Note to the Teacher

Please read this before you duplicate the Student Workbook.

This Smart Skies™ FlyBy Math™ Student Workbook contains activities for one Air Traffic Control problem. In particular, the Workbook includes **worksheets for 6 different calculation methods** your students can use to solve the problem:

—Count feet and seconds
—Plot points on a grid
—Draw and stack blocks
—Use the distance-rate-time formula
—Plot points on two vertical lines
—Graph two linear equations

You will most likely want to **assign only 1 or 2 of the calculation methods**. So we recommend that you follow these steps before you duplicate the Workbook.

1. **Select and Keep:**
   Choose the calculation method(s) you want your students to use.

2. **Recycle the Rest:**
   Remove the calculation worksheets you do not wish to assign.

   To find the calculation worksheets, look for the Calculations footers at the page bottoms.

Other activity worksheets follow the Calculation pages. Be sure to remove the Calculation worksheets ONLY.

For more information about Smart Skies™ workbooks, please see the Smart Skies™ teacher materials available online at:
https://www.nasa.gov/smartskies/flybymath

FlyBy Math™

Math & Science for Air Traffic Control

STUDENT WORKBOOK

Investigator: _____

Problem #2

- Aircraft are on merging routes
- Aircraft are traveling at the same speed
- Aircraft start at different distances from where the routes meet

Investigator: _____

READ THE PROBLEM

Begin Your Challenge: Will two planes, flying on merging jet routes, meet where the routes intersect?

Flight WAL27

- WAL27 is 16 feet (5.0 meters) from where the routes come together.
- The WAL27 speed is ½ foot per second (0.15 meters/second).

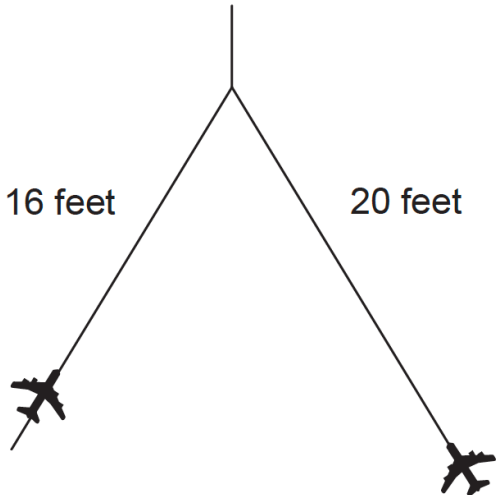
1 Write the speed of WAL27 in the box below its picture.

2 How far does WAL27 travel in 1 second?

_____ feet

3 How far does WAL27 travel in 10 seconds?

_____ feet



WAL27 _____ ft/sec

NAL63 _____ ft/sec

Flight NAL63

- NAL63 is 20 feet (6.1 meters) from where the routes come together.
- The NAL63 speed is ½ foot per second (0.15 meters/second).

4 Write the speed of NAL63 in the box below its picture.

5 How far does NAL63 travel in 1 second?

_____ feet

6 How far does NAL63 travel in 10 seconds?

_____ feet

To meet your Challenge, you will:

- Conduct an experiment.
- Do some math calculations.
- Analyze your results

Then, you will use your results to answer this question:

- Will the planes meet at the point where the routes intersect?
- If not, how many feet apart will the planes be when the first plane reaches the point where the routes intersect?

End of Worksheet

Investigator: _____

SET UP and DO THE EXPERIMENT

Begin Your Task: Experiment to see what happens when the first plane reaches the point where the routes meet.

Flight WAL27

1. Begin at the point where the routes meet.

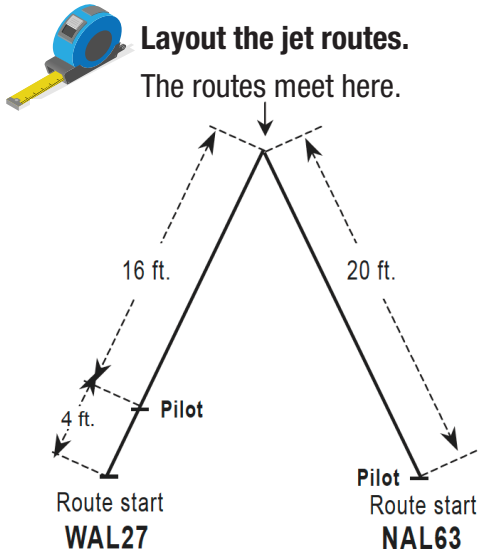
   Use chalk or masking tape to mark off a line 20 feet long.

   - The WAL27 pilot has a 4-foot headstart.

2. Measure 4 feet from the route start and make a mark.

   Place a "Pilot" label next to the mark.

Layout the jet routes.



Flight NAL63

3. Begin at the point where the routes meet.

   Mark off a line 20 feet long.

4. At the jet route start, make a mark.

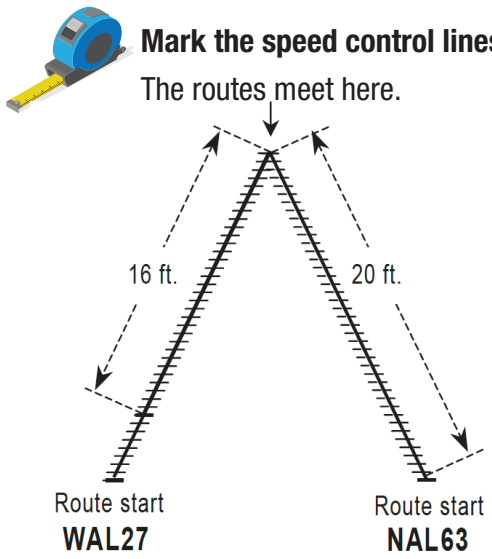
   Place a "Pilot" label next to the mark.

Flight WAL27

5. Begin at the start of the jet route.

   Place a mark (or piece of tape) every ½ foot (6 inches) along the jet route all the way to the point where the routes meet.

Mark the speed control lines.



Flight NAL63

6. Begin at the start of the jet route.

   Place a mark (or piece of tape) every ½ foot (6 inches) along the jet route all the way to the point where the routes meet.

Investigator: _____

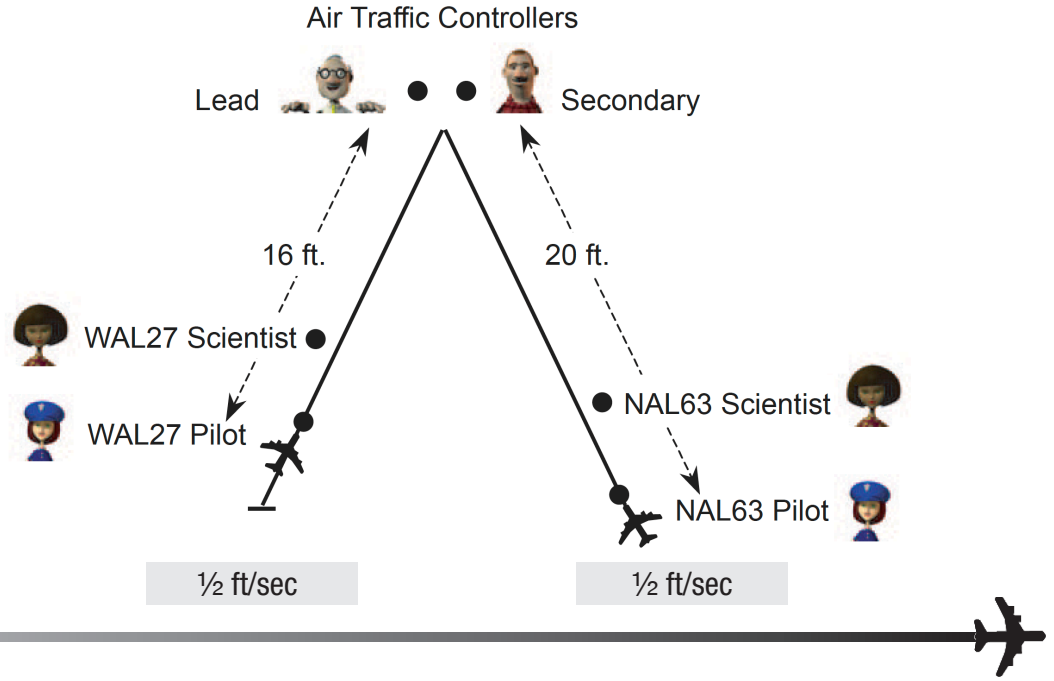
SET UP and DO THE EXPERIMENT

Record the starting conditions for the experiment.

7 Fill in this table with the starting conditions.

Flight Number	Speed (ft/sec)	Distance from the point where the routes meet (ft)
WAL27		
NAL63		

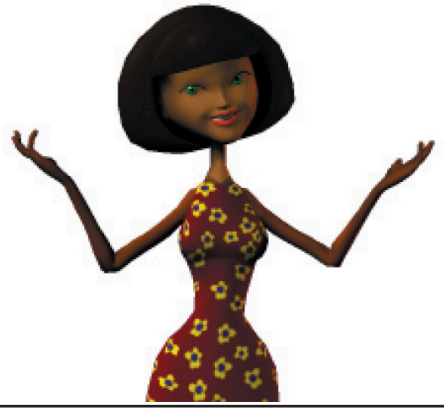
8 Circle your assigned role on the route diagram on the right.



Make your prediction

9 Do you think the planes will meet at the point where the routes come together?

Yes ☐ No ☐



10 Why or why not?

_____

_____

_____

_____

Investigator: _____

SET UP and DO THE EXPERIMENT

11 Circle your role and the steps below it. Then, do the experiment 3 times using the steps for your role.

Do the experiment.

STEPS	 Lead Air Traffic Controller	 Pilot	 Secondary Air Traffic Controller	 NASA Scientist
1 Take Your Position	Give the command: "Take your positions."	Stand at the Pilot mark on your jet route. Put one foot on each side of your route.	Go to your position.	Take your measuring tape to your jet route. Stand a few feet ahead of your pilot.
2 Get Ready to Start	Give the command: "Ready."	Practice stepping down your route.		
3 Start the Experiment	Start your stopwatch. Count seconds: "1", "2",...	Take your first step on "1". On each count, take a step to the next mark.	Wait for the first pilot to reach the point where the routes meet.	Move ahead of the pilot.
4 Stop the Experiment	When you hear "Halt", stop counting seconds.	When you hear "Halt", stop where you are.	When the first pilot arrives, say "Halt". Record the seconds.	After your hear "Halt", measure and record the distance between pilots.

Record the data

12 Record the Halt Time measured by the Secondary Controller.

13 Record the Separation Distance measured by the NASA Scientist.

14 Use the results of your three experiments to choose the best answer to this question:

What is the separation distance where the routes meet?

Experiment:	1	2	3
Halt Time (seconds)			
Did the planes meet?			

End of Worksheet

Investigator: _____

DO THE CALCULATIONS–Count Feet and Seconds

Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

Count feet and seconds along the jet route.

Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart").
- In 1 second, it moves ½ foot.
- In 2 seconds, it moves 1 foot—from 4 feet to 5 feet.

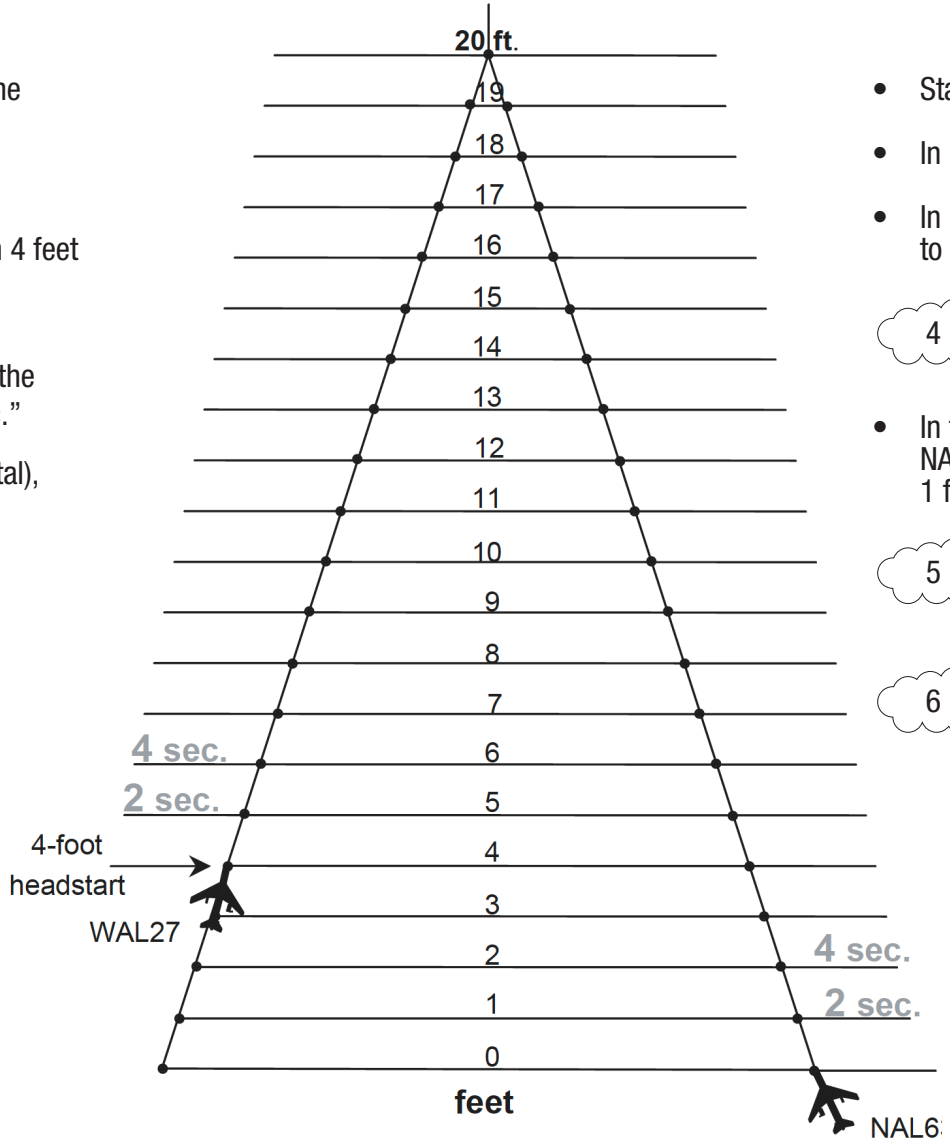
1 On the WAL27 jet route, find the 5-foot mark and trace "2 sec."

- In the next 2 seconds (4 seconds total), WAL27 moves another foot—from 5 feet to 6 feet.

2 On the WAL27 jet route, find the 6-foot mark and trace "4 sec."

3 Keep going on the WAL27 jet route, one foot at a time, until you reach the point where the routes meet.

At each foot-mark, write the total number of seconds to reach that mark.



Flight NAL63

- Starts where its route begins (at 0 feet)
- In 1 second, it moves ½ foot.
- In 2 seconds, it moves 1 foot—from 0 feet to 1 foot.

4 On the NAL63 jet route, find the 1-foot mark and trace "2 sec."

- In the next 2 seconds (4 seconds total), NAL63 moves another foot—from 1 foot to 2 feet.

5 On the NAL63 jet route, find the 2-foot mark and trace "4 sec."

6 Keep going on the NAL63 jet route, one foot at a time, until you reach the point where the routes meet.

At each foot-mark, write the total number of seconds to reach that mark.

Investigator: _____

DO THE CALCULATIONS–Count Feet and Seconds

Answer the questions.

7 How many seconds did it take each plane to arrive at the point where the routes intersect?

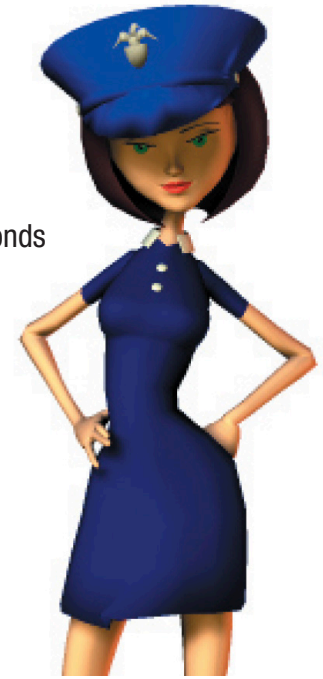
WAL27 _____ seconds

NAL63 _____ seconds

8 Did the planes meet at the point where the two routes intersect? Yes ☐ No ☐

9 If No, which plane arrived first? WAL27 ☐ NAL63 ☐

10 How many seconds did it take this plane to travel to the point where the two routes intersect? _____ seconds



11 At that time, how far away was the other airplane? _____ feet
(Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?)

12 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

_____

_____

13 You moved along each jet route, one foot at a time, to find the number of seconds it took each plane to travel to the point where the routes meet. Can you think of a faster way to find the number of seconds? If so, describe the faster way.

_____

_____

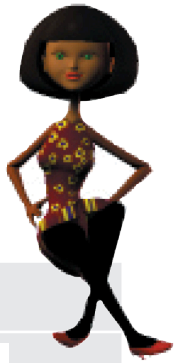
End of Worksheet

Investigator: _____

DO THE CALCULATIONS–Draw Blocks

Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

Use blocks to picture feet and seconds.

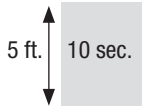


1 In this table, fill in the distance each plane will travel in 10 seconds.

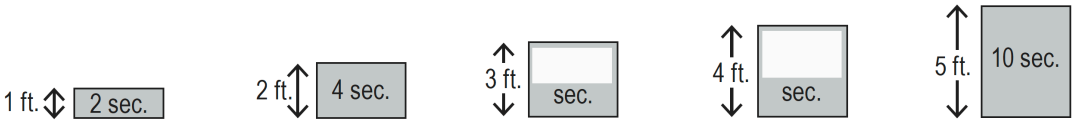
The speed of each plane is ½ foot per second.

Each plane takes...	1 second	2 seconds	4 seconds	10 seconds
to travel...	½ foot	1 foot	2 feet	

- Next, you will use this 10-second block to represent 5 feet of travel.
- The block height represents 5 feet, the distance the plane travels in 10 seconds.



2 Fill in the number of seconds it will take each plane to travel the distance shown.



DO THE CALCULATIONS–Draw Blocks

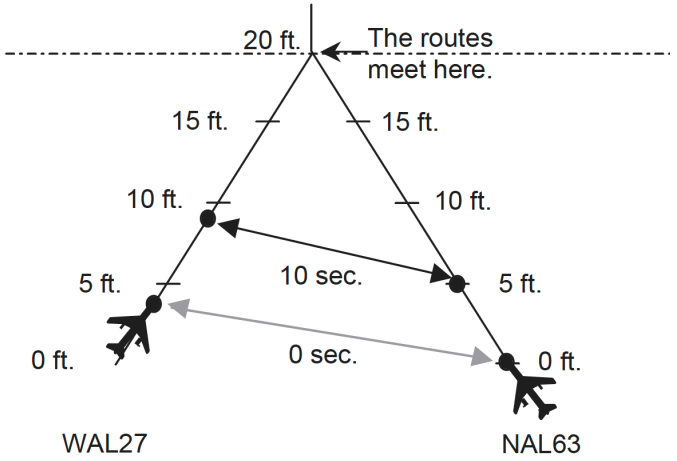
Follow along with this example of how to plot points and stack blocks.

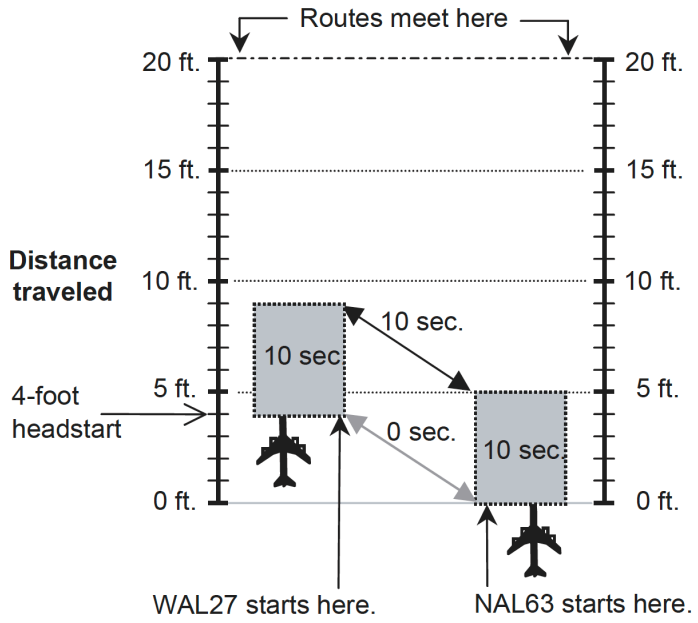
Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart").
- In 10 seconds, it moves 5 feet.
- Now it is 9 feet along its route.

3 Circle the 9-foot point on the WAL27 jet route.

4 Trace the 10-second block for WAL27.





Flight NAL63

- Starts where its route begins (at 0 feet)
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along it route.

5 Circle the 5-foot point on the NAL63 jet route.

6 Trace the 10-second block for NAL63.

DO THE CALCULATIONS–Draw Blocks

Now it's your turn to draw and connect.

Flight WAL27 and Flight NAL63

7 On the route, draw a dot to show the position of WAL27 after 20 seconds.

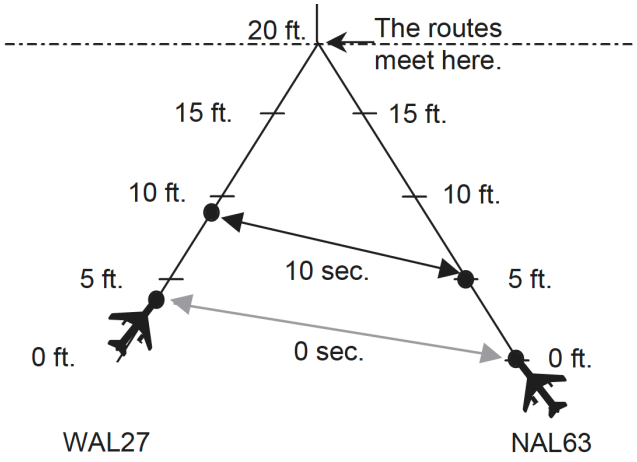
8 Trace the block on the graph below that shows the position of WAL27 after 20 seconds.

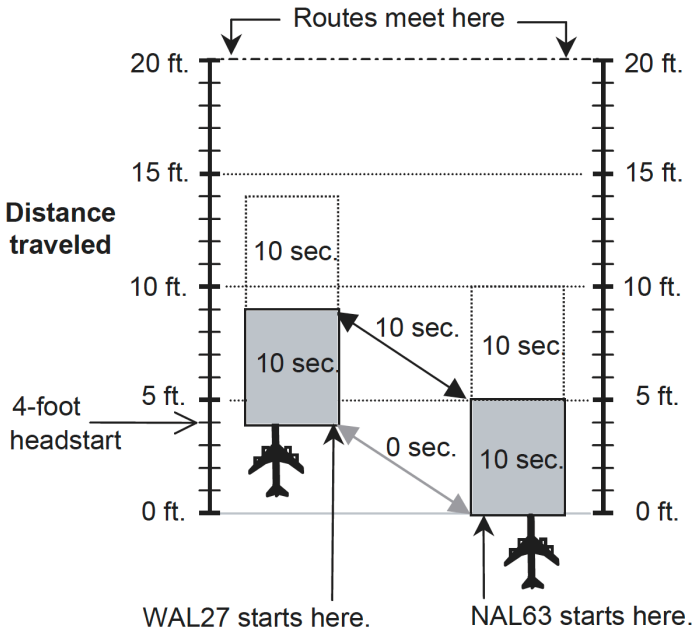
9 On the route, draw a dot to show the position of NAL63 after 20 seconds.

10 Trace the block on the graph that shows the position of NAL63 after 20 seconds.

11 Connect your dots with a line marked "20 sec."

12 Connect your blocks with a line marked "20 sec."





Now draw and connect at 30 seconds.

13 Draw dots and blocks at 30 seconds.

14 Connect the dots and connect blocks at 30 seconds

Keep going...

15 Keep going until the first plane reaches the point where the routes meet. You may need to use a smaller block.

Be sure to use this smaller block for BOTH planes.

16 How many feet long must the smaller block be?

_____ feet

17 How many seconds does this block represent?

_____ seconds

DO THE CALCULATIONS–Draw Blocks

Answer the questions.

18 Did the planes meet at the point where the two routes intersect?

Yes ☐ No ☐

19 If No, which plane arrived first? WAL27 ☐ NAL63 ☐

20 How many seconds did it take this plane to travel to the point where the two routes intersect? _____ seconds

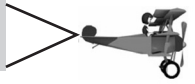
21 At that time, how far away was the other airplane?
(Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?) _____ feet

22 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

_____

_____

End of Worksheet



Investigator: _____

DO THE CALCULATIONS–Plot Points on Lines

Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

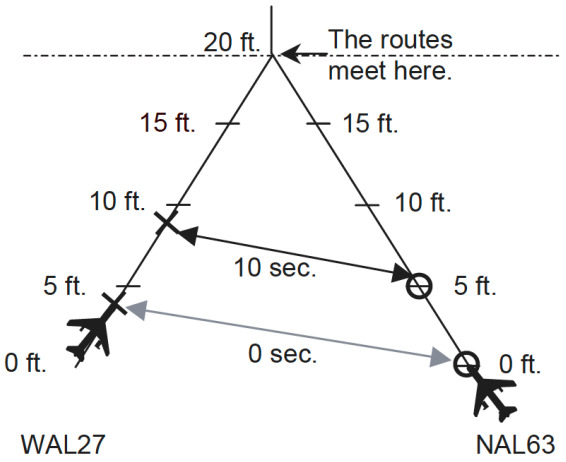
Follow along with this example of how to plot points.

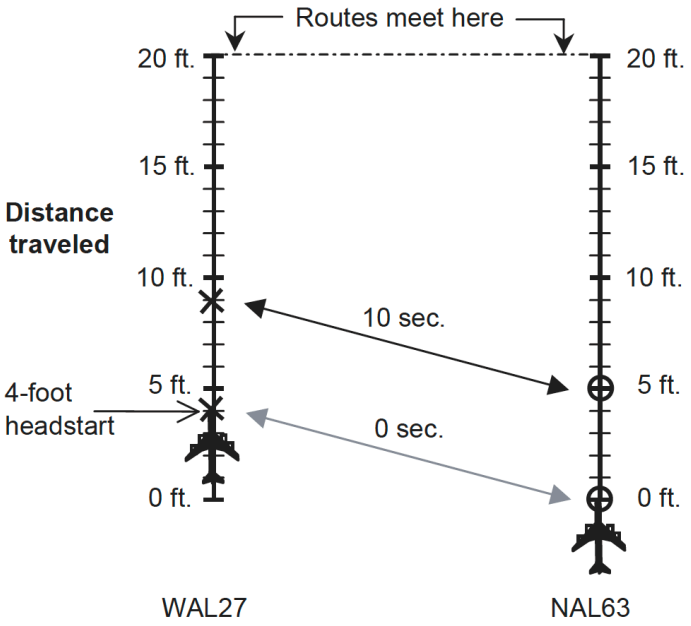
Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart").
- In 10 seconds, it moves 5 feet.
- Now it is 9 feet along its route.

1 Circle the **X** at the 9-foot point on the WAL27 jet route.

2 Circle the **X** at the 9-foot point on the WAL27 line graph.





Flight NAL63

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

3 Circle the **O** at the 5-foot point on the NAL63 jet route.

4 Circle the **O** at the 5-foot point on the NAL63 line graph.

Investigator: _____

DO THE CALCULATIONS–Plot Points on Lines

Now it's your turn to draw and connect.

Flight WAL27 and Flight NAL63

5 On the route, draw an **X** to show the position of WAL27 after 20 seconds

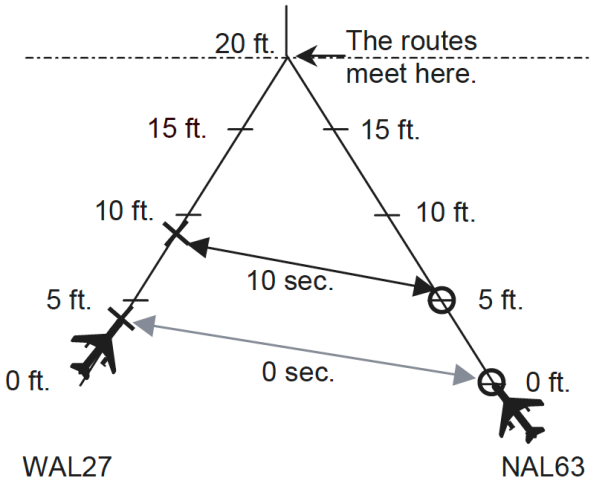
6 On the line graph, draw an **X** to show the position of WAL27 after 20 seconds

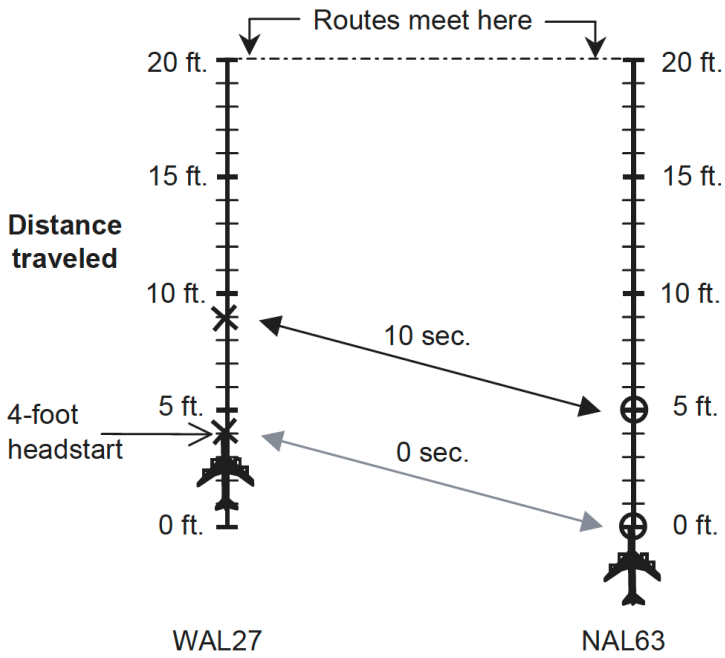
7 On the route, draw an **O** to show the position of NAL63 after 20 seconds

8 On the line graph, draw an **O** to show the position of NAL63 after 20 seconds

9 On the routes, connect your **X** and **O** with a line marked "20 sec".

10 On the line graph, connect your **X** and **O** with a line marked "20 sec".





Now draw and connect at 30 seconds.

11 On the routes, draw, connect, and label an **X** and an **O** at 30 seconds.

12 On the graph, draw, connect, and label an **X** and an **O** at 30 seconds.

Keep going...

13 Keep going until each plane reaches the point where the routes meet.

- At its last step, the first plane may need to fly a distance shorter than 5 feet.

14 What is that shorter distance?

_____ feet

15 How many seconds does it represent? (Each plane travels 1 foot in 2 seconds.)

_____ seconds

16 How far does the second plane travel in that many seconds?

_____ feet

Investigator: _____

DO THE CALCULATIONS–Plot Points on Lines

Answer the questions.

17 Did the planes meet at the point where the two routes intersect?

Yes ☐ No ☐

18 If No, which plane arrived first? WAL27 ☐ NAL63 ☐

19 How many seconds did it take this plane to travel to the point where the two routes intersect? _____ seconds

20 At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?) _____ feet

21 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

_____

_____

End of Worksheet

Investigator: _____

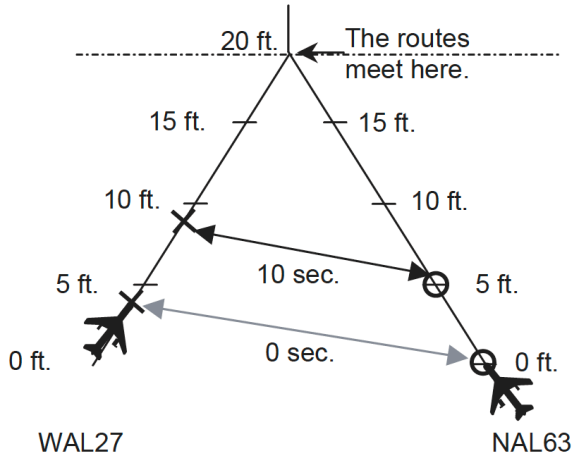
DO THE CALCULATIONS–Plot Points on a Grid

Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

Follow along with this example of how to plot points.

Flight WAL27

- Starts 4 feet from the point where the route begins (a 4-foot "headstart").
- In 10 seconds, it moves 5 feet.
- Now it is 9 feet along its route.



Flight NAL63

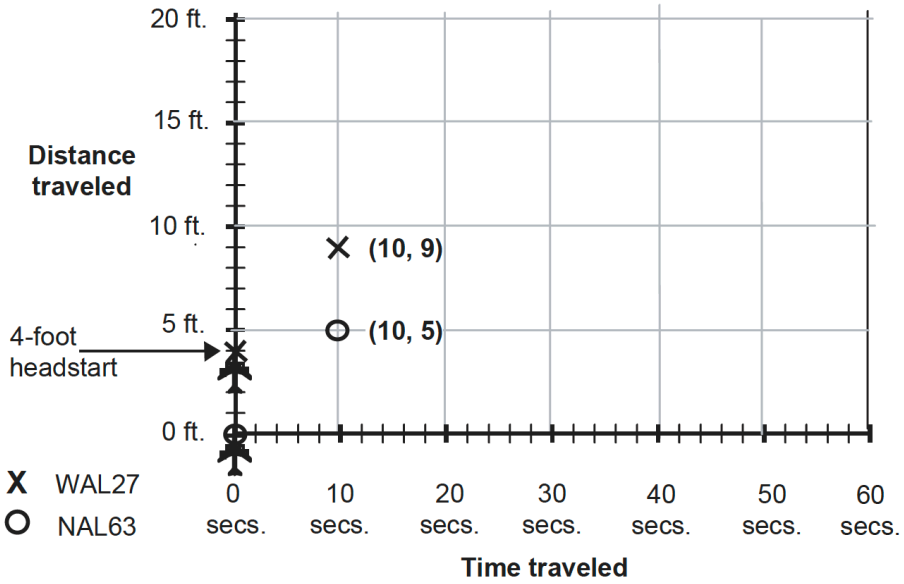
- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

1 Circle the **X** at the 5-foot point on the WAL27 jet route.

3 Circle the **O** at the 5-foot point on the NAL63 jet route.

2 Circle the **X** at the point (10, 9) on the grid.

4 Circle the **O** at the 5-foot point (10, 5) on the grid.



Investigator: _____

DO THE CALCULATIONS–Plot Points on a Grid

Now it's your turn to plot points.

Flight WAL27 and Flight NAL63

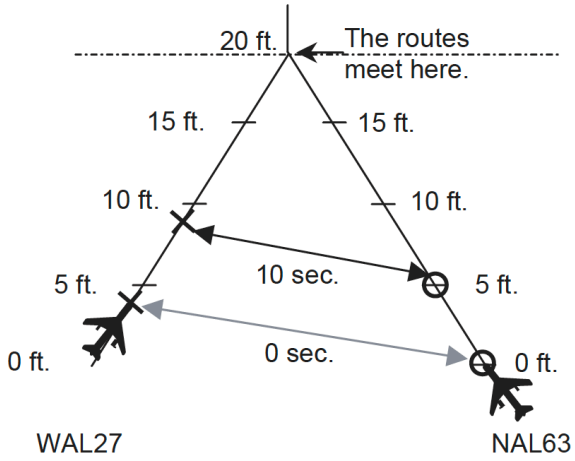
5 On the route, draw an **X** to show the position of WAL27 after 20 seconds

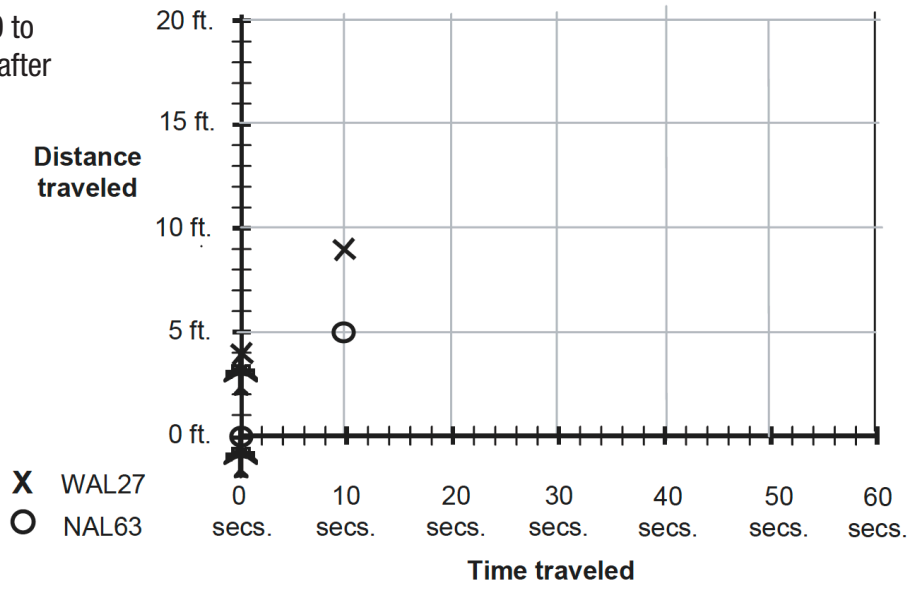
6 On the line graph, draw an **X** to show the position of WAL27 after 20 seconds

7 On the route, draw an **O** to show the position of NAL63 after 20 seconds

8 On the line graph, draw an **O** to show the position of NAL63 after 20 seconds

9 On the routes, connect your **X** and **O** with a line marked "20 sec".





Now draw and connect at 30 seconds.

10 On the routes, draw, connect, and label an **X** and an **O** at 30 seconds.

11 On the grid, draw an **X** and an **O** at 30 seconds.

Keep going...

12 Keep going until each plane reaches the point where the routes meet.

- At its last step, the first plane may need to fly a distance shorter than 5 feet.

13 What is that shorter distance?

_____ feet

14 How many seconds does it represent? (Each plane travels 1 foot in 2 seconds.)

_____ seconds

15 How far does the second plane travel in that many seconds?

_____ feet

DO THE CALCULATIONS–Plot Points on a Grid

Answer the questions.

17 Did the planes meet at the point where the two routes intersect?

Yes ☐ No ☐

18 If No, which plane arrived first? WAL27 ☐ NAL63 ☐

19 How many seconds did it take this plane to travel to the point where the two routes intersect? _____ seconds

20 At that time, how far away was the other airplane? (Hint: At that time, how many feet had the second plane traveled? How many feet was it from the intersection?) _____ feet

21 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

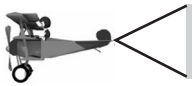
_____

_____

End of Worksheet

Investigator: _____

DO THE CALCULATIONS–Use a Formula



Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

Find a pattern.

- The speed of each plane is 0.5 feet per second.

1 Use multiplication to fill in this table.

In this many seconds...	Each plane travels this many feet...
1	0.5 feet/second x 1 second = 0.5 feet
2	0.5 feet/second x 2 second = 1.0 feet
3	0.5 feet/second x 3 second = 1.5 feet
4	____ feet/second x ____ second = ____ feet
5	____ feet/second x ____ second = ____ feet
6	____ feet/second x ____ second = ____ feet

2 How could you use multiplication to find the distance each plane travels in 14 seconds?

_____

_____

- The pattern in the table suggests this rule:
  "To find the distance traveled, multiply the speed by the time traveled."
- In math and science, we often say "rate" instead of "**s**peed."
- So we can write a rule like this:

  distance = **r**ate x **t**ime
- This relationship is call the Distance-Rate-Time Formula.
- We often write it like this:

Distance-Rate-Time Formula

d = r • t

3 Use the formula to answer this question:

How many feet does each plane travel in 20 seconds? ____ feet

Investigator: _____

DO THE CALCULATIONS–Use a Formula

Use the Formula.

Distance-Rate-Time Formula

$$d = r \bullet t$$

If we divide both sides of the equation by r...

...then, we get a formula for time traveled.

$$t = \frac{d}{r}$$

Here are the steps:

$$\frac{d}{r} = \frac{1r \bullet t}{r1}$$

$$\frac{d}{r} = t$$

4 Use this formula to find the number of seconds for WAL27 to travel 16 feet to the point where the routes meet.

$$t = \frac{16 \text{ feet}}{0.5 \text{ feet per second}} = \square \text{ seconds}$$

5 Use the same formula to find the number of seconds for NAL63 to travel 20 feet to the point where the routes meet.

$\square$ seconds

6 Will the planes meet at the point where the two routes intersect?

Yes ☐ No ☐

7 If No, which plane will arrive first? WAL27 ☐ NAL63 ☐

8 How many seconds will it take this plane to travel to the point where the two routes intersect? ☐ seconds

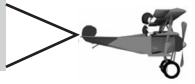
9 At that time, how far away is the other airplane? (Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?) ☐ feet

10 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

_____

_____

End of Worksheet



DO THE CALCULATIONS–Graph Linear Equations

Your Task: Calculate if two planes flying on merging routes will meet where the routes intersect.

Find an equation that describes the distance traveled by each plane.

Flight WAL27

- We can use the Distance-Rate-Time formula

  d = r • t

  to find d, the distance in feet WAL27 travels in t seconds.

- The WAL27 rate is 0.5 feet / second.

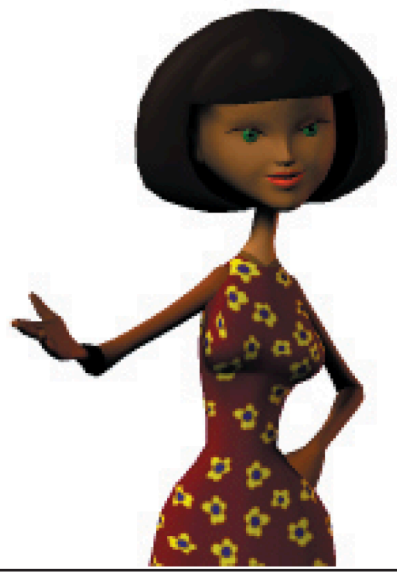
- WAL27 has a 4-foot headstart. (When you start your stopwatch at t = 0, WAL27 has already traveled 4 feet.)

  So...

  d = 0.5 t + 4

We can write the formula with the multiplication dot
d = r • t

or without the dot
d = rt.



Flight NAL63

- We can use the Distance-Rate-Time formula

  d = r • t

  to find d, the distance in feet NAL63 travels in t seconds.

- The NAL63 rate is 0.5 feet / second. So...

  d = 0.5 t

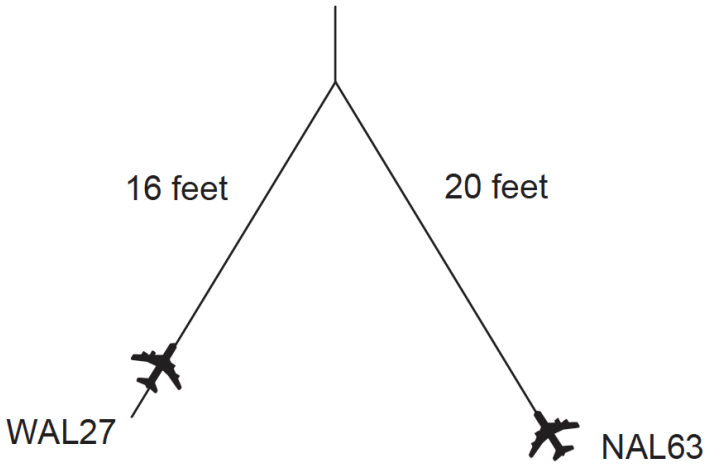
DO THE CALCULATIONS–Graph Linear Equations

Fill in each table.

1 Fill in the table for WAL27.

d = 0.5t + 4

t seconds	d feet
0	
10	
20	
30	
40	



2 Fill in the table for NAL63.

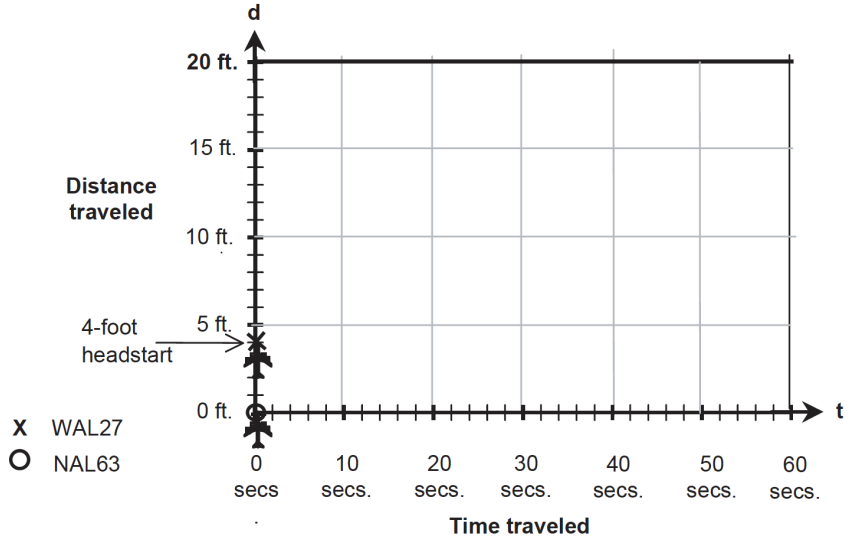
d = 0.5t

t seconds	d feet
0	
10	
20	
30	
40	

Graph each line.

3 Use an **X** to graph each point in the WAL27 table.

Use a solid line _____ to connect the points.



4 Use an **O** to graph each point in the NAL63 table.

Use a dotted line …….. to connect the points.

Investigator: _____

DO THE CALCULATIONS–Graph Linear Equations

Answer the questions.

5 How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27 _____ seconds

NAL63 _____ seconds

6 Will the planes meet at the point where the two routes intersect?

Yes ☐ No ☐

7 If NO, which plane will arrive first?

WAL27 ☐ NAL63 ☐

8 How many seconds will it take this plane to travel to the point where the two routes intersect?

_____ seconds

9 At that time, how far away is the other plane?
(Hint: At that time, how many feet has the second plane traveled? How many feet is it from the intersection?)

_____ feet

10 Write the number that is the slope of the solid line representing WAL27.

_____

11 Write the number that is the slope of the solid line representing WAL27. ☐

12 Write the number that is the slope of the dotted line representing NAL63. ☐

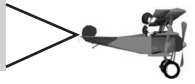
13 What information does the slope of the line tell you about each plane?

_____

_____

_____

End of Worksheet



Investigator: _____

ANALYZE YOUR RESULTS

Your Task: Analyze and explain your results. Then apply what you learned to another problem.

Compare your math results with your experimental data.

1 Use your experimental data and math results to fill in this table:

	Experiment	Mathematics
Did the planes meet where the routes meet? (Yes or No)		

2 Do your experimental and your math results match?

Yes ☐ No ☐

3 If No, why do you think they don't match?

_____

_____

4 Which is correct? Why? _____

_____

_____

5 Use your experimental data and math results to fill in this table:

	Experiment	Mathematics
Separation distance (feet) where the routes meet.		

6 Do your experimental and your math results match?

Yes ☐ No ☐

7 If No, why do you think they don't match?

_____

_____

8 Which is correct? Why? _____

_____

_____

Investigator: _____

ANALYZE YOUR RESULTS

Compare the speeds and compare the distances.

9 Are the planes' speeds the same or different? Same ☐ Different ☐

10 What is the "difference in the planes' starting distances?" (That is, what is the "headstart?") ☐ feet

11 What is the difference in the planes' final positions? (What is the separation distance where the routes meet?) ☐ feet

12 Are the distances the same or different? Same ☐ Different ☐

Consider the general problem.

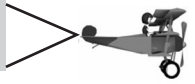
Two planes are flying at the same speed on two different routes. The planes start at different distances from the point where the routes meet.

13 When the first plane reaches the point where the routes meet, the separation distance will be:

the same time. ☐ different times. ☐

the "difference" in the plane's starting distances (the "headstart").

End of Worksheet



Investigator: _____ **EXTENSION**

Your Task: Will the two planes violate the separation standard where the routes meet?

Add a safety requirement.

- For safety reasons, the planes must be separated by at least 5 feet at the point where the routes meet.

Compare the distances.

1 Based upon your calculation, what is the difference in the planes' final positions? (That is, what is the planes' separation distance where the routes meet?) _____ feet

2 Does this distance satisfy the separation requirement?

Yes ☐ No ☐

3 If No, what would you tell the air traffic controller to do to meet the separation requirement?

_____

_____

_____

SHORTCUT **Analyze an alternate route.**

- To avoid a separation violation, the air traffic controller gives WAL27 a new route.

4 How far will WAL27 travel to reach the **new** point where the routes meet?

_____ feet

5 How far will NAL63 travel to reach the **new** point where the routes meet?

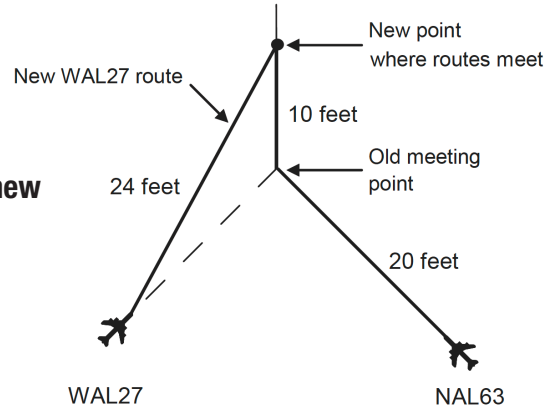
_____ feet

6 Which plane will fly a shorter distance?

WAL27 ☐ NAL63 ☐

7 How much shorter?

_____ feet



- The planes are traveling at the same speed.
  WAL27 Speed = ½ foot per second
  NAL63 Speed = ½ foot per second

8 When WAL27 arrives at the **new** point where the routes meet, how many feet away is NAL63 on its route?

_____ feet

- Here's another way to think about this.

9 When WAL27 has traveled 24 feet to the **new** point where the routes meet, how far has NAL63 traveled?

_____ feet

10 At the time, how many feet away is NAL63 on its route?

_____ feet

11 Is the separation standard violated?

Yes ☐ No ☐

End of Worksheet