

This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6.0 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image?
Step 2: Use clues in the image description to determine a physical distance or length.
Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?
Question 2: What is the width, in centimeters, of the largest feature?
Question 3: What is the size of the smallest feature you can see?
Question 4: How big is the stone shown by the arrow?

## Answer Key:

This NASA, NEAR image of the surface of the asteroid Eros was taken on February 12, 2001 from an altitude of 120 meters (Credit: Dr. Joseph Veverka/ NEAR Imaging Team/Cornell University)). The image is 6 meters wide.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the image width is 6 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters long is the image? Answer: 144 millimeters

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 6.0 meters
Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in centimeters per millimeter.
Answer: 6.0 meters / $144 \mathrm{~mm}=600 \mathrm{~cm} / 144$ millimeters $=4.2 \mathrm{~cm} / \mathrm{mm}$
Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in centimeters.

Question 1: What are the dimensions, in meters, of this image?
Answer: Height $=80 \mathrm{~mm}=336 \mathrm{~cm}$ or 3.4 meters so area is $6.0 \mathrm{~m} \times 3.4 \mathrm{~m}$

Question 2: What is the width, in centimeters, of the largest feature?
Answer: The big rock at the top of the image is about 60 mm across or 2.5 meters.

Question 3: What is the size of the smallest feature you can see?
Answer: The small pebbles are about 0.5 millimeters across or 2.1 centimeters (about 1 inch).

Question 4: How big is the stone shown by the arrow?
Answer: 4 millimeters or 17 centimeters (about 7 inches).

## Space Math



Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

Problem 2 - At a distance of $3,162 \mathrm{~km}, \mathrm{~A}$ ) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth ( 0.5 degrees), how much larger was Lutetia?

Problem 3 - If the Rosetta spacecraft traveled at a speed of $15 \mathrm{~km} / \mathrm{s}$ on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Image credit: ESA 2010 MPS for OSIRIS Team
MPS/UPD/LAM/IAA/RSSD/INTA/UPM/DASP/IDA. European Space Agency's Rosetta spacecraft, with NASA instruments aboard, flew past asteroid Lutetia on Saturday, July 10, 2010. Asteroid diameter about 130 km . This view is from a distance of $3,162 \mathrm{Km}$. The probe spent several hours shooting images of the irregular shaped space rock, circling more than 450 million km ( 280 million miles) out from the sun. The space agency says its OSIRIS camera was able to capture detail down to just a few dozen meters.

Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

Answer: 130 km / 152 mm = 855 meters/mm.

Problem 2 - At a distance of $3,162 \mathrm{~km}, \mathrm{~A}$ ) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth ( 0.5 degrees), how much larger was Lutetia?

Answer: A) $\operatorname{Tan}(\theta)=130 / 3162$ so $\theta=2.3$ degrees.
B) Asteroid is $2.3 / 0.5=4.6$ times the diameter of the full moon.

Problem 3 - If the spacecraft traveled at a speed of $15 \mathrm{~km} / \mathrm{s}$ on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Answer: To have an angular diameter of 0.5 degrees ,the spacecraft has to be 4.6 times farther away than at closest approach, or a distance of $4.6 \times 3,162=14,545 \mathrm{~km}$. From the Pythagorean theorem, the distance from the closest approach point is just d $=\left(14545^{2}-3162^{2}\right)^{1 / 2}=14,197 \mathrm{~km}$. To travel this distance takes $\mathrm{T}=14197 / 15=946$ seconds or 15.8 minutes.


On July 15, 2011 the NASA spacecraft Dawn completed a 2.8 billion kilometer journey taking four years, and went into orbit around the asteroid Vesta. Vesta is the second largest asteroid in the Asteroid Belt. Its diameter is 530 kilometers. After one year in orbit, Dawn departed in 2012 for an encounter with asteroid Ceres in 2015. Meanwhile, from its orbit around Vesta, it will map the surface and see features less than 1 kilometer across.

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Problem 3 - Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Problem 4 - The Space Shuttle traveled at a speed of $28,000 \mathrm{~km} / \mathrm{hr}$ in its orbit around Earth. How many times faster than the Shuttle does the Dawn spacecraft travel?

Problem 1 - Use a millimeter ruler and the diameter information for this asteroid to determine the scale of this image in kilometers per millimeter.

Answer: When printed using a standard printer, the width of the asteroid is about 123 millimeters. Since the true diameter of the asteroid is 530 km , the scale is then $\mathrm{S}=530$ $\mathrm{km} / 123 \mathrm{~mm}=4.3$ kilometers per millimeter.

Problem 2 - What is the diameter of the largest and smallest features that you can see in this image?

Answer: Students can find a number of small features in the image that are about 1 mm across, so that is about 4.3 kilometers. Among the largest features are the 9 large craters located along the middle region of Vesta from left to right. Their diameters are about 4 to 7 millimeters or 17 to 30 kilometers across. The large depression located in the upper left quadrant of the image is about 40 mm long and 15 mm wide in projection, which is equivalent to $172 \mathrm{~km} \times 65 \mathrm{~km}$ in size.

Problem 3-Based on the distance traveled, and the time taken by the Dawn satellite, what was the speed of this spacecraft in A) kilometers per year? B) kilometers per hour?

Answer: Time $=4$ years, distance $=2.8$ billion km , so the speed is $A) \mathrm{S}=2.8$ billion km / 4 years $=700$ million km/year. B) Converting to an hourly rate, $S=700$ million $\mathrm{km} / \mathrm{yr} \times$ (1 year/365 days) $\times(1$ day/24 hours) $=79,900 \mathrm{~km} / \mathrm{hour}$.

Problem 4 - The Space Shuttle traveled at a speed of $28,000 \mathrm{~km} / \mathrm{hr}$ in its orbit around Earth. How many times faster than the Shuttle does the Dawn spacecraft travel?

Answer: $\quad$ Ratio $=(79,900 \mathrm{~km} / \mathrm{hr}) /(28,000 \mathrm{~km} / \mathrm{hr})=2.9$. So Dawn is traveling at an average speed that is $\mathbf{2 . 9}$ times faster than the Space Shuttle!

Note: The Space Shuttle is traveling 8 times faster that a bullet (muzzle velocity) from a high-powered M16 rifle! So Dawn is traveling 23 times faster than such a bullet!

## Asteroids Between Mars and the Sun

Astronomers have catalogued and determined orbits for 30,000 minor planets in the solar system (asteroids, comets etc). Over 150,000 bodies larger than a few hundred meters across have been spotted and remain to have their orbits exactly determined. Below is a plot made on November 30, 2005 of the locations of all known objects (white dots) within the orbit of Mars whose path skirts the inner edge of the asteroid belt (green dots).


Question 1 - How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars?

Question 2 - If the radius of Earth's orbit is 150 million km, what is the scale of this figure in millions of km per millimeter?

Question 3 - About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel?

Question 4 - How many asteroids crossed Earth's orbit on November 30, 2005 ?

The plot of the minor planets was obtained from the IAU, Minor Planets Center (http://cfa-www.harvard.edu/iau/lists/InnerPlot2.html). It shows the location of the known asteroids, comets and other 'minor planets' for November 30, 2005. The plot shows the orbits of Mercury, Venus, Earth and Mars. Objects that have parahelias (closest orbit location to the sun) less than 1.3 AU are shown in white circles. More details can be found at http://www.space.com/scienceastronomy/solarsystem/asteroid_toomany_011019-1.html

Question 1 - How many minor planets are located inside the orbit of Mercury? Venus? Earth? Mars? Answer: Students should count the plotted symbols within (or on) the first inner ring (Mercury's orbit) and get 13 symbols ( don't include the sun!). For the space between Venus and Mercury, I count 119 spots which makes the total 132 minor planets inside the orbit of Venus. Between Earth and Venus there are about 280 for a total of 412 minor planets inside Earth's orbit. Between Mars and Earth, a careful student may be able to count about 833 which means there are $833+412=1245$ minor planets inside the orbit of Mars.

Question 2 - What is the scale of this figure in millions of km per millimeter? Answer: The radius of Earth's orbit is 150 million kilometers, which corresponds to 70 millimeters, so the scale is 2.1 million km per millimeter.

Question 3 - About how far apart are the minor planets from each other on this particular day? Would they be a hazard for space travel? Answer: Although the asteroids are only plotted as though they are located in the same 2-D plane, we can estimate from the average 'eyeball' separation between asteroids of about 2 millimeters, that they are about 4.2 million kilometers apart. A spacecraft would not collide with a typical asteroid unless it was directed to specifically target an asteroid for investigation...or impact. It is a popular myth about space travel that astronauts have to dodge asteroids when traveling to Mars or the outer solar system. Interplanetary dust grains and micro-meteoroids are, however, a much bigger hazard!!

Question 4 - How many asteroids crossed Earth's orbit on November 30, 2005? Answer: Just count the number of white spots that touch the line that represents the orbit of Earth. There are about 70 spots that touch the orbit line.

Could the Earth collide with them? Each dot is about 1 mm in radius, so this represents a distance of 2 million kilometers. Since Earth is only $12,000 \mathrm{~km}$ across and a typical asteroid is only 1 km across, collision is extremely unlikely even when the diagram seems to show otherwise. There is another way that this diagram makes the situation look worse than it is. Because the asteroid orbits can be several million miles above or below the orbit of Earth as the asteroids cross this location, there are very few close calls between Earth and any given asteroid in the current catalog. Astronomers call the ones that get close 'Near-Earth Asteroids' and there are about 700 of these known. Only one of these known NEAs may get close enough to Earth in the next 30 years to be a potential collision problem, but astronomers are still finding dozens more NEAs every year.


The Asteroid 2005 YU55 passed inside the orbit of our moon sometime between November 8 and November 9, 2011. The diagram shows the lunar orbit as a circle centered on Earth. The diagonal line is the orbit of Earth around the sun. The line segment $A B$ is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point $A$ is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438 , where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time ( 9.438 days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. What was the date and time when the asteroid was at Point A?

The line segment $A B$ is a portion of the orbit of the asteroid. The horizontal line at the bottom of the page is 1 million kilometers long at the scale of the figure. Point $A$ is the location of the asteroid on November 8.438. Point B is its location one day later on November 9.438, where we have used digital days to indicate a precise hour and minute within each endpoint date in terms of Universal Time.

Problem 1 - The figure above shows the location of the Moon on November 9 at 10:13 Universal Time (9.438days) in its counter-clockwise journey around earth in a circular orbit. The period of the orbit is 27.3 days. What was the date and time when the asteroid was at Point A?

Answer: The time between Point A and Point B is exactly 1.0 days, so we need to determine the position of the moon 1 day earlier that its location in the diagram.

The moon travels 360 degrees in 27.3 days for a speed of 13.2 degrees per day, so we need to find the position of the moon 13.2 degrees clockwise of its position in the diagram. Using a protractor, the figure below shows this position.



It was only 300 meters in diameter, but on June 18, 2013 it was discovered by the Pan-STARRS-1 survey telescope in Hawaii (top image), making it the $10,000^{\text {th }}$ Near Earth Object (NEO).

NASA estimates that there are 15,000 asteroids that are 140-meters or larger with orbits close to Earth. We have now discovered $30 \%$ of them according to statistical estimates based on the rate of discovery of these NEOs. Objects of this size or larger are considered severe hazards should they impact near cities. The February 2013 'Russian' meteor was 17 meters in diameter, yet it injured over 3000 people, though it left no crater. A 140-meter asteroid impact would produce a 25 -meter crater and the atmospheric shock wave would damage thousands of buildings.

The orbit is shown in the left. The figure to the left. The orbit is tilted nearly $45^{\circ}$ to Earth's orbit plane.

The distance from Earth to 2013 MZ5 is given in the table below, every 10 days, and in units of millions of kilometers.

| Date | Distance | Date | Distance | Date | Distance |
| :--- | :---: | :--- | :---: | :--- | :---: |
| May 22 | 115 | July 1 | 71 | August 9 | 82 |
| June 1 | 100 | July 10 | 69 | August 19 | 89 |
| June 11 | 88 | July 20 | 71 | August 29 | 97 |
| June 21 | 77 | July 30 | 76 | September 8 | 104 |

Problem 1 - Graph the tabulated distance function and connect the points with a smooth curve.

Problem 2 - On what date is the asteroid closest to Earth?

Problem 3 - The approximate formula for the approach speed of the asteroid to Earth in $\mathrm{km} / \mathrm{hr}$ is given by $\mathrm{S}=1100 \mathrm{~T}-62000$, where T is the elapsed days from May 22 . How fast was it approaching Earth on June 18, the day of its discovery?

Orbit - $\underline{\text { http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2013\%20MZ5;orb=1;cov=0;log=0;cad=0\#orb }}$ Data - http://ssd.jpl.nasa.gov/sbdb.cgi?sstr=2013\ MZ5
http://www.nasa.gov/centers/jpl/news/neo20130624.html
NASA - Ten Thousandth Near-Earth Object Unearthed in Space
June 24, 2013.
Problem 1 - Graph the tabulated distance function and connect the points with a smooth curve. Answer: See graph below.


Problem 2 - On what date is the asteroid closest to Earth?
Answer: The table shows that on July 10 (Elapsed day 50) it reached its minimum distance of 69 million kilometers.

Problem 3 - The approximate formula for the approach speed of the asteroid to Earth in $\mathrm{km} / \mathrm{hr}$ is given by $S=1100 \mathrm{~T}-62000$, where T is the elapsed days from May 22. How fast was it approaching Earth on June 18, the day of its discovery?

Answer: June 18 is elapsed day 27, so $T=27$ and so $S=1100(27)-6200=\mathbf{- 2 3 , 5 0 0} \mathbf{k m} / \mathrm{hr}$. The negative sign means that the distance to Earth was decreasing.


The Catalina Sky Survey near Tucson, Ariz., discovered two small asteroids on the morning of Sunday, September 5, 2010 during a routine monitoring of the skies.

Asteroid 2010RX30 is about 15 meters in diameter and will pass within 248,000 kilometers of Earth. Asteroid 2010RF12, about 10 meters in diameter, will pass within 79,000 kilometers of Earth.

Both asteroids should be observable near closest approach to Earth with moderate-sized amateur telescopes.

Neither of these asteroids has a chance of hitting Earth. A 10-meter- sized near-Earth asteroid from the undiscovered population of about 50 million would be expected to pass almost daily within a lunar distance, and one might strike Earth's atmosphere about every 10 years on average. The last asteroid that was observed to enter Earth's atmosphere in this size range was the Great Daylight Fireball of 1972 which streaked above the Grand Tetons. It was about 5 meters in diameter but skipped out of the atmosphere and never struck ground.

Small asteroids appear very faint in the sky, not only because they are small in size, but because their surfaces are very dark and reflect very little sunlight. The formula for the brightness of a typical asteroid that is spotted within a few million kilometers of Earth is given by:

$$
R=0.011 d 10^{-\frac{1}{5}(m)}
$$

where: R is the asteroid radius in meters, d is the distance to the asteroid from Earth in kilometers, and $m$ is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about $m=+6.5$. The planet Venus when it is brightest in the evening sky has a magnitude of $m=-2.5$. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid (10 $\mathrm{km} / \mathrm{sec}$ ) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

## Answer Key

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?
Answer:
2010RX30, $R=15$ meters, $d=248,000$ kilometers
$15=0.011(248,000) 10^{-.2 m}$ then $0.0055=10^{-.2(m)}$
$\log (0.0055)=-0.2 m \quad$ so $\mathbf{m}=+11.3$ magnitudes

2010RF12, $R=10$ meters, $d=79,000$ kilometers
$\begin{array}{lr}10=0.011(79,000) 10^{-.2 m} & \text { then } 0.011=10^{-.2(m)} \\ \log (0.011)=-0.2 \mathrm{~m} & \text { so } \mathrm{m}=+9.7 \text { magnitudes }\end{array}$

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid ( $10 \mathrm{~km} / \mathrm{sec}$ ) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

Answer: First evaluate the equation for $\mathrm{d}=1$ million km and solve for $\mathrm{m}(\mathrm{R})$
$R=0.011\left(1.0 \times 10^{6}\right) 10^{-.2 \mathrm{~m}}$
$R=1.1 \times 10^{4} 10^{-.2 \mathrm{~m}}$
so $m(R)=-5 \log 10(0.000091 R)$
For $\mathrm{R}=1$ to 500 meters, $\mathbf{m}=+\mathbf{2 0 . 2}$ to $\boldsymbol{+ 6 . 7}$

The most common asteroids have sizes between 1 meter and 50 meters, so the detection of such small, faint, and rapidly moving asteroids with ground based telescopes is a major challenge and may be a matter of luck in most cases.

For more information, read the NASA press release at
"Two Asteroids to Pass By Earth Wednesday"
http://www.nasa.gov/topics/solarsystem/features/asteroid20100907.html


Near Earth Objects (NEOs) are asteroids that have orbits very close to Earth's orbit in the solar system. That means that, over time, they might collide with Earth. The most devastating asteroids are larger than 1 km and can cause world-wide extinction events. Smaller bodies from 10 to 300-meters can damage cities but are too small to affect continent-sized areas.

Astronomers have, for decades detected and tracked these smaller bodies as they come near Earth, and from this determine their orbits. The graphs to the left show the progress of these searches since 1995.

Problem 1 - The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Problem 2 - What is the total number of asteroids discovered in 2012?

Problem 3 - What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B)Smaller than 1 kilometer?

Problem 4 - From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Problem 5 - Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Problem 1 - The top graph shows the number of NEOs detected each year. Asteroids that are a kilometer or more in size can cause extinction events. How many of these were discovered in 2012?

Answer: The last bar in the graph shows that 9 were discovered in 2012.

Problem 2 - What is the total number of asteroids discovered in 2012?

Answer: The second graph shows that for 2012 the column indicates about 470 were discovered that year.

Problem 3 - What percentage of the asteroids discovered in 2012 were A) larger than 1 kilometer? B) Smaller than 1 kilometer?

Answer: A) $P=100 \% \times(9 / 470)=1.9 \%$.
B) $\mathrm{P}=100 \% \mathrm{x}(461 / 470)=\mathbf{9 8 . 1 \%}$

Problem 4 - From the bottom figure of total discovered asteroids, what percentage are a) smaller than 1 kilometer? B) Larger than 1 kilometer?

Answer: A) Adding up all 5 columns gives a total of $1350+2300+2500+3000+850=10,000$ Then Small asteroids $P=100 \% \times(10,000-850) / 10000=91.5 \%$
B) Large asteroids $P=100 \%-91.5 \%=8.5 \%$

Problem 5 - Compare the top two figures. What can you conclude about the number of 1 kilometer or larger asteroids that are yet to be discovered, compared to those smaller than 1 kilometer?

Answer: The top graph shows that the detection rate for these large asteroids has declined steadily since 2000. This means that each year the surveys are finding fewer and fewer large asteroids that they did not previously know about. This means that the surveys have almost completely detected all of the NEO asteroids that are this large. Because these large NEOS are only $2 \%$ of the detected asteroids, the majority of NEO asteroids are much smaller than the large ones, and more numerous. The middle graph shows that the numbers we are detecting continues to grow each year with no sign of decreasing. This means that there are many more of these to be discovered than we have found already. By some estimates, we have only discovered about $5 \%$ of all that there are near Earth, which is why we have to keep searching. Once the number of new discoveries begins to follow the profile of the top figure, we will know that we have discovered the vast majority of the small asteroids that remain.

