Two planes on merging routes are:
-- same distances from the intersection
-- traveling at different speeds.

SMART SKIES™ FLYBY MATH™

Distance-Rate-Time Problems in Air Traffic Control for Grades 5–9

AIR TRAFFIC CONTROL PROBLEM 3

Teacher Guide with Answer Sheets

Overview of Air Traffic Control Problem 3

In this Air Traffic Control (ATC) Problem, students will determine if two airplanes traveling on different merging routes will conflict with (meet) one another at the intersection of their flight routes.

The airplanes are each **the same distance from the point of intersection.**
The airplanes are traveling at the **different constant (fixed) speeds.**

On the five *FlyBy Math*™ ATC Problems, this scenario is considerably more challenging than the simpliest case (ATC Problem 1) in which the speeds, as well as the distances, are the same. This is the first ATC problem in which the planes travel at *different* speeds.

Objectives

Students will determine the following:

- If two planes are traveling at different constant (fixed) speeds on two different routes and the planes are each the same distance from the point where the two routes come together, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.
- Also, since the planes are traveling at a different constant (fixed) speed, their separation distance at the intersection is directly proportional to the difference in speeds. So, for example, if the difference in speeds were twice as great, then the separation at the intersection would also be twice as great.
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Materials

Students handouts:

ATC Problem 3 Student Workbook
ATC Problem 3 Assessment Package (optional)

The student handouts are available on the *FlyBy Math*™ website:

https://www.nasa.gov/smart-skies/flyby-math

Materials for the experiment:

- sidewalk chalk or masking tape or cashier's tape or a knotted rope
- measuring tape or ruler
- marking pens (optional)
- 1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
- pencils
- signs (available on the *FlyBy Math*™ website) identifying pilots, controllers, and NASA scientists
- clipboard (optional)

Introducing Your Students to the ATC Problem

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, plan to meet at the movies. Ana lives 20 blocks from the theater. Alex also lives 20 blocks from the theater. Ana and Alex will each leave their homes at the same time and walk at different constant (fixed) speeds.

You can ask your students these questions:

Will Ana and Alex arrive at the movie theater at the same time? Why or why not?

In particular, if your students think Ana and Alex will arrive at the same time, ask them to explain their reasoning.

Since the students are walking at different constant (fixed) speeds and each must travel the same distance to the theater, students may realize that the students will *not* arrive at the theater at the same time.

Student Workbook

For a detailed description of the Student Workbook features found in each ATC Problem, see the *FlyBy Math*™ Educator Guide.

The following section addresses *special features* of the ATC Problem 3 Workbook.

It may help your students to think of the plane speeds in inches per second:
6 inches/second
4 inches/second

Read the Problem
The speed of each airplane is ½ foot/second. The speed of the other airplane is ⅓ foot/second. Both airplanes are 20 feet from the point of intersection.

Note: The speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.

As a problem enhancement, you may want to ask your students to solve the problem using real-world data.

In a real-world scenario, one plane might be traveling at 400 nautical miles per hour. The other plane might be traveling at 320 nautical miles per hour. Each plane might be 40 nautical miles from the point of intersection.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a "knot".

Set Up and Do the Experiment
A complete description of this section is contained in the *FlyBy Math*™ Educator Guide.

Do the Calculations
Each of the six calculation methods is described in the *FlyBy Math*™ Educator Guide.

One method, Graph Two Linear Equations, is described in greater detail below.

- **Graph Two Linear Equations**
  Caution: Students may confuse the path of a plane with the graph of the plane's distance traveled as a function of time. In particular, the intersection of the graphs at t = 0 indicates that each plane is the same distance, 20 feet, from the intersection of the routes.

Analyze Your Results
As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a movie theater. (See the *Introducing Your Students to the ATC Problem* section of this document.)

Now, you might suggest they consider two cars traveling in parallel lanes on the same road, with the two lanes merging into one lane. Each car is traveling at different constant (fixed) speed. The cars are each the same distance from the merge. Students should realize that the cars will arrive at the merge at different times.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would *not* be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

Extension

The extension introduces a separation requirement at the point where the routes intersect. For safety reasons, when the first plane reaches the intersection, the planes must be separated by a distance greater than or equal to a given standard separation distance. If their separation is less than this standard, a separation violation will occur.

Students are asked to review their calculations to determine the separation distance between the planes at the intersection of the routes. They are then asked whether that distance meets the separation requirement.

Answers and Expectations

The first part of this section summarizes the answers to the key questions posed in the ATC Problem. The remainder of this section is organized by activity and includes graphs, diagrams, and answers to individual questions, as well as discussions of the general problems posed in the analysis activity and the posttest.

The speed of Flight WAL27 is ½ foot/second, so the plane travels
½ foot in 1 second.
The speed of Flight NAL63 is ⅓ foot/second, so the plane travels
⅓ foot in 1 second.
Flight WAL27 is 20 feet from the point of intersection.
Flight NAL63 is 20 feet from the point of intersection.

Since the planes are traveling at different constant (fixed) speeds and must travel the same distance to the point of intersection, the planes will arrive at the intersection at different times. Students must calculate or graph to determine the number of seconds for the faster plane to reach the intersection

In particular:

- It will take 40 seconds for Flight WAL27 to travel 20 feet to the point where the routes come together.
- It will take 60 seconds for Flight NAL63 to travel 20 feet to the point where the routes come together.
- At 40 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be approximately 6.7 feet from the intersection. So the planes will be approximately 6.7 feet apart when the first plane arrives at the intersection.

Answer by Activity

Answers are provided for all worksheets including the Pretest, Student Workbook, and Posttest. Please see the following pages.

Note: Answers are given only for the numbered activity steps that require students to provide a numerical or verbal response. For example, if a step requires a student to trace or circle a portion of a diagram, that step is *not* included in the Answers.

ANSWERS BY ACTIVITY

Pretest

1. Do you think the planes will arrive at the same time at the point where the two routes intersect?

   No

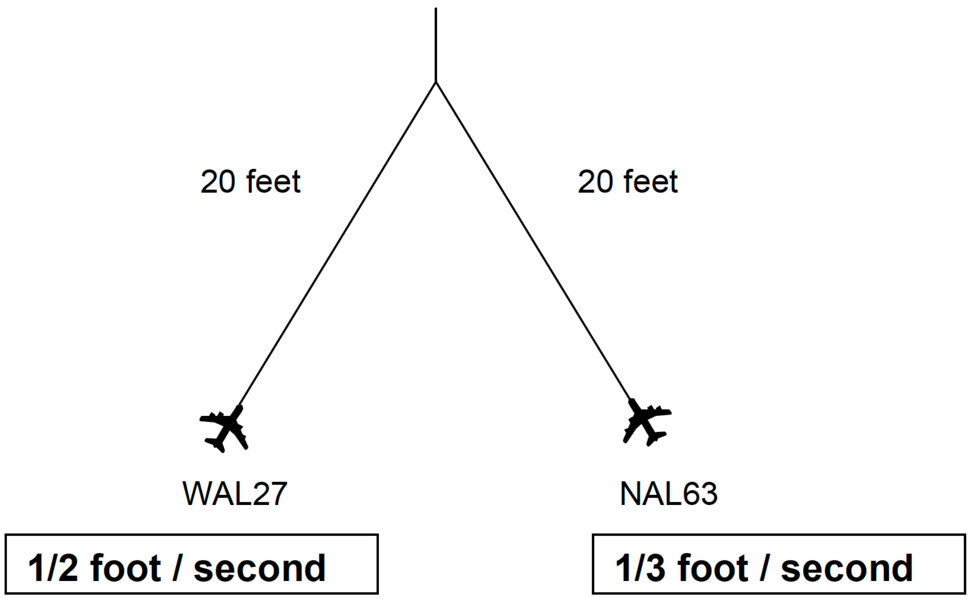
   Why or why not?

   The planes are traveling at different constant speeds and each plane must travel the same distance to the point where the routes intersect. So the planes will *not* meet at the intersection.

2. If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes intersect?

   Approximately 6.7 feet

Student Workbook: Read the Problem



2. How far does WAL27 travel in 1 second? **1/2 foot**

3. How far does WAL27 in 10 seconds? **5 feet**

5. How far does NAL63 travel in 1 second? **1/3 foot**

6. How far does NAL63 travel in 10 seconds? **Approximately 3.3 feet**

Student Workbook: Do the Calculations—Count Feet and Seconds

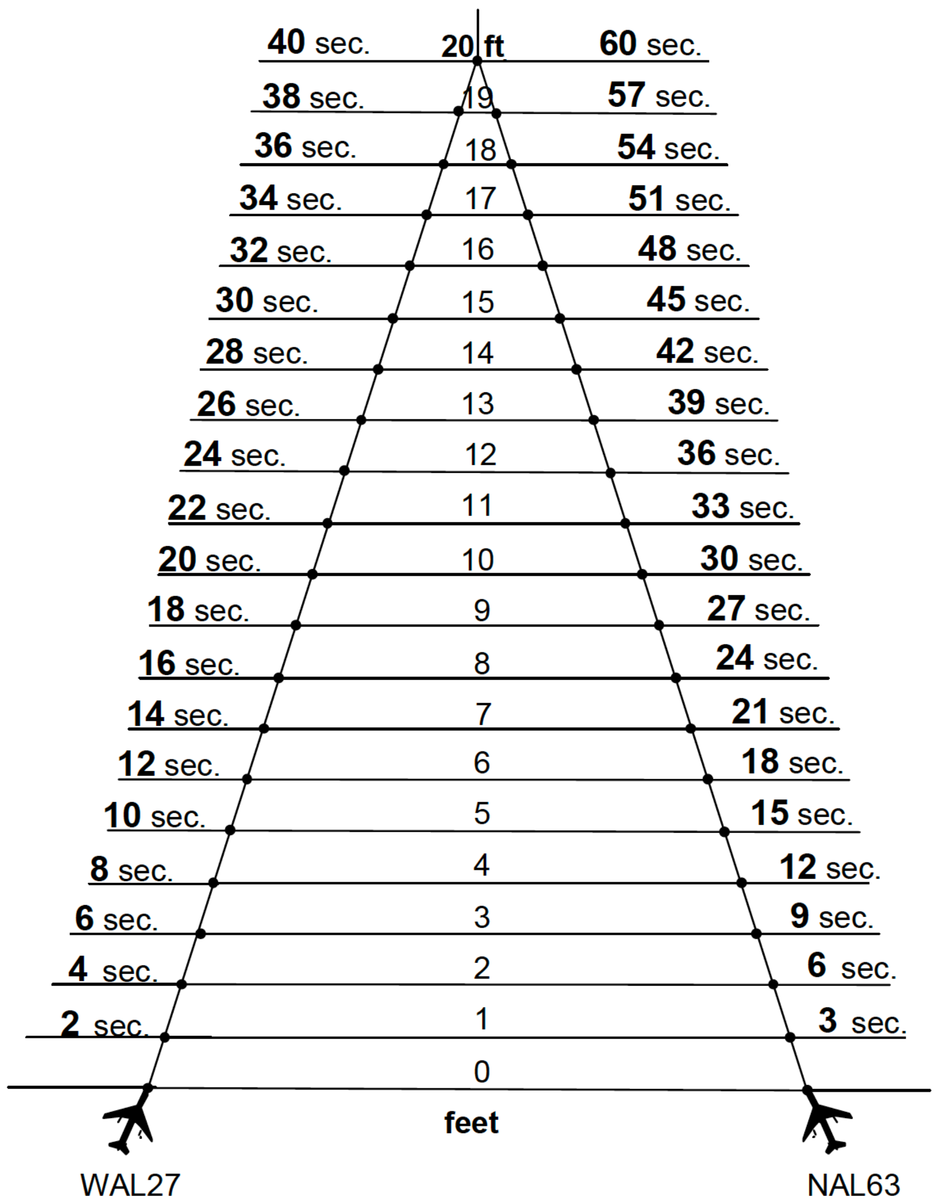
Discussion: Flight WAL27 travels 1 foot in 2 seconds. Count by 2s to fill in the seconds in the blanks along the jet route.

Flight NAL63 travels 1 foot in 3 seconds. Count by 3s to fill in the seconds in the blanks along the jet route.

Flight WAL27 will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.)

Flight NAL63 will travel 20 feet in 60 seconds. (Students can also multiply 3 seconds per foot by 20 feet to obtain 60 seconds.)

So Flight WAL27 arrives at the intersection 20 seconds ahead of Flight NAL63.



Student Workbook: Do the Calculations—Count Feet and Seconds (cont.)

7. How many seconds did it take each plane to arrive at the point where the routes intersect?
   WAL27 **40** seconds   NAL63 **60** seconds

8. Did the planes meet at the point where the two routes intersect?
   No

9. If No, which plane arrived first?
   WAL27

10. How many seconds did it take this plane to travel to the point where the two routes intersect?
    40 seconds

11. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)
    Flight NAL63 approximately 6.7 feet away.
    After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet and was approximately 6.7 feet from the intersection. So the planes do not meet.

12. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?
    Change the speed or change the route of one of the planes.

13. You moved along each jet route, one foot at a time, to find the number of seconds it took each plane to travel to the point where the routes meet. Can you think of a faster way to find the number of seconds? If so, describe the faster way.
    For Flight WAL27, multiply 2 seconds per foot by 20 feet to obtain 40 seconds. For Flight NAL63, multiply 3 seconds per foot by 20 feet to obtain 60 seconds.

Student Workbook: Do the Calculations—Draw Blocks

Discussion:
Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go 3⅓ feet (approximately 3.3 feet).

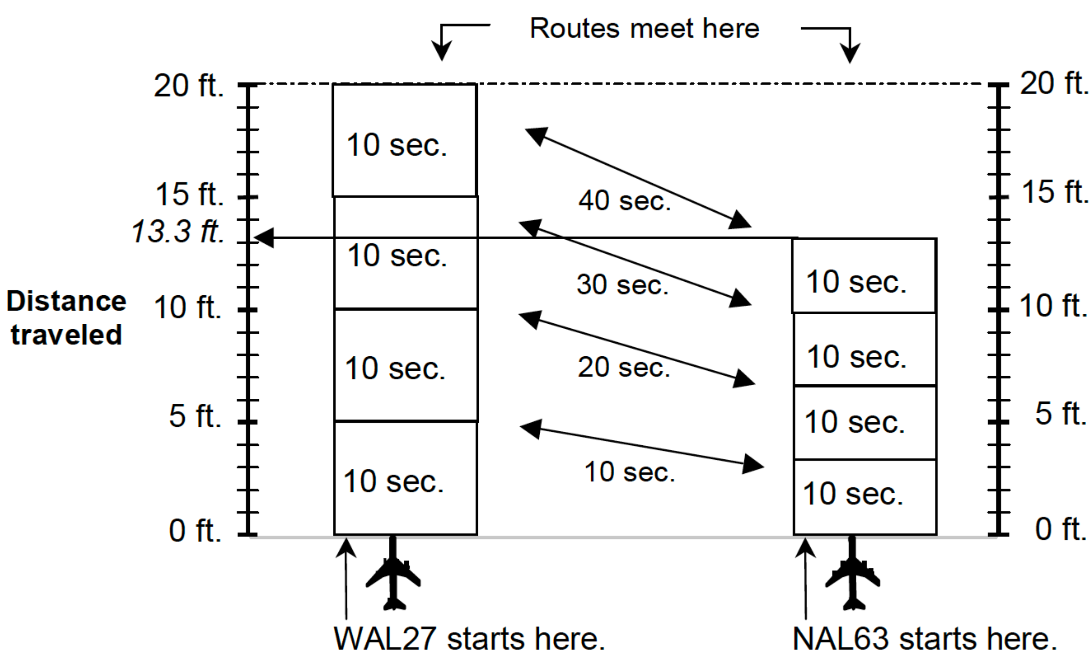
The following diagram shows a stack of 10-second blocks for each plane.

In the stack corresponding to Flight WAL27, each block represents 5 feet.

In the stack corresponding to Flight NAL63, each block represents approximately 3.3 feet.

As the blocks for Flight WAL27 are stacked, it will become clear that the flight arrives at the intersection in 40 seconds.

At this time, Flight NAL63 has traveled approximately 13.3 feet and is approximately 6.7 feet from the intersection. So a conflict will not occur.



1. Each plane takes 10 seconds to travel ...
   5 feet

2. Each plane will travel
   Approximately 3.3 feet
   4 feet in ___8___ seconds

Student Workbook: Do the Calculations—Draw Blocks (cont.)

3. Did the planes meet at the point where the two routes intersect?

   No

4. If No, which plane arrived first?

   WAL27

5. How many seconds did it take this plane to travel to the point where the two routes intersect?

   40 seconds

6. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)

   Flight NAL63 was 6.7 feet away.
   After 40 seconds, Flight NAL63 had approximately 13.3 feet and was approximately 6.7 feet from the intersection. So the planes do not meet.

9. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

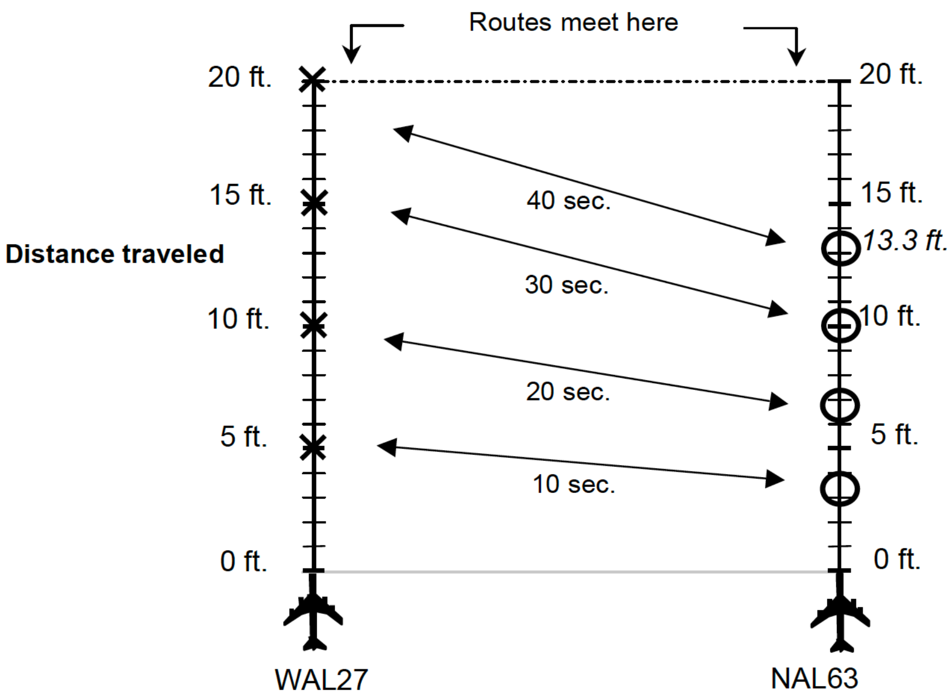
   Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Plot Points on Lines

Discussion:
Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go 3⅓ feet (approximately 3.3 feet).

The following diagram shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the flight arrives at the intersection in 40 seconds. At this time, Flight NAL63 is approximately 13.3 feet from the start and is therefore 6.7 feet from the intersection. So a conflict will not occur.



1. Did the planes meet at the point where the two routes intersect?
   No

2. If No, which plane arrived first?
   WAL27

Student Workbook: Do the Calculations—Plot Points on Lines (cont.)

3. How many seconds did it take this plane to travel to the point where the two routes intersect?

   40 seconds

4. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)

   Flight NAL63 was 6.7 feet away.
   After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet and was approximately 6.7 feet from the intersection. So the planes do not meet.

5. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collison?

   Change the speed or change the route of one of the planes.

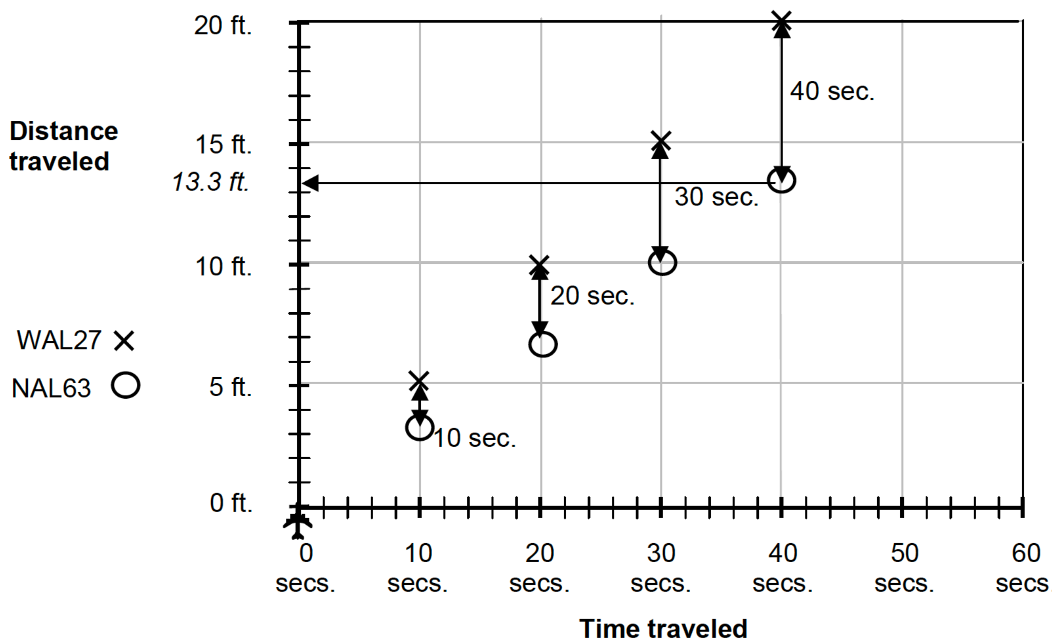
Student Workbook: Do the Calculations—Plot Points on a Grid

Discussion:
Flight WAL27 travels 1 foot in 2 seconds. So in 10 seconds, Flight WAL27 will go 5 feet.
Flight NAL63 travels 1 foot in 3 seconds. So in 10 seconds, Flight NAL63 will go 3⅓feet (approximately 3.3 feet).

The following graph shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the flight arrives at the intersection in 40 seconds.

At this time, Flight NAL63 is approximately 13.3 feet from the start and is therefore 6.7 feet from the intersection. So a conflict will not occur.



1. Did the planes meet at the point where the two routes intersect?

   No

Student Workbook: Do the Calculations—Plot Points on a Grid (cont.)

2. If No, which plane arrived first?
   WAL27

3. How many seconds did it take this plane to travel to the point where the two routes intersect?
   40 seconds

4. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)
   Flight NAL63 was 6.7 feet away.
   After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet and was approximately 6.7 feet from the intersection. So the planes do not meet.

5. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?
   Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Use a Formula

Discussion:Flight WAL27 arrives at the intersection first, in 40 seconds.

Flight NAL63 arrives at the intersection 20 seconds later, at 60 seconds.

Since the times are different, a conflict will not occur.

1. —In 4 seconds, Flight WAL27 travels **0.5 feet/second** × **4 seconds** = **2.0** feet.

   —In 5 seconds, Flight WAL27 travels **0.5 feet/second** × **5 seconds** = **2.5** feet.

2. How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds?
   Multiply 0.5 feet/second by 14 seconds.

3. Use the formula

   $$d = r \bullet t$$

   to answer this question.
   How many feet does Flight WAl27 travel in 20 seconds? **10 feet**

4. Use the formula

   $$d = r \bullet t$$

   to answer this question.

   How many feet does Flight NAL63 travel in 20 seconds?
   Approximately 6.7 feet

Student Workbook: Do the Calculations—Use a Formula (cont.)

5. Use the formula

$$t = \frac{d}{r}$$

to find the number of seconds for WAL27 to travel 20 feet to the point where the routes meet

$$t = \frac{\text{20 feet}}{\text{0.5 feet per second}} = \underline{\ 40\ } \text{ seconds}$$

6. Use the same formula to find the number of seconds for NAL63 to travel 20 feet to the point where the routes meet.

$$t = \frac{\text{20 feet}}{\text{1/3 foot per second}} = \underline{\ 60\ } \text{ seconds}$$

7. Will the planes meet at the point where the two routes intersect?

No

8. If No, which plane will arrive first?

WAL27

Student Workbook: Do the Calculations—Use a Formula (cont.)

9. How many seconds will it take this plane to travel to the point where the two routes intersect?

   40 seconds

10. At that time, how far away is the other airplane? (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)

    Flight NAL63 was 6.7 feet away.
    After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet and is approximately 6.7 feet from the intersection. So the planes do not meet.

11. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

    Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Graph Linear Equations

Discussion: Flight WAL27 arrives at the intersection first, in 40 seconds.

Flight NAL63 arrives at the intersection 20 seconds later, at 60 seconds.

Since the times are different, a conflict will not occur.

1. Fill in the table for **WAL27**.

d = (1/2)t

t seconds	d feet
0	**0**
10	**5**
20	**10**
30	**15**
40	**20**

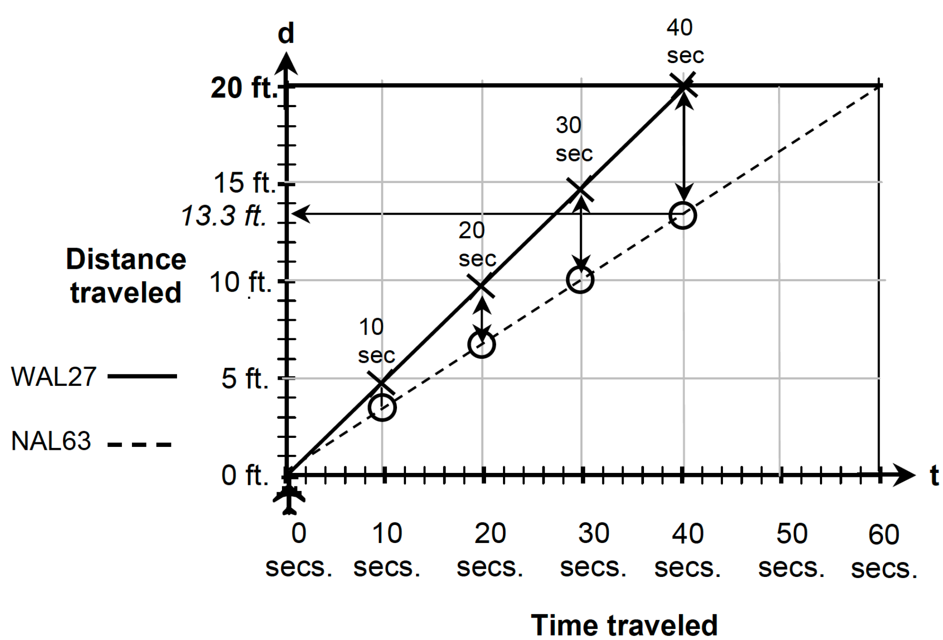
2. Fill in the table for **NAL63**.

d = (1/3)t

t seconds	d feet
0	**0**
10	**3.3**
20	**6.7**
30	**10**
40	**16.7**

3. Use an **X** to graph each point in the WAL27 table. Use a solid line to connect the points.

4. Use an O to graph each point in the NAL63 table. Use a dotted line to connect the points.



Student Workbook: Do the Calculations—Graph Linear Equations (cont.)

5. How many seconds will it take each plane to arrive at the point where the routes intersect?

   WAL27 **40 seconds** NAL63 **60 seconds**

6. Will the planes meet at the point where the two routes intersect?
   No

7. If no, which plane will arrive first?
   WAL27

8. How many seconds will it take this plane to travel to the point where the two routes intersect?
   40 seconds.

9. At that time, how far away is the other airplane? (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)
   Flight NAL63 was 6.7 feet away.
   After 40 seconds, Flight NAL63 had traveled approximately 13.3 feet and is approximately 6.7 feet from the intersection. So the planes do not meet.

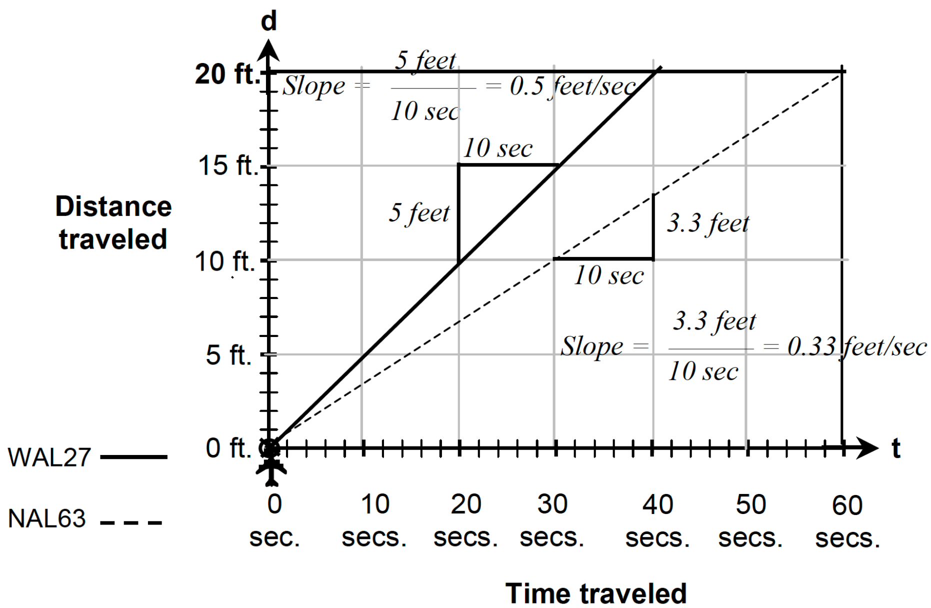
10. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?
    Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Graph Linear Equations (cont.)

Discussion: For each line, its slope represents the speed of the corresponding plane.

For the line corresponding to Flight WAL27, the slope of the line is 0.5 feet/sec, the speed of Flight WAL27.

For the line corresponding to Flight NAL63, the slope of the line is approximately 0.33 feet/sec, the speed of Flight NAL63.



11. Write the number that is the slope of the line representing WAL27.
    1/2 feet/second

12. Write the number that is the slope of the line representing NAL63.
    1/3 feet/second

13. What information does the slope of the line tell you about each plane?.
    For Flight WAL27, the slope of the line is 1/2 foot/second, the speed of Flight WAL27. For Flight NAL63, the slope of the line is 1/3 foot/second, the speed of Flight NAL63.

Student Workbook: Analyze Your Results

Discussion: Suppose two planes are traveling at different constant (fixed) speeds on two different routes and the planes are the same distance from the point where the two routes come together.

Since the planes are traveling at different constant (fixed) speeds and must each travel the same distance to the point of intersection, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.

If the difference in speeds were twice as great, then the separation distance at the intersection would also be twice as great. Students may understand this relationship based upon the activities they have completed.

To obtain a more formal explanation, one can use the Distance-Rate-Time formula as follows:

Let d_F, r_F, *and* t_F *represent the distance, rate, and time of the Faster plane.*

Let d_S, r_S, *and* t_S *represent the distance, rate, and time of the Slower plane.*

Examine the separation distance, $d_F - d_S$, *at the time when the Faster plane reaches the intersection.*
That time is t_F.

At time t_F, *the distance traveled by the Faster plane is:* $d_F = r_F \cdot t_F$

At time t_F, *the distance traveled by the Slower plane is:* $d_S = r_S \cdot t_F$
(Notice that the time is the arrival time of the Faster plane.)

The separation distance is:

$$d_F - d_S = r_F \cdot t_F - r_S \cdot t_F$$

$$= t_F\,(r_F - r_S)$$

So the separation distance is proportional to the difference in speeds.

If the difference in speeds is doubled, the new separation distance $= t_F \cdot 2(r_F - r_S)$

Therefore, the separation distance is doubled.

Student Workbook: Analyze Your Results (cont.)

1. Are the plane speeds the same or different?
   Different

2. Are the planes' starting distance the same or different?
   Same

3. Did the planes meet where the routes meet?
   No

Two planes are flying at the same speed on two different routes. The planes start at different distances from the point where the routes meet.

4. The two planes will reach the point where the routes meet at **different times.**

- Suppose the difference in the plane speeds is twice as great.

5. Then the difference in separation distance will be **twice as great.**

Student Workbook: Extension

1. Based upon your calculations, what is the difference in the planes' final positions? (That is, what is the planes' separation distance where the routes meet?)
   Approximately 6.7 feet

2. Does this distance satisfy the separation requirement?
   Yes

3. If No, what would you tell the air traffic controller to do to meet the separation requirement?
   Change the speed or change the route of one of the planes.

Posttest

1. Do you think the planes will meet at the point where the two routes intersect?
   No

   Why or why not?
   The planes are traveling at different speeds and each plane must travel the same distance to the point where the routes intersect. So the planes will *not* meet at the intersection.

2. If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes intersect?
   10 feet

3. Does your answer to Question 2 meet the 5-foot separation standard?
   Yes

4. If you think two planes will *not* meet the 5-foot separation standard, what could you tell the traffic controller to do to make sure that the separation standard will be met?
   Change the speed or change the route of one of the planes.

Now consider this general problem.

Two planes are traveling at different speeds on two different routes.

The planes are the same distance from the point where the two routes intersect.

5. Will the planes meet at the point where the routes intersect?
   No

Posttest (cont.)

6. Suppose the difference in speeds is twice as great. What would you expect to happen to the separation distance at the point where the routes come together? Why?

   The separation at the intersection would be twice as great. The planes are traveling at different constant (fixed) speeds, so their separation distance at the intersection is directly proportional to the difference in speeds. If the difference in speeds is twice as great, then the separation at the intersection will be twice as great.