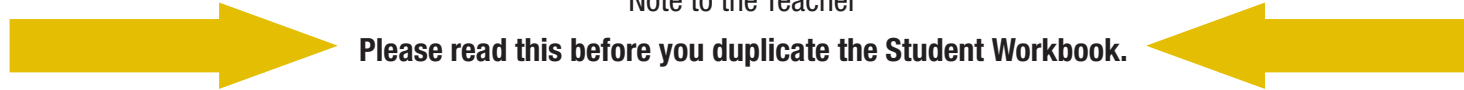


Note to the Teacher



Please read this before you duplicate the Student Workbook.

This Smart Skies™ FlyBy Math™ Student Workbook contains activities for one Air Traffic Control problem. In particular, the Workbook includes **worksheets for 6 different calculation methods** your students can use to solve the problem:

- Count feet and seconds
- Plot points on a grid
- Draw and stack blocks
- Use the distance-rate-time formula
- Plot points on two vertical lines
- Graph two linear equations

You will most likely want to **assign only 1 or 2 of the calculation methods**. So we recommend that you follow these steps before you duplicate the Workbook.



1. Select and Keep:

Choose the calculation method(s) you want your students to use.



2. Recycle the Rest:

Remove the calculation worksheets you do not wish to assign.

To find the calculation worksheets, look for the Calculations footers at the page bottoms.

Smart

Skies™ FlyBy Math™

ATC #1 – Calculations – Blocks – Page 1 of 4

EG-2004-11-111-ARC



Other activity worksheets follow the Calculation pages. Be sure to remove the Calculation worksheets **ONLY**.

For more information about Smart Skies™ workbooks, please see the Smart Skies™ teacher materials available online at:

<https://www.nasa.gov/smartskies/flybymath>

FlyBy Math™

Math & Science for Air Traffic Control

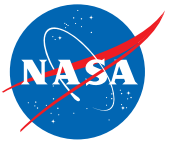


Problem #5

- Aircraft are on the same route
- Aircraft are traveling at different speeds
- Aircraft start at different distances along the route
- The trailing aircraft is traveling faster than the leading aircraft

STUDENT WORKBOOK

Investigator: _____



Begin Your Challenge: When will two planes, flying on the same jet route, meet?

Flight NAL63

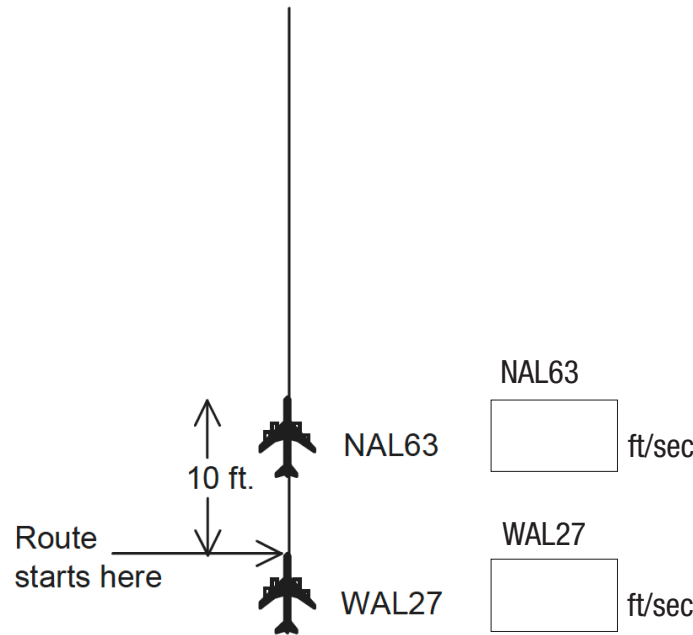
- NAL63 is 10 feet (3.0 meters) ahead of the route start.
- The NAL63 speed is $\frac{1}{4}$ foot per second (0.08 meters/second).

1 Write the speed of NAL63 in the box next to its picture.

2 How far does NAL63 travel in 1 second?

 feet

3 How far does NAL63 travel in 10 seconds?

 feet


Flight WAL27

- WAL27 is 1st at the start of the route (0 feet).
- The WAL27 speed is $\frac{1}{2}$ foot per second (0.15 meters/second).

4 Write the speed of WAL27 in the box next to its picture.

5 How far does WAL27 travel in 1 second?

 feet

6 How far does WAL27 travel in 10 seconds?

 feet

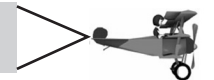
To meet your Challenge, you will:

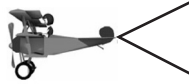
- Conduct an experiment.
- Do some math calculations.
- Analyze your results

Then, you will use your results to answer this question:

- How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63?

End of Worksheet

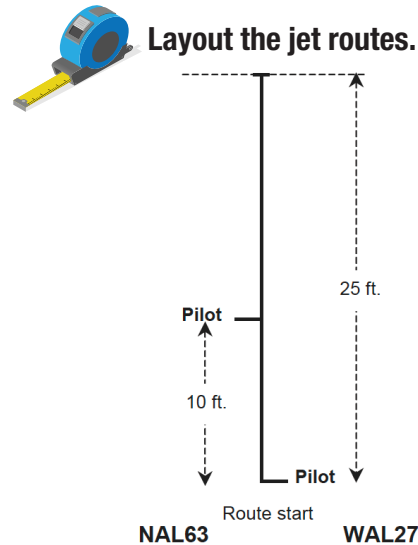




Begin Your Task: Experiment to see when the trailing plane will catch the leading plane.

Flight NAL63

- 1 Begin at the start of the jet route.
Use chalk or masking tape to mark off a line 25 feet long.
 - The NAL63 pilot has a 10-foot headstart.
- 2 Measure 10 feet from the route start and make a mark on the **left** side of the route. Place a "Pilot" label next to the mark.

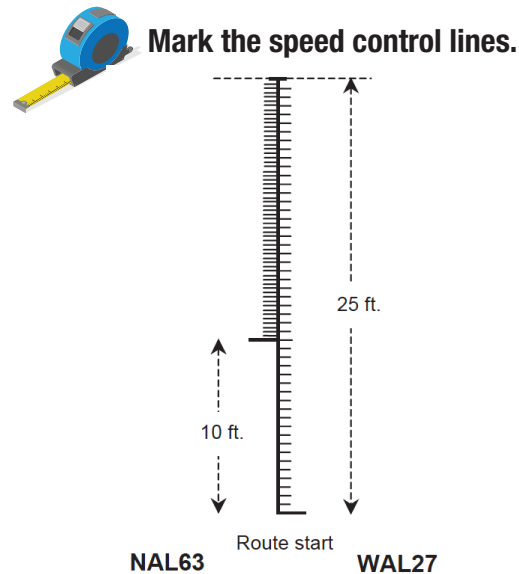


Flight WAL27

- 3 At the jet route start, make a mark on the **right** side of the route.
Place a "Pilot" label next to the mark.

Flight NAL63

- 4 Begin at the 10-foot mark next to the "Pilot" label on the **left** side of the jet route.
Place a mark (or piece of tape) every $\frac{1}{4}$ foot (3 inches) along the **left** side of the jet route all the way to the end of the jet route.
You need to make the mark (or piece of tape) at least 1 foot long.

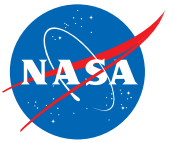


Flight WAL27

- 5 Begin at the start of the jet route.
Place a mark (or piece of tape) every $\frac{1}{2}$ foot (6 inches) along the **right** side of the jet route all the way to the end of the jet route.
You need to make the mark (or piece of tape) at least 1 foot long.

Investigator: _____

SET UP and DO THE EXPERIMENT

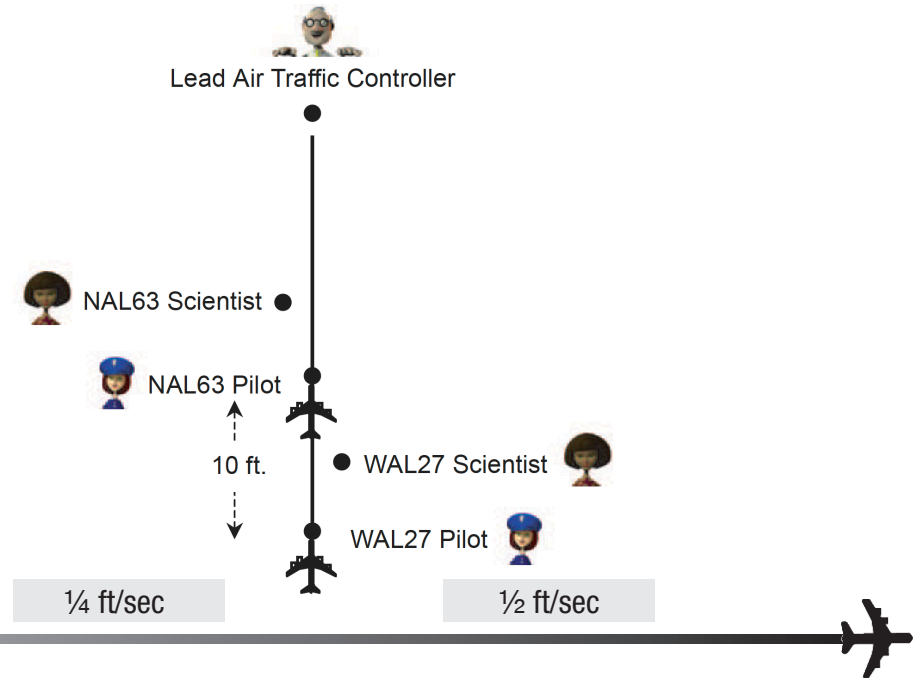


Record the starting conditions for the experiment.

6 Fill in this table with the starting conditions.

Flight Number	Speed (ft/sec)	Distance from the start of the jet route (ft)
WAL27		
NAL63		

7 Circle your role on the route diagram on the right.

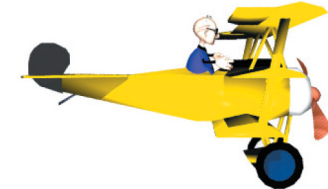


Make your prediction



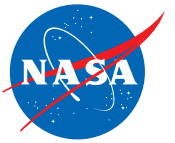
8 How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63?

seconds



Investigator: _____

SET UP and DO THE EXPERIMENT



9

Circle your role and the steps below it. Then, do the experiment 3 times using the steps for your role.



Do the experiment.

	Lead Air Traffic Controller	Pilot	NASA Scientist
1 Take Your Position	Give the command: "Take your positions."	Go to your position on your side of the jet route.	Go to your position.
2 Get Ready to Start	Give the command: "Ready."	Practice stepping down your route.	
3 Start the Experiment	Start your stopwatch. Count seconds: "1", "2",...	Take your first step on "1". On each count, take a step to the next mark.	Wait for WAL27 to catch up to NAL63
4 Stop the Experiment	When you hear "Halt", stop counting seconds.	When you hear "Halt", stop where you are.	When WAL27 catches up, say "Halt". Record the seconds.



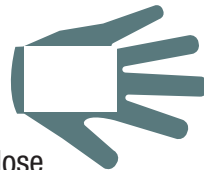
Record the data

10

Record the Halt Time measured by the NASA Scientist.

11

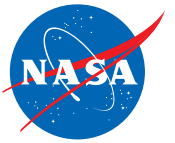
Use the results of your three experiments to choose the best answer to this question:



How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63?

Experiment:	1	2	3
Halt Time (seconds)			

End of Worksheet



Your Task: Calculate when two planes flying on merging routes will meet.



Count feet and seconds along the jet route.

Flight NAL63

- Starts 10 feet from the point where the route begins (a 10-foot headstart).
- In 1 second, it moves $\frac{1}{4}$ foot.
- In 4 seconds, it moves 1 foot—from 10 feet to 11 feet.

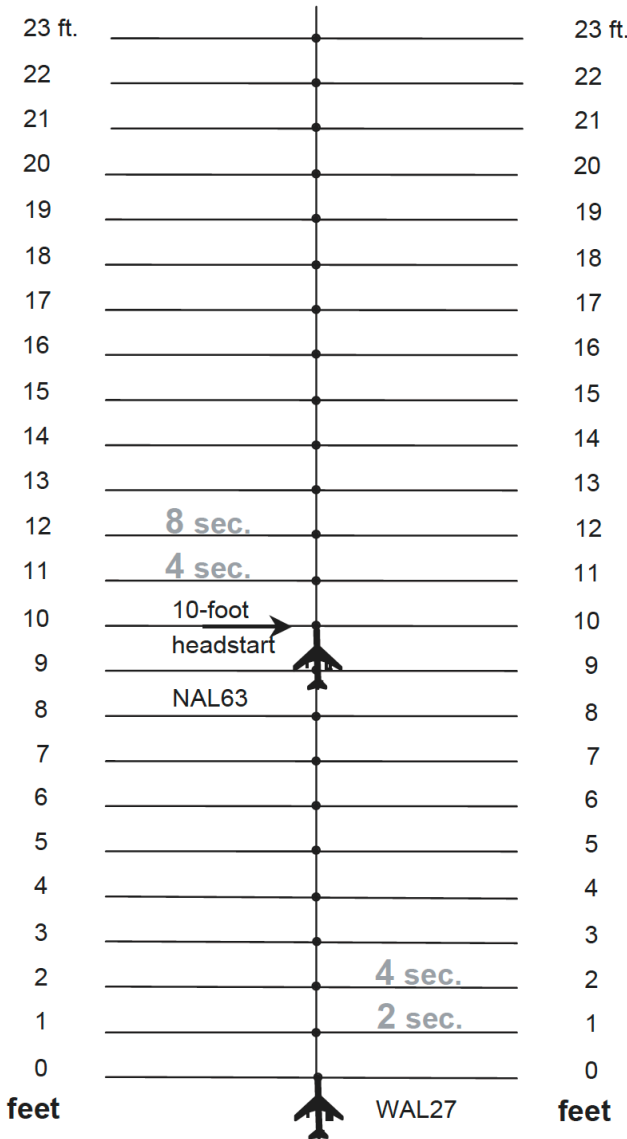
1 On the left side of the jet route, find the 11-foot mark and trace "4 sec."

- In the next 4 seconds (8 seconds total), NAL63 moves another foot—from 11 feet to 12 feet.

2 On the left side of the jet route, find the 12-foot mark and trace "8 sec."

3 Keep going on the left side of the jet route, one foot at a time, until you reach the end of the jet route.

At each foot-mark, write the total number of seconds to reach that mark.



Flight WAL27

- Starts where the route begins (at 0 feet).
- In 1 second, it moves $\frac{1}{2}$ foot.
- In 2 seconds, it moves 1 foot—from 0 feet to 1 foot.

4 On the right side of the jet route, find the 1-foot mark and trace "2 sec."

- In the next 2 seconds (4 seconds total), WAL27 moves another foot—from 1 foot to 2 feet.

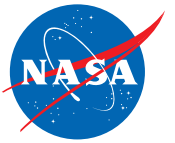
5 On the right side of the jet route, find the 2-foot mark and trace "4 sec."

6 Keep going on the right side of the jet route, one foot at a time, until you reach the end of the jet route.

At each foot-mark, write the total number of seconds to reach that mark.

Investigator: _____

DO THE CALCULATIONS—Count Feet and Seconds



Answer the questions.

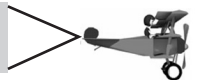
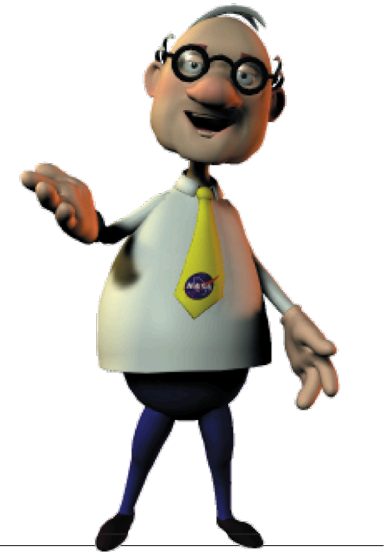
7

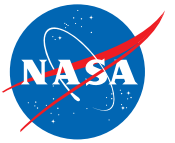
How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63? That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time?

seconds

8

What would you tell the air traffic controller to do to avoid a collision?





Your Task: Calculate when two planes flying on the same route will meet.



Use blocks to picture feet and seconds.

1

In this table, fill in the distance Flight WAL27 will travel in 10 seconds.

The speed of each plane is $\frac{1}{2}$ foot per second.

Flight WAL27 takes...	1 second	2 seconds	10 seconds
to travel...	$\frac{1}{2}$ foot	1 foot	<input type="text"/> feet

2

In this table, fill in the distance Flight NAL63 will travel in 10 seconds.

The speed of Flight NAL63 is $\frac{1}{4}$ foot per second.

Flight NAL63 takes...	1 second	2 seconds	10 seconds
to travel...	$\frac{1}{4}$ foot	1 foot	<input type="text"/> feet

- Next, you will use this 10-second block to represent 5 feet of travel.
- The block height represents 5 feet, the distance Flight WAL27 travels in 10 seconds.

A diagram showing a grey rectangular block. To its left, a vertical double-headed arrow is labeled "5 ft.". To the right of the block, the text "10 sec." is written.

- Next, you will use this 10-second block to represent $2\frac{1}{2}$ feet of travel.
- The block height represents $2\frac{1}{2}$ feet, the approximate distance Flight NAL63 travels in 10 seconds.

A diagram showing a grey rectangular block. To its left, a vertical double-headed arrow is labeled "2 1/2 ft.". To the right of the block, the text "10 sec." is written.



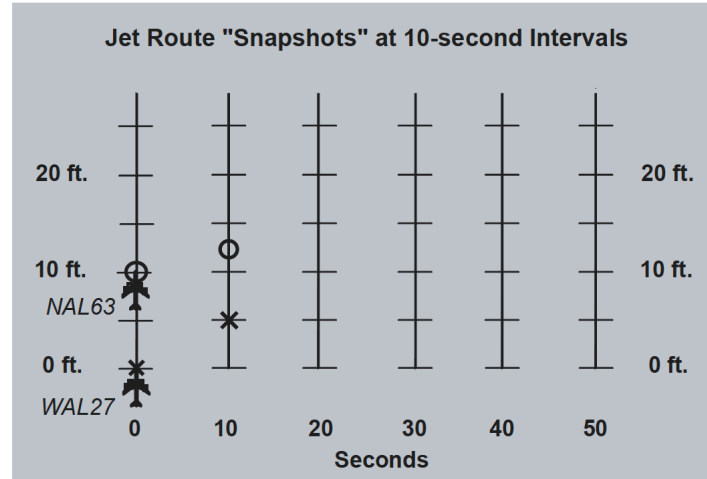
Follow along with this example of how to plot points and stack blocks.

Flight NAL63

- Starts 10 feet from the point where the route begins (a 10-foot headstart).
- In 1 second, it moves $2\frac{1}{2}$ feet.
- Now it is $12\frac{1}{2}$ feet along the route.

3 Circle the **0** for NAL63 at the $12\frac{1}{2}$ foot point on the "10-second" jet route.

4 Trace the 10-second block for NAL63.

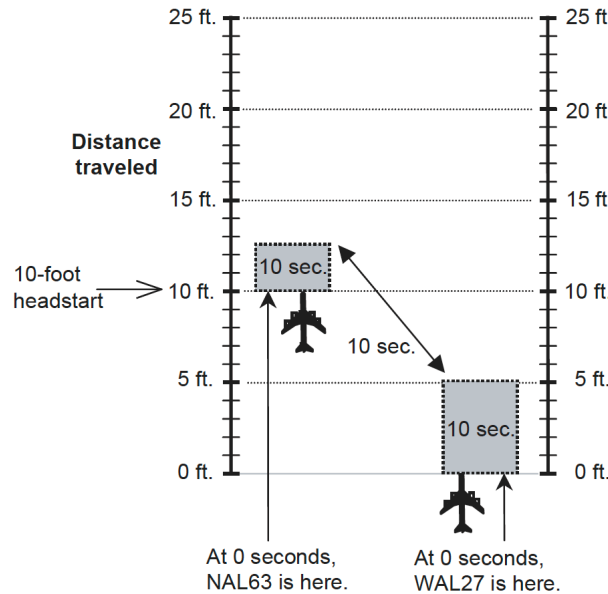


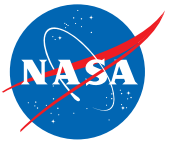
Now draw and connect at 30 seconds.

- Starts where the route begins (at 0 feet).
- In 10 seconds, it moves **5** feet.
- Now it is **5** feet along the route.

5 Circle the **X** for WAL27 at the 5-foot point on the "10-second" jet route

6 Trace the 10-second block for WAL27.

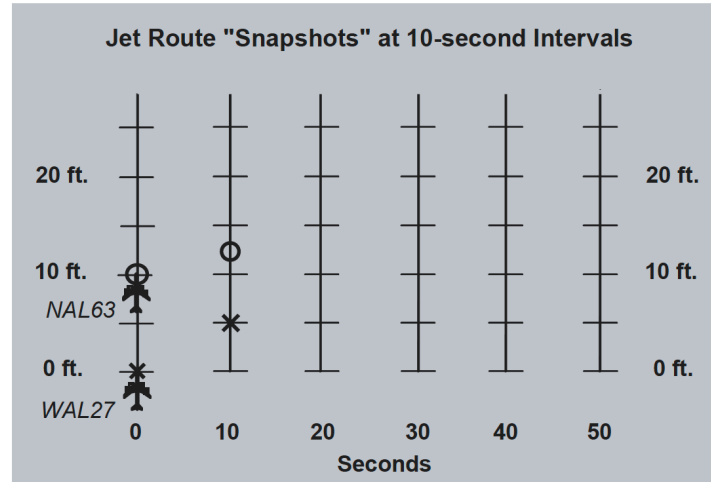




Follow along with this example of how to plot points and stack blocks.

Flight NAL63 and Flight WAL27

- 7 On the "20-second" route, draw an **O** to show the position of NAL63 after 20 seconds.
- 8 Trace the block on the graph below that shows the position of NAL63 after 20 seconds.
- 9 On the "20-second" route, draw an **X** to show the position of WAL27 after 20 seconds.
- 10 Trace the block on the graph below that shows the position of WAL27 after 20 seconds.
- 11 Connect your blocks with a line marked "20 sec."



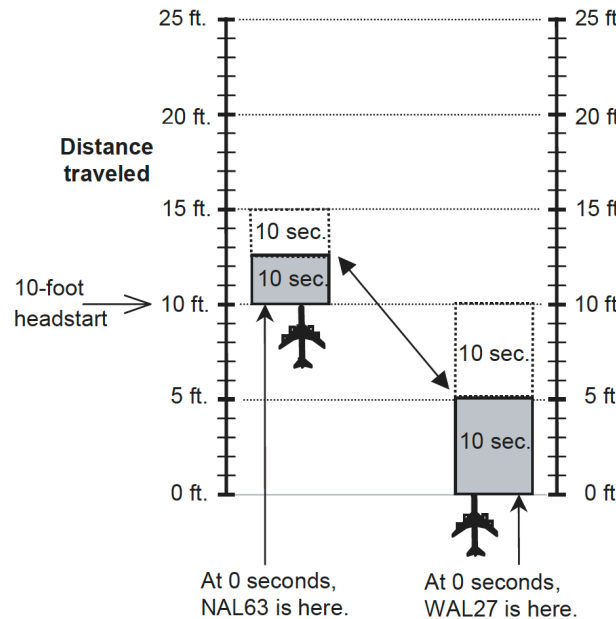
Now draw and connect at 30 seconds.

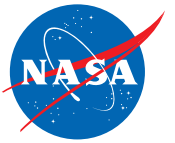
- 12 On the "30-second" route, plot an **O** and an **X** to show the position of each plane at 30 seconds.
- 13 On the graph, draw blocks to show the position of each plane at 30 seconds.
- 14 Connect the blocks at 30 seconds.

Keep going...

- 15 Keep going until WAL27 closes the gap and catches up with NAL63.

Be sure to connect your blocks.





Investigator: _____

DO THE CALCULATIONS—Draw Blocks



Answer the questions.

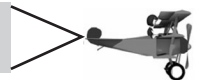
18

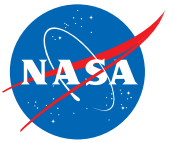
How many seconds did it take WAL27 to close the 10-foot gap and catch up with NAL63?

seconds

20

What would you tell the air traffic controller to do to avoid a collision?





Your Task: Calculate when two planes flying on the same route will meet.



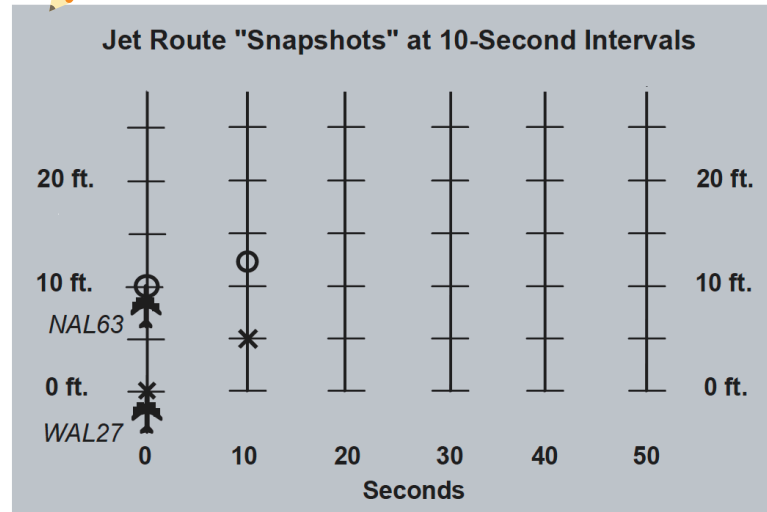
Follow along with this example of how to plot points.

Flight NAL63

- Starts 10 feet from the point where the route begins (a 10-foot headstart).
- In 10 seconds, it moves 2 ½ feet.
- Now it is 12 ½ feet along its route.

1 Circle the **0** for NAL63 at the 12 ½-foot point on the "10-second" jet route.

2 Circle the **0** at the 12 ½-foot point on the NAL63 line graph.

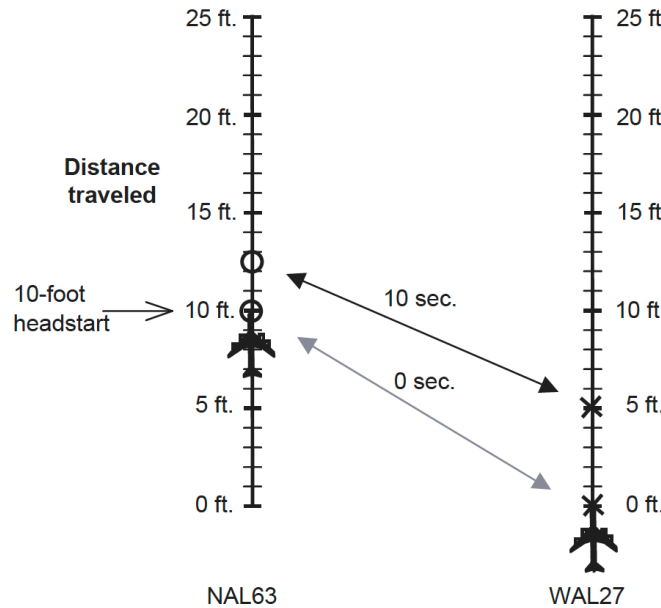


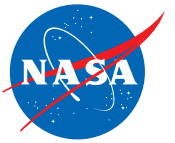
Flight WAL27

- Starts where the route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

3 Circle the **X** at the 5-foot point on the "10-second" jet route.

4 Circle the **X** at the 5-foot point on the WAL27 line graph.





Now it's your turn to draw and connect.

Flight WAL27 and Flight NAL63

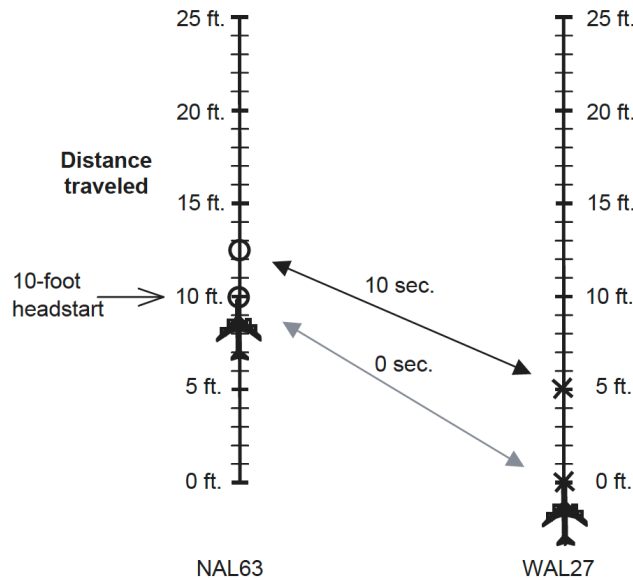
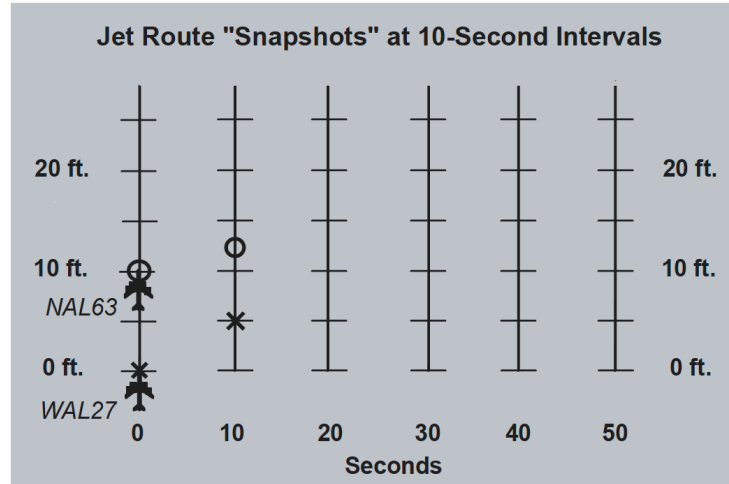
5 On the "20-second" route, draw an **O** to show the position of NAL63 after 20 seconds

6 On the line graph, draw an **O** to show the position of NAL63 after 20 seconds

7 On the "20-second" route, draw an **X** to show the position of WAL27 after 20 seconds

8 On the line graph, draw an **X** to show the position of WAL27 after 20 seconds

9 On the line graph, connect your **X** and **O** with a line marked "20 sec".



Now draw and connect at 30 seconds.

10 On the "30-second" route, plot an **O** and an **X** to show the position of each plane.

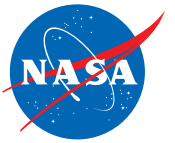
11 On the graph, draw, connect, and label an **X** and an **O** at 30 seconds.

Keep going...

12 Keep going until WAL27 closes the gap and catches up with NAL63.

Be sure to connect an **X** and an **O** on the graph.





Investigator: _____

DO THE CALCULATIONS—Plot Points on Lines



Answer the questions.

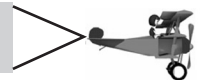
13

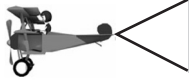
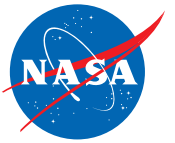
How many seconds did it take WAL27 to close the 10-foot gap and catch up with NAL63?

seconds

14

What would you tell the air traffic controller to do to avoid a collision?





Your Task: Calculate when two planes flying on the same routes will meet.



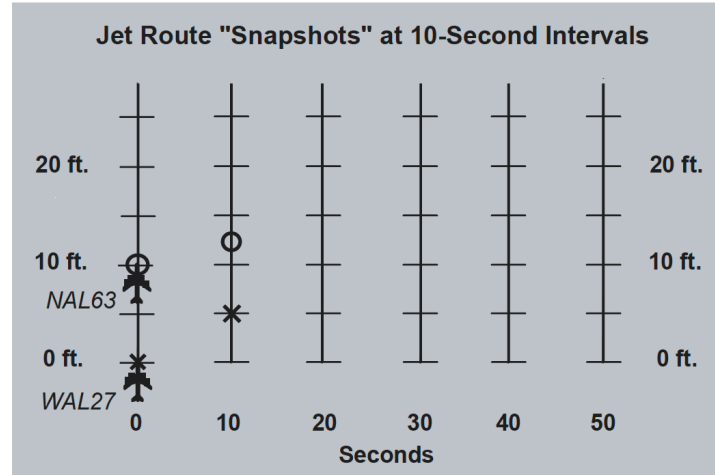
Follow along with this example of how to plot points.

Flight NAL63

- Starts 10 feet from the point where the route begins (a 10-foot headstart).
- In 10 seconds, it moves 2 ½ feet.
- Now it is 12 ½ feet along its route.

1 Circle the **O** for NAL63 at the 12 ½ foot point on the "10-second" jet route.

2 Circle the **O** at the point (10, 12 ½) on the grid.

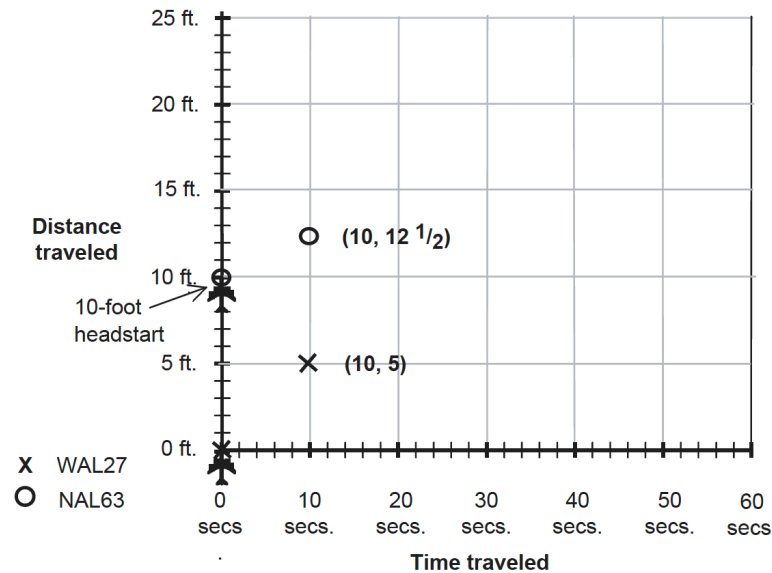


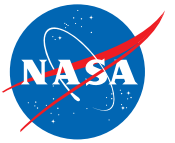
Flight WAL27

- Starts where the route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

3 Circle the **X** for the WAL27 at the 5-foot point on the "10-second" jet route.

4 Circle the **X** at the point (10, 5) on the grid.

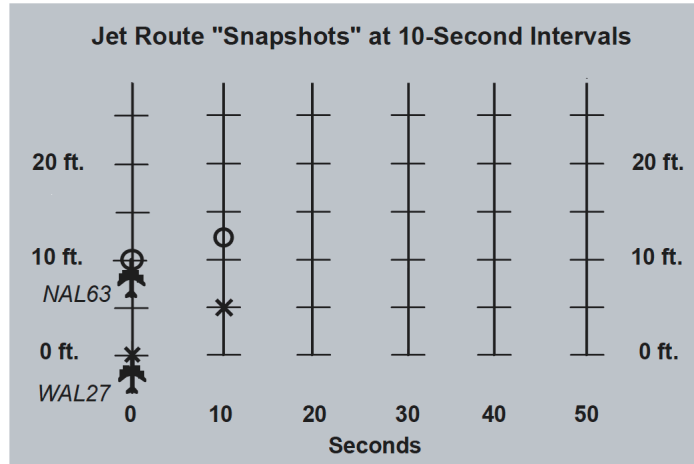




Now it's your turn to plot points.

Flight NAL63 and Flight WAL27

- 5 On the "20-second" route, draw an **O** to show the position of NAL63 after 20 seconds
- 6 On the line graph, draw an **O** to show the position of NAL63 after 20 seconds
- 7 On the "20-second" route, draw an **X** to show the position of WAL27 after 20 seconds
- 8 On the grid, draw an **X** to show the position of WAL27 after 20 seconds

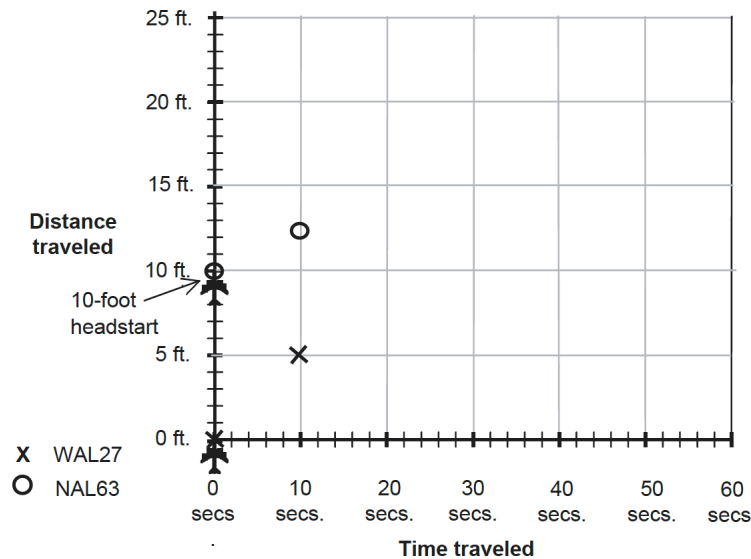


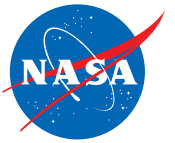
Now draw and connect at 30 seconds.

- 9 On the "30-second" route, plot an **O** and an **X** to show the position of each plane.
- 10 On the grid, draw an **X** and an **O** at 30 seconds.

Keep going...

- 11 Keep going until WAL27 closes the gap and catches up with NAL63.





Investigator: _____

DO THE CALCULATIONS—Plot Points on a Grid

 Answer the questions.

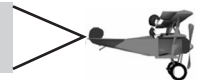
12

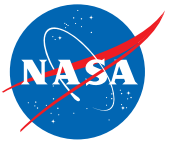
How many seconds did it take WAL27 to close the 10-foot gap and catch up with NAL63?

seconds

13

What would you tell the air traffic controller to do to avoid a collision?





Your Task: Calculate when two planes flying on the same route will meet.



Find a pattern.

- The speed of Flight WAL27 is 0.5 feet per second.

1 Use multiplication to fill in this table.

In this many seconds...	Each plane travels this many feet...
1	0.5 feet/second x 1 second = 0.5 feet
2	0.5 feet/second x 2 second = 1.0 feet
3	0.5 feet/second x 3 second = 1.5 feet
4	<input type="text"/> feet/second x <input type="text"/> second = <input type="text"/> feet
5	<input type="text"/> feet/second x <input type="text"/> second = <input type="text"/> feet
6	<input type="text"/> feet/second x <input type="text"/> second = <input type="text"/> feet

2 How could you use multiplication to find the distance Flight WAL27 travels in 14 seconds?

- The pattern in the table suggests this rule:
“To find the distance traveled, multiply the speed by the time traveled.”
- In math and science, we often say “rate” instead of “speed.”
- So we can write a rule like this:
$$\text{distance} = \text{rate} \times \text{time}$$
- This relationship is call the Distance-Rate-Time Formula.
- We often write it like this:

Distance-Rate-Time Formula

$$d = r \cdot t$$

3 Use the formula to answer this question:

How many feet does Flight WAL27 feet travel in 20 seconds?

- The speed of Flight NAL63 is $\frac{1}{4}$ foot per second.

4 Use the formula to answer this question:

How many feet does Flight NAL63 feet travel in 20 seconds?

Investigator: _____

DO THE CALCULATIONS—Use a Formula



Use the Formula.

We can use the formula to find an equation that describes the distance traveled by each plane.

Flight NAL63

The NAL63 rate is $\frac{1}{4}$ foot per second.

- NAL63 has a 10-foot headstart. (When you start your stopwatch at $t = 0$, NAL63 has already traveled 10 feet).

So

$$d = \frac{1}{4}t + 10$$

Distance-Rate-Time Formula

$$d = r \cdot t$$

Flight WAL27

The WAL27 rate is $\frac{1}{2}$ foot per second.

So

$$d = \frac{1}{2}t$$

5 For each plane, use its equation to fill in the table.

Experiment:	0 sec.	10 sec.	20. sec	30 sec.	40 sec.
Distance traveled by Flight NAL63	10 ft.				
Distance traveled by Flight WAL27	0 ft.				

6 How many seconds did it take WAL27 to close the 10-foot gap and catch up with NAL63? seconds

7 What would you tell the air traffic controller to do to avoid a collision? _____

End of Worksheet



Your Task: Calculate when two planes flying on the same route will meet.



Find an equation that describes the distance traveled by each plane.

Flight NAL63

- We can use the Distance-Rate-Time formula

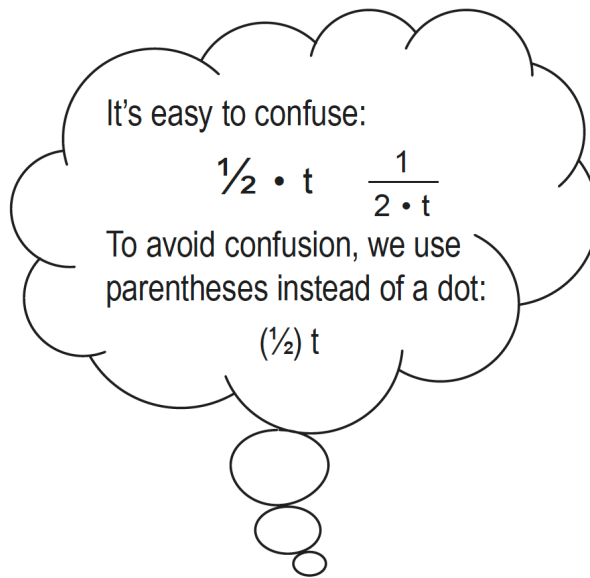
$$d = r \cdot t$$

to find d , the distance in feet NAL63 travels in t seconds.

- The NAL63 rate is $\frac{1}{4}$ foot per second.
- NAL63 has a 10-foot headstart. (When you start your stopwatch at $t = 0$, NAL63 has already traveled 10 feet.)

So...

$$d = \left(\frac{1}{4}\right) t + 10$$



Flight WAL27

- We can use the Distance-Rate-Time formula

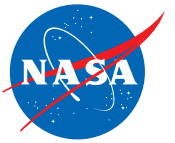
$$d = r \cdot t$$

to find d , the distance in feet NAL63 travels in t seconds.

- The WAL27 rate is $\frac{1}{2}$ feet per second.
- So...

$$d = \left(\frac{1}{2}\right) t$$





Fill in each table.

1 Fill in the table for NAL63.

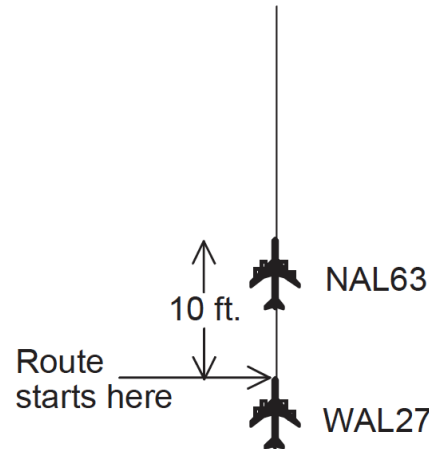
$$d = \frac{1}{4}t + 10$$

t seconds	d feet
0	
10	
20	
30	
40	

2 Fill in the table for WAL27.

$$d = \frac{1}{2}t$$

t seconds	d feet
0	
10	
20	
30	
40	



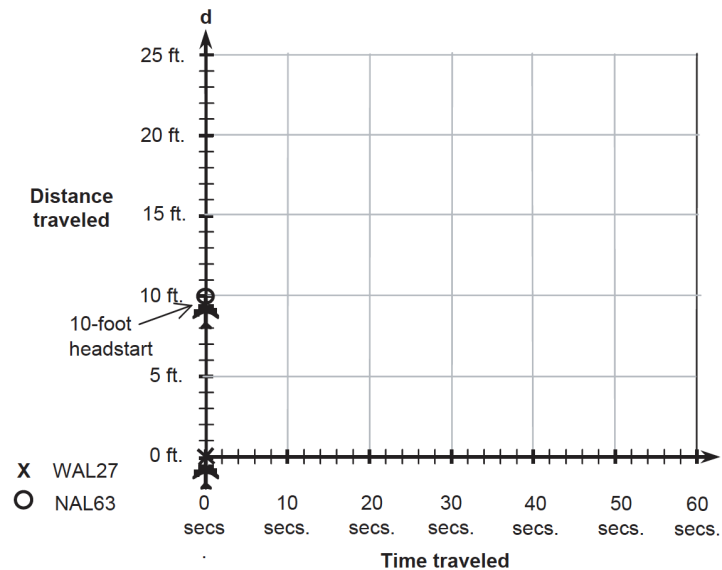
Graph each line.

3 Use an **O** to graph each point in the NAL63 table.

Use a solid line _____ to connect the points.

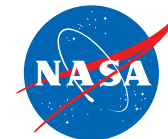
4 Use an **X** to graph each point in the WAL27 table.

Use a dotted line to connect the points.



Investigator: _____

DO THE CALCULATIONS—Graph Linear Equations



Answer the questions.

5

How many seconds will it take WAL27 to close the 10-foot gap and catch up with NAL63?

That is, after how many seconds will WAL27 and NAL63 be in the same place at the same time?

seconds

6

What would you tell the air traffic controller to do to avoid a collision?

7

Write the number that is the slope of the solid line representing NAL63.

8

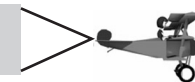
Write the number that is the slope of the dotted line representing WAL27.

9

What information does the slope of the line tell you about each plane?

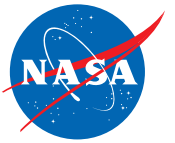


End of Worksheet



Investigator: _____

ANALYZE YOUR RESULTS



Your Task: Analyze and explain your results. Then apply what you learned to another problem.



Compare your math results with your experimental data.

1

Use your experimental data and math results to fill in this table:

	Experiment	Mathematics
Number of seconds for WAL27 to catch up with NAL63		

2

Do your experimental and your math results match?

Yes

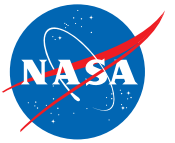
No

3

If No, why do you think they don't match?

4

Which is correct? Why? _____



Compare the speeds and compare the distances.

- The WAL27 rate is $\frac{1}{2}$ foot per second.
- The NAL63 rate is $\frac{1}{4}$ foot per second.

5 What is the difference in speed between WAL27 and NAL63? That is, how many feet per second faster is the speed of the trailing plane than the speed of the leading plane?

feet per second

6 How fast is WAL27 closing the gap between the planes? That is, the difference between the planes changes how many feet each second?

feet per second

7 What is the relationship between the difference in speeds and the speed at which WAL27 is closing the gap?

- Suppose the speed of WAL27 were $\frac{3}{4}$ foot per second.

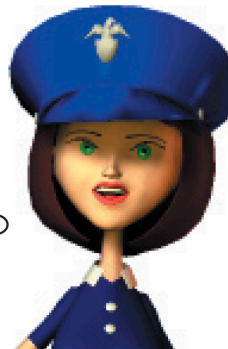
8 With this new faster speed, at how many feet per second would WAL27 close the gap?

feet per second

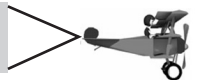
9 With the new faster speed, how many seconds will it take to close the gap?

seconds

The difference in speeds is called **relative speed**.



End of Worksheet





? Consider the general problem.

Two planes are flying at different speeds on the same route. The planes start at the same distance from the beginning of the route.

10 Do you have enough information to predict whether the trailing plane will catch up to the leading plane?

Yes No

11 If No, what other information do you need?

- Now suppose the trailing plane is traveling faster than the leading plane.
- Also, suppose the difference in speeds is twice as great.

12 What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane?

Why? _____

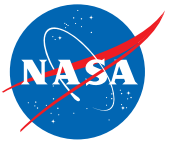
- Suppose the planes each travel at their original speeds, but the distance between the planes is twice as great.

13 What would you expect to happen to the amount of time it would take the trailing plane to catch up to the leading plane?

Why? _____

Investigator: _____

EXTENSION



Your Task: When will the two planes violate the separation standard?



Add a safety requirement.

- For safety reasons, the planes must be separated by at least 5 feet at the point where the routes meet.



Answer the questions.

- Use the results of your calculations to answer this question.

1

How many seconds will it take WAL27 to close the gap from 10 feet to 5 feet?

seconds

2

What would you tell the air traffic controller to do to be sure the planes always meet the separation requirement?



End of Worksheet

