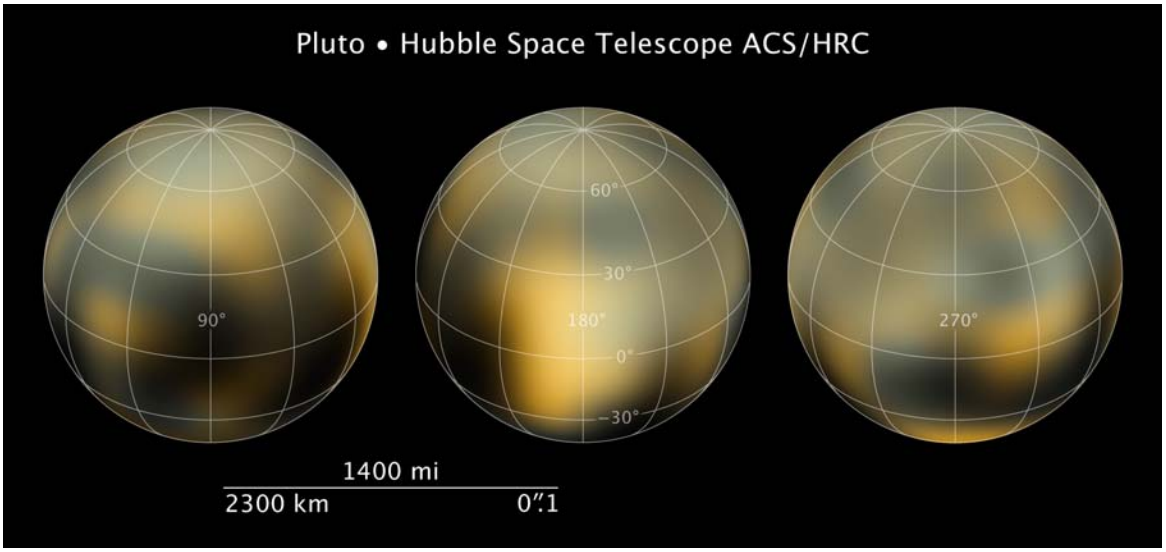
Seeing a Dwarf Planet Clearly: Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. The images, created at the very limits of Hubble's ability to see small details (sometimes called a telescope's resolving power), show enigmatic light and dark regions that are probably organic compounds (dark areas) and methane or water-ice deposits (light areas). Since these photos are all that we are likely to get until NASA's New Horizons spacecraft arrives in 2015, let's see what we can learn from the image!

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter?

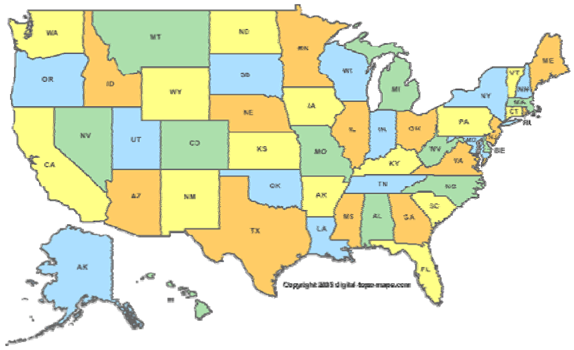
Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?

Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3×10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)?

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter? Answer: The Legend bar indicates 2,300 km and is 43 millimeters long so the scale is 2300/43 = **53 km/mm**.

Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?
Answer; Student's selection will vary, but on the first image to the lower right a feature measures about 8 mm in diameter which is 8 mm x (53 km/1mm) = **424 kilometers wide**. This is about the same size as the **state of Utah**!

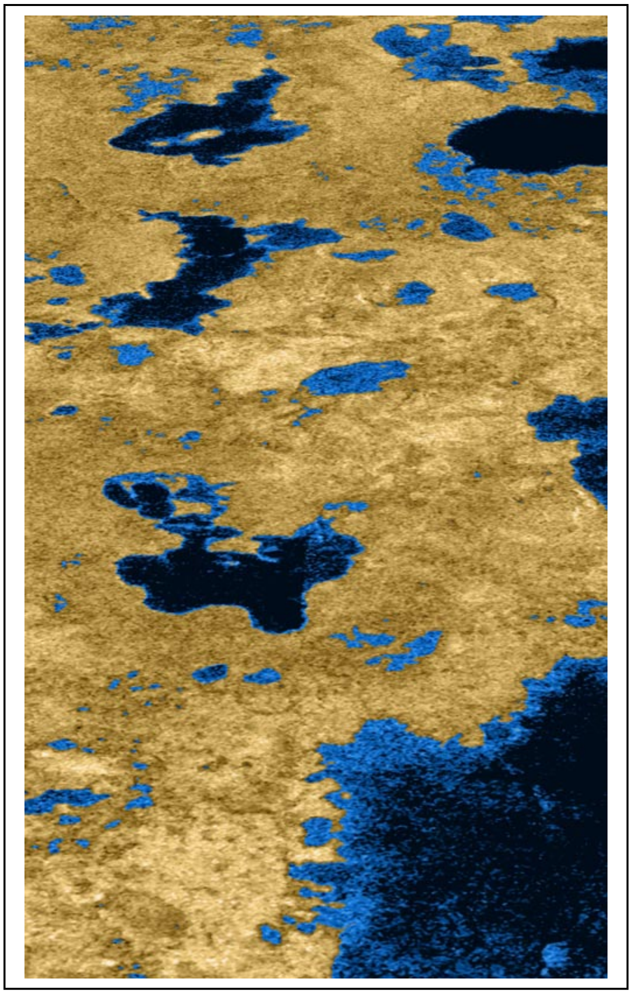


Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3×10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)? Answer: From the image, Pluto is a sphere with a diameter of 2,300 km, so its volume will be $V = 4/3\ \pi\ (1{,}250{,}000)^3 = 8.2 \times 10^{18}$ meters^3. Then its density is just D = M/V = (1.3×10^{22} kilograms)/ (8.2×10^{18} meters^3) so **D = 1,600 kg/m^3**. This would be about the density of **a mixture of rock and water-ice**.

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Answer: We want to match the density of Pluto (1,600 km/m^3) by using ice (917 kg/m^3) and rock (2300 kg/m^3). Suppose we made Pluto out of half-rock and half-ice **by mass**. The volume this would occupy would be V = ($0.5 * 1.3 \times 10^{22}$ kilograms/917 kg/m^3) = 7.1×10^{18} meters^3 for the ice, and V = ($0.5 * 1.3 \times 10^{22}$ kilograms/3000 kg/m^3) = 2.2×10^{18} meters^3 for the rock, for a total volume of 9.3×10^{18} meters^3 for both. This is a bit larger then the actual volume of Pluto (8.2×10^{18} meters^3) so we have to increase the mass occupied by ice, and lower the 50% by mass occupied by the rock component. The result, from student trials and errors should yield after a few iterations **about 40% ice and 60% rock.** This can be done very quickly using an Excel spreadsheet. For advanced students, it can also be solved exactly using a bit of algebra.

Lakes of Methane on Titan



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Answer: 150 km / 77 millimeters = **1.9 km/mm**.

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Answer: Combining the areas over the rectangular field of view gives about 1/4 of the area covered. The field of view measures 77 mm x 130mm or 150 km x 247 km or an area of 37,000 km^2. The dark areas therefore cover about 1/4 x 37,000 km^2 or **9,300 km^2.**

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

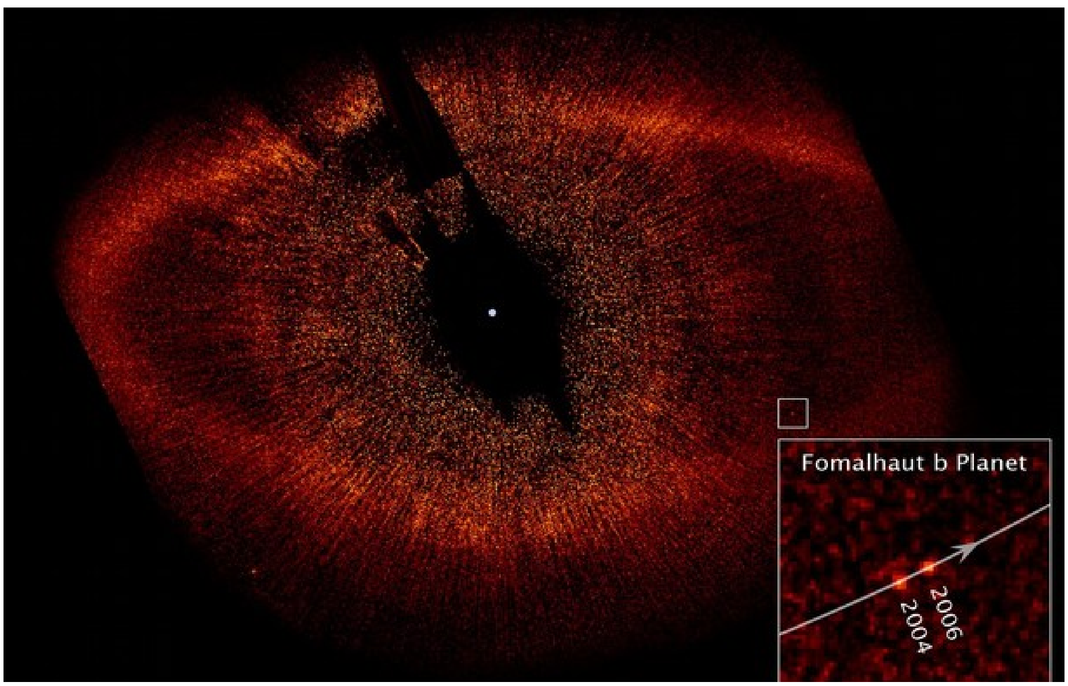
Answer: Volume = area x height = 9,300 km^2 x (0.02 km) = **190 km^3**.

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?
.
Answer: 190 km^3 / 150 km^3 = **1.3 Lake Tahoes**.

Hubble Sees a Distant Planet

The bright star Fomalhaut, in the constellation Piscis Austrinus (The Southern Fish) is only 25 light years away. It is 2000^{o} K hotter than the Sun, and nearly 17 times as luminous, but it is also much younger: Only about 200 million years old. Astronomers have known for several decades that it has a ring of dust (asteroidal material) in orbit 133 AU from the star and about 25 AU wide. Because it is so close, it has been a favorite hunting ground in the search for planets beyond our solar system. In 2008 such a planet was at last discovered using the Hubble Space Telescope. It was the first direct photograph of a planet beyond our own solar system.

In the photo below, the dusty ring can be clearly seen, but photographs taken in 2004 and 2006 revealed the movement of one special 'dot' that is now known to be the star's first detected planet. The small square on the image is magnified in the larger inset square in the lower right to show the location of the planet in more detail.



Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer Key

Problem 1 – The scale of the image is 2.7 AU/millimeter. If 1.0 AU = 150 million kilometers, how far was the planet from the star in 2006?

Answer: The distance from the center of the ring (location of star in picture) to the center of the box containing the planet is 42 millimeters, then 42 x 2.7 AU/mm = 113 AU. Since 1 AU = 150 million km, the distance is 113 x 150 million **= 17 billion kilometers.**

Problem 2 – How many kilometers had the planet moved between 2004 and 2006?

Answer: On the main image, the box has a width of 4 millimeters which equals 4 x 2.7 = 11 AU. The inset box showing the planet has a width of 36 mm which equals 11 AU so the scale of the small box is 11AU/36 mm = 0.3 AU/mm. The planet has shifted in position about 4 mm, so this corresponds to 4 x 0.3 = **1.2 AU or 180 million km**.

Problem 3 – What was the average speed of the planet between 2004 and 2006 if 1 year = 8760 hours?

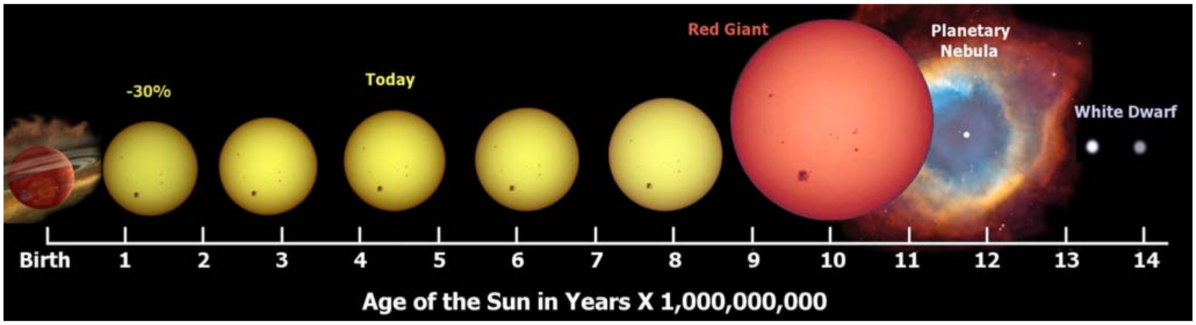
Answer: The average speed is 180 million km/17520 hours = **10,273 km/hr.**

Problem 4 – Assuming the orbit is circular, with the radius found from Problem 1, about how many years would it take the planet to make a full orbit around its star?

Answer: The radius of the circle is 113 AU so the circumference is $2 \pi R$ = 2 (3.141) (113 AU) = 710 AU. The distance traveled by the planet in 2 years is, from Problem 2, about 1.2 AU, so in 2 years it traveled 1.2/710 = 0.0017 of its full orbit. That means a full orbit will take 2.0 years/0.0017 = **1,176 years.**

Note - Because we are only seeing the 'projected' motion of the planet along the sky, the actual speed could be faster than the estimate in Problem 3, which would make the estimate of the orbit period a bit smaller than what students calculate in Problem 4.

A careful study of this system by its discoverer, Dr. Paul Kalas (UC Berkeley) suggests an orbit distance of 119 AU, and an orbit period of 872 years.



Our sun was formed 4.6 billion years ago. Since then it has been steadily increasing its brightness. This normal change is understood by astronomers who have created detailed mathematical models of the sun's complex interior. They have considered the nuclear physics that causes its heating and energy, gravitational forces that compress its dense core, and how the balance between these processes change in time. The diagram above shows the major stages in our sun's evolution from birth to end-of-life after 14 billion years. A simple formula describes how the power of our sun changes over time:

$$L = \frac{L_0}{1+\frac{2}{5}(1-x)}$$

where $x = t/t_0$ t_0 = 4.6 billion yrs, L_0 = 1.0 for the luminosity of the sun today.

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years

Problem 2 – By what percentage will L increase when it is 2 billion years older than it is today?

Problem 3 – A simple formula for the temperature, in kelvins, of Earth is given by:

$$T = 284[(1-A)L]^{\frac{1}{4}}$$

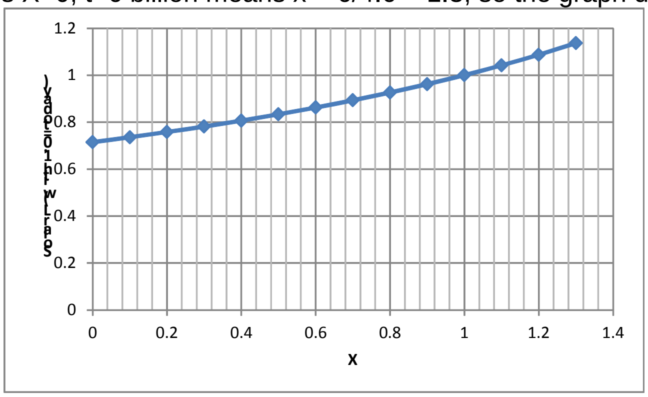
where L is the solar luminosity (today L=1.0), and A is the surface albedo, which is a number between 0 and 1, where asphalt is A=0 and A=1.0 is a perfect mirror A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

Problem 4 - Combine the two formulae above to define a new formula that gives Earth's temperature in kelvins only as a function of time, t, and albedo, A.

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 150^{o} F (339 kelvins)? (Note: it is currently 60^{o} F)

$$L = \frac{L_0}{1 + \frac{2}{5}(1 - x)}$$

Problem 1 – Graph the function L(x) for the age of the sun between 0 and 6 billion years.
Answer: t = 0 means X=0, t=6 billion means x = 6/4.6 = 1.3, so the graph domain is [0,1.3]



Problem 2 - By what percentage will L increase when it is 2 billion years older than it is today?

Answer: X = (4.6 + 2.0)/4.6 = 1.43, then L = 1 / (1+0.4(1-1.43)) so L = 1.20 this is **20% brighter** than today.

Problem 3 – A) What is the estimated current temperature of Earth if its average albedo is 0.4? B) What will be the estimated temperature of Earth when the sun is 5% brighter than today assuming that the albedo remains the same?

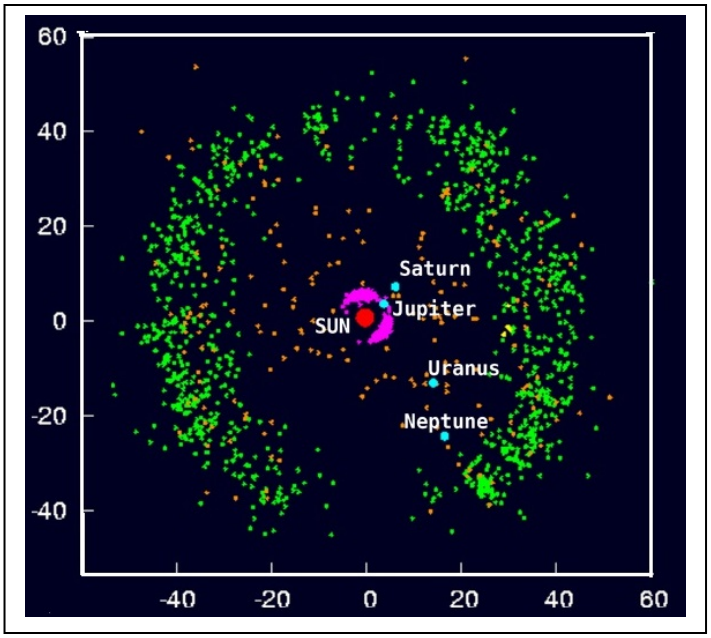
Answer: A) L = 1.0 today so T = $284((1-0.4)\times1.0)^{1/4}$ = **250 kelvins** (or -23^o Celsius)
B) L = 1.05 so T = $284(0.6\times1.05)^{1/4}$ = **253 kelvins** (or -20^o Celsius)

Problem 4 - Combine the formulae for L(x) and T to define a new formula, T(x,A) that gives Earth's temperature only as a function of time, x, and albedo, A, and assumes that L_0=1.0 today.

$$T(x, A) = 425\left(\frac{1 - A}{7 - 2x}\right)^{\frac{1}{4}}$$

Problem 5 – If the albedo of Earth increases to 0.6, what will be the age of the sun when Earth's average temperature reaches 100^o F (310 kelvins)? (Note: it is currently 60^o F)

Answer: $310 = 425\,(0.4/(7-2x))^{1/4}$
$0.4\,(425/310)^4 = 7-2x$
1.4 = 7 – 2x
2x = 5.6 so x = 2.8
and the age of the sun will be t = 2.8 x 4.6 billion = **12.9 billion years**.
This occurs 12.9 – 4.6 = 8.3 billion years in the future.



Since the belt was discovered in 1992, the number of known Kuiper belt objects (KBOs) has increased to over a thousand, and more than 100,000 KBOs over 100 km (62 mi) in diameter are believed to exist.

Pluto is the largest known member of the Kuiper belt. Originally considered a planet, Pluto's status as part of the Kuiper belt caused it to be reclassified as a "dwarf planet" in 2006

This figure shows the locations of the known KBOs with the X and Y positions given in terms of Astronomical Units (AUs), where 1 AU equals the distance from Earth to the Sun (93 million miles or 149 million km).

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by

$$V = 2\pi^2 r^2 R,$$

where R is the Kuiper Belts average distance from the sun and r is its radius?

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km^3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

Problem 1 – The Kuiper Belt stretches from 30 to 60 AU and has a torus shape. What is the volume of the Kuiper Belt in cubic kilometers if the volume of a torus is given by $V = 2\pi^2 r^2 R$, where R is the Kuiper Belts average distance from the sun and r is its radius?

Answer: $R = (60+30)/2 = 45$ AU or $45 \times 149 \times 10^6$ km = 6.7×10^9 km
$r = (60-30)/2 = 15$ AU or $15 \times 149 \times 10^6$ km = 2.2×10^9 km

$V = 2\,(3.141)2\,(2.2\times10^9)^2\,(6.7\times10^9) =$ **6.4×10^{29} km^3**

Problem 2- It is estimated that over 100,000 objects larger than 100 km reside in the Kuiper Belt, of which 1,200 have been discovered by 2013. What is the density of the estimated 100,000 objects in objects/km3 if they are uniformly distributed throughout the toroidal volume of the Kuiper Belt?

Answer: $N = 10^5/6.4\times10^{29}$ km^3 = **1.6×10^{-25} objects/km^3**.

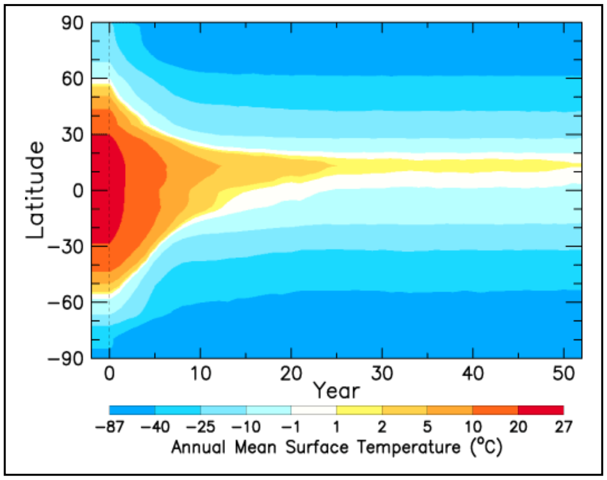
Problem 3 – Based upon the average density calculated in Problem 2, about what is the average distance between the Kuiper Belt Objects compared to the distance between Earth and Sun?

Answer: We just need to calculate the cube root of the density to get the reciprocal of this distance:

$D = 1 / (1.6\times10^{-25})^{1/3}$ = **183 million kilometers**.

Since the Earth-Sun distance is 149 million kilometers, the average distance between KBOs is about 1.2 AU or **1.2 times the Earth-Sun distance**.

The Temperature of Earth without Carbon Dioxide



A computer model developed by NASA scientists at the Goddard Institute for Space Science shows that without carbon dioxide, the terrestrial greenhouse would collapse and plunge Earth into an icebound state. Today, the average temperature is +15°C. Within 50 years the average temperature would drop to -21°C without the warming provided by atmospheric carbon dioxide. The delicate link between the planet's temperature and carbon dioxide has also been proved by geologic records of CO_2 levels during ice ages and interglacial periods. The temperature difference between an ice age period and an interglacial period is only 5°C. During previous ice ages, CO_2 levels were near 180 parts per million (ppm). During the warm interglacial periods the levels were near 280 ppm. Today we are living in an interglacial period that started 12,000 years ago and may last another 40,000 years. Scientists continue to worry that, as CO_2 levels approach 400 ppm, we are in uncharted territory with no historical precedent as far back as 1 million years.

Although there is no known process that would instantly remove all CO_2 from the atmosphere, this computer model is important for another reason. It helps us predict how warm a planet would be if it had no greenhouse gases, even though it is close to its star.

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where T < 0°C?

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

http://www.giss.nasa.gov/research/news/20101014/
How Carbon Dioxide Controls Earth's Temperature
October 14, 2010

Problem 1 - The surface area of the Earth above a latitude of θ degrees is given by

$$A = 4\pi R^2(1-\sin\theta)$$

From the computer model, after how many years will exactly half of the surface of Earth be covered by ice caps where $T < 0^oC$?

Answer: From the formula, we need $A/4\pi R^2 = ½$
so $½=1-\sin\theta$ and so $\theta = 30^o$ latitude

From the model, the zone where $T = -1^oC$ to $+1^o$ C reaches a latitude of 30^o occurs after a time of **5 years.**

Problem 2 – The albedo of Earth is a number between 0 and 1 that indicates how much sunlight it reflects back into space. The higher the albedo, the more light is reflected back into space and the less heating occurs. An albedo of A = 1.0 is a perfect mirror so that all sunlight is reflected and none is absorbed to heat the planet. An albedo of A=0 is similar to asphalt and reflects no light back into space and absorbs all the light energy to heat the planet. Ice has an albedo of A=0.7 and ocean water has A=0.2. After how many years will its albedo increase to 0.6 according to the computer models?

Answer: The average albedo is found by averaging the albedo of the area covered by the ice caps (A=0.7) with the albedo of the area covered by the ocean (a=0.2).

The area covered by ice caps = $4\pi R^2(1-\sin\theta)$
The remaining area covered by water = $4\pi R^2 - 4\pi R^2(1-\sin\theta) = 4\pi R^2(\sin\theta)$

$$0.6\ (4\pi R^2) = 0.7\ (4\pi R^2)(1-\sin\theta) + 0.2(4\pi R^2)[\sin\theta]$$

Simplifying:
$0.6 = 0.7(1-\sin\theta) + 0.2\sin\theta$
$0.6 = 0.7 - 0.7\sin\theta + 0.2\sin\theta$
$-0.1 = -0.5\sin\theta$
$\sin\theta = 0.2$
So $\theta = 12^o$

So, when the zone $-1^oC < T < +1^oC$ reaches a latitude of $+12^o$, the albedo of Earth will have increased from its current value of 0.4 to a much higher reflectivity of 0.6. According to the model graph, this happens after 10 years.

Note: Although CO_2 loss is an important cooling process, the rapid increase in albedo is a significant cause for cooling and an important 'feed back' in the process. As the planet cools, more ice appears and the albedo increases, causing more cooling and more ice to appear…