National Aeronautics and Space Administration

STEM LEARNING:

Advanced Air Mobility: Flight Control Math 5 (Line Equation(s) and Point of Intersection) Student Guide

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DETECT AND AVOID

NASA is leading the nation to quickly open a new era in air travel called Advanced Air Mobility, or AAM. The vision of AAM is that of a safe, automated, and affordable air transportation system for passengers and cargo in urban and rural locations.

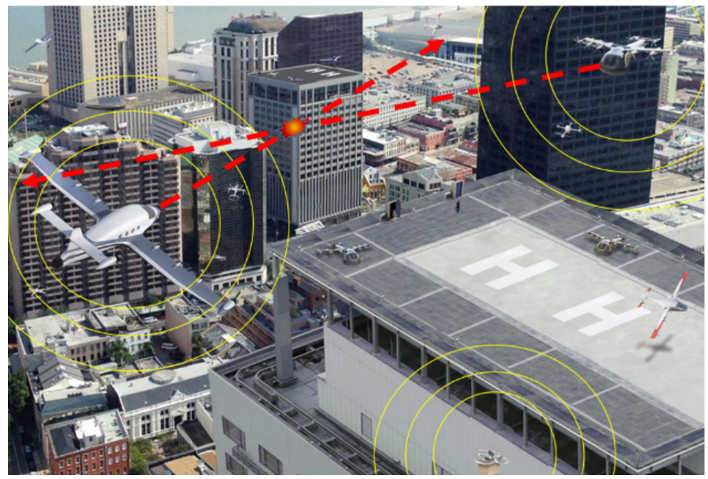

Figure 1. While flying, UAVs need to detect other aircraft and avoid them.

Many of the Unmanned Aerial Vehicles, or UAVs, do not have an onboard pilot. In order to stay safe, these autonomous aircraft monitor where they are at all times. At the same time, they need to know where other aircraft are and what these aircrafts' projected flightpaths are. This information enables the UAV to detect if there is a danger of colliding with another aircraft. If there is, the flightpath of the UAVs can be adjusted to avoid the collision. This process is called Detect and Avoid (DAA).

A microcomputer on the UAV that controls the UAV's flight also controls the DAA process. As a UAV flies, the microcomputer determines its current location and intended course. This information is broadcast to other UAVs located nearby so they can calculate whether there is a danger of collision. If there is no danger, the UAVs continue along their current flightpath. If the flightpaths will be too close to one another, the microcomputer determines which UAV has priority and adjusts the course and speed accordingly.

Several mathematical concepts are required for DAA to work. First, the microcomputer calculates its own course. Mathematically, this means that it uses two points to find an equation that describes its course. Next, the microcomputer uses the equation for its course and the equation for another UAV's course to determine whether they will meet at a point. This is done by solving a system of two equations. Finally, the microcomputer uses the speeds of the two UAVs to figure out what time each will arrive at the meeting point of their courses. If they both arrive there simultaneously, there is a collision risk.

FINDING THE EQUATION OF A LINE

For this activity, we will assume that flightpaths are linear, meaning that the flightpath can be expressed using the slope-intercept formula:

$$y = mx + b$$

If two points are given, the first thing to do is to find the slope using the slope formula:

$$m = \frac{(y2-y1)}{(x2-x1)}$$

The calculated slope and the (x) and (y) coordinates of one of the points are substituted in slope-intercept form and the y-intercept (b) is calculated.

Finally, the slope (m) and y-intercept (b) are substituted into slope-intercept form. This gives you the equation for the line.

Example: Find the equation for a line from (-5, -5) to (7, 19):

$$y = mx + b \quad \rightarrow \quad 19 = (2)(7) + b = 14 + b$$

Substitute the slope and one of the coordinates:

$$5 = b$$

Substitute the slope and one of the coordinates:

$$y = 2x + 5$$

FINDING THE POINT OF INTERSECTION FOR TWO LINES

Once you have the equation for the flightpath of each UAV, the next step is to determine where the two flightpaths intersect. Thinking of the two linear equations as a system of equations, the point of intersection can be found by solving the system of equations. Solving a system of equations determines a point that lies on both lines. If such a point exists, that is the point of intersection.

Example: One UAV has the equation $y = 2x + 5$ and the other has the equation $y = -x + 8$

This can be solved using several methods, but the easiest is substitution since the slope-intercept format already has the equation solved for (y).

$$y = 2x + 5 \text{ and } y = -x + 8$$

$$2x + 5 = -x + 8$$

Add (x) to each side and subtract 5 from each side:

$$3x = 3$$

Divide each side by 3:

$$x = 1$$

Substitute this into one of the original equations to find the y-coordinate of the point of intersection:

$$y = -1 + 8 = 7$$

The point of intersection is: (1, 7)

Important notes:

- Once the point of intersection is found, check to see if the (x) and (y) values are in the range of the UAVs flightpath. If it is out of the range of either variable, the two UAVs will not meet.

  Example: A UAV takes off from (-8, -2), lands at (3, 9), and the point of intersection is (5, 5). Notice that the (x) value is higher than any point the UAV will ever fly to. This means that there is no collision risk because the UAV lands before getting to what would be the point of intersection.

- If there is no solution, the UAVs will not intersect because they are on parallel courses.

DISTANCE FORMULA

Once it is determined that there is a point of intersection, the microcomputer calculates how long it will be until each UAV reaches that point. If they reach there at (or very close to) the same time, there is a collision risk. If they reach the point at significantly different times, there is no collision risk. The distance from the starting point to the point of intersection and the speed being traveled are required to determine the arrival time.

The distance formula can be used to determine the distance between two points on a graph.

Distance between two points: $P_1 (x_1, y_1)$ and $P_2 (x_2, y_2)$

$$distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Determine the distance between the starting point (-5, -5) and the point of intersection (1, 7) where each point on the graph represents 0.5 kilometers (km).

$$distance = \sqrt{(1-(-5))^2 + (7-(-5))^2}$$

$$distance = \sqrt{6^2 + 12^2}$$

$$distance = \sqrt{36 + 144}$$

$$distance = \sqrt{180}$$

$$distance = 13.4\ squares$$

Convert to km: $(13.4)(0.5) = 6.7\ km$

SPEED FORMULA

The speed formula is used to determine the distance an object moves in a certain amount of time. To calculate the speed, divide the distance by the time.

$$speed = \frac{distance}{time}$$

By mathematically rearranging the formula, you can determine the formula for calculating the time it takes to reach the intercept point.

$$time = \frac{distance}{speed}$$

Example: The UAV travels from the starting point (-5, -5) to the point of intersection (1, 7), which is a distance of 6.7 km at a speed of 1.3 km per minute (km/min). How long does it take to arrive?

$$time = \frac{6.7\ km}{1.3\ km/min} = 5.2\ min$$

The other UAV starts at (8, 0) and travels to the point of intersection (1, 7) at a speed of 1.5 km/min. Each point on the graph represents 0.5 km.

$$distance = \sqrt{(8-1)^2 + (0-7)^2}$$

$$distance = \sqrt{7^2 + (-7)^2} = \sqrt{49+49} = \sqrt{98} = 9.9\ squares$$

Convert to km: $(9.9)(0.5) = 5\ km$

$$time = \frac{5\ km}{1.5\ km/min} = 3.3\ min$$

Since they arrive at the point of intersection 1.9 minutes apart, there is no collision risk.

STUDENT ACTIVITY

- A coordinate system has been set up over the city to help control UAV flights.
- The origin for the coordinate system is a point in the city, as shown in figure 2.
- All distances should be in kilometers (km) and all times should be in minutes (min).
- All speeds are in kilometers per minute (km/min).
- Each square of the graph is equal to 0.5 km.

There are three UAVs that all leave at the same time. They all travel at **1.2 km/min**. Given the speed of the UAVs, arriving at a point **within 30 seconds of each other** is considered a collision risk.

The three UAVs starting and ending points are:
- UAV 1: starts at (-10, -5) and ends at (12, 6);
- UAV 2: starts at (1, -4) and ends at (7, 8);
- UAV 3: starts at (6, 7) and ends at (-18, -5).

PART 1: FIND THE EQUATIONS FOR EACH LINE

Calculate the equation, in slope-intercept format, for each of the three UAVs. Show your work.

1. UAV 1: (-10, -5) → (12, 6)

2. UAV 2: (1, -4) → (7, 8)

3. UAV 3: (6, 7) → (-18, -5)

PART 2: FIND THE POINTS OF INTERSECTION

You are in charge of UAV 1. Determine where the points of intersection with each of the other two UAVs are. Show your work.

1. Point of intersection for UAV 1 and UAV 2:

2. Point of intersection for UAV 1 and UAV 3:

PART 3: CALCULATING TIME TO THE POINT OF INTERSECTION

Given that each UAV travels at 1.2 km/min, calculate the time until each UAV reaches the point of intersection. You will first need to calculate the distance from the starting point to the point of intersection, then calculate the time it takes to get there.

Based on your calculations, determine if there is a collision risk. Remember that there is a risk if the UAVs arrive at the point of intersection **within 30 seconds of each other**.

Remember to convert your answers to km (1 square = 0.5 km).

1. Time to the point of intersection for UAV 1 and UAV 2:

2. Time to the point of intersection for UAV 1 and UAV 3:

PART 4: EXTRA PRACTICE

For each of the following, calculate:

- The equation, in slope-intercept format, of each UAV
- The point of intersection for the two UAVs
- The distance and time until reaching the point of intersection
- Assess whether there is a collision risk

1. UAV 1 flies from (-2, -2) to (2, 6) and UAV 2 flies from (2, 0) to (5, 9).

2. UAV 1 flies from (-7, 0) to (3, 10) and UAV 2 flies from (-2, 0) to (-8, 9).

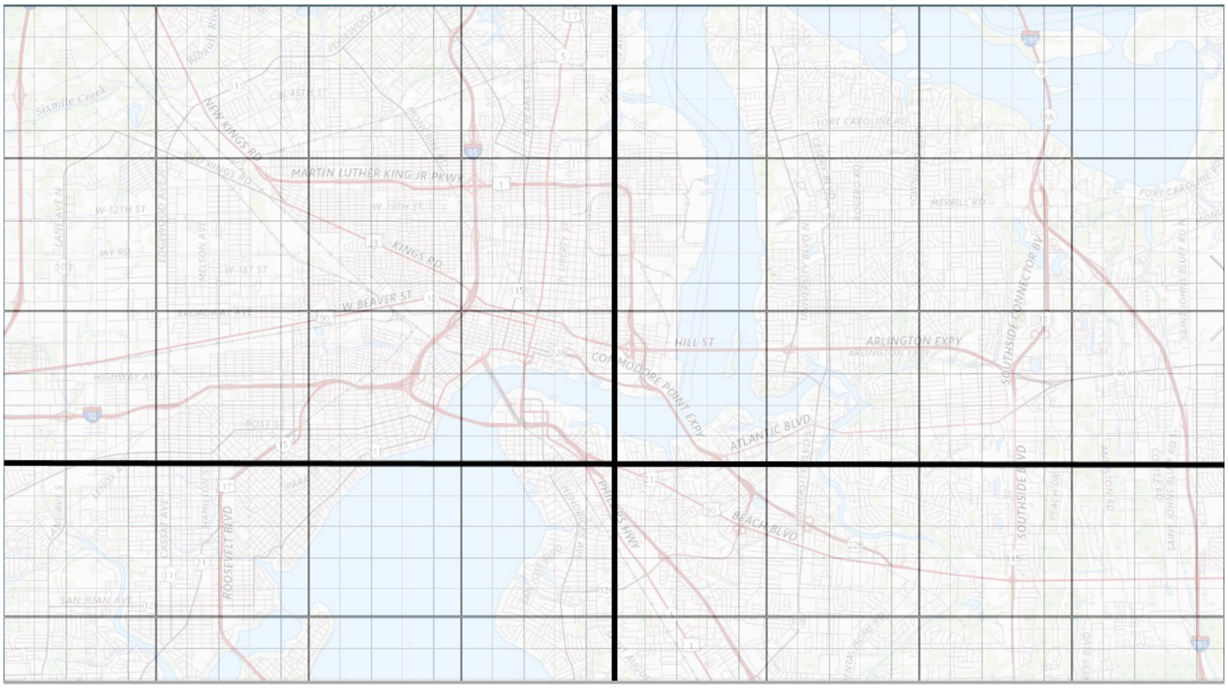

Figure 2. Coordinate system over the map. Credit: USGS

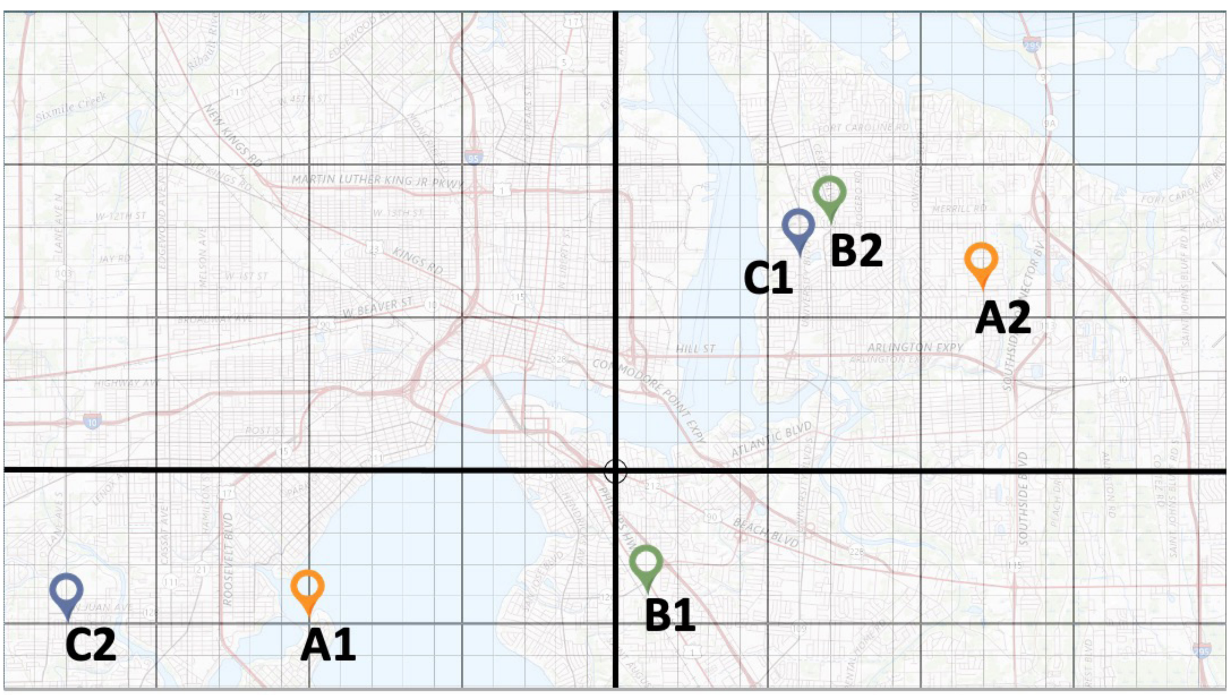

Figure 3. Map with route starting and ending points plotted. Credit: USGS

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