →	.1417 →	.7253 →	82.99	.4954 →	1828 →	2.4486013	127.00	.6903	.3781	8037	5.996	1341	.9113 →	.1127 →
84.00	.9448 →	.1334 →	.7081 →	83.99	.4667 →	1940 →	2.4486223	127.05	.6821	.3781	8232	5.996	1373	.8585 →	.1101 →
85.00	.8697 →	.1258 →	.6916 →	84.99	.4399 →	2059 →	2.4486426	127.09	.6741	.3781	8429	5.996	1406	.8092 →	.1075 →
86.00	.8014 →	.1186 →	.6756 →	85.99	.4149 →	2182 →	2.4486622	127.12	.6662	.3781	8629	5.996	1439	.7632 →	.1050 →
87.00	.7391 →	.1120 →	.6602 →	86.99	.3916 →	2312 →	2.4486811	127.16	.6586	.3781	8830	5.996	1473	.7204 →	.1026 →
88.00	.6823 →	.1058 →	.6452 →	87.99	.3699 →	2448 →	2.4486994	127.20	.6511	.3781	9035	5.996	1507	.6804 →	.1003 →
89.00	.6305 →	.9995 →	.6308 →	88.99	.3496 →	2590 →	2.4487170	127.24	.6438	.3781	9241	5.996	1541	.6431 →	.9805 →
90.00	.5831 →	.9452 →	.6169 →	89.99	.3306 →	2739 →	2.4487341	127.27	.6366	.3781	9450	5.996	1576	.6082 →	.9588 →
91.00	.5397 →	.8944 →	.6034 →	90.99	.3129 →	2894 →	2.4487506	127.31	.6296	.3781	9661	5.996	1611	.5755 →	.9378 →
92.00	.5000 →	.8469 →	.5904 →	91.99	.2962 →	3057 →	2.4487666	127.34	.6228	.3781	9875	5.997	1647	.5450 →	.9175 →
93.00	.4636 →	.8023 →	.5778 →	92.99	.2807 →	3226 →	2.4487820	127.38	.6160	.3781	1009 →	5.997	1683	.5163 →	.8978 →
94.00	.4302 →	.7606 →	.5656 →	93.99	.2661 →	3403 →	2.4487970	127.41	.6095	.3781	1031 →	5.997	1719	.4894 →	.8790 →
95.00	.3995 →	.7214 →	.5537 →	94.99	.2524 →	3588 →	2.4488115	127.44	.6031	.3781	1053 →	5.997	1756	.4642 →	.8605 →
96.00	.3712 →	.6846 →	.5422 →	95.99	.2395 →	3781 →	2.4488255	127.47	.5968	.3781	1075 →	5.997	1793	.4405 →	.8427 →
97.00	.3453 →	.6501 →	.5311 →	96.99	.2274 →	3982 →	2.4488392	127.50	.5907	.3781	1096 →	5.997	1831	.4183 →	.8254 →
98.00	.3214 →	.6176 →	.5204 →	97.99	.2161 →	4191 →	2.4488524	127.53	.5847	.3781	1121 →	5.997	1869	.3974 →	.8087 →
99.00	.2993 →	.5870 →	.5099 →	98.99	.2054 →	4410 →	2.4488652	127.56	.5787	.3781	1143 →	5.997	1907	.3778 →	.7923 →
100.00	.2790 →	.5583 →	.4998 →	100.00	.1953 →	4637 →	2.4488776	127.59	.5730	.3781	1167 →	5.997	1945	.3593 →	.7765 →

NOTATIONS FOR TABLES I AND II

- $M$  or  $M_1$  local Mach number or Mach number upstream of a normal shock wave
- $\frac{p}{p_1}$  ratio of static pressure to total pressure
- $\frac{\rho}{\rho_1}$  ratio of static density to total density
- $\frac{T}{T_1}$  ratio of static temperature to total temperature
- $\beta$   $\sqrt{M^2 - 1}$
- $\frac{q}{p_1}$  ratio of dynamic pressure,  $\frac{1}{2} \rho V^2$ , to total pressure
- $\frac{A}{A_*}$  ratio of local cross-sectional area of an isentropic stream tube to cross-sectional area at the point where  $M=1$
- $\frac{V}{a_*}$  ratio of local speed to speed of sound at the point where  $M=1$
- $\nu$  Prandtl-Meyer angle (angle through which a supersonic stream is turned to expand from  $M=1$  to  $M>1$ ), deg
- $\mu$  Mach angle,  $\sin^{-1} \frac{1}{M}$ , deg
- $M_2$  Mach number downstream of a normal shock wave
- $\frac{p_2}{p_1}$  static pressure ratio across a normal shock wave
- $\frac{\rho_2}{\rho_1}$  static density ratio across a normal shock wave
- $\frac{T_2}{T_1}$  static temperature ratio across a normal shock wave
- $\frac{p_{t2}}{p_{t1}}$  total pressure ratio across a normal shock wave
- $\frac{p_1}{p_{t2}}$  ratio of static pressure upstream of a normal shock wave to total pressure downstream



CHARTS

The charts that follow present numerical values of certain physical quantities that are functions of two variables and hence are cumbersome to tabulate. These charts are designed to provide accuracy to three significant figures.

Charts 1 through 8 and chart 25 are for a perfect gas. The values presented in charts 1 through 4 and chart 25 were calculated for a ratio of specific heats of 7/5. The values presented in charts 5 through 8 were taken from references 6 and 14 and are for a ratio of specific heats of 1.405.

Charts 9 through 24 provide correction factors to account for the effects of caloric imperfections on the quantities tabulated in tables I and II and plotted in charts 2, 3, and 4.

On many charts, points corresponding to static temperatures of 5000° R and 100°R (−360° F) have been indicated. These temperatures represent very approximately the limits of validity of the charts. Exact limits cannot be stated simply as they depend on pressure as well as temperature. At temperatures near 5000° R dissociation effects, which were neglected in the calculations, can be significant at high altitudes though perhaps not at sea level. At temperatures less than about 100° R, air may condense at the pressures encountered in many wind tunnels.

On the Reynolds number chart (chart 25), points corresponding to a static temperature of 180° R (−280° F) also are indicated since this is the lowest temperature for which experimental viscosity data have been obtained. At temperatures much lower than −280° F, Sutherland's equation (A2) may significantly underestimate the true viscosity.

The contents of the charts are as follows:

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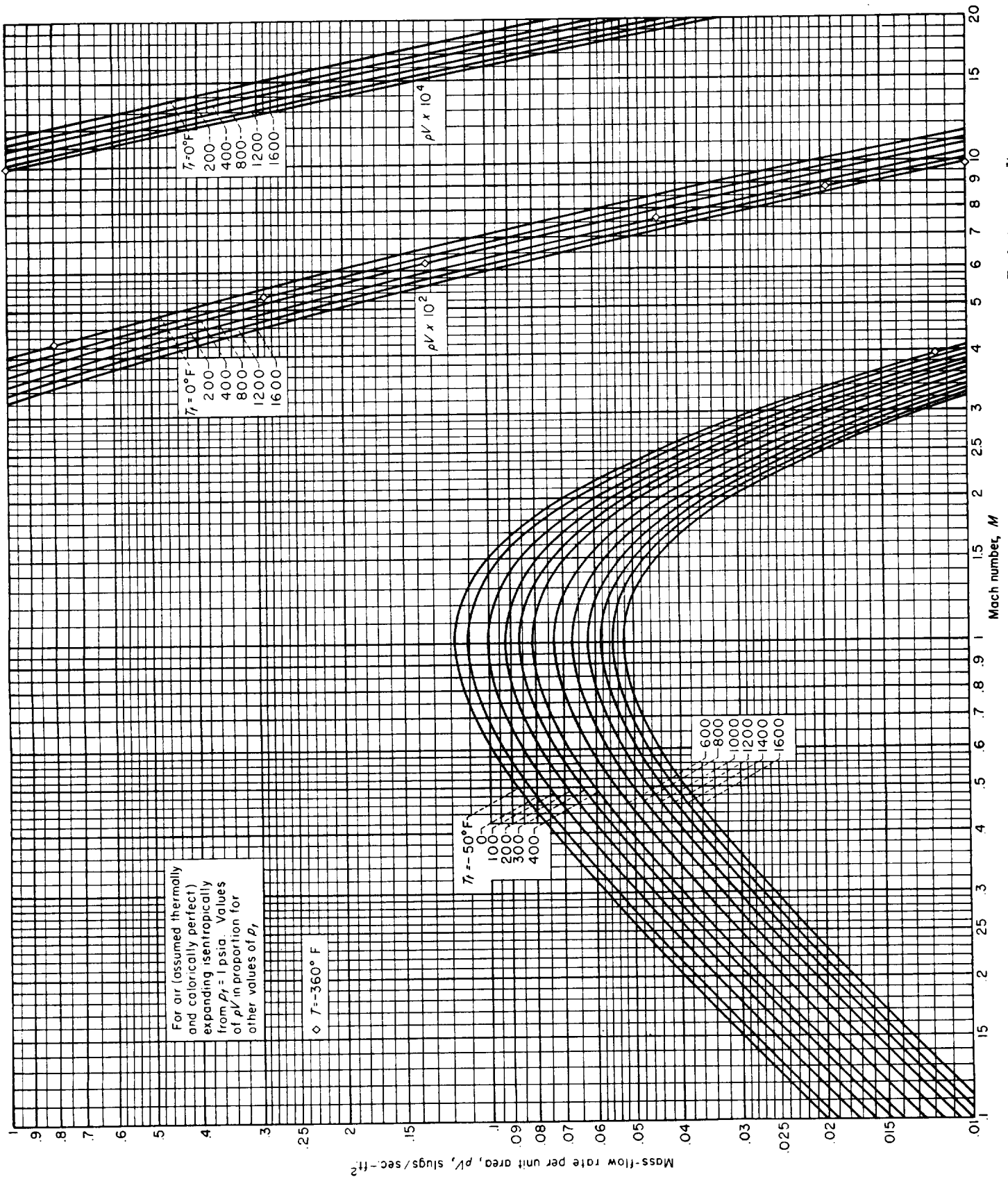


CHART 1.—Variation of mass-flow rate per unit area with Mach number for various total temperatures Perfect gas,  $\gamma = \frac{7}{5}$ .

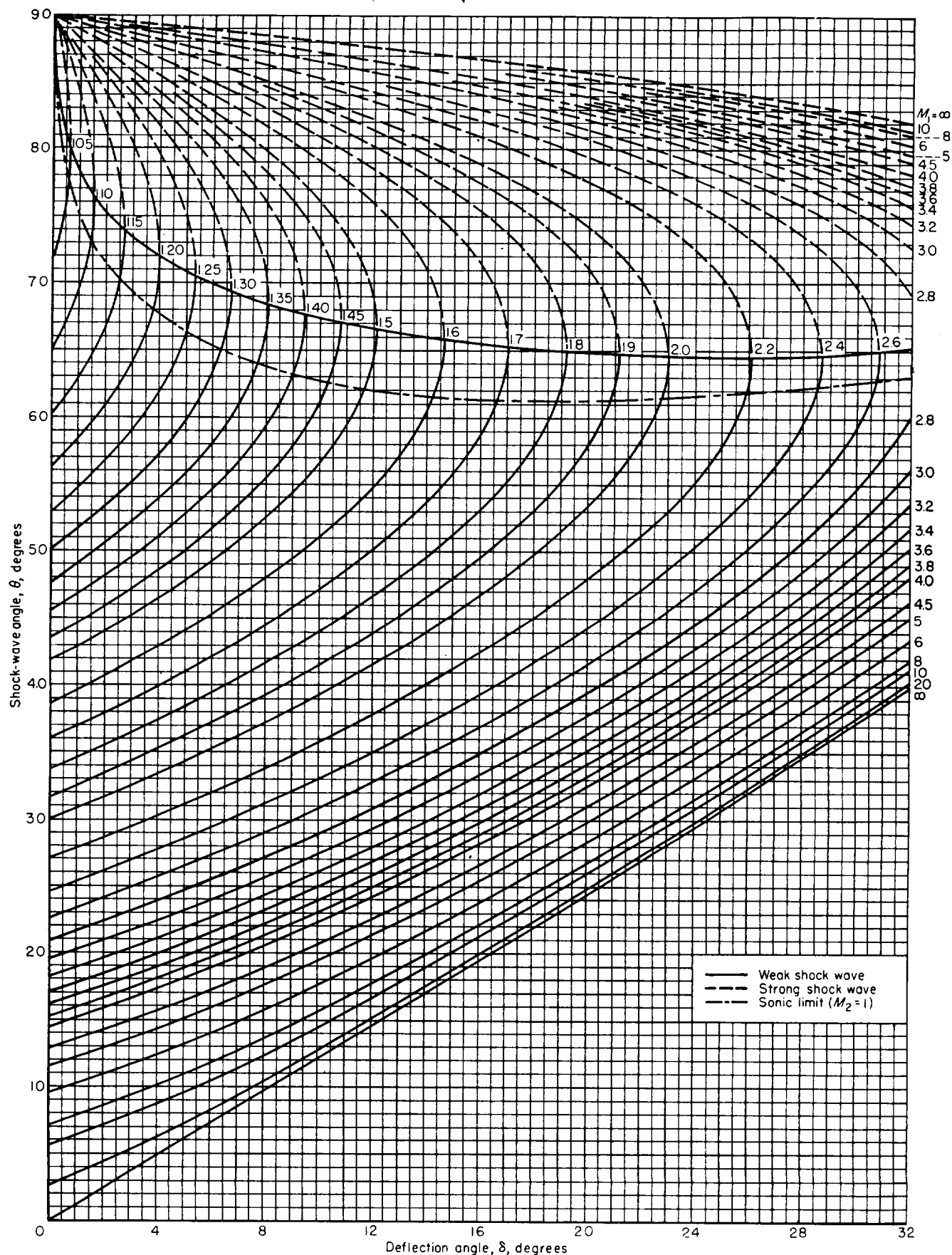


CHART 2.—Variation of shock-wave angle with flow-deflection angle for various upstream Mach numbers Perfect gas,  $\gamma = \frac{7}{5}$ .

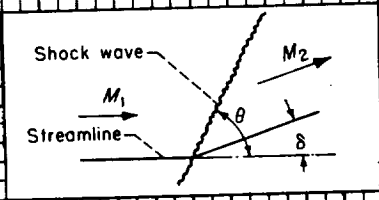
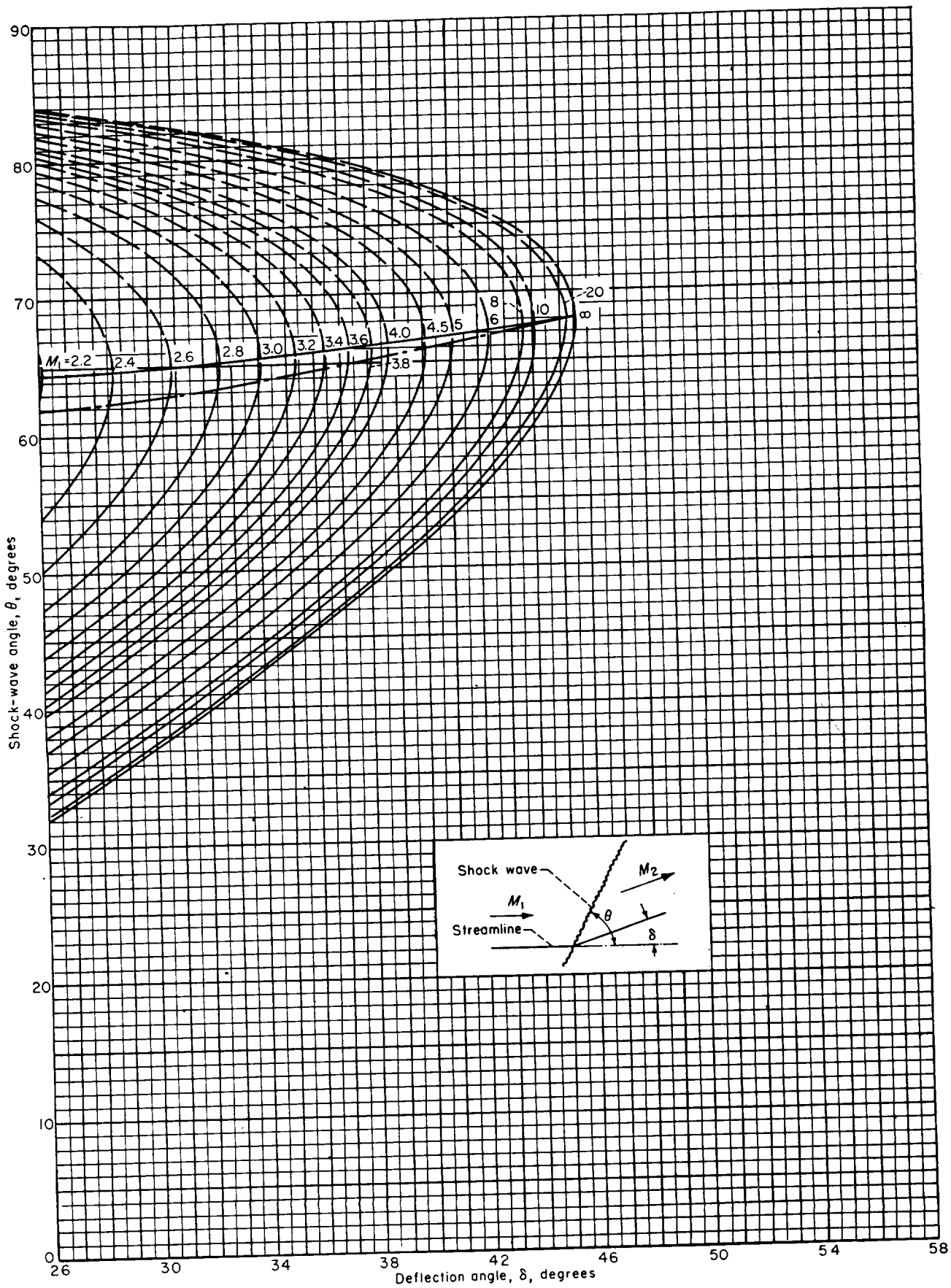


CHART 2.—Concluded

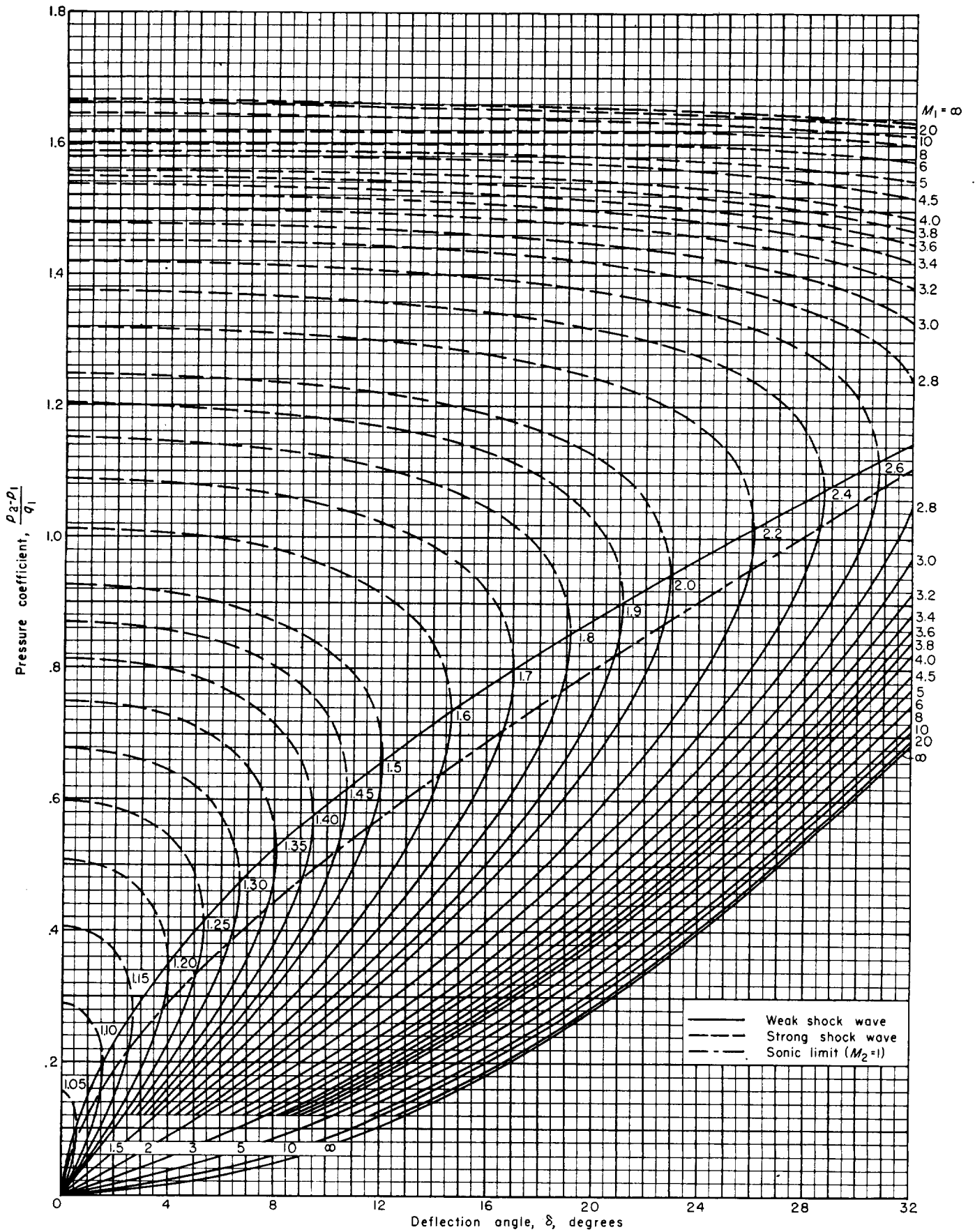


CHART 3.—Variation of pressure coefficient across shock waves with flow-deflection angle for various upstream Mach numbers. Perfect gas,  $\gamma = \frac{7}{5}$ .

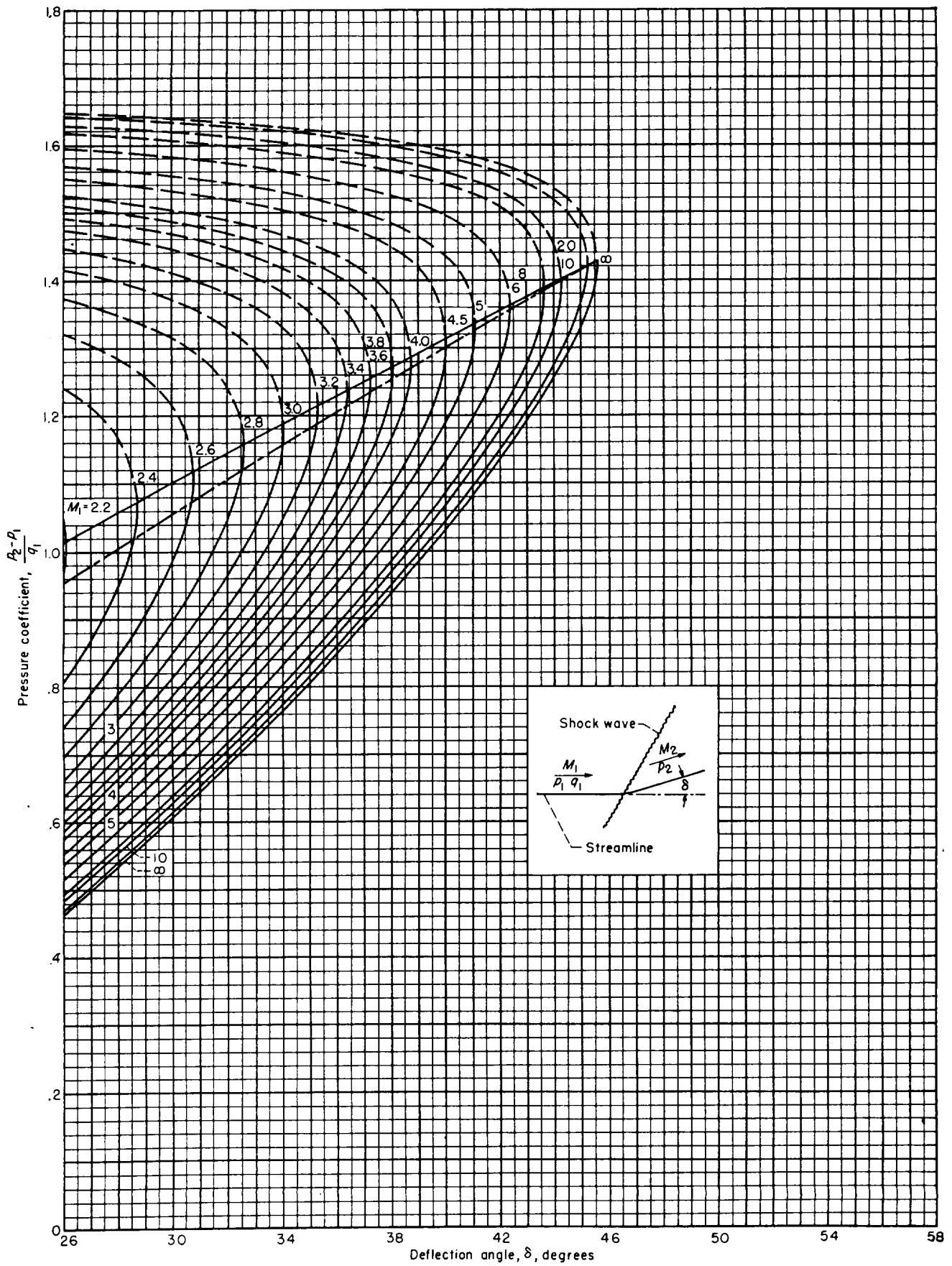


CHART 3.—Concluded

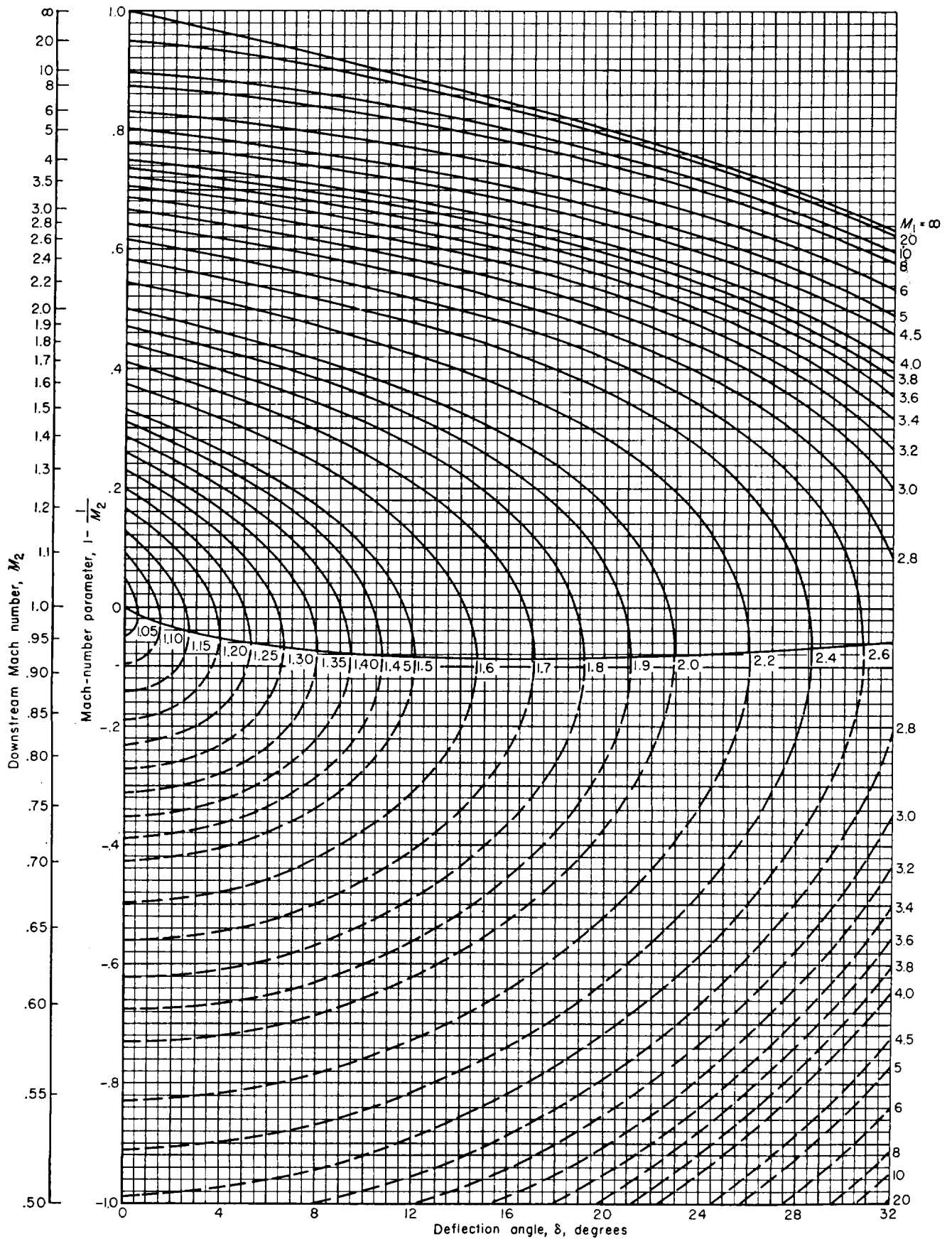


CHART 4.—Variation of Mach number downstream of a shock wave with flow-deflection angle for various upstream Mach numbers. Perfect gas,  $\gamma = \frac{7}{5}$ .

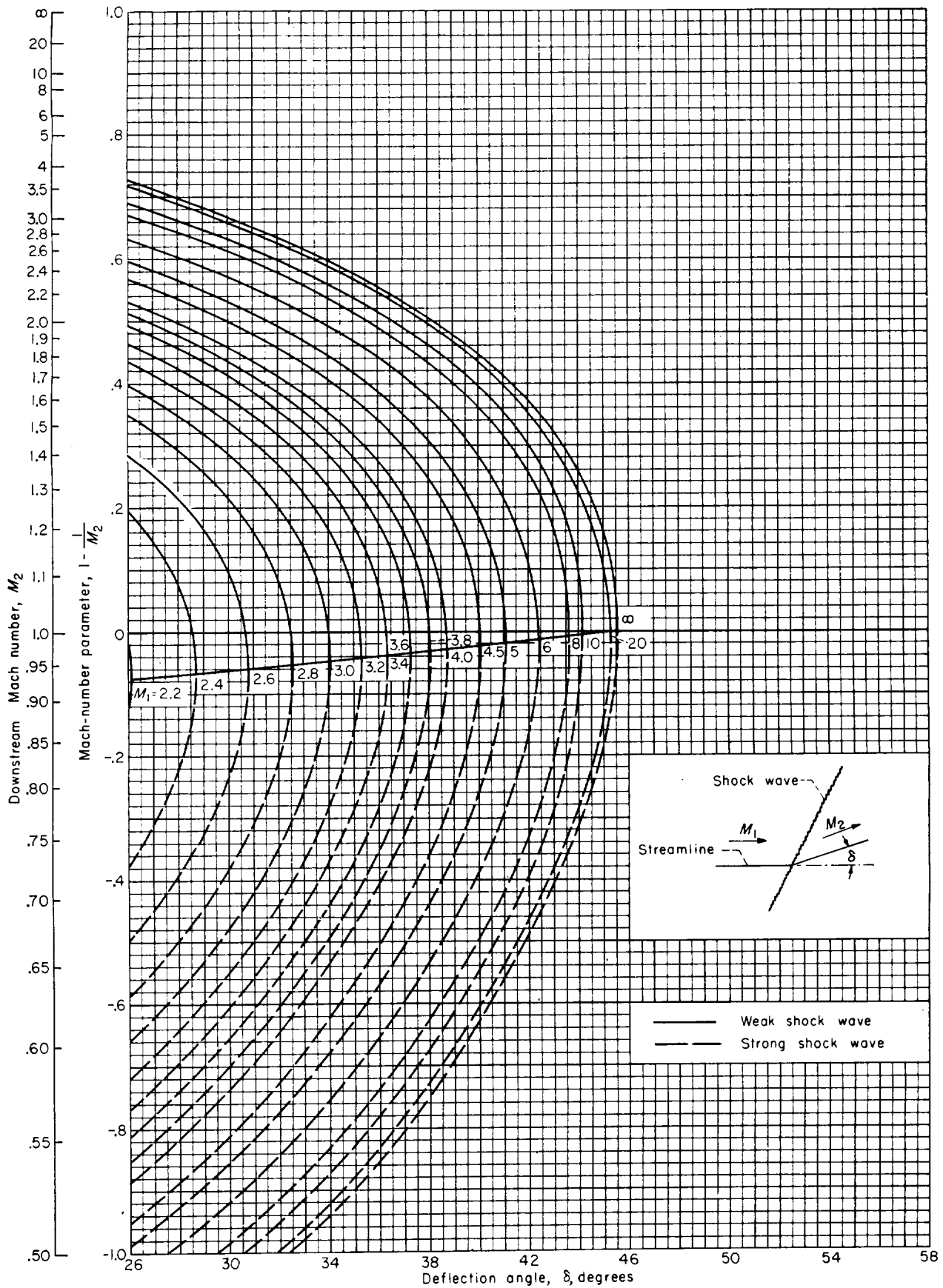


CHART 4.—Concluded



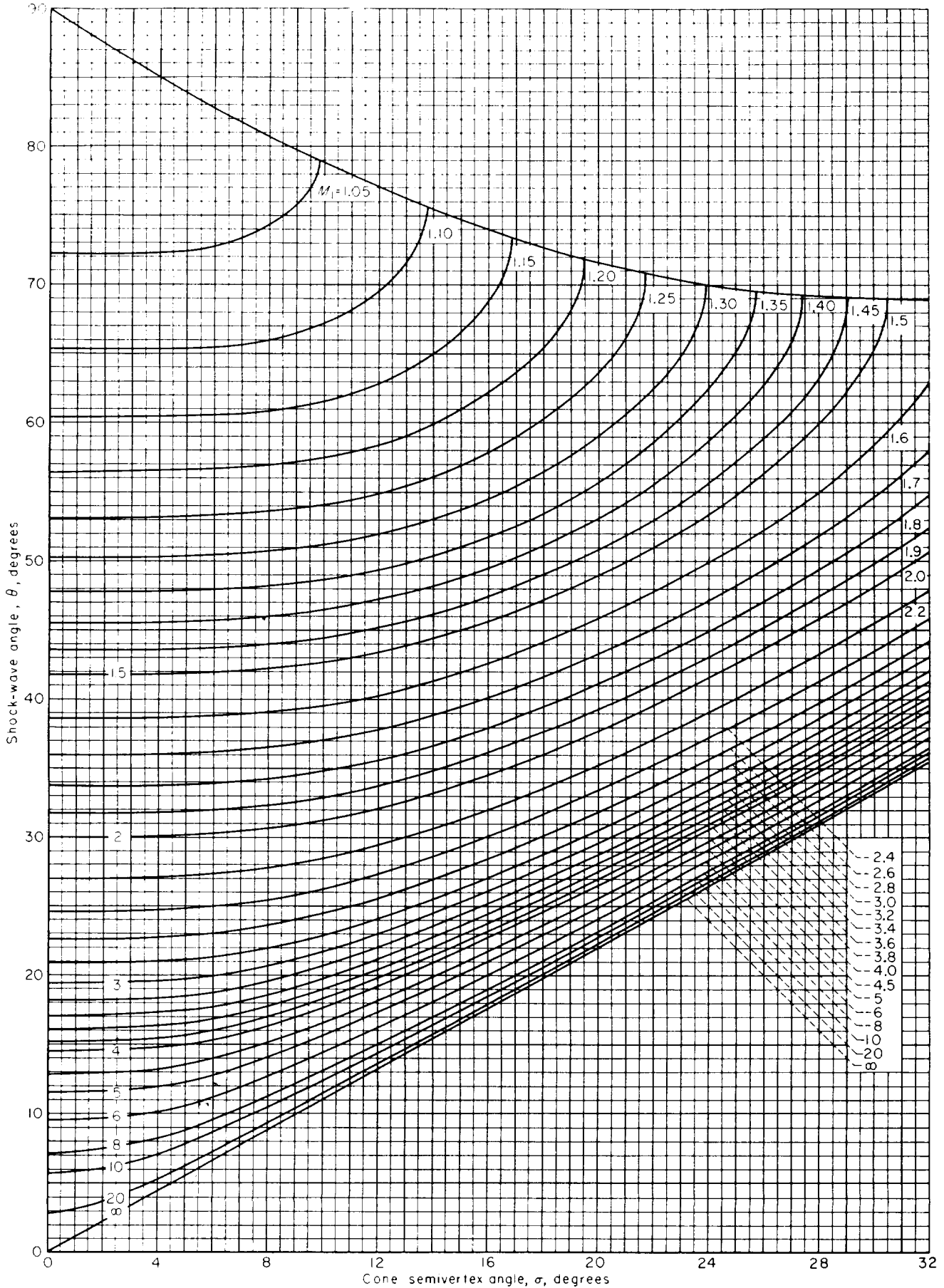


CHART 5.—Variation of shock-wave angle with cone semivertex angle for various upstream Mach numbers. Perfect gas,  $\gamma = 1.405$ .

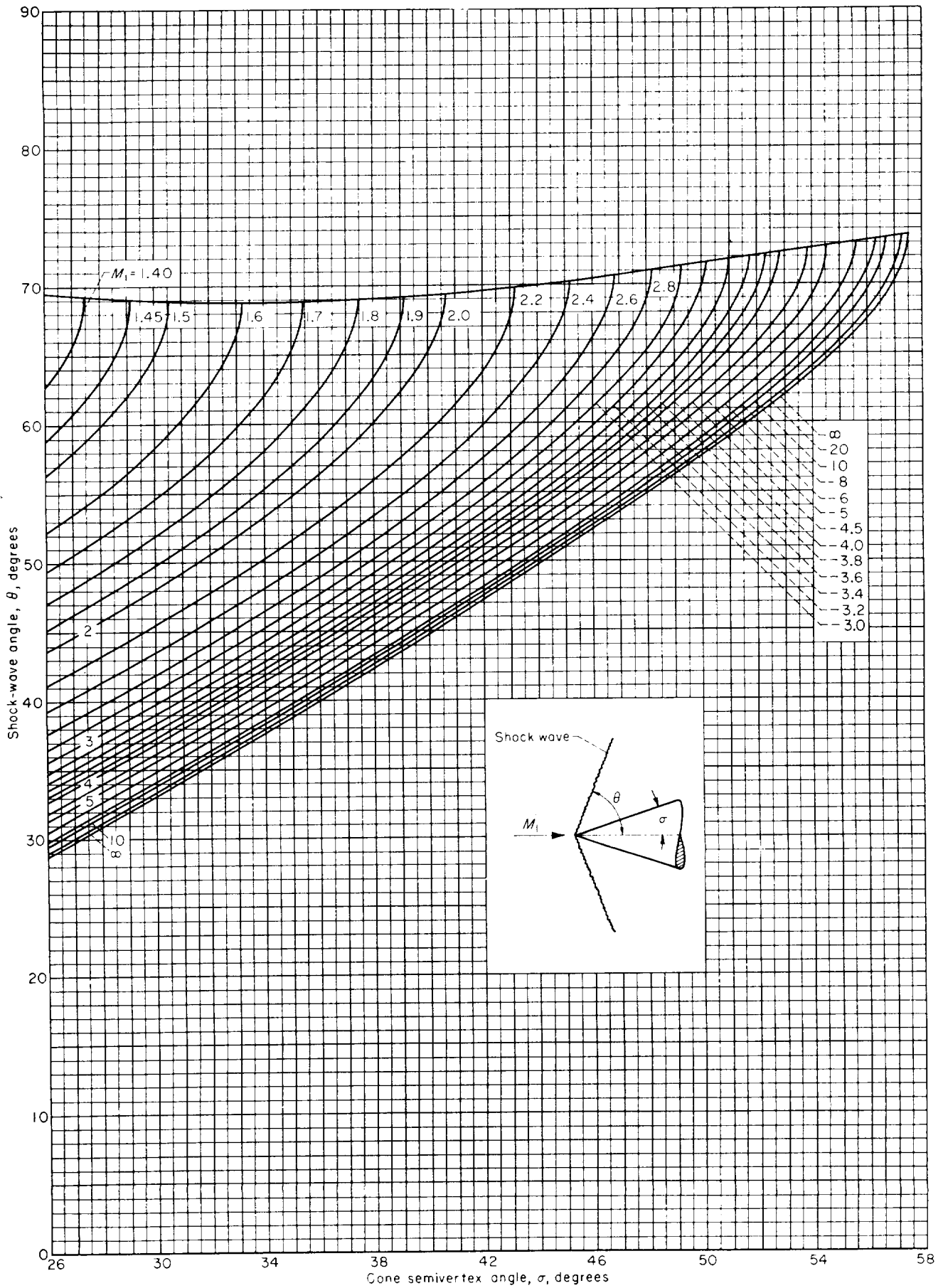


CHART 5.—Concluded



CHART 6.--Variation of surface pressure coefficient with cone semivertex angle for various upstream Mach numbers. Perfect gas,  $\gamma = 1.405$ .

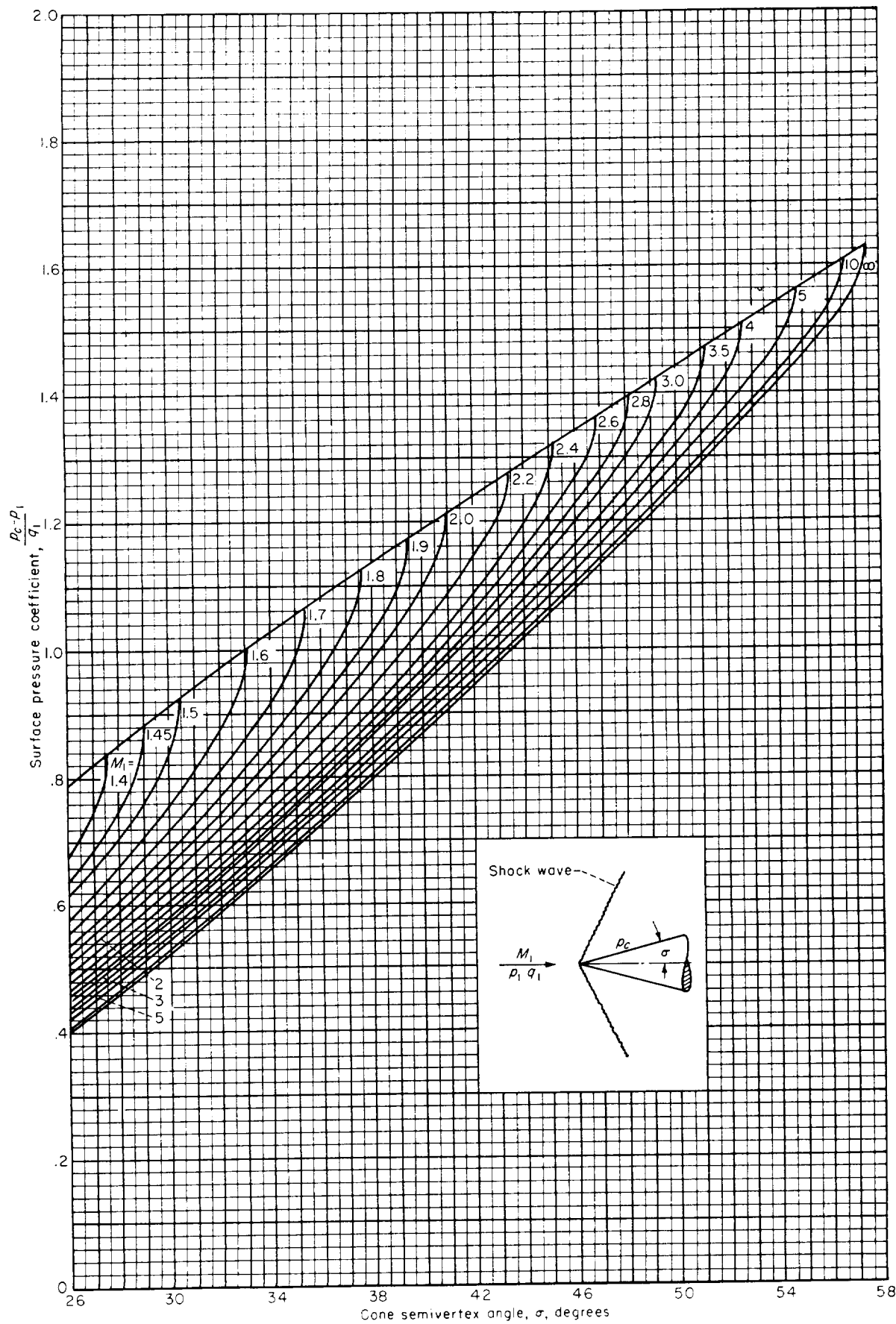


CHART 6.—Concluded

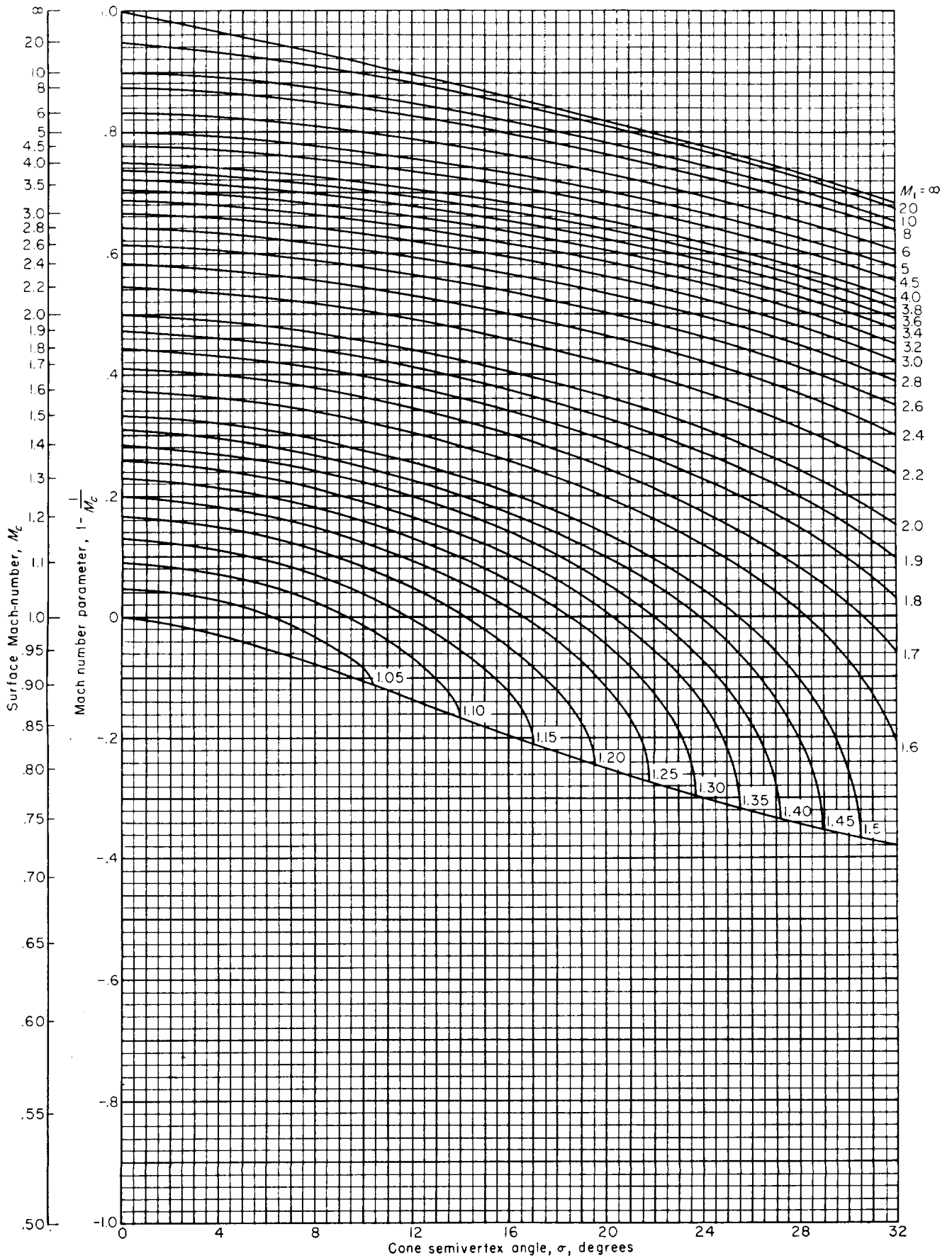


CHART 7.—Variation of Mach number at the surface of a cone with cone semivertex angle for various upstream Mach numbers. Perfect gas.  $\gamma = 1.405$ .

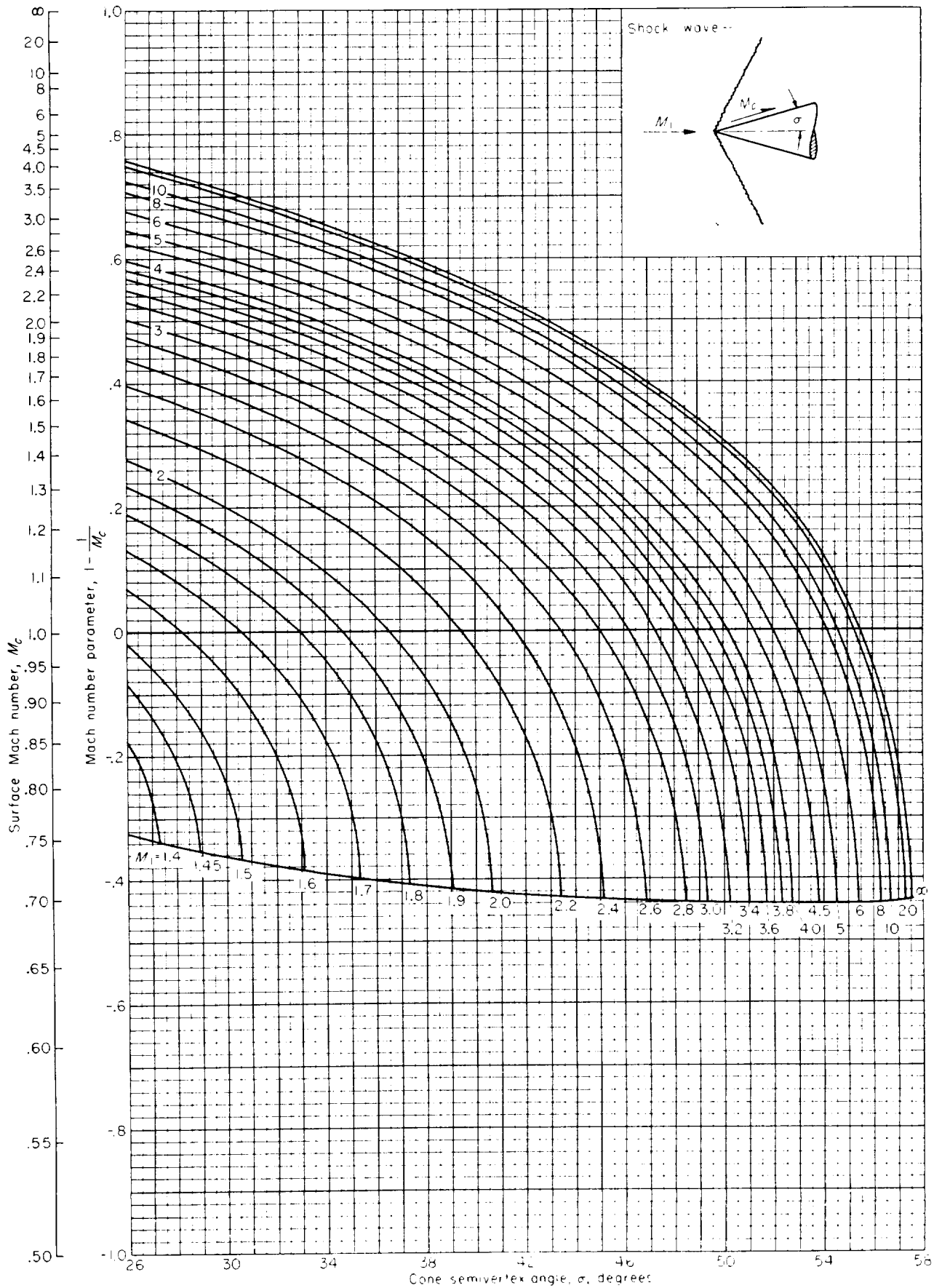


CHART 7.—Concluded

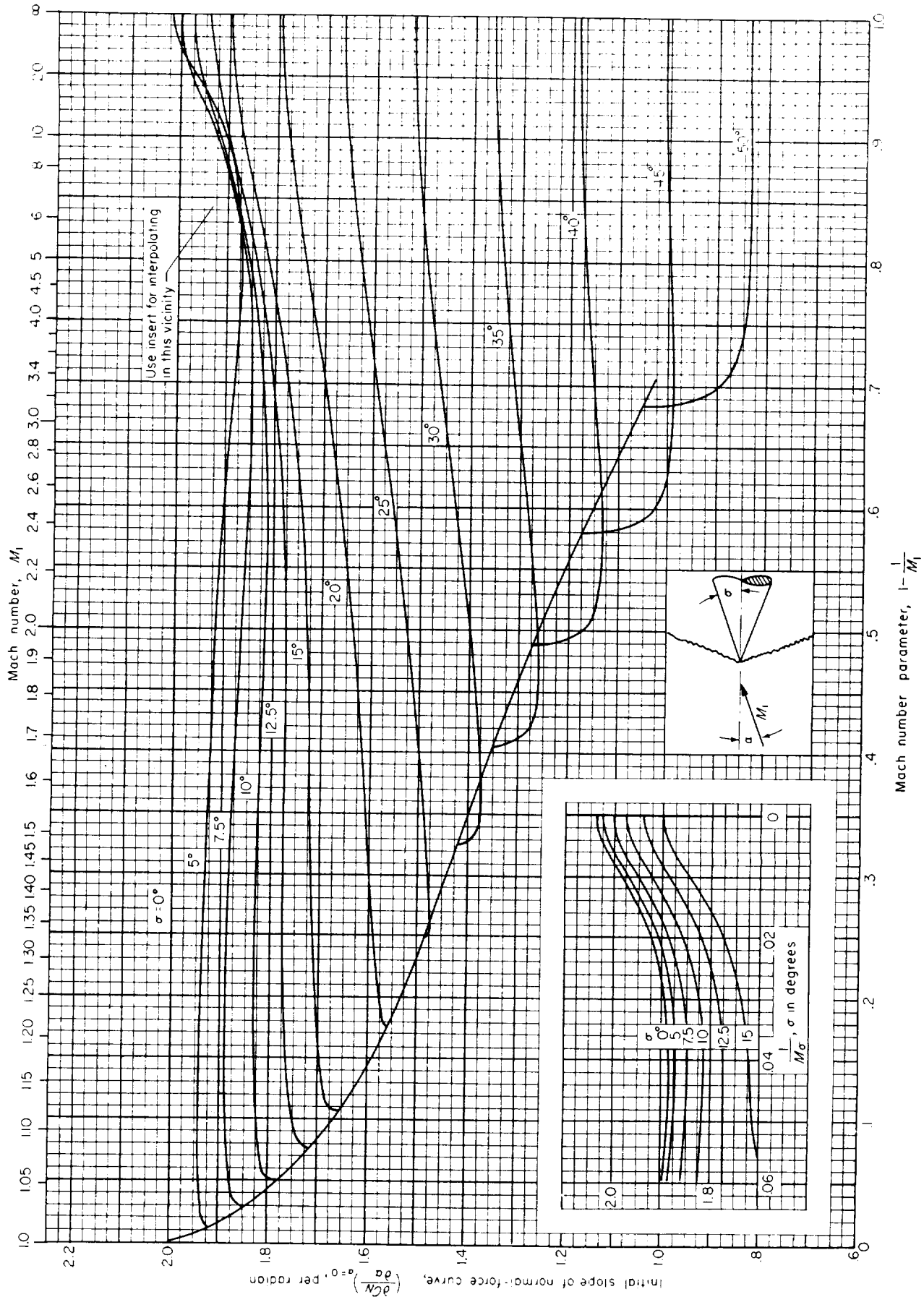


CHART 8.—Variation of the initial slope of the normal-force curve with upstream Mach number for various cone semivertex angles. Perfect gas,  $\gamma = 1.405$ .

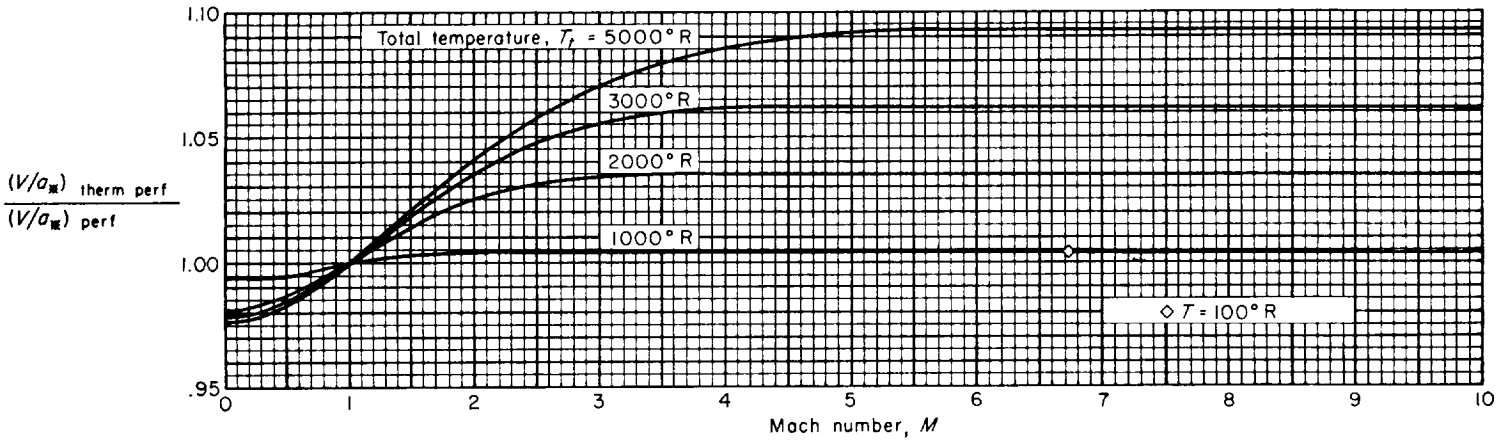


CHART 9.—Effect of caloric imperfections on the ratio of local speed to speed of sound at the point where  $M=1$ .

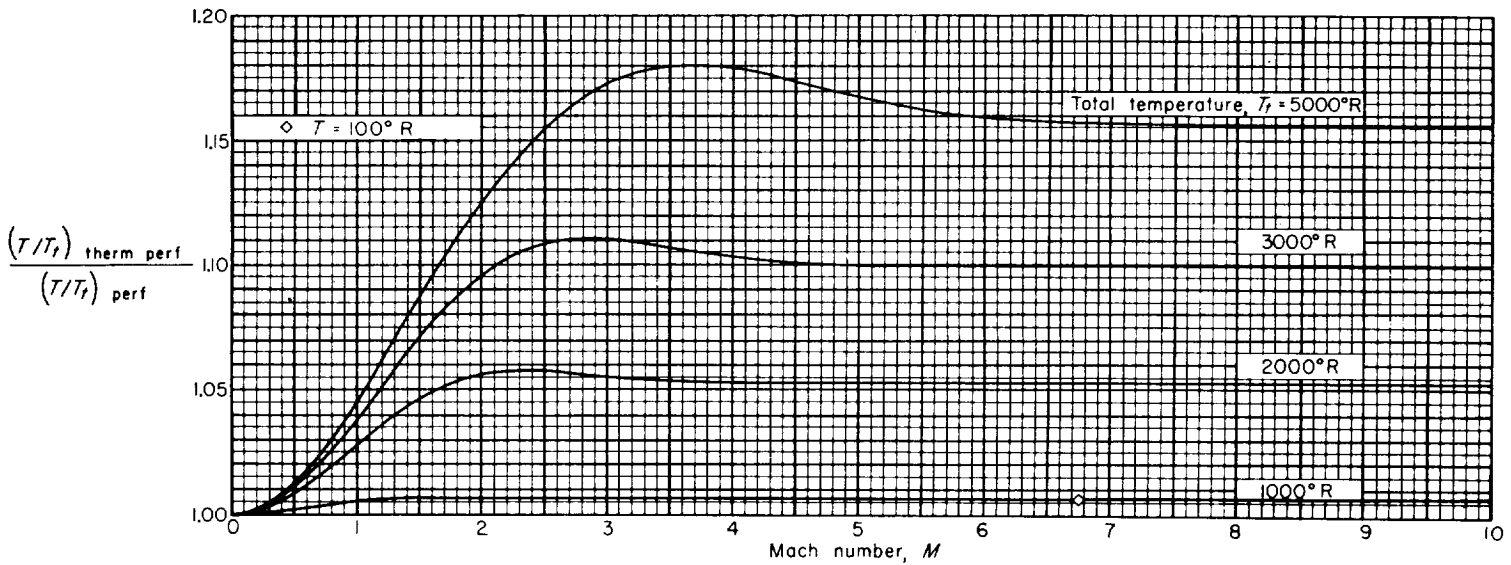


CHART 10.—Effect of caloric imperfections on the ratio of static temperature to total temperature.

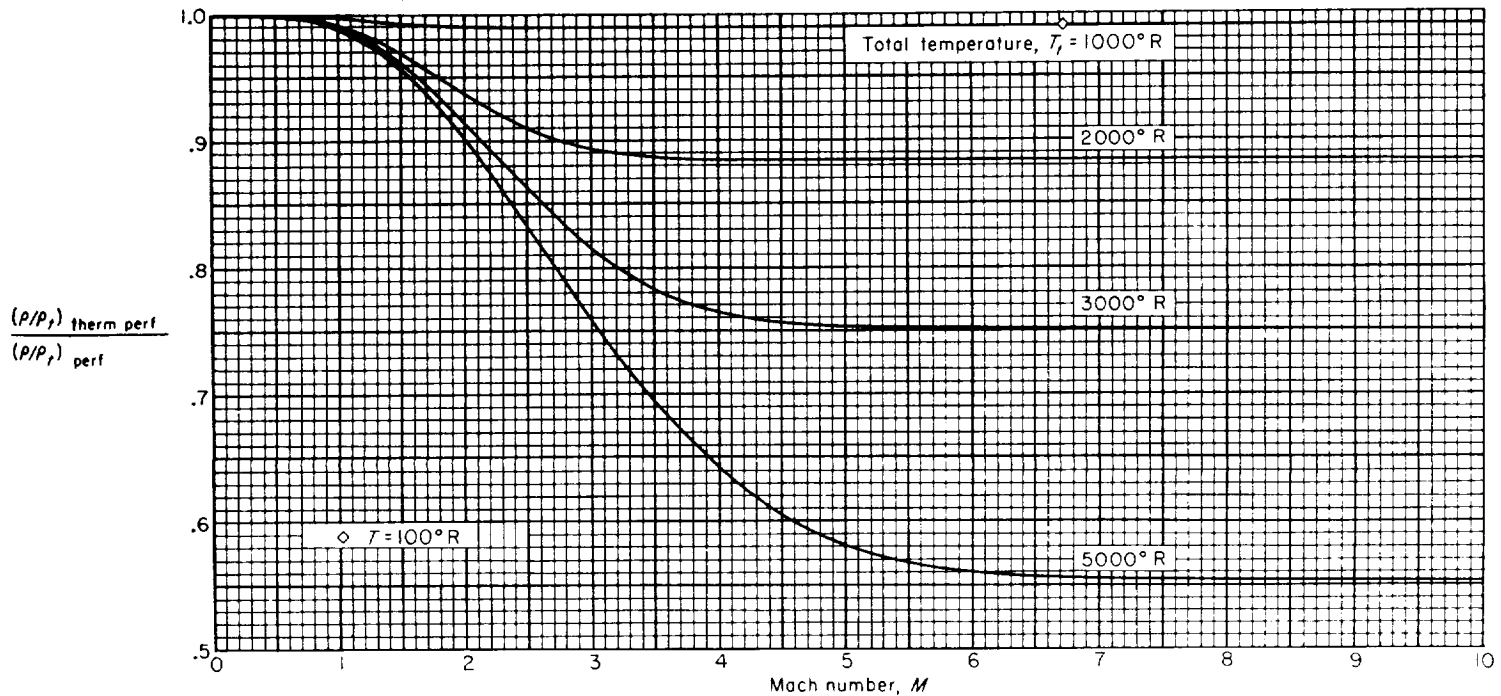


CHART 11.—Effect of caloric imperfections on the ratio of static density to total density.



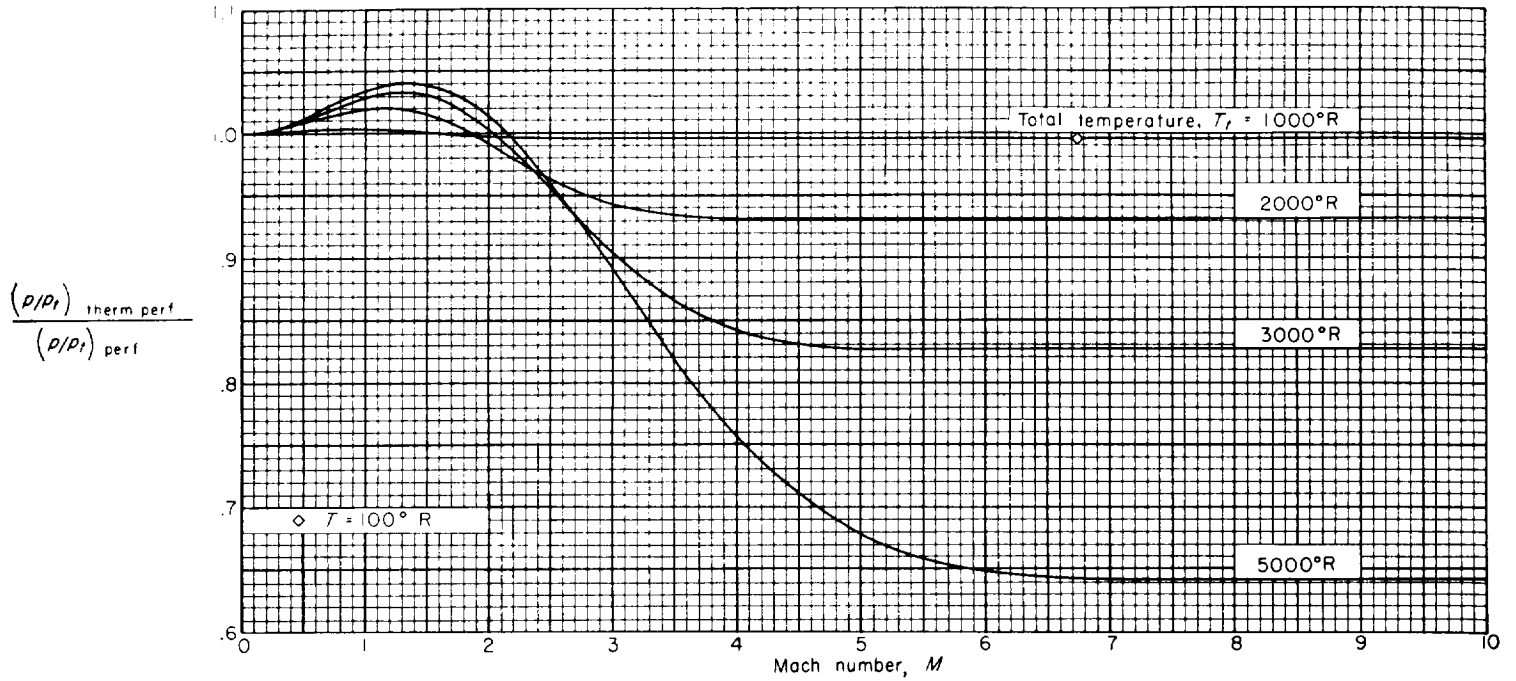


CHART 12.—Effect of caloric imperfections on the ratio of static pressure to total pressure.

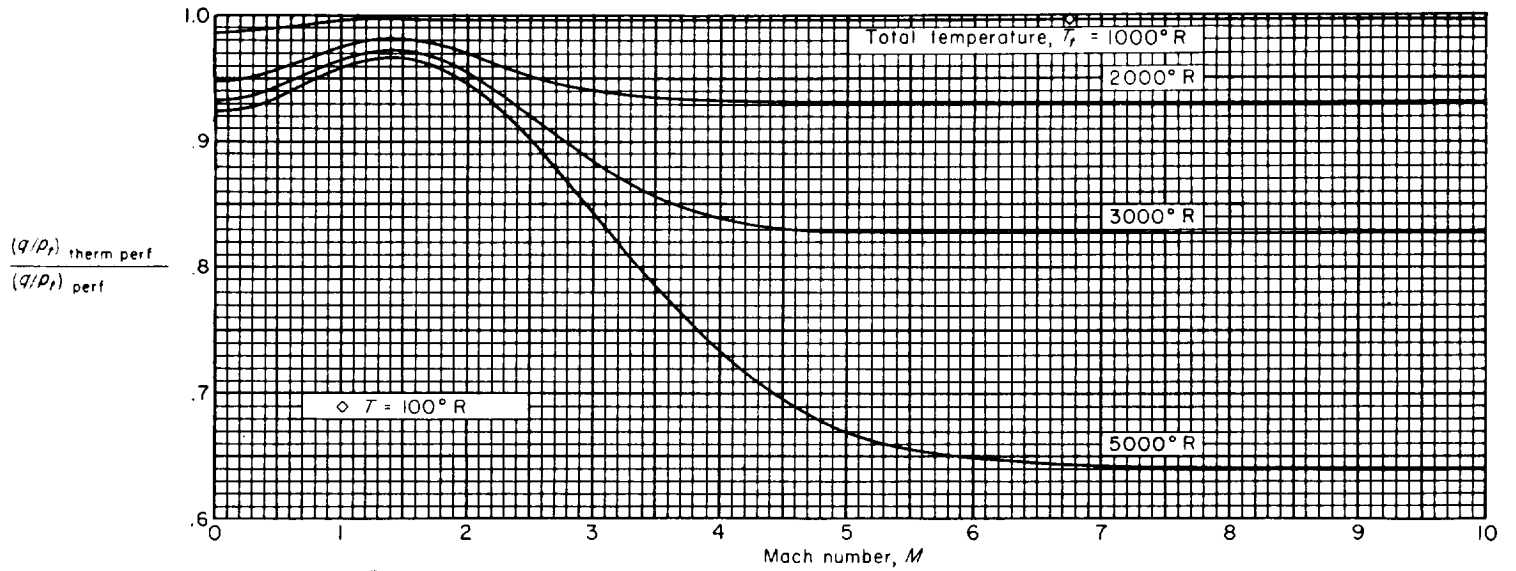


CHART 13.—Effect of caloric imperfections on the ratio of dynamic pressure to total pressure.

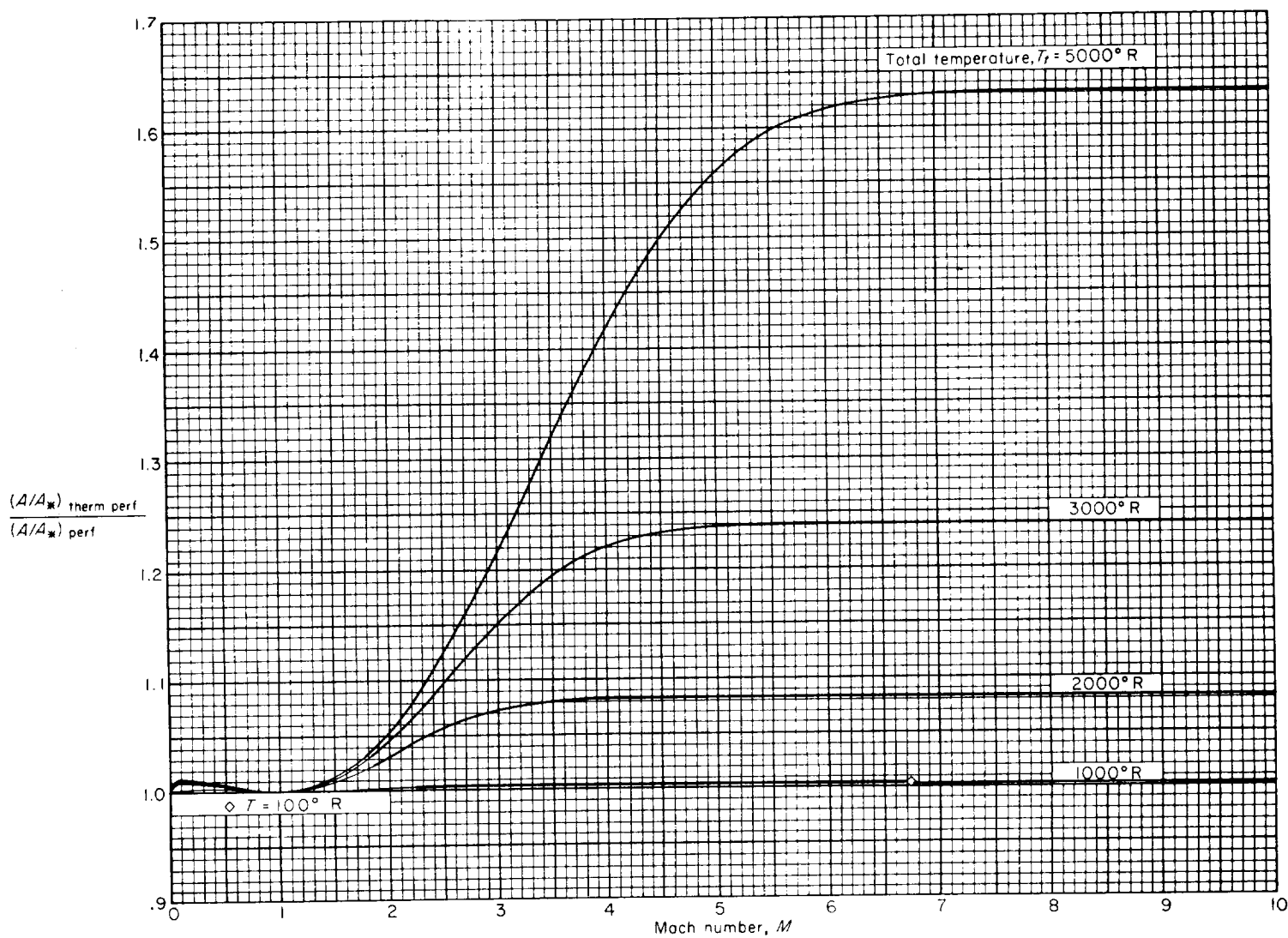


CHART 14.—Effect of calorie imperfections on the ratio of local cross-sectional area of a stream tube to the cross-sectional area at the point where  $M=1$ .

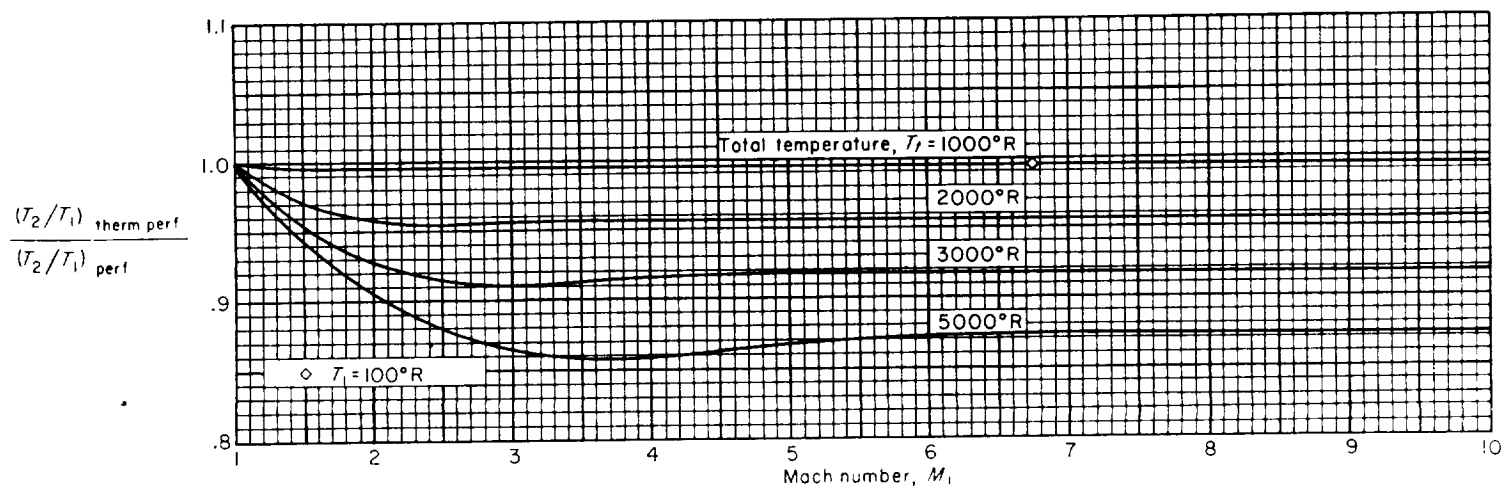


CHART 15.—Effect of calorie imperfections on the static-temperature ratio across a normal shock wave.

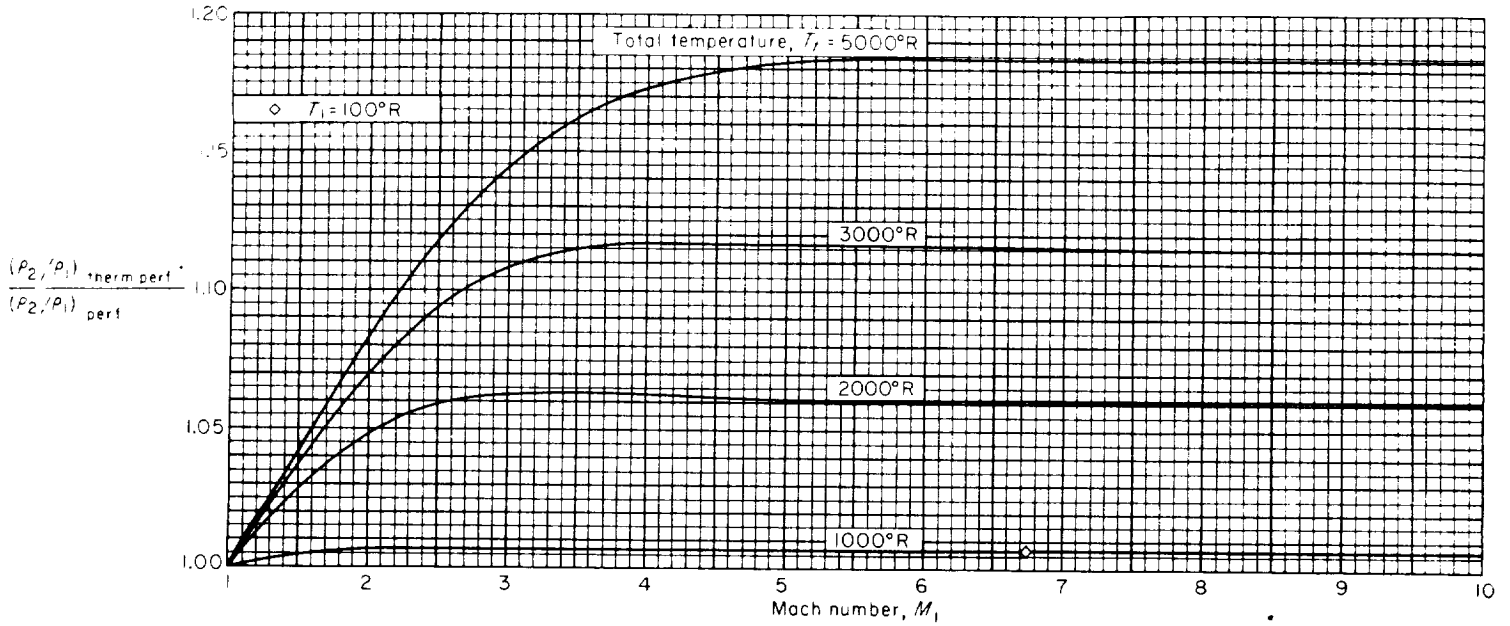


CHART 16.—Effect of caloric imperfections on the static-density ratio across a normal shock wave.

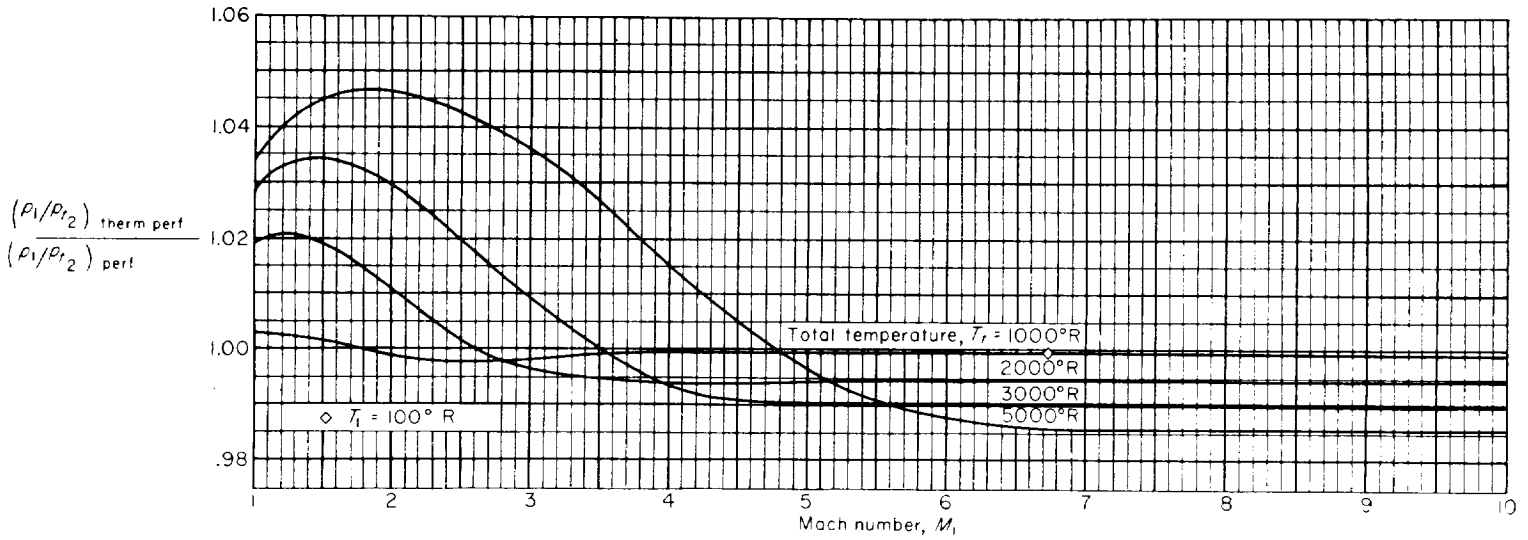


CHART 17.—Effect of caloric imperfections on the ratio of static pressure upstream of a normal shock wave to total pressure downstream.

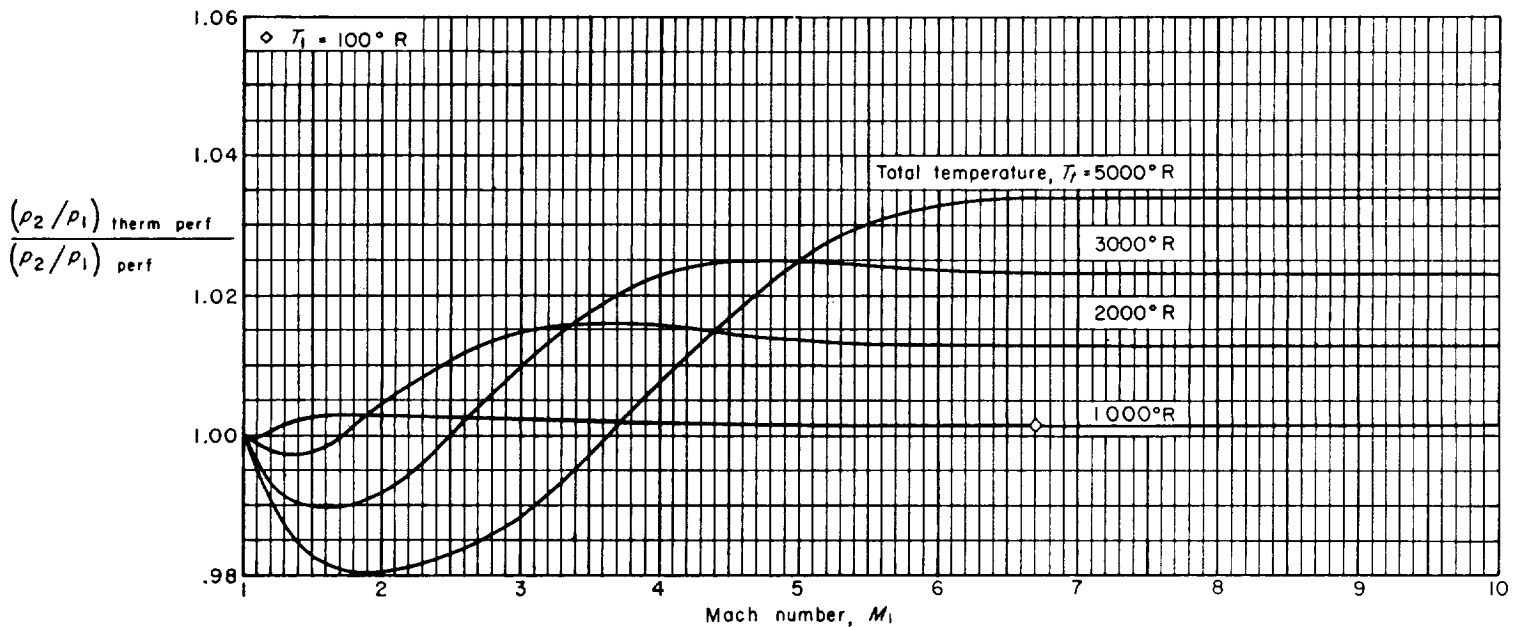


CHART 18.—Effect of caloric imperfections on the static-pressure ratio across a normal shock wave.

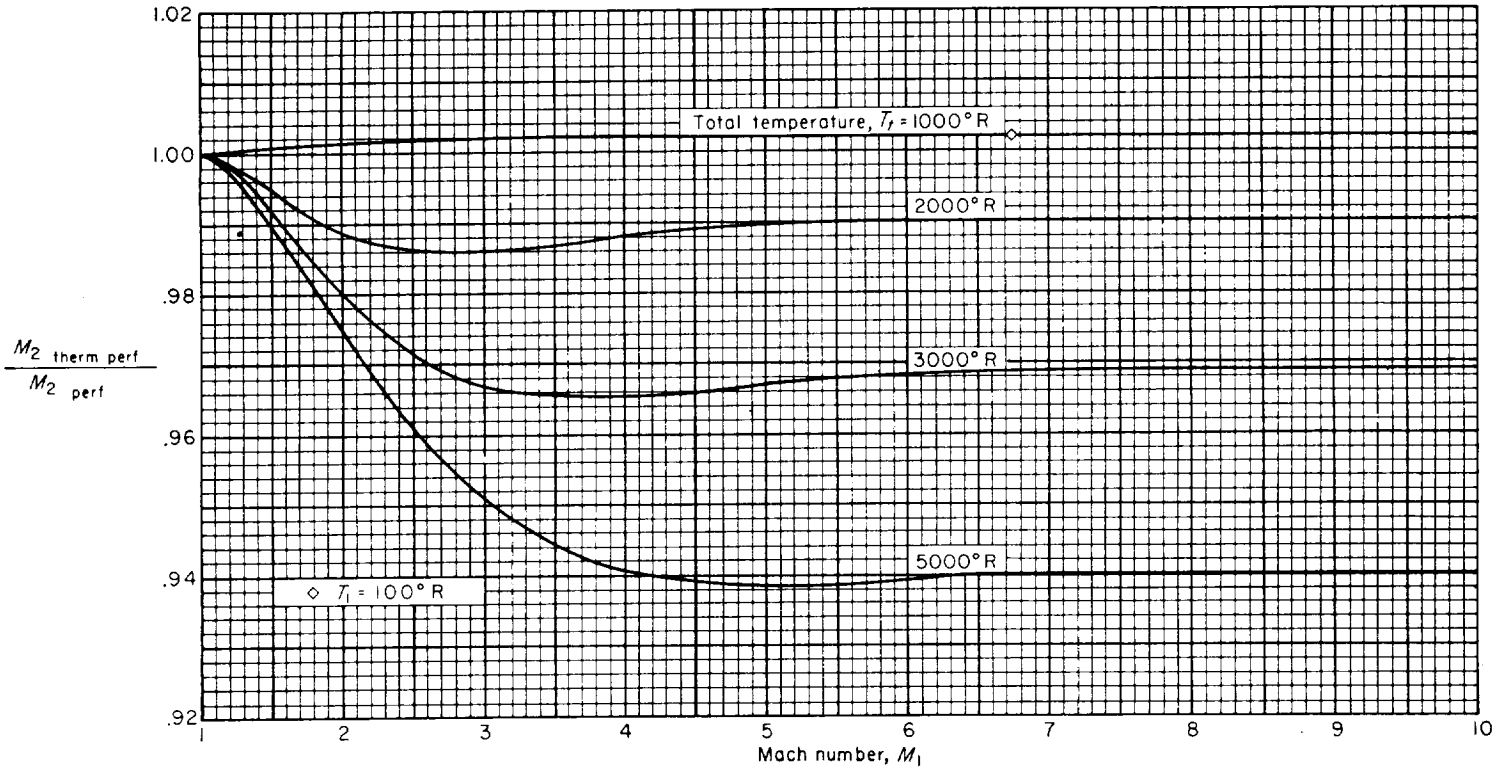


CHART 19.—Effect of caloric imperfections on the Mach number downstream of a normal shock wave.

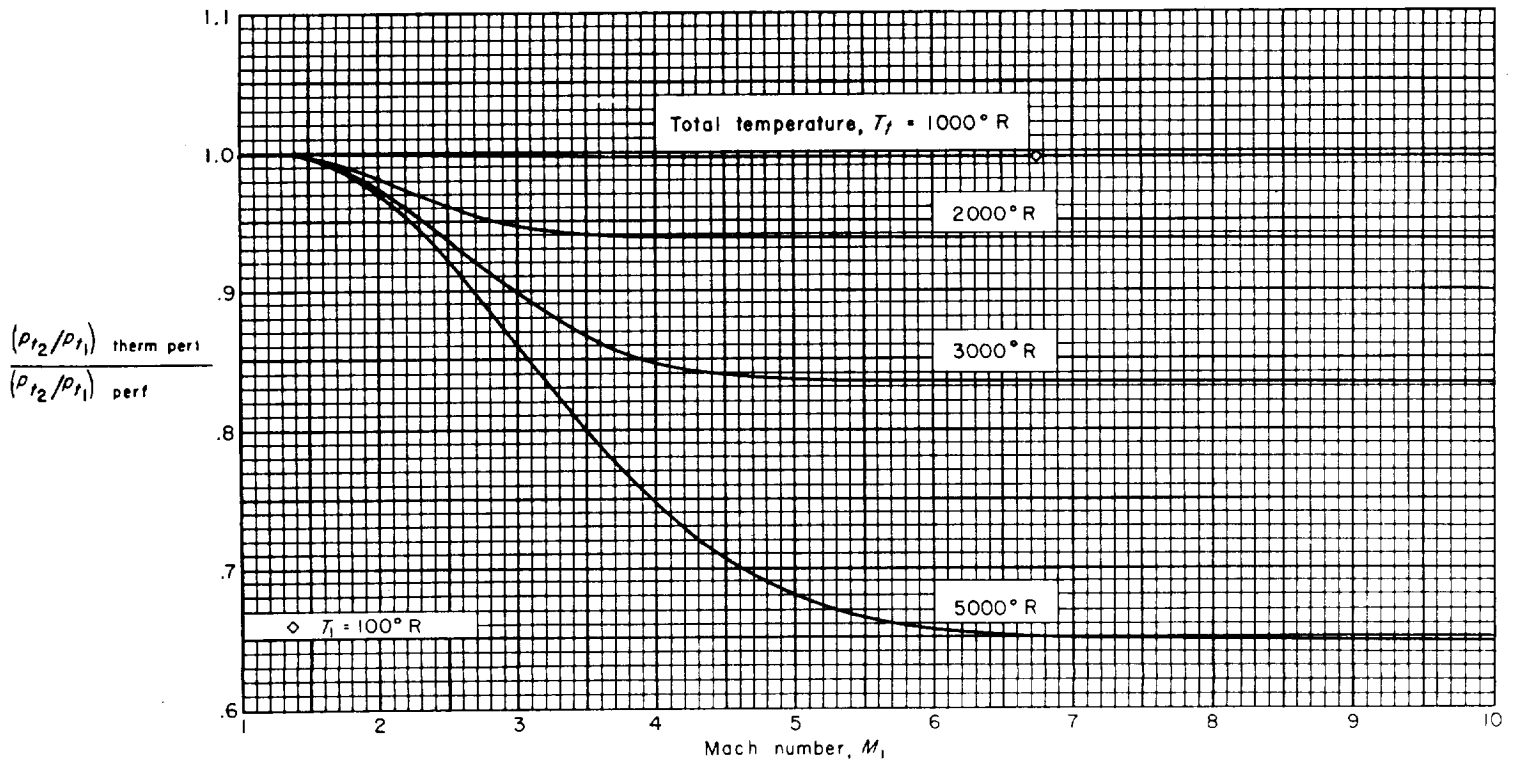
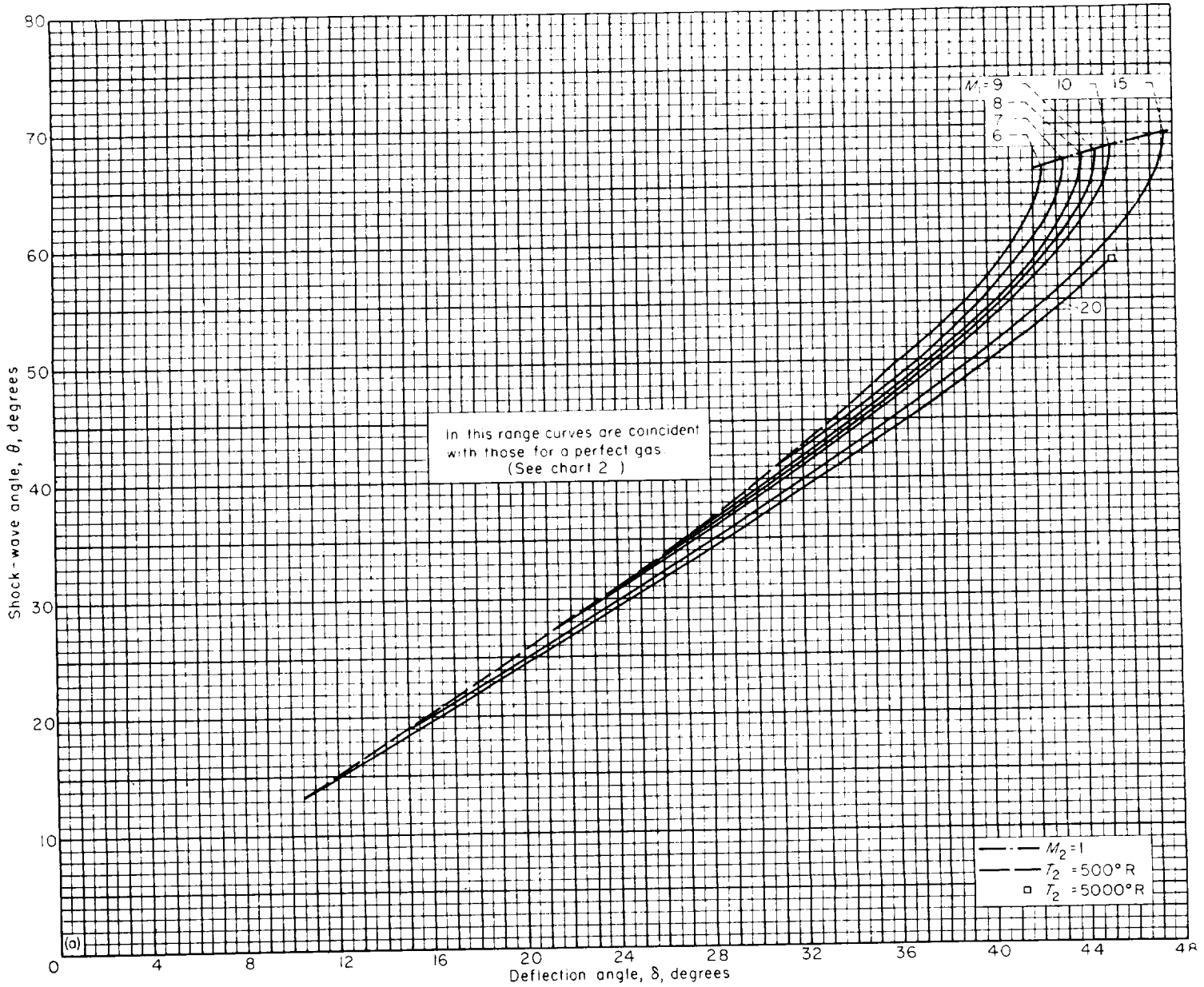
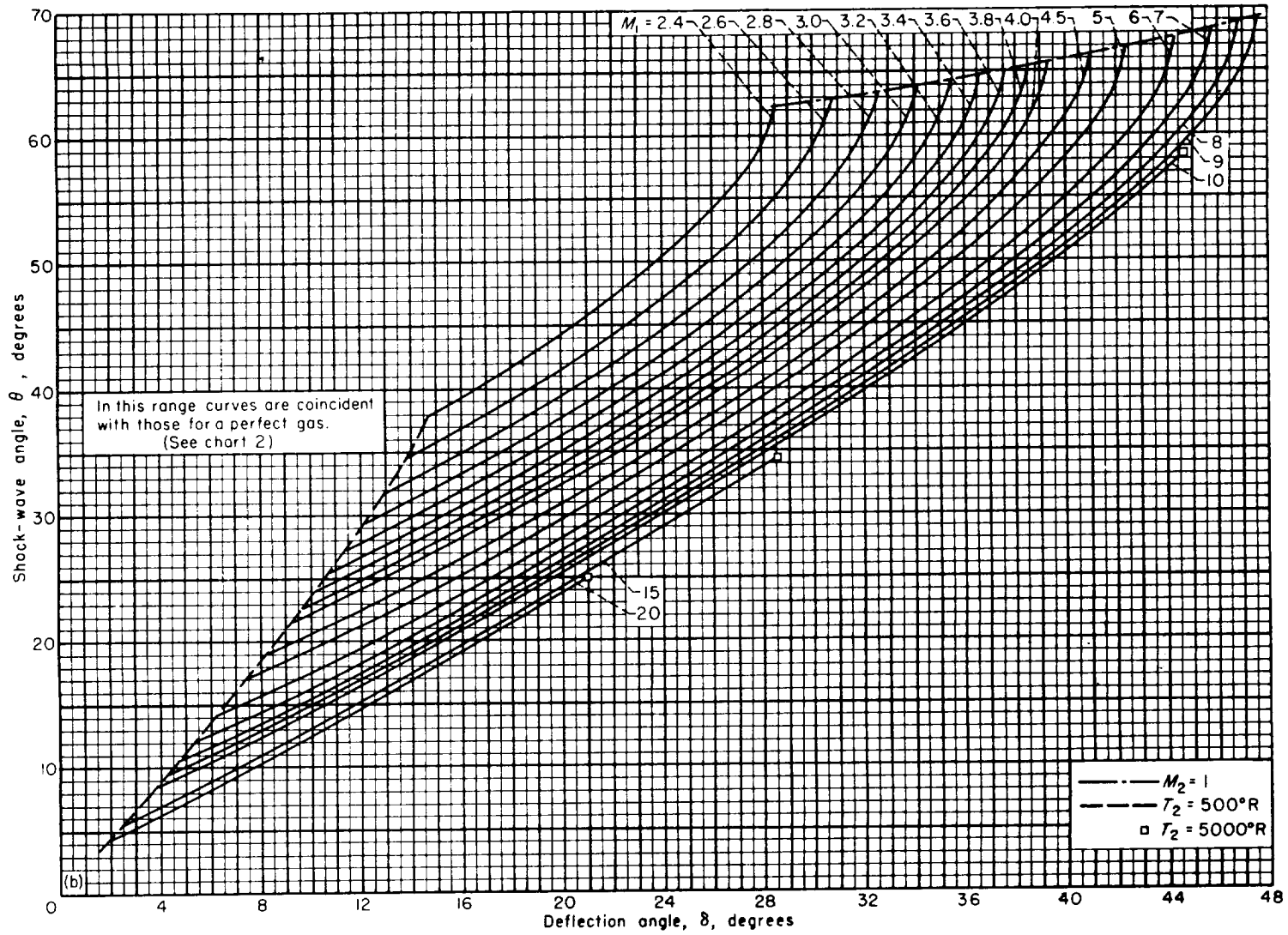


CHART 20.—Effect of caloric imperfections on the total-pressure ratio across a normal shock wave.



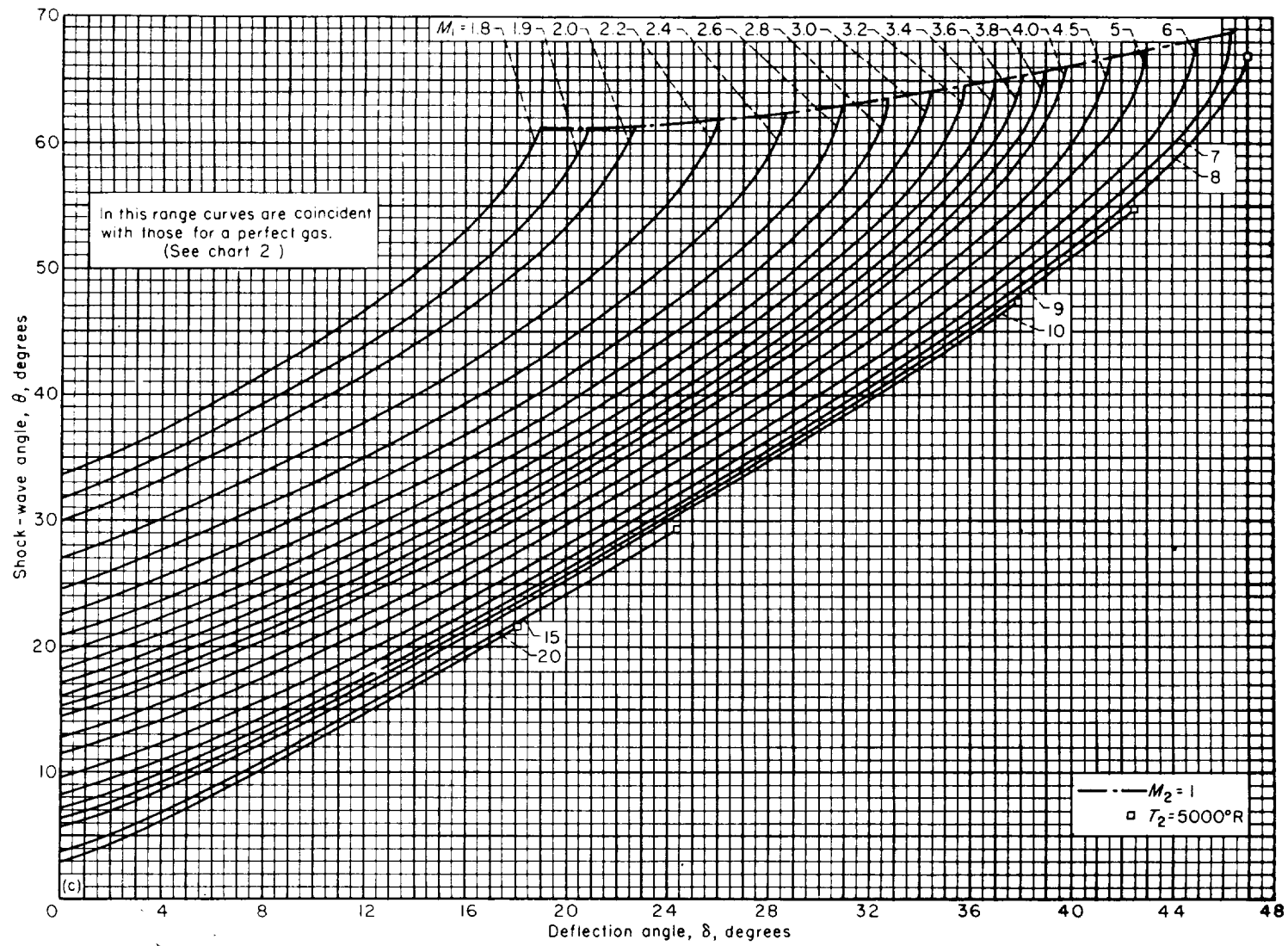
(a)  $T_1=100^\circ\text{R}$

CHART 21.—Effect of caloric imperfections on the variation with flow-deflection angle of the shock-wave angle for a weak oblique shock wave

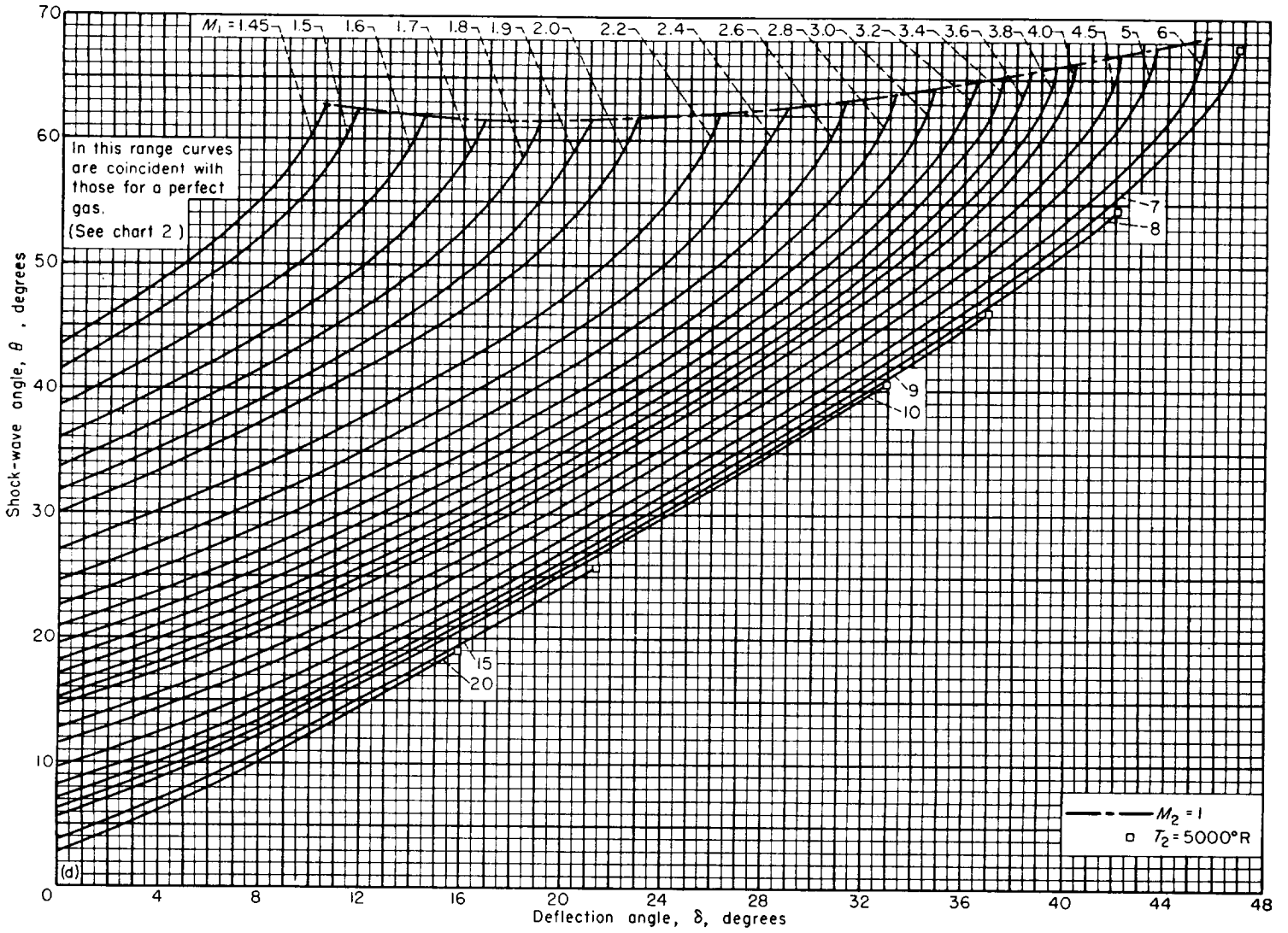


(b)  $T_1 = 390^\circ\text{R}$

CHART 21. -Continued



(c)  $T_1 = 500^\circ \text{R}$   
CHART 21.—Continued



(d)  $T_1 = 630^\circ\text{R}$

CHART 21.—Concluded



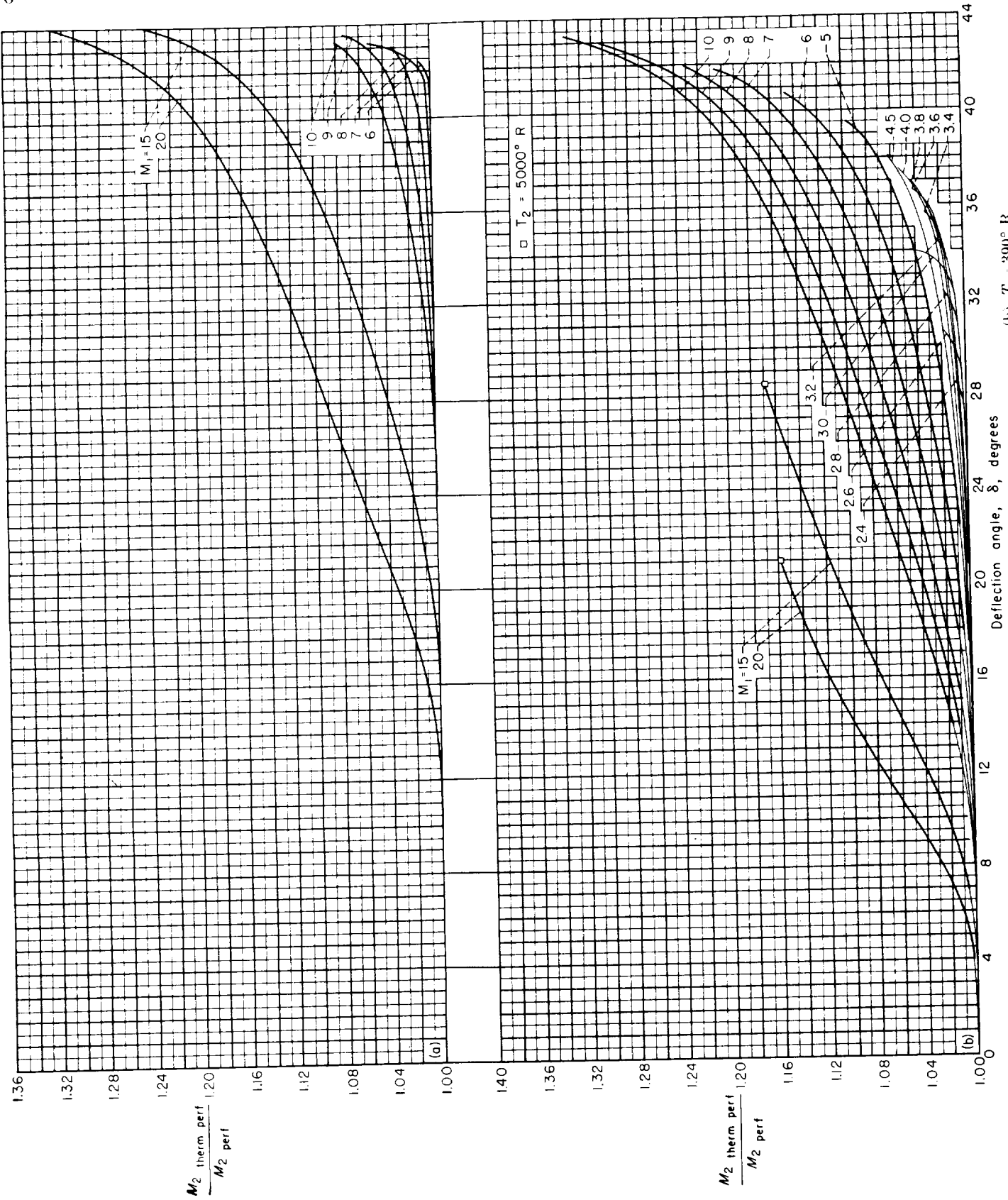
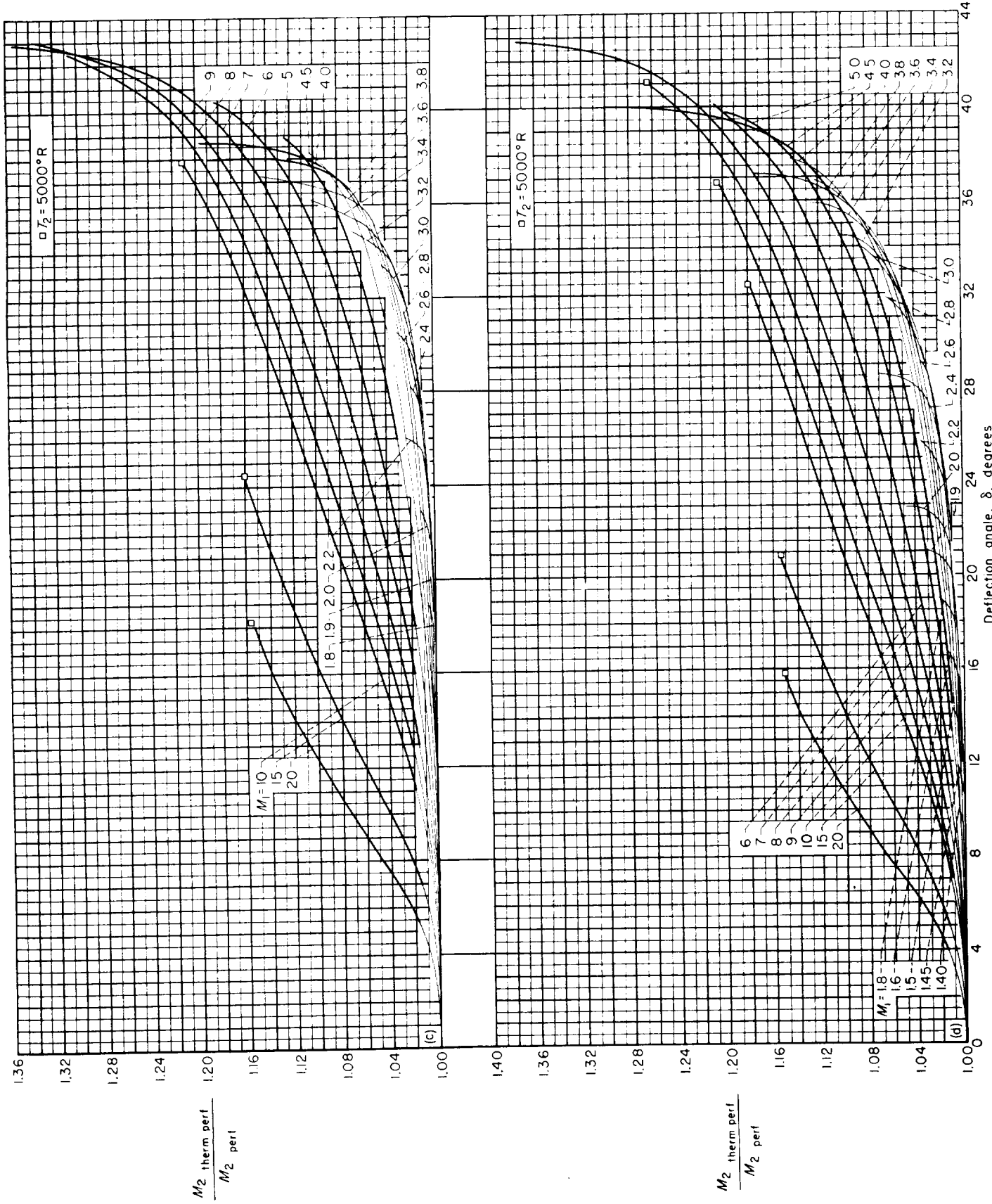


Chart 22.- Effect of calorific imperfections on the variation with flow-deflection angle of the Mach number downstream of a weak oblique shock wave.



(d)  $T_1 = 630^\circ R$

(c)  $T_1 = 5000^\circ R$

CHART 22---Concluded

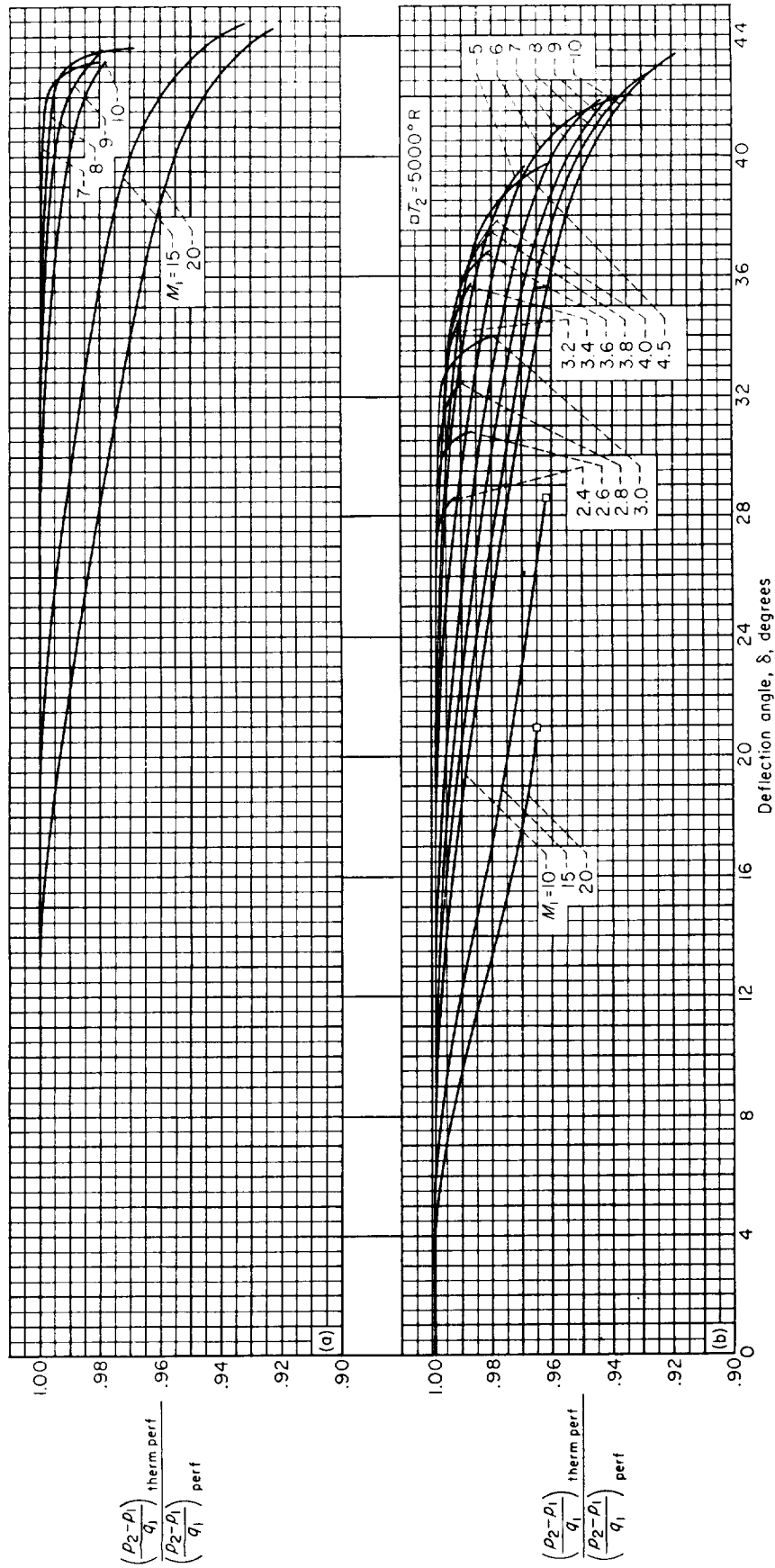
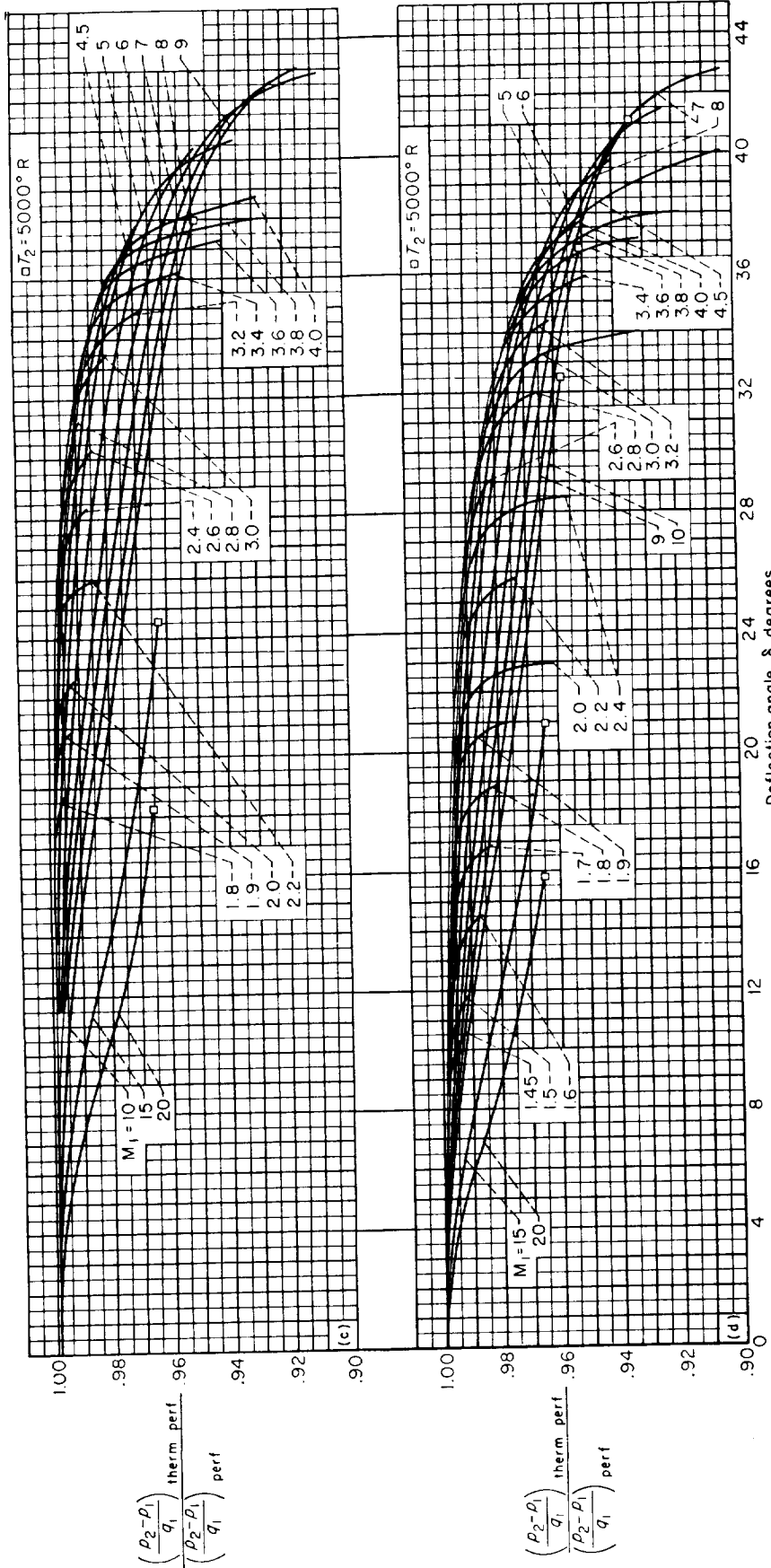


CHART 23.—Effect of caloric imperfections on the variation with flow-deflection angle of the pressure coefficient across a weak oblique shock wave.



(c)  $T_1 = 500^\circ R$

(d)  $T_1 = 630^\circ R$

CHART 23.—(Concluded)

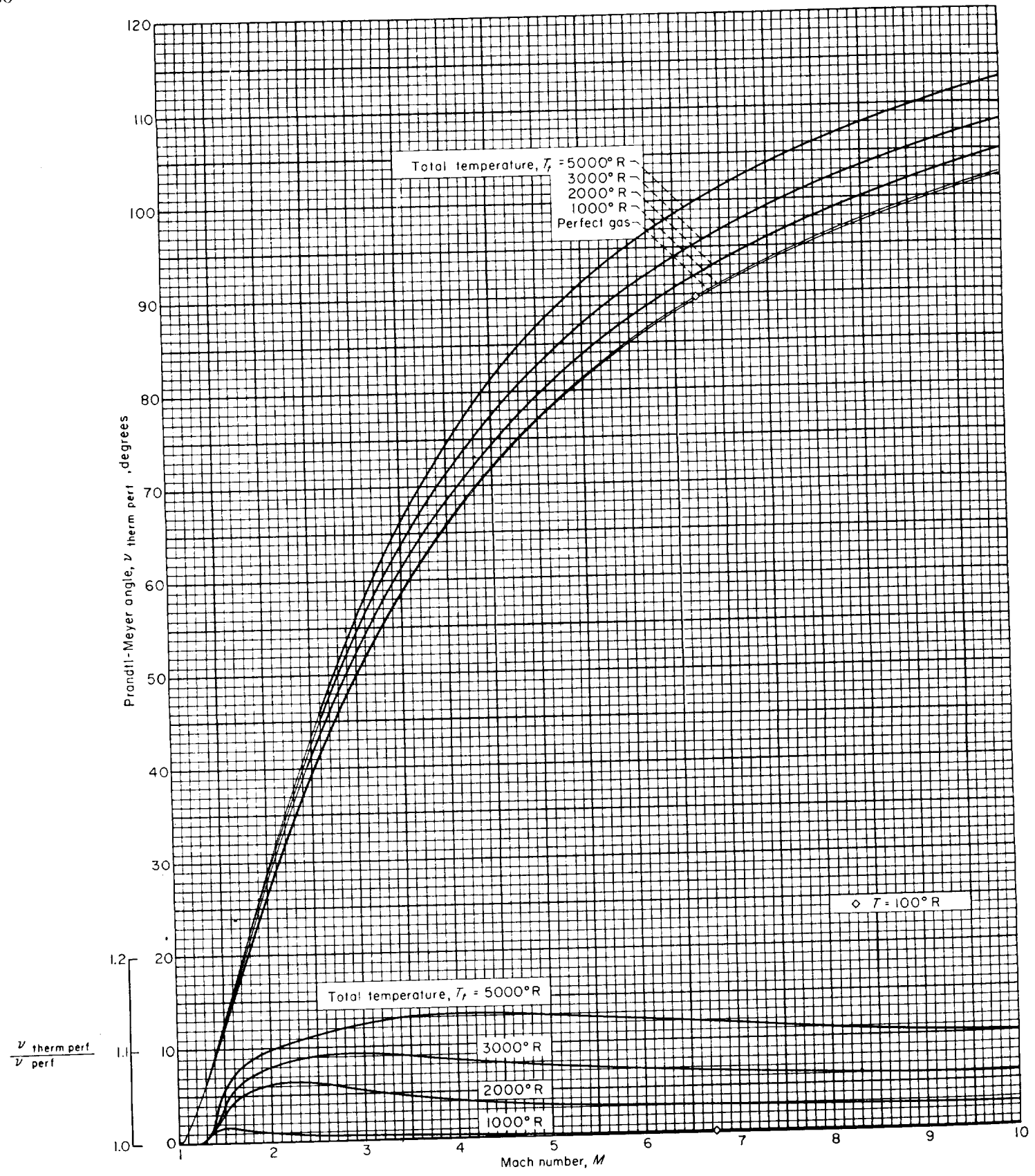


CHART 24.—Effect of caloric imperfections on the Prandtl-Meyer angle.

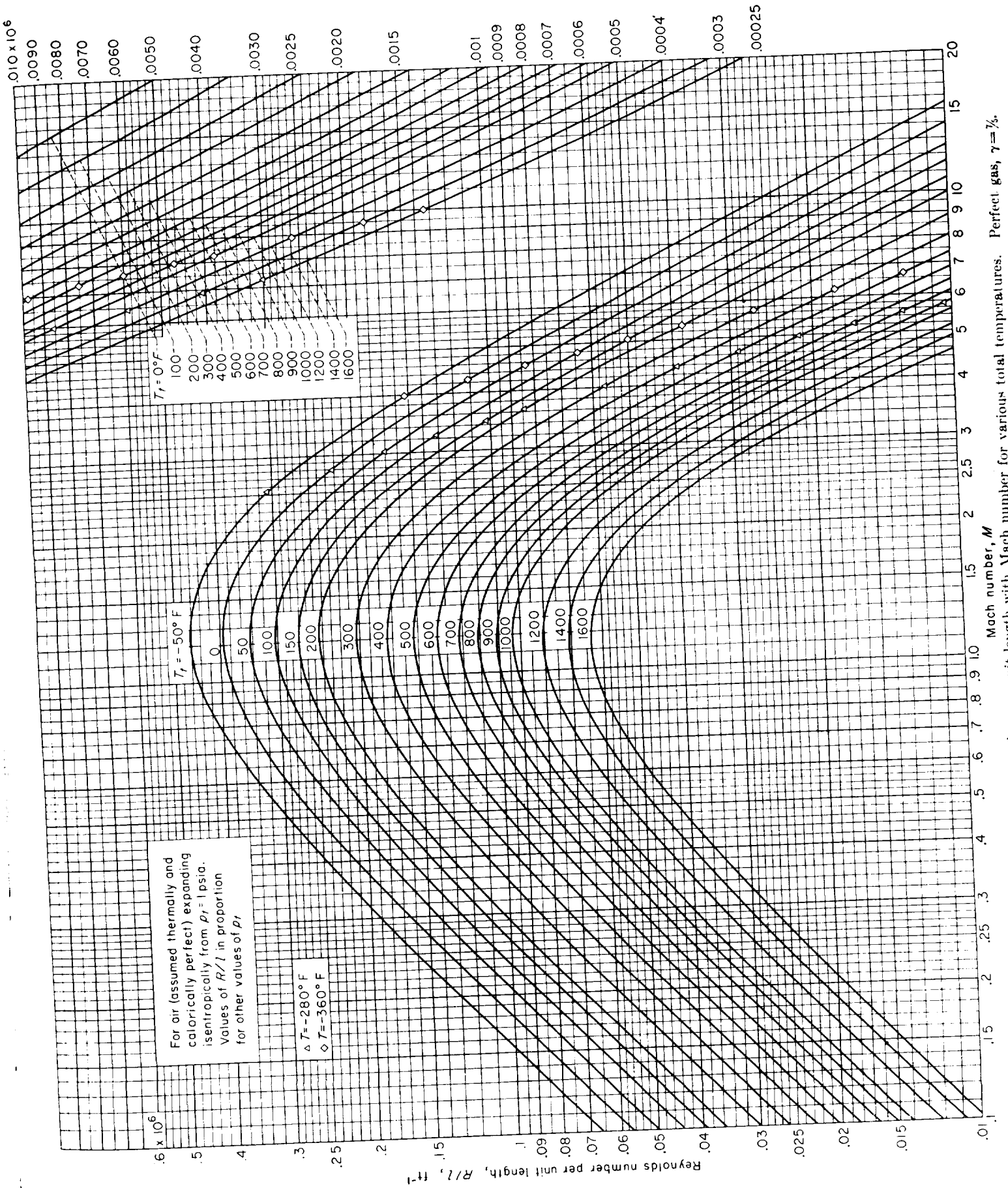


CHART 25.—Variation of Reynolds number per unit length with Mach number for various total temperatures. Perfect gas,  $\gamma = 1.4$ .