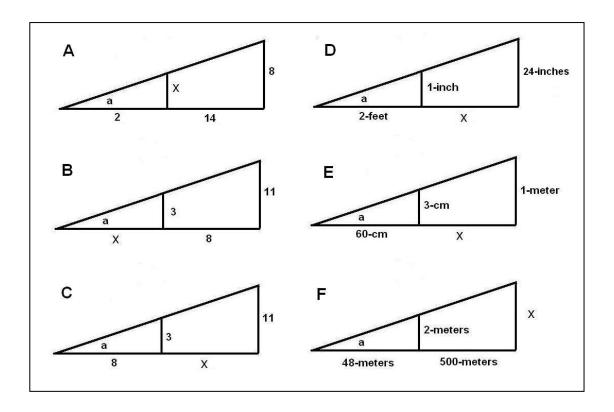


The corresponding sides of similar triangles are proportional to one another as the illustration to the left shows. Because the vertex angle of the triangles are identical measure, two objects at different distances from the vertex will subtend the same The corresponding angle, a. side to 'X' is '1' and the corresponding side to '2' is the combined length of '2+4'.

Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

Problem 2: Which triangles must have the same measure for the indicated angle **a**?

Problem 3: The sun is 400 times the diameter of the moon. Explain why they appear to have about the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?



Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

- A) X/2 = 8/16 so X = 1
- B) 3/X = 11/(X+8) so 3(X+8) = 11X; 3X + 24 = 11X; 24 = 8X and so X = 3.
- C) 3/8 = 11/(x+8) so 3(x+8) = 88; 3X + 24 = 88; 3X = 64 and so $X = 21 \frac{1}{3}$
- D) 1-inch / 2-feet = 24 inches / (D + 2 feet); First convert all units to inches; 1 / 24 = 24 / (D + 24); then solve (D + 24) = 24x24 so D = 576 24; D = 552 inches or 46 feet.
- E) 3 cm / 60 cm = 1 meter / (X + 60 cm). 3/60 = 1 meter / (X + 0.6 m) then 3(X + 0.60) = 60; 3X + 1.8 = 60; 3X = 58.2 meters so X = 19.4 meters.
- F) 2 meters / 48 meters = X / 548 meters; 1/24 = X/548; X = 548 / 24; so X = 22.8.

Problem 2: Which triangles must have the same measure for the indicated angle a?

Answer: Because the triangle (D) has the side proportion 1-inch /24-inches = 1/24 and triangle (F) has the side proportion 2 meters / 48 meters = 1/24 these two triangles, **D** and **F**, have the same angle measurement for angle a

Problem 3: The Sun is 400 times the diameter of the Moon. Explain why they appear to have the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?

Answer: From one of our similar triangles, the long vertical side would represent the diameter of the sun; the short vertical side would represent the diameter of the moon; the angle **a** is the same for both the sun and moon if the distance to the sun from Earth were 400x farther than the distance of the moon from Earth. Since the lunar distance is 384,000 kilometers, the sun must be at a distance of 154 million kilometers, which is close to the number given.



The larger of the two moons of Mars, Phobos, passes directly in front of the other, Deimos, in a new series of sky-watching images from NASA's Mars rover Curiosity. Large craters on Phobos are clearly visible in these images from the surface of Mars. No previous images from missions on the surface caught one moon eclipsing the other.

Deimos (small image), and Phobos (large image), are shown together as they actually were photographed by the Mast Camera (Mastcam) on NASA's Mars rover Curiosity on August 1, 2013.

How do we figure out how big something will look when it's far away? We draw a scale model of the object showing its diameter and its distance as the two sides of a triangle. The angle can then be measured. As the distance to the object increases, the 'angular size' of the object will decrease proportionately. A simple proportion can then be written that relates the angles to the lengths of the sides:

Let's see how this works for estimating the sizes of the moons of Mars as viewed from the Curiosity rover!

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

NASA Rover Gets Movie as a Mars Moon Passes Another http://www.nasa.gov/mission_pages/msl/news/msl20130815.html August 15, 2013

Problem 1 – Earth's moon is located 370,000 km from the surface of earth, and has a diameter of 3476 km. About how many degrees across does the lunar disk appear in the sky?

Answer: Apparent size = $57.3 \times (3476/370000) = 0.5 \text{ degrees}$.

Problem 2 - Deimos has a diameter of 7.5 miles (12 kilometers) and was 12,800 miles (20,500 kilometers) from the rover at the time of the image. Phobos has a diameter 14 miles (22 kilometers) and was 3,900 miles (6,240 kilometers) from the rover at the time of the image. What are the angular diameters of Phobos and Diemos as seen by the Curiosity rover?

Answer: Diemos: $57.3 \times (12/20500) = 0.033 \text{ degrees}$

Phobos: $57.3 \times (22/6240) = 0.2$ degrees.

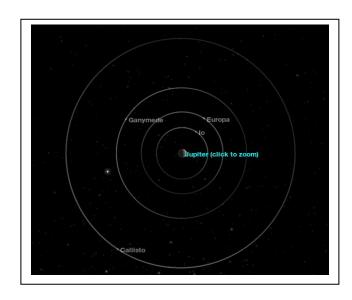
Note: Diemos appears to be about .033/.2 = 1/6 the diameter of Phobos in the sky.

Problem 3 – Mars is located 227 million kilometers from the sun, and the sun has a diameter of 1,400,000 kilometers. What is the angular diameter of the sun as viewed from Mars?

Answer: $57.3 \times (1400000/227,000,000) = 57.3 \times (1.4/227) = 0.35 \text{ degrees}.$

Problem 4 – Occasionally, Phobos and Diemos pass across the face of the sun as viewed from the surface of Mars. Will the moons create a full eclipse of the sun in the same way that Earth's moon covers the full face of the sun as viewed from Earth?

Answer: Diemos is only 0.033/0.35 = 1/11 the diameter of the sun in the sky as viewed from Mars, so it does not cover the full disk of the sun. As it passes across the sun it would look like a large dark spot. Phobos is $0.2/0.35 = \frac{1}{2}$ the diameter of the sun in the sky and it would not produce an eclipse like our moon does. It would look like a large black spot $\frac{1}{2}$ the diameter of the sun.



In the future, rovers will land on the moons of Jupiter just as they have on Mars. Rover cameras will search the skies for the disks of nearby moons. One candidate for landing is Europa with its ocean of water just below its icy crust.

The figure to the left shows the orbits of the four largest moons near Europa. How large will they appear in the Europan sky compared to the Earth's moon seen in our night time skies? The apparent angular size of an object in arcminutes is found from the proportion:

Apparent size	Diameter (km)
=	
3438 arcminutes	Distance (km)

The table below gives the diameters of each 'Galilean Moon' together with its minimum and maximum distance from Europa. Jupiter has a diameter of 142,000 km. Europa has a diameter of 2960 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun hass a diameter of 1.4 million km. Our Moon has a diameter of 3476 km.

	Shortest Distance (km)	Size (arcminutes)	Longest Distance (km)	Size (arcminutes)
Europa to Callisto	1.2 million		2.6 million	
Europa to lo	255,000		1.1 million	
Europa to Ganymede	403,000		1.7 million	
Europa to Jupiter	592,000		604,000	
Europa to Sun	740 million		815 million	
Earth to Moon	356,400		406,700	

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa.

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. . In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Problem 1 – From the information in the table, calculate the maximum and minimum angular size of each moon and object as viewed from Europa. Jupiter has a diameter of 142,000 km. Io has a diameter of 3620 km. Callisto's diameter is 4720 km and Ganymede's diameter is 5200 km. The sun has a diameter of 1.4 million km. Our Moon has a diameter of 3476 km. Answer: see below.

	Shortest Distance	Size (arcminutes)	Longest Distance	Size (arcminutes)
	(km)	(0.101111111111111111111111111111111111	(km)	(0.101111111111111111111111111111111111
Europa to Callisto	1.2 million	13.5	2.6 million	6.2
Europa to Io	255,000	48.8	1.1 million	11.3
Europa to Ganymede	403,000	44.4	1.7 million	10.5
Europa to Jupiter	592,000	824.7	604,000	808.3
Europa to Sun	740 million	6.5	815 million	5.9
Earth to Moon	356,400	33.5	406,700	29.4

Problem 2 – Compared to the angular size of the sun as seen from Jupiter, are any of the moons viewed from Europa able to completely eclipse the solar disk?

Answer: The solar disk has an angular size between 5.9 and 6.5 arcminutes. Only Callisto at its longest distance has an angular diameter (6.2 arcminutes) close to the solar diameter and so a complete eclipse is possible.

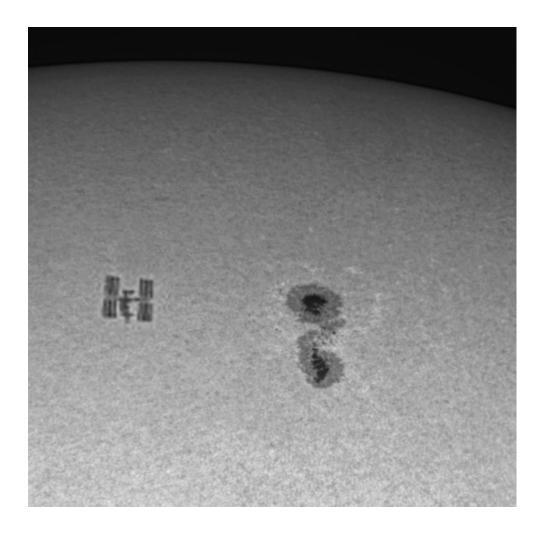
Problem 3 – Which moons as viewed from Europa would have about the same angular diameter as Earth's moon viewed from Earth?

Answer: Io and Ganymede have angular diameters between 11 and 49 arcminutes. Our moon has a range of sizes between 29.4 and 33.5 arcminutes, so at some points in the orbits of lo and Ganymede, they will appear about the same size as our moon dies in our night sky.

Problem 4 – Io is closer to Jupiter than Europa. That means that Io will be able to pass across the face of Jupiter as viewed from Europa. In terms of the maximum and minimum sizes, about how many times smaller is the apparent disk of Io compared to the disk of Jupiter?

Answer: When Jupiter appears at its largest (824 arcminutes) and lo at its smallest (11.3 arcminutes) Jupiter will be 73 times bigger than the disk of lo. When Jupiter is at its smallest (808 arcminutes) and lo is at its largest (48.8 arcminutes) Jupiter will appear to be 16.6 times larger than the disk of lo.

ISS and a Sunspot - Angular Scales



Photographer John Stetson took this photo on March 3, 2010 by carefully tracking his telescope at the right moment as the International Space Station passed across the disk of the sun.

The angular size, θ , in arcseconds of an object with a length of L meters at a distance of D meters is given by

$$\theta = 206265 \frac{L}{D}$$

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Answer: As viewed from the ground, the ISS subtends an angle of

Angle = $206265 \times (108 \text{ meters}/350,000 \text{ meters})$ so

Angle = 63 arcseconds.

At the distance of the sun, which is 150 million kilometers, the angular size of the ISS corresponds to a physical length of

L = 150 million kilometers x (63/206265) so

L = 46,000 kilometers.

The sunspot is comparable in width to that of the ISS and has a length about twice that of the ISS so its size is about 46,000 km x 92,000 km.

As a comparison, Earth has a diameter of 13,000 km so the sunspot is about 3 times the diameter of Earth in width, and 6 times the diameter of Earth in length.

Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Answer: From the ground, convert the speed of the ISS in km/sec to an angular speed in arcseconds/sec.

In one second, the ISS travels 10 km along its orbit. From the ground this corresponds to an angular distance of

Angle = $206265 \times (10 \text{ km}/350 \text{ km})$

= 5900 arcseconds.

The speed is then 5900 arcseconds/sec. The diameter of the sun is 0.5 degrees which is 30 arcminutes or 1800 arcseconds. To cover this angular distance, the ISS will take

T = 1800 arcseconds / (5900 arcseconds/s) so

T = 0.3 seconds!

Eclipses, Transits and Occultations



The images above show a variety of transits, eclipses and occultations. The images are labeled from left to right as (Top Row) A, B, C, D, E; (Middle Row) F, G, H, I, J, (Bottom Row), K, L, M, N, O. Using the definitions of these three astronomical events, identify which images go along with each of the three types of events. One example is shown below.

A) Deimos and the Sun ______ I) Moon and Star Cluster ______

B) Moon and Earth ______ J) Sun and Phobos ______

C) Sun and Mercury ______ K) Sun and Venus ______

D) Sun and Moon ______ L) Moon and Saturn ______

E) Rhea and Saturn _____ M) Sun and Moon ______

F) Rhea and Dione _____ N) Sun and Space Station _____

G) Jupiter and lo _____ O) Moon and galaxy ______

H) Earth and Moon ______

- A) <u>Transit</u> of Deimos across the Sun seen by Opportunity Rover
- B) **Occultation** of Earth by the Moon seen by Apollo-8 astronauts
- C) <u>Transi</u>t of Mercury across sun seen by TRACE?
- D) Transit of Moon across Sun seen by STEREO satellite
- E) Transit of satellite Rhea across Saturn seen by Cassini spacecraft
- F) Rhea occulting Dione seen by Cassini spacecraft near Saturn
- G) Io transiting Jupiter seen by Galileo spacecraft
- H) Earth occulting moon seen by Space Shuttle astronauts
- I) Moon occulting the Pleiades star cluster
- J) Phobos transiting sun seen by Opportunity Rover on Mars.
- K) Venus **transiting** the sun seen by TRACE satellite.
- L) Moon occulting Saturn
- M) Eclipse of the sun by the moon.
- N) Transit of the space station across sun
- O) Hypothetical occultation of the andromeda galaxy by the moon.

Special Image credits:

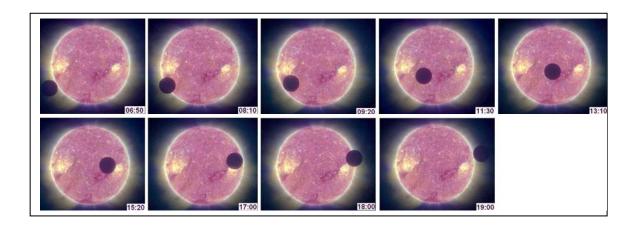
Moon occulting Saturn - Pete Lawrence (http://www.digitalskyart.com/) pete.lawrence@pbl33.co.uk.

Space Station transiting sun - John Stetson

 ${\bf Moon\ and\ Andromeda\ -\ Adam\ Block\ (ngc1535@caelumobservatory.com}$

- Tim Puckett (tpuckett@mindspring.com)

Moon and Star Cluster - Jerry Lodriguss (jerry5@astropix.com) http://www.astropix.com/



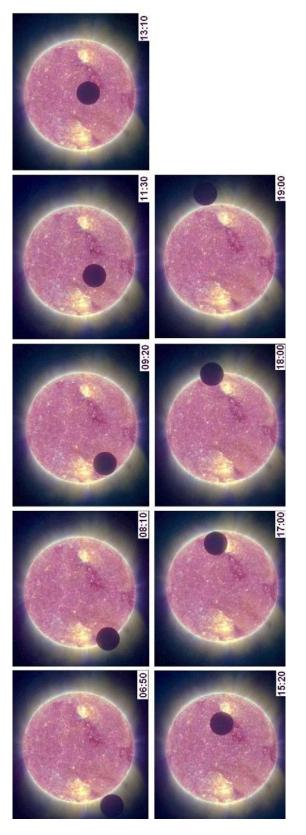
NASA's STEREO-B satellite is in an orbit around the sun at the same distance as Earth. On February 25, 2007 it took a series of pictures of the sun during the time when the moon was in transit across the sun, and when the satellite was 1.7 million km from the moon. The normal Earth-Moon distance is 380,000 km. By comparison, the distance to the sun from Earth is 149 million km.

Problem 1 - If the angular diameter of the sun was 2100 arcseconds at the time of the transit as viewed by STEREO-B, what was the diameter of the moon as viewed by STEREO-B in A) arcseconds? B) degrees?

Problem 2 - By what percentage was the sun's light dimmed during the times when the full, circular, lunar disk covered the solar surface in these images?

Problem 3 - Based on the sequence of images in the above series, with the Universal Time (hour:minutes) indicated in the lower right corner of each image, draw the light curve of this lunar transit from start to finish in terms of the percentage of sunlight visible by the STEREO-B satellite, from 93% to 100%, and the Universal Time in decimal hours since 06:00.

Answer Key 9.6



Problem 1 - Method 1: The diameter of the solar disk in each image is about 21 millimeters, so the scale of the images is 2100 arcseconds/21 mm = 100 arcseconds/mm. The lunar disk measures about 4mm, so the angular diameter is 4mm (100 asec/mm) = **400 arcseconds**.

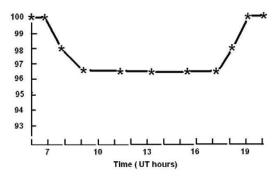
Method 2: Using the trigonometric formula $Tan(\theta) = D/R$ $Tan(\theta) = 3400/1,700,000 = 0.002$ So $\theta = 0.11$ degrees = **410 arcseconds**.

So the diameter of the moon is about **400** arcseconds

Problem 2 - By what percentage was the sun's light dimmed during the times when the full, circular, lunar disk covered the solar surface in these images?

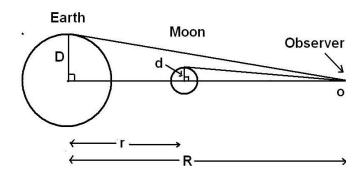
Answer: By the ratio of their circular areas. $100\% \times (400/2100)^2 = 3.6\%$

Problem 3 - Answer: See graph below:



Movies of the transit can be found at the STEREO website

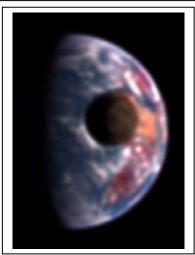
http://stereo.gsfc.nasa.gov/gallery/item.php?id=selects&iid=8



In space, your perspective can change in complicated ways that sometimes go against Common Sense unless you 'do the math'. This happens very commonly when we are looking at one object pass across the face of another. Even though the Moon is 1/4 the diameter of Earth, the simple ratio of their apparent diameters will depend on how far from them YOU are when you see them.

An important 'skinny triangle' relationship for triangles states that, if the angle is less than 1 degree (< 0.017 radians), the angle measure in radians equals very nearly the sine of the angle, which is just the ratio of the opposite side to the hypotenuse: θ = D/R where D is the radius of the object in kilometers, R, is the distance to the object in kilometers, and θ is the angular radius of the object in radians. For instance, the Moon is located r = 384,000 km from Earth and it has a radius of d = 1,738 km, so its angular radius is 1,738/384,000 = 0.0045 radians. Since 1 radian = 57.3 degrees, the angular radius of the Moon is 0.0045 x 57.3 = 0.26 degrees, so its diameter is 0.52 degrees as viewed from Earth.





Problem 1 - In the figure above, assume that the diameter of the Moon is less than 1 degree when spotted by the spacecraft located at 'O'. What is the angular diameter of: A) the Moon, θ_{m} , in terms of d and R? B) The Earth, θ_{e} , in terms of D and R? and C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R?

Problem 2 - A spacecraft is headed directly away from the Moon along the line connecting the center of Earth and the Moon. At what distance will the angular diameter of the Moon equal the angular diameter of Earth?

Problem 3 - The figure to the top left shows the martian satellite Phobos passing across the disk of the sun as viewed from the surface of Mars by the Rover Opportunity. If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken?

Problem 4 - The Deep Impact spacecraft observed the Moon pass across the disk of Earth as shown in the photo to the bottom left. The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft?

Problem 5 - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1?

Space Math

Problem 1 - What is the angular diameter of:

A) the Moon, θ_m , in terms of d and R? Answer:

$$\theta m = \frac{2\pi d}{R - r}$$

B) The Earth, θ_e , in terms of D and R? Answer:

$$\theta e = \frac{2\pi D}{R}$$

C) What is the ratio of the angular diameter of the Moon to the Earth in terms of d, D, r and R? Answer:

$$\frac{\theta m}{\theta e} = \frac{R}{R - r} \frac{d}{D}$$

Problem 2 - At what distance will the angular diameter of the Moon equal the angular diameter of Earth? Answer:

$$1 = \frac{R}{R - 384,000} \frac{(1,786)}{(6,378)}$$
 so 3/4 R = 384,000 km, and so R = 512,000 km.

Problem 3 - If the ratio of the diameters is 1/2, and if r = 228 million km, d = 10 km, and D=696,000 km, about how far from Phobos was Opportunity at the time the photo was taken? Answer:

$$\frac{1}{2} = \frac{R}{R - 228 million} \frac{(10 km)}{(696,000 km)}$$
 so 34799 R = 228 million km, and so R = 6,552 km.

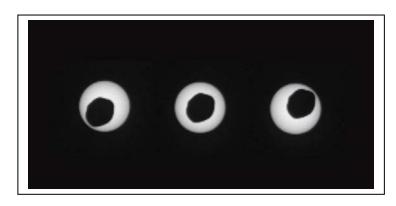
Problem 4 - The ratio of the disk diameters is 1/3.9, and if r = 384,000 km, d=1,786 km and D = 6,378 km, about how far from Earth, R, was the spacecraft? Answer:

$$\frac{1}{3.9} = \frac{R}{R - 384,000km} \frac{(1,786km)}{(6,378km)}$$
 so $0.02 \text{ R} = 384,000 \text{ km}$, and so R = 19.2 million km. Note: the actual distance was about 30 million km for the photo shown in this problem.

Problem 5 - As the distance, R, becomes very large, in the limit, what does the angular ratio of the disk approach in the equation defined in Problem 1? Answer: As R becomes much, much larger than r (e.g the limit of r approaches infinity), then the equation approaches

$$\frac{\theta m}{\theta e} = \frac{d}{R} \frac{R}{D}$$

and since the 'R' terms cancel, we get the angular ratio approaching the physical ratio d/D of the diameters of the two bodies. In other words, although the apparent angular sizes change rapidly when you are very close to the bodies and the value of R is comparable to 'r', at very great distances, the angular ratio approaches a constant value d/D. This has many practical consequences in the search for planets around other stars as they' transit' their stars.



This set of three images shows views three seconds apart as the larger of Mars' two moons, Phobos, passed directly in front of the sun as seen by NASA's Mars rover Curiosity.

Curiosity photographed this annular eclipse with the rover's Mast Camera on August 17, 2013 or 'Sol 369' by the Mars calendar.

Curiosity paused during its drive to Mount Sharp to take a set of observations that the camera team carefully calculated to record this celestial event. Because this eclipse occurred near mid-day at Curiosity's location on Mars, Phobos was nearly overhead. This timing made Phobos' silhouette larger against the sun -- as close to a total eclipse of the sun as is possible from Mars.

Angular size is given by
$$\Theta = 57.3 \text{ x}$$
 ------ degrees Distance (km)

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Annular Eclipse of the Sun by Phobos, as Seen by Curiosity http://www.nasa.gov/mission_pages/msl/news/msl20130828.html Aug. 28, 2013

Problem 1 – At the time of the transit, Phobos which has a diameter of 11 km, was 6000 km from the surface of Mars, and Mars was 235 million km from the Sun. What are the angular diameters of the Sun and Phobos viewed from the surface of Mars if the diameter of the Sun is 1.4 million km? How large are these angles in minutes of arc?

Answer: Phobos: $57.3 \times (11/6000) = 0.10 \text{ degrees}$

or 0.1 degrees x 60 = 6 minutes of arc

Sun: $57.3 \times (1.4/235) = 0.34 \text{ degrees}$ or $0.34 \times 60 = 20 \text{ minutes of arc}$.

Problem 2 – Phobos orbits at a distance of 9,400 km from the center of Mars at a speed of 2.1 km/sec. As viewed from the surface of Mars (6000 km), how fast is it traveling across the sky in arcminutes/second?

Answer: If the object is located 6000 km from the surface and moves 2.1 km, it will appear to cover an angle of $57.3 \times (2.1/6000) = 0.02$ degrees. There are 60 arcminutes in 1 degree, so this angle is 1.2 arcminutes. Since this distance is traveled in 1 second, the angular speed is **1.2 arcminutes /second**.

Problem 3 – To the nearest second, how long will it take for Phobos to travel completely across the disk of the sun?

Answer: The diameter of the sun is 20 arcminutes and the diameter of Phobos is 6 arcminutes. When the center of the disk of Phobos is 3 arcminutes from the eastern edge of the sun, it is just touching the solar disk and about to start its transit. When it is 3 arcminutes from the western edge of the sun, it has just finished its transit, so the total distance it has to travel is 3 arcminutes + 20 arcminutes + 3 arcminutes or 26 arcminutes. It travels at a speed of 1.2 arcminutes per second, to it will cover 26 arcminutes in about 26/1.2 = 22 seconds.