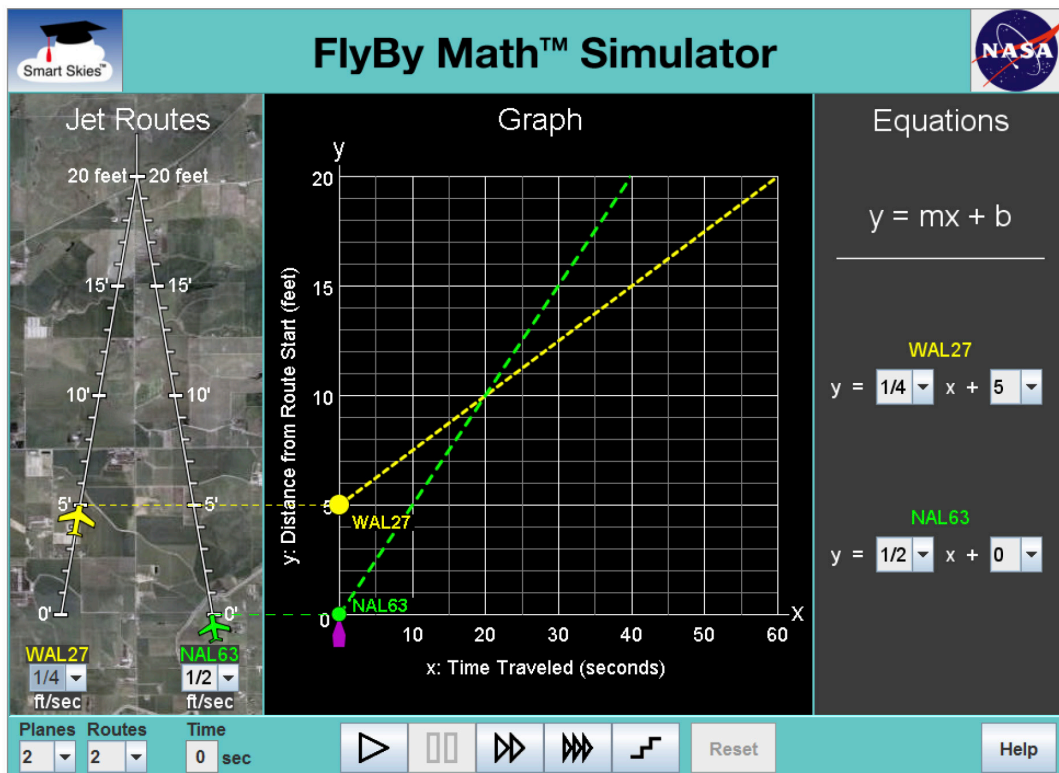




# A Teacher Guide to FlyBy Math™ Simulator

## Linked Representations of Distance-Rate-Time Relationships in Air Traffic Control Grades 6–9



### A Smart Skies™ Product

<https://www.nasa.gov>

<https://www.nasa.gov/flybymath>

Educational Product	
Educators & Students	Grades 6-9
EG-2004-11-111-ARC	



Smart Skies™ FlyBy Math™ is available in electronic format through the NASA Education Portal—NASA’s electronic resources specifically developed for the educational community. This publication and other educational products may be accessed at the following address:

<http://www.nasa.gov/audience/foreducators/topnav/materials/about/index.html>

The FlyBy Math™ simulator and classroom materials are free and available to download from the Smart *Skies FlyBy Math* teacher website:

<https://www.nasa.gov/flyby-math>

Student access to the simulator is available at the following address:

<https://atcsim.nasa.gov/simulator/sim2/sector33.html>

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# TEACHER GUIDE FOR THE FLYBY MATH™ SIMULATOR

## BECOMING ACQUAINTED WITH FLYBY MATH™ SIMULATOR

### A Smart Skies™ Educational Product

#### What is the FlyBy Math Simulator?

The FlyBy Math simulator is an **online visualization tool** that offers multiple linked dynamic representations to help students understand **distance-rate-time** relationships in the **real-world context** of air traffic control. The side-by-side layout enables students to observe and manipulate three views: the planes on their routes, the corresponding distance vs. time graph, and the equation of each line on the graph. The linked visualization tool is supported by five print student worksheets with accompanying assessments.

#### Access to Materials

The FlyBy Math simulator and print materials are free and available online in the FlyBy Math simulator window housed within the FlyBy Math website:

<https://www.nasa.gov/smart-skies/flyby-math>

Direct student access to the FlyBy Math simulator is available via the website:

<https://atcsim.nasa.gov/simulator/sim2/sector33.html>

#### Access to FlyBy Math™

The FlyBy Math simulator was developed under Smart Skies, a key part of the NASA Airspace Systems Program's educational efforts. Smart Skies supports the Program's goal to conduct advanced research on technologies to help pilots and air traffic controllers operate the nation's air transportation system with reduced flight delays and improved efficiency and access.

#### What is the connection between the simulator and FlyBy Math?

The FlyBy Math simulator is a standalone activity, but it may also be used in conjunction with other FlyBy Math activities. In these other activities, students conduct experiments and use a variety of math methods to analyze and solve traditional distance-rate-time problems in air traffic control. One of the math methods is graphing a system of linear equations. For each plane, students are given a linear equation in the form  $y = mx + b$ , where  $x$  is the plane's travel time in minutes and  $y$  is the distance traveled in feet.

The FlyBy Math simulator builds upon that graphing activity by providing students with a linked visualization tool that enables them to reproduce the experiments, equations, and graphs in a side-by-side interactive online format. Students can manipulate the planes on their routes, the corresponding distance vs. time graph, and the equation of each line on the graph. As students change one representation, the other representations are updated accordingly.



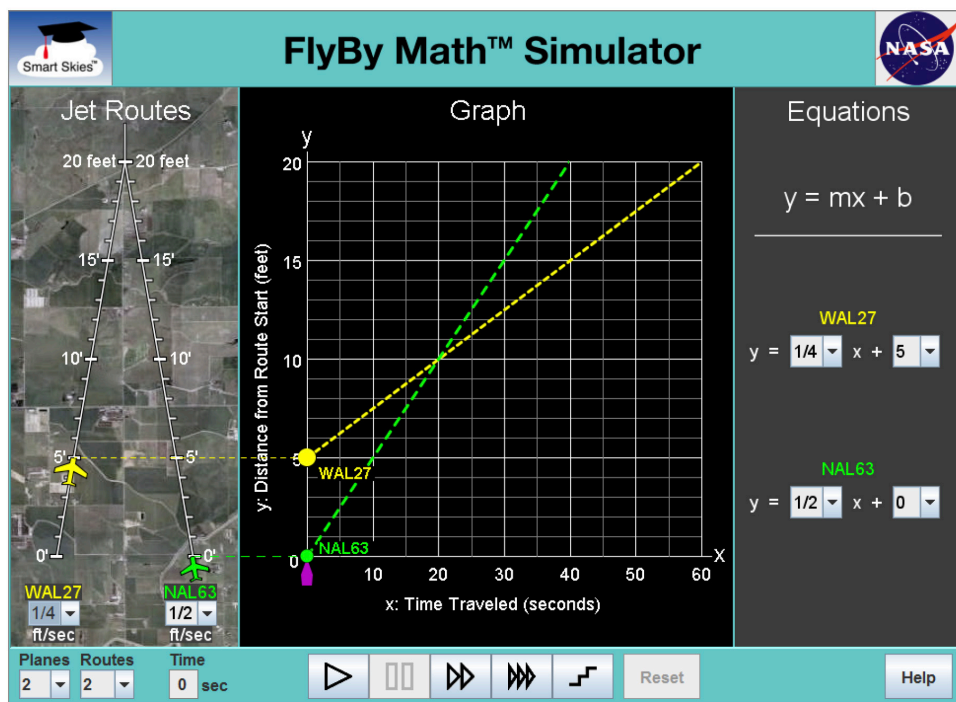
## A Linked Visualization Tool and Five Problem Sets

The FlyBy Math simulator instructional materials consist of:

- An interactive linked visualization tool (the online simulator)
- 5 Problem Sets, each comprised of a Student Worksheet and an Assessment (Note: There is no Assessment document for Worksheet A, the Introduction to the FlyBy Math Simulator .)
- Answer Guides for the Student Worksheets; Answer Guides for the Assessments
- A 2-minute QuickTime video “Animation of 24 hours of flights in the US.” (This video compresses 24-hours of US flight paths to 2 minutes. It illustrates the world’s biggest distance-rate-time problem and motivates the study of air traffic control.)
- A 2-minute QuickTime video “An Introduction to the FlyBy Math Simulator.” (This video introduces students to the simulator and its three linked panels.)

## Three Linked Visualization Tool Panels and the Control Bar

The FlyBy Math simulator is a linked visualization tool with three panels (the Jet Route Panel, the Graph Panel, and the Equation Panel) to help students view and analyze the movement of airplanes on their jet routes. The planes fly at the same altitude. Once a plane’s speed has been selected for a simulation, the plane flies at that constant speed for the duration of the simulation.





*The two flights shown are  
WAL27 (World Airlines flight 27)  
and NAL63 (National Airlines  
flight 63).*

At the same time, a student can:

- View the planes on their **JET ROUTES**.
- See a distance vs. time **GRAPH** for each plane.
- See an **EQUATION** for each line on the graph.

A student can set a plane's speed and/or starting position in any panel.

To run a problem, students use the **Control Bar** at the bottom of the simulator.

Note: To be allowed to change a plane's speed or starting position, a student may first need to click the Reset button or drag the time slider (▲) to 0 on the x-axis.

In the **Jet Route Panel**, students can:

- Show two jet routes with one plane on each route.
- Show one jet route with one plane or with two planes.
- Choose a plane's starting position by dragging a plane between 0 ft and 10 ft on its route.
- Choose a plane's speed using the drop down menu.

Note: The speeds in the Jet Route Panel are not real-world plane speeds. Rather, the Panel speeds correspond to the speeds used in the FlyBy Math experiments. To access the free FlyBy Math activities, including the experiments, go to

<https://www.nasa.gov/smart-skies/flyby-math>

In the **Graph Panel**, students can:

- Drag a dot (•) up and down along the y-axis, between 0 ft and 10 ft.
- Grab a dashed line and rotate it to change (and view) the slope of the line.
- Drag the time slider (▲) along the x-axis.

In the **Equation Panel**, students can:

- Choose the slope of a line using the drop down menu.
- Choose the y-intercept of a line using the drop down menu.



## Each Panel Relates Time and Distance

With the Control Bar, students can:

- Run the problem.
- Pause the problem.
- Run the simulation 4 times faster than normal speed.
- Run the simulation 10 times faster than normal speed.
- Step the time slider (▲) one second at a time.
- Reset the problem to 0 seconds.

Each panel shows the relationship between two key quantities, time and distance. “Time” indicates how long a plane has been flying. “Distance” indicates how far a plane is from the start of its route.

In the Jet Route Panel, time is shown on the clock, from 0 to 60 seconds. Distance is shown along the routes, from 0 to 20 feet.

In the Graph Panel, time is shown along the x-axis, from 0 to 60 seconds. Distance is shown along the y-axis, from 0 to 20 feet.

(Note: The clock displays the time to the nearest whole second. Similarly, the time slider in the Graph Panel can be paused at discrete intervals. So students can use the clock and/or the graph to find the approximate solution of a system of two linear equations. To find the precise solution, students can solve the system algebraically.)

In the Equation Panel, each equation is in slope-intercept form

$$y = mx + b .$$

Time is represented by the variable x. Distance is represented by the variable y.

The slope, m, represents the speed of the corresponding plane.

The y-intercept, b, represents the starting position of the corresponding plane on its jet route. For a particular problem, the plane may start at the beginning of its jet route (0 feet). Then the equation  $y = mx + b$  becomes  $y = mx + 0$  which can be written

$$y = mx .$$

Note the correspondence between this equation and the formula  
distance = rate · time .





## Students Worksheets, Assessments, and Answer Documents

The five Problem Sets provide a structured learning environment for exploring key concepts with the FlyBy Math simulator. The Problem Sets are described briefly below and in more detail in the FlyBy Math Simulator Problem Sets Table that follows this section of this Teacher Guide document.

- Problem Set A: students explore the FlyBy Math simulator features and functionality.
- Problem Set B: students investigate the relationship between a plane's starting position and the vertical-intercept of the corresponding linear distance vs. time graph and equation.
- Problem Set C: students investigate the relationship between a plane's speed and the slope of the corresponding linear distance vs. time graph and equation.
- Problem Set D: students investigate the effect of a plane's "headstart" when two planes are traveling at constant speed. Students learn how to recognize and represent "headstart" using the jet routes and the corresponding linear distance vs. time graphs and equations.
- Problem Set E: students investigate the relationship between the intersection of lines on a distance vs. time graph and the corresponding position of two planes on a single jet route or on two separate jet routes.

Each Problem Set has a print Student Worksheet with an Answer document that contains answers and solutions to each of the Worksheet problems.

Worksheet A consists of a step-by-step introduction to the FlyBy Math simulator.

Worksheets B through E are investigations, each structured as follows:

- First, students use the FlyBy Math simulator to engage in a guided open-ended exploration of the featured worksheet concept.
- Next, students use the FlyBy Math simulator to gather and analyze specified data.
- Finally, students are guided to summarize the results of the previous activities and invited to "go beyond" those activities to extend their current understanding.

Each of Problem Sets B through E also has a Student Assessment with an Answer document. We recommend that students complete the assessment without access to the FlyBy Math simulator.



## Instructional Goals

The FlyBy Math simulator activities have two overarching goals:

- To enable students to use mathematical reasoning and multiple representations to investigate, model, and solve distance-rate-time problems in air traffic control.
- To offer students a variety of problem solving approaches, linked visualization tools, skills, and experiences in the context of challenging, yet accessible, real-world problems.

## Standards

For a detailed alignment to the NCTM and NSTA standards, as well as to the grades 5–9 mathematics standards for all 50 states, see the FlyBy Math website: <https://www.nasa.gov/smart-skies/flyby-math>

The FlyBy Math simulator activities support the following National Council of Teachers of Mathematics (NCTM) Standards and Expectations. Particular emphasis is placed on content standards and expectations from Algebra and Representation.

### (Algebra 6-8)

*Represent and analyze mathematical situations and structures using algebraic symbols.*  
Expectation: Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.

### (Algebra 6-8)

*Use mathematical models to represent and understand quantitative relationships.*  
Expectation: Model and solve contextualized problems using various representations such as graphs, tables, and equations.

### (Algebra 6-8)

*Analyze change in various contexts.*  
Expectation: Use graphs to analyze the nature of changes in quantities in linear relationships.

### (Representation K-12)

*Select, apply, and translate among mathematical representations to solve problems.*  
Use representations to model and interpret physical, social, and mathematical phenomena.

## Prerequisites

In order for students to benefit fully from the FlyBy Math simulator activities, they should satisfy the following prerequisites:

- Be currently enrolled in or have already completed Algebra 1.
- Know how to graph a linear equation given in slope-intercept form,  $y = mx + b$ .
- Know how to find the slope of a non-vertical line given two points on the line.
- For Worksheet E, it is helpful (but not required) for students to know how to solve a system of two linear equations by graphing.

## Worksheet Objectives

The following table lists the materials and specific learning objectives for each FlyBy Math simulator Worksheet.



## The FlyBy Math™ Simulator Worksheet Objectives

Problem Set (Estimated Completion Time)	Number of Planes	Number of Routes	Objectives
<b>A</b> Introduction to the FlyBy Math Simulator  (30 - 45 minutes)	2	2	Students will learn the features and functionality of : <ul style="list-style-type: none"> <li>– the simulator Jet Route Panel</li> <li>– the simulator Graph Panel</li> <li>– the simulator Equation Panel</li> <li>– the simulator Control Bar.</li> </ul>
<b>B</b> Analyzing the Starting Position of One Plane  (45 - 60 minutes)	1	1	Students will learn that for a plane traveling at a constant speed, the initial position of the plane on its jet route corresponds to: <ul style="list-style-type: none"> <li>– the vertical intercept of the corresponding line on the distance vs. time graph</li> <li>– the y-intercept, b, in the corresponding equation <math>y = mx + b</math>.</li> </ul>
<b>C</b> Analyzing the Speed of One Plane  (45 - 60 minutes)	1	1	Students will learn that for a plane traveling at a constant speed, the speed of the plane on its jet route corresponds to: <ul style="list-style-type: none"> <li>– the slope of the corresponding line on the distance vs. time graph</li> <li>– the slope, m, in the corresponding equation <math>y = mx + b</math>.</li> </ul>
<b>D</b> Analyzing Headstart: Two Planes Flying at the Same Speed  (45 - 60 minutes)	2	2	Students will learn that for two planes, each traveling at the <b>same</b> constant speed, one plane having a “headstart:” <ul style="list-style-type: none"> <li>– the plane with the “headstart” maintains that “headstart” throughout the problem</li> <li>– the corresponding distance vs. time graphs are parallel lines</li> <li>– the vertical distance between the parallel lines is the same as the “headstart.”</li> <li>– each line has the same slope.</li> </ul>
<b>E</b> Analyzing a Distance vs. Time Graph for Two Planes  (45 - 60 minutes)	2	1 or 2	Students will learn that for two planes, traveling at <b>different</b> constant speeds with the trailing plane traveling faster than the leading plane: <ul style="list-style-type: none"> <li>– the corresponding distance vs. time graphs will intersect (although that intersection may not be shown on the graph)</li> <li>– if the planes are traveling on the same route, the planes will collide at the time and distance that correspond to the intersection of the lines on the graph (although that intersection may not be shown on the graph)</li> <li>– if the planes are traveling on different routes, the planes do not necessarily collide at the time and distance that correspond to the intersection of the graphs; the planes can collide only at the intersection of the jet routes and only if they arrive at that intersection at the same time.</li> </ul>



## PREPARING TO TEACH THE FLYBY MATH SIMULATOR™

### Select Materials

*If you do not want to print the document, you can simply preview it online.*

### Review Materials

To prepare to teach the FlyBy Math simulator:

1. Go to the FlyBy Math teacher website:

<https://www.nasa.gov/flyby-math>

2. Click the link to the FlyBy Math simulator and classroom materials.
3. Select the Problem Set you will use to introduce your students to the FlyBy Math simulator. We strongly recommend starting with Problem Set A. Download the Student Worksheet for the Problem Set you have selected.
4. Download the Assessment document that accompanies the Worksheet you have selected. (Note: There is no Assessment document for Worksheet A, the introduction to the FlyBy Math simulator.)
5. With the Student Worksheet in hand, and with online access to the FlyBy Math simulator, read the following Implementation section in this document.
6. Duplicate the Student Worksheet and Student Assessment to provide each student with one print copy.
7. Now you're cleared for takeoff! ✈



# IMPLEMENTING THE FLYBY MATH SIMULATOR™ WITH YOUR STUDENTS

## Instructional Activities

*Estimated time:  
5 minutes*

The following sequence of instructional activities is recommended for implementing each FlyBy Math Simulator Student Worksheet. As you read this section, you may find it helpful to have a copy of the Student Worksheet and Answer document.

1. If your students are new to the FlyBy Math simulator, begin with the video:
  - Animation of 24 hours of flights in the US.

This 2-minute QuickTime movie is available from the FlyBy Math simulator teacher window.

*Estimated time:  
5 minutes*

2. If your students are new to the FlyBy Math simulator, show the introductory video:
  - An Introduction to the FlyBy Math Simulator.

This 2-minute QuickTime movie is available from the FlyBy Math simulator teacher window.

*Estimated time: 20 minutes for  
Worksheet A*

3. If your students are new to the FlyBy Math simulator, assign Student Worksheet A to introduce the simulator. You may choose to have the students work independently or in pairs on individual computers. (Or you may choose to project the simulator and the videos for a whole class activity.)

Provide time for your students to explore the FlyBy Math simulator and the assigned worksheet activities that include exploration, data gathering, and analysis.

*20- 30 minutes for Worksheets B  
through E*

Students should access the FlyBy Math simulator from the website:

<https://atcsim.nasa.gov/simulator/sim2/sector33.html>

*Estimated time:  
10 minutes*

4. After your students have completed the Worksheet, discuss the activities with them. For Worksheets B through E, make sure that students have correctly answered the summary questions that follow the data gathering. These summary questions lead students to generalize the data they have collected and guide students to an understanding of the key Worksheet objectives.



*Estimated time:*  
*15–20 minutes*

## Materials

5. Assign the Assessment that accompanies the completed Worksheet. (Note: There is no Assessment document for Worksheet A, the introduction to the FlyBy Math simulator.) The Assessment questions feature a series of sketches of the linked FlyBy Math simulator panels in which some of the information is missing. Instruct your students to use the given data to “fill in the missing information” and complete the multiple representations.

You may want to let your students know that each of the questions in the Assessment can be answered without using the FlyBy Math simulator. In order to encourage your students to apply what they have learned in the Worksheet, we recommend that your students complete this section without access to the simulator. (In fact, for some of the problems, there is not enough information given for students to recreate the starting conditions on the FlyBy Math simulator.)

### **Classroom Materials:**

- QuickTime movie: Animation of 24 hours of flights in the US
- QuickTime movie: An Introduction to the FlyBy Math Simulator
- FlyBy Math simulator
- Student Worksheets A through E
- Student Assessments B through E

### **Teacher Materials:**

- Teacher Guide for the FlyBy Math Simulator (this document)
- FlyBy Math Simulator Quick Start Guide
- Worksheet Answer document for each Worksheet
- Assessment Answer document for each Assessment