

Once you have discovered a planet, you need to figure out whether liquid water might be present. In our solar system, Mercury and Venus are so close to the sun that water cannot remain in liquid form. It vaporizes! For planets beyond Mars, the sun is so far away that water will turn to ice. Only in what astronomers call the Habitable Zone (shown in green in the figure above) will a planet have a chance for being at the right temperature for liquid water to exist in large quantities (oceans) on its surface!

The Table on the following page lists the 54 planets that were discovered by NASA's Kepler Observatory in 2010. These planets come in many sizes as you can see by their radii. The planet radii are given in terms of the Earth, where ' 1.0 ' means a planet has a radius of exactly 1 Earth radius ( 1.0 Re ) or 6,378 kilometers. The distance to each planet's star is given in multiples of our Earth-Sun distance, called an Astronomical Unit, so that '1.0 AU' means exactly 150 million kilometers.

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth?

Problem 2 - About what is the average temperature of the planets for which $\mathrm{R}<4.0 \mathrm{Re}$ ?
Problem 3 - About what is the average temperature of the planets for which $\mathrm{R}>4.0 \mathrm{Re}$ ?
Problem 4-Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with $\mathrm{R}>4.0 \mathrm{Re}$. Histogram-2 for planets with $\mathrm{R}<4.0 \mathrm{Re}$. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets?

Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature ( $270-290 \mathrm{~K}$ ) if 157,453 stars were searched for the Kepler survey?

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth? Answer: There are 28 planets for which $R<4.0$ re, so $P=100 \% \times(28 / 54)=52 \%$

Problem 2-About what is the average temperature of the planets for which $\mathrm{R}<4.0 \mathrm{Re}$ ? Answer; Students will identify the 28 planets in the table that have $R<4.0$, and then average the planet's temperatures in Column 6. Answer: 317 K.

Problem 3 - About what is the average temperature of the planets for which $\mathrm{R}>4.0 \mathrm{Re}$ ? Students will identify the 26 planets in the table that have $R>4.0$, and then average the planet's temperatures in Column 6. Answer: $\mathbf{3 0 6}$ K.

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with $\mathrm{R}>4.0 \mathrm{Re}$. Histogram-2 for planets with $\mathrm{R}<4.0 \mathrm{Re}$. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets? Answer; They tend to be found slightly closer to their stars, which is why in Problem 2 their average temperatures were slightly hotter than the larger planets.


Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature ( $270-290 \mathrm{~K}$ ) if 157,453 stars were searched for the Kepler survey?

Answer: Students may come up with a number of different strategies and estimates. For example, they might create Venn Diagrams for the data in the table that meet the criteria given in the problem. Then, from the number of planets in the intersection, find their proportion in the full sample of 54 planets, then multiply this by the ratio of 40 billion to 157,453 . Estimates near 1 million are in the right range.

|  | Planet Name (KOI) | Orbit Period (days) | Distance To Star (AU) | Planet Radius (Re) | Planet Temp. (K) | Star Temp. (K) |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 683.01 | 278 | 0.84 | 4.1 | 239 | 5,624 |
| 2 | 1582.01 | 186 | 0.63 | 4.4 | 240 | 5,384 |
| 3 | 1026.01 | 94 | 0.33 | 1.8 | 242 | 3,802 |
| 4 | 1503.01 | 150 | 0.54 | 2.7 | 242 | 5,356 |
| 5 | 1099.01 | 162 | 0.57 | 3.7 | 244 | 5,665 |
| 6 | 854.01 | 56 | 0.22 | 1.9 | 248 | 3,743 |
| 7 | 433.02 | 328 | 0.94 | 13.4 | 249 | 5,237 |
| 8 | 1486.01 | 255 | 0.80 | 8.4 | 256 | 5,688 |
| 9 | 701.03 | 122 | 0.45 | 1.7 | 262 | 4,869 |
| 10 | 351.01 | 332 | 0.97 | 8.5 | 266 | 6,103 |
| 11 | 902.01 | 84 | 0.32 | 5.7 | 270 | 4,312 |
| 12 | 211.01 | 372 | 1.05 | 9.6 | 273 | 6,072 |
| 13 | 1423.01 | 124 | 0.47 | 4.3 | 274 | 5,288 |
| 14 | 1429.01 | 206 | 0.69 | 4.2 | 276 | 5,595 |
| 15 | 1361.01 | 60 | 0.24 | 2.2 | 279 | 4,050 |
| 16 | 87.01 | 290 | 0.88 | 2.4 | 282 | 5,606 |
| 17 | 139.01 | 225 | 0.74 | 5.7 | 288 | 5,921 |
| 18 | 268.01 | 110 | 0.41 | 1.8 | 295 | 4,808 |
| 19 | 1472.01 | 85 | 0.37 | 3.6 | 295 | 5,455 |
| 20 | 536.01 | 162 | 0.59 | 3.0 | 296 | 5,614 |
| 21 | 806.01 | 143 | 0.53 | 9.0 | 296 | 5,206 |
| 22 | 1375.01 | 321 | 0.96 | 17.9 | 300 | 6,169 |
| 23 | 812.03 | 46 | 0.21 | 2.1 | 301 | 4,097 |
| 24 | 865.01 | 119 | 0.47 | 5.9 | 306 | 5,560 |
| 25 | 351.02 | 210 | 0.71 | 6.0 | 309 | 6,103 |
| 26 | 51.01 | 10 | 0.06 | 4.8 | 314 | 3,240 |
| 27 | 1596.02 | 105 | 0.42 | 3.4 | 316 | 4,656 |
| 28 | 416.02 | 88 | 0.38 | 2.8 | 317 | 5,083 |
| 29 | 622.01 | 155 | 0.57 | 9.3 | 327 | 5171 |
| 30 | 555.02 | 86 | 0.38 | 2.3 | 331 | 5,218 |
| 31 | 1574.01 | 115 | 0.47 | 5.8 | 331 | 5,537 |
| 32 | 326.01 | 9 | 0.05 | 0.9 | 332 | 3,240 |
| 33 | 70.03 | 78 | 0.35 | 2.0 | 333 | 5,342 |
| 34 | 1261.01 | 133 | 0.52 | 6.3 | 335 | 5,760 |
| 35 | 1527.01 | 193 | 0.67 | 4.8 | 337 | 5,470 |
| 36 | 1328.01 | 81 | 0.36 | 4.8 | 338 | 5,425 |
| 37 | 564.02 | 128 | 0.51 | 5.0 | 340 | 5,686 |
| 38 | 1478.01 | 76 | 0.35 | 3.7 | 341 | 5,441 |
| 39 | 1355.01 | 52 | 0.27 | 2.8 | 342 | 5,529 |
| 40 | 372.01 | 126 | 0.50 | 8.4 | 344 | 5,638 |
| 41 | 711.03 | 125 | 0.49 | 2.6 | 345 | 5,488 |
| 42 | 448.02 | 44 | 0.21 | 3.8 | 346 | 4,264 |
| 43 | 415.01 | 167 | 0.61 | 7.7 | 352 | 5,823 |
| 44 | 947.01 | 29 | 0.15 | 2.7 | 353 | 3,829 |
| 45 | 174.01 | 56 | 0.27 | 2.5 | 355 | 4,654 |
| 46 | 401.02 | 160 | 0.59 | 6.6 | 357 | 5,264 |
| 47 | 1564.01 | 53 | 0.28 | 3.1 | 360 | 5,709 |
| 48 | 157.05 | 118 | 0.48 | 3.2 | 361 | 5,675 |
| 49 | 365.01 | 82 | 0.37 | 2.3 | 363 | 5,389 |
| 50 | 374.01 | 173 | 0.63 | 3.3 | 365 | 5,829 |
| 51 | 952.03 | 23 | 0.12 | 2.4 | 365 | 3,911 |
| 52 | 817.01 | 24 | 0.13 | 2.1 | 370 | 3,905 |
| 53 | 847.01 | 81 | 0.37 | 5.1 | 372 | 5,469 |
| 54 | 1159.01 | 65 | 0.30 | 5.3 | 372 | 4,886 |

Space Math

# Discovering Earth-like Worlds by their Color 



Earth is invitingly blue. Mars is angry red. Venus is brilliant white. NASA astronomer Lucy McFadden, and UCLA research assistant in geochemistry Carolyn Crow, have now discovered that a planet's "true colors" can reveal important details.

Mars is red because its soil contains rusty red stuff called iron oxide. Our planet, the "blue marble" has an atmosphere that scatters blue light rays more strongly than red ones. This suggests that astronomers could use color information to identify Earthlike worlds. Their colors will tell us which ones to study in more detail.

As NASA's Deep Impact spacecraft cruised through space, its High Resolution Instrument (HRI) measured the intensity of Earth's light. HRI is a 30-cm telescope that feeds light through seven different color filters. Each filter samples the incoming light at a different portion of the visible-light spectrum, from ultraviolet and blue to near-infrared. A table showing the reflectivity of each body is shown below. The numbers indicate the percentage of light reflected by the planet at 350, 550 and 850 nanometers (nm). For example, compared to the light that it reflects at 550 nm , Venus reflects 116\% more light at 850 nm .

| Object | 350 nm | 550 nm | 850 nm | Object | 350 nm | 550 nm | 850 nm |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Mercury | 47 | 100 | 142 | Jupiter | 60 | 100 | 64 |
| Venus | 58 | 100 | 109 | Saturn | 45 | 100 | 78 |
| Earth | 152 | 100 | 110 | Titan | 34 | 100 | 88 |
| Moon | 67 | 100 | 169 | Uranus | 98 | 100 | 15 |
| Mars | 34 | 100 | 203 | Neptune | 125 | 100 | 13 |

Note: Table based upon data published by the astronomers in the Astrophysical Journal (March 10, 2011).
Problem 1 - One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where $X=R(850) / R(550)$ and $X=R(350) / R(550)$. For example, for the Moon, where $R(350)=61 \%, R(550)=100 \%$, and $R(850)=155 \%$ we have $X=155 / 100=1.55$, and $Y=61 / 100=0.61$. Using this method, calculate $X$ and $Y$ for each object and then plot the ( $X, Y$ ) points on a graph.

Problem 2 - The planetary data in the table can be written as an ordered triplet. For example, for Mercury the reflectivities in the table above would be written as (56, 100, 177). Using the definition for X and Y in Problem 1, which of the planets below would you classify as Earth-like, Jupiter-like, or Moon-like, if the planetary reflectivities are: Planet A (61, 82, 156), Planet B (45, 35, 56), Planet C (90, 120, 67).

Problem 3 - Can you create a different plot for the planets that makes their differences stand out even more?

Problem 1 - One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where $X=R(850) / R(550)$ and $X$ $=R(350) / R(550)$. For example, for the Moon, where $R(350)=61 \%, R(550)=100 \%$, and $R(850)=155 \%$ we have $X=155 / 100=1.55$, and $Y=61 / 100=0.61$. Using this method, calculate $X$ and $Y$ for each object and then plot the $(X, Y)$ points on a graph. See graph below.

| Object | 350 nm | 550 nm | 850 nm | X | Y |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Mercury | 47 | 100 | 142 | $\mathbf{1 . 4 2}$ | $\mathbf{0 . 4 7}$ |
| Venus | 58 | 100 | 109 | $\mathbf{1 . 0 9}$ | $\mathbf{0 . 5 8}$ |
| Earth | 152 | 100 | 110 | $\mathbf{1 . 1 0}$ | $\mathbf{1 . 5 2}$ |
| Moon | 67 | 100 | 169 | $\mathbf{1 . 6 9}$ | $\mathbf{0 . 6 7}$ |
| Mars | 34 | 100 | 203 | $\mathbf{2 . 0 3}$ | $\mathbf{0 . 3 4}$ |
| Jupiter | 60 | 100 | 64 | $\mathbf{0 . 6 4}$ | $\mathbf{0 . 6 0}$ |
| Saturn | 45 | 100 | 78 | $\mathbf{0 . 7 8}$ | $\mathbf{0 . 4 5}$ |
| Titan | 34 | 100 | 88 | $\mathbf{0 . 8 8}$ | $\mathbf{0 . 3 4}$ |
| Uranus | 98 | 100 | 15 | $\mathbf{0 . 1 5}$ | $\mathbf{0 . 9 8}$ |
| Neptune | 125 | 100 | 13 | $\mathbf{0 . 1 3}$ | $\mathbf{1 . 2 5}$ |

Problem 2 - The planetary data in the table can be written as an ordered triplet. For example, for Mercury the reflectivities in the table above would be written as (56, 100, 177). Using the definition for X and Y in Problem 1, which of the planets below would you classify as Earthlike, Jupiter-like, or Moon-like, if the planetary reflectivities are: Planet A (61, 82, 156), Planet B (45, 35, 56), Planet C (90, 120, 67). Answer: Planet A has X and Y values similar to the moon; Planet B is more like Earth; and Planet C is more like Jupiter.

Problem 3 - Can you create a different plot for the planets that makes their differences stand out even more? Answer: There are more than 100 different ways in which students may decide to create new definitions for $X$ and $Y$ such as $X=R(350)-R(850) ; Y=$ $R(850) / R(350)$ and so on. Some will not visually let you see a big difference between the planet 'colors' while other may. Astronomers try many different combinations, usually with some idea of the underlying physics and how to enhance what they are looking for. There is no right or wrong answer, only ones that make the analysis easier or harder!



All of the planets in our solar system, and some of its smaller bodies too, have an outer layer of gas we call the atmosphere. The atmosphere usually sits atop a denser, rocky crust or planetary core. Atmospheres can extend thousands of kilometers into space.

The table below gives the name of the kind of gas found in each object's atmosphere, and the total mass of the atmosphere in kilograms. The table also gives the percentage of the atmosphere composed of the gas.

| Object | Mass <br> (kilograms) | Carbon <br> Dioxide | Nitrogen | Oxygen | Argon | Methane | Sodium | Hydrogen | Helium | Other |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Sun | $3.0 \times 10^{30}$ |  |  |  |  |  |  | $71 \%$ | $26 \%$ | $3 \%$ |
| Mercury | 1000 |  |  | $42 \%$ |  |  | $22 \%$ | $22 \%$ | $6 \%$ | $8 \%$ |
| Venus | $4.8 \times 10^{20}$ | $96 \%$ | $4 \%$ |  |  |  |  |  |  |  |
| Earth | $1.4 \times 10^{21}$ |  | $78 \%$ | $21 \%$ | $1 \%$ |  |  |  |  | $<1 \%$ |
| Moon | 100,000 |  |  |  | $70 \%$ |  | $1 \%$ |  | $29 \%$ |  |
| Mars | $2.5 \times 10^{16}$ | $95 \%$ | $2.7 \%$ |  | $1.6 \%$ |  |  |  |  | $0.7 \%$ |
| Jupiter | $1.9 \times 10^{27}$ |  |  |  |  |  |  | $89.8 \%$ | $10.2 \%$ |  |
| Saturn | $5.4 \times 10^{26}$ |  |  |  |  |  |  | $96.3 \%$ | $3.2 \%$ | $0.5 \%$ |
| Titan | $9.1 \times 10^{18}$ |  | $97 \%$ |  |  | $2 \%$ |  |  |  | $1 \%$ |
| Uranus | $8.6 \times 10^{25}$ |  |  |  |  | $2.3 \%$ |  | $82.5 \%$ | $15.2 \%$ |  |
| Neptune | $1.0 \times 10^{26}$ |  |  |  |  | $1.0 \%$ |  | $80 \%$ | $19 \%$ |  |
| Pluto | $1.3 \times 10^{14}$ | $8 \%$ | $90 \%$ |  |  | $2 \%$ |  |  |  |  |

Problem 1 - Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth.

Problem 2 - Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto.

Problem 3 - Which planet has the atmosphere with the greatest percentage of Oxygen?

Problem 4 - Which planet has the atmosphere with the greatest number of kilograms of oxygen?

Problem 5 - Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Problem 1 - Draw a pie graph (circle graph) that shows the atmosphere constituents for Mars and Earth. Answer: Mars (left), Earth (middle)


Problem 2 - Draw a pie graph that shows the percentage of Nitrogen for Venus, Earth, Mars, Titan and Pluto. Answer: First add up all the percentages for Nitrogen in the column to get $271.7 \%$. Now divide each of the percentages in the column by $271.7 \%$ to get the percentage of nitrogen in the planetary atmospheres that is taken up by each of the planets: Venus $=(4 / 271)$ $=1.5 \%$; Earth $=(78 / 271)=28.8 \%$, Mars $=(2.7 / 271)=1.0 \%$, Titan $=(97 / 271)=35.8 \%$, Pluto=(90/271)=33.2\%. Plot these new percentages in a pie graph (see above right). This pie graph shoes that across our solar system, Earth, Titan and Pluto have the largest percentage of nitrogen. In each case, the source of the nitrogen is from similar physical processes involving the chemistry of the gas methane (Titan and Earth) or methane ice (Pluto).

Problem 3 - Which planet has the atmosphere with the greatest percentage of Oxygen? Answer: From the table we see that Mercury has the greatest percentage of oxygen in its atmosphere.

Problem 4 - Which planet has the atmosphere with the greatest number of kilograms of oxygen? Answer: Only two planets have detectable oxygen: Earth and Mercury. Though mercury has the highest percentage of oxygen making up its atmosphere, the number of kilograms of oxygen is only $1000 \mathrm{~kg} \times 0.42=420$ kilograms. By comparison, Earth has a smaller percentage of oxygen (21\%) but a vastly higher quantity: $1.4 \times 10^{21} \mathrm{~kg} \times 0.21=$ $2.9 \times 10^{20}$ kilograms. (That's $290,000,000,000,000,000,000 \mathrm{~kg}$ )

Problem 5 - Compare and contrast the objects with the greatest percentage of hydrogen, and the least percentage of hydrogen.

Answer: The objects with the highest percentage of hydrogen are the sun, Mercury, Jupiter, Saturn, Uranus and Neptune. The objects with the least percentage are Venus, Earth, Moon, Mars, Titan, Pluto. With the exception of Mercury, which has a very thin atmosphere, the highpercentage objects are the largest bodies in the solar system. The planet Jupiter, Saturn, Uranus and Neptune are sometimes called the Gas Giants because so much of the mass of these planets consists of a gaseous atmosphere. These bodies generally lie far from the sun. The low-percentage objects are among the smallest bodies in the solar system. They are called the 'Rocky Planets' to emphasize their similarity in structure, where a rocky core and mantel are surrounded by a thin atmosphere. Most of these bodies lie close to the sun.

## Organic Molecules Detected on Distant Planet!



The basic chemistry for life has been detected in a second hot gas planet, HD 209458b, depicted in this artist's concept. Two of NASA's Great Observatories - the Hubble Space Telescope and Spitzer Space Telescope, yielded spectral observations that revealed molecules of carbon dioxide, methane and water vapor in the planet's atmosphere. HD 209458b, bigger than Jupiter, occupies a tight, 3.5-day orbit around a sun-like star about 150 light years away in the constellation Pegasus. (NASA Press release October 20, 2009)

Some Interesting Facts: The distance of the planet from the star HD209458 is 7 million kilometers, and its orbit period (year) is only 3.5 days long. At this distance, the temperature of the outer atmosphere is about $1,000 \mathrm{C}(1,800 \mathrm{~F})$. At these temperatures, water, methane and carbon dioxide are all in gaseous form. It is also known to be losing hydrogen gas at a ferocious rate, which makes the planet resemble a comet! The planet itself has a mass that is $69 \%$ that of Jupiter, and a volume that is $146 \%$ greater than that of Jupiter. The unofficial name for this planet is Osiris.

Problem 1 - The mass of Jupiter is $1.9 \times 10^{\mathbf{3 0}}$ grams. The radius of Jupiter is 71,500 kilometers. A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere? B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?

Problem 2 - From the information provided; A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere? B) What is the mass of Osiris in grams? C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?

Problem 3 - The densities of some common ingredients for planets are as follows:

| Rock |  | 3.0 grams/cc |
| :---: | :---: | :---: |
| Iron |  | 9.0 grams/cc |
| Water |  | 5.0 grams/cc |
| Ice |  | 1.0 gram/cc |
| Mixtur | f hydrogen + helium | 0.7 grams/cc |

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Problem 1 - The mass of Jupiter is $1.9 \times 10^{\mathbf{3 0}}$ grams. The radius of Jupiter is 71,500 kilometers.
A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere?

Answer: The radius of Jupiter, in centimeters, is
$R=71,500 \mathrm{~km} \times(100,000 \mathrm{~cm} / 1 \mathrm{~km})$

$$
=7.15 \times 10^{9} \mathrm{~cm} .
$$

For a sphere, $V=4 / 3 \pi R^{3}$ so the volume of Jupiter is
$V=1.33 \times(3.141) \times\left(7.15 \times 10^{9}\right)^{3}$
$\mathrm{V}=1.53 \times 10^{30} \mathrm{~cm}^{3}$
B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?
Answer: Density = Mass/Volume so the density of Jupiter is $D=\left(1.9 \times 10^{30} \mathrm{grams}\right) /(1.53 \mathrm{x}$ $10^{30} \mathrm{~cm}^{3}$ ) $=1.2 \mathrm{gm} / \mathrm{cc}$

Problem 2 - From the information provided;
A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere?

Answer: The information says that the volume is $146 \%$ greater than Jupiter so it will be $\mathrm{V}=$ $V=1.46 \times\left(1.53 \times 10^{30} \mathrm{~cm}^{3}\right)$
$=2.23 \times 10^{30} \mathrm{~cm}^{3}$
B) What is the mass of Osiris in grams?

Answer: the information says that it is $69 \%$ of Jupiter so
$M=0.69 \times\left(1.9 \times 10^{30}\right.$ grams $)$
$=1.3 \times 10^{30}$ grams
C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?
Answer: D = Mass/volume

$$
\begin{aligned}
& =1.3 \times 10^{30} \text { grams } / 2.23 \times 10^{30} \mathrm{~cm}^{3} \\
& =0.58 \text { grams } / \mathrm{cc}
\end{aligned}
$$

Problem 3 - The densities of some common ingredients for planets are as follows:
Rock 3 grams/cc Ice 1 gram/cc Iron $\quad 9$ grams/cc $\quad$ Mixture of hydrogen + helium $\quad 0.7$ grams/cc Water 5 grams/cc

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Answer: Because the density of Osiris is only about 0.6 grams/cc, the closest match would be a mixture of hydrogen and helium. This means that, rather than a solid planet like earth, which is a mixture of higher-density materials such as iron, rock and water, Osiris has much in common with Jupiter which is classified by astronomers as a Gas Giant!


In December 2009, astronomers announced the discovery of the transiting super-Earth planet GJ 1214b located 42 light years from the sun, and orbits a reddwarf star. Careful studies of this planet, which orbits a mere 2 million km from its star and takes 1.58 days to complete 'one year'. Its mass is known to be 6.5 times our Earth and a radius of about 2.7 times Earth's. Its day-side surface temperature is estimated to be $370^{\circ} \mathrm{F}$, and it is locked so that only one side permanently faces its star.

When a planet passes in front of its star, light from the star passes through any atmosphere the planet might contain and travels onwards to reach Earth observers. Although the disk of the planet will temporarily decrease the brightness of its star by a few percent, the addition of an atmosphere causes an additional brightness decrease. The amount depends on the thickness of the atmosphere, the presence of dust and clouds, and the chemical composition. By studying the light dimming at many different wavelengths, astronomers can distinguish between different atmospheric constituents by using specific spectral 'fingerprints'. They can also estimate the thickness of the atmosphere in relation to the diameter of the planet.

Problem 1 - Assuming the planet is a sphere, from the available information, to two significant figures, what is the average density of the planet in $\mathrm{kg} /$ meter $^{3}$ ? (Earth mass $=$ $6.0 \times 10^{24} \mathrm{~kg}$; Diameter $=6378 \mathrm{~km}$ ).

Problem 2 - The average density of Earth is $5,500 \mathrm{~kg} / \mathrm{m}^{3}$. Suppose that GJ 1214 b has a rocky core with Earth's density and a radius of R , and a thin atmosphere with a density of D. Let $\mathrm{R}=1.0$ at the top of the atmosphere, and $\mathrm{R}=0$ at the center of the planet, and assume the core is a sphere, and that the atmosphere is a spherical shell with inner radius R and outer radius $\mathrm{R}=1.0$. The formula relating the atmosphere density, D , to the core radius, $R$, is given by:

$$
1900=(5500-D) R^{3}+D
$$

A) Re-write this equation by solving for $D$.
B) Graph the function $D(R)$ over the domain $R:[0,1]$.
C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which $D=100 \mathrm{~kg} / \mathrm{m}^{3}$, what fraction of the radius of the planet is occupied by the Earthlike core, and what fraction is occupied by the atmosphere?

Problem 1 - Answer: Volume $=4 / 3(3.141)(2.7 \times 6378000)^{3}=2.1 \times 10^{22}$ meter $^{3}$. Density $=$ Mass/Volume

$$
\begin{aligned}
& =\left(6.5 \times 6.0 \times 10^{24} \mathrm{~kg}\right) /\left(2.1 \times 10^{22} \text { meter }^{3}\right) \\
& =1,900 \mathrm{~kg} / \text { meter }^{3}
\end{aligned}
$$

Problem 2 - A) Answer: $1900=(5500-D) R^{3}+D$

$$
\begin{aligned}
& 1900-D=5500 R^{3}-D R^{3} \\
& D\left(R^{3}-1\right)=5500 R^{3}-1900 \\
& D=\frac{5500 R^{3}-1900}{R^{3}-1}
\end{aligned}
$$

B) See below:


Note that for very thin atmospheres where D: [0, 100] the function predicts that the core has a radius of about $\mathrm{R}=0.7$ or $70 \%$ of the radius of the planet. Since the planet's radius is 2.7 times Earth's radius, the core is about $0.7 \times 2.7=1.9 \times$ Earth's radius. Values for $\mathrm{D}<0$ are unphysical even though the function predicts numerical values. This is a good opportunity to discuss the limits of mathematical modeling for physical phenomena.
C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which $D=100 \mathrm{~kg} / \mathrm{m}^{3}$, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Answer: For $D=100, R=0.71$, so the core occupies the inner $71 \%$ of the planet, and the surrounding atmospheric shell occupies the outer $29 \%$ of the planet's radius.


We can check the numbers in the information box ourselves. Here are a few of the measurements made of the star's speed:

| Time <br> (hours) | Speed <br> (cm/sec) | Time <br> (hours) | Speed <br> (cm/sec) |
| :--- | :--- | :--- | :--- |
| 6 | 170 | 48 | 50 |
| 10 | 150 | 56 | 70 |
| 21 | 110 | 71 | 130 |
| 33 | 60 | 83 | 170 |

Problem 1 - Graph the speed data. Draw a smooth curve through the data (which need not go through all the points) and estimate the period (in days) of the speed curve to get the orbit period of the proposed planet.

Problem 2 - Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star (D in Astronomical Units) using the formula

$$
T^{2}=D^{3}
$$

where 1 Astronomical Unit equals the distance from Earth to our sun ( 150 million km). Using your estimated planet period, what is the orbit distance of the new planet from Centauri B in A) Astronomical Units? B) kilometers?

Problem 3 - What is the temperature $T$ (in kelvins) of the new planet if its average temperature at a distance of $D$ Astronomical Units is given by the formula:

$$
T=\frac{310}{\sqrt{D}}
$$

Alpha Centauri is a binary star system located 4.37 light years from our sun. The two stars, A and B, are both sun-like stars, but they are older than our sun by about 1.5 billion years.

Astronomers have used the European Space Agency's 3.6 meter telescope at La Silla in Chile to detect the tell-tail motion of Alpha Centauri B caused by an earth-sized planet in close orbit around this star.

The planet, called Alpha Centauri Bb, orbits at a distance of only six million kilometers from its parent star - closer than Mercury is to the sun. The planet is bathed in unbearable heat, and has a surface temperature of 1,200 Celsius ( 2,200 F or 1,500 Kelvin). This is hot enough that its surface must be mostly molten lava. Its tight orbit means a year passes in only 3.2 Earth days.

The astronomers made hundreds of measurements of the speed of the Alpha Cen B star to search for a periodic change it its speed through space. They found a change (amplitude) of about $50 \mathrm{~cm} / \mathrm{sec}$ that increased and decreased with a precise period, which would only be expected from an orbiting object.

This discovery is still being confirmed through independent observations by other astronomers.

Problem 1 - Graph the speed data. Draw a smooth curve through the data and estimate the period of the speed curve to get the orbit period of the proposed planet. Answer should be about 77 hours or 3.2 days.


Problem 2 - Kepler's Third Law can be used to relate the period of the planet's orbit (T in years) to its distance from its star ( $D$ in Astronomical Units) using the formula

$$
\mathrm{T}^{2}=\mathrm{D}^{3}
$$

where 1 Astronomical Unit equals the distance from the earth to our sun ( 150 million km ). Using your estimated planet period, what is the orbit distance from Centauiri B in A) Astronomical Units? B) kilometers? Answer: $T=3.2$ days or in terms of earth-years $T=3.2 / 365=0.00877$ years. Then $D^{3}=$ $(0.00877)^{2}, \quad D^{3}=7.69 \times 10^{-5} \quad D=\left(7.69 \times 10^{-5}\right)^{1 / 3} \quad D=0.043$ Astronomical Units. B) In kilometers, this is $0.043 \mathrm{AU} \times(150$ million $\mathrm{km} / 1 \mathrm{AU})=6.4$ million kilometers.

Problem 3 - What is the temperature T (in kelvins) of the planet if its average temperature at a distance of $D$ Astronomical Units. Answer: $T=310 /(0.043)^{1 / 2}$ so $T=\mathbf{1 , 5 0 0}$ kelvins.

The actual graph of the data published by the astronomers is shown below:


In 1961, astronomer Frank Drake devised an ingenious equation that has helped generations of scientists estimate how many intelligent civilizations may exist in the Milky Way. The 'Drake Equation' looks like this:
$N=S \times P \times E \times C \times I \times A \times L$
Where:
$S=$ Number of stars in the Milky Way
$P=$ Fraction of stars with planets
$E=$ Number of planets per star in the right temperature zone
$C=$ Fraction of planets in E actually able to support life
$I=$ Fraction of planets in $C$ where intelligent life evolves
$A=$ Fraction of planets in I that communicate with radio wave technology
$\mathrm{L}=$ Fraction of a stars lifetime when communicating civilization exists


On Earth, bacteria have existed for nearly 4 billion years. Insects for 500 million years. Modern humans for 20,000 years. Which lifeform is the most likely to be found on a distant planet?

```
Known values:
\(S=500\) billion
\(P=0.1\)
For our solar system:
\(\mathrm{E}=2\) (Earth and Mars)
\(\mathrm{C}=0.5\) (Earth)
I = 1.0 (Earth)
A = 1.0 (Earth)
\(\mathrm{L}=0.00000002\) ( \(100 \mathrm{yrs} / 4.5\) billion yrs )
```

The great challenge is to determine from direct or indirect measurements what each of these factors might be. Fortunately, there are at least a few of these factors that we have pretty good ideas about, especially for our own planet. The estimate based on the solar system as a model is the sum of the products in the sample table $\mathrm{N}=500$ billion $\times 0.1 \times 2 \times 0.5 \times 1.0 \times 1.0 \times 0.00000002=1000$ civilizations existing right now.

Question 1: Based on your internet research, what do you think are the possible ranges for the factors P, E, and C? Remember to cite your sources and use only primary sources by astronomers or other scientists, not opinions by non-scientists.

Question 2: Which factors are the most uncertain and why?
Question 3: What kinds of astronomical observations might help decide what the values for $E$ and $C$ ?
Question 4: Using the evolution of life on Earth as a guide, what could you conclude about I? What is the most likely form of life in the Milky Way?

Question 5: How would you try to estimate A using a radio telescope?
Question 6: What would the situation have to be for the value of $L$ to be 0.000002 or 0.000000002 ?
Question 7: Using the Drake Equation, and a Milky Way in which 1 million intelligent civilizations exist, work backwards to create a 'typical' scenario for each factor so that a) $\mathrm{N}=1$ million. B) $\mathrm{N}=1$. Make sure you can defend your choices of the different factors.

Question 1: Based on your internet research, what do you think are the possible ranges for the factors $\mathrm{P}, \mathrm{E}$, and C ?

Answer: Students may cite the following approximate ranges, or plausible variations after providing the appropriate bibliographic reference: P probably between $1 \%$ and $10 \%$ based on local planet surveys; E between 1 and 3 based on planet surveys and our own solar system; $C$ between 0.1 and 0.5 ;

Question 2: Which factors are the most uncertain and why?
Answer: I , A and L. These depend on the details of evolution on non-Earth planets and it is basically anyone's guess what these numbers might be. Students may consult various internet resources or essays on the Drake Equation to get an idea of what ranges are the most talked about.

Question 3: What kinds of astronomical observations might help decide what the values for E and C? Answer: Our best tool is to conduct surveys of nearby stars and attempt to detect earth-sized planets, not the much larger Jupiter-sized bodies that astronomers currently study.

Question 4: Using the evolution of life on Earth as a guide, what could you conclude about I? What is the most likely form of life in the Milky Way?

Answer: Bacteria were the first life forms on Earth and have survived for nearly 4 billion years. Statistically, they should be the most common life forms in the universe. Also, alien life forms are much more likely to be simple rather than complex. This is favored by the distribution of life on Earth, in which there are very few complex organisms compared to simple ones, especially if you rank them by the total mass of the species and use this to set probabilities. For example, for every billion pounds of bacteria, there are perhaps only 1 thousand pounds of species larger and more complex than insects.

Question 5: How would you try to estimate A using a radio telescope?
Answer, by measuring the radio output of thousands of stars every year and looking for signs of intelligent 'modulation' like a morse-code signal. Stars don't normally emit radio signals that vary in a precise way in time. The SETI program is continuing to conduct these kinds of surveys.

Question 6: What would the situation have to be for the value of $L$ to be 0.000002 or 0.000000002 ? Answer: A star like the sun lives for about 10 billion years. L would then be 10 billion $\times 0.000002=$ 20,000 years or as little as 10 billion $\times 0.000000002=20$ years. In the first case, a civilization has learned to survive its technological 'Childhood' and prosper. In the second case, the civilization may have perished after learning how to use radio technology, or it may still be a thriving civilization that no longer uses radio communication.

Question 7: Using the Drake Equation, and a Milky Way in which 1 million intelligent civilizations exist, work backwards to create a 'typical' scenario for each factor so that a) $\mathrm{N}=1$ million. B) $\mathrm{N}=1$.

Answer: This will depend on the values that students assign to the various factors. Make sure that each student can defend their choice. This may also be opened to class discussion as a wrap-up.

## Extracting Oxygen from Moon Rocks



About $85 \%$ of the mass of a rocket is taken up by oxygen for the fuel, and for astronaut life support. Thanks to the Apollo Program, we know that as much as $45 \%$ of the mass of lunar soil compounds consists of oxygen. The first job for lunar colonists will be to 'crack' lunar rock compounds to mine oxygen.

NASA has promised $\$ 250,000$ for the first team capable of pulling breathable oxygen from mock moon dirt; the latest award in the space agency's Centennial Challenges program.

Lunar soil is rich in oxides of silicon, calcium and iron. In fact, $43 \%$ of the mass of lunar soil is oxygen. One of the most common lunar minerals is ilmenite, a mixture of iron, titanium, and oxygen. To separate ilmenite into its primary constituents, we add hydrogen and heat the mixture. This hydrogen reduction reaction is given by the 'molar' equation:

$$
\mathrm{FeTiO}_{3}+\mathrm{H}_{2}---\mathrm{Fe}+\mathrm{TiO}_{2}+\mathrm{H}_{2} \mathrm{O}
$$

A Bit Of Chemistry - This equation is read from left to right as follows: One mole of ilmenite is combined with one mole of molecular hydrogen gas to produce one mole of free iron, one mole of titanium dioxide, and one mole of water. Note that the three atoms of oxygen on the left side $\left(\mathrm{O}_{3}\right)$ is 'balanced' by the three atoms of oxygen found on the right side (two in $\mathrm{TiO}_{2}$ and one in $\mathrm{H}_{2} \mathrm{O}$ ). One 'mole' equals $6.02 \times 10^{23}$ molecules.

The 'molar mass' of a molecule is the mass that the molecule has if there are 1 mole of them present. The masses of each atom that comprise the molecules are added up to get the molar mass of the molecule. Here's how you do this:

For $\mathrm{H}_{2} \mathrm{O}$, there are two atoms of hydrogen and one atom of oxygen. The atomic mass of hydrogen is 1.0 AMU and oxygen is 16.0 AMU , so the molar mass of $\mathrm{H}_{2} \mathrm{O}$ is 2 $(1.0)+16.0=18.0 \mathrm{AMU}$. One mole of water molecules will equal $\mathbf{1 8}$ grams of water by mass.

Problem 1 -The atomic masses of the atoms in the ilmenite reduction equation are $\mathrm{Fe}=$ 55.8 and $\mathrm{Ti}=47.9$. A) What is the molar mass of ilmenite? B) What is the molar mass of molecular hydrogen gas? C) What is the molar mass of free iron? D) What is the molar mass of titanium dioxide? E) Is mass conserved in this reaction?

Problem 2 - If 1 kg of ilmenite was 'cracked' how many grams of water would be produced?

Inquiry Question - If 1 kg of ilmenite was 'cracked' how many grams of molecular oxygen would be produced if the water molecules were split by electrolysis into

$$
2 \mathrm{H}_{2} \mathrm{O}-->2 \mathrm{H}_{2}+\mathrm{O}_{2} ?
$$

## Problem 1 -

A) What is the molar mass of ilmenite?
$1(55.8)+1(47.9)+3(16.0)=151.7$ grams/mole
B) What is the molar mass of molecular hydrogen gas? 2(1.0) $=\mathbf{2 . 0}$ grams/mole
C) What is the molar mass of free iron? $1(55.8)=55.8$ grams/mole
D) What is the molar mass of titanium dioxide? $1(47.9)+2(16.0)=79.9$ grams $/ \mathrm{mole}$
E) Is mass conserved in this reaction? Yes. There is one mole for each item on each side, so we just add the molar masses for each constituent. The left side has $151.7+2.0=153.7$ grams and the right side has $55.8+79.9+18.0=153.7$ grams so the mass balances on each side.

## Problem 2 -

Step 1 - The reaction equation is balanced in terms of one mole of ilmenite ( 1.0 x $\mathrm{FeTiO}_{3}$ ) yielding one mole of water ( $1.0 \times \mathrm{H}_{2} \mathrm{O}$ ). The molar mass of ilmenite is 151.7 grams which is the same as 0.1517 kilograms, so we just need to figure out how many moles is needed to make one kilogram.

Step 2 - This will be 1000 grams/151.7 grams $=6.6$ moles. Because our new reaction is that we start with $6.6 \times \mathrm{FeTiO}_{3}$ that means that for the reaction to remain balanced, we need to produce $6.6 \times \mathrm{H}_{2} \mathrm{O}$, or in other words, 6.6 moles of water.

Step 3 - Because the molar mass of water is 18.0 grams $/ \mathrm{mole}$, the total mass of water produced will be $6.6 \times 18.0=\mathbf{1 1 9}$ grams of water.

Inquiry Question - The reaction is: $2 \mathrm{H}_{2} \mathrm{O}-->2 \mathrm{H}_{2}+\mathrm{O}_{2}$
This means that for every 2 moles of water, we will get one mole of $\mathrm{O}_{2}$. The ratio is 2 to 1 . From the answer to Problem 2 ,we began with 6.59 moles of water not 2.0 moles. That means we will produce $6.6 / 2=3.3$ moles of water. Since 1 molecule of oxygen has a molar mass of $2(16)=32$ grams $/$ mole, the total mass of molecular oxygen will be 3.3 moles $\times 32$ grams $/$ mole $=\mathbf{1 0 6}$ grams. So, $\mathbf{1}$ kilogram of ilmenite will eventually yield 106 grams of breathable oxygen.


Microphotograph of the new bacterium GFAJ-1 that subsists on the toxic element arsenic.

NASA researchers exploring extremophile bacteria in Mono Lake, California claimed to have discovered a new strain of bacterium GFAJ-1 in the Gammaproteobacteria group, which not only feeds on the poisonous element arsenic, but incorporates this element in its DNA as a replacement for normal phosphorus. All other known life forms on Earth use 'standard' DNA chemistry based upon the common elements carbon, oxygen, nitrogen and phosphorus.

In the search for life on other worlds, knowing that 'life' can exist that is fundamentally different than Earth life now broadens the possible places to search for the chemistry of life in the universe.


This diagram shows the elements that make up a small section of normal DNA containing the four bases represented from top to bottom by the sequence 'CACT'. They are held together by a 'phosphate backbone' consisting of a phosphor atom, P , bonded to four oxygen atoms, O. Each phosphor group (called a phosphodiester) links together two sugar molecules (dioxyribose), which in turn bond to each of the bases by a nitrogen atom, N .

Problem 1 - The atomic mass of phosphor $\mathrm{P}=31$ AMU, arsenic As= 75 AMU, hydrogen $\mathrm{H}=1 \mathrm{AMU}$ and Oxygen $\mathrm{O}=16 \mathrm{AMU}$. A) What is the total atomic mass of one phosphodiester molecule represented by the formula $\mathrm{PO}_{4}$ ? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula $\mathrm{AsO}_{4}$ ?

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the $1,166,000$ phosphodiester molecules contribute about $30 \%$ of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Problem 1 - The atomic mass of phosphor $P=31 \mathrm{AMU}$, arsenic $A s=75 \mathrm{AMU}$, hydrogen $\mathrm{H}=1 \mathrm{AMU}$ and Oxygen $\mathrm{O}=16 \mathrm{AMU}$. A) What is the total atomic mass of one phosphodiester molecule represented by the formula $\mathrm{PO}_{4}$ ? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula $\mathrm{AsO}_{4}$ ?

$$
\begin{aligned}
& \text { Answer: A) } \mathrm{PO}_{4}=1 \text { Phosphorus }+4 \text { Oxygen } \\
& =1 \times 31 \mathrm{AMU}+4 \times 16 \mathrm{AMU} \\
& =95 \mathrm{AMU} \\
& \text { B) } \mathrm{AsO}_{4}=1 \text { Arsenic }+4 \text { Oxygen } \\
& =1 \times 75 \mathrm{AMU}+4 \times 16 \mathrm{AMU} \\
& =139 \mathrm{AMU}
\end{aligned}
$$

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the $1,166,000$ phosphodiester molecules contribute about 30\% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Answer: The arsenic-substituted ester has a mass of 139 AMU compared to the phosphorus-based ester with 95 AMU , so the new molecule $\mathrm{AsO}_{4}$ is $100 \% \mathrm{x}$ $(95 / 139)=68 \%$ more massive than $\mathrm{PO}_{4}$.

Since in the normal DNA the $\mathrm{PO}_{4}$ contributes $30 \%$ of the total DNA mass, the non$\mathrm{PO}_{4}$ molecules contribute $70 \%$ of the normal mass.

This is added to the new arsenic-based molecule mass for $\mathrm{AsO}_{4}$ of $30 \% \times 1.68=$ $50 \%$ to get a new mass that is $70 \%+50 \%=120 \%$ heavier than the original, 'normal' DNA based on $\mathrm{PO}_{4}$.

So we would predict that the DNA of the twin arsenic-based organism is only $20 \%$ more massive than the DNA of the original phosphate-based organism.

Note: Students may have a better sense of the calculation if they start with a concrete amount of 100 grams of normal DNA. Then 70 grams are in the non- $\mathrm{PO}_{4}$ molecules and 30 grams is in the $\mathrm{PO}_{4}$ molecules. Because $\mathrm{AsO}_{4}$ is $68 \%$ more massive than $\mathrm{PO}_{4}$, its contribution would be 30 grams $\times 1.68=50$ grams. Then adding this to the 70 grams you get 120 grams with is 20 grams more massive than normal DNA for a gain of $120 \%$.

New research published in 2012 now disputes the claim that the organism is truly an arsenic-based life form.

