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DERIVING EVENT THRESHOLDS AND COLLISION PROBABILITY FOR AUTOMATED CONJUNCTION ASSESSMENT AT MARS AND THE MOON

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It is well known that conjunction assessment is necessary in the Earth orbiting environment to prevent spacecraft collisions and reduce debris in orbit. Although there are many fewer spacecraft in orbit around Mars and the Moon, the number of missions to these bodies is increasing rapidly and the consequences of creating debris riskier due to the inability to track debris objects in those environments. The Multimission Automated Deepspace Conjunction Assessment Process (MADCAP) is used at the Jet Propulsion Laboratory (JPL) to perform conjunction assessment at Mars, the Moon, and Sun/Earth libration points. Previous papers have described this process and its development. Conjunction assessment of spacecraft trajectories produces many close approaches which must be screened in order to inform users when a specific event may be of concern. Probability of Collision (PC) is the main attribute used for screening conjunctions in the Earth orbiting environment. Trajectory uncertainty information in the form of covariance data is required to calculate the PC. However, covariance data is not as easily obtained for objects in the Lunar and Martian environments. Instead, various attributes of the close approach must be calculated and compared against thresholds based on typical expected orbit uncertainty. These thresholds are best informed by the individual spacecraft navigation teams who are most knowledgeable about the spacecraft's orbital uncertainties. This paper will explore the development of the thresholds used in MADCAP and provide a guideline for navigation teams to derive them from their orbit determination analysis. A method for the derivation of close approach attribute thresholds from covariance data, when it is available, is also described. Finally, collision probability calculations are examined, with exploration of methods for approximating probabilities when covariance data are limited.

INTRODUCTION

The ongoing problem with space debris in the Earth orbital regime has been well documented. There are currently no known debris fields at the Moon and Mars, but the creation of such debris fields would be highly undesirable due to the infeasibility of tracking such objects from Earth. There are a growing number of missions in the Martian and Lunar orbital environments, with many

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more planned in the near future. The creation of debris fields in these environments would greatly jeopardize future operations in those orbital regimes for both robotic and human missions. Preventing such debris fields is imperative to safely continue spacecraft operations.

NASA currently expends resources to monitor the Earth's orbital debris environment via its Conjunction Assessment and Risk Analysis (CARA) program located at the Goddard Space Flight Center (GSFC). [1, 2, 3] The Multimission Automated Deepspace Conjunction Assessment Process (MADCAP) has been used at the Jet Propulsion Laboratory to perform conjunction assessment at Mars and the Moon since 2011, and Sun/Earth libration points since 2020. The MADCAP process has been described in previous publications. [4, 5, 6] The general process is reviewed here, before entering a more detailed explanation of how thresholds are used to categorize conjunction events. The derivation of these thresholds from orbit uncertainty data delivered by spacecraft navigation teams is investigated. Updates in covariance processing capabilities and Probability of Collision (PC) calculations are also covered. Areas for future work are also identified, with some enhancements already in progress.

MADCAP GENERAL DESCRIPTION

The fundamental design concepts of MADCAP have not materially changed since their conception in 2011. A parameter file is setup for each orbital environment to be analyzed, which allows conjunction analyses in any orbital environment without modifying the underlying software. The parameters fall into a few general classes: environment (central body, coordinate system); bodies within the environment (active spacecraft, inactive spacecraft, natural bodies); thresholds used to classify conjunction events and control report generation; options for detailed reports and plots; and email lists for report participants. The main parameters that establish the orbital environment are the specification of the central body and a list of at least two spacecraft (or other bodies including natural satellites or debris).

MADCAP requires information about the trajectory of the orbiting bodies in order to find close conjunctions, and information about the uncertainty of those trajectories in order to evaluate the risk of collision. In Earth orbit, there are many ways to compute the orbits of spacecraft, including an onboard GPS receiver providing continual updates to the position, and active and passive tracking using radar, optical, and radiometric methods. Passive radar methods can also be used for inactive spacecraft, and orbital elements for all non-classified spacecraft are available through Two-Line Element (TLE) files provided by the Department of Defense. At lunar distances and beyond, however, the options are fewer, and ephemerides of these spacecraft are obtained primarily from the project's ground-based navigation teams computing their orbits using active radiometric tracking. The number of antennas that can perform this tracking are limited; the primary ones used by NASA are those of the Deep Space Network (DSN). At the time of MADCAP's conception, all spacecraft in these environments used DSN tracking data. Thus, the navigation teams for these spacecraft regularly upload updated trajectory ephemeris files to the DSN Service Preparation Subsystem (SPS) portal; a central repository convenient for the DSN and other users to easily access the orbital information. The SPS portal also allows for autonomous downloads of these files, which makes it a convenient place for MADCAP to collect these files from in order to conduct conjunction assessment. In cases where SPS is not used, trajectory information can be loaded manually or through specialized automated interfaces that can be worked on a case-by-case basis.

Ephemeris data is stored on SPS in two formats: SPICE (Spacecraft Planet Instrument C-matrix Event) SPK (Spacecraft and Planet Kernel) [10] and CCSDS (Consultative Committee for Space Data Systems) OEM (Orbit Ephemeris Message) [11]. The SPK is a standardized format which allows any user with the publicly available SPICE software to access the trajectory information. The OEM stores ephemeris information in a standardized text format that is both machine and human readable. A third format, which is occasionally used internally at JPL if ephemeris information is provided independently and not through the SPS portal, is the MONTE (Mission Design and Operations Navigation Toolkit Environment) BOA[12] (Binary Object Archive) format. MONTE is JPL's signature astrodynamics computing platform, used for space mission design and in-flight navigation, with ephemeris information stored in the binary BOA file format.

In addition to the ephemerides of the spacecraft, it is important for calculating collision risks to include the uncertainty associated with the position estimates stored in the ephemeris files. This is normally computed by the spacecraft's navigation team as part of the orbit determination (OD) process. Ideally, MADCAP would use this uncertainty estimate as it represents the best estimate of the position errors, represented by a 3x3 position covariance matrix which can be evaluated at any time along the orbit. However, because the DSN does not utilize this information, navigation teams do not include it in the files submitted to SPS. Furthermore, there is no mechanism in the SPK or BOA formats to provide the covariance – only the OEM format allows a user to include this. When the formal covariance data is not available, an approximation of the uncertainty can be computed using time-varying polynomials of orbit radial and timing uncertainties relative to the trajectory creation time. These are also specified by spacecraft navigation teams, with assistance from the MADCAP team, and used to determine the uncertainties of the state at a given conjunction. The formal covariance information is preferred because it is a more accurate representation of the true errors. However, to date, covariance information has not been consistently delivered, so polynomials are generally being used to evaluate collision risk. Future work is planned to automate the delivery of trajectory and covariance files to MADCAP outside of SPS in order to handle the increasing number spacecraft arriving in the Martian and Lunar environments which are not tracked by the DSN.

MADCAP Process

MADCAP is automatically initiated and downloads the latest ephemerides from the DSN SPS portal that were prepared by the navigation teams for tracking purposes. Two basic types of files are downloaded from SPS: "predicts grade" ephemerides and "scheduling grade" ephemerides. The predicts grade ephemerides represent the navigation team's best estimate of the spacecraft trajectory; these ephemerides are used in the generation of DSN pointing and frequency predicts. The scheduling grade ephemerides may represent a lower fidelity predict used for scheduling antenna time or a "reference trajectory". The reference ephemeris files are usually longer duration files that represent a reference, baseline, or nominal trajectory, and often include some future planned maneuvers. The reference ephemeris files are typically updated less frequently than predicts grade ephemerides.

The SPS metadata is parsed to find the most recent predicts grade SPK and OEM files available. Files are checked against those downloaded in the last run to determine if they are new. If the most recent OEM and SPK files are based on the same submission, then the OEM file will be downloaded and later checked for covariance data (position covariance matrices at specified times). The SPK file is used for conjunction assessment analysis while the OEM file is used to obtain covariance data to calculate thresholds. If no matching predicts grade OEM file is found, MADCAP will then search for a matching "ProjectUse" grade OEM file to use. The "ProjectUse" grade files are not used by the DSN for tracking purposes, but are intended for projects to be able to provide supplemental information . Alternatively, a manually specified trajectory file can be used for conjunction assessment instead of downloading the latest file from SPS. One additional ephemeris is allowed to be specified per spacecraft. This can be either the latest scheduling grade SPK file on

SPS, or a local file. Covariance data are not considered for additional ephemerides. A flow chart showing these possible trajectory and covariance file sources is shown in Figure 1.

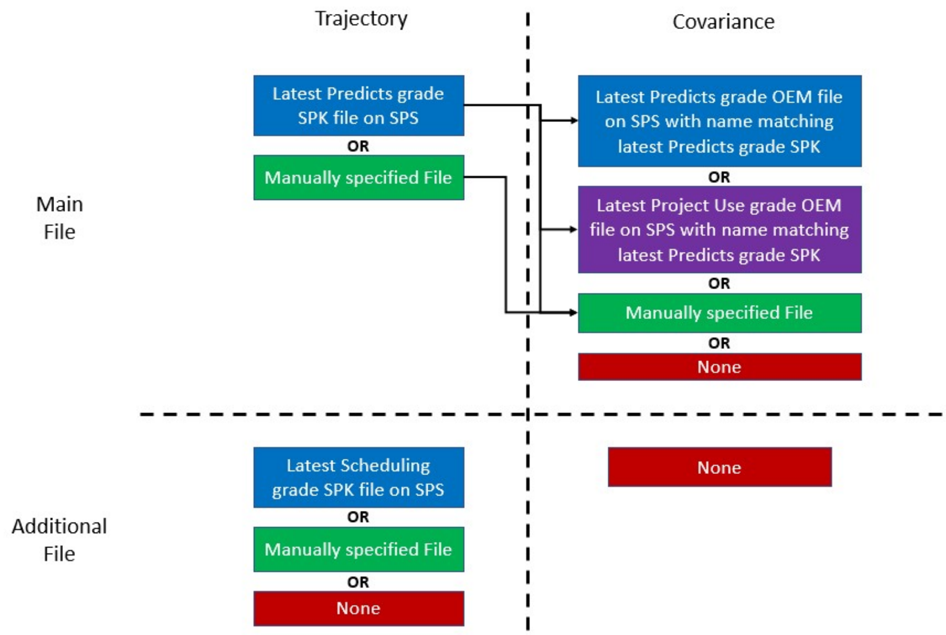

Figure 1. MADCAP Trajectory and Covariance File Sources for a Given Spacecraft

Given the ephemerides of two orbiting objects, the next step is to find the close approaches between them. MADCAP performs pairwise comparisons of the ephemerides of the bodies listed in the input parameter file. Analysis for all combinations of two objects is performed (e.g., for spacecraft A, B and C, results are calculated for A-B, A-C, and B-C). Comparisons are made over the duration of the overlapping time period of the two ephemeris files analyzed (with a reduced overlap during times when the objects are nearly coplanar or very long duration). The pairwise analysis will be performed once using the nominal ephemeris file specified for each body and then again for all possible combinations of the additional files specified (1 additional per body, thus a total of 4 analyses per pair is possible). For each analysis, a search is conducted for local minimum relative distances between the two bodies analyzed; each relative minimum is considered a "Close Approach Event". Times of the events and various orbit attributes are printed to an output table, plotted, and used to classify events into two main categories based on impact risk: 'Red' and 'All'. These are described in detail in the next section.

For the case of two non-coplanar orbits, an impact can only occur where the two orbit planes intersect. Therefore, the impact risk can be characterized by two uncertainties: that of the time at which the primary spacecraft crosses the orbit plane of the secondary spacecraft, and that of the radius from the central point of the orbit at the time of that orbit plane crossing. With a known time and radius of both spacecraft at that point, the feasibility of an impact can be determined based on whether the difference in timing and the difference in radius is within a certain number of standard deviations of the uncertainty, typically three. Given this, there are three main close approach event attributes used to analyze the events: close approach distance (CAD), orbit crossing distance (OXD), and orbit crossing timing (OXT). The CAD is the relative distance between the two bodies at the time of the closest approach (t_{CA}). The OXD is the minimum distance between the orbits of the two bodies. For non-coplanar orbits, this occurs at the two points where the orbits cross each other. Since the orbits are slowly changing over time, the orbit crossings that are within one orbital period of the t_{CA} are evaluated, and the one with the minimum OXD is reported for the close approach event. The OXD value can be represented by the following equation:

$$OXD = r_1(t_{OX1}) - r_2(t_{OX2}) \tag{1}$$

Where $r(t_{OX})$ represents the orbital radius of each body evaluated at the time it is at the orbit crossing location. The "sign" of this number is meant to convey information about which orbit is higher at the point at which the orbits cross. So, for a "Primary-Secondary" body pairing, if the orbit crossing distance is positive, then the primary orbit is above the secondary orbit at the crossing time. If it's negative, then the primary orbit is below the secondary orbit. A visual representation of CAD and OXD is shown in Figure 2, where $\vec{r}_{OX}$ represents the body's orbit radial direction at the time of the crossing. The third attribute, OXT, is defined as the difference between the times that the two bodies are at the crossing. This can be represented as:

$$OXT = t_{OX1} - t_{OX2} \tag{2}$$

If the value is positive, then the primary arrives later than secondary; if it is negative then the reverse is true.

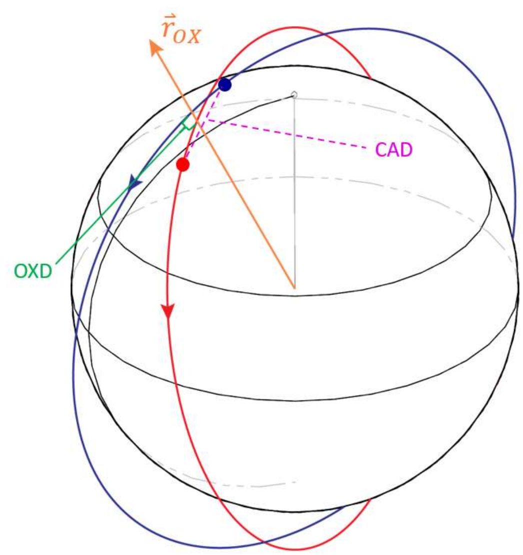

Figure 2. Main Close Approach Event Attributes

In the case the two spacecraft are in coplanar or nearly coplanar (hereafter referred to as coplanar) obits, an updated algorithm was developed to calculate minimum orbit distances. This algorithm uses a brute force method to compare distances at different points along both orbits to find the minimum distance between them. It has an increased run time, but provides more accurate results when orbits are coplanar. Thus, a check has been inserted to determine when the orbits being compared are coplanar (angular momentum vectors within a parameter specified angle). For each close approach event, if the orbits are determined to be coplanar, the updated algorithm is used to calculate minimum orbit distances and timing which are reported as orbit crossing distances and timing in both the detailed tables and summary reports. To avoid long run times for bodies which are often coplanar, the coplanar algorithm is only used for events within 60 days from the current time. Another way MADCAP avoids long run times is to not analyze natural bodies against each other (e.g. Phobos v Deimos is not analyzed unless specifically requested).

A summary report is generated for each MADCAP run which highlights important results from the conjunction analysis. The report is generated in HTML (Hyper Text Markup Language) format and sent out via email to a list of interested parties. Text messages which highlight conjunctions of concern can also be sent to specified users. More detailed plots and tables of the close approach events are also generated and can be sent out via email for each body pairing. An example summary report is included in the Appendix.

THRESHOLDS

The close approach attributes mentioned in the previous section are checked against thresholds to place events into two categories: 'Red' and 'All'. The 'Red' event list is comprised of significant, near-term events which may require further investigation, while the 'All' event list is composed of any future events which may be of interest. These thresholds are specified by the individual spacecraft navigation teams based on the criteria explained in the following sections.

'Red' Events

The category of 'Red' Events is intended to represent significant, near-term predicted conjunction events. This category includes data for all of the events that meet the 'Red' thresholds and occur within 14 days of the analysis time (defined as the time that a MADCAP run was initiated). Only pairings that involve at least one active spacecraft using the primary specified ephemeris files (predicts grade for those on SPS) are included in the 'Red' Event category. Pairings involving inactive spacecraft are not categorized as 'Red'.

As mentioned above, the three types of threshold categories are orbit crossing distance (OXD), orbit crossing timing (OXT), and close approach distance (CAD) thresholds. A conjunction event is considered 'Red' when the OXD and OXT for that event are less than the 'Red' thresholds. The CAD threshold is not considered for 'Red' Events, but the CAD value is still listed for reference in the summary report. The 'Red' thresholds correspond to the 3-sigma uncertainties as follows: OXD-radial position uncertainty, OXT-downtrack timing uncertainty. When covariance files become available, they will be used to calculate these values based on an interpolation and mapping of the position covariance matrices which bracket the event in time (see the next section for detailed description of covariance processing). The threshold values for both bodies in a pairing are calculated for each event, and the Root Sum Square (RSS) of the two values is then used as the event threshold. In the absence of formal covariance files, 'Red' thresholds are based on a quadratic fit of the 3-sigma uncertainty values (as provided by the spacecraft navigation team) as a function of time to the event. Figure 3 shows a flowchart of this process, where the threshold is represented by τ.

The polynomial equations used to calculate 'Red' threshold values for bodies without covariance data specified are described as follows:

$$\text{RED OXD Threshold} = 3\sigma_r = OXD0 + OXD1 * t + OXD2 * t^2 \quad \text{[km]} \tag{3}$$

$$\text{RED OXT Threshold} = 3\sigma_t = OXT0 + OXT1 * t + OXT2 * t^2 \quad \text{[km]} \tag{4}$$

$$\text{Where t} = Close\ Approach\ Time - Ephemeris\ Submit\ Time\ (in\ days) \tag{5}$$

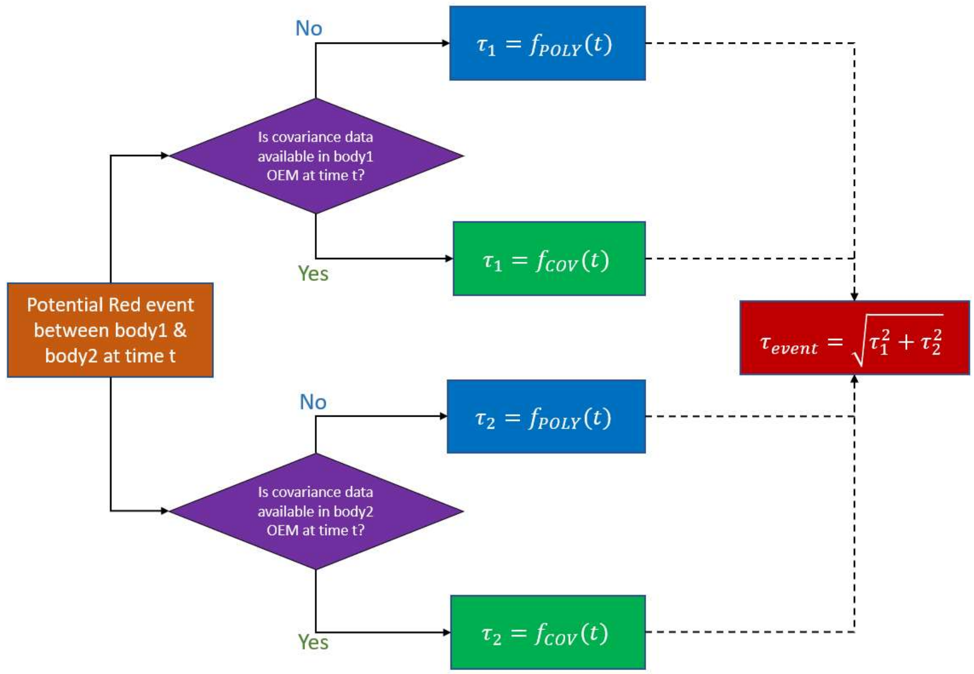

Figure 3. Threshold Sources

The six coefficients above must be specified for each body (except for inactive spacecraft which are not considered for 'Red' Events). Bodies using constant thresholds simply have zero values for OXD1, OXD2, OXT1, and OXT2. The time in days, t, stated above can be any positive real number (not limited to integer value). In order to obtain the values of these coefficients, spacecraft navigation teams must fit a second order polynomial to their 3σ worst case nominal radial (for OXD) and timing (for OXT) uncertainties over the 14 days after last available tracking data. "Worst case nominal" is meant to reflect the largest 3σ uncertainties experienced during nominal spacecraft operations (Ex: certain attitude configuration or pointing mode, not safe mode). Alternatively, spacecraft navigation teams can deliver their worst case nominal radial and timing uncertainty data to MADCAP analysts to produce a curve fit. In certain cases, the polynomial fit may be altered from "best-fit" of the data to a curve that better reflects the intentions of its use (Ex: avoid negative thresholds, better fit in first 10 days vs last 4 days). In such cases, the spacecraft navigation teams will be notified of this alteration and must approve of the final fit. Figure 4 shows four examples of worst case nominal orbit uncertainty data that was fit to a second order polynomial to obtain MADCAP thresholds.

These fits are an estimate of the uncertainty of the predicted trajectory based on the available data when the prediction was made. The uncertainty of the prediction is based on the amount of time that has passed since the last available tracking data used for the prediction, regardless of when the analysis was run. The ephemeris submit time to SPS provides a satisfactory general approximation of the data cutoff time since the actual data cutoff is not available in the ephemeris file. If a manually specified ephemeris file is used instead of one submitted to SPS, the analysis time will be used in place of the submit time.

The threshold coefficients for a body are colored blue and noted with an "*" in the summary report if they have changed since the last MADCAP run. The polynomial coefficients used are listed in a table in the report, and the threshold source for each 'Red' event is also reported in body pair order (Ex: P-P, P-C, etc. where P=polynomial and C=covariance). The coefficients are also colored blue if they have changed since the last run.

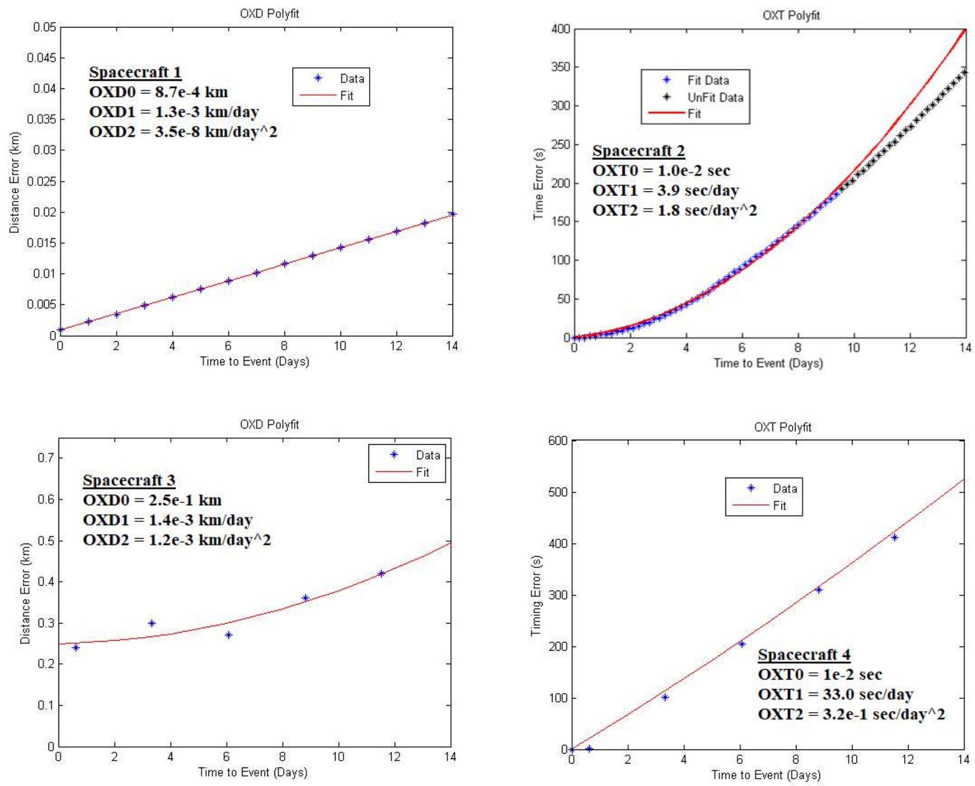

Figure 4. 2nd Order Polynomial Fits of 3σ Uncertainties of Select Mars Orbiters

‘All’ Events

The ‘All’ Events category is intended to represent any future events which may be of interest to the recipients of the summary report. It includes data for all of the events identified in the analysis that have orbit crossing and close approach distances less than the constant ‘All’ OXD and ‘All’ CAD thresholds for all pairings of active spacecraft. The OXT threshold is not used for the ‘All’ events, but the timing values are still listed in the summary report for reference. For any particular pairing of bodies, the analysis is performed for the overlap of the two ephemeris files used, from the present time through the end of the overlap, or for a specified maximum number of days (as decided by the appropriate input parameters). The notes on sign convention and epoch time in the above ‘Red’ Events section apply to the ‘All’ Events as well. The ‘All’ Events thresholds are provided by the spacecraft teams in order to cover any upcoming events they would like to see listed in the summary report. The ‘Red’ Thresholds are assumed to be tighter than the ‘All’ thresholds, such that only the events in the ‘All’ Events category are screened for inclusion in the ‘Red’ Events category.

Data for active spacecraft and natural bodies are included in this category. Data for inactive spacecraft are not included, but events from analysis using ‘additional’ trajectories are encompassed. However, the ‘additional’ trajectory events are only displayed if the event comes after the end of the time span covered by the 'main' trajectory file (see Figure 1 for source of 'main' vs 'additional' files). This is done to give precedence to the 'main' file and not confuse users with multiple listings for the same event.

DERIVING THRESHOLDS FROM COVARIANCE DATA

The polynomials described in the previous section are approximations taken from computations of expected uncertainties, and are not updated frequently, so necessarily represent a worst case nominal scenario. This causes events to be flagged that may be much safer than this approach would imply. For that reason, there has been increased interest in the delivery of covariances generated by the navigation team's OD filter as part of the standard delivery that would allow these uncertainties to be computed in a more robust way.

Characterizing the Desired Uncertainties

The first component to consider is the "timing" error, more explicitly defined as the uncertainty of the time of the event at which the primary body crosses the orbit plane of the secondary body. Typically, because it is assumed that variation in the anomaly of the orbit is the most significant error in any orbit prediction, this has been approximated by projecting the position covariance into the velocity direction, to determine the downtrack position error, and dividing by the nominal speed to convert to timing. However, this computation is an approximation, since off-axis position errors and velocity errors also affect the geometry of this crossing, though in practice the effect of this is very small. It is important to clarify that the position covariance that is projected into the velocity direction is the covariance at the epoch of the nominal plane crossing; that is, it is not constrained to the plane crossing event, which would by definition reduce the uncertainty drastically by eliminating the velocity-direction position errors from the uncertainty.

The second component is the "radial" error. More explicitly, there is interest in the uncertainty of the radius of the primary body at the time it crosses the orbit plane of the secondary body. This is calculated by projecting the position covariance at the event of the plane crossing onto the nominal radius vector. The computation from the constrained covariance is important, because the radius of the orbit will shift with the downtrack variation of the orbit according to the flight path angle, so that there is a high degree of correlation between the two components. Only when the downtrack portion of the covariance is constrained to the plane of the other orbit does the radial component correctly reflect the value that potentially poses a danger to the two bodies.

Throughout the rest of this document, the term "constrained" covariance will refer to the covariance of the state at the time-varying plane-crossing event, while the term "unconstrained" covariance will refer to the variation of the state at the epoch of the nominal plane crossing.

Proposed Algorithm

In the previous section, a tension stands out. The timing error is naturally defined by the unconstrained covariance, while the radial error is naturally defined by the constrained. Previous efforts attempted to project a single covariance, either constrained or unconstrained, onto the position and velocity vectors to determine the radial and timing uncertainties, but this method was destined for failure, either drastically increasing the radial errors or drastically reducing the timing errors. The navigation teams will not necessarily know the trajectory of the other conjunction object in advance, so constraining covariances to the plane-crossing event is not practical. Fixed-anomaly constraint mappings would be possible, though asking teams to deliver two sets of covariances does not seem practical either, and unconstrained covariances are easier to both generate and explain. Therefore, it is recommended for projects to deliver unconstrained covariances of the position and velocity in EME2000 at fixed times, which can be converted to radial and timing errors using the formulae described here.[7] EME2000 is the commonly used Earth Centered Inertial (ECI) coordinate frame defined with the Earth's Mean Equator and Mean Equinox (MEME) at 12:00 Terrestrial Time on 1 January 2000. The x-axis is pointed toward the mean equinox, the z-axis pointed toward the Earth's rotation axis (at that time), and y-axis is the cross product of x and z.

Timing Error. In general, an "event" is the time at which some function $f(\vec{\boldsymbol{x}},t) = 0$ where $\vec{\boldsymbol{x}}$ are the constant orbit parameters defining a trajectory, such as the state at a specific time. Given a nominal trajectory and the epoch that meets this constraint, defined by $\hat{\vec{\boldsymbol{x}}}$ and $\hat{t}$, the event function can be linearized about that point as

$$f\left(\hat{\vec{\boldsymbol{x}}} + \vec{\boldsymbol{x}}, \hat{t} + \Delta t\right) = f\left(\hat{\vec{\boldsymbol{x}}}, \hat{t}\right) + \frac{\partial f}{\partial \vec{x}}\Delta\vec{\boldsymbol{x}} + \frac{\partial f}{\partial t}\Delta t = 0 + \frac{\partial f}{\partial \vec{x}}\Delta\vec{\boldsymbol{x}} + \frac{\partial f}{\partial t}\Delta t \tag{6}$$

Solving this linearized form for when it equals zero yields the partial derivative of the event time relative to the parameters:

$$\frac{\partial t}{\partial \vec{x}} = \frac{\Delta t}{\Delta \vec{x}} = -\frac{\partial f/\partial \vec{x}}{\partial f/\partial t} \tag{7}$$

Now, we can define the function for the plane crossing by recognizing that the plane crossing event is defined by the epoch at which $\vec{\boldsymbol{r}}_1$, the position of the primary body, is perpendicular to $\vec{\boldsymbol{h}}_2$, the angular momentum vector of secondary body, which is assumed to be constant for the duration of interest for a given conjunction (so that we avoid considering its time derivatives). Defining the function and taking the derivative gives the partial derivative of the event time relative to the state at the nominal epoch of the plane-crossing

$$f_{OX} = \vec{\boldsymbol{r}}_1 \cdot \vec{\boldsymbol{h}}_2 \tag{8}$$

$$\frac{\partial t_{OX}}{\partial \vec{\boldsymbol{r}}_1} = -\frac{\partial f_{OX}/\partial \vec{\boldsymbol{r}}_1}{\partial f_{OX}/\partial t} = -\frac{\vec{\boldsymbol{h}}_2}{\vec{\boldsymbol{v}}_1 \cdot \vec{\boldsymbol{h}}_2} \tag{9}$$

where $\vec{\boldsymbol{v}}_1$ represents the velocity vector of the primary body. Applying this transform to the position covariance gives the uncertainty of the timing event:

$$\sigma_{t1} = \frac{\sqrt{\vec{\boldsymbol{h}}_2^T \boldsymbol{P}_1 \vec{\boldsymbol{h}}_2}}{\vec{\boldsymbol{v}}_1 \cdot \vec{\boldsymbol{h}}_2} \tag{10}$$

where $\boldsymbol{P}_1$ represents the unconstrained position covariance of the primary body. The timing error can be understood as the unconstrained covariance projected onto the normal vector of the orbital plane, scaled by the velocity projected onto that same normal vector. When the largest principal axis of the covariance is also aligned with the velocity vector, this is equivalent to our previously-defined approximation. Note that this can easily be computed from the unconstrained covariance and the nominal trajectories of the two spacecraft.

Radial Error. The radial error can be understood as the constrained position covariance projected onto the position vector direction. In order to compute the transition matrix from the unconstrained state to a constrained state:

$$\frac{\partial \vec{\boldsymbol{x}}_c}{\partial \vec{\boldsymbol{q}}} = \frac{\partial \vec{\boldsymbol{x}}_u}{\partial \vec{\boldsymbol{q}}} + \frac{\partial \vec{\boldsymbol{x}}_u}{\partial t}\frac{\partial t_{event}}{\partial \vec{\boldsymbol{q}}} \tag{11}$$

where $\vec{\boldsymbol{x}}_c$ represents the constrained state, $\vec{\boldsymbol{x}}_u$ represents the unconstrained state. Assuming that $\vec{\boldsymbol{q}}$ is the unconstrained state at the epoch of the node crossing $\vec{\boldsymbol{x}}_u$, we can get a transition matrix from the unconstrained to the constrained state:

$$\frac{\partial \vec{x}_c}{\partial \vec{x}_u} = \frac{\partial \vec{x}_u}{\partial \vec{x}_u} + \frac{\partial \vec{x}_u}{\partial t}\frac{\partial t_{OX}}{\partial \vec{x}_u} = \boldsymbol{I}_{6x6} + \begin{bmatrix} \vec{\boldsymbol{v}}_1 \\ \vec{\boldsymbol{a}}_1 \end{bmatrix} \begin{bmatrix} \frac{-\vec{\boldsymbol{h}}_2^T}{\vec{v}_1 \cdot \vec{\boldsymbol{h}}_2} & \boldsymbol{0}^T \end{bmatrix} \tag{12}$$

where $\boldsymbol{I}_{6X6}$ represents the six by six identity matrix. The acceleration, $\vec{\boldsymbol{a}}_1$, is multiplied by zero and has no impact. From this transformation, the radial uncertainty can be computed by projecting the constrained position covariance onto the radial direction, for a final formulation (eliminating zero terms) where:

$$\sigma_{r1} = \sqrt{\vec{\Phi}\boldsymbol{P}_1\vec{\Phi}^T} \tag{13}$$

$$\text{where } \vec{\Phi} = \frac{\vec{\boldsymbol{r}}_1^T}{r_1}\left(\boldsymbol{I}_{3x3} - \frac{\vec{v}_1\vec{\boldsymbol{h}}_2^T}{\vec{v}_1 \cdot \vec{\boldsymbol{h}}_2}\right) \tag{14}$$

Correlations. While it is tempting, and mathematically straightforward, to compute the correlations of these values and potentially compute the Mahalanobis norm to use as a threshold, that is not recommended at this time. Because the radial and timing errors become highly correlated after a few orbits, the semiminor axis of this joint covariance is very small. JPL navigation computes these covariances using a linearized state transition matrix propagation, which leads to small errors in these values, as can be demonstrated with simple two-body propagation Monte Carlo simulations. With small correlations this would be fine, but as the correlations grow, small linearization errors lead to large areas that do not overlap between the 3σ ellipses represented by linearized and true covariances. Therefore, while we trust the 'corners' of the bounding rectangle of the covariance in the radial/timing space to be accurate to within a few percent, the width and orientation of the ellipse within that rectangle are extremely sensitive and cannot be accurately used for computation.

Results. The radial and timing errors were computed using these equations for a Mars orbiter over a three-week period of time with results shown in Figure 5. The computed errors are compared to those obtained via the polynomial fits of uncertainty described in the previous section showing that the polynomial fit used was slightly conservative in this case.

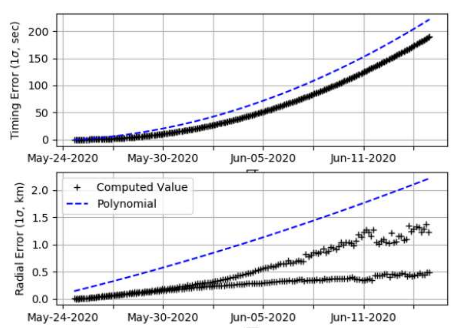

Figure 5. Covariance derived timing and radial errors compared to polynomials for a Mars orbiter

Uncertainty in Secondary Orbit Plane. The timing and radial error equations above were derived without considering the uncertainty in the angular momentum vector of the secondary spacecraft,

$\vec{\boldsymbol{h}}_2$. The uncertainty in angular momentum for orbiters is typically very small, but can be accounted for if the information is available. However, covariance data is currently still rarely reported to MADCAP and information on orbit normal uncertainty has not historically been requested of missions by the MADCAP team. Testing based on data from a current Mars orbiter has shown that induced uncertainties in the primary spacecraft's radial error by the secondary body's normal error can be up to a few hundred meters in certain cases (induced timing uncertainty is much smaller, sub-second level). Though not negligible, these errors represent a small fraction of the overall uncertainty and are currently unaccounted for by MADCAP. Future methods for inclusion could involve obtaining the angular momentum uncertainty of the secondary body from covariance data in OEM files, requesting a constant worst case normal uncertainty from spacecraft navigation teams, or assuming a constant worst case normal uncertainty for all bodies analyzed by MADCAP. These methods are still under consideration for future MADCAP enhancements.

Interpolation of Covariances. In order to obtain the unconstrained covariances at the close approach time from data delivered at arbitrary intervals, MADCAP must interpolate the covariance data delivered via OEM file. A method based on the two-body state transition matrix (STM) mapping is used to perform this interpolation. The covariances at epochs which bracket the close approach time are linearly mapped to this time using the two-body STM:

$$\boldsymbol{P}(t_e) = \boldsymbol{\Psi}_{ei}\boldsymbol{P}(t_i)\boldsymbol{\Psi}_{ei}^T \tag{15}$$

Where $\boldsymbol{\Psi}_{\boldsymbol{ei}}$ represents the STM calculated to transform the states from the time being interpolated from, t_i, to the event time, t_e:

$$\boldsymbol{\Psi}_{ei} = \frac{d\vec{\boldsymbol{x}}(t_e)}{d\vec{\boldsymbol{x}}(t_i)} \tag{16}$$

The weighted average of these mappings is then computed based on the relative proximity of the entry times to the close approach time:

$$\boldsymbol{P}(t_e) = \omega\boldsymbol{P}(t_e)_a + [1-\omega]\boldsymbol{P}(t_e)_b \tag{17}$$

Where

$$\omega = (t_e - t_a)/(t_b - t_a) \tag{18}$$

And t_a and t_b represent the interpolated from times.

This method was compared to three other candidate interpolation methods for two orbiters in the Mars environment: linear interpolation of EME2000 cartesian covariances, linear interpolation of covariances in a velocity-aligned frame (y-axis pointed in the spacecraft velocity direction, z-axis pointed in the angular momentum direction, and x-axis defined by the cross-product of y and z), and linear interpolation of EME2000 covariances of equinoctial conic elements. Comparisons were made to truth values at known plane-crossing events, interpolated from covariances generated at time steps from 1 minute to 30 minutes. The maximum error in computed 1σ uncertainties for conjunctions involving the analyzed orbiters are shown in Table 1 for covariances reported at these intervals. The STM mapping interpolation performs best among the tested methods and should hold valid across a range of orbits and missions. The two-body gravitational acceleration used in this method is dominant over other forces by orders of magnitude, and the interpolation smooths over small differences associated with higher fidelity models. If this approximation proves insufficient for certain time periods or missions, the performance can be improved by reducing the time step. MADCAP does not require covariance data to be delivered in fixed time steps, so covariance data can be reported at finer spacing for different parts of an orbit if needed.

Table 1. Maximum Interpolation Error in 1σ Uncertainty

Interpolation Method	Error Type	1-min Interval	2-min Interval	5-min Interval	10-min Interval	30-min Interval
EME2000 Cartesian	Radial (km)	1.3e+01	2.8e+01	6.9e+01	1.4e+02	4.1e+02
	Timing (s)	5.1e-02	2.1e-01	1.3e+00	5.2e+00	4.4e+01
Velocity-Aligned	Radial (km)	1.0e-04	1.0e-04	2.0e-03	6.0e-03	5.9e-02
	Timing (s)	7.0e-03	3.0e-02	1.9e-01	7.6e-01	7.4e+00
EME2000 Conic	Radial (km)	3.0e-03	1.1e-02	7.2e-02	2.9e-01	1.1e+00
	Timing (s)	7.0e-03	1.8e-02	1.1e-01	3.9e-01	1.3e+00
EME2000 STM Map	Radial (km)	2.7e-06	2.7e-06	5.1e-05	2.8e-06	1.8e-05
	Timing (s)	1.5e-07	1.9e-07	1.1e-04	2.3e-06	2.0e-05

The above equations were derived with the assumption of plane-crossing events and do not hold in the case of coplanar orbiting bodies. A separate method was derived for use in such cases, the details of which are described in Reference 7. The performance of this method compares reasonably well with errors obtained via the plane-crossing method described above for non-coplanar events analyzed for a specific set of Mars orbiters. Future analysis is planned to ensure acceptable performance across a range of orbital environments and uncertainties.

PROBABILITY OF COLLISION

In the past, collision probability calculations were not performed for a few reasons. The main factor in not calculating PC is that covariance data has not been provided for most bodies in the Martian and Lunar environments. This is slowly changing as missions will begin to deliver covariance information for use in MADCAP. However, this covariance data is self-reported and thus of unknown methodology. It is possibly less accurate than the radar-based uncertainty information available for Earth orbiters. For example, a mission reporting overly conservative covariance data could lead to artificially low PC values. Nevertheless, mission teams are the most knowledgeable of their own uncertainties and will be more likely to accurately report information when the usage is explained. A final factor in the previous absence of reporting PC values is to avoid collision avoidance decisions being made based on one number without considering the larger context of the conjunction being analyzed. However, this can be prevented by clarifying to users that the PC is to be used as an additional conjunction event metric, and not the sole determining number to base decisions on.

MADCAP now has the capability to report PC values when the appropriate covariance data is available from at least one of the bodies involved in a conjunction pair. In this section, we describe how PC is computed for the cases where covariance data is available for both spacecraft or for only one spacecraft (PC will not be computed if neither spacecraft has covariance information). The flowchart in Figure 7 shows the process followed. We will first describe how covariance information is obtained from polynomials in the case that the formal covariance matrix is not available.

Constructing Covariance from Polynomials

When covariance data is not available, a pseudo-covariance matrix can be constructed from the 'Red' threshold polynomials previously described and used for PC calculations. This can provide a more realistic probability than that for the "worst case" which can be artificially large, especially for large miss distances. However, it may conflict with the primary purpose of the polynomials: to be used as thresholds. Missions tend to choose conservative polynomials to reflect their worst case nominal uncertainties so that any conjunction events of concern will be flagged 'Red'. Yet, artificially conservative covariances lead to artificially low PC values. To avoid inaccurate covariance data produced via polynomials, the default MADCAP setting is to not convert the polynomials into pseudo-covariance data since not reporting PC is considered better than reporting inaccurate information. The conversion of polynomials into pseudo-covariances can be turned on for each body independently. This allows the feature to only be used when the corresponding polynomials are known to provide more accurate PC information than the alternative.

The pseudo-covariance matrix is constructed by using Equations (19-21) to compute the position uncertainties in the body velocity aligned frame. This frame is composed of a y-axis defined in the body velocity direction, z-axis in the angular momentum vector direction, and x-axis defined by the cross product of y and z (x - crosstrack, y - downtrack, z - normal). The uncertainty in the normal direction is assumed to be reasonably approximated by that in the crosstrack direction. This may not hold for all missions, in which case the pseudo-covariance matrix should not be used. Future MADCAP enhancements could allow for direct use of polynomial orbit-normal uncertainties provided by the spacecraft navigation teams.

$$3\sigma_{CROSSTRACK} = OXD0 + OXD1 * t + OXD2 * t^2 \tag{19}$$

$$3\sigma_{DOWNTRACK} = [OXT0 + OXT1 * t + OXT2 * t^2]\bar{\bar{v}} \tag{20}$$

$$\sigma_{NORMAL} = \sigma_{CROSSTRACK} \tag{21}$$

The position errors can then be used to create a diagonal pseudo-covariance matrix as follows:

$$\boldsymbol{P} = \begin{bmatrix} \sigma^2_{Crosstrack} & 0 & 0 \\ 0 & \sigma^2_{Downtrack} & 0 \\ 0 & 0 & \sigma^2_{Normal} \end{bmatrix} \tag{22}$$

This pseudo-covariance matrix can then be rotated into the Collison Frame and added to the position covariance matrix from the secondary body. The Collison Frame is defined by x-axis pointed in the relative miss direction (body1 to body2), y-axis pointed in the relative velocity direction, and z-axis defined by the cross-product of x and y. The x-z plane is known as the "collision plane".

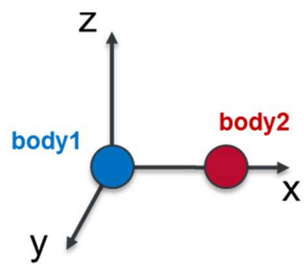

Figure 6. Collision Frame

Calculating Probability of Collision

In the case where covariance data for both spacecraft are available (either the formal or pseudo-covariance constructed as described above), the covariance matrices and hard body radii from both bodies are combined through addition. Depending on the source of the covariance data, the matrices can either be added together in EME2000 frame and then rotated into the Collison Frame (see Figure 6), or rotated into the Collision Frame and then combined. Once in the Collision Frame, the problem is reduced into the 2 dimensions of the collision plane. This allows for the use of Foster's method to calculate the probability of collision (described in detail in the referenced paper). [8]

If covariance data is only available for one of the bodies, then the Frisbee Method is used to calculate the worst case 2D collision probability. [9] An upper bound on the PC can be obtained by assuming the 1σ error ellipse for the missing body is a degenerate ellipse (straight line) with semi-major axis equal to the miss distance. When linearly combined with the known covariance in the 2D collision frame (x-z), this results in the worst case PC:

$$\boldsymbol{P}_{COMBINED} = \boldsymbol{P}_1 + \begin{bmatrix} x_0^2 & 0 \\ 0 & 0 \end{bmatrix} \tag{23}$$

Where x_0 represents the miss distance. This problem is not well formed for large miss distances, $(x_0 \gg \sigma_{x1})$. MADCAP checks for this condition and does not calculate a PC if the miss distance is too large. The current value used for "large miss distance" in MADCAP is $(x_0 > 10{,}000\sigma_{x1})$. This value was arrived at heuristically with future work necessary to determine more specifically when the assumption breaks down.

The probability of collision is printed in the summary report for 'Red' events as an additional conjunction parameter. The probability value is followed by a descriptor pair of letters which explain what method was used to calculate the probability. If covariance data is available for both bodies, then Foster's Method is used to calculate a 2D collision probability value, and "C-C" will be displayed. If covariance data is only available for one of the bodies, then the Frisbee Method is used to calculate the worst case 2D collision probability, and "C-N" or N-C" will be displayed. The order corresponds to the order the pair is specified in the first column, with "C" indicating that covariance data was available and used, while "N" indicates that no data was available. If no covariance data is available for either body, then the probability of collision cannot be calculated and "No Data" is displayed. If a polynomial derived covariance matrix is used, the letter "P" will be displayed in place of "C". This process is shown graphically in Figure 7.

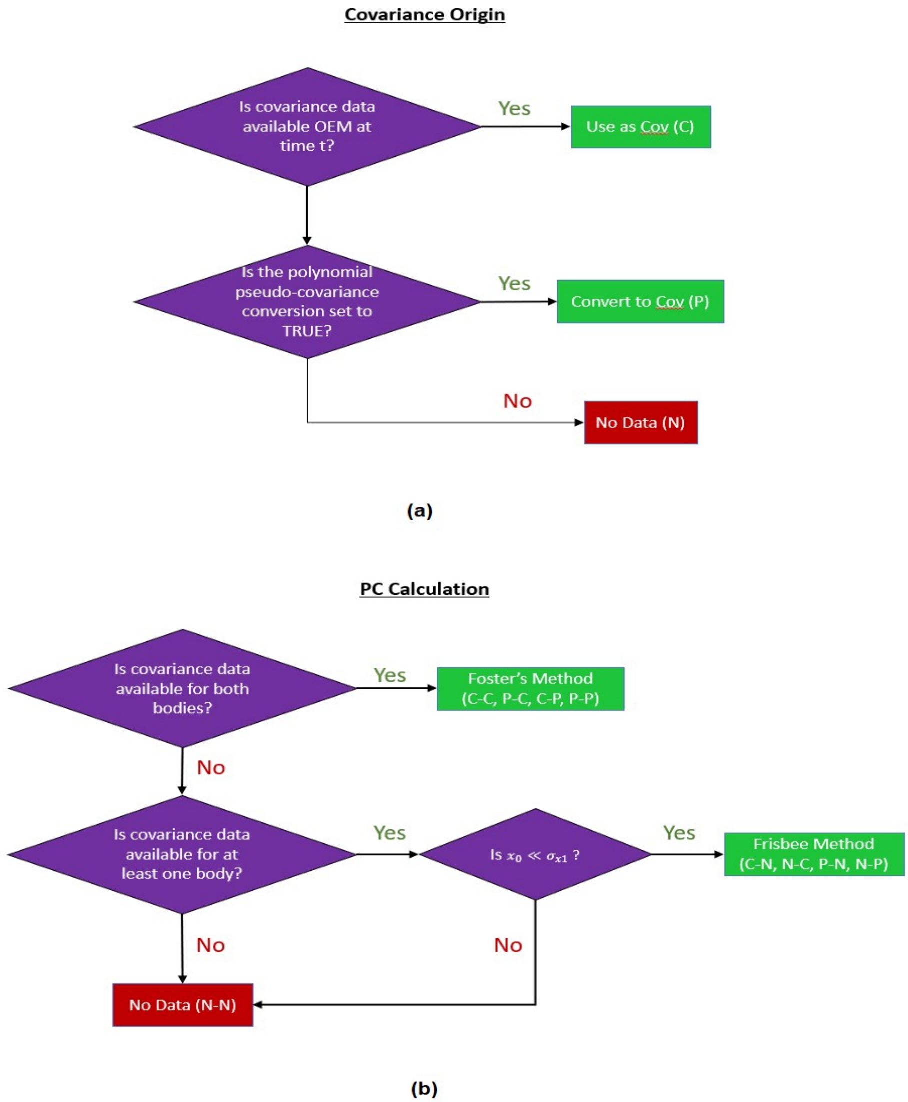

Figure 7. Flow Charts for a) Covariance Origin and b) PC Calculation

CONCLUSION

This paper has presented an update to the automated process used at NASA/JPL to conduct conjunction assessment at Mars, the Moon, and Sun-Earth Libration points. The process of deriving thresholds and collision probabilities from orbit uncertainties to screen conjunctions in these environments has been thoroughly described. The MADCAP screening and reporting process can only work as well as the inputs used to generate these thresholds and probabilities. The methods described here for threshold formation will enable current and future spacecraft navigation teams to provide the relevant data required for effective conjunction screening. This will ensure a safer orbital environment for all spacecraft operating in these increasingly crowded environments.

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APPENDIX: EXAMPLE MOON SUMMARY REPORT

An example summary report is displayed below (the ‘All’ table has been abridged for length):

Analysis Time: 2021-12-31 16:47:32 UTC

RED Threshold Updates:	0
ALL Threshold Updates:	0
Ephemeris Updates:	4

Conjunction Assessment Bodies and Types

Body	Name	Type
1	LRO	Active
1r	LRO	Active/Reference
2	ARTEMIS-P1	Active
2r	ARTEMIS-P1	Active/Reference
3	ARTEMIS-P2	Active
3r	ARTEMIS-P2	Active/Reference
4	CH2O	Active
5	Ouna	Inactive
6	CH1	Inactive

Red (Conjunction Data < 'Red' Thresholds and Event < 14 days from Analysis Time)

Bodies	OXD value/limit (km)			OXT value/limit (sec)			CAD value/limit (km)			Collision Probability	CA Epoch (UTC-SCET)
1-4	-0.4	3.2	4P	3.2	8.3	1P	163.4	-	-	no data	2022-01-13 07:31:48

1-4	-0.4	3.2	4P	3.2	8.3	1P	4.1	-	-	no data	2022-01-13 08:31:26

All (Conjunction Data < 'All' Thresholds for <= 100 days)

Bodies	OXD (km)	OXT (sec)	CAD (km)	CA Epoch (UTC-SCET)
1-2	-152.5	-212.8	305.8	2022-01-03 10:52:35
1-4	-9.7	367.1	472.8	2022-01-03 23:57:37
1-4	-9.4	307.8	485.5	2022-01-04 00:55:55
1-4	-9.4	307.8	396.8	2022-01-04 01:55:18
1-4	-9.2	248.3	409.1	2022-01-04 02:53:36

Notes

OXD means "Orbit Crossing Distance". OXT means "Orbit Crossing Timing". CAD means "Close Approach Distance".

Data for active spacecraft and natural bodies are displayed in the tables above. Data for inactive spacecraft are not displayed, but they are available in the conjunction metric tables and plots, which have been stored in the output directory listed below. Data for reference trajectories are not considered for Red events, but are considered in the All section for events not covered by the predicts file. Reference trajectories use the same thresholds as the nominal trajectories.

The note after the probability value listed for RED events refers to the source of covariance data used for the probability calculation, with the following definitions:

C - Covariance data provided by the mission in OEM format.
P - A covariance converted from the RED threshold polynomial coefficients listed below.
N - No covariance data, worst case covariance assumed for this body.
No Data - Probability could not be calculated due to a lack of covariance data from both bodies.

For more information, please see the point of contact listed below.

Analysis time:	2021-12-31 16:47:32 UTC
Active spacecraft:	LRO, ARTEMIS-P1, ARTEMIS-P2, CH2O
Natural bodies:	None
Inactive spacecraft:	Ouna, CH1
Output directory:	/nav/home/jplmdnav/MADCAP/Moon/archive
Point of contact:	MADCAP_Moon@jpl.nasa.gov
MADCAP build:	3.1.2

Red Thresholds -- Polynomial Coefficients

Body	Name	OXD0 (km)	OXD1 (km/t)	OXD2 (km/t^2)	OXT0 (sec)	OXT1 (sec/t)	OXT2 (sec/t^2)
1	LRO	0.1500	0.0125	0.0005	1.8750	0.2671	0.0184
2	ARTEMIS-P1	0.1370	-0.0024	0.0089	0.0100	2.8154	0.0045
3	ARTEMIS-P2	0.1370	-0.0024	0.0089	0.0100	2.8154	0.0045
4	CH2O	0.0000	0.2509	0.0000	0.0000	0.1490	0.0005

Red OX Distance Threshold = OXD0 + (OXD1 * t) + (OXD2 * t^2) [km]
Red OX Timing Threshold = OXT0 + (OXT1 * t) + (OXT2 * t^2) [sec]
where t = CA Epoch - Ephemeris File Submit Time (in days)

Red thresholds are based on 3-sigma values. Thresholds listed as "P" are based on a quadratic fit of the 3-sigma values as a function of time to the event. The polynomial coefficients used are listed in the table above. Lines for coefficients which have been updated since the last run are colored blue, and each line's body is marked with an "*". Thresholds listed as "C" are based on 3-sigma covariance data provided by the mission.

All Thresholds -- Constants

Body	Name	OXD (km)	CAD (km)
1	LRO	1	40
2	ARTEMIS-P1	500	500
3	ARTEMIS-P2	500	500
4	CH2O	500	500

All OX Distance Threshold = OXD
All CA Distance Threshold = CAD

All thresholds are always constants. The constants used are listed in the table above. Lines for constants which have been updated since the last run are colored blue, and each line's body is marked with an "*".

Ephemerides

Body	Ephemeris	Submitted	Begin	End
1*	28day_20211231_01.bsp	2021-12-31 12:34:41 UTC	31-DEC-2021 00:00:00 UTC	28-JAN-2022 00:00:00 UTC
1r	558day_20211215_01.bsp	2021-12-15 13:39:10 UTC	15-DEC-2021 00:00:00 UTC	26-JUN-2023 00:00:00 UTC

2*	192.THEMIS_B.SHORT_TERM.2021_365.oem.bsp_V0.1	2021-12-31 10:41:59 UTC	31-DEC-2021 00:00:00 UTC	30-JAN-2022 00:00:00 UTC
2r	192.THEMIS_B.LONG_TERM.2021_104.oem.bsp_V0.1	2021-04-14 17:57:45 UTC	14-APR-2021 00:00:00 UTC	13-APR-2027 00:00:00 UTC
3*	193.THEMIS_C.SHORT_TERM.2021_365.oem.bsp_V0.1	2021-12-31 10:43:08 UTC	31-DEC-2021 00:00:00 UTC	30-JAN-2022 00:00:00 UTC
3r	193.THEMIS_C.LONG_TERM.2021_104.oem.bsp_V0.1	2021-04-14 18:01:40 UTC	14-APR-2021 00:00:00 UTC	13-APR-2027 00:00:00 UTC
4*	ISRO-CH2-2021-12-29-OD893-365-v1.xsp.bsp	2021-12-31 12:08:16 UTC	29-DEC-2021 22:00:00 UTC	19-JAN-2022 00:00:00 UTC
5	ouna_191201_230101_150608_SMM0710031456-jpl-ekl.bsp	Analysis Time	01-DEC-2019 00:59:23 UTC	31-DEC-2022 23:58:23 UTC
6	traj_ch1_010920-011022.bsp	Analysis Time	31-AUG-2020 23:58:50 UTC	30-SEP-2022 23:58:50 UTC

Ephemeris files for the bodies analyzed are listed in the table above. Lines for files which have been updated since the last run are colored blue, and each line's body is marked

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