Two planes on merging routes are:
-- different distances from the intersection
-- traveling at the same speed.

SMART SKIES™ FLYBY MATH ™

Distance-Rate-Time Problems in Air Traffic Control for Grades 5–9

AIR TRAFFIC CONTROL PROBLEM 2

Teacher Guide with Answer Sheets

Overview of Air Traffic Control Problem 2

In this Air Traffic Control (ATC) Problem, students will determine if two airplanes traveling on different merging routes will conflict with (meet) one another at the intersection of their flight routes.

The airplanes are each a **different distance from the point of intersection.** The airplanes are traveling at the **same constant (fixed) speeds.**

On the five *FlyBy Math*™ ATC Problems, this scenario is one step removed from the simpliest case (ATC Problem 1) in which the distances, as well as the speeds, are the same.

Objectives

Students will determine the following:

- If two planes are traveling at the same constant (fixed) speed on two different routes and the planes are different distances from the point where the two routes come together, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.
- Also, since the planes are traveling at the same constant (fixed) speed, the plane closest to the intersection will mainatin its "headstart." So at the intersection, the separation between the planes will be the same as the "headstart" of the plane that was closest to the intersection at the beginning of the problem.

Materials

Students handouts:

ATC Problem 1 Student Workbook
ATC Problem 1 Assessment Package (optional)

The student handouts are available on the *FlyBy Math*™ website:

https://www.nasa.gov/smart-skies/flyby-math

Materials for the experiment:

- sidewalk chalk or masking tape or cashier's tape or a knotted rope
- measuring tape or ruler
- marking pens (optional)
- 1 stopwatch or 1 watch with a sweep second hand or 1 digital watch that indicates seconds
- pencils
- signs (available on the *FlyBy Math*™ website) identifying pilots, controllers, and NASA scientists
- clipboard (optional)

Introducing Your Students to the ATC Problem

You may want to show the FlyBy Math™ video clips to introduce your students to the air traffic control system.

(For more detail, see the FlyBy Math™ Educator Guide.)

To help your students understand the problem, you can ask them to consider this related problem that is set in a more familiar context:

Two students, Ana and Alex, plan to meet at the movies. Each student lives 20 blocks from the theater. Alex lives 16 blocks from the theater. Ana and Alex will each leave their homes at the same time and walk at the same constant (fixed) speed.

You can ask your students these questions:

Will Ana and Alex arrive at the movie theater at the same time? Why or why not?

In particular, if your students think Ana and Alex will arrive at the same time, ask them to explain their reasoning.

Since the students are walking at the same constant (fixed) speed and each must travel a different distance to the theater, students may realize that the students will not arrive at the theater at the same time. Alex, who has a 4-block "headstart," will arrive first.

Student Workbook

For a detailed description of the Student Workbook features found in each ATC Problem, see the *FlyBy Math*™ Educator Guide.

The following section addresses *special features* of the ATC Problem 2 Workbook.

Read the Problem
The speed of each airplane is ½ foot/second. One airplane is 20 feet from the point of intersection. The other airplane is 16 feet from the point of intersection. (This plane has a 4-foot "headstart.")

Note: The speeds and distances were chosen to reflect the classroom experiment that the students will conduct and are not related to real-world parameters.

As a problem enhancement, you may want to ask your students to solve the problem using real-world data.

In a real-world scenario, each plane might be traveling at 400 nautical miles per hour. Each plane might be 40 nautical miles from the point of intersection. The other plane might be 36 nautical miles from that point.

An international nautical mile is 1,852 meters.

A nautical mile per hour is called a "knot".

Set Up and Do the Experiment
A complete description of this section is contained in the *FlyBy Math*™ Educator Guide.

Do the Calculations
Each of the six calculation methods is described in the *FlyBy Math*™ Educator Guide.

One method, Graph Two Linear Equations, is described in greater detail below.

– **Graph Two Linear Equations**
  Caution: Students may confuse the path of a plane with the graph of the plane's distance traveled as a function of time. In particular, students need to understand that the routes meet, but the planes and the graphs do not meet.

Analyze Your Results
As part of the Analysis, you may also want to ask your students to create a similar problem in a different setting. They have already considered a problem in which two students walk from their respective homes to a movie theater. (See the *Introducing Your Students to the ATC Problem* section of this document.)

Now, you might suggest they consider two cars traveling in parallel lanes on the same road, with the two lanes merging into one lane. Each car is traveling at the same constant (fixed) speed. The cars are each a different distance from the merge. Students should realize that the cars will arrive at the merge at different times.

Note: To be consistent with the airspace scenarios, it is important that for each problem created by you or your students, you choose a fixed (constant) speed for each vehicle or person. (For example, a rocket launch scenario would not be appropriate because a launched rocket typically accelerates and therefore its speed is not constant.)

Extension

The extension introduces a separation requirement at the point where the routes intersect. For safety reasons, when the first plane reaches the intersection, the planes must be separated by a distance greater than or equal to a given standard separation distance. If their separation is less than this standard, a separation violation will occur.

Students are asked to review their calculations to determine the separation distance between the planes at the intersection of the routes. They are then asked whether that distance meets the separation requirement. Students will determine that the separation standard will be violated.

To avoid a separation violation, the air traffic controller assigns a new route to the plane that is 16 feet from the original intersection. Students are asked to determine whether the new route averts the separation violation.

Answers and Expectations

The first part of this section summarizes the answers to the key questions posed in the ATC Problem. The remainder of this section is organized by activity and includes graphs, diagrams, and answers to individual questions, as well as discussions of the general problems posed in the analysis activity and the posttest.

Answer Summary

The speed of Flight WAL27 is ½ foot/second, so the plane travels ½ foot in 1 second.

The speed of Flight NAL63 is ½ foot/second, so the plane travels ½ foot in 1 second.

Flight WAL27 is 16 feet from the point of intersection.
Flight NAL63 is 20 feet from the point of intersection.

Since the planes are traveling at the same constant (fixed) speed and must travel a different distance to the point of intersection, a conflict will not occur at the intersection.

Answers and Expectations

In particular:

- It will take 32 seconds for Flight AL27 to travel 16 feet to the point where the routes come together.
- It will take 40 seconds for Flight NAL63 to travel 20 feet to the point where the routes come together.
- Since the times are different for each airplane, they will arrive at different times and a conflict will not occur.
- Since the planes are traveling at the same constant (fixed) speed, Flight WAL27 will maintain its 4-foot "headstart." So at the intersection, the separation between the planes will be 4 feet, the same as the Flight WAL27 4-foot "headstart."
- At 32 seconds, when Flight WAL27 arrives at the intersection, Flight NAL63 will be 4 feet from the intersection. So the planes will be 4 feet apart when the first plane arrives at the intersection.

Answers and Expectations

Answers are provided for all worksheets including the Pretest, Student Workbook, and Posttest. Please see the following pages.

Note: Answers are given only for the numbered activity steps that require students to provide a numerical or verbal response. For example, if a step requires a student to trace or circle a portion of a diagram, that step is not included in the Answers.

ANSWERS BY ACTIVITY

Pretest

1. Do you think the planes will meet at the point wher the two routes intersect?

   No

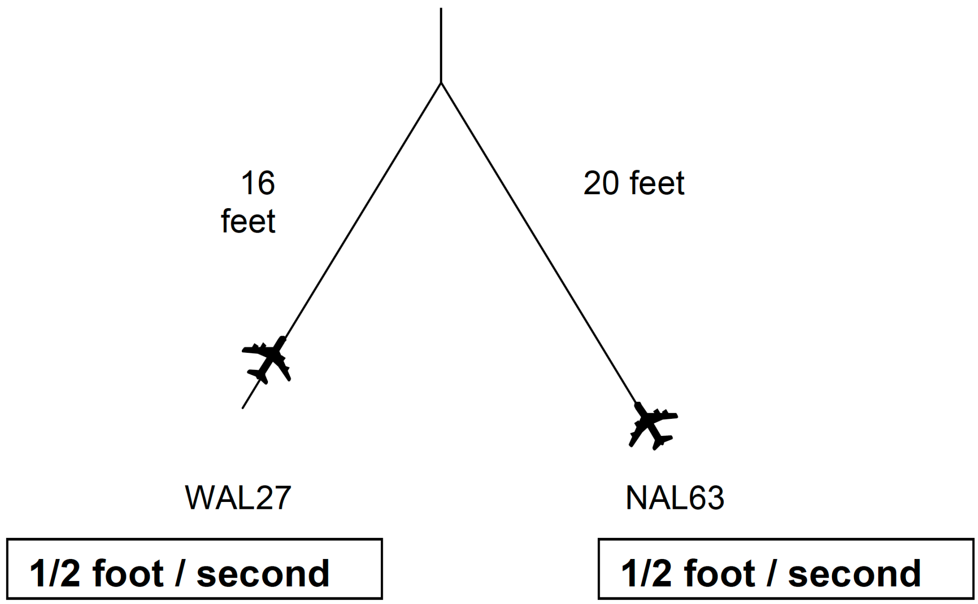
   Why or why not?

   The planes are traveling at the same constant speed and each plane must travel the same distance to the point where the routes intersect. So the planes will not meet at the intersection.

2. If not, how many feet apart do you think the planes will be when the first plane reaches the point where the routes intersect?

   4 feet

Student Workbook: Read the Problem



2. How far does WAL27 travel in 1 second? <u>**1/2 foot**</u>

3. How far does WAL27 in 10 seconds? <u>**5 feet**</u>

5. How far does NAL63 travel in 1 second? <u>**1/2 foot**</u>

6. How far does NAL63 travel in 10 seconds? <u>**5 feet**</u>

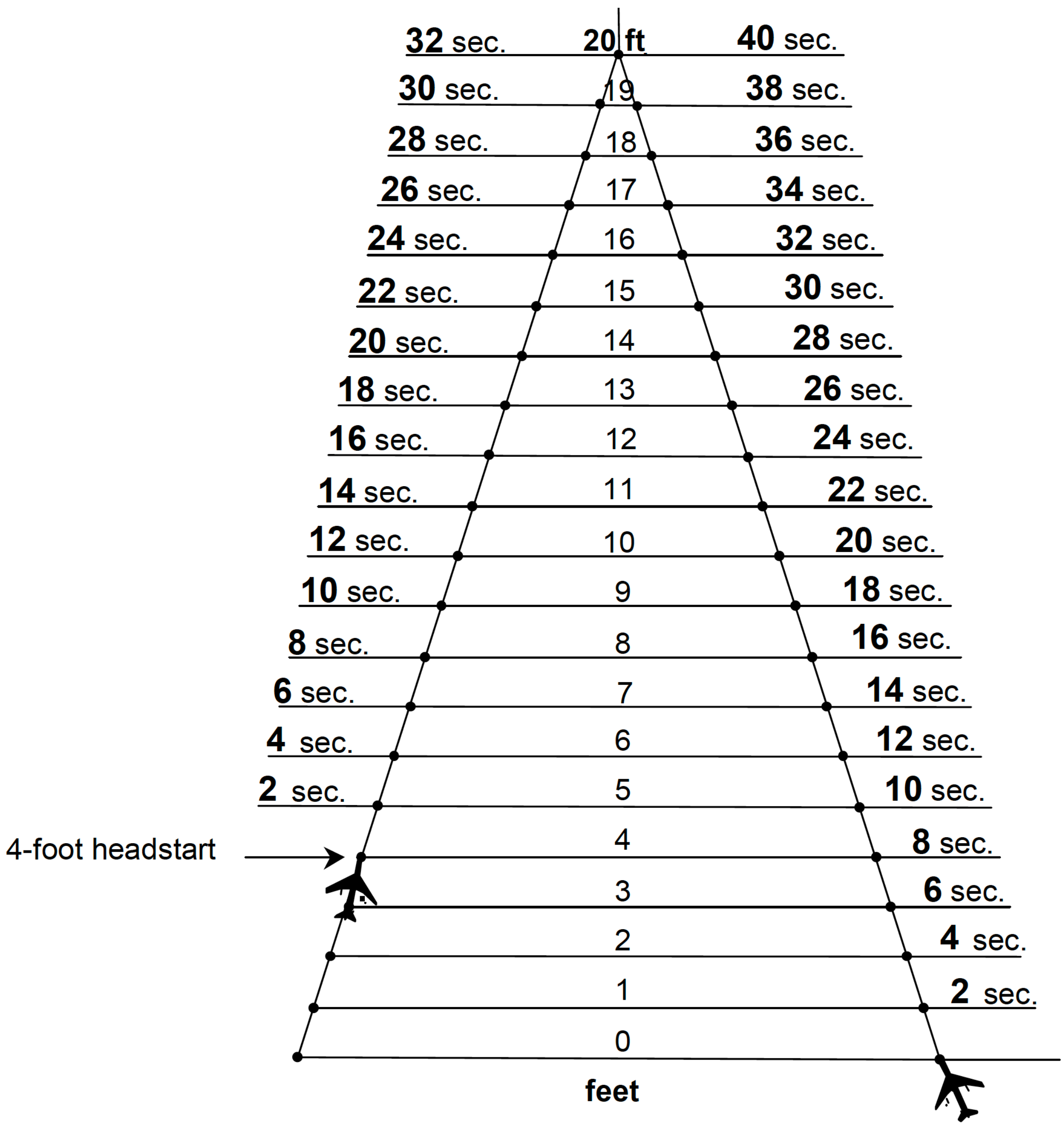
Student Workbook: Do the Calculations—Count Feet and Seconds

Discussion: Each plane travels 1 foot in 2 seconds. Count by 2s to fill in the seconds in the blanks along the jet route.

Flight WAL27 will travel 16 feet in 32 seconds. (Students can also multiply 2 seconds per foot by 16 feet to obtain 32 seconds.)

Flight NAL63 will travel 20 feet in 40 seconds. (Students can also multiply 2 seconds per foot by 20 feet to obtain 40 seconds.)

Since the times are different for each airplane, they will arrive at different times and a conflict will not occur.



Student Workbook: Do the Calculations—Count Feet and Seconds (cont.)

7. How many seconds did it take each plane to arrive at the points where the routes intersect?
   WAL27 **32** seconds NAL63 **40** seconds

8. Did the planes meet at the point where the two routes intersect?
   No

9. If No, which plane arrived first?
   WAL27

10. How many seconds did it take this plane to travel to the point where the two routes intersect?
    32 seconds

11. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)
    Flight NAL63 was 4 feet away. After 32 seconds, Flight NAL63 traveled 16 feet was was 4 feet from the intersection. So the plane do not meet.

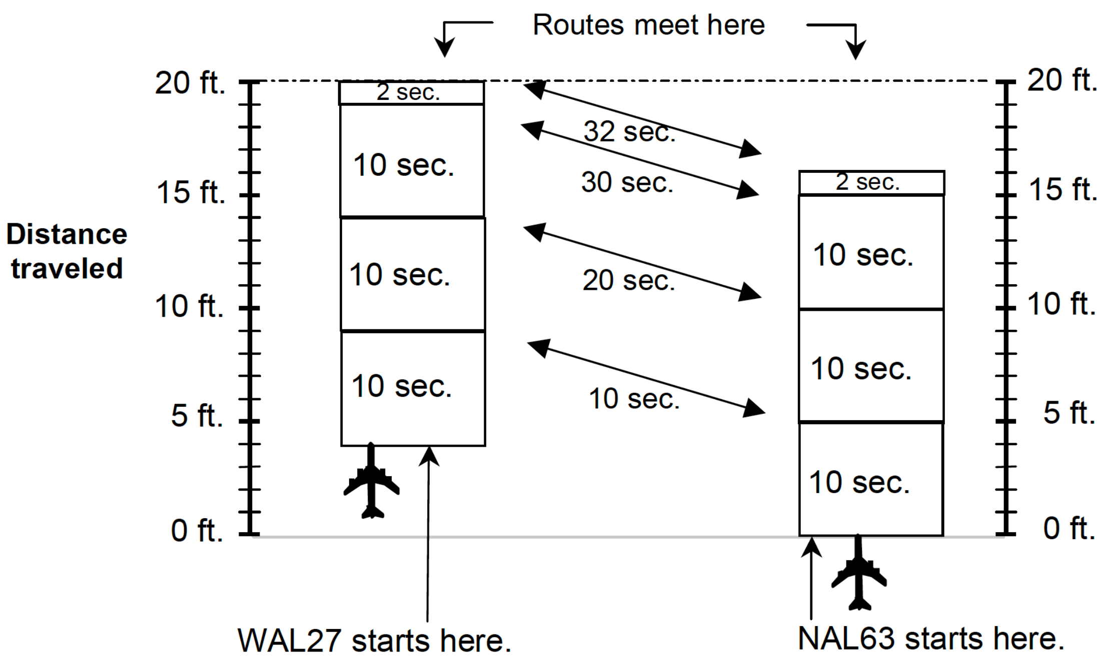
12. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?
    Change the speed or change the route of one of the planes.

13. You moved along each jet route, one foot at a time, to find the number of seconds it took each plane to travel to the point where the routes meet. Can you think of a faster way to find the number of seconds? If so, describe the faster way.
    For Flight WAL27, multiply 2 seconds per foot by 16 feet to obtain 32 seconds. For Flight NAL63, multiply 2 seconds per foot by 20 feet to obtain 40 seconds.

Student Workbook: Do the Calculations—Draw Blocks

Discussion: Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the 5-foot blocks added together. To get the answer in seconds, add the 10-second blocks.

As the blocks for Flight WAL27 are stacked, it will become clear that the answer is a little more than 30 seconds. In particular, Flight WAL27 has to go only 1 foot past 30 seconds. This foot corresponds to 2 seconds. So Flight WAL27 will arrive in 32 seconds. At this time, Flight NAL63 is still 4 feet from the intersection. So a conflict will not occur.



1. Each plane takes 10 seconds to travel ...
   <u>5 feet</u>

2. Each plane will travel
   3 feet in ____**6**____ seconds
   4 feet in ____**8**____ seconds

Student Workbook: Do the Calculations—Draw Blocks (cont.)

3. How many feet long must the smaller block be?
   1 foot

4. How many seconds does this block represent?
   2 seconds

5. Did the planes meet at the point where the two routes intersect?
   No

6. If No, which plane arrived first?
   WAL27

7. How many seconds did it take this plane to travel to the point where the two routes intersect?
   32 seconds

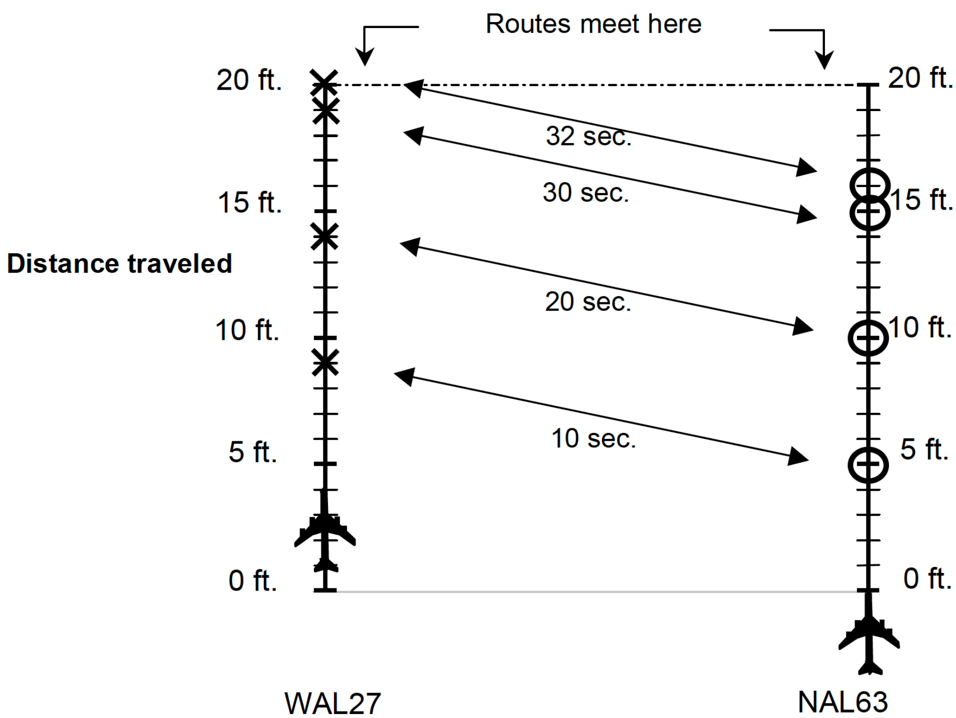
8. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)
   Flight NAL63 was 4 feet away.
   After 32 seconds, Flight NAL63 had traveled 16 feet and was 4 feet from the intersection. So the planes do not meet.

9. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?
   Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Plot Points on Lines

Discussion: Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following diagram shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the answer is a little more than 30 seconds. In particular, Flight WAL27 has to go only 1 foot past 30 seconds. This foot corresponds to 2 seconds. So Flight WAL27 will arrive in 32 seconds. At this time, Flight NAL63 is still 4 feet from the intersection. So a conflict will not occur.



1. Keep going until the first plane reaches the point where the routes meet. Hint: At its last step, the first plane may need to fly a distance shorter than 5 feet to reach the point where the routes meet.

2. What is the shorter distance?
   1 foot

3. How many seconds does it represent?
   2 seconds

4. How far does the second plane travel in that many seconds?
   1 foot

Student Workbook: Do the Calculations—Plot Points on Lines (cont.)

5. Did the planes meet at the point where the two routes intersect?
   No

6. If No, which plane arrived first?
   WAL27

7. How many seconds did it take this plane to travel to the point where the two routes intersect?
   32 seconds

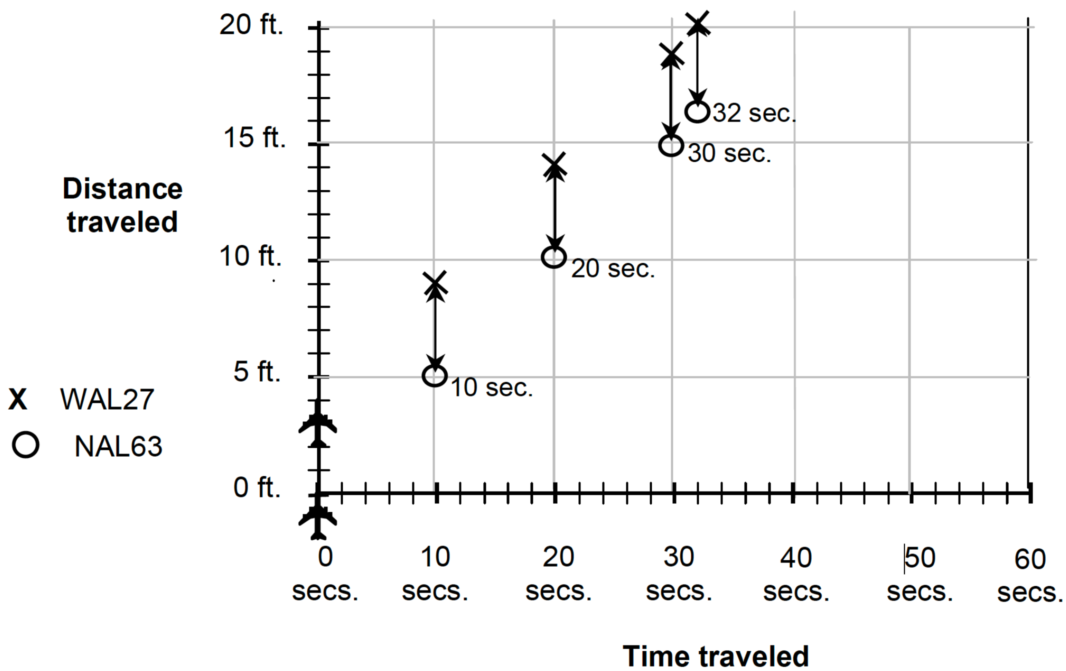
8. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)
   Flight NAL63 was 4 feet away.
   After 32 seconds, Flight NAL63 had traveled 16 feet and was 4 feet from the intersection. So the planes do not meet.

9. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collison?
   Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Plot Points on a Grid

Discussion: Each plane travels 1 foot in 2 seconds. So in 10 seconds, each plane will go 5 feet. The following graph shows the position of each plane at 10-second intervals.

As the points for Flight WAL27 are plotted, it will become clear that the answer is a little more than 30 seconds. In particular, Flight WAL27 has to go only 1 foot past 30 seconds. This foot corresponds to 2 seconds. So Flight WAL27 will arrive in 32 seconds. At this time, Flight NAL63 is still 4 feet from the intersection. So a conflict will not occur.



1. Keep going until the first plane reaches the point where the routes meet. Hint: At its last step, the first plane may need to fly a distance shorter than 5 feet to reach the point where the routes meet.

2. What is that shorter distance?
   1 foot

3. How many seconds does it represent?
   2 seconds

4. How far does the second plane travel in that many seconds?
   1 foot

Student Workbook: Do the Calculations—Plot Points on a Grid (cont.)

5. Did the planes meet at the point where the two routes intersect?
   No

6. If No, which plane arrived first?
   WAL27

7. How many seconds did it take this plane to travel to the point where the two routes intersect?
   32 seconds

8. At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)
   Flight NAL63 was 4 feet away.
   After 32 seconds, Flight NAL63 had traveled 16 feet and was 4 feet from the intersection. So the planes do not meet.

9. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?
   Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Use a Formula

Discussion: Flight WAL27 arrives at the intersection in 32 seconds.

Flight NAL63 arrives at the intersection 8 seconds later at 40 seconds.

Since the times are the same, a conflict will occur.

1. —In 4 seconds, each plane travels **0.5 feet/second** × **4 seconds** = **2.0** feet.

   —In 5 seconds, each plane travels **0.5 feet/second** × **5 seconds** = **2.5** feet.

   —In 6 seconds, each plane travels **0.5 feet/second** × **6 seconds** = **3.0** feet.

2. How could you use multiplication to find the distance each plane travels in 14 seconds?
   Multiply 0.5 feet/second by 14 seconds.

3. Use the formula

$$d = r \bullet t$$

   to answer this question.
   How many feet does each plane travel in 20 seconds? **10 feet**

4. Use the formula

$$t = \frac{d}{r}$$

   to find the number of seconds for WAL27 to travel 16 feet to the point where the routes meet. (Hint: Divide 16 by 0.5).

$$t = \frac{\text{16 feet}}{\text{0.5 feet per second}} = \underline{\ 32\ } \text{ seconds}$$

Student Workbook: Do the Calculations—Use a Formula (cont.)

5. Use the same formula to find the number of seconds for NAL63 to travel 20 feet to the point where the routes meet.

$$t = \frac{\text{20 feet}}{\text{0.5 feet per second}} = \underline{\ 40\ } \text{ seconds}$$

6. Did the planes meet at the point where the two routes intersect?
   No

7. If No, which plane will arrive first?
   WAL27

8. How many seconds will it take this plane to travel to the point where the two routes intersect?
   32 seconds

7. At that time, how far away is the other airplane? (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)
   Flight NAL63 was 4 feet away.
   After 32 seconds, Flight NAL63 had traveled 16 feet and was 4 feet from the intersection. So the planes do not meet.

8. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?
   Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Graph Linear Equations

Discussion: Flight WAL27 arrives at the intersection first, in 32 seconds.

Flight NAL63 arrives at the intersection 8 seconds later, at 40 seconds.

Since the times are the same, a conflict will occur.

1. Fill in the table for **WAL27**.

d = 0.5 t + 4

t seconds	d feet
0	**4**
10	**9**
20	**14**
30	**19**
40	**24**

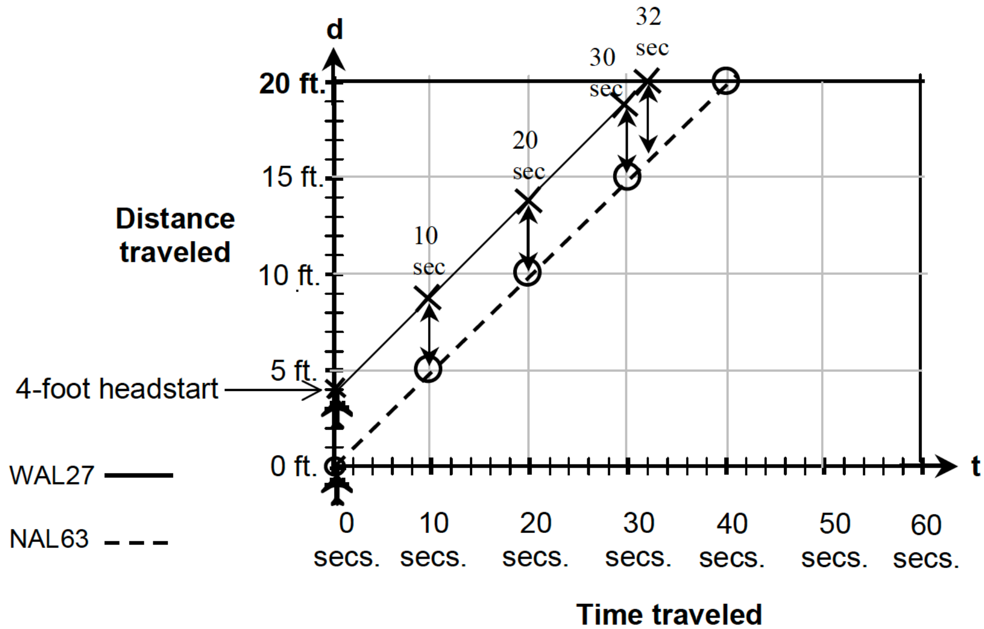
2. Fill in the table for **NAL63**.

d = 0.5 t

t seconds	d feet
0	**0**
10	**5**
20	**10**
30	**15**
40	**20**

3. Use an **X** to graph each point in the WAL27 table. Use a solid line to connect the points.

4. Use an O to graph each point in the NAL63 table. Use a dotted line to connect the points.



Student Workbook: Do the Calculations—Graph Linear Equations (cont.)

5. How many seconds will it take each plane to arrive at the point where the routes intersect?

   WAL27 **32 seconds** NAL63 **40 seconds**

6. Will the planes meet at the point where the two routes intersect?

   No

7. If no, which plane will arrive first?

   WAL27

8. How many seconds will it take this plane to travel to the point where the two routes intersect?

   32 seconds.

9. At that time, how far away is the other airplane? (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)

   Flight NAL63 was 4 feet away.
   After 32 seconds, Flight NAL63 had traveled 16 feet and was 4 feet from the intersection. So the planes do not meet.

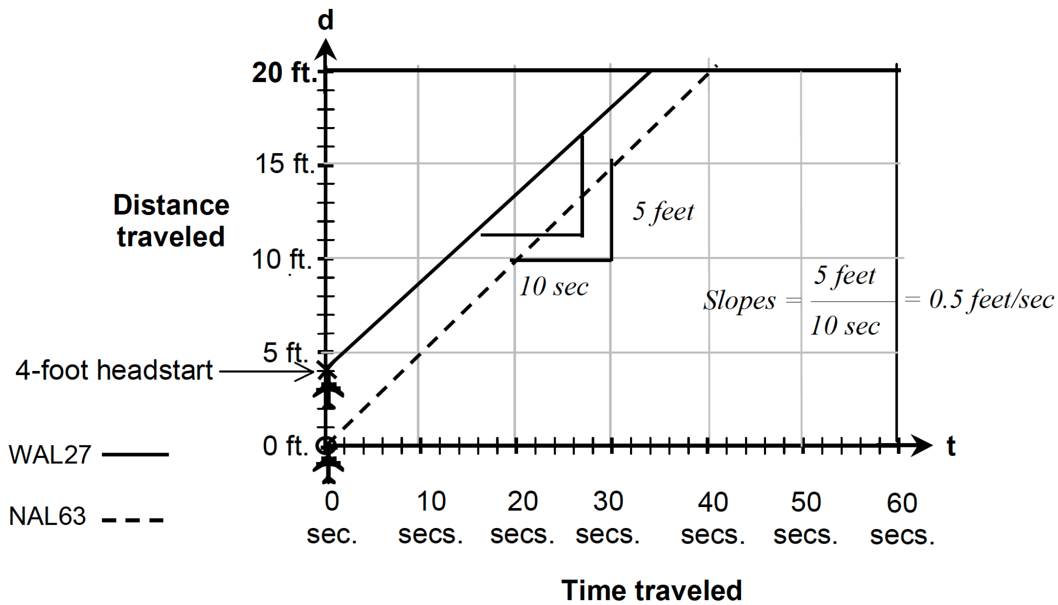
10. If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

    Change the speed or change the route of one of the planes.

Student Workbook: Do the Calculations—Graph Linear Equations (cont.)

Discussion: For each line, its slope represents the speed of the corresponding plane. Since the planes have the same speed, their lines have the same slope.

The vertical distance between the lines represents the initial WAL27" headstart." Since the planes are traveling at th same constant speed, the "headstart" remains the same and the vertical distance between the lines remains the same.



11. Write the number that is the slope of the line representing WAL27.
    <u>0.5 feet/second</u>

12. Write the number that is the slope of the line representing NAL63.
    <u>0.5 feet/second</u>

13. What information does the slope of the line tell you about each plane?.
    <u>The slope of each line is 0.5 feet/second, the speed of the plane.</u>

Student Workbook: Analyze Your Results

Discussion: Suppose two planes are traveling at the same constant (fixed) speed on two different routes and the planes are different distances from the point where the two routes come together.

Since the planes are traveling at the same constant (fixed) speed and must each travel a different distance to the point of intersection, the planes will arrive at the intersection at different times. So the planes will not meet at the point where the routes come together.

Also, since the planes are traveling at the same constant (fixed) speed, their separation remains the same. So at the intersection, the separation between the planes will be the same as their initial separation.

1. Are the planes' speeds the same or different?
   Same

2. What is the "difference" in the planes' starting distances? (That is, what is the "headstart"?)
   4 feet

3. What is the difference in the planes' final positions? (What is the separation distance where the routes meet?)
   4 feet

4. Are these distances the same or different?
   Same

Two planes are flying at the same speed on two different routes. The planes start at different distances from the point where the routes meet.

5. When the first plane reaches the point where the routes meet, the separation distance will be **the same as** the "*difference*" in the planes' starting distances (the "headstart").

Student Workbook: Extension

1. Based upon your calculations, what is the difference in the planes' final positions? (That is, what is the planes' separation distance where the routes meet?)
   4 feet

2. Does this distance satisfy the separation requirement?
   No

3. If No, what would you tell the air traffic controller to do to meet the separation requirement?
   Change the speed or change the route of one of the planes.

4. How far will WAL27 travel to reach the **new** point where the routes meet?
   24 feet

5. How far will NAL63 travel to reach the **new** point where the routes meet?
   30 feet

6. Which plane will fly a shorter distance?
   WAL27

7. How much shorter?
   6 feet

8. When WAL27 arrives at the new point where the routes meet, how many feet away is NAL63 on its route?
   6 feet

Student Workbook: Extension (cont.)

9. What WAL27 has traveled 24 feet to the new point where the routes meet, how far has NAL63 traveled?

   24 feet

10. At that time, how many feet away is NAL63 on its route?

    6 feet

11. Is the separation standard violated?

    No

Posttest

1. Do you think the planes will meet at the point where the two routes intersect?
   No

   Why or why not?
   The planes are traveling at the same constant speed and each plane must travel a different distance to the point where the routes intersect. So the planes will *not* meet at the intersection.

2. If not, how far apart do you think the planes will be when the first plane reaches the point where the routes intersect?
   4 feet

3. Does your answer to Question 2 meet the 5-foot separation standard?
   No

4. If you think two planes will *not* meet the 5-foot separation standard, what could you tell the traffic controller to do to make sure that the separation standard will be met?
   Change the speed or change the route of one of the planes.

Now consider this general problem.

Two planes are traveling at the same speed on two different routes.

The planes are different distances from the point where the two routes intersect.

5. Will the planes meet at the point where the routes intersect?
   No

Posttest (cont.)

6. If not, how far apart will the planes be when the first plane reaches the point where the routes intersect?
   The planes will be the same distance apart as they were at the beginning of the problem.

7. Explain your answers.
   The planes are traveling at the same constant speed, so the distance between them stays the same.