Fundamental Error in NASA/TM—2004–213283 experiment, detrimental to both Israel and to the USA national interests

Eytan Suchard,

Previous correspondence to NASA was ignored: <u>JSC-Partnerships@mail.nasa.gov</u> on Fri, Jan 3, 2014, 11:07, my time.

If NASA/TM—2004–213283 used a barrel shaped capacitor was tested also for the so called "Biefeld-Brown" effect in vacuum and reported no thrust,

Under the title, "6. VACUUM TESTING AND RESULTS" it reads, "... In hard vacuum even with potentials above 50 kV, there was no measurable performance observed ...".

The experiment is fundamentally wrong because in vacuum, thrust is not expected to appear due to high electric field gradients but due to a totally different effect. What causes an electro-gravitational warp drive is charge separation, see my paper, that is about to be published, on the next page.

The barrel shaped capacitor has maybe only few Pico-Farads of capacitance and even under 50000 volts not enough charges form an electric dipole to yield a measurable electro-gravitational effect.

My assessment is of an effect that resembles Alcubierre warp drive and also accounts for the well know Dark Matter and Dark Energy effects in large astronomical scale due to slight charge imbalance or due to magnetic fields.

The virtual mass appears on the two sides of the dipole and can be calculate according to the non-quantum limit equation (7) in my paper and according to (6.4) as

Virtual_Mass =
$$\frac{\pm Q}{\sqrt{2K\varepsilon_0}}$$
 such that *K* is the gravity constant, ε_0 is the permittivity and *Q*

denotes the charges. The Sciama-like Inertial Induction conservation law can be seen in (25).

By using very special two multi-layered boards, a capacitance of 20 Micro-Farads over a gap of 1 meter and with 1 Mega-volts voltage can achieve high performance gravity which at optimistic assessment, without analytic solution of (7) and (25), implies a space-craft. With the correct geometry, and cockpit just above the positive board/plate, zero g will be experienced by the pilot.

Kind and warm regards, wishing you Eytan Suchard.

Eytan Suchard

EYTAN H. SUCHARD

R & D Algorithms Department, ANB/ANT – Applied Neural Biometrics/Technology Netanya Israel Abstract: Einstein equation of Gravity has on one side the momentum energy density tensor and on the other, Einstein tensor which is derived from Ricci curvature tensor. A better theory of gravity will have both sides geometric. One way to achieve this goal is to develop a new measure of time that due to multiple curve intersections can't be a coordinate. One natural nominee for such time is the upper limit of measurable proper time measured along the longest geodesic curve from near the "big bang", either as a set of events or as a singularity, to any event. By this, the author constructs a scalar field of an upper limit of measurable time. Time, however, is measured by material clocks. What is the maximal time, that can be measured by a small microscopic clock, when our curve starts near the "big bang" event or events - and ends at an event within the nucleus of an atom ? Will our tiny clock move along geodesic curves or will it move along a non geodesic curve within matter ? It is almost paradoxical that a test particle in General Relativity will always move along geodesic curves but the motion of matter within the particle, may not be geodesic at all. For example, the ground of the Earth does not move at geodesic speed. Where there is no matter, we choose a curve from near "big bang" event or events, to an event such that the time measured is maximal. The gravitational field causes that more than one such curves intersect at events, which could result in discontinuity of the gradient of the scalar field of time. The discontinuity can be avoided only if geodesic motion is prohibited in material fields.

Keywords: Time;Foliation;Field Curvature;General Relativity;Quantum Gravity;Ashtekar Variables

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1. Introduction

A) The principles of the presented theory

All the following paper is an improvement of an early paper [1].

- If spacetime is homotopic to either a single creation event or to a set of creation events from which we can say the cosmos started its expansion, then maximal proper time curves can be drawn between that event and any other event and therefore attach a time value to any event in space time. Intersections of such curves, however, prevent the global use of a single curve as a time coordinate but does not prevent definition of such time as a scalar field. Although the big bang is either a singularity event or singularity initial events, it is impossible that unbound time can be measured from an event backwards to near big bang. Such unbound measurement is inconsistent with any physical reality. Along these curves, we can imagine a tiny clock that travels and measures time. It is an important point that not the time is a field, but its upper measurable value by a clock particle is, see [2] for Sam Vaknin's idea of Chronon.
- It is essential to understand that the time discussed in this paper is not a coordinate. Many curves along which the upper limit on time form big bang event or manifold

of events is measured, can cross the same event. Therefore the gradient of the upper limit is a gradient of a scalar field and not of any coordinate. This paper uses this gradient to describe curvature which is closely related to acceleration. It is the curvature of clock trajectories as they interact with matter fields. In other words, matter prohibits geodesic 4-motion even of stationary particles due to gravity and not only of the ones measuring the upper limit of measurable proper time. A new principle of equivalence and an anecdote of gradient continuity: The gradient of the upper limit of measurable time from an event, back to near big bang - event or events - is continuous. Clock tick is different under space location due to gravity. The scalar field therefore, has a significant gradient by space. Where there is matter, however, different upper-limit-of-measurable-time curves may intersect. Therefore, at intersection events, the gradient of the scalar field can't be parallel to all of the curves. The result of this short argument is that a test particle moving along an-upper-limit-of-measurable-time-curves will not undergo parallel translation and will be non-geodesic near matter. This fact is a strong motivation to offer an intrinsic curvature operator of the gradient of the upper limit of measurable time as equivalent to matter.

• The definition of event: The paper does not deal with a description of the coordinate of time. The coordinate of time is not subject to any equations! The upper limit on measurable time from the past to an event is subject to such equations because it is measured by material clocks and material clocks are influenced by space-time curvature and by forces. The main problem is that any

particle that has mass, experiences a different trajectory under the same force. To say that the geometry of the trajectory has a physical meaning, we must accept that either there is a unique particle that can interact with any material field e.g. [2] or to give a definition of an event that is consistent with this theory as related to non-gravitational acceleration.

Definition: We will define an event as a non-gravitational interaction or more precisely as a collision. A satellite Fly-by interaction is therefore not considered as an event unless non-inertial acceleration is detected.

- The laws of Nature will never directly involve any absolute upper time limit. Instead, we will be coerced to define them by using the gradient of such time, which is indeed local. This point is crucial to the understanding of the paper.
- The apparent shortcoming of this paper is that on one hand it talks about particles but on the other, it tries to avoid discussion of quantum field theory, however, the main subject of the paper is the phenomenological non geodesic movement of an interacting particle that measures an upper limit of time to an event. The classical description of the geometry of the particles trajectory and its relation to the existence of mass does not require Quantum Mechanics. Nevertheless, the idea of quantum coupling between an upper limit of measurable time, and how much matter is present where this upper limit is measured, is discussed as a natural possibility of the presented theory along with its outcome.

 A nice, though less important issue, is that local foliation of space time into 3+1 dimensions requires time orthogonality unlike in Kerr solution. This idea will also be addressed though it is a bit speculative.

Classical or quantum matter: The gradient need not be parallel to any geodesic curve due to force interactions, avoiding singularities:

Our strategy now will be to understand, why are forces needed where there is mass? We will see that without such forces, the previously mentioned "anecdote of gradient continuity " can't hold. The direction in space time of the maximum proper time forms a geodesic curve but not necessarily the gradient of the field is parallel to a geodesic curve because 1) more than one curve can reach the same event 2) at that event, force will cause any test particle clock to move along non geodesic curves (see Appendix B for understanding the role of forces) and 3) A real world particle clock will not move along geodesic curves within matter, otherwise its measurement will result in discontinuities or singularities of the gradient of the upper limit of measurable time. The idea of such particle clocks is not quite new [2] and is important for the action operator that will be presented in order to have physical meaning. Good examples of discontinuity are the center and edge of a hollowed ball of mass. Due to General Relativity, the clock ticks in the gravitational field of the ball are slower than far from the ball. As a result, max proper time geodesic curves from say "big bang" - event or events - must come from outside the ball into the ball. The time at the center of the ball is also a geodesic curve but it is in the time direction in Schwarzschild coordinates due to symmetry. The vector field of the lines is therefore discontinuous

and we may have a non zero [3] Euler number of the gradient of the upper limit of measurable time. As was already mentioned, one way to resolve such singularities is that our particle clock will experience force. Space-time in a hollow ball of mass is conic i.e. the Schwarzschild metric coefficient g_{rr} of dr is greater than 1, and is flat but with a zero measure singularity of the Gaussian curvature at the center which is the tip of 4 dimensional cone. In any case, such force has to be negligible though it should exist and should disallow geodesic movement in the sub atomic scale that will otherwise manifest the gradient singularity.

(Fig. 1) The line of the max proper time field from "big bang" - event or events - is discontinuous in the middle of a hollowed ball of mass and therefore a real world clock will not move along geodesic curves at such points no matter how negligible is such an effect.



The "anecdote of gradient continuity" can hold in the sub-atomic level by forces that prevent any microscopic real world clock from moving along geodesic curves. Our field sets an upper limit to measurable time by any such tiny clock particle. The conflict without the existence of forces is apparent also on the edges of the ball because matter is granular, that is to say that the mass is not evenly distributed. Particles measuring absolute maximum proper time from "big bang" - event or events - along curves that enter the ball, must pass through the walls of the ball or

hyper-cylinder in 4 dimensions. Therefore, gravitational lenses are formed and events in the hollowed part of the ball are accessible by more than one curve. These singularities can be resolved too if any real world particle-like clock will not move along geodesic curves in the microscopic vicinity of matter, e.g. due to Casimir/Casimir Lifshitz [4] and thus the gradient will be smoothed.

(Fig. 1A) On the edges, gravitational lenses due to granularity cause the geodesic conflicts. The particles form an obstacle that is bypassed by the entering curves.



Also in this case, the new set of gradient conflicts (at absolute maximum proper time intersection events) can be avoided by forces exerted on the clock. If we say that matter is measured by such conflicts/intersections of time gradients, then the fact we also have a microscopic though negligible geodesic conflict in the center of the ball and the "anecdote of gradient continuity" attest to some non-locality of the energy of matter. Weak force fields out of known boundaries of matter are yet to be experimentally found. We will call them "Secondary Dark Matter", "secondary", because it is not the regular notion of warm or cold dark matter that is often mentioned in astrophysics. In big words, the theory that is behind this paper says that indeed, laws of physics are local but the entire geometric context has influence on a new effect we have just named, "Secondary Dark Matter". Contrary to the absolute

maximal proper time from "big bang" - event or events - measured by a microscopic clock, most geodesic curves - though they also use travelling clocks - usually measure only local maximum proper time. The curves along which the upper limit on measurable time is measured usually have tangents that point to a 4-direction in space time along which the time changes. In free of matter space, in perpendicular to the (Lorentzian) direction of the gradient, the differential should be zero. Therefore, in geodesic coordinates such that the time is parallel to one of the absolute maximum proper time curves, the mixed terms of the metric tensor vanish. Locally, the separation between space and time works also in metrics such as the Kerr metrics and time appears perpendicular to 3D space manifolds along the maximum proper time curves, e.g. in a set of rotating reference frames. Separation of space and time is important and can be achieve at least locally also along closed time-like curves, however, this paper has a much higher priority motivation, to get an equation that depends only on geometry. To show that time is an emergent dimension is not the issue of this paper.

B) Open questions – local emergent time unsolved issue The question is: Can inverted logic work ? By minimizing an action operator on three dimensional manifolds, can a degree of freedom yield multiple solutions for the metric tensor, such that:

1) The action can serve as a local homotopy [5] parameter.

2) The action will be invariant under Lorentz - like rotations in the resulting four dimensional manifold. This question, to the author's opinion justifies further research although foliation of space-time works only locally. We would like to describe the

curvature of the gradient of the upper limit of measurable time that is measured by our material microscopic particle-like clocks and show its possible relation to Ricci curvature and to Einstein's tensor. The idea is that the gradient of the upper limit of time from an event back to near "big bang" - event or events - forms curves that have non vanishing curvature where there is matter. Again, it is important to say these gradients are local and that our time is an upper limit on all possible tiny test particles. We now proceed to the measurement of the trajectory curvature of our test particles. We need to develop tools to deal with such curvature as was already presented in a previous paper [1]. In appendix C it will be shown that this curvature can be seen as non-gravitational acceleration.

Intuitive discussion about the second power of curvature of a conserving vector field and about "bending energy".

In special relativity, the square norm of a normalized 4-velocity of a particle is constant $N^2 = u_i u^i = 1$ written in tensor formalism. Also

$$N^2_{,k} = (u_i u^i)_{,k} = \frac{d(u_i u^i)}{dx^k} = 0$$
 such that $u^i = \frac{dx^i}{Cd\tau}$ such that x^i denotes the

coordinates and C is the speed of light. If we want to express the curvature of a particle's trajectory when force is exerted on the particle then we can't use derivatives of $N^2 = u_i u^i = 1$ to express that curvature. However, if τ is a scalar field, measuring the upper limit of measurable proper time from near "big bang" singularity event or starting events, to any event, then its derivatives form a vector field. This vector field will not be always geodesic if there are locations in space time where real forces are exerted on any particle, e.g. in matter's vicinity. So $N^2 = \tau_i \tau^i$ such that

 $\tau_i = \frac{d\tau}{dr^i}$ may offer a way to achieve $N^2_{,k} \neq 0$. The reader can find the origin of this work in [6] and especially of the use of an early "Bending Energy" operator. So let us begin. We will now define what the Square Curvature or conserving-field-curvature of a vector field V in \mathbb{R}^n , with positive definite Euclidean -geometry, is. The formalism will not be tensor but don't worry about that because in most of the paper, full tensor formalism is inevitable. The same formalism is easily extended to Riemannian geometry. We also define Bending Energy as the Square Curvature multiplied by the square norm of the gradient of a scalar field. We would like the field V to reduce or increase its differential in directions that are perpendicular to the direction of the field. This requirement is also comprehensible when the metric tensor of a manifold with coordinates in R^n has only positive eigenvalues in local orthogonal coordinates and we shall see that the operator that describes Field Curvature has quite the same formalism in Riemannian manifolds. We will start with an intuitive description of the operator and later give a proof it is the square curvature of the conserving (here it simply means a gradient of a scalar field) vector field. Given two infinitesimally close points in \mathbb{R}^n , q1 and q2 = q1 + hV for some infinitesimal h, we would like that $V(q_2) - V(q_1)$ will be as parallel as possible to the field $V(q_1)$. By Pythagoras it can be written as the following problem to locally minimize

$$\left(\left(\frac{V(q2) - V(q1)}{h}\right) \bullet \left(\frac{V(q2) - V(q1)}{h}\right) - \left(\left(\frac{V(q2) - V(q1)}{h}\right) \bullet \frac{V(q1)}{|V(q1)|}\right)^2\right) h^2$$
(1)

When • is the inner product in R^n .

Here $(V(q2) - V(q1)) \bullet \frac{V(q1)}{|V(q1)|}$ represents the projection of the derivative matrix of

the vector field V(q) multiplied by the field direction in space.

In other words, since h^2 is arbitrarily small, our objective is to minimize,

$$(\nabla V \bullet \frac{V}{|V|}) \bullet (\nabla V \bullet \frac{V}{|V|}) - \left(\left(\frac{V}{|V|}\right)^{t} \bullet \nabla V \bullet \frac{V}{|V|}\right)^{2} = \frac{(\nabla V \bullet V) \bullet (\nabla V \bullet V)}{V \bullet V} - \left(\frac{V^{t} \bullet \nabla V \bullet V}{V \bullet V}\right)^{2}$$
(2)

Here ∇V means the matrix $a_{ij} = \frac{\partial V_i}{\partial X^j}$ in its non covariant form.

(Fig. 2) – "Bending Energy" which is the Field Curvature multiplied by the squared norm of the gradient and its Euclidean geometric meaning – informal description is, how much the field changes in direction perpendicular to itself.



The following next figure shows us two curves one on the left for which "Bending Energy" *BE* is zero and one on the right for which *BE* is positive:

(Fig. 3) - Parallel deviation on the right.



2. Tensor formalism of the Square Curvature of a conserving field

How can we measure, how much the gradient of the upper limit on measurable time from an event back to near "big bang" event or events, bends, i.e. force on particles measuring such time, is exerted in matter ? As will be discussed, in tensor formalism, derivatives are replaced by covariant derivatives and are denoted by semi colon ";" and derivatives by comma ",".

For example the covariant derivative of the vector field V_{λ} by the coordinate dx^{μ} ,

is written, V_{λ} ; $_{\mu} = \frac{dV_{\lambda}}{dx^{\mu}} - \Gamma_{\lambda\mu}^{k}V_{k} = V_{\lambda},_{\mu} - \Gamma_{\lambda\mu}^{k}V_{k}$ where $\Gamma_{\lambda\mu}^{k}$ denotes the upper Christoffel symbols . Upper and lower indices represent the covariant and contra-variant properties and upper and lower indices sum according to Einstein convention so (2) can be written as a tensor density with local coordinates in R^{n} . In this paper, often the gradient of a scalar field P by the coordinate x^{μ} , $\frac{dP}{dx^{\mu}}$ will be replaced by P_{μ} . Regarding the square root of the determinant of the metric tensor $\sqrt{-g}$, so following are tensor densities [7], that yield tensor equations [8],

$$SquareCurvature = \frac{1}{4} \left(\frac{(P^{\lambda}P_{\lambda})_{,m} (P^{s}P_{s})_{,k} g^{mk}}{(P^{i}P_{i})^{2}} - \frac{((P^{\lambda}P_{\lambda})_{,m} P^{m})^{2}}{(P^{i}P_{i})^{3}} \right) \sqrt{-g} \text{ or } (3)$$

that can be written as $\frac{1}{4} \left(V_m V^m - (\frac{1}{t})^2 \right) \sqrt{-g}$ for $V_m = \frac{(\mathbf{P}^\lambda \mathbf{P}_\lambda)_{m}}{\mathbf{P}^i \mathbf{P}_i}$ and

 $\frac{1}{t} = \frac{(\mathbf{P}^{\lambda} \mathbf{P}_{\lambda})_{m} \mathbf{P}^{m}}{(\mathbf{P}^{i} \mathbf{P}_{i})^{3/2}}$. Don't' confuse the scalar function t with maximum proper time.

We choose a simpler expression for our absolute maximum proper time from "big bang" event or events, namely τ , and where there is no matter in space-time, we expect one of the following to be true.

$$P = \tau, P \text{ is real} \tag{4}$$

or

$$PP^* = \tau^2 \psi \psi^*, P = \tau \psi \text{ is complex}$$
(5)

Important: Please note that SquareCurvature(P) = SquareCurvature(kP) for constant k. This fact attests to the intrinsic geometric trajectory curvature that (3) represents. Please note that in the model presented in (5), the time τ is coupled with a wave function ψ and there is only a need for $P = \tau \psi$ to have 3rd order derivatives but not for τ alone. For the reason of coupling please refer to T. Banks, Willy Fischler page 7, [9]. An intuitive idea behind the coupling $\tau \psi$ is that ψ tells us in Quantum Mechanics language, how much matter there is where the upper limit τ can be measured. This approach is different than the philosophy behind Richard Feynman's (3) following: path integrals. Development from the of be can $\begin{bmatrix}
L = \frac{1}{4} \left(\frac{(P^{\lambda}P_{\lambda})_{,m}(P^{s}P_{s})_{,k}g^{mk}}{(P^{i}P_{i})^{2}} - \frac{((P^{\lambda}P_{\lambda})_{,m}P^{m})^{2}}{(P^{i}P_{i})^{3}} \right) = \frac{1}{4} U^{j} U_{j} \\
U_{m} = \frac{(P^{\lambda}P_{\lambda})_{,m}}{P^{i}P_{i}} - \frac{(P^{\lambda}P_{\lambda})_{,\mu}P^{\mu}}{(P^{i}P_{i})^{2}} P_{m}$ (6)

The unique properties of (6) as an intrinsic geometric operator that does not depend on the parameterization of the curves formed by P_{λ} , can be seen in the following,

we can write
$$U_m = \frac{N^2_{,m}}{N^2} - \frac{N^2_{,\mu}P^{\mu}}{N^4}P_m$$
 s.t. $N^2 \equiv P^i P_i$ (also found as Z in this

paper) we can sloppily omit the comma for the sake of brevity the same way we write

P_i instead of P_i and instead of
$$\frac{dP}{dx^i}$$
 and write $U_m = \frac{N^2_m}{N^2} - \frac{N^2_\mu P^\mu}{N^4} P_m$. Suppose

that we replace P by f(P) such that f is increasing with p, then

$$f(P)_i \equiv \frac{df(P)}{dx^i} = \frac{df(p)}{dp}\frac{dP}{dx^i} = f_p(P)P_i$$
. Let $N^2 \equiv P^\lambda P_\lambda$ then we can write

$$\hat{N}^2 \equiv f(P)_{\lambda} f(P)^{\lambda} = N^2 f_p(P)^2$$
 and $\frac{\hat{N}^2_k}{\hat{N}^2} = \frac{N^2_k}{N^2} + \frac{2f_{pp}(p)}{f_p(p)} p_k$ but also

$$\hat{U}_{k} = \frac{\hat{N}_{k}^{2}}{\hat{N}^{2}} - \frac{\hat{N}_{s}^{2}}{\hat{N}^{2}} \frac{f_{p}(p)p^{s}f_{p}(p)p_{k}}{\hat{N}^{2}} = \frac{N_{k}^{2}}{N^{2}} + \frac{2f_{pp}(p)}{f_{p}(p)}p_{k} - (\frac{N_{s}^{2}}{N^{2}} + \frac{2f_{pp}(p)}{f_{p}(p)}p_{s})\frac{f_{p}(p)p^{s}f_{p}(p)p_{k}}{N^{2}f_{p}(p)^{2}} = \frac{N_{k}^{2}}{N^{2}} - \frac{N_{\mu}^{2}\mu^{\mu}}{N^{4}}P_{k} = U_{k}$$
(6.1)

and $L = \frac{1}{4}U^{j}U_{j} = \frac{1}{4}\hat{U}^{j}\hat{U}_{j}$ and

 $SquareCurvature(P) = L\sqrt{-g} = SquareCurvature(f(P))$

Obviously $U_m P^m = 0$. The vector U_m describes the direction and intensity of the curvature of the field P_{λ} which is a change perpendicular to P^m .

3. Classical non-relativistic limit - passive acceleration energy difference

The purpose of the following is to approximate the energy of the weak gravitational field. It will be shown in appendix C that the non-relativistic classical limit of our square curvature of the gradient of upper limit of measurable time back to near "big bang" event or events should be α^2/C^4 where

 $\alpha^2 = (a_x \pm g_x)^2 + (a_y \pm g_y)^2 + (a_z \pm g_z)^2$ where (g_x, g_y, g_z) denotes the classical gravitational acceleration g and (a_x, a_y, a_z) , denotes average acceleration by

material fields, C is the speed of light. x, y, z are the three dimensional coordinates. If we consider α^2 / K , K is the gravity constant, as an energy density we include the gravitational field energy in our calculation. However, (3) describes a

non-geodesic acceleration field which has its classical limit as $\frac{a^2}{C^4} = \frac{a_x^2 + a_y^2 + a_z^2}{C^4}$.

We do not know the average non-gravitational acceleration that a clock particle undergoes in matter so a^2 is unknown but due to Einstein's principle of equivalence we do know that a particle resting on the ball or in the ball is actually accelerated. An average non-relativistic classical acceleration a, can be with the direction of gravitational acceleration, opposite to gravity or perpendicular, so averaging these accelerations, we can write an approximation

 $\alpha^{2} = ((a_{x} - g_{x})^{2} + (a_{y} - g_{y})^{2} + (a_{z} - g_{z})^{2} + (a_{x} + g_{x})^{2} + (a_{y} + g_{y})^{2} + (a_{z} + g_{z})^{2} + 4(a^{2} + g^{2}))/6 = (a_{x}^{2} + a_{y}^{2} + a_{z}^{2}) + (g_{x}^{2} + g_{y}^{2} + g_{z}^{2})$ Such that $a^{2} + g^{2}$ accounts for perpendicular $a = (a_{x}, a_{y}, a_{z}), g = (g_{x}, g_{y}, g_{z}).$ Or in a more illuminating language $\alpha^{2} = ((a + g)^{2} + (a - g)^{2})/2 = a^{2} + g^{2}$. If the integration over volume of α^{2} is preserved and $g^{2} < a^{2}$ then g^{2} should be proportional to the negative potential gravitational pseudo energy.

 $(g_x^2 + g_y^2 + g_z^2)/C^4$ is a non-relativistic limit of curvature of the gradient of time because in the special theory of relativity 4-acceleration is a curvature vector and the time component of that vector is very small in the non-relativistic limit. Let us integrate $(g_x^2 + g_y^2 + g_z^2)/C^4$ in our ball. Suppose our ball has a radius r_0 and a volume $V = \frac{4\pi}{3}r_0^3$ and a mass *M* and that our gravitational constant is *K*. So it's

density is
$$\frac{M}{V} = \frac{M}{\frac{4\pi}{3}r_0^3}$$
 then the integration yields:

$$\frac{1}{C^4} \int_0^{r_0} \left(\frac{K(\frac{M}{V} \pi r^3)}{r^2} \right)^2 4\pi r^2 dr = \frac{K}{C^4} \left(\frac{3}{5} \frac{KM^2}{r_0} \right) \quad \text{, however, if we integrate the negative}$$

gravitational potential energy of the ball we have

$$\int_{0}^{r_{0}} \left(\frac{K(\frac{M}{V}\pi r^{3})}{r} \right) 4\pi r^{2} \frac{M}{V} dr = \left(\frac{3}{5} \frac{KM^{2}}{r_{0}} \right) = -E_{g} \text{ so we have}$$

$$\int_{volume} \frac{(g_{x}^{2} + g_{y}^{2} + g_{z}^{2})}{C^{4}} dVolume = -\frac{K}{C^{4}} E_{g} \text{ but that suggests the following:}$$

$$L = \frac{1}{4} \left(\frac{(P^{\lambda}P_{\lambda})_{,m} (P^{s}P_{s})_{,k} g^{mk}}{(P^{i}P_{i})^{2}} - \frac{((P^{\lambda}P_{\lambda})_{,m} P^{m})^{2}}{(P^{i}P_{i})^{3}} \right) = \frac{1}{4} U^{j} U_{j} = \frac{K}{C^{4}} \rho C^{2}$$
(6.2)

Where ρC^2 is the energy density and ρ is the mass density.

In general, $\frac{\alpha^{i}}{C^{2}} = \frac{d^{2}x^{i}}{(C^{2}d\tau)^{2}}$ which is the 4-acceleration expressed in length units.

4. Classical non-relativistic limit – small uncharged particles in an electric field

The energy of matter is expressible as non-geodesic motion of particles measuring the upper limit of measurable proper time but by (6.1) also of other particles. We can use the classical limit which is a non-gravitational acceleration field also in an electric field. We have to explain how an uncharged particle that seems to be inertial actually interacts with the field and accelerates, possibly from the positive charges to the negative ones. As we shall see when we formalize the equation of gravity, charged particles change the metric of space time in a very special way. The energy density of

a static electric field is $\frac{\varepsilon_0}{2}E^2$ such that ε_0 is the permittivity constant and E^2 is the square norm of the electric field.

$$\frac{\varepsilon_0}{2} \int_{\Omega} E^2 dVolume = Energy \qquad (6.3)$$

We can easily see that our curvature 4-vector $U_m = \frac{(P^{\lambda}P_{\lambda})_{,m}}{P^{i}P_{i}} - \frac{(P^{\lambda}P_{\lambda})_{,\mu}P^{\mu}}{(P^{i}P_{i})^2}P_m$ such

that $P = \tau$ the upper limit of measurable time to an event, is perpendicular to the unit 4-vector $\frac{P_{\lambda}}{\sqrt{P^{i}P_{i}}}$ and that $\frac{P_{\lambda}}{\sqrt{P^{i}P_{i}}}$ is expressible by derivatives by length so we have an approximation in the non-relativistic classical limit by $K\rho C^{2} = \frac{1}{4}U_{m}U^{m} \approx \frac{\alpha^{2}}{C^{4}}$ as we saw. Where ρC^{2} is the energy density, K is the gravitational constant and α is an acceleration by a force field, expressing a

gravitational constant and α is an acceleration by a force field, expressing a non-geodesic motion of a particle. So we can write

$$\frac{K\varepsilon_0}{2C^4}E^2 \approx \frac{\alpha^2}{C^4} \Rightarrow \sqrt{\frac{K\varepsilon_0}{2}}E \approx \alpha \quad (6.4)$$

(6.4) Yields $1.71888777*10^{-11}$ Metre² * Volt⁻¹ * Second⁻². It takes 1000000 Volts over a gap of 1mm to expose an acceleration field of 1.71888777 cm/sec². Since it is an acceleration field, also a neutral particle starting at rest must respect the conservation of (3) and accelerate unless the force field is changed by other force fields.

As will be described in the equation of gravity there is more to (6.4). A parallel plates capacitor charged at high enough voltage will manifest local gravity. There are ways that such gravity will respect the conservation laws by interaction with the inertial space-time, see James F. Woodward paper from 1997 [10], and D.W. Sciama, 1952

[11]. A problem maintaining high voltage in a parallel-plates-capacitor is the creation of pairs of charged particles [12]. It is possible to create very strong electric field densities by using relatively low voltage and by using conducting board and a cone to achieve a high gradient of the square norm of the electric field $\frac{d}{dx^i}E^2$ and this field is known to achieve the phenomenon of Dielectrophoresis [13]. By subtracting the force F_{DEP} caused by Dielectrophoresis from the total force F on a ball of matter in a strong average electric field E we would expect

$$\sqrt{\frac{K\varepsilon_0}{2}}Em_0 \approx F - F_{DEP}$$
 (6.5)

where m_0 is the mass of a ball in the electric field. It is much more difficult to achieve high values of E in homogenous fields even when electric potentials of millions of volts are applied. (6.5) has evidence in the experiment by T. Datta, M. Yin, A. Dimofte [14], however, this experiment was done with metal balls in which the surrounding field caused induced dipoles to appear. The Fly-By Anomaly [15] is probably also related to a very same interaction with the Earth magnetic field.

5. Classical non-relativistic limit – tidal force

If a metal rod is suddenly exposed to a very strong non-uniform gravitational field, the rod may break due to tidal forces. So we may think that our definition of an event as a non-gravitational interaction is wrong. However, the rod experiences tidal forces due to chemical and covalent bonds which are the reason for its non - geodesic motion and therefore the tidal rod experiment doesn't violate our definition of an event as a non-gravitational interaction.

6. The equation of gravity

Although QFT is not the subject of this paper, it is worth mentioning [9] and especially that the meaning of $U_m \neq 0$ is the Unruh effect. Also see Appendix C in this paper which presents the link between the pair U_m and $\frac{P_{\lambda}}{\sqrt{P^i P_i}}$ and Minkowsky

rotations. We continue from the minimum action of

$$Z = N^{2} = P_{\mu}P^{\mu} \text{ and } U_{\lambda} = \frac{Z_{\lambda}}{Z} - \frac{Z_{k}P^{k}P_{\lambda}}{Z^{2}} \text{ and } L = \frac{1}{4}U^{k}U_{k}$$

$$R = Ricci \text{ curvature.}$$

$$Min Action = Min \int_{\Omega} \left(\frac{1}{2}R - 8\pi L\right) \sqrt{-g} d\Omega$$

See Appendix A, for the most general equation, using Einstein Tensor

$$L = \frac{1}{4} U_{i} U^{i} \text{ and } Z = P^{k} P_{k}$$

$$\frac{8\pi}{4} \left(+ 2\left(\left(\frac{(P^{\lambda} P_{\lambda})_{,m} P^{m}}{Z^{3}} P^{k}\right);_{k} - 2\left(\frac{Z^{m}}{Z^{2}}\right);_{m}\right) P_{\mu} P_{\nu} + \frac{8\pi}{2} \left(\frac{(P^{\lambda} Z_{\lambda})^{2}}{Z^{3}} \frac{P_{\mu} P_{\nu}}{Z} - 2\frac{Z^{\lambda} Z_{\lambda}}{Z^{2}} \frac{P_{\mu} P_{\nu}}{Z} + \frac{1}{2} U_{\mu} U_{\nu} - \frac{1}{2} U_{k} U^{k} g_{\mu\nu} \right) = R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu}$$

$$\frac{8\pi}{4} (U_{\mu} U_{\nu} - \frac{1}{2} U_{k} U^{k} g_{\mu\nu} - 2U^{k};_{k} \frac{P_{\mu} P_{\nu}}{Z}) = R_{\mu\nu} - \frac{1}{2} Rg_{\mu\nu}$$
(7)

as we shall see later, if we consider variation by P^{μ} and by $g^{\mu\nu}$ and their derivatives and do not explicitly regard $P = \tau$ then a simple solution

$$W^{\mu\nu} = U^k;_k \frac{P_{\mu}P_{\nu}}{Z} = 0$$
 to the Euler Lagrange equations yields

$$\left(\frac{Z^{m}}{Z^{2}}\right);_{m} - \left(\frac{(P^{\lambda}P_{\lambda}),_{m}P^{m}}{Z^{3}}P^{k}\right);_{k} = -\left(\frac{Z^{\lambda}Z_{\lambda}}{Z^{3}} - \frac{(Z_{s}P^{s})^{2}}{Z^{4}}\right) \text{ and}$$

$$\left(\frac{Z^{k}}{Z}\right);_{k} - \left(\frac{(P^{\lambda}P_{\lambda}),_{m}P^{m}}{Z^{2}}P^{k}\right);_{k} = U^{k};_{k} = 0$$
(8)

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and therefore (7) becomes

$$\frac{8\pi}{4} (U_{\mu}U_{\nu} - \frac{1}{2}U^{k}U_{k}g_{\mu\nu}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$$
(9)

and we have -L = -R, such that $R_{\mu\nu}$ is the Ricci curvature tensor [16]. [17]. If we ignore (6), and the coming (18)-(24),(34),(35) and (36), it is a bit disappointing that after all the efforts we simply get [16] and [17] which look like an ordinary General Relativity matter-geometry equation. Up to a sign, there is always a way to solve the following equation $\frac{8\pi}{4}(U^{\mu}U^{\nu}-\frac{1}{2}U^{k}U_{k}g^{\mu\nu})=\frac{8\pi K}{C^{4}}T^{\mu\nu}$ in General Relativity constants for an ordinary dust energy momentum tensor and therefore (9) is consistent with existing theories and is an important link to well established work on General Relativity. (9) suggests $\frac{8\pi}{4}U^{\mu}U^{\nu} = R^{\mu\nu} \Rightarrow R^{\mu\nu}P_{\mu}P_{\nu} = 0$ so for sufficiently small geodesic normal exponential coordinates y_{μ} such that $\frac{dy_{\mu}}{d\tau} = p_{\mu}$, the volume direction is like the one of flat space cone in y_{μ} since $d\Omega_{SpaceTime} = (1 - \frac{1}{6}R_{ij}y^{i}y^{j} + O(|y|^{3}))d\Omega_{Euclidean} \text{ so } d\Omega_{SpaceTime} = (1 + O(|y|^{3}))d\Omega_{Euclidean}.$ To see how (6) is related to spinors [18] see Appendix C. However, (7) offers more interesting fields and (6) is a purely geometric term. We see that curvature of the gradient of upper limit on measurable time is equivalent to Ricci curvature. If that can be true then we have an equation that is based solely on geometry. We have a simple action (without spinors [18] and other advanced mathematical technology) of the form: $V^{\lambda}V_{\lambda} - \frac{1}{t^2}$ such that V_{λ} is a vector field and $\frac{1}{t}$ is also a scalar field. If the definition is in 3 dimensions, it hints at 4 dimensional Lorentzian metric geometry. U_{μ} is in units of $\frac{1}{Length}$. For the complex square (second power of) curvature operator we

set the curvature vector
$$\hat{U}_{\mu} = \frac{P_{\mu}; P^{*i}}{\sqrt{(P_k P^{*k})(P_L^* P^L)}} - \frac{P_k; P^{*i} P^{*k}}{(P_k P^{*k})(P_L^* P^L)}.$$
 Obviously

 $U_{\mu}P^{*\mu} = 0$. Bearing in mind that in our case $P_kP^{*k} = \frac{1}{2}(P_kP^{*k} + P^*_kP^k)$ we calculate

$$\frac{1}{2}(\hat{U}_{k}\hat{U}^{*k} + \hat{U}^{*}_{k}\hat{U}^{k})\sqrt{-g} =$$

$$SquareCurvature = \begin{pmatrix} \frac{1}{2}((P_{i}; P^{*j})(P^{*}_{k}; P^{L}) + (P^{*}_{i}; P^{j})(P_{k}; P^{*L}))g^{ik} \\ (P^{*i}P_{i})(P^{\lambda}P^{*}_{\lambda}) \\ (\frac{(P_{i}; P^{*i}P^{*j})(P^{*}_{\mu}; P^{\mu}P^{\nu})}{((P^{*i}P_{i})(P^{\lambda}P^{*}_{\lambda}))^{\frac{3}{2}}} \end{pmatrix} \sqrt{-g}$$

(10)

7. Quantum Gravity in a nutshell

The idea that τ is meaningful where a material reference frame can interact with a particle measuring that upper limit of time, requires a formalism of how much matter there is to "contribute" or influence that measurement as a wave function ψ and that is the idea behind the coupling $PP^* = \tau^2 \psi \psi^*$ to denote observer- time measurement coincidence. We would also like to discuss the calculative outcome of this philosophical idea. \hat{U}_k can be written in a more illuminating way as

$$\hat{U}_{k} = \left(\frac{\hat{N}^{2}_{k}}{\hat{N}^{2}} - \frac{\hat{N}^{2}_{j}(\tau\psi)^{*j}}{(\hat{N}^{2})^{2}}(\tau\psi)_{k}\right)$$
(10.1)

Where the index k, means derivative by the coordinate x^k , $\hat{N}^2 = (\tau \psi)_k (\tau \psi^*)^k$ and for the sake of simplicity $N^2 = \tau_k \tau^k$.

We can replace ψ by an eigen-function that depends on τ and write

$$\psi = e^{\frac{-iE\tau}{\hbar}} \text{ s.t. } i = \sqrt{-1}$$
(10.2)

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and where E plays the role of energy of a coupled wave function, so we have

$$(\tau\psi)_{k} = \tau_{k}\psi + \tau\psi_{k} = \tau_{k}\psi(1 - \frac{i\tau E}{\hbar})$$
(10.3)

$$\hat{N}^2 = \tau_k \tau^k \left(1 + \frac{\tau^2 E^2}{\hbar^2}\right) = N^2 \left(1 + \frac{\tau^2 E^2}{\hbar^2}\right)$$
(10.4)

and

$$\frac{\hat{N}_{s}^{2}}{\hat{N}^{2}} = \frac{N_{s}^{2}}{N^{2}} \frac{(1 + \frac{\tau^{2}E^{2}}{\hbar^{2}})}{(1 + \frac{\tau^{2}E^{2}}{\hbar^{2}})} + \frac{2\tau\tau_{s}E^{2}N^{2}/\hbar^{2}}{(1 + \frac{\tau^{2}E^{2}}{\hbar^{2}})N^{2}} = \frac{N_{s}^{2}}{N^{2}} + \frac{2\tau\tau_{s}E^{2}}{(\hbar^{2} + \tau^{2}E^{2})}$$
(10.5)

Now we want to calculate $\frac{\hat{N}_{j}^{2}(\tau\psi)^{*j}}{(\hat{N}^{2})^{2}}(\tau\psi)_{k}$ so we have

$$\frac{\hat{N}_{j}^{2}(\tau\psi)^{*j}}{(\hat{N}^{2})^{2}}(\tau\psi)_{k} = \left(\frac{N^{2}_{j}}{N^{2}} + \frac{2\tau\tau_{j}E^{2}}{(\hbar^{2} + \tau^{2}E^{2})}\right)\frac{(\tau^{j}\psi^{*}(1 + \frac{i\tau E}{\hbar}))}{(1 + \frac{\tau^{2}E^{2}}{\hbar^{2}})N^{2}}\tau_{k}\psi(1 - \frac{i\tau E}{\hbar})) = \left(\frac{N^{2}_{j}}{N^{2}} + \frac{2\tau\tau_{j}E^{2}}{(\hbar^{2} + \tau^{2}E^{2})}\right)\frac{\tau^{j}}{N^{2}}\tau_{k} = \frac{N^{2}_{j}\tau^{j}\tau_{k}}{(N^{2})^{2}} + \frac{2\tau\tau_{k}E^{2}}{(\hbar^{2} + \tau^{2}E^{2})}$$
(10.6)

Now from (10.1), (10.5) and (10.6) we have the result

$$\hat{U}_{k} = \left(\frac{\hat{N}_{k}^{2}}{\hat{N}^{2}} - \frac{\hat{N}_{j}^{2}(\tau\psi)^{*j}}{(\hat{N}^{2})^{2}}(\tau\psi)_{k}\right) = \\ \left(\left(\frac{N_{k}^{2}}{N^{2}} + \frac{2\tau\tau_{k}E^{2}}{(\hbar^{2} + \tau^{2}E^{2})}\right) - \left(\frac{N_{j}^{2}\tau^{j}\tau_{k}}{(N^{2})^{2}} + \frac{2\tau\tau_{k}E^{2}}{(\hbar^{2} + \tau^{2}E^{2})}\right)\right) = \\ \left(\frac{N_{k}^{2}}{N^{2}} - \frac{N_{j}^{2}\tau^{j}\tau_{k}}{(N^{2})^{2}}\right) = \left(\frac{N_{k}^{2}}{N^{2}} - \frac{N_{j}^{2}P^{j}P_{k}}{(N^{2})^{2}}\right) = U_{k}$$
(10.7)

Therefore if the wave function depends solely on the upper limit of measurable time, (10) is reduced to (3). Now recall (6.1) and we have that replacing $\tau \rightarrow \tau \psi \rightarrow f(\tau)\psi$ renders (3),(6),(7),(8),(9),(10) invariant. Recall that the upper limit of measurable time can be defined as a limit backwards from any event to near the "big bang" singularity

event or events and that τ starts from zero. Now suppose a particle appears as a fluctuation in space time then when looking at the (10.5) additive,

$$\frac{2\tau\tau_k E^2}{(\hbar^2 + \tau^2 E^2)} \cong \frac{2\tau\tau_k E^2}{(\pm\delta\tau)^2 (\delta E)^2}$$
(10.8)

we can see that the denominator can't be smaller than \hbar^2 for small enough τ and Eand since the denominator is in Joule * Second units, the physical meaning of such a denominator can be

$$(\pm \delta \tau)^{2} (\delta E)^{2} = (2\delta \tau)^{2} (\delta E)^{2} = \hbar^{2} + \tau^{2} E^{2} \Rightarrow$$

$$4(\delta \tau)^{2} (\delta E)^{2}) = \hbar^{2} + \tau^{2} E^{2} \Rightarrow$$

$$(\delta \tau)(\delta E) \ge \frac{\hbar}{2}$$
(10.9)

which is a form of the principle of uncertainty. The change in the direction of the gradient of the time field is due to the need to avoid discontinuity of gradient measurement by particle clocks in max proper time curves intersections .Discontinuity of the gradient is avoided by uncertainty of the intersection events/strings .Then $\psi\psi$ * could be the probability of the 4-location of such avoided geodesic conflict in the middle of a constellation of particles. The coupling of τ and ψ has an extra important meaning which is that quantum uncertainty resolves the discontinuities of the gradient of τ and prevents its measurement. In the classical model of gravity in this theory, τ is "smoothened out" by the equation (7) or (9) which are an approximation or a limit of a Quantum effect. The classical model is sufficient for a giving a new description of matter, however, ψ is required for resolving gradient singularities of τ that do not exist in the classical model.

8. Noether's theorem

Zero divergence if we consider variations of P_{μ} not as derivatives of P.

Another proof of the divergence conservation is based on the invariance of ζ under scaling of $P_{\mu} \rightarrow P_{\mu}(1 + \varepsilon)$ so $\delta P_{\mu} = \varepsilon P_{\mu}$ and by Noether's

theorem
$$\frac{\frac{d}{dx^{\nu}}\left(\varepsilon P_{\mu}\left(\frac{4P^{\mu}Z^{\nu}}{Z^{2}}-4P_{k}Z^{k}\frac{P^{\mu}P^{\nu}}{Z^{3}}\right)\sqrt{-g}\right)}{\varepsilon \frac{d}{dx^{\nu}}\left(\frac{4Z^{\nu}}{Z}-4P_{k}Z^{k}\frac{P^{\nu}}{Z^{2}}\right)\sqrt{-g}} = \varepsilon 4U^{\nu};_{\nu} = \varepsilon 4Div(U^{\nu}) = 0$$

conservation of the non-gravitational acceleration field exactly as will be shown in (8),(23).

9. Vaknin's Chronon fields

A natural question is, given a solution to (7) when is $\zeta = \frac{1}{4}U^k U_k \sqrt{-g}$ invariant under rotations $A_{\mu}^{\ j}$ of P_k ? By definition, a rotation can be seen as isometrics in Minkowsky space. Less than full isometrics is actually required. We require at first:

$$A_{\mu}^{\ j}P_{j}A_{\nu}^{\ k}P_{k}g^{\mu\nu} = P_{\mu}P_{\nu}g^{\mu\nu} = P_{k}P^{k}$$
(10.10)

So $Z = N^2 = P_{\lambda}P^{\lambda}$ is invariant and therefore also $Z_k = \frac{\partial Z}{\partial x^k}$ is invariant under $\hat{P}_{\mu} = A_{\mu}^{\ j}P_j$. We continue by recalling that,

$$U_{k} = \frac{N^{2}, }{N^{2}} - \frac{N^{2}, P^{j}P_{k}}{N^{4}} = \frac{Z_{k}}{Z} - \frac{Z_{j}P^{j}P_{k}}{Z^{2}}$$
(10.11)

We need to find a "rotation" $A_{\mu}^{\ j}$ for which $\zeta = \frac{1}{4}U^{k}U_{k}\sqrt{-g}$ remains invariant.

Since $U^k U_k = \frac{Z_k Z^k}{Z} - \frac{(Z_j P^j)^2}{Z^3}$ and since Z and Z_k remain invariant, we only need our "rotation" $A_{\mu}^{\ j}$ to keep $Z_j P^j$ invariant. It is sufficient for $A_{\mu}^{\ j}$ to be a "rotation" about the Z_j axis that leaves Z_j invariant i.e. $Z_{\mu} = Z_j A_{\mu}^{\ j}$ and

 $Z_{j}P^{j}$ invariant. For $Z^{\lambda}Z_{\lambda} \neq 0$, the vector that is rotated is

$$Y_{k} = \frac{P_{k}}{\sqrt{Z}} - \frac{Z_{\mu}Z_{k}}{Z^{\lambda}Z_{\lambda}} \frac{P^{\mu}}{\sqrt{Z}}$$
(10.12)

And

$$Y_{k} \frac{Z^{k}}{Z} = \frac{P_{k}}{\sqrt{Z}} \frac{Z^{k}}{Z} - \frac{Z_{\mu}Z_{k}}{Z^{\lambda}Z_{\lambda}} \frac{P^{\mu}}{\sqrt{Z}} \frac{Z^{k}}{Z} = 0$$
(10.13)

From the invariance of $Z_{\mu} = Z_j A_{\mu}^{\ j}$ and $Z_j P^j$ under the "rotation" $A_{\mu}^{\ j}$,

$$\hat{Y}_{k} \frac{Z^{k}}{Z} = \hat{Y}_{k} \frac{\hat{Z}^{k}}{\hat{Z}} = A_{k}^{\mu} Y_{\mu} A_{s}^{j} \frac{Z_{j}}{Z} g^{ks} =$$

$$\frac{\hat{P}_{k} Z^{k}}{\sqrt{Z} Z} - \frac{Z_{\mu} Z_{k}}{Z^{\lambda} Z_{\lambda}} \frac{P^{\mu}}{\sqrt{Z}} \frac{Z^{k}}{Z} = \frac{P_{k} Z^{k}}{\sqrt{Z} Z} - \frac{Z_{\mu} Z_{k}}{Z^{\lambda} Z_{\lambda}} \frac{Z^{k}}{Z} \frac{P^{\mu}}{\sqrt{Z}} = 0$$
(10.14)

An example condition for rotation is

$$-Y_{k} = A_{k}^{\ \mu} A_{\mu}^{\ j} Y_{j}$$
(10.15)

which reminds of spinors.

Vaknin's Chronon field A_k^{μ} as defined here can't exist if $P_j = KZ_j$ for some K

Because then $Y_k = \frac{KZ_k}{\sqrt{Z}} - \frac{Z^{\mu}Z_k}{Z^{\lambda}Z_{\lambda}} \frac{KZ_{\mu}}{\sqrt{Z}} = 0$ and the rotation is degenerated.

$$P_j = KZ_j$$
 also means $U_k = \frac{Z_k}{Z} - \frac{Z_j P^J P_k}{Z^2} = \frac{P_k}{KZ} - \frac{P_j P^J P_k}{KZ^2} = 0$.

A necessary condition for Vaknin's Chronon field to exist is therefore

$$A_k^{\mu} \neq \delta_k^{\mu} \Longrightarrow U_k \neq 0 \tag{10.16}$$

Such that δ_k^{μ} is the Kronecker delta.

In 1982 Dr. Sam Vaknin laid the foundations of the existence of a Chronon field [2] and also to possible irreversible cosmic expansion. Replacement of P_k by $A_k^{\ \mu}P_{\mu}$ leaves ζ invariant but not the curvature vector and therefore influences gravity.

There are other ways to use symmetries in $\zeta = \frac{1}{4}U^k U_k \sqrt{-g}$ to show spin, however, curves along which the upper limit of measurable time is measured and that enter a ball of hollow mass, cause Z_j at the center of the ball point in the Schwarzschild time axis. These clocks must come from the outside because clock ticks are slowed by gravity. In such a case rotation of P_k around Z_j makes physical sense.

(Fig. 5) – Conic rotation



10. Electro-gravitational engine

The idea of electro-gravity is based on both (6.4) and (7). From (6) and (6.4),

In a weak gravitational Schwarzschild field,

$$\frac{1}{2}U_{m} = \frac{1}{2}\left(\frac{(P^{\lambda}P_{\lambda})_{m}}{P^{i}P_{i}} - \frac{(P^{\lambda}P_{\lambda})_{\mu}P^{\mu}}{(P^{i}P_{i})^{2}}P_{m}\right) \approx \frac{a}{C^{2}} = \sqrt{\frac{K\varepsilon_{0}}{2}}\frac{E_{m}}{C^{2}}$$
(10.17)

C is the speed of light, *a* is the weak acceleration of an uncharged particle, ε_0 is the permittivity constant in vacuum, *K* is the gravitational constant and *E* is the static field such that $E_0 = 0$ and also $E^k;_0 = 0$.

The conservation law that governs non-zero divergence U^k ; $_k \neq 0$ will be described in (25) in the "Chameleon Fields, Pressure or Electro-gravity ? " section. Also see Inertial Induction in "The origin of Inertia", D.W. Sciama [11].

In an electrostatic capacitor the electric field is stationary so we have, up to a sign,

$$-\frac{1}{4}U^{k};_{k} \approx \frac{1}{2}\sqrt{\frac{K\varepsilon_{0}}{2}}\frac{E^{k};_{k}}{C^{2}}$$
(10.18)

Now looking at (7), $\frac{8\pi}{4} (U_{\mu}U_{\nu} - \frac{1}{2}U_{k}U^{k}g_{\mu\nu} - 2U^{k};_{k}\frac{P_{\mu}P_{\nu}}{Z}) = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ The term $\frac{8\pi}{4} (U_{\mu}U_{\nu} - \frac{1}{2}U_{k}U^{k}g_{\mu\nu}) \text{ can be approximated in the electrostatic field by}$ $\frac{1}{4} (U_{\mu}U_{\nu} - \frac{1}{2}U_{k}U^{k}g_{\mu\nu}) \approx \frac{K\varepsilon_{0}}{2}\frac{1}{C^{4}}(E_{\mu}E_{\nu} - \frac{1}{2}E_{k}E^{k}g_{\mu\nu}) \text{ and since our electric field}$

is stationary we can write,

$$-\frac{1}{4}2U^{k};_{k}\frac{P_{\mu}P_{\nu}}{Z}\approx\sqrt{\frac{K\varepsilon_{0}}{2}}\frac{E^{k};_{k}}{C^{2}}\frac{P_{\mu}P_{\nu}}{Z}$$
(10.19)

The gradient of the upper limit of time will be discussed later in the Schwarzscheild solution in Appendix B, following are derivatives by the radius and time,

$$P_t = \frac{dP}{dt} = (1 - \frac{R}{r}), P_r = \frac{dP}{dr} = \sqrt{\frac{r}{R}}, Z = \frac{r}{R} + \frac{R}{r} - 2$$
 such that R is

Schwarzschild radius. The most significant term is $\frac{P_r P_r}{Z} = \frac{r/R}{r/R + R/r - 2} \approx 1$.

In reality we have to take Friedmann–Lemaître–Robertson–Walker metric into account because there is not only one body of mass in the entire cosmos and due to symmetry of mass distribution $P_r/Z \rightarrow 0$ and the $P_t/Z \rightarrow 1$ term is also significant. Now sticking to the somewhat unrealistic idealized one body of mass, the contribution to the gravity equation is mainly to the $G_{rr} = R_{rr} - \frac{1}{2}g_{rr}R$ term.

$$8\pi \frac{r/R}{r/R + R/r - 2} \sqrt{\frac{K\varepsilon_0}{2}} \frac{E^k;_k}{C^2} = G_{rr}$$
(10.20)

From the electro-magnetic theory we have E^k ; $_k = \frac{\rho}{\varepsilon_0}$ such that ρ is the charge

density. We have

$$8\pi \frac{r/R}{r/R + R/r - 2} \sqrt{\frac{K}{2\varepsilon_0}} \frac{\rho}{C^2} = G_{rr} = G_{11}$$
(10.21)

And we see that the sign of the charge density ρ greatly influences gravity. In reality in a weak gravitational field we will have

$$8\pi \sqrt{\frac{K}{2\varepsilon_0}} \frac{\rho}{C^2} \approx G_{tt} = G_{00}$$
(10.22)

We yet do not know if the acceleration field in (6.4) is from minus to plus or vice versa but a single experiment is enough. Here is an example in which using different size of plates controls the charge density and therefore enables to design the geometry of the gravitational field. A usual parallel board capacitor will not work. The boards should be layered into thousands of disk layers gapped with slabs of an insulator or dielectric material in order for each one of the two disk boards to have high enough capacitance.

(Fig. 6) – Electro-gravity thrust engine that requires very high charge densities



By local integration over space, (10.22) is expressible as a virtual electro-gravitational mass, such that Q is the charge on each board or plate,

$$8\pi \sqrt{\frac{K}{2\varepsilon_0}} \frac{Q}{C^2} = \pm 8\pi K \frac{Virtual_Mass}{C^2}$$
 which yields,

$$Virtual_Mass = \frac{\pm Q}{\sqrt{2K\varepsilon_0}}$$
(10.23)

Let us see what we can do with one gram of ionized hydrogen.

The number of atoms by Avogadro's number is $n = 6.02214129 \times 10^{23}$. The charge of the electron is $e = 1.602176565 \times 10^{-19}$ Coloumbs so

 $Q = \pm 9.64853364595686885 \times 10^4$ Coloumbs $K = 6.67384 \times 10^{-11} \text{ m}^3 \text{kg}^{-1} \text{sec}^{-1}$ and $\varepsilon_0 = 8.8541878176... \times 10^{-12} \text{ F/m}$ so (10.23) reaches a virtual mass of

Virtual_Mass $\approx \pm 2.8066228550259684867784287266634 \times 10^{15} Kg$. That is far less than the mass of the Earth $M_{Earth} = 5.97219 \times 10^{24} Kg$ but the distance between the capacitor boards is much less than the average Earth radius and therefore a field that overcomes the Earth gravitational field is feasible.

11. Capacitance requirements for a practical electro-gravitational engine

We will calculate *Virtual_Mass* = $\frac{\pm Q}{\sqrt{2K\varepsilon_0}}$ for ± 20 Coulombs.

$$\frac{\pm 1 Coulomb}{\sqrt{2K\varepsilon_0}} = \pm 2.908859478561344421131641706156 \times 10^{10} Kg$$

Multiplied by 20 we have

$$\frac{\pm 20 \, Coulombs}{\sqrt{2K\varepsilon_0}} = 5.8177189571226888422632834123121 \times 10^{11} Kg$$

Consider a distance of one meter and Newton's gravity law as an approximation although this gravity resembles an electric dipole and not a single source,

$$\frac{K \cdot Virtual_Mass}{radius^2} = 5.8177189571226888422632834123121 \times 10^{11} Kg/1^2 = 38.826525484803685703050391368425 \frac{Meter}{Second^2}$$

That is a little less than 4g. Over a gap of one meter and within one square meter under one million volts we need by the following:

 $Capacitance_C * Voltage_V = Charge_Q$ so

 ± 20 Coulombs = 20 Micro_Farads *1000000 Volts but the capacitance of an ordinary parallel plates capacitor is $\frac{\varepsilon_0 A}{d}$ such that A is the boards area and d is the distance between them so because ε_0 is small, an impractically big area A is required. The solution to that problem can be that the boards in (Fig. 6) will be as offered, i.e. layered into many thin disks, possibly spaced by an insulator or by a dielectric material and thus each single board will have high capacitance.

12. Proof that SquareCurvature is the square (to the second power of)

conserving field curvature

The square curvature of a conserving vector field is defined by an arc length parameterization t along the curves it forms

$$Curv^{2} = \frac{d}{dt} \left(\frac{V_{\lambda}}{\sqrt{V^{k}V_{k}}}\right) \frac{d}{dt} \left(\frac{V_{\mu}}{\sqrt{V^{k}V_{k}}}\right) g^{\lambda\mu}$$
(11)

such that $g^{\lambda\mu}$ is a diagonal unit matrix. For convenience we will write $Norm \equiv \sqrt{V^k V_k}$ and $\dot{V}_{\lambda} \equiv \frac{d}{dt} V_{\lambda}$. For arc length parameter t. Let W_{λ} denote: $W_{\lambda} = \frac{d}{dt} \left(\frac{V_{\lambda}}{\sqrt{V^k V_k}} \right) = \frac{\dot{V}_{\lambda}}{Norm} - \frac{V_{\lambda}}{Norm^3} V_k \dot{V}_{\nu} g^{k\nu}$ (12)

Obviously

$$W_{\lambda}V_{k}g^{\lambda k} = \frac{\dot{V}_{\lambda}V_{k}g^{\lambda k}}{Norm} - \frac{V_{\lambda}V_{s}g^{\lambda s}}{Norm^{3}}V_{k}\dot{V}_{\nu}g^{k\nu} = \frac{\dot{V}_{\lambda}V_{k}g^{\lambda k}}{Norm} - \frac{V_{k}\dot{V}_{\nu}g^{k\nu}}{Norm} = 0$$
(13)

Thus

$$Curv^{2} = W_{\lambda}W^{\lambda} = \frac{\dot{V}_{\lambda}\dot{V}_{\nu}g^{\lambda\nu}}{Norm^{2}} - \frac{V_{\lambda}\dot{V}_{s}g^{\lambda s}}{Norm^{4}}V_{k}\dot{V}_{\nu}g^{k\nu} = \frac{\dot{V}_{\lambda}\dot{V}^{\lambda}}{Norm^{2}} - (\frac{V_{\lambda}\dot{V}^{\lambda}}{Norm^{2}})^{2}$$
(14)

Since $\frac{V_{\lambda}}{Norm}$ is the derivative of the normalized curve or normalized "speed", using

the upper Christoffel symbols,
$$\frac{d}{dt}V_{\lambda} = (\frac{d}{dx^r}V_{\lambda} - V_s\Gamma_{\lambda r}^s)\frac{dx^r}{dt} = (V_{\lambda};_r)\frac{V^r}{Norm}$$
 such

that x^{r} denotes the local coordinates. If V_{λ} is a conserving field then $V_{\lambda};_{r} = V_{r};_{\lambda}$ and thus $V_{\lambda},_{r}V^{r} = \frac{1}{2}Norm^{2},_{\lambda}$ and $Curv^{2} = \frac{\dot{V}_{\lambda}\dot{V}^{\lambda}}{Norm^{2}} - (\frac{V_{\lambda}\dot{V}^{\lambda}}{Norm^{2}})^{2} = \frac{1}{4}(\frac{Norm^{2},_{\lambda}Norm^{2},_{k}g^{\lambda k}}{Norm^{4}} - (\frac{Norm^{2},_{s}V_{r}g^{sr}}{Norm^{3}})^{2})$ (15)

Writing the last term in Riemannian geometry is the same field curvature operator that we chose on a conserving vector field.

13. Locally separable coordinates

The following is a bit speculative but may be important. It can't work globally because different upper limit time curves may intersect at single events. We see that 3 dimensions hint at 4 dimensional action. This is done by looking at the action (3) in three dimensions and observing the following way to write it,

$$\begin{pmatrix} (P^{\lambda}P_{\lambda})_{,m}P^{m} & (P^{\lambda}P_{\lambda})_{,0} \\ (P^{i}P_{i})^{\frac{3}{2}} & (P^{\lambda}P_{\lambda})_{,0} \\ P^{i}P_{i} & P^{i}P_{i} \end{pmatrix} \begin{pmatrix} (P^{\lambda}P_{\lambda})_{,1} & (P^{\lambda}P_{\lambda})_{,2} \\ P^{i}P_{i} & P^{i}P_{i} \end{pmatrix} \begin{pmatrix} (P^{\lambda}P_{\lambda})_{,0} & (P^{\lambda}P_{\lambda})_{,2} \\ 0 & g^{00} & g^{01} & g^{02} \\ 0 & g^{20} & g^{21} & g^{22} \end{pmatrix} \begin{pmatrix} (P^{\lambda}P_{\lambda})_{,0} & (P^{\lambda}P_{\lambda})_{,0} \\ (P^{\lambda}P_{\lambda})_{,0} & (P^{\lambda}P_{\lambda})_{,0} \\ P^{i}P_{i} & (P^{\lambda}P_{\lambda})_{,1} \\ P^{i}P_{i} & (P^{\lambda}P_{\lambda})_{,2} \\ (P^{\lambda}P_{\lambda})_{,2} & (P^{\lambda}P_{\lambda})_{,2} \\ 0 & g^{20} & g^{21} & g^{22} \end{pmatrix}$$

$$g^{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & g^{00} & g^{01} & g^{02} \\ 0 & g^{10} & g^{11} & g^{12} \\ 0 & g^{20} & g^{21} & g^{22} \end{pmatrix} \text{ and } q^{ij} = \begin{pmatrix} g^{00} & g^{01} & g^{02} \\ g^{10} & g^{11} & g^{12} \\ g^{20} & g^{21} & g^{22} \end{pmatrix}$$

$$(16)$$

Where $g^{\mu\nu}$ is the metric tensor in 4 dimensions and q^{ij} is in 3. q^{ij} implicitly refers to a local submersion [19] where time is locally held constant.

Can we do the opposite, look at 4 dimensions and reduce the problem to 3 without violating the principle of covariance ?

First, our maximum proper time curves are intrinsic and do not depend on the coordinates. We can therefore agree that the absolute maximum proper time curves are different than ordinary geodesic curves on which only local maxima of proper time can be measured. We choose to describe (3) on our space-time in our special coordinates. Under correct choice of coordinates, the direction in space time of the maximum proper time is an eigenvector of the metric tensor with the biggest eigenvalue, our metric tensor is of the form presented in (16) for which the mixed space time terms are zero. Also,

$$P = \tau \implies P_0 = 1 \implies P_{\mu}P^{\mu} = -1 + P_{\lambda}P^{\lambda}, \lambda = 1, 2, 3 \text{ and}$$
$$P_{0,1} = P_{0,2} = P_{0,3} = P_{1,0} = P_{2,0} = P_{3,0} = 0$$

We can assume as possible $P_1 \neq 0, P_2 \neq 0, P_3 \neq 0$ especially if multiple maximum proper time curves to the same event 'e' exist. So instead of (3) we reduce the action to become three dimensional,

Tweaked SquareCurvature
$$\equiv \frac{1}{4} \left(\frac{(P^{\lambda}P_{\lambda})_{,m} (P^{s}P_{s})_{,k} g^{mk}}{(-1+P^{i}P_{i})^{2}} - \frac{((P^{\lambda}P_{\lambda})_{,m} P^{m})^{2}}{(-1+P^{i}P_{i})^{3}} \right) \sqrt{g}$$
or
$$T \text{ weaked } BE \equiv \frac{1}{4} \left(\frac{(P^{\lambda}P_{\lambda})_{,m} (P^{s}P_{s})_{,k} g^{mk}}{(-1+P^{i}P_{i})} - (\frac{(P^{\lambda}P_{\lambda})_{,m} P^{m}}{(-1+P^{i}P_{i})})^{2} \right) \sqrt{g} = (-1+P^{i}P_{i}) \cdot SquareCurvature$$

$$(17)$$

This means that on our three dimensional sub-manifolds ("Leaves of a foliation"), there is a corresponding action operator that is free of derivative dependence on time. Solving the Euler Lagrange equations for the Tweaked Square Curvature and receiving a plurality of solutions is indeed a promising direction of research.

14. Unsynchronizability

Since P is not constant on the 3 dimensional sub-manifolds perpendicular to the upper-limit-of-measurable-time-from-near-big-bang curves, these manifolds are not synchronizable and are therefore not the ideal inflating S(3) i.e. Friedmann – Robertson - Walker.

15. History of the paper's concept of time

The idea of an unreachable time, such as maximum proper time from a common event or set of events from which we can say the cosmos had started, i.e. "big bang", is not new [20], [21] and it appears in Hebrew writing such as the Book of Principles by the philosopher Rabbi Josef Albo 1380-1444. Rabbi Josef Albo wrote about time that can be measured by devices and another aspect of time which he termed immeasurable which he considered as an absolute inaccessible time because it does not depend on subjective measurement. The maximum proper time can't be measured by any massive devices because due to General Relativity, clock ticks are slowed down by the gravitational field of any mass.

16. Conservation of known matter from the Euler Lagrange Equations

Finally we get the following zero divergence:

$$\frac{d}{dx^{\mu}}\left(\frac{\partial L}{\partial P_{\mu}} - \frac{d}{dx^{\nu}}\frac{\partial L}{\partial P_{\mu,\nu}}\right)\left(U_{k}U^{k}\sqrt{-g}\right) = W^{\mu};_{\mu}\sqrt{-g} = 0 \text{ where}$$

 W^{μ} is obtained from the subtraction of (35) from (36), see Appendix A. Variation by P_{μ} and its derivatives is a special case

$$\left(\frac{\partial L}{\partial P_{\mu}} - \frac{d}{dx^{\nu}} \frac{\partial L}{\partial P_{\mu,\nu}}\right) (U_{k}U^{k}\sqrt{-g}) = 0.$$

$$\left(-4\left(\frac{Z^{\nu}}{Z^{2}}\right);_{\nu} - 4\frac{Z_{m}Z^{m}}{Z^{3}}\right)P^{\mu} + 4\left(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}}\right);_{\nu}P^{\mu}$$

$$-2\frac{Z_{m}P^{m}Z^{\mu}}{Z^{3}} + 6\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu} =$$

$$-4\left(\frac{Z^{\nu}}{Z^{2}}\right);_{\nu}P^{\mu} - 4\frac{Z_{m}Z^{m}}{Z^{3}}P^{\mu} +$$

$$+4\left(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}}\right);_{\nu}P^{\mu} + 4\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu}$$

$$-2\frac{Z_{m}P^{m}}{Z^{2}}\left(\frac{Z^{\mu}}{Z} - \frac{Z_{m}P^{m}P^{\mu}}{Z^{2}}\right) =$$

$$-4\left(\left(\frac{U^{k}}{Z}\right);_{k} + \frac{U^{k}U_{k}}{Z}\right)P^{\mu} - 2\frac{Z_{m}P^{m}}{Z^{2}}U^{\mu} = 0$$
(18)

Recall that $U^k P_k = 0$, multiplication by $\frac{-P_{\mu}}{4}$ and contraction yields,

$$\left(\left(\frac{Z^{\nu}}{Z^{2}}\right);_{\nu}-\left(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}}\right);_{\nu}\right)Z+\frac{Z_{m}Z^{m}}{Z^{2}}-\frac{(Z_{m}P^{m})^{2}}{Z^{3}}=0$$
(19)

$$\left(\frac{Z^{\nu}}{Z^{2}}\right);_{\nu}-\left(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}}\right);_{\nu}+\frac{1}{Z}\left(\frac{Z_{m}Z^{m}}{Z^{2}}-\frac{(Z_{m}P^{m})^{2}}{Z^{3}}\right)=0$$
(20)

and as a result of (20) the following term from (7) vanishes,

$$-2(U^{k});_{k} \frac{P^{\mu}P^{\nu}}{Z} = -2(\frac{U^{k}}{Z});_{k} P^{\mu}P^{\nu} - 2U^{k}U_{k} \frac{P^{\mu}P^{\nu}}{Z} = -2(\frac{(P^{\lambda}P_{\lambda}),_{m}P^{m}}{Z^{3}}P^{k});_{k} P^{\mu}P^{\nu} - 2\frac{(Z_{s}P^{s})^{2}}{Z^{3}}\frac{P^{\mu}P^{\nu}}{Z} + 2(\frac{Z^{m}}{Z^{2}});_{m} P^{\mu}P^{\nu} + 2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}\frac{P^{\mu}P^{\nu}}{Z} = -2(\frac{(P^{\lambda}P_{\lambda}),_{m}P^{m}}{Z^{3}}P^{k});_{k} P^{\mu}P^{\nu} - 2\frac{(Z_{s}P^{s})^{2}}{Z^{3}}\frac{P^{\mu}P^{\nu}}{Z} + 2(\frac{Z^{m}}{Z^{2}});_{m} P^{\mu}P^{\nu} + 2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}\frac{P^{\mu}P^{\nu}}{Z} = 2\left((\frac{Z^{m}}{Z^{2}});_{m} - (\frac{(P^{\lambda}P_{\lambda}),_{m}P^{m}}{Z^{3}}P^{k});_{k} + \frac{1}{Z}(\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}} - \frac{(Z_{s}P^{s})^{2}}{Z^{3}})\right)P^{\mu}P^{\nu} = 0$$
(21)

Which yields a simpler equation (9). Recall that $U^{\nu} = \frac{Z^{\nu}}{Z} - \frac{(Z_s P^s)P^{\nu}}{Z^2}$,

And that
$$\frac{Z_{\nu}}{Z}U^{\nu} = U_{\nu}U^{\nu}$$

 $(\frac{Z^{\nu}}{Z^{2}});_{\nu} - (\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu} + \frac{1}{Z}(\frac{Z_{m}Z^{m}}{Z^{2}} - \frac{(Z_{m}P^{m})^{2}}{Z^{3}}) =$
 $(\frac{U^{\nu}}{Z});_{\nu} + \frac{1}{Z}(U_{m}U^{m}) = \frac{1}{Z}(U^{\nu});_{\nu} - \frac{1}{Z^{2}}U^{\nu}Z_{\nu} + \frac{1}{Z}(U_{m}U^{m}) =$ (22)
 $\frac{1}{Z}(U^{\nu});_{\nu} = 0$

Which proves (8)

$$(U^{\nu});_{\nu} = 0$$
 (23, see 8)

And proves the simple representation of the field equations

That we saw in (7)

$$\frac{8\pi}{4} (U^{\mu}U^{\nu} - \frac{1}{2}U_{k}U^{k}g^{\mu\nu}) = R^{\mu\nu} - \frac{1}{2}Rg^{\mu\nu}$$
(24, see 9)

17. Chameleon Fields, Pressure or Electro-gravity ?

As we can see, the more general case is,

$$W^{\mu};_{\mu} = \begin{pmatrix} 4(\frac{(Z_{s}P^{s})P^{\nu}}{Z^{3}});_{\nu}P^{\mu} + 4\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu} + \\ -4(\frac{Z^{\nu}}{Z^{2}});_{\nu}P^{\mu} - 4\frac{Z_{m}Z^{m}}{Z^{3}}P^{\mu} + \\ 2\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu} - 2\frac{Z_{m}P^{m}Z^{\mu}}{Z^{3}} \end{pmatrix};_{\mu} = 0$$
(25)
$$\frac{d}{z^{\mu}}(\frac{\partial L}{\partial P} - \frac{d}{z^{\mu}}\frac{\partial L}{\partial P})(U_{k}U^{k}\sqrt{-g}) = W^{\mu};_{\mu}\sqrt{-g} = 0$$
(26)

$$\frac{d}{dx^{\mu}}\left(\frac{\partial L}{\partial P_{\mu}} - \frac{d}{dx^{\nu}}\frac{\partial L}{\partial P_{\mu,\nu}}\right)\left(U_{k}U^{k}\sqrt{-g}\right) = W^{\mu};_{\mu}\sqrt{-g} = 0$$
(2)

instead of

$$\left(\frac{\partial L}{\partial P_{\mu}} - \frac{d}{dx^{\nu}}\frac{\partial L}{\partial P_{\mu},\nu}\right)\left(U_{k}U^{k}\sqrt{-g}\right) = 0$$
(27)

An effect which is contrary to gravity will add a positive delta to the Ricci curvature and therefore from (7), multiplication by the metric tensor $g_{\mu\nu}$ and contraction yields,

+2(
$$\frac{(P^{\lambda}P_{\lambda})_{,m}P^{m}}{Z^{3}}P^{k}$$
);_kZ+2 $\frac{(Z_{s}P^{s})^{2}}{Z^{3}}$ -2($\frac{Z^{m}}{Z^{2}}$);_mZ-2 $\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}$ <0 (28)

An effect which adds gravity, will add negative delta to Ricci curvature and therefore,

+2(
$$\frac{(P^{\lambda}P_{\lambda})_{,m}P^{m}}{Z^{3}}P^{k}$$
);_kZ+2 $\frac{(Z_{s}P^{s})^{2}}{Z^{3}}$ -2($\frac{Z^{m}}{Z^{2}}$);_mZ-2 $\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}$ >0 (29)

Known matter will be simpler

$$+2(\frac{(P^{\lambda}P_{\lambda}), P^{m}}{Z^{3}}P^{k}); Z+2\frac{(Z_{s}P^{s})^{2}}{Z^{3}}-2(\frac{Z^{m}}{Z^{2}}); Z-2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}=0$$
(30)

It is possible that either (28) or (29) is mathematically not valid. Additional terms can't violate the vanishing of the divergence of Einstein tensor. High order derivatives

of the metric tensor and pressure were studied by Deser and Tekin [22]. For application of (28) and/or (29) to space-time warp drive please refer to [23].

18. Acknowledgement

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19. Conclusions – test for the theory

General

Maximal time from an event back to near "big bang" event or events, is measured by particle clocks. This time sets an upper limit on measurable time quite similar to the way the speed of light sets an upper limit on speed. Physics, however is local and therefore only the gradient of this upper limit has a true physical meaning.

Since time is measured by material clocks, these material clocks are influenced by forces. The particle that can measure the maximal possible time from the "big bang"

event or events to an event within matter will therefore be influenced by such forces along with the influence of its own gravitational field. Non geodesic motion as uniform acceleration is well defined by Friedman-Scarr representation (although the author does not agree with all of their claims) and has a linear interpretation by an anti-symmetric tensor [24] which also indirectly describes an intuitive relation to spinors (well at least if we can blissfully afford to ignore wave functions, and group representation, see Appendix C), however, the most interesting effects that this theory offers, are beyond the scope of Friedman-Scarr representation of acceleration $A_{\mu\nu}W^{\nu} = a_{\mu}$, speed $W_{\mu}W^{\mu} = 1$ and acceleration matrix $A_{\mu\nu} = -A_{\nu\mu}$. As a down-to-earth summary, the proof of this theory will start by experimental evidence that there is a lower limit of acceleration $\frac{1}{C^4} \frac{du^i}{d\tau} \frac{du_i}{d\tau}$ as a particle interacts with a constant material force field. This will show that the idea of event that is discussed in this paper has a physical meaning. There is experimental evidence regarding high gradient of an electric field in which force that acts on metal balls is represented as the ordinary force on the induced dipole as in ordinary dielectrophoresis [13] plus an unexpected "force" that depends on mass [14]. On one hand [14] can attest to the existence of true force field, which is not gravity, that depends on mass but can also be gravity. This is one way to achieve a unique trajectory of the maximally measured proper time by any massive test particle including zero mass Chronons [2] which are perfect theoretical clocks. In this case, [14] can be an exposure of a fundamental field, more basic than the electro-magnetic forces and it can be expressed in the non relativistic classical limit as

 $\sqrt{\frac{K\varepsilon_0}{2}}Em_0 \approx F - F_{DEP}$. Even in a homogenous static electric field we would expect a very weak force field, $1.71888777 * 10^{-11}$ Metre² * Volt⁻¹ * Second⁻². It takes 1000000 Volts over a gap of 1mm to expose an acceleration field of 1.71888777 cm/sec²

which results in a force that depends on mass.

This property offers a new technology as an electro-gravitational engine which is realized as an Alqubierre Warp Drive [23] and respects a more general conservation law (25) which could be D. W. Sciama's Inertial Induction [11]. Further research and even government funding is highly important.

20. Appendix A: The Euler Lagrange Equations of the SquareCurvature action

We will not solve the entire system

$$Z = P_{\mu}P^{\mu} \text{ and } U_{\lambda} = \frac{Z_{\lambda}}{Z} - \frac{Z_{k}P^{k}P_{\lambda}}{Z^{2}} \text{ and } L = \frac{1}{4}U^{k}U_{k}$$

$$R = Ricci \ curvature.$$

$$\delta \int_{\Omega} \left(\frac{1}{2}R - \frac{8\pi}{4}U^{k}U_{k}\right) \cdot \sqrt{-g} d\Omega = 0$$
(31)

But rather focus on $\int_{\Omega} U^k U_k \sqrt{-g} d\Omega$

$$\begin{split} & L = \frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}} \ s.t. \ Z = P_{\mu}P^{\mu} \\ & \frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}} - \frac{d}{dx^{m}} \frac{\partial(L\sqrt{-g})}{\partial(g^{\mu\nu},_{m})} = \\ & \left(-2(\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}} P_{\mu}P_{\nu}P^{m});_{m} - 2\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}} (\Gamma_{\mu}^{\ i}{}_{m}P_{i}P_{\nu}P^{m} + \Gamma_{\nu}^{\ i}{}_{m}P_{\mu}P_{i}P^{m}) + \right) \\ & + 2(\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}} P_{\mu}P_{\nu});_{m}P^{m} + 2\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}} (\Gamma_{\mu}^{\ i}{}_{m}P_{i}P_{\nu}P^{m} + \Gamma_{\nu}^{\ i}{}_{m}P_{\mu}P_{i}P^{m}) + \right) \\ & + 2(\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}} P_{\mu}P_{\nu});_{m}P^{m} + 2\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{4}})P_{\mu}P_{\nu} - \frac{1}{2}\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}g_{\mu\nu} \\ & = \left(-2(\frac{(P^{\lambda}P_{\lambda}),_{m}P^{m}}{Z^{3}} P^{k});_{k}P_{\mu}P_{\nu} - 2\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{Z} + 2(\frac{(P^{\lambda}P_{\lambda}),_{s}P^{s}}{Z^{3}})Z_{\mu}P_{\nu} + \right) \sqrt{-g} \\ & = \left(-\frac{1}{2}\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}g_{\mu\nu} - \frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{Z} \right) \\ \end{array}$$

$$\begin{split} & L = \frac{Z^{\lambda} Z_{\lambda}}{Z^{2}} \ s.t. \ Z = P_{\mu} P^{\mu} \\ & \frac{\partial (L\sqrt{-g})}{\partial g^{\mu\nu}} - \frac{d}{dx^{m}} \frac{\partial (L\sqrt{-g})}{\partial g^{\mu\nu},_{m}} = \\ & \left(-2(\frac{Z^{m} P_{\mu} P_{\nu}}{Z^{2}});_{m} - 2\frac{(\Gamma_{\mu}^{\ i} m P_{i} P_{\nu} Z^{m} + \Gamma_{\nu}^{\ i} m P_{\mu} P_{i} Z^{m})}{Z^{2}} \right) + \\ & + 2\frac{(P_{\mu} P_{\nu});_{m} Z^{m}}{Z^{2}} + 2\frac{(\Gamma_{\mu}^{\ i} m P_{i} P_{\nu} Z^{m} + \Gamma_{\nu}^{\ i} m P_{\mu} P_{i} Z^{m})}{Z^{2}} \right) + \\ & + \frac{Z_{\mu} Z_{\nu}}{Z^{2}} - 2\frac{Z_{s} Z^{s}}{Z^{3}} P_{\mu} P_{\nu} - \frac{1}{2}\frac{Z_{m} Z^{m}}{Z^{2}} g_{\mu\nu} \\ & \left(-2(\frac{Z^{m}}{Z^{2}});_{m} P_{\mu} P_{\nu} - 2\frac{Z^{\lambda} Z_{\lambda}}{Z^{2}} \frac{P_{\mu} P_{\nu}}{Z} - \frac{1}{2}\frac{Z_{k} Z^{k}}{Z^{2}} g_{\mu\nu} + \frac{Z_{\mu} Z_{\nu}}{Z^{2}} \right) \sqrt{-g} \end{split}$$

(33)

$$\begin{split} & Z = P_{\mu}P^{\mu} \text{ and } U_{\lambda} = \frac{Z_{\lambda}}{Z} - \frac{Z_{k}P^{k}P_{\lambda}}{Z^{2}} \text{ and } L = U^{\kappa}U_{k} \\ & \frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}} - \frac{d}{dx^{m}} \frac{\partial(L\sqrt{-g})}{\partial g^{\mu\nu}} = \\ & \left(+ 2(\frac{(P^{\lambda}P_{\lambda})_{\cdot m}P^{m}}{Z^{3}} P^{k})_{i_{k}}P_{\mu}P_{\nu} + 2\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}} \frac{P_{\mu}P_{\nu}}{Z} - 2(\frac{(P^{\lambda}P_{\lambda})_{\cdot v}P^{v}}{Z^{3}})Z_{\mu}P_{\nu} + \\ & + \frac{1}{2}\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}g_{\mu\nu} + \frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{2}} \frac{P_{\mu}P_{\nu}}{Z} + \\ & \left(-2(\frac{Z^{m}}{Z^{2}})_{i_{m}}P_{\mu}P_{\nu} - 2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}} \frac{P_{\mu}P_{\nu}}{Z} - \frac{1}{2}\frac{Z_{k}Z^{k}}{Z^{2}}g_{\mu\nu} + \frac{Z_{\mu}Z_{\nu}}{Z^{2}} \right) \\ & \left(+ 2((\frac{(P^{\lambda}P_{\lambda})_{i_{m}}P^{m}}{Z^{3}} P^{k})_{i_{k}} - 2(\frac{Z^{m}}{Z^{2}})_{i_{m}})P_{\mu}P_{\nu} + \\ & + \frac{2(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{Z} - 2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}\frac{P_{\mu}P_{\nu}}{Z} + \\ & + \frac{1}{2}\frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}} g_{\mu\nu} - \frac{1}{2}\frac{Z_{k}Z^{k}}{Z^{2}}g_{\mu\nu} + \\ & \left(+ \frac{Z_{\mu}Z_{\nu}}{Z^{3}} - 2(\frac{(P^{\lambda}P_{\lambda})_{i_{s}}P^{s}}{Z})Z_{\mu}P_{\nu} + \frac{(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{Z} \right) \\ & \left(+ 2((\frac{(P^{\lambda}P_{\lambda})_{i_{m}}P^{m}}{Z^{3}} P^{k})_{i_{k}} - 2(\frac{Z^{m}}{Z^{2}})_{i_{m}})P_{\mu}P_{\nu} + \\ & \left(+ \frac{2(P^{\lambda}Z_{\lambda})^{2}}{Z^{3}}\frac{P_{\mu}P_{\nu}}{Z} - 2\frac{Z^{\lambda}Z_{\lambda}}{Z^{2}}\frac{P_{\mu}P_{\nu}}{Z} + \\ & \left(+ \frac{U_{\mu}U_{\nu}} - \frac{1}{2}U_{k}U^{k}g_{\mu\nu} - 2U^{k}i_{k}\frac{P_{\mu}P_{\nu}}{Z^{2}} \right)\sqrt{-g} \end{array} \right), \sqrt{-g} = \\ & \left(U_{\mu}U_{\nu} - \frac{1}{2}U_{k}U^{k}g_{\mu\nu} - 2U^{k}i_{k}\frac{P_{\mu}P_{\nu}}{Z} \right)\sqrt{-g} \end{aligned} \right). \end{split}$$

(34)

$$\begin{bmatrix} L = \frac{(Z^{s}P_{s})^{2}}{Z^{3}} & \text{s.t. } Z = P^{\lambda}P_{\lambda} \text{ and } Z_{m} = (P^{\lambda}P_{\lambda})_{,m} \\ \frac{\partial(L\sqrt{-g})}{\partial P_{\mu}} - \frac{d}{dx^{\nu}} \frac{\partial(L\sqrt{-g})}{\partial P_{\mu,\nu}} = \\ \begin{pmatrix} -4\frac{(Z_{s}P^{s})}{Z^{3}}(P^{\mu}P^{\nu})_{;\nu} - 4\frac{(Z_{s}P^{s})}{Z^{3}}\Gamma_{i}^{\mu}\nu^{\mu}P^{\nu} + \\ +4\frac{(Z_{s}P^{s})}{Z^{3}}P^{\mu}_{;\nu}P^{\nu} + 4\frac{(Z_{s}P^{s})}{Z^{3}}\Gamma_{i}^{\mu}\nu^{\mu}P^{i}P^{k} + \\ +2\frac{Z_{m}P^{m}Z^{\mu}}{Z^{3}} - 6\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu} \end{bmatrix} \sqrt{-g} = \\ (-4(\frac{(Z_{s}P^{s})P^{\nu})}{Z^{3}})_{;\nu}P^{\mu} + 2\frac{Z_{m}P^{m}Z^{\mu}}{Z^{3}} - 6\frac{(Z_{m}P^{m})^{2}}{Z^{4}}P^{\mu})\sqrt{-g} \end{bmatrix} \\ \frac{L = \frac{Z^{s}Z_{s}}{Z^{2}} \text{ s.t. } Z = P^{\lambda}P_{\lambda} \text{ and } Z_{m} = (P^{\lambda}P_{\lambda})_{,m} \\ \frac{\partial(L\sqrt{-g})}{\partial P_{\mu}} - \frac{d}{dx^{\nu}} \frac{\partial(L\sqrt{-g})}{\partial P_{\mu,\nu}} = \\ \begin{pmatrix} -4(\frac{P^{\mu}Z^{\nu}}{Z^{2}})_{;\nu} - \frac{4}{Z^{2}}\Gamma_{i}^{\mu}kP^{i}Z^{k} + \\ + \frac{4}{Z^{2}}P^{\mu}_{;\nu}Z^{\nu} + \frac{4}{Z^{2}}\Gamma_{i}^{\mu}kP^{i}Z^{k} + \\ -4\frac{Z_{m}Z^{m}}{Z^{3}}P^{\mu}\sqrt{-g} \end{pmatrix} \sqrt{-g} \end{bmatrix}$$
(36)

21. Appendix B: The scalar time field of the Schwarzschild solution

We would like to calculate
$$\left(\frac{(P^{\lambda}P_{\lambda}), (P^{s}P_{s}), g^{mk}}{(P^{i}P_{i})^{2}} - \frac{((P^{\lambda}P_{\lambda}), P^{m})^{2}}{(P^{i}P_{i})^{3}}\right)$$
 in

Schwarzschild coordinates for a freely falling particle. This theory predicts that where there is no matter, the result must be zero. The result also must be zero along any geodesic curve but in the middle of a hollowed ball of mass the gradient of the absolute maximum proper time from "Big Bang" event or events, derivatives by space must be zero due to symmetry which means the curves come from different directions to the same event at the center. Close to the edges, gravitational lenses due to granularity of matter become crucial. The speed U of a falling particle as measured by an observer in the gravitational field is

$$V^{2} = \frac{U^{2}}{C^{2}} = \frac{R}{r} = \frac{2GM}{rC^{2}}$$
(37)

Where *R* is the Schwarzschild radius. If speed *V* is normalized in relation to the speed of light then $V = \frac{U}{C}$. For a far observer, the deltas are denoted by dt', dr' and, $\dot{r}^2 = (\frac{dr}{dt})^2 = V^2(1 - \frac{R}{r})$ (38)

because $dr = dr' / \sqrt{1 - R/r}$ and $dt = dt' \sqrt{1 - R/r}$.

$$P = \int_{0}^{t} \sqrt{\left(1 - \frac{R}{r}\right) - \frac{\left(\frac{dr}{dt}\right)^{2}}{\left(1 - \frac{R}{r}\right)}} dt = \int_{0}^{t} \sqrt{\left(1 - \frac{R}{r}\right) - \frac{\frac{R}{r}\left(1 - \frac{R}{r}\right)^{2}}{\left(1 - \frac{R}{r}\right)}} dt = \int_{0}^{t} \sqrt{\left(1 - \frac{R}{r}\right)^{2}} dt = \int_{0}^{t} \sqrt{\left(1 - \frac{R}{r}\right)^{2}} dt$$

Which results in,

$$P_t = \frac{dP}{dt} = (1 - \frac{R}{r}) \tag{39}$$

Please note, here t is not a tensor index and it denotes derivative by t !!!

On the other hand

$$P = \int_{\infty}^{r} \sqrt{\left(1 - \frac{R}{r}\right)\frac{1}{\dot{r}^{2}} - \frac{1}{\left(1 - \frac{R}{r}\right)}} dr = \int_{\infty}^{r} \sqrt{\frac{\left(1 - \frac{R}{r}\right)\frac{r}{R}}{\left(1 - \frac{R}{r}\right)^{2}} - \frac{1}{\left(1 - \frac{R}{r}\right)}} dr = \int_{\infty}^{r} \sqrt{\frac{\frac{r - R}{R}}{\frac{r - R}{r}}} dr = \int_{\infty}^{r} \sqrt{\frac{r}{R}} dr$$

Which results in

$$P_r = \frac{dP}{dr} = \sqrt{\frac{r}{R}}$$
(40)

Please note, here r is not a tensor index and it denotes derivative by r !!!

For the square norms of derivatives we use the inverse of the metric tensor,

So we have
$$(1-\frac{R}{r}) \rightarrow \frac{1}{(1-\frac{R}{r})}$$
 and $\frac{1}{(1-\frac{R}{r})} \rightarrow (1-\frac{R}{r})$

So we can write

$$N^{2} = P^{\lambda}P_{\lambda} = (1 - \frac{R}{r})P_{r}^{2} - \frac{1}{1 - \frac{R}{r}}P_{t}^{2} = (1 - \frac{R}{r})(\frac{r}{R} - 1) = \frac{r}{R} + \frac{R}{r} - 2$$

$$N^{2} = \frac{r}{R} + \frac{R}{r} - 2$$
 (41)

$$N^2_{\ \lambda} = \frac{dN^2}{dx^{\ \lambda}}$$
 And we can calculate

$$\frac{N^{2}{}_{\lambda}N^{2^{\lambda}}}{(N^{2})^{2}} = \frac{(1-\frac{R}{r})^{2}(\frac{1}{R}-\frac{R}{r^{2}})^{2}}{(\frac{r}{R}+\frac{R}{r}-2)^{2}}$$
(42)

We continue to calculate

$$N_{t}^{2}P_{t} = (1 - \frac{R}{r})^{2} (\frac{1}{R} - \frac{R}{r^{2}}) \sqrt{\frac{R}{r}} \text{ and}$$

$$\frac{N_{t}^{2}P_{t}}{(1 - \frac{R}{r})} = (1 - \frac{R}{r}) (\frac{1}{R} - \frac{R}{r^{2}}) \sqrt{\frac{R}{r}}$$
(43)

Please note, here t is not a tensor index and it denotes derivative by t !!!

$$(1 - \frac{R}{r})N^2 P_r = (1 - \frac{R}{r})(\frac{1}{R} - \frac{R}{r^2})\sqrt{\frac{r}{R}}$$
(44)

Please note, here r is not a tensor index and it denotes derivative by r !!!

$$N^{2}{}_{\lambda}P^{\lambda} = (1 - \frac{R}{r})(\frac{1}{R} - \frac{R}{r^{2}})(\sqrt{\frac{r}{R}} - \sqrt{\frac{R}{r}}) \text{ And}$$
$$(N^{2}{}_{\lambda}P^{\lambda})^{2} = (1 - \frac{R}{r})^{2}(\frac{1}{R} - \frac{R}{r^{2}})^{2}(\frac{r}{R} + \frac{R}{r} - 2)$$
(45)

So

$$\frac{(N^2{}_{\lambda}P^{\lambda})^2}{(N^2)^3} = \frac{(1-\frac{R}{r})^2(\frac{1}{R}-\frac{R}{r^2})^2}{(\frac{r}{R}+\frac{R}{r}-2)^2}$$
(46)

And finally, from (42) and (46) we have,

$$\left(\frac{(\mathbf{P}^{\lambda}\mathbf{P}_{\lambda})_{,\mathrm{m}} (\mathbf{P}^{s}\mathbf{P}_{s})_{,\mathrm{k}} g^{mk}}{(\mathbf{P}^{i}\mathbf{P}_{i})^{2}} - \frac{((\mathbf{P}^{\lambda}\mathbf{P}_{\lambda})_{,\mathrm{m}} \mathbf{P}^{m})^{2}}{(\mathbf{P}^{i}\mathbf{P}_{i})^{3}}\right) = \frac{N^{2}_{\lambda}N^{2^{\lambda}}}{(N^{2})^{2}} - \frac{(N^{2}_{\lambda}P^{\lambda})^{2}}{(N^{2})^{3}} = (47)$$

$$\frac{(1-\frac{R}{r})^{2}(\frac{1}{R}-\frac{R}{r^{2}})^{2}}{(\frac{r}{R}+\frac{R}{r}-2)^{2}} - \frac{(1-\frac{R}{r})^{2}(\frac{1}{R}-\frac{R}{r^{2}})^{2}}{(\frac{r}{R}+\frac{R}{r}-2)^{2}} = 0$$

which shows that indeed the gradient of time measured, by a falling particle until it hits an event in the gravitational field, has zero curvature as expected.

The term $N^2 = \frac{r}{R} + \frac{R}{r} - 2$ is slightly disturbing because at very far distances, $\frac{r}{R}$ becomes significant. Moreover, if *R* has a lower atomic limit, then for such *R* the term $\frac{r}{R}$ is a whole number! We now return to the discussion about a hollowed ball of mass. It is clear that the maximum proper time from "Big Bang" - event or events - curves entering the ball are symmetrical in relation to the center and therefore

$$P_r(0) = 0 = \frac{dP}{dr} \neq \sqrt{\frac{r_0}{R}}$$
 where r_0 is the radius in the far coordinate system of the

hollowed ball of mass. However, $P_t = \frac{dP}{dt} = (1 - \frac{R}{r_0})$. Writing the gradient in two

dimensions in t, r, ignoring the gravitational lenses due to mass granularity, and ignoring quantum uncertainties of coordinates and of energy momentum, we have

$$(P_{t}, P_{r}) = \begin{cases} r > r_{0} \Rightarrow ((1 - \frac{R}{r}), \sqrt{\frac{r}{R}}) \\ 0 < r < r0 \Rightarrow ((1 - \frac{R}{r_{0}}), \sqrt{\frac{r_{0}}{R}})??? \\ r = 0 \Rightarrow ((1 - \frac{R}{r_{0}}), 0)??? \end{cases}$$
(48)

The last result $P_r(0) = 0 = \frac{dP}{dr} \neq \sqrt{\frac{r_0}{R}}$ is an inevitable outcome of the symmetry in the

center of the ball. The gradient by the space coordinates must be zero and the change of direction in the gradient means that curvature is inevitable.

Center analysis if there is an atom of translation length

Without even negligible forces acting on a test particle and without quantum center location uncertainty, in the middle of a hollowed ball of mass the gradient of absolute maximal proper time is discontinuous due to symmetry. Suppose that the difference between the gradient at the center where r = 0 and where, $r = \delta r$, such that δr is small, results in $\delta N^2(0)$. We want to measure the second power of the curvature of the gradient of absolute maximum proper time due to that difference. Suppose that the change happens smoothly within a small radius from the center, measured around r = 0. We assume that such curvature measures how much the gradient is not geodesic due to curve intersections. Consider $g^{3,3} = (1 - \frac{R}{r_0})$ and $g^{0,0} = 1/(1 - \frac{R}{r_0})$

$$(P_t(0), P_r(0)) = ((1 - \frac{R}{r_0}), 0)$$
(49)

$$(P_t(\delta r'), P_r(\delta r')) = ((1 - \frac{R}{r_0}), \sqrt{\frac{r_0}{R}})$$
 (50)

$$\delta N^{2}(0) = \frac{\left(1 - \frac{R}{r_{0}}\right)^{2}}{\left(1 - \frac{R}{r_{0}}\right)} - \left(\left(\frac{\left(1 - \frac{R}{r_{0}}\right)^{2}}{\left(1 - \frac{R}{r_{0}}\right)} - \frac{r_{0}}{R}\left(1 - \frac{R}{r_{0}}\right)\right) = \frac{r_{0}}{R} - 1$$
(51)

$$\frac{\delta N^{2}(0)}{\delta r}g^{3,3} = \frac{\delta N^{2}(0)}{\delta r'\sqrt{1-\frac{R}{r_{0}}}}g^{3,3} =$$

$$\frac{\frac{r_0}{R} - 1}{\delta r' \sqrt{1 - \frac{R}{r_0}}} (1 - \frac{R}{r_0}) = (\frac{r_0}{R} - 1) \sqrt{1 - \frac{R}{r_0}}$$
(52)

$$\frac{\delta N^{2}(0)}{\delta t}g^{0,0} = \frac{\delta N^{2}(0)\sqrt{1-\frac{R}{r_{0}}}}{\delta t'}g^{3,3} = \frac{\delta N^{2}(0)\sqrt{1-\frac{R}{r_{0}}}}{\delta r'\sqrt{\frac{r_{0}}{R}}}g^{3,3} =$$
(53)

$$\frac{\delta N^{2}(0)}{\delta r' \sqrt{1 - \frac{R}{r_{0}}} \sqrt{\frac{r_{0}}{R}}} = \frac{\frac{r_{0}}{R} - 1}{\delta r' \sqrt{1 - \frac{R}{r_{0}}} \sqrt{\frac{r_{0}}{R}}}$$

$$N^{2}{}_{\lambda} N^{2^{\lambda}} = \frac{\left(\frac{r_{0}}{R} - 1\right)^{2}}{\left(\delta r'\right)^{2} \frac{r_{0}}{R}} - \frac{\left(\frac{r_{0}}{R} - 1\right)^{2}}{\left(\delta r'\right)^{2}} = -\frac{\left(\frac{r_{0}}{R} - 1\right)^{2}}{\left(\delta r'\right)^{2}} \left(1 - \frac{R}{r_{0}}\right)$$
(54)

$$\frac{N^{2}_{\lambda}N^{2^{\lambda}}}{(N^{2})^{2}} = -\frac{(\frac{r_{0}}{R}-1)^{2}(1-\frac{R}{r_{0}})}{(\delta r')^{2}(\frac{r_{0}}{R}+\frac{R}{r_{0}}-2)^{2}}$$
(55)

$$\frac{\delta N^{2}(0)}{\delta t} g^{0,0} P_{t} = \frac{(\frac{r_{0}}{R} - 1)\sqrt{\frac{R}{r_{0}}}\sqrt{1 - \frac{R}{r_{0}}}}{\delta r'}$$
(56)

$$\frac{\delta N^{2}(0)}{\delta r} g^{3,3} P_{r} = \frac{(\frac{r_{0}}{R} - 1)\sqrt{\frac{r_{0}}{R}}\sqrt{1 - \frac{R}{r_{0}}}}{\delta r'}$$
(57)

$$N^{2}{}_{\lambda}P^{\lambda} = \frac{(\frac{r_{0}}{R} - 1)\sqrt{1 - \frac{R}{r_{0}}}}{\delta r'} (\sqrt{\frac{R}{r_{0}}} - \sqrt{\frac{r_{0}}{R}})$$
(58)

$$(N^{2}{}_{\lambda}P^{\lambda})^{2} = \frac{(\frac{r_{0}}{R} - 1)^{2}(1 - \frac{R}{r_{0}})}{(\delta r')^{2}}(\frac{r_{0}}{R} + \frac{R}{r_{0}} - 2)$$
(59)

$$\frac{(N^{2}{}_{\lambda}P^{\lambda})^{2}}{(N^{2})^{3}} = \frac{\frac{(\frac{r_{0}}{R}-1)^{2}(1-\frac{R}{r_{0}})}{(\delta r')^{2}}(\frac{r_{0}}{R}+\frac{R}{r_{0}}-2)}{(\frac{r_{0}}{R}+\frac{R}{r_{0}}-2)^{3}} =$$
(60)

$$\frac{(\frac{r_0}{R}-1)^2(1-\frac{R}{r_0})}{(\delta r')^2(\frac{r_0}{R}+\frac{R}{r_0}-2)^2}$$

$$\frac{N^{2}{}_{\lambda}N^{2^{\lambda}}}{(N^{2})^{2}} - \frac{(N^{2}{}_{\lambda}P^{\lambda})^{2}}{(N^{2})^{3}} = -2\frac{(\frac{r_{0}}{R}-1)^{2}(1-\frac{R}{r_{0}})}{(\delta r')^{2}(\frac{r_{0}}{R}+\frac{R}{r_{0}}-2)^{2}}$$
(61)

$$\frac{N_{\lambda}^{2}N^{2^{\lambda}}}{(N^{2})^{2}} - \frac{(N_{\lambda}^{2}P^{\lambda})^{2}}{(N^{2})^{3}} = -2\frac{(\frac{r_{0}-R}{R})^{2}(\frac{r_{0}-R}{r_{0}})}{(\delta r')^{2}(\frac{(r_{0}-R)^{2}}{r_{0}R})^{2}} = \frac{-2}{(\delta r')^{2}(r_{0}-R)} = \frac{-2}{(\delta r')^{2}(1-\frac{R}{r_{0}})}$$
(62)

Given the radius $\delta r'$ that is seen within the gravitational field, the surface of a small ball around the center is smaller than expected in flat space-time,

$$Vol' = \frac{4\pi}{3} (\delta r')^3 (1 - \frac{R}{r_0})$$
(63)

We now calculate the curvature and then multiply it by the volume of the ball in which the direction of the gradient changes towards the center as seen in (49),(50),

$$SquareCurvature = \frac{1}{4} \left(\frac{N_{\lambda} N^{2^{\lambda}}}{(N^{2})^{2}} - \frac{(N^{2}_{\lambda} P^{\lambda})^{2}}{(N^{2})^{3}} \right) \sqrt{-g}$$
$$\frac{1}{4} \left(\frac{N^{2}_{\lambda} N^{2^{\lambda}}}{(N^{2})^{2}} - \frac{(N^{2}_{\lambda} P^{\lambda})^{2}}{(N^{2})^{3}} \right) Vol' = \frac{4\pi}{3} (\delta r')^{3} \left(\frac{r_{0} - R}{r_{0}} \right) \frac{1}{4} \frac{-2r_{0}}{(\delta r')^{2} (r_{0} - R)} = \frac{-2\pi}{3} \delta r'$$
(64)

(64) is very interesting because it depends only on δr and not on the mass of the gravitational source.

22. Appendix B2: Approximated validity test at the Planck scale

(64) imposes some strict limits on the offered theory. For the following, we assume that $\delta r = \delta r' (\sqrt{1 - R/\delta r})$ is big enough in comparison to the Schwarzschild radius *R* otherwise none of the following calculations will be valid. Suppose that all the matter we have is due to force field acting along the distance $\delta r'$. Then by (9) and the following conclusion that -L = -R and by Einstein equation of Gravity $\frac{8\pi K}{c^4}T_{\mu\nu} = G_{\mu\nu}$, and from (64) we have the following, $\frac{-(8\pi)2\pi}{3}\delta r' = -\frac{8\pi K}{c^4}\delta E$ (65)

Where δE is achieved via integration of energy on space. K is the known Gravity constant $K = 6.67384(80) * 10^{-11} m^3 kg^{-1} s^{-2}$ and $c = 2.99792458 * 10^8 m^1 s^{-1}$. So we can divide the equation by $\delta r'$

So we have
$$\frac{2\pi c^4}{3K} = \frac{\delta E}{\delta r'} = Force$$
 (66)

That is quite a strong force, about 2.5349249979452157571914167314415*10⁴⁴ Newtons. On the other hand if our energy is within a ball of radius $\delta r'$ and $\delta r'$ is also the uncertainty of the space coordinate of the center then we have by the law of uncertainty of Quantum Mechanics

$$\delta P \delta r' \ge \frac{\hbar}{2} \tag{67}$$

 $\hbar \simeq 1.05457172647 * 10^{34} J \cdot s$ and in the inequality extremity of equality,

$$\delta P \delta r' = \frac{\hbar}{2} \tag{68}$$

Now consider (66) which is a very strong force, acting on a small enough particle so virtually we can say that the speed of the particle is approximated by an average speed which the speed of light. So

$$c\,\partial P\,\partial r' \cong \partial E \Longrightarrow \partial r' = \frac{3K}{2\pi\,c^3}\,\partial P = \frac{3K\hbar}{4\pi\,c^3\,\partial r'} \Longrightarrow \partial r' = \sqrt{\frac{3K\hbar}{4\pi\,c^3}}$$
 (69)

Recall the definition of work as Force multiplied by length on which the force acted, we have from (69) and from (66)

Force *
$$\delta r' \cong \frac{2\pi c^4}{3K} \sqrt{\frac{3K\hbar}{4\pi c^3}} = \sqrt{\frac{\pi \hbar c^5}{3K}}$$
 (70)

This value is quite close to the Planck Energy $\sqrt{\frac{\hbar c^5}{K}}$.

23. Appendix C: Ashtekar variables in the description of a non-gravitational acceleration field

We would like to extract a general force from the relation between U_{μ} and P_{λ} .

This work is related to both Sam Vaknin's work from 1982 [2] and to Fridman - Scarr force representation [24]. Please note that the theory is of a non-gravitational acceleration that causes space-time curvature rather than force theory.

The curvature vector is
$$U_{\mu} = \frac{P_{\mu}; P^{*i}}{\sqrt{(P_k P^{*k})(P_L^* P^L)}} - \frac{P_k; P^{*i} P^{*k} P_{\mu}}{(P_k P^{*k})(P_L^* P^L)}$$
 or

$$U_m = \frac{(\mathbf{P}^{\lambda} \mathbf{P}_{\lambda})_m}{\mathbf{P}^{\mathrm{i}} \mathbf{P}_{\mathrm{i}}} - \frac{(\mathbf{P}^{\lambda} \mathbf{P}_{\lambda})_{,\mu} \mathbf{P}^{\mu}}{(\mathbf{P}^{\mathrm{i}} \mathbf{P}_{\mathrm{i}})^2} P_m. \quad \text{Obviously} \quad U_{\mu} P^{*\mu} = 0.$$

It measures how a trajectory of a particle measuring the gradient of upper limit of measurable time $P_k = \frac{dP}{dx^k} = \frac{d\tau}{dx^k}$, (or coped with probability function $P_k = \frac{d(\tau \psi)}{dx^k}$), is not geodesic. We now want to link P_k and U_{μ} to force in order to predict force on any test particle (with some conditions on its direction). By the principle of parsimony we would like to find a matrix $A_{k\mu} = -A_{\mu k}$ such that

$$A_{\mu k} \frac{P^{*k}}{\sqrt{\sqrt{(P_k P^{*k})(P^{*}_L P^L)}}} = U^{*}_{\mu}$$
(71)

The simplest representation needs only three complex variables, a, b, c

$$A_{\mu k} = \begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix}$$
(72)

Please note that the matrix is orthogonal in the Euclidean sense and that the following is Unitary in the Euclidean sense,

$$U_{\mu k} = \frac{1}{\sqrt{a^2 + b^2 + c^2}} \begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix}$$
(73)

We need to prove that if
$$W^{k} = \begin{pmatrix} r_{0} \\ r_{1} \\ r_{2} \\ r_{3} \end{pmatrix} = \frac{P^{*k}}{\sqrt{\sqrt{(P_{k}P^{*k})(P^{*}_{L}P^{L})}}}$$
 (74)

then indeed we can describe any perpendicular vector by spanning the 3D perpendicular space to (4),

$$\begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix} \begin{pmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{pmatrix} = a \begin{pmatrix} r_1 \\ -r_0 \\ -r_3 \\ r_2 \end{pmatrix} + b \begin{pmatrix} -r_2 \\ -r_3 \\ r_0 \\ r_1 \end{pmatrix} + c \begin{pmatrix} -r_3 \\ r_2 \\ -r_1 \\ r_0 \end{pmatrix}$$
(75)

The latter is a linear combination of vectors perpendicular to (73). Therefore (71) defines a force theory. Obviously $AA^{t} = (a^{2} + b^{2} + c^{2})I$ where I is the identity matrix and the determinant is $Det(A) = (a^{2} + b^{2} + c^{2})^{2} = (\frac{1}{2}(U_{i}U^{*i} + U^{*}_{i}U^{i}))^{2}$. Which quite reminds the determinant used by Abhay Ashtekar [25].

$$W^{k} = \begin{pmatrix} r_{0} \\ r_{1} \\ r_{2} \\ r_{3} \end{pmatrix} \text{ is a contravariant vector and } V_{i}(1) = \begin{pmatrix} r_{1} \\ -r_{0} \\ -r_{3} \\ r_{2} \end{pmatrix}, V_{i}(2) = \begin{pmatrix} -r_{2} \\ -r_{3} \\ r_{0} \\ r_{1} \end{pmatrix}, V_{i}(3) = \begin{pmatrix} -r_{3} \\ r_{2} \\ -r_{1} \\ r_{0} \end{pmatrix}$$

are covariant vectors. By our choice,

$$V_{i}(1)W^{i} = V_{i}(2)W^{i} = V_{i}(3)W^{i} = 0,$$

$$V_{i}(1)V_{j}(2)g^{ij} \neq 0, V_{i}(1)V_{j}(3)g^{ij} \neq 0, V_{i}(2)V_{j}(3)g^{ij} \neq 0$$
(76)

The latter differs from Ashtekar vectors [25] because only three orthogonality conditions are required. An open question: Is $A_{\mu k}$ applicable to any W^k direction ? A clue seems to prove that $V_i(1), V_i(2), V_i(3)$ are space-like and for the non relativistic limit to show that,

$$\begin{pmatrix} 0 & a & -b & -c \\ -a & 0 & c & -b \\ b & -c & 0 & -a \\ c & b & a & 0 \end{pmatrix} \begin{pmatrix} \gamma \beta_x \\ \gamma \beta_y \\ \gamma \beta_z \end{pmatrix} =$$

$$\begin{pmatrix} \gamma \beta_x \\ -1 \\ -\gamma \beta_z \\ \gamma \beta_y \end{pmatrix} + b \begin{pmatrix} -\gamma \beta_y \\ -\gamma \beta_z \\ 1 \\ \gamma \beta_x \end{pmatrix} + b \begin{pmatrix} -\gamma \beta_z \\ \gamma \beta_y \\ -\gamma \beta_z \\ 1 \end{pmatrix} \approx \begin{pmatrix} 0 \\ -a \\ b \\ c \end{pmatrix} \approx \begin{pmatrix} 0/C^2 \\ (a_x \pm g_x)/C^2 \\ (a_z \pm g_z)/C^2 \\ (a_z \pm g_z)/C^2 \end{pmatrix}$$
(77)

Such that $\gamma = \frac{1}{\sqrt{1 - v^2 / C^2}}, \beta_x = \frac{dx/dt}{C} = \frac{v_x}{C}$ etc. where C is the speed of light,

v the speed, as acceptable usual annotations in the special theory of relativity and accelerations are $a_x a_y$ and a_z . The classical limit of our square curvature of the gradient of upper limit of measurable time back to near big bang event or events, should be α^2/C^4 where a^2 is $\alpha^2 = (a_x \pm g_x)^2 + (a_y \pm g_y)^2 + (a_z \pm g_z)^2$ where (g_x, g_y, g_z) denotes the classical gravitational acceleration. If we want to measure acceleration in length units then, $\frac{d^2x^k}{(Cd\tau)^2}$ is simply the expression of the acceleration vector by differentiation by length, which is the source of the term α^2/C^4 .

Conflict of Interests

The author declares that there is no conflict of interests.

EYTAN H. SUCHARD

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