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Entanglement-assisted Communication System for NASA's Deep-Space Missions

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Final Summary of Research

Abstract: For this project we have studied various forms of quantum communication, and quantum-enhanced classical communication. In particular, we have performed the first realization of a novel quantum protocol, superdense teleportation. We have also showed that in some cases, the advantages of superdense coding (which enhances classical channel capacity by up to a factor of two) can be realized without the use of entanglement. Finally, we considered some more advanced protocols, with the goal to realize 'superactivation' – two entangled channels have capabilities beyond the sum of the individual channels—and conclude that more study is needed in this area.

Introduction

As NASA continues to push into deeper space, and to ever more detailed investigations of our "local" celestial objects, there is a need for increased communication data rates. This has prompted many investigators to look into what quantum phenomena can offer. Communication at optical wavelengths offers some advantages over radio frequencies due to reduced diffraction. In NASA's conventional optical communication scenario, a laser source is used to transmit a coherent ensemble of photons obeying Poisson number statistics distribution. At astronomical distances, the communication signal arrives one photon at a time and thus quantum mechanical effects prevail. For a full quantum account of a deep-space communication scenario, we consider communication systems where the transmitter is an ultra-bright single-photon (or entangled-photon) source and the receiver employs one or more single-photon detectors. The coding alphabet can be imprinted on different degrees-of-freedom (DOF) of a single photon (polarization, orbital angular momentum, time-energy....). In a communication channel, information can be a digitized classical bit (cbit: {0, 1}) or a quantum bit, a superposition of 0 and 1 described by two continuous parameters*: $|\psi\rangle = \cos(\theta)|0\rangle + e^{i\phi} \sin(\theta)|1\rangle$. Fortunately, most commonly used communication channels, such as optical fibers, air, and vacuum are in fact *quantum* information channels – they are able to preserve quantum superpositions, and thus may be used to transmit both bits and qubits, a fact that we can exploit to enable the transfer of quantum information, and to improve the capacity of classical information transfer. These were the central goals of our proposed research.

The generation, manipulation, and measurement of quantum states lie at the heart of quantum information science. Also known as nonseparable states, entangled states represent the properties of particles that have become linked in such a way that it is impossible to express the quantum states of the two particles independently. Entangled

* Larger alphabets are also possible by using degrees of freedom with more than 2 orthogonal states. For instance, pulse-position modulation is a classical example; at the quantum level we then have qudits (for a d-level system).

particles have been shown to display stronger non-local correlations than allowed by classical mechanics and have been demonstrated in various physical systems. Photons, in particular, have several characteristics which make photonic qubits (quantum bits) well-suited to deep-space quantum communication protocols, as they can transmit quantum information over long distances much more quickly and easily than other quantum systems. Furthermore, through the process of spontaneous parametric down-conversion, entangled photon states can be produced much more easily than entangled states in other physical systems [1]. Finally, because photons interact only very weakly with other particles, they do not decohere by becoming entangled with their surrounding environment as many other physical quantum systems do.

Even using photons, however, it can be very challenging to transport quantum information between two remote parties. For example, signals transmitted though long lengths of optical fiber experience a high amount of loss. Classical signals transmitted through long distances using optical fiber are periodically amplified; however, as stated by the no-cloning theorem, a quantum state cannot be duplicated, making direct quantum state transportation over long distances in fibers unfeasible. Similar arguments may be applied to free-space communication. While it is possible to correct errors introduced by the quantum channel, such as unwanted bit, phase flips, and loss, quantum error correcting protocols are very resource-intensive.

Quantum technology has made *revolutionary* advances in the broad areas of quantum-enhanced information processing, metrology, and communication. Most promising for the latter are the nonlocal quantum mechanical correlations present when two (possibly very remote) systems are *entangled*. For example, maximum polarization entanglement between two photons is represented by $(|H>|H>+|V>|V>)$ where $|H>|H>$ $(|V>|V>)$ is the quantum state of the |first>|second> photons with horizontal (vertical) polarizations [2,3]. In this entangled state each photon alone has *no* definite polarization, and yet measurement of one photon immediately determines the state of the other one, no matter how distant. Photon pairs can be simultaneously "hyper-entangled" [4,5], e.g., $\{(|H\rangle|H\rangle+|V>|V>)\otimes(|\Updownarrow>|\Updownarrow>+|\Leftrightarrow>|\Leftrightarrow>)\}$, formed from polarization (|H>, |V>) and spatial-mode $(|\Updownarrow>, |\Leftrightarrow>)$ DOFs. Also different single-photon DOFs can be "hybrid-entangled" [6], e.g.$(|H>|\Updownarrow>+|V>|\Leftrightarrow>)$.

Our research in this NIAC proposal has focused on three main areas:

1. Using quantum mechanics to enhance the capacity of classical communication channels;
2. Implementing a novel quantum communication protocol: superdense teleportation;
3. Investigating more general communication protocols see what quantum advantage there might be, and how they might be realized.

The majority of experimental work was devoted to item #2 on this list, as will be the bulk of this final report.

A. Quantum-Enhanced Classical Communication

It has been known for some time (in fact, this was one of the first quantum information protocols ever proposed) that the use of quantum entanglement enables one to send more classical information per photon than would otherwise be possible. In particular, if two parties share a pair of entangled quantum bits, then the first can transmit one of four messages to the second, a factor of two greater than would be possible if they did not share

entanglement. And in fact, a number of experiments have been done to show that there is a quantum advantage. Unfortunately, although in principle one can encode four messages in a single pair of qubits, in fact one cannot reliably distinguish all four if one is limited to optical qubits, because there is no simple way to achieve the strong photon-photon interactions required for the measurement protocol. However, it was shown that if one used photons that were hyperentangled – simultaneously entangled in multiple degrees of freedom – then one could, in fact, distinguish all four of the quantum states that the sender might prepare [7]. Our group had previously performed such an experiment, and achieved the world-record for quantum-enhanced classical communication capacity [8]. And in the first months of this NIAC project we repeated some of those early experiments, with the goal of realizing a quantum state analyzer that uses entanglement in spatial mode to help distinguish the following four polarization "Bell states":
$|\phi\rangle^{+} = |HH\rangle + |VV\rangle$, $|\phi\rangle^{-} = |HH\rangle - |VV\rangle$, $|\psi\rangle^{+} = |HV\rangle + |VH\rangle$, and $|\psi\rangle^{-} = |HV\rangle - |VH\rangle$, where H and V represent horizontal and vertical polarizations of the two photons, respectively. By deterministically distinguishing these states, it is possible for a sender to transmit twice as much information to a receiver as would be seemingly be possible using classical polarization encoding.

As part of this NIAC proposal, we wanted to look more deeply at this proposal, to see how it might be improved. There are two main conclusions:

1. The first, and most significant, result is that we discovered a method whereby one could achieve the same factor of two enhancement, i.e., sending four messages on a single qubit, *without* the need for shared entanglement. Specifically, we developed a scheme whereby one could send a single photon in what we might call a "hybrid" entangled state, i.e., a state possessing correlations between two degrees of freedom; details are given in the Appendix of this report. In brief, in order to easily implement the superdense coding, one need only prepare a particular superposition of polarization and spatial mode state. Then, by acting only on the polarization, the sender is able to convert this state into one of four possible other states, which the receiver can distinguish by making precisely the sort of measurement that was used in our previous superdense-coding experiment. The realization that there is a classical way to achieve the same goal is quite significant, as it is generally much easier to prepare single-photon states (even ones with hybrid entanglement between the different degrees of freedom) than it is to prepare separate entangled photons.
2. We also showed that the more advanced schemes, by which one is able to transmit more than four messages (at the cost of requiring a more complicated analysis on the receiving end) do *not* have a similar sort of classical implementation. Therefore, in future experiments, it will be very important to realize these more sophisticated classical communication encoding protocols. One such example is shown in Fig. 1.

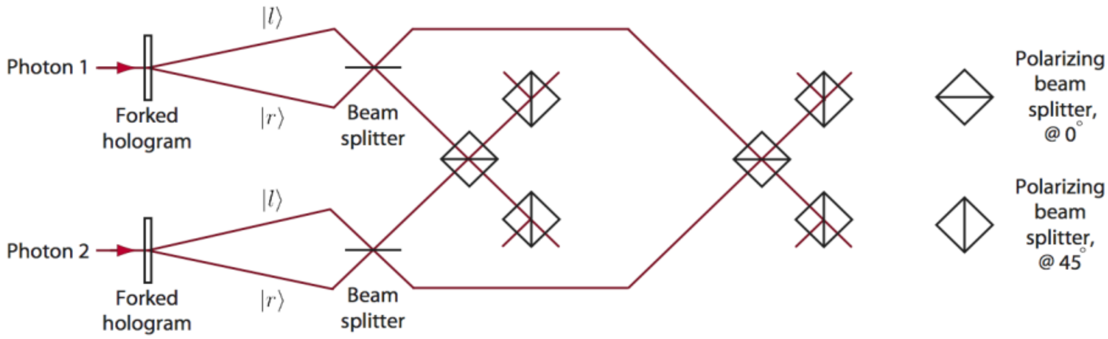


Fig. 1: The experimental setup for embedded Bell-state analysis. This design uses two-photon interference to discriminate between various classes of hyper-Bell states. Without photon number resolving detectors, it is possible to discriminate 6 distinct classes; with such detectors, up to 7 can be discriminated.

B. *Novel Quantum Communication: Superdense Teleportation*

The previous task dealt with attempts to use quantum states to improve the *classical* information capacity of a channel. Here we describe our efforts to implement the first realization of a novel *quantum* communication protocol—superdense teleportation—and relate to the more common protocols, teleportation and remote-state preparation. If a sender (Alice) and a receiver (Bob) share a two-particle entangled state, they can use quantum teleportation to indirectly communicate an arbitrary qubit state encoded on a third particle with two classical bits of information [9]. Generalizing to higher dimen–sions, Alice may use quantum teleportation to send an n-dimensional quantum state, parameterized by $2n - 2$ continuous variables, to Bob with $\log_2 n^2$ bits [10]. However, the difficulty of Alice's measurements significantly increases as n increases. This is especially problematic considering that even *qubit* $(n = 2)$ teleportation already cannot currently be imple–mented deterministically with photons and is difficult with other quantum systems [11].

Remote state preparation (RSP) is an alternative strategy to transmit quantum information between two remote parties who share an entangled state [12]. However, instead of trying to teleport the parameters of a quantum state encoded on a third particle, Alice uses RSP to send parameters encoded on her half of the entangled state by a state "chooser" (Charles). She accomplishes this task by performing a measurement on her photon and transmitting the (classical) results to Bob. Then, just as in quantum teleportation, Bob performs a unitary transformation on his half of the (now collapsed) entangled state, thus transforming his photon to the state Charles chose. RSP is easier to implement deterministically than traditional quantum teleportation, because it does not require a measurement in the Bell state basis. Probabilistic RSP of an arbitrary qubit has been implemented [13] and requires Alice to communicate only one bit of classical information to Bob (as opposed to two classical bits required for quantum teleportation). *Deterministic* RSP of an arbitrary qubit, however, requires a complicated type of generalized measurement, and Alice has to communicate two classical bits to Bob [14]. In general, deterministic RSP of an n-dimensional arbitrary quantum state requires difficult measurements and requires Alice to communicate the same number of bits to Bob as quantum teleportation [8], i.e., there is no advantage in terms of classical communication resource requirements.

However, the situation changes considerably if we partially constrain the states to be transmitted i.e., to limit the space of remotely prepared states to a specific class of states. The resource requirements of RSP in particular subsets of a Hilbert spaces have been theoretically examined [8]. In particular, our collaborator Herb Bernstein of Hampshire College developed a specific strategy, known as super-dense teleportation (SDT), to remotely prepare a particular class of states with fewer measurement and communication resources than arbitrary RSP [10]. Specifically, he showed that two-party maximally entangled states of the form $|\Theta^n\rangle = \frac{1}{\sqrt{n}}\left(|11\rangle + e^{i\varphi_1}|22\rangle + e^{i\varphi_2}|33\rangle + \cdots + e^{i\varphi_{n-1}}|nn\rangle\right)$ could be used to teleport the n-1 phases of an equimodular state $\frac{1}{\sqrt{n}}\left(|1\rangle + e^{i\varphi_1}|2\rangle + e^{i\varphi_2}|3\rangle + \cdots + e^{i\varphi_{n-1}}|n\rangle\right)$ with only $\log_2 n$ bits of classical information. Not only does this technique teleport more continuous parameters per classical bit transmitted than arbitrary RSP, it requires a much simpler measurement required for deterministic RSP. To demonstrate these benefits, we have constructed an experimental setup which uses photons hyperentangled in polarization and orbital angular momentum to implement four-dimensional SDT.

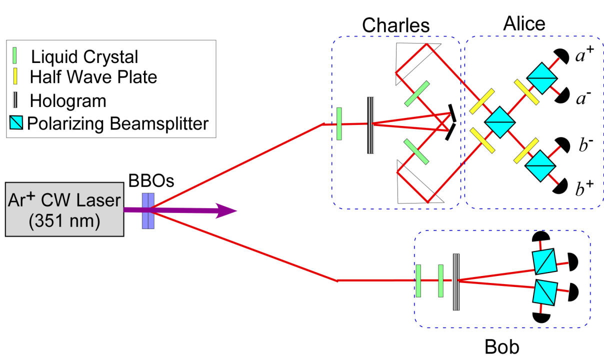


Fig. 2: The experimental setup we use to perform SDT. Charlie and Bob both receive one half of an entangled state. Charlie applies phases to photon using liquid crystals to transform the total state to Eqn. 2 and then sends the state to Alice. Alice then measures in a particular basis using a combination of holograms, wave plates and polarizing beam splitters, thereby preparing Bob's photon in the desired state. By measuring the photons in coincidence, it is possible to characterize the remotely prepared states.

In our SDT implementation, we pump two orthogonally oriented nonlinear BBO crystals with a diagonally polarized 351-nm Ar^+ laser, producing a pair of polarization entangled photons (see Fig. 1). Moreover, because this downconversion process conserves orbital angular momentum, the two photons will also be entangled in orbital angular momentum [5]. The state may be written:

$$|\Theta^4\rangle_{spin-orbit} = \frac{1}{2}\left(|H \circlearrowleft\rangle|H \circlearrowright\rangle + |H \circlearrowright\rangle|H \circlearrowleft\rangle + |V \circlearrowleft\rangle|V \circlearrowright\rangle + |V \circlearrowright\rangle|V \circlearrowleft\rangle\right) \quad (1$$

One photon of the resulting hyperentangled state is sent to Bob and the other is sent to Charles. Charles uses liquid crystals to control the inter-term phases, transforming the global state to:

$$|\Theta^4\rangle_{spin-orbit} = \frac{1}{2}\left(|1\rangle|1\rangle + e^{i\varphi_1}|2\rangle|2\rangle + e^{i\varphi_2}|3\rangle|3\rangle + e^{i\varphi_3}|4\rangle|4\rangle\right) \quad (2$$

where $|1\rangle = |H \circlearrowleft\rangle, |2\rangle = |H \circlearrowright\rangle, |3\rangle = |V \circlearrowleft\rangle,$ and $|4\rangle = |V \circlearrowright\rangle$. Charles also uses a special hologram to transform the two ±1 orbital angular momentum states into Gaussian beams with different momenta [11]. He then sends the photon to Alice, who combines the two spatial modes on a polarizing beam splitter, allowing her to make a measurement in the $|a^\pm\rangle = \frac{1}{\sqrt{2}}(|D \circlearrowleft\rangle \pm |A \circlearrowright\rangle), |b^\pm\rangle = \frac{1}{\sqrt{2}}(|D \circlearrowright\rangle \pm |A \circlearrowleft\rangle)$ basis. Based on the outcome of Alice's measurement, Bob's photon will be projected into one of four uni-modular states:

$$|\Theta\rangle_{spin-orbit} = \frac{1}{4}\Big[|a^+\rangle\big(|H \circlearrowright\rangle + e^{i\varphi_1}|H \circlearrowleft\rangle + e^{i\varphi_2}|V \circlearrowright\rangle - e^{i\varphi_3}|V \circlearrowleft\rangle\big) + |a^-\rangle\big(|H \circlearrowright\rangle - e^{i\varphi_1}|H \circlearrowleft\rangle + e^{i\varphi_2}|V \circlearrowright\rangle + e^{i\varphi_3}|V \circlearrowleft\rangle\big) + |b^+\rangle\big(|H \circlearrowright\rangle + e^{i\varphi_1}|H \circlearrowleft\rangle - e^{i\varphi_2}|V \circlearrowright\rangle + e^{i\varphi_3}|V \circlearrowleft\rangle\big) + |b^-\rangle\big(-|H \circlearrowright\rangle + e^{i\varphi_1}|H \circlearrowleft\rangle + e^{i\varphi_2}|V \circlearrowright\rangle + e^{i\varphi_3}|V \circlearrowleft\rangle\big)\Big]. \quad (3)$$

Thus, based on the two bits that encode Alice's measurement outcome, Bob has enough information to transform his state into the target state by applying a 180º phase shift to the relevant term.

To demonstrate how a measurement on Alice's photon affects the state of Bob's pho– ton (as shown in Eqn. 3), we varied each phase by applying incrementally increasing vol– tages to the liquid crystals shown in Fig. 1, and the making joint measurements on both Alice and Bob's photons. When making measurements that were sensitive to the corres– ponding phase, we observed the predicted high-visibility oscillations in the rate of Alice and Bob's coincidence counts (see Fig. 2). For example, when varying φ_1, $|a^+\rangle|H \frac{(\circlearrowright+\circlearrowleft)}{\sqrt{2}}\rangle$ and $|b^+\rangle|H \frac{(\circlearrowright+\circlearrowleft)}{\sqrt{2}}\rangle$ showed 86% visibility fringes. The resulting oscillations demonstrate that measurements on Alice's photon affect the state of Bob's photon. However, differ– rent outcomes of Alice's measurement correspond to different resulting states of Bob's photon. It is for this reason that Alice must communicate her measurement outcome to Bob for him to be able to transform his photon into the state Charles chose.

Prior to applying the actual state transformation phase shifts, we determine how successfully we could remotely prepare Bob's photon into the target state, by performing quantum state tomography on each of the four different states that Alice's measurements herald. This is performed by making a complete set of measurements on Bob's photons in coincidence with Alice's measurements. The quantum states which most closely fit these coincidence measurements are then determined through the use of maximum-likelihood state reconstruction [15]. Because the remotely prepared states are polarization and orbital angular momentum two-qubit states, a complete set of measurements must to be performed on both polarization and spatial mode qubits. This requires a minimum of 16 measurements and may be accomplished using liquid crystals, a polarizing beam splitter, and a tomography hologram used to make projections into a complete set of spatial modes [11]. Initial measurements show that these states may be reconstructed (see Fig. 3), though we are still working to understand the underlying system imperfections and implement methods to correct them.

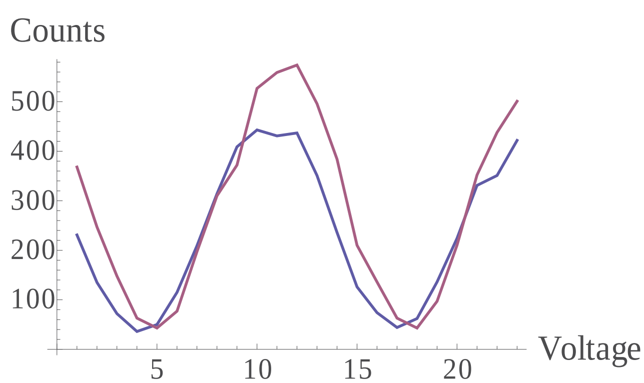


Fig. 3: This plot shows the coincidence count as a function of voltage applied to phase shifter. The high-visibility fringes demomstrate that Alice has remotedly pre– pared a particular uni-modular state at Bob.

After improving the fidelity of our remotely prepared states, we will explore the possibility of teleporting a more general class of states than those given in Equation 3, without increasing the resource requirements. Specifically, we wish to determine to what extent it is possible to teleport states if there is only partial coherence between the terms in Equation 3. If possible, this will vastly increase the known space of states that can be teleported using SDT and increase the applicability of the technique.

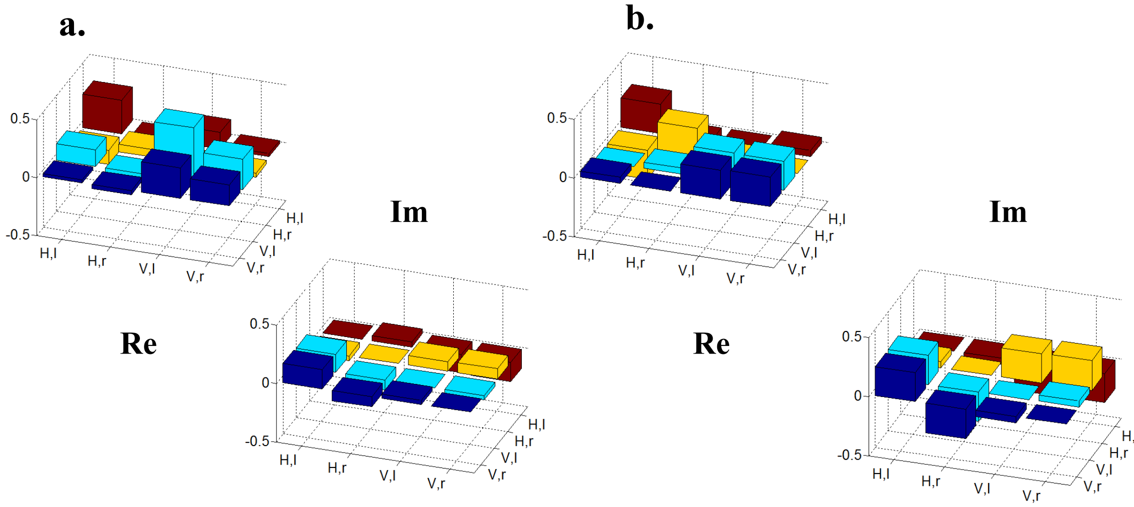


Fig. 4: The reconstructed state that Alice's measurement prepares Bob's photon in (for $\varphi_1 = 270°$, $\varphi_2 = 347°$, $\varphi_3 = 258°$). The difference between experiment **(a)** and theory **(b)** is mostly due to the different magnitudes of the diagonal elements (fidelity ≈ 80%). We believe that this incongruity arises from different efficiencies in the spatial-mode tomography holograms and measurement crosstalk, which we are not working to correct.

C. Other Quantum-Enhanced Communication Protocols

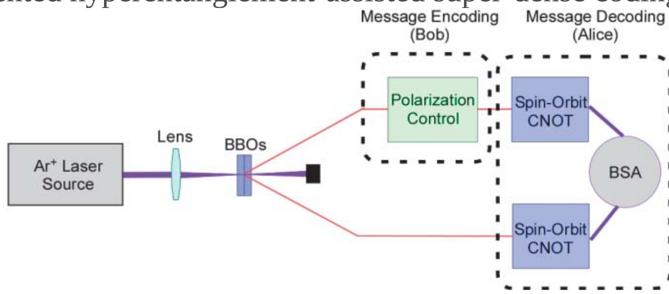
Finally, we also spent some time looking at various theoretical proposals for other quantum-enabled communications. In particular, there have been some recent ideas in the area of "activation" by which one is able to combine two channels, that combined (via entanglement) allow more noiseless communication than the sum of the individual channels. In fact, one can even have 'superactivation', where the subchannels individually have absolutely no capacity for noiseless communication, while the combined channel does. Unfortunately, we were thus far not able to uncover schemes that would be readily implementable.

Summary

Our research into quantum communication, and quantum-enhanced classical communication for deep-space applications has had several significant advances. In particular, we performed the first realization quantum superdense teleportation, remotely preparing up to three independent coherent phases. We have also showed that the advantages of superdense coding can sometimes be realized without the use of entanglement; in other cases, only true multipartite entanglement seems to provide an advantage. Finally, we considered some more advanced protocols, with the goal to realize entangled channels that have capabilities beyond the sum of the individual channels; we conclude that more study is needed to identify practical implementations.

Appendix: Single-photon Superdense Coding

We have recently been investigating if there is any true quantum advantage to our previously implemented hyperentanglement-assisted super-dense coding (HSDC):



1. Photons are prepared in hyperentangled state in polarization (spin) and orbital angular momentum (OAM): $\Phi^{+}_{\text{spin}} \otimes \Psi^{+}_{\text{OAM}} = \frac{1}{2}(|HH\rangle + |VV\rangle) \otimes (|lr\rangle + |rl\rangle)$. One of the photon pair is then sent to Bob and Alice.
2. Bob encodes two bits by performing one of the following four operations on the polarization on the photon: identity, bit-flip, phase-flip, bit-flip + phase-flip. These operations transform the initial polarization state into one of the four Bell states and do not affect the orbital angular momentum. Bob then sends his photon to Alice
3. Alice decodes Bob's message by performing a Bell state measurement on the photon pair. She accomplishes this by decomposing each of the four polarization Bell state into a superposition of single-photon Bell States in polarization and OAM:

$$\Phi^{\pm}_{\text{spin}} \otimes \Psi^{+}_{\text{OAM}} = \frac{1}{2}(\pm\phi^{+}_{1} \otimes \phi^{\pm}_{2} \mp \phi^{-}_{1} \otimes \phi^{\mp}_{2} \pm \psi^{+}_{1} \otimes \psi^{\pm}_{2} \mp \psi^{-}_{1} \otimes \psi^{\mp}_{2})$$

$$\Psi^{\pm}_{\text{spin}} \otimes \Psi^{+}_{\text{OAM}} = \frac{1}{2}(\phi^{+}_{1} \otimes \psi^{\pm}_{2} + \phi^{-}_{1} \otimes \psi^{\mp}_{2} + \psi^{+}_{1} \otimes \phi^{\pm}_{2} + \psi^{-}_{1} \otimes \phi^{\mp}_{2}),$$

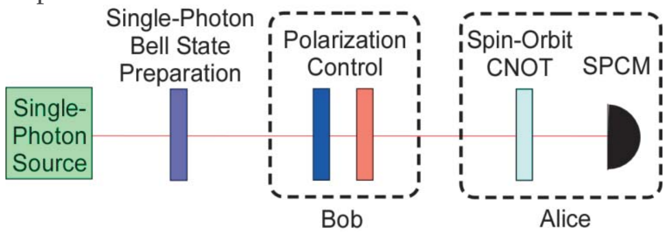
where

$$\phi^{\pm} = \frac{1}{\sqrt{2}}(|Hl\rangle \pm |Vr\rangle) \text{ and } \psi^{\pm} = \frac{1}{\sqrt{2}}(|Hr\rangle \pm |Vl\rangle).$$

Alice can distinguish all four single-photon Bell states using a spin-orbit CNOT gate. Since each term of the polarization Bell state has a unique coincidence signature, Alice can distinguish all four polarization Bell states by detecting single-photon Bell states in coincidence. Thus, Alice can decode both bits of information encoded by Bob.

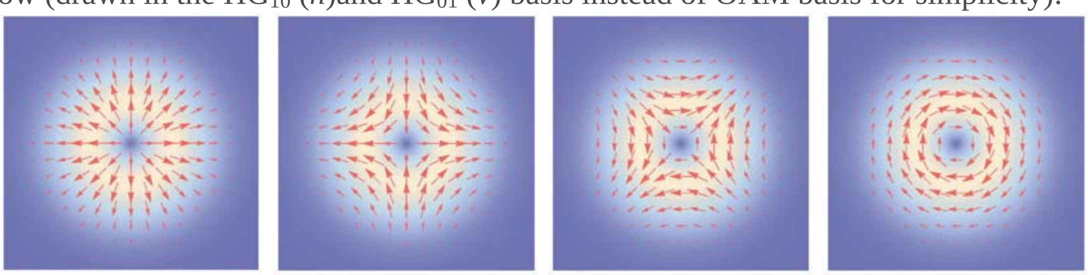
Specifically, we have been concerned about the way Alice performs her Bell-state measurement in step 3. Because the polarization Bell state measurement which Alice performs requires no interference between the two photons, she need not measure both photons at the same time. In fact, Alice could perform a measurement on her photon before Bob even receives his photon without affecting the results of the experiment. In this situation Bob's photon is collapsed into one of four possible single-photon Bell states shown in the decomposition of $\Phi^{+}_{\text{spin}} \otimes \Psi^{+}_{\text{OAM}}$ above. Bob then performs his transformations on a single-photon Bell state (which has a classical description) and sends the resulting state to Alice. Because Alice performed a measurement on her photon before Bob got his photon, she knows what singe-photon Bell state Bob's photon was projected into. Therefore, Alice can identify the transformation Bob made by detecting which single-photon Bell state Bob sends her. Through this argument we observe that the only role Alice's photon has is to provide information about which single-photon Bell state Bob's photon is in. However, if this information were already known, then there would be no need for Alice's photon at all (or the entanglement between the photons).

Thus, the channel enhancement which was seen in our HSDC experiment can be dupli–ßcated using classical encoding (requiring no entanglement between photons). An example of such an implementation is shown below:



1. A single photon is prepared in one of four single-photon Bell states (e.g. $\phi^+ = \frac{1}{\sqrt{2}}(|Hl\rangle + |Vr\rangle)$).
2. The photon is then sent to Bob, who encodes two bits by performing one of the following four operations on the polarization on the photon: identity, bit-flip, phase-flip, bit-flip + phase-flip. These operations will transform Bob's single-photon Bell state into one of the four following states: $\phi^\pm = \frac{1}{\sqrt{2}}(|Hl\rangle \pm |Vr\rangle)$ or $\psi^\pm = \frac{1}{\sqrt{2}}(|Hr\rangle \pm |Vl\rangle)$.
3. Bob sends the photon to Alice who decodes the two bit message using a spin-orbit CNOT gate.

The single-photon Bell states referred to above are spatially varying polarization states shown below (drawn in the HG_{10} (h)and HG_{01} (v) basis instead of OAM basis for simplicity):



$\frac{1}{\sqrt{2}}(|Hh\rangle + |Vv\rangle)$ $\frac{1}{\sqrt{2}}(|Hh\rangle - |Vv\rangle)$ $\frac{1}{\sqrt{2}}(|Hv\rangle + |Vh\rangle)$ $\frac{1}{\sqrt{2}}(|Hv\rangle - |Vh\rangle)$

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