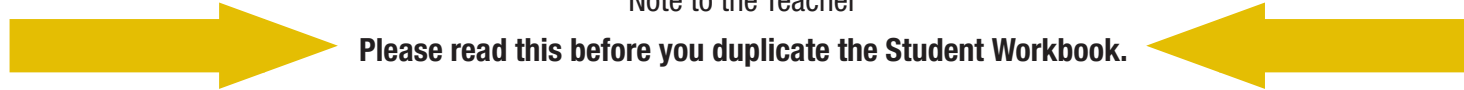




Note to the Teacher



**Please read this before you duplicate the Student Workbook.**

This Smart Skies™ FlyBy Math™ Student Workbook contains activities for one Air Traffic Control problem. In particular, the Workbook includes worksheets for 6 different calculation methods your students can use to solve the problem:

- Count feet and seconds
- Plot points on a grid
- Draw and stack blocks
- Use the distance-rate-time formula
- Plot points on two vertical lines
- Graph two linear equations

You will most likely want to assign only 1 or 2 of the calculation methods. So we recommend that you follow these steps before you duplicate the Workbook.



1. **Select and Keep:**  
Choose the calculation method(s) you want your students to use.



2. **Recycle the Rest:**  
Remove the calculation worksheets you do not wish to assign.

To find the calculation worksheets, look for the Calculations footers at the page bottoms.

**Smart Skies™** FlyBy Math™

ATC #1 – Calculations – Blocks – Page 1 of 4

EG-2004-11-111-ARC



Other activity worksheets follow the Calculation pages. Be sure to remove the Calculation worksheets **ONLY**.

For more information about Smart Skies™ workbooks, please see the Smart Skies™ teacher materials available online at:

<https://www.nasa.gov/smartskies/flybymath>

# FlyBy Math™

## Math & Science for Air Traffic Control



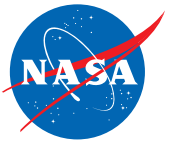
### Problem #1

- Aircraft are on merging routes
- Aircraft are traveling at the same speed
- Aircraft start at the same distance from where the routes meet

## STUDENT WORKBOOK

Investigator: \_\_\_\_\_

# READ THE PROBLEM



**Begin Your Challenge:** Will two planes, flying on merging jet routes, meet where the routes intersect?

## Flight WAL27

- WAL27 is 20 feet (6.1 meters) from where the routes come together.
- The WAL27 speed is  $\frac{1}{2}$  foot per second (0.15 meters/second).

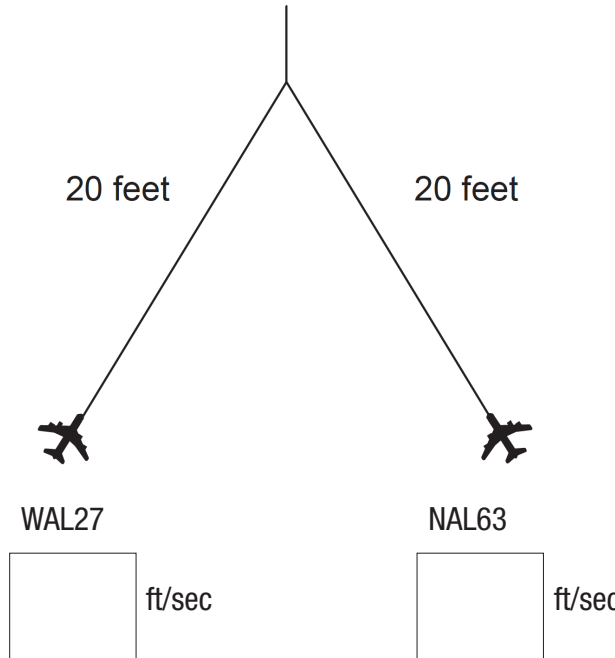
1 Write the speed of WAL27 in the box below its picture.

2 How far does WAL27 travel in 1 second?

feet

3 How far does WAL27 travel in 10 seconds?

feet



## Flight NAL63

- NAL63 is 20 feet (6.1 meters) from where the routes come together.
- The NAL63 speed is  $\frac{1}{2}$  foot per second (0.15 meters/second).

4 Write the speed of NAL63 in the box below its picture.

5 How far does NAL63 travel in 1 second?

feet

6 How far does WAL27 travel in 10 seconds?

feet

### To meet your Challenge, you will:

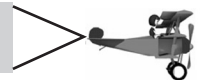
- Conduct an experiment.
- Do some math calculations.
- Analyze your results

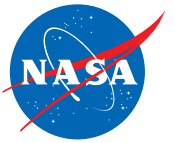
### Then, you will use your results to answer this question:

- Will the planes meet at the point where the routes intersect?



End of Worksheet

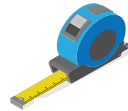




**Begin Your Task:** Experiment to see what happens when the first plane reaches the point where the routes meet.

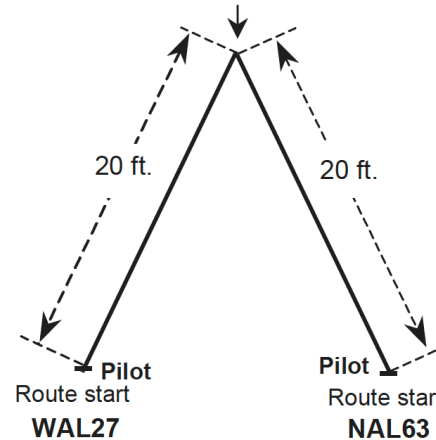
## Flight WAL27

- 1 Begin at the point where the routes meet.  
Use chalk or masking tape to mark off a line 20 feet long.
- 2 At the jet route start, make a mark. Place a "Pilot" label next to the mark.



### Layout the jet routes.

The routes meet here.



## Flight NAL63

- 3 Begin at the point where the routes meet.  
Mark off a line 20 feet long.
- 4 At the jet route start, make a mark.  
Place a "Pilot" label next to the mark.

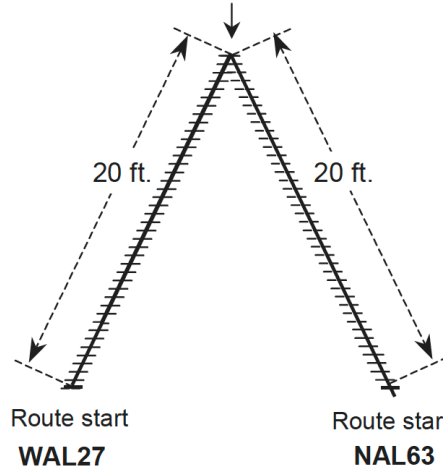
## Flight WAL27

- 5 Begin at the start of the jet route.  
Place a mark (or piece of tape) every 1/2 foot (6 inches) along the jet route all the way to the point where the routes meet.



### Mark the speed control lines.

The routes meet here.



## Flight NAL63

- 6 Begin at the start of the jet route.  
Place a mark (or piece of tape) every 1/2 foot (6 inches) along the jet route all the way to the point where the routes meet.





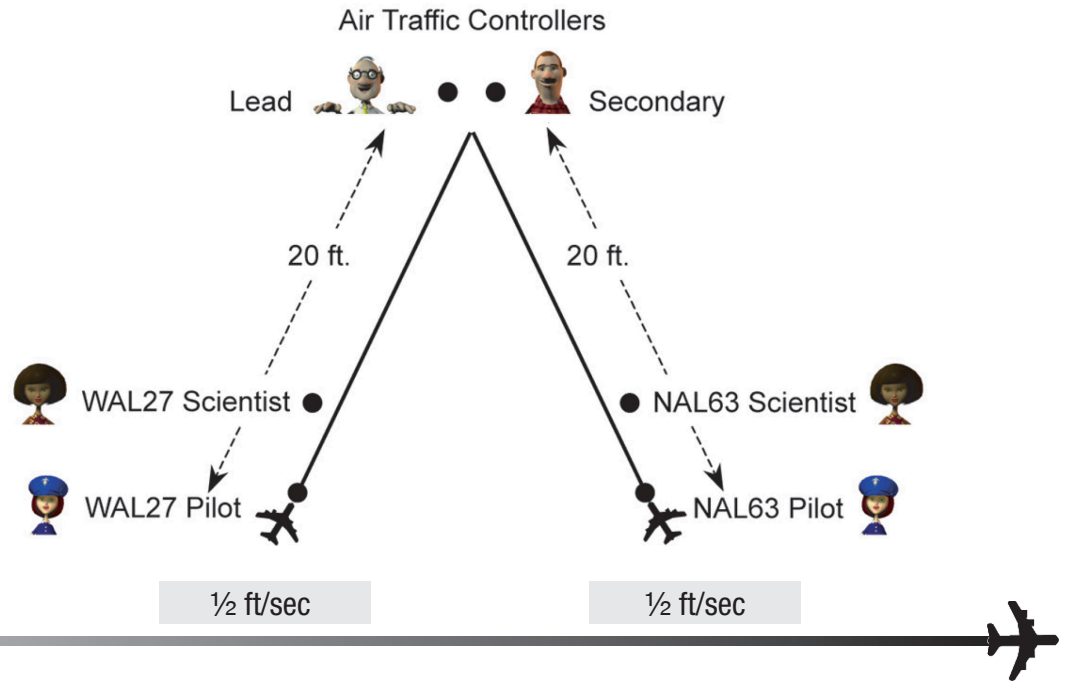
Record the starting conditions for the experiment.

## Flight WAL27

7 Fill in this table with the starting conditions.

Flight Number	Speed (ft/sec)	Distance from the point where the routes meet (ft)
WAL27		
NAL63		

8 Circle your assigned role on the route diagram on the right.



Make your prediction

9 Do you think the planes will meet at the point where the routes come together?

Yes

No

10 Why or why not?

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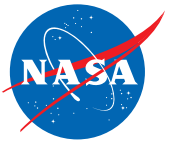


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Investigator: \_\_\_\_\_

## SET UP and DO THE EXPERIMENT



11 Circle your role and the steps below it. Then, do the experiment 3 times using the steps for your role.

 **Do the experiment.**

	 <b>Lead Air Traffic Controller</b>	 <b>Pilot</b>	 <b>Secondary Air Traffic Controller</b>	 <b>NASA Scientist</b>
<b>STEPS</b>				
<b>1</b> Take Your Position	Give the command: "Take your positions."	Stand at the Pilot mark on your jet route. Put one foot on each side of your route.	Go to your position.	Take your measuring tape to your jet route. Stand a few feet ahead of your pilot.
<b>2</b> Get Ready to Start	Give the command: "Ready."	Practice stepping down your route.		
<b>3</b> Start the Experiment	Start your stopwatch. Count seconds: "1", "2",...	Take your first step on "1". On each count, take a step to the next mark.	Wait for the first pilot to reach the point where the routes meet.	Move ahead of the pilot.
<b>4</b> Stop the Experiment	When you hear "Halt", stop counting seconds.	When you hear "Halt", stop where you are.	When the first pilot arrives, say "Halt". Record the seconds.	After you hear "Halt", measure and record the distance between pilots.

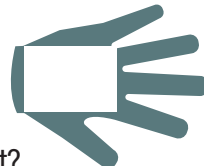
 **Record the data**

12 Record the Halt Time measured by the NASA Scientist.

13 Record whether or not the planes meet where the routes meet.

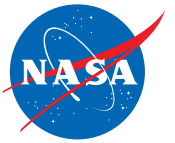
14 Use the results of your three experiments to choose the best answer to this question:

Did the planes meet where the routes meet?



Experiment:	1	2	3
Halt Time (seconds)			
Did the planes meet?			

End of Worksheet 



**Your Task:** Calculate if two planes flying on merging routes will meet where the routes intersect.



Count feet and seconds along the jet route.

**Flight WAL27**

- Starts where its route begins (at 0 feet).
- In 1 second, it moves  $\frac{1}{2}$  foot.
- In 2 seconds, it moves 1 foot—from 0 feet to 1 foot.

1 On the WAL27 jet route, find the 1-foot mark and trace “2 sec.”

- In the next 2 seconds (4 seconds total), WAL27 moves another foot—from 1 foot to 2 feet.

2 On the WAL27 jet route, find the 2-foot mark and trace “4 sec.”

3 Keep going on the WAL27 jet route, one foot at a time, until you reach the point where the routes meet.

At each foot-mark, write the total number of seconds to reach that mark.

**Flight NAL63**

- Starts where its route begins (at 0 feet)
- In 1 second, it moves  $\frac{1}{2}$  foot.
- In 2 seconds, it moves 1 foot—from 0 feet to 1 foot.

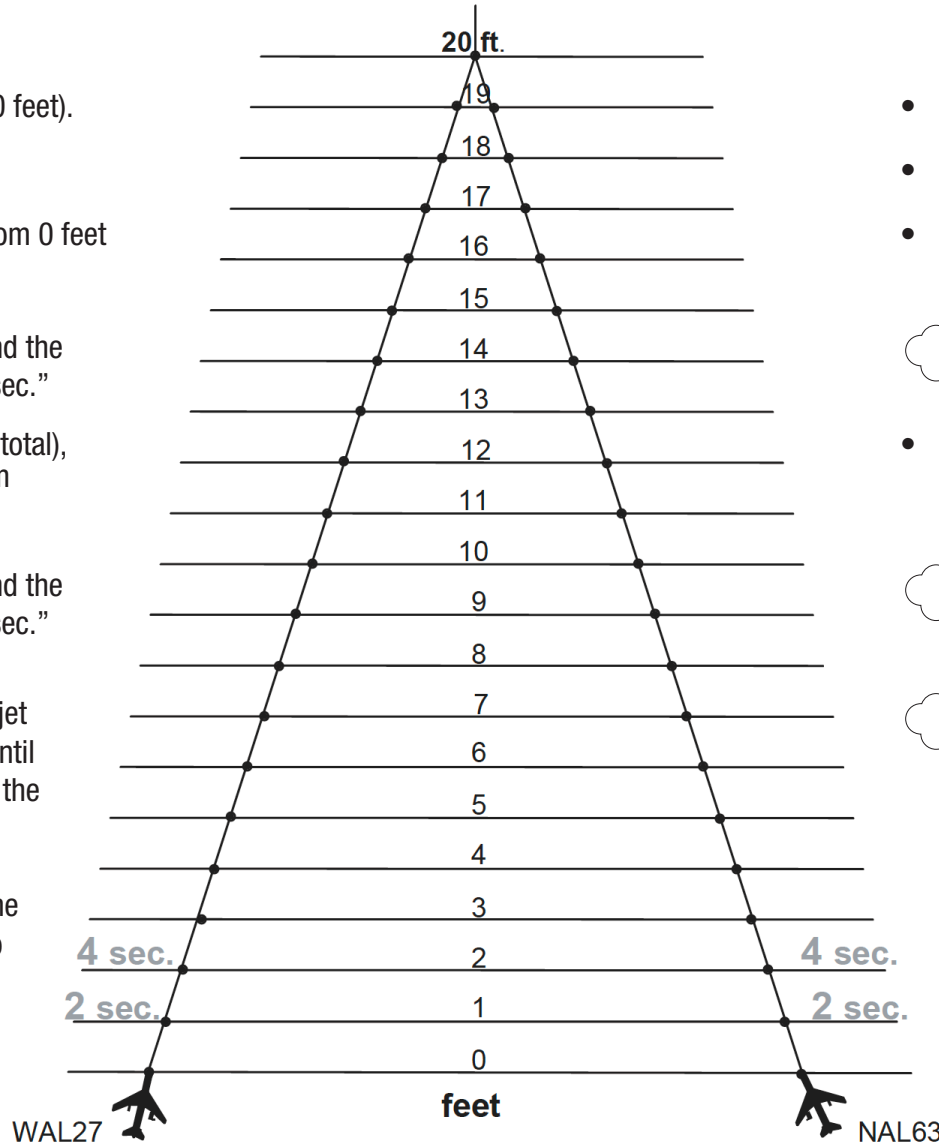
4 On the NAL63 jet route, find the 1-foot mark and trace “2 sec.”

- In the next 2 seconds (4 seconds total), NAL63 moves another foot—from 1 foot to 2 feet.

5 On the NAL63 jet route, find the 2-foot mark and trace “4 sec.”

6 Keep going on the NAL63 jet route, one foot at a time, until you reach the point where the routes meet.

At each foot-mark, write the total number of seconds to reach that mark.



Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Count Feet and Seconds



Answer the questions.

7 How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27

seconds

NAL63

seconds

8 Did the planes meet at the point where the two routes intersect?

Yes

No

9 Why or why not?

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10 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

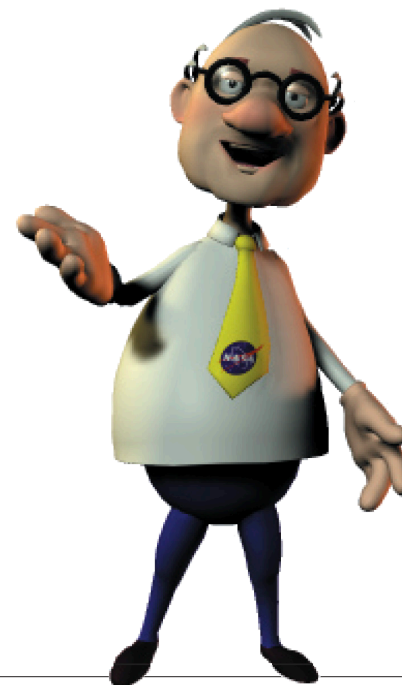
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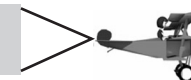
11 You moved along each jet route, one foot at a time, to find the number of seconds it took each plane to travel to the point where the routes meet. Can you think of a faster way to find the number of seconds? If so, describe the faster way.

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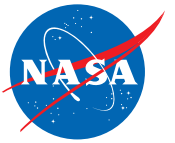
End of Worksheet





Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Draw Blocks



**Your Task:** Calculate if two planes flying on merging routes will meet where the routes intersect.



Use blocks to picture feet and seconds.

1

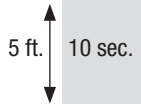
In this table, fill in the distance each plane will travel in 10 seconds.

The speed of each plane is  $\frac{1}{2}$  foot per second.

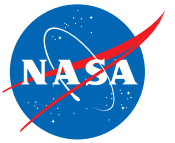
Each plane takes...	1 second	2 seconds	4 seconds	10 seconds
to travel...	1/2 foot	1 foot	2 feet	<input type="text"/>



- Next, you will use this 10-second block to represent 5 feet of travel.
- The block height represents 5 feet, the distance the plane travels in 10 seconds.



A diagram showing a gray rectangular block. To its left, a vertical double-headed arrow indicates a height of 5 ft. Inside the block, the text '10 sec.' is written.



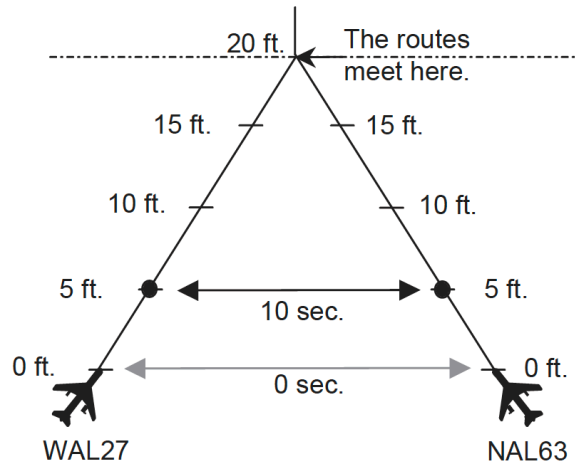
Follow along with this example of how to plot points and stack blocks.

**Flight WAL27**

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

2 Circle the 5-foot point on the WAL27 jet route.

3 Trace the 10-second block for WAL27.

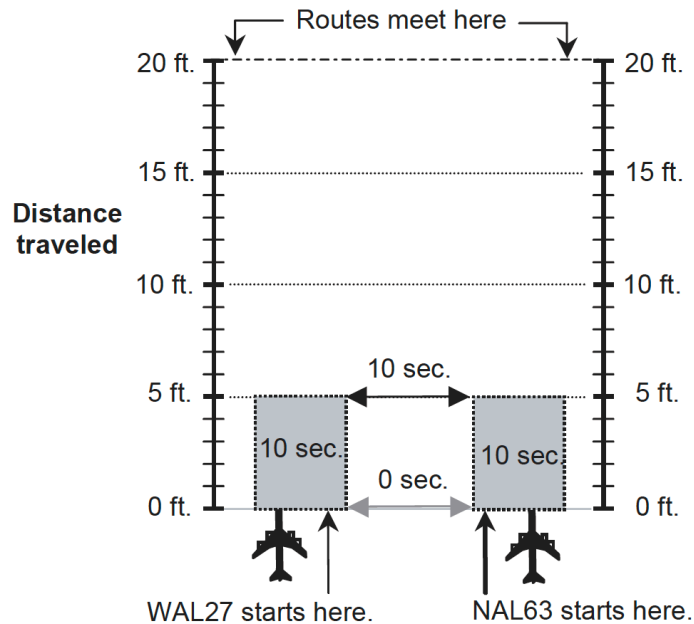


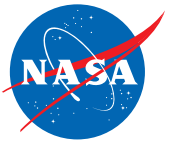
**Flight NAL63**

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

4 Circle the 5-foot point on the NAL63 jet route.

5 Trace the 10-second block for NAL63.

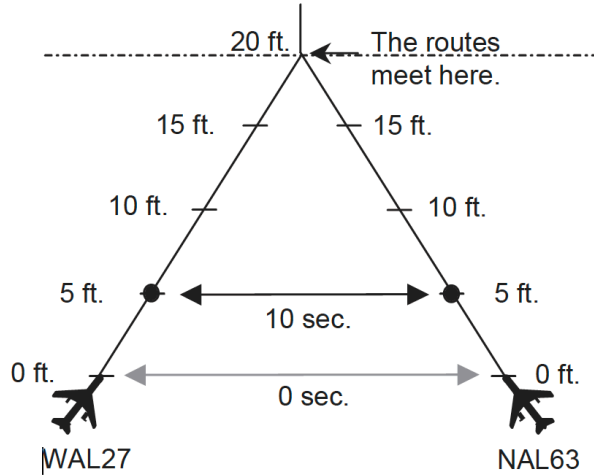




Now it's your turn to draw and connect.

**Flight WAL27 and Flight NAL63**

- 6 On the route, draw a dot to show the position of WAL27 after 20 seconds.
- 7 Trace the block on the graph below that shows the position of WAL27 after 20 seconds.
- 8 On the route, draw a dot to show the position of NAL63 after 20 seconds.
- 9 Trace the block on the graph that shows the position of NAL63 after 20 seconds.
- 10 Connect your dots with a line marked "20 sec."
- 11 Connect your blocks with a line marked "20 sec."

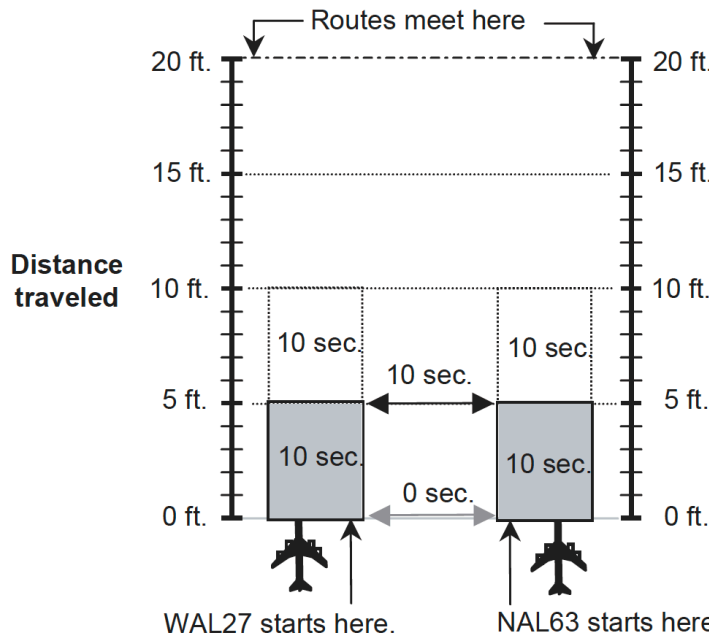


**Now draw and connect at 30 seconds.**

- 12 Draw dots and blocks at 30 seconds.
- 13 Connect the dots and connect blocks at 30 seconds

**Keep going...**

- 14 Keep going until each plane reaches the point where the routes meet.
- 15 How many seconds did it take each plane to arrive at the point where the routes intersect?

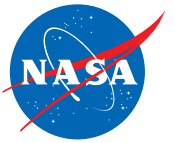


WAL27  seconds

NAL63  seconds

Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Draw Blocks



Answer the questions.

16

Did the planes meet at the point where the two routes intersect?

Yes

No



17

Why or why not?

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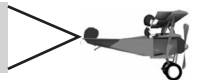
18

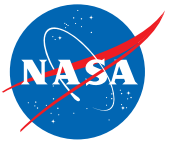
If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

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Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Plot Points on Lines



**Your Task:** Calculate if two planes flying on merging routes will meet where the routes intersect.



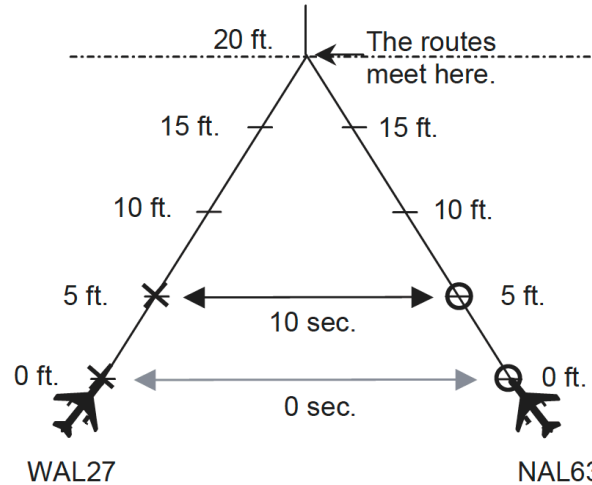
Follow along with this example of how to plot points.

### Flight WAL27

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

1 Circle the **X** at the 5-foot point on the WAL27 jet route.

2 Circle the **X** at the 5-foot point on the WAL27 line graph.

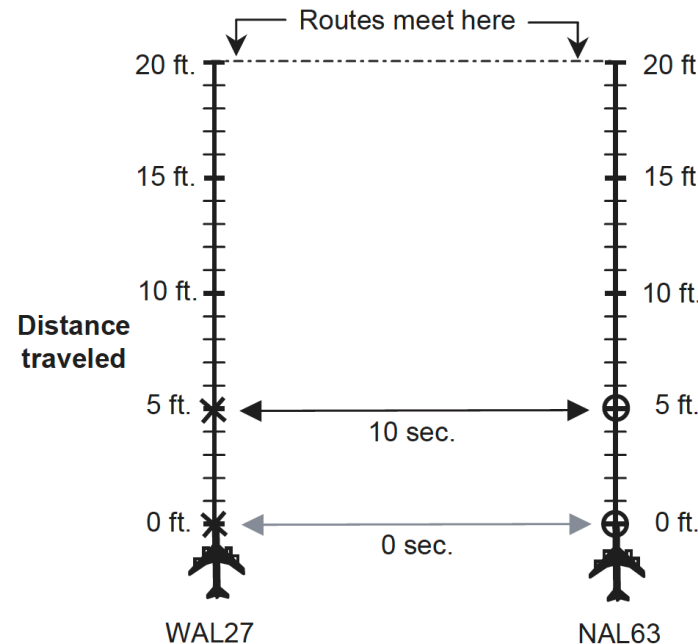


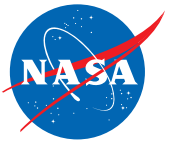
### Flight NAL63

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

3 Circle the **O** at the 5-foot point on the NAL63 jet route.

4 Circle the **O** at the 5-foot point on the NAL63 line graph.

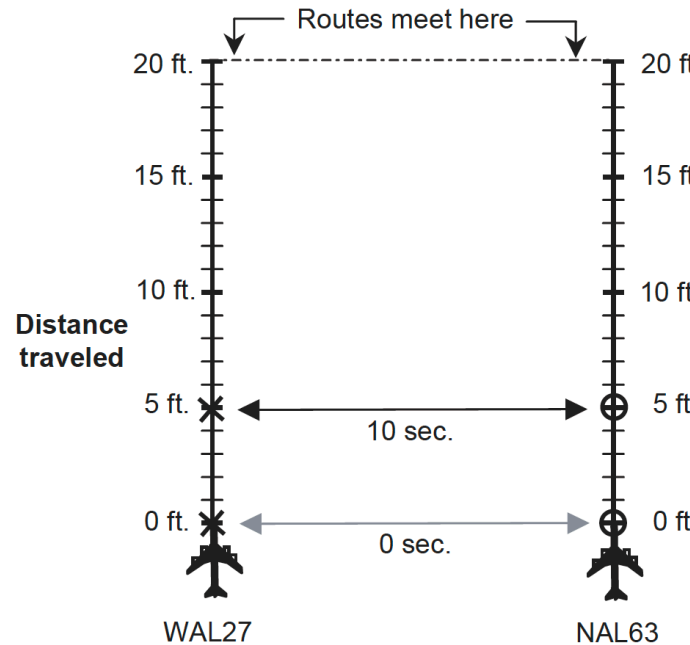
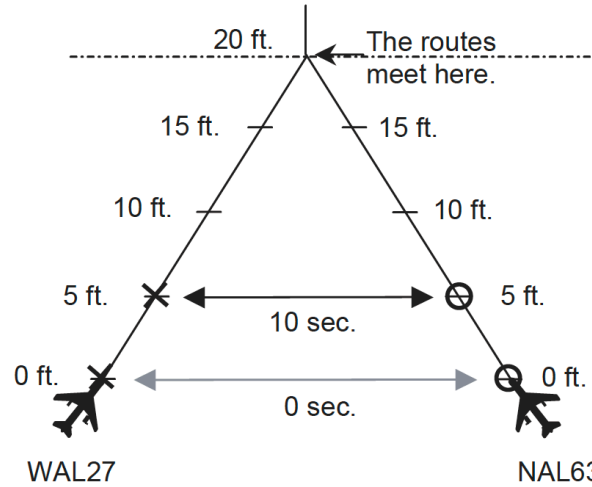




Now it's your turn to draw and connect.

**Flight WAL27 and Flight NAL63**

- 5 On the route, draw an **X** to show the position of WAL27 after 20 seconds
- 6 On the line graph, draw an **X** to show the position of WAL27 after 20 seconds
- 7 On the route, draw an **O** to show the position of NAL63 after 20 seconds
- 8 On the line graph, draw an **O** to show the position of NAL63 after 20 seconds
- 9 On the route, connect you **X** and **O** with a line marked "20 sec".
- 10 On the line graph, draw an **X** and **O** with a line marked "20 sec".



**Now draw and connect at 30 seconds.**

- 11 On the routes, draw, connect, and label an **X** and an **O** at 30 seconds.
- 12 On the graph, draw, connect, and label an **X** and an **O** at 30 seconds.

**Keep going...**

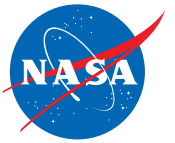
- 13 Keep going until each plane reaches the point where the routes meet.
- 14 How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27  seconds

NAL63  seconds

Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Plot Points on Lines



Answer the questions.

15

Did the planes meet at the point where the two routes intersect?

Yes

No

16

Why or why not?

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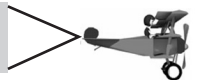
17

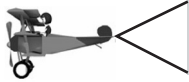
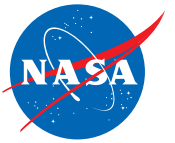
If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

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**Your Task:** Calculate if two planes flying on merging routes will meet where the routes intersect.



Follow along with this example of how to plot points.

**Flight WAL27**

- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

1 Circle the **X** at the 5-foot point on the WAL27 jet route.

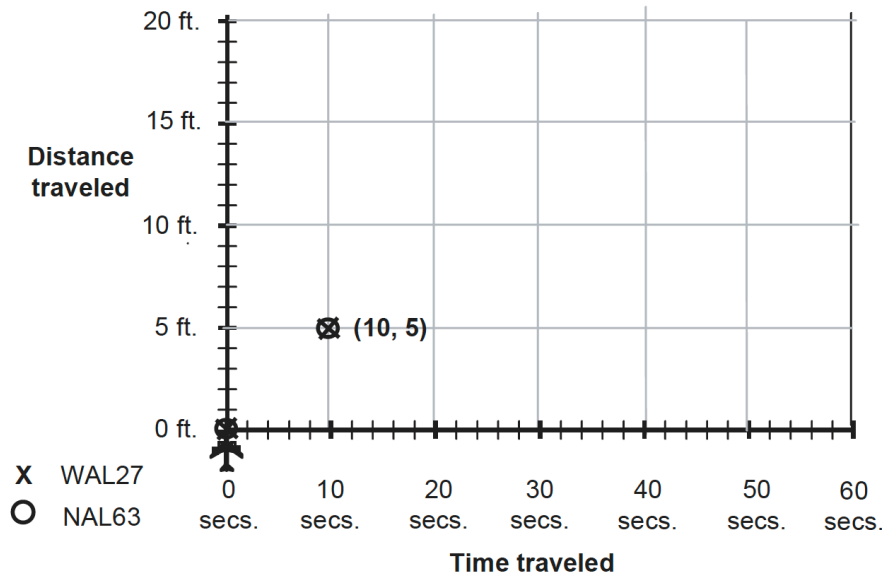
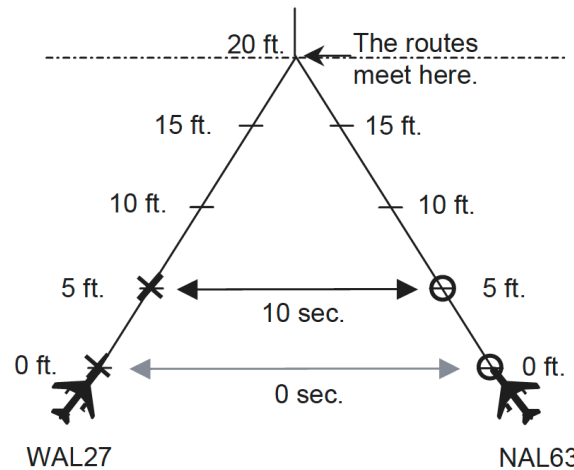
2 Circle the **X** at the point (10, 5) on the grid.

**Flight NAL63**

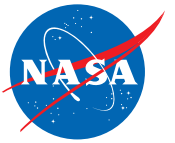
- Starts where its route begins (at 0 feet).
- In 10 seconds, it moves 5 feet.
- Now it is 5 feet along its route.

3 Circle the **O** at the 5-foot point on the NAL63 jet route.

4 Circle the **O** at the 5-foot point (10, 5) on the grid.







Now it's your turn to plot points.

**Flight WAL27 and Flight NAL63**

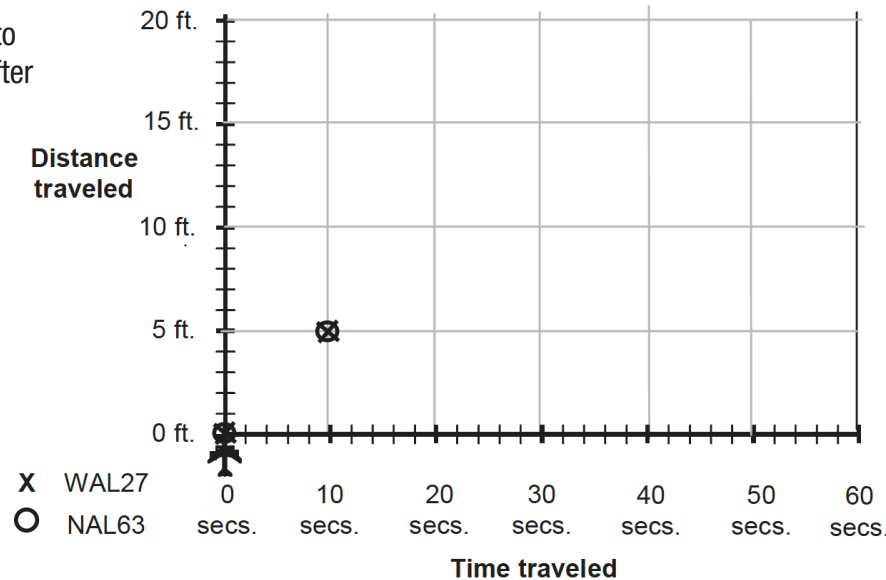
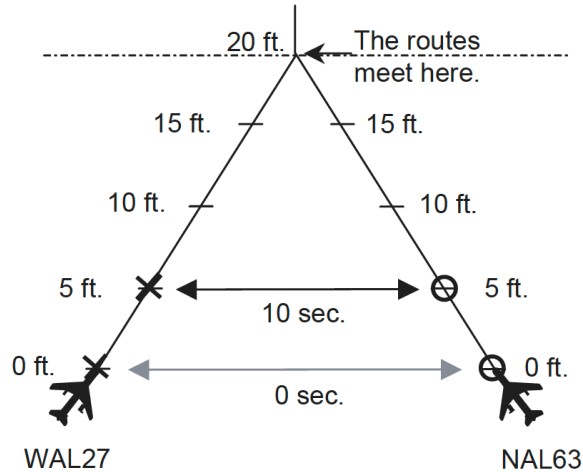
5 On the route, draw an **X** to show the position of WAL27 after 20 seconds

6 On the line graph, draw an **X** to show the position of WAL27 after 20 seconds

7 On the route, draw an **O** to show the position of NAL63 after 20 seconds

8 On the line graph, draw an **O** to show the position of NAL63 after 20 seconds

9 On the routes, connect your **X** and **O** with a line marked "20 sec".



**Now draw and connect at 30 seconds.**

10 On the routes, draw, connect, and label an **X** and an **O** at 30 seconds.

11 On the grid, draw an **X** and an **O** at 30 seconds.

**Keep going...**

12 Keep going until each plane reaches the point where the routes meet.

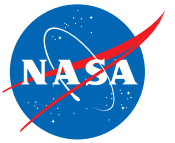
13 How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27  seconds

NAL63  seconds

Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Plot Points on a Grid



Answer the questions.

14

Did the planes meet at the point where the two routes intersect?

Yes

No

15

Why or why not?

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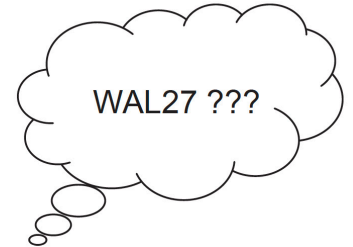
16

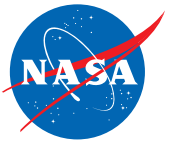
If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

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**Your Task:** Calculate if two planes flying on merging routes will meet where the routes intersect.



Find a pattern.

- The speed of each plane is 0.5 feet per second.

1 Use multiplication to fill in this table.

In this many seconds...	Each plane travels this many feet...
1	0.5 feet/second x 1 second = 0.5 feet
2	0.5 feet/second x 2 second = 1.0 feet
3	0.5 feet/second x 3 second = 1.5 feet
4	<input type="text"/> feet/second x <input type="text"/> second = <input type="text"/> feet
5	<input type="text"/> feet/second x <input type="text"/> second = <input type="text"/> feet
6	<input type="text"/> feet/second x <input type="text"/> second = <input type="text"/> feet

2

How could you use multiplication to find the distance each plane travels in 14 seconds?

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- The pattern in the table suggests this rule:  
“To find the distance traveled, multiply the speed by the time traveled.”
- In math and science, we often say “rate” instead of “speed.”
- So we can write a rule like this:  
$$\text{distance} = \text{rate} \times \text{time}$$
- This relationship is call the Distance-Rate-Time Formula.
- We often write it like this:

**Distance-Rate-Time Formula**

$$d = r \cdot t$$

3

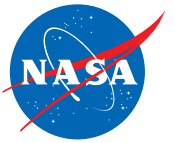
Use the formula to answer this question:

How many feet does each plane travel in 20 seconds?

 feet

Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Use a Formula



Use the Formula.

### Distance-Rate-Time Formula

$$d = r \cdot t$$

If we divide both sides of the equation by r...

...then, we get a formula for time traveled.

$$t = \frac{d}{r}$$

Here are the steps:

$$\frac{d}{r} = \frac{\cancel{r} \cdot t}{\cancel{r} 1}$$

$$\frac{d}{r} = t$$

4 Use this formula to find the number of seconds for WAL27 to travel 20 feet to the point where the routes meet.

$$t = \frac{20 \text{ feet}}{0.5 \text{ feet per second}} = \boxed{\phantom{000}} \text{ seconds}$$

5 Use the same formula to find the number of seconds for NAL63 to travel 20 feet to the point where the routes meet.

$$\boxed{\phantom{000}} \text{ seconds}$$

6 Will the planes meet at the point where the two routes intersect?

Yes

No

7 Why or why not?

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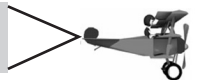
8 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

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End of Worksheet





**Your Task:** Calculate if two planes flying on merging routes will meet where the routes intersect.



Find an equation that describes the distance traveled by each plane.

### Flight WAL27

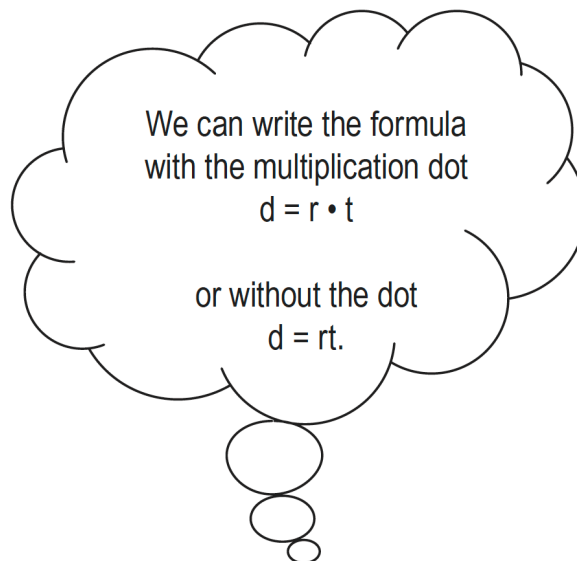
- We can use the Distance-Rate-Time formula

$$d = r \cdot t$$

to find  $d$ , the distance in feet WAL27 travels in  $t$  seconds.

- The WAL27 rate is 0.5 feet / second. So...

$$d = 0.5 t$$



### Flight NAL63

- We can use the Distance-Rate-Time formula

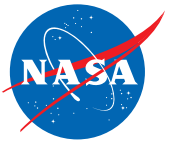
$$d = r \cdot t$$

to find  $d$ , the distance in feet NAL63 travels in  $t$  seconds.

- The NAL63 rate is 0.5 feet / second. So...

$$d = 0.5 t$$





Fill in each table.

1 Fill in the table for WAL27.

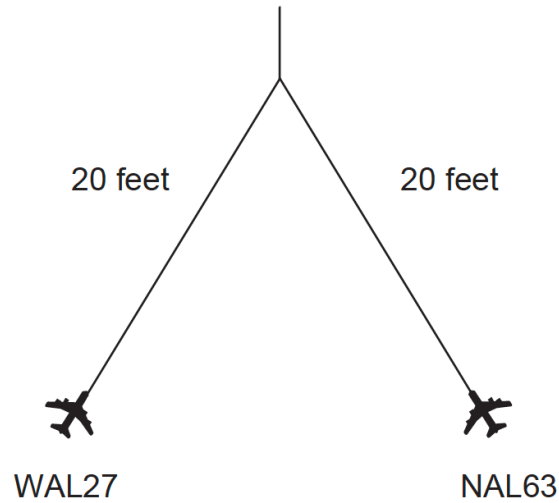
$$d = 0.5t$$

t seconds	d feet
0	
10	
20	
30	
40	

2 Fill in the table for NAL63.

$$d = 0.5t$$

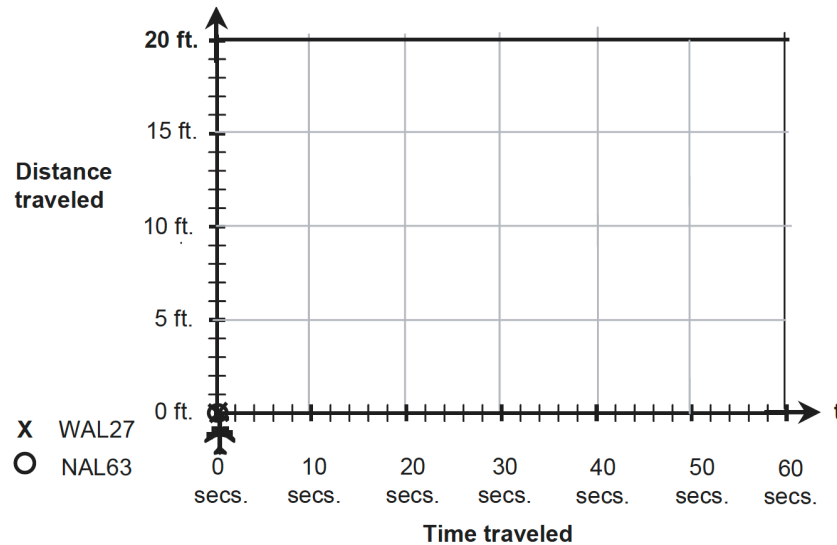
t seconds	d feet
0	
10	
20	
30	
40	



Graph each line.

3 Use an **X** to graph each point in the WAL27 table.

Use a solid line \_\_\_\_\_ to connect the points.

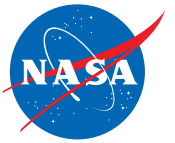


4 Use an **O** to graph each point in the NAL63 table.

Use a dotted line ..... to connect the points.

Investigator: \_\_\_\_\_

## DO THE CALCULATIONS—Graph Linear Equations



Answer the questions.

5 How many seconds did it take each plane to arrive at the point where the routes intersect?

WAL27	NAL63
<input type="text"/> seconds	<input type="text"/> seconds

6 Will the planes meet at the point where the two routes intersect?

Yes  No

7 Why or why not?

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8 If you think two planes will meet, what would you tell the air traffic controller to do to avoid a collision?

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9 Write the number that is the slope of the solid line representing WAL27.

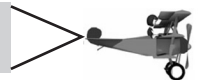
10 Write the number that is the slope of the dotted line representing NAL63.

11 What information does the slope of the line tell you about each plane?

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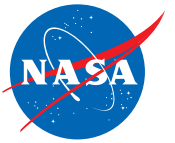
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Investigator: \_\_\_\_\_

## ANALYZE YOUR RESULTS



**Your Task:** Analyze and explain your results. Then apply what you learned to another problem.

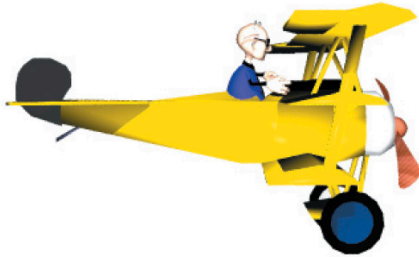


**Compare your math results with your experimental data.**



1 Use your experimental data and math results to fill in this table:

	Experiment	Mathematics
Did the planes meet where the routes meet? (Yes or No)		



2 Do your experimental and your math results match?

Yes

No



3 If No, why do you think they don't match?

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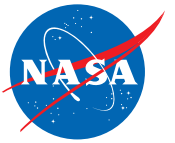


4 Which is correct? Why? \_\_\_\_\_

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Compare your math results with your experimental data.

5 Are the planes' speeds the same or different?

Same

Different

6 Do your experimental and your math results match?

Same

Different

7 Do the planes meet where the routes meet?

Yes

No

 Consider the general problem.

Two planes are flying at the same speed on two different routes.

The planes start at the same distance from the point where the routes meet.

8 The two planes will reach the point where the routes meet at

the same time.  different times.