How Telescopes Work

You have probably seen a telescope before, and wondered how it works!

Telescopes are important in astronomy because they do two things extremely well. Their large lenses and mirrors can collect much more light than the human eye, which make it possible to see very faint things. This is called Light Gathering Ability. They also make distant things look much bigger than what the human eye can see so it is easier to study details. This is called magnification.

The human eye at night is a circle about 7 millimeters in diameter, called the pupil, which lets light pass through its lens and onto the retina. A telescope can have a main mirror or lens that can be many meters in diameter.

How do you figure out how much Light Gathering Ability a telescope has compared to the human eye? Just calculate the area of the two circles and form their ratio!

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of π = 3.145, what is the area of the human pupil in square millimeters?

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

Answer Key

Problem 1 – The human eye can have a pupil diameter of as much as 7 millimeters. Using the formula for the area of a circle, and a value of π = 3.145, what is the area of the human pupil in square millimeters?

Answer: A = 3.14 $(7/2)^2$ = **0.78 mm^2**

Problem 2 - The Hubble Space Telescope mirror has a diameter of 2.4 meters, which equals 2400 millimeters. What is the area of the Hubble mirror in square millimeters?

Answer: A = 3.14 $(2400/2)^2$ = **4,521,600 mm^2**

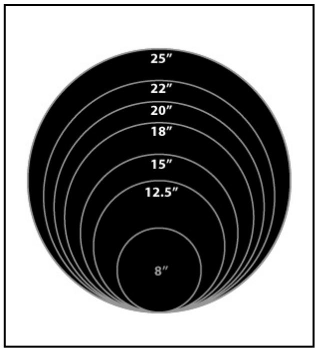
Problem 3 – What is the ratio of the area of the Hubble mirror to the human pupil? This is called the Light Gathering Ability of the Hubble Space Telescope!

Answer: 4,521,600 / 0.78 = **5,796,923 times the human eye**

Problem 4 - The faintest stars in the sky that the human eye can see are called magnitude +6.0 stars. To see magnitude +11 stars, you need a telescope that can see 100 times fainter than the human eye. What is the diameter of the mirror or lens that will let you see these faint stars?

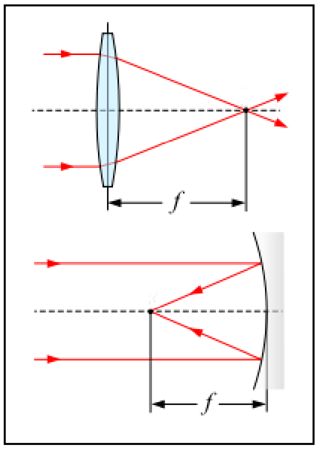
Answer: 100 = $\pi R^2/0.78$ so R^2 = 24.8 and so R = 5.0 and **D = 10.0 millimeters**.

Focal Lengths, Apertures and F/ Numbers



Any optical system, such as a telescope, camera or microscope, can be described by a just few basic numbers.

Aperture is the main lens or mirror that gathers the light to a focus. Aperture diameter, D, is commonly measured in inches, millimeters, centimeters or even meters. The larger the aperture, the more light the system gathers and the finer details it can see. The top figure shows various aperture diameters for telescopes that can be bought.



Focal length is the distance between the center of the aperture and the point in space where distant light rays come to a focus. In the figure, both a lens and a properly-curved mirror can have focal points. The symbol, f, represents the focal length.

F/ number is a measure of the speed and clarity of the optical system. It is the ratio of the focal distance to the aperture size. Fast systems have small F/numbers such as F/1, F/2 or F/3. Slow systems have large F/ numbers such as F/8, F/15 or even F/20. In photography these are also called F-stops.

$$F/ = f / D$$

Problem 1 – An astronomer wants to design a telescope that takes up the least amount of space in a research satellite. The aperture has to be 254 millimeters in order to gather the most light possible and provide the clearest images. The light path between the mirror center and the focus can be folded 3 times between mirrors separated by 500 millimeters. What is the focal length of this system and the F/ number? Is this a fast or slow system?

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

System 1 :	F/2.0		f = 100 mm
System 2 :		D= 10-inches,	f = 1270 mm
System 3 :	F/15.0	D = 50 mm	

Fill-in the missing quantities and describe the pros and cons of each system.

Problem 1 – An astronomer wants to design a telescope that takes up the least amount of space in a research satellite. The aperture has to be 254 millimeters in order to gather the most light possible and provide the clearest images. The light path between the mirror center and the focus can be folded 3 times between mirrors separated by 500 millimeters. What is the focal length of this system and the F/ number? Is this a fast or slow system?

Answer: The focal length is 3 x 1500 millimeters = 1500 millimeters, and F/ = 1500/254 = 5.9. It is a slow system because F/ > 3.0.

Problem 2 – An amateur astronomer wants to buy a telescope and has a choice between three different systems that cost about the same:

System 1 :	F/2.0	D=200 mm	f = 100 mm
System 2 :	F/5.0	D= 254 mm	f = 1270 mm
System 3 :	F/15.0	D = 50 mm	f = 750 mm

Fill-in the missing quantities and describe the pros and cons of each system.

Answer: System 1: D = 100 mm x 2.0 = 200 mm. System 2: D = 10 inches x 25.4 mm/inch = 254 millimeters so F = 1270/254 = 5.0; System 3: f = 50 mm x 15.0 = 750 mm.

System 3 is the slowest optical system in the group, and has the smallest aperture, which means that it gathers the least amount of light and so images will appear fainter and show less detail.

System 1 and 2 are very similar in aperture so they gather about the same amount of light, however, System 1 is nearly 3 times faster and so will provide the clearest images. System 1 is also shorter that System 2 (100 mm vs 1270 mm) so it would be easier and lighter to operate.

How do Telescopes Magnify?

Early depiction of a 'Dutch telescope' from the "Emblemata of zinne-werck" (Middelburg, 1624) of the poet and statesman Johan de Brune (1588-1658). The print was engraved by Adriaen van de Venne, who, together with his brother Jan Pieters van de Venne, printed books not far from the original optical workshop of Hans Lipperhey.

Telescopes can magnify the sizes of distant objects so that the eye can see them more clearly. This is very handy for astronomers who want to study distant planets, stars and galaxies to figure out what they are!

A simple telescope, called a refractor, has two lenses. The large one collects the light from a distant objects and amplifies it so that the image is much brighter than what the eye normally sees. This is called the Objective Lens, or for reflecting telescopes, the Objective Mirror. A second lens is placed at the focus of the Objective and provides the magnification you need to study the objects.

Both the Objective and the eye lens (called the Eyepiece) have their own focus points. The distance between the lens and this focus point is called the focal length. The magnification of the telescope is just the ratio of the Objective focal length to the eyepiece focal length!

$$M = \frac{\text{Objective focal length}}{\text{Eyepiece focal length}}$$

Note, the units for the focal lengths both have to be the same units…inches…millimeters….etc.

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 milimeters. What was the magnification of this telescope?

Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces? (1 inch = 25.4 mm)

Problem 1 – Galileo's first telescope consisted of two lenses attached to the inside of a tube. The Objective had a focal length of 980 millimeters and the eye lens had a focal length of 50 milimeters. What was the magnification of this telescope?

Answer. M = 980/50 = **19.6 times**.

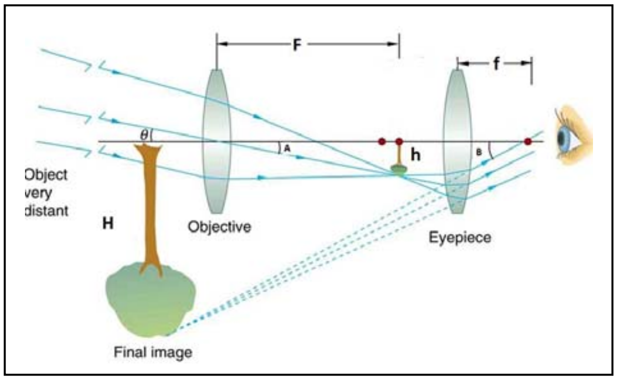
Problem 2 – In 1686, astronomer Christian Huygens built an 8-inch refractor with a 52 meter focal length. If he used the same magnifying eyepiece that Galileo had used, what would be the magnification of this 'long tube refractor'?

Answer: 52 meters = 52000 millimeters, then for a 50mm eyepiece, the magnification is M = 52000/50 = **1040 times**.

Problem 3 – An amateur builds a 20-inch reflector that has a focal length of 157 inches. He already owns three very expensive eyepieces with focal lengths of 4mm, 20mm and 35 mm. What magnification will he get from each of these eyepieces?

Answer: 157 inches x 25.4 mm/inch = 3988 mm, then

M = 3988/4 = 997x,
M = 3988/20 = 199 x
M = 3988/35 = 114 x



A telescope consists of an objective mirror or lens and an eyepiece. The role of the eyepiece is to change the angle, A, of the rays from the objective as they enter the eye. As the figure shows, when B > A, it appears as though the image of the tree is bigger than its actual image at the focus of the telescope objective. A simple proportion relates the image sizes to the focal lengths of the lenses:

$$\frac{H}{h} = \frac{F}{f}$$

For example, if the telescope objective has a focal length of 2000 millimeters and the eyepiece has a focal length of 4 millimeters, H/h = 2000/4 = 500, so the image h has been magnified by 500 times. The quantity F/f is the magnification.

Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information. Focal lengths and aperture dimensions are given in millimeters.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203		1198	10	
Obsession-20	Reflector	508	5.0		8	
1-meter	Reflector	1000	17.0			850
David Dunlop	Reflector	1880	17.3		100	
Hubble	Reflector	2400		57600		2880
Mt Palomar	Reflector		3.3	16830	28	
Yale	Refractor	1020	19.0		4	
Subaru	Reflector	8200		15000		7500
Keck	Reflector		1.75	17500	1	

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

Answer Key

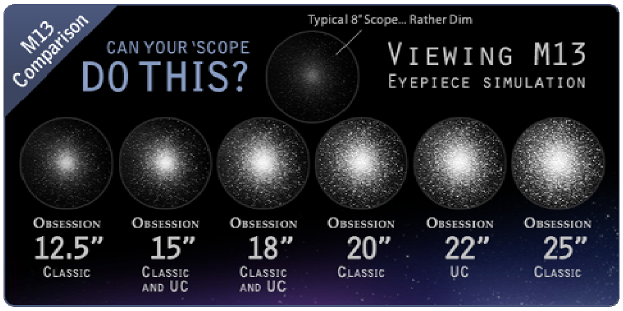
Problem 1 – The table below gives the optical data for some large telescopes. Use this data to calculate the magnification for each indicated lens. Also fill in all other missing information.

Telescope	Type	Aperture	F/	Focal Length	Eyepiece F.L.	Magnification
8-inch Orion	Reflector	203	**5.9**	1198	10	**120**
Obsession-20	Reflector	508	5.0	**2540**	8	**317**
1-meter	Reflector	1000	17.0	**17000**	**20**	850
David Dunlop	Reflector	1880	17.3	32,524	**100**	**325**
Hubble	Reflector	2400	**24**	57600	**20**	2880
Mt Palomar	Reflector	**5100**	3.3	16830	28	**601**
Yale	Refractor	1020	19.0	**19400**	4	**4850**
Subaru	Reflector	8200	**1.83**	15000	**2**	7500
Keck	Reflector	**10000**	1.75	17500	1	**17500**

Problem 2 – Suppose that the eyepiece was eliminated and the human eye was used as the eyepiece instead. If the focal length of the human eye is 25 cm, what is the magnification for the Obsession-20 telescope operating in this way? (Note: this is called Prime Focus observing).

Answer: The focal length of the Obsession-20 mirror is 2540 mm, and the eye's focal length is 250 mm, so the magnification is only about **10 times**. This means that if you look at the moon in this way, it will appear 10 times bigger 'in the sky' than without the telescope.

Note: A rule-of-thumb is that you do not use a higher magnification than 2 times the aperture size in millimeters. At higher magnifications the image remains blurry and you do not see additional details. In the table in Problem 1, the only eyepiece that violates this rule is the one selected for the Yale Telescope, and so an eyepiece with a longer focal length and lower magnification is the best to use.



This figure shows a comparison of the same faint star cluster seen with telescopes of increasing aperture size and LGA. Notice the big change between an 8-inch and am 18-inch!

The human eye is a small lens that lets-in only a small amount of light. This is useful when you are looking at a bright daytime scene, but when you are studying faint stars this becomes a problem.

A telescope has a much larger aperture than the eye and allows more light to be brought to a focus to study. This means that even stars too faint to be detected by the eye can easily be 'brightened' by the telescope so that they are easy to detect and study.

Light Gathering Ability is the property of an optical system that tells you how much brighter things will appear than what the human eye can see. It is the ratio of the area of the objective to the area of the human eye lens.

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Answer Key

Problem 1 – A pair of binoculars has a lens with a diameter of 50 mm. If the human eye lens has a diameter of 7mm, how much more light do the binoculars gather than the human eye?

Answer: LGA = $(50/7)^2$ = 51 times more light.

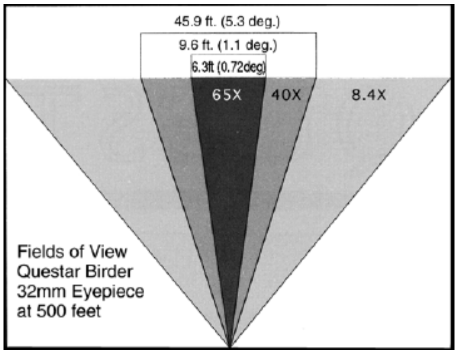
Problem 2 – Star brightness is measured on the magnitude scale where each magnitude represents an increase in intensity by a factor of 2.514. What is the brightness difference between a star with m = +1.0 and m = +6.0?

Answer: The magnitude difference is m = 5.0, so the brightness difference is a factor of $(2.512)^5$ = 100 times.

Problem 3 – The human eye can see stars as faint as m = +6.0. What size mirror will be needed so that stars as faint as +16.0 can be seen?

Answer: The magnitude difference is +16.0 - +1.0 = +10.0, which is a brightness factor of 100x100 = 10,000. You need a telescope that provides a LGA of 10000 so
10000 = $(D/7mm)^2$ and so D = 700 millimeters (27-inches in diameter).

Telescope Field of View - How much can you see?



Although magnifying an object makes it appear larger, the view also gets smaller as the diagram to the left shows. Every combination of telescope and eyepiece produces its own Field of View (FOV), usually stated in angular terms. For example, a combination that gives an angular FOV of 10^{o} in diameter will easily let you see the entire full moon, which is only 0.5^{o} in diameter. But if you use a lens with 40 times more magnification, the FOV is now only $1/4^{o}$ so you only see ½ of the full disk of the moon in the eyepiece.

By itself, an eyepiece allows incoming light to be brought to a focus for the human eye or camera. The incoming light rays can come from many different directions within a cone whose vertex is the focus point for the lens. The angle of the cone's vertex defines the FOV for the eyepiece. The table below shows the FOVs for various eyepieces that are used with telescopes:

Vendor	Model	Focal Length	Apparent FOV (°)	Actual FOV (°)	Magnifi-cation	Price
Orion	Optilux 2"	40	60	1.18	51	$140
Televue	Panoptic 2"	35	68	1.17	58	$370
Orion	FMC Plössl 2"	50	45	1.11	41	$120
Orion	DeepView 2"	42	52	1.07	48	$70
Edmund Optics	RKE Erfle 2"	32	68	1.07	64	$225
Meade	SWA 2"	32	67	1.06	64	$240
Orion	Optilux 2"	32	60	0.94	64	$140
Teleview	Panoptic 2"	27	68	0.90	75	$330
Teleview	Plössl 1.25"	40	43	0.85	51	$110
Celestron	Ultima 1.25"	35	49	0.84	58	$108

The apparent FOV for each eyepiece ranges from 45^{o} to 68^{o} and is a result of how the eyepiece is designed. When used in this example with an 8-inch telescope with a focal length of 2032 millimeters, the magnifications range from 41x to 75x. The resulting telescope FOV is then just FOV = Eyepiece FOV/magnification. For the Optilix 2" eyepiece, the FOV is then $60^{o}/51 = 1.18^{o}$.

Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60^{o}. What magnification will give him the desired FOV?

Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50^{o}. His old telescope provided a 2.0^{o} FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

Answer Key

Problem 1 – An astronomer wants to design a system so that the full moon fills the entire FOV of the telescope. He uses an eyepiece with a FOV of 60^{o}. What magnification will give him the desired FOV?

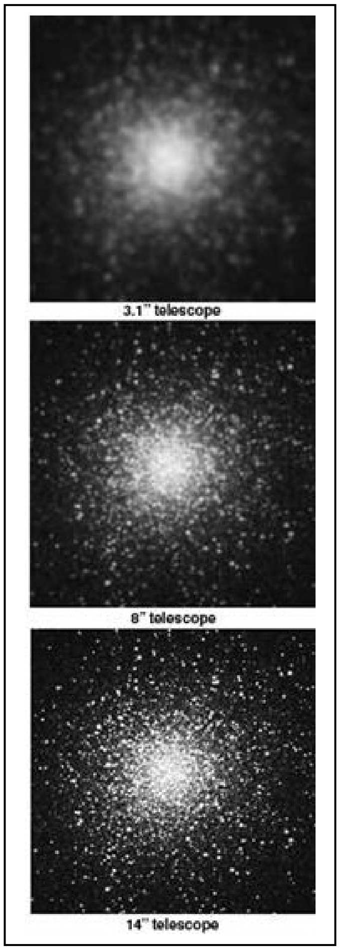
Answer: $0.5^{o} = 60^{o} / M$ so the magnification is M = **120x**.

Problem 2 – An amateur astronomer upgrades to a larger telescope and keeps his old eyepieces, which have FOVs of 50^{o}. His old telescope provided a 2.0o FOV for his most expensive eyepiece. Because the focal length of the new telescope is twice that of his older telescope, all magnifications on the new telescope will be twice as high. What will the FOV be for his most expensive eyepiece on the new telescope?

Answer: Because FOV = eyepiece FOV/magnification, if the new telescope provides magnifications that are twice the older system, then the FOV will be half as large for this eyepiece or 1.0^{o}.

Telescope Resolution – How much detail can you see?

16.7



The images to the left show what the star cluster Messier-13 would look like to three different telescopes with apertures of 3.1, 8.0 and 14.0 inches. Notice that as the aperture increases, the fuzzy smudges seen by the smallest telescope become increasingly more clear to see as the aperture increases. This is an example of Optical Resolution, which is sometimes called the <u>Resolving Power</u> of a telescope.

To make the clearest photographs of stars, planets, or even people, it helps to use the largest lens or aperture to make crisp, clear images. Astronomers also want the highest resolutions possible so that they can study the smallest details on a planet surface, or in a distant galaxy.

Telescope resolution at optical wavelengths can be calculated using the simple formula:

$$R = \frac{134}{D}$$

where D is the diameter of the objective in millimeters, and R is the resolution in seconds of arc. (There are 3600 seconds of arc in 1 angular degree).

For example, a pair of binoculars with D = 50 mm, provides a resolution limit of R = 2.8 arcseconds. A small 8-inch telescope for which D = 200 mm, provides R = 0.67 arcseconds.

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a single telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Problem 1 – An astronomer wants to design a system that will let him study craters on the moon that are about 0.1 arcseconds in diameter as seen from Earth. What is the minimum-sized aperture he needs to conduct his study?

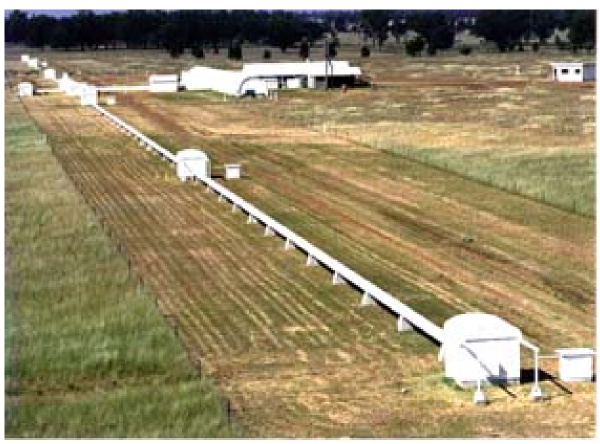
Answer: 0.1 = 134/D so D = **1340 millimeters (53 inches)**.

Problem 2 – The Hubble Space Telescope has a diameter of 2.4 meters. What is its maximum resolution?

Answer: R = 134/2400 = **0.06 arcseconds** or 60 milliarcseconds

Problem 3 – Two telescopes are combined in an instrument called an interferometer, which creates a telescope with a diameter of 640 meters. What is the maximum resolution of this system?

Answer: R = 134/640000 = **0.0002 arcseconds or 0.2 milliarcseconds**.



A photo of the Sydney University Stellar Interferometer (SUSI) is a long-baseline optical interferometer located approximately 20km west of the town of Narrabri in northern New South Wales, Australia. The equivalent diameter of the optical aperture for this instrument is 640 meters.

Buying a Telescope!

OK…So those wonderful pictures of planets, star clusters and galaxies have got your curiosity on fire. You want to have your own telescope so that you can see the universe for yourself! All you have to do is spend a few minutes on the Internet and you will see a bewildering number of choices for telescopes you can buy. Some are pretty inexpensive and cost less than $70.00, but others can cost $500.00 or more. How do you decide which one is right for you?

Remember, the bigger the objective lens or mirror, the fainter you can see stars in the sky. The longer the focal length, the higher will be the magnification. The only limit to either of these is that you should not use magnifications higher than 50x the diameter of the objective, or 2 times its diameter in millimeters. Higher magnifications only make images look worse!

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost
Reflector	7.6	300		$64.95
Refractor	6.0	700		$54.95
Refractor	8.9	910		$300.00
Reflector	11.4	900		$129.95
Reflector	15.2	610		$319.95
Refractor	10.0	900		$749.95
Reflector	20.3	1000		$699.95
Refractor	15.2	1219		$1,199.00
Reflector	50.8	2032		$4,400.00

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Answer Key

Type	Objective (cm)	Focal Length (millimeters)	Maximum Magnification	Cost	Cost per area
Reflector	7.6	300	152x	$64.95	1.4
Refractor	6.0	700	120x	$54.95	1.9
Refractor	8.9	910	178x	$300.00	4.7
Reflector	11.4	900	228x	$129.95	1.3
Reflector	15.2	610	304x	$319.95	1.8
Refractor	10.0	900	200x	$749.95	9.6
Reflector	20.3	1000	406x	$699.95	2.2
Refractor	15.2	1219	304x	$1,199.00	6.6
Reflector	50.8	2032	1016x	$4,400.00	2.2

Problem 1 – You have a set of eyepieces with focal lengths of 2mm, 4mm and 28mm. If

$$\text{magnification} = \frac{\text{telescope focal length}}{\text{eyepiece focal length}}$$

would you be able to use all of these eyepieces with the telescopes in the table above?

Answer: The limit would be set by the 2mm eyepiece. For the telescope focal lengths in the table, this eyepiece could not be used with the telescopes shaded in yellow in the table, Telescopes 2, 3, 4, 6, 7 and 8. The 4mm would not be used on telescopes 2, 3, 4, and 6.

Problem 2 - In terms of cost per objective area, which type of telescopes seem to be the best value: reflectors or refractors?

Answer: You can get a reflector with a larger area than you can a refractor. Refractors are more expensive per unit area. From the table, reflectors cost 1.3 to 2.2 dollars per square centimeter, while refractors cost 1.9 to 9.6 dollars per square centimeter.

Problem 3 – About how much would you expect to pay for a 50.8-cm refractor?

Answer: A 15.2 cm refractor costs about 6.6 dollars per square centimeter, so a 50.8-cm refractor would cost $\pi (50.8/2)^2 \times 6.6 =$ **$13,374.**

Designing a Telescope System

Telescope diameter = D
Telescope focal length = F
Eyepiece focal length = f
Eyepiece field of view = FOVe
Magnification = M
Resolution = R

D, F, f are in millimeters
FOVe is in degrees
R is in arcseconds

$$M = \frac{F}{f}$$

$$F/number = \frac{F}{D}$$

$$R = \frac{134}{D}$$

Astronomers don't just go out and buy a telescope and then use it. Whether it is for use on a satellite orbiting Saturn, or in an observatory, telescopes are designed 'from the ground up' by starting from a set of goals that the research needs to accomplish. The telescope is mathematically designed to meet these research goals.

The table to the left gives the basic quantities and formulae for designing a simple telescope system. Let's see how two different research goals can lead to very different telescope systems!

System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossl with a FOV of 68^{o}, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

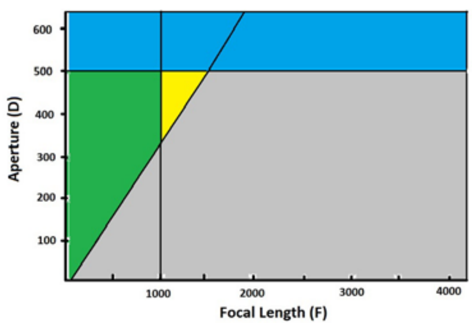
To design these systems, create a graph with the aperture diameter (vertical) and the focal length (horizontal).

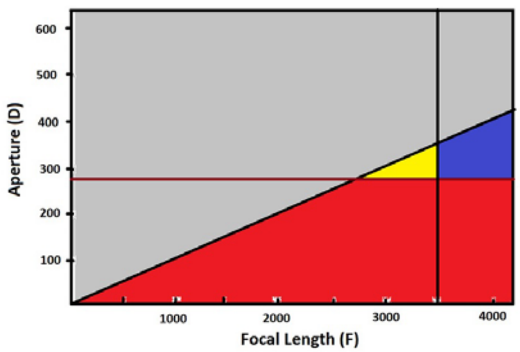
System 1 – Veronica has been an amateur astronomer for 20 years and especially enjoys photographing faint galaxies and nebulae. She has owned three telescopes and plans to sell them to fund her next system. She can afford a telescope with an aperture no larger than 20-inches (500 mm), and needs it to be a fast optical system with an F-number less than 3.0. She has a set of expensive eyepieces that she will keep. Her favorite one is a 20mm Plossel with a FOV of 68^{o}, and for best results she wants this eyepiece to have a magnification of no more than 50x. What is the best combination of aperture size and focal length for the telescope that will satisfy all of her needs?

Answer: D < 20 inches (500 mm); F/number < 3.0; f = 20 mm, FOVe=68^{o}, M> 50x

On the D vs F graph, draw a line representing F/D = 3.0 or F = 3.0D and shade the excluded region below this line (grey), which represents F/n > 3.0. Now draw a horizontal line for D = 500mm and shade (blue) the region above this line which represents D > 500mm. Magnification = F/f so we have F/f > 50 and F > 50f. For f = 20mm, the constraint is F > 1000 mm. Draw a vertical line at F = 1000mm and shade (green) all points to the left as the excluded region. The permitted regions is the one shown in yellow. The final plot (below left) should look like the one below.

An optimal system is near the middle of the yellow permitted region for which D = 450 mm and F = 1250mm. We then have F/number = 2.8, a magnification of 62, a telescope FOV of 68/62 = 1 degree, and a resolution of 134/450 = 0.3 arcseconds.



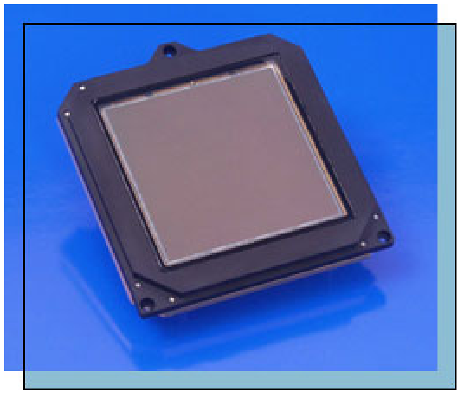


System 2 – Leonard has a program of observing Saturn to keep track of its equatorial belt system. He needs a telescope with F/number > 10 and a resolution between 1/3 and 1/2 arcseconds. He has three eyepieces with focal lengths of f = 5mm, 10mm and 20mm that have provided him with high, medium and low magnification on his previous telescope, which got damaged in a house fire. He wants the 5mm eyepiece to provide no more than a magnification of 700x. What is the best system that meets his needs?

Answer: F/number > 10; R = 1/3 to ½ arcseconds; M < 700x.
Draw a line representing F/number = 10 and shade (grey) the excluded area above this line which indicates F/number <10. For R = ½ arcseconds, draw a horizontal line at D = 270mm and shade the region (red) below this line that represents R > ½. The magnification M = F/5mm so for M=700 we have F < 3500mm. Draw a vertical line at F = 3500 and shade (blue) the excluded area to the right. The allowed region is in yellow in the diagram on the upper right.

The best system lies in the center of the triangle with D = 300mm and F = 3250 mm. This gives F/10.8; Resolution = 134/300 = 0.4 arcseconds; Lens magnifications of 650, 325 and 162.

Digital Camera Math!



This is a 9200x9200 image sensor for a digital camera. The grey area contains the individual 'pixel' elements. Each square pixel is about 8 micrometers (8 microns) wide, and is sensitive to light. The pixels accumulate electrons as light falls on them, and computers read out each pixel and create the picture.

Digital cameras are everywhere! They are in your cell phones, computers, iPads and countless other applications that you may not even be aware of.

In astronomy, digital cameras were first developed in the 1970s to replace and extend photographic film techniques for detecting faint objects. Digital cameras are not only easy to operate and require no chemicals to make the images, but the data is already in digital form so that computers can quickly process the images.

Commercially, digital cameras are referred to by the total number of pixels they contain. A '1 megapixel camera' can have a square-shaped sensor with 1024x1024 pixels. This says nothing about the sensitivity of the camera, only how big an image it can create from the camera lenses. Although the largest commercial digital camera has 80 megapixels in a 10328x7760 format, the largest astronomical camera developed for the Large Synoptic Survey Telescope uses 3200 megapixels (3.2 gigapixels)!

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Answer Key

Problem 1 – An amateur astronomer purchases a 6.1 megapixel digital camera. The sensor measures 20 mm x 20 mm. What is the format of the CCD sensor, and about how wide are each of the pixels in microns?

Answer: This is a square array, so s2 = 6100000 pixels and so s = 2469 pixels. The format is **2469 x 2469 pixels**. Since the width of a side is 20 mm, each pixel is about 20 mm/2469 = 0.0000081 meters or **8.1 microns** on a side.

Problem 2 – Suppose that with the telescope optical system, the entire full moon will fit inside the square CCD sensor. If the angular diameter of the moon is 1800 arcseconds, about what is the resolution of each pixel in the camera?

Answer: 1800 arcseconds/2469 pixels = **0.7 arcseconds/pixel**.

Problem 3 – The LSST digital camera is 3.2 gigapixels in a 10328x7760 format. If the long side of the field covers an angular range of 3.5 degrees, what is the angular resolution of this CCD camera in arcseconds/pixel?

Answer: 1 degree = 3600 arcseconds, so 3.5 degrees = 12600 arcseconds. Then 12600 arcseconds/10328 pixels = **1.2 arcseconds/pixel.**


Life-size model of the Webb Space Telescope.

In 2018, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, θ(R), for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on its surface is given by

$$T(R) = \frac{250}{\sqrt{R}} \text{ Kelvins}$$

where R is the distance from the sun in AU. Create a graph that shows T(R) vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

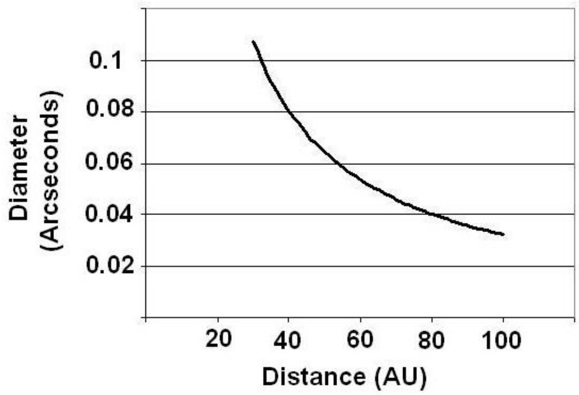
Problem 3 - A body in other outer solar system with an angular size θ(R) emits most of its light energy in the infrared and has a temperature given by T(R) in Kelvins. Its brightness in units of Janskys, F, at a wavelength of 20 microns will be given by:

$$F(T) = \frac{120000}{(e^x - 1)}\theta(R)^2 \text{ Janskys} \qquad \text{where} \qquad x = \frac{720}{T(R)}$$

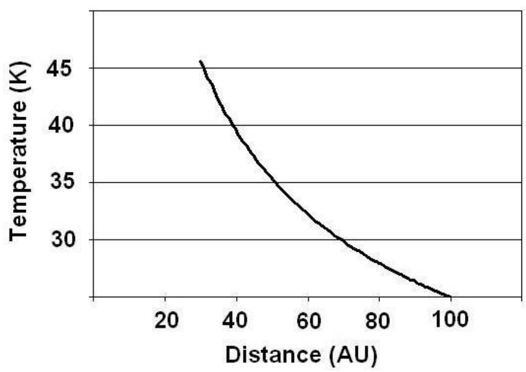
From the formula for θ(R) and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

Answer Key

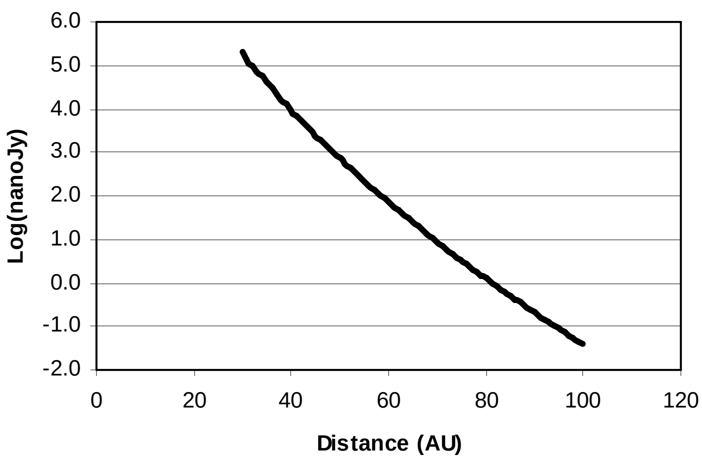
Problem 1 - Answer: At 90 AU, the disk of a Pluto-sized body will be 0.035 arcseconds in diameter.



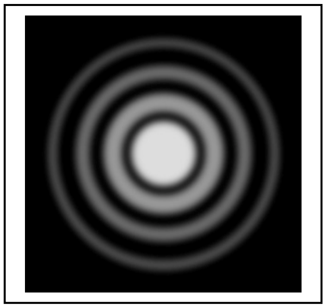
Problem 2 Answer: At 90 AU, the predicted temperature will be about 28 K.

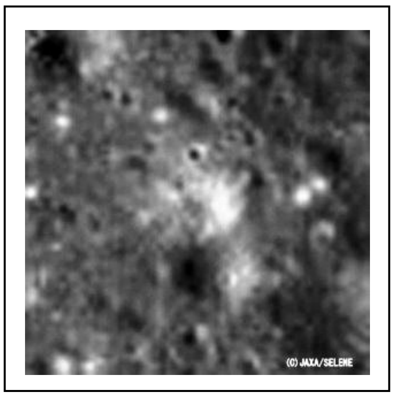


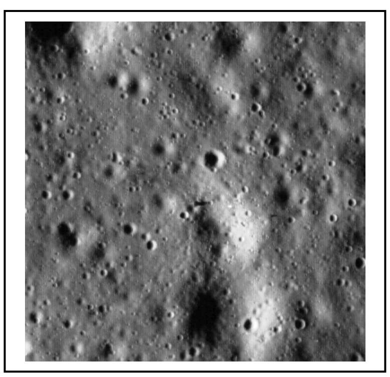
Problem 3 - Answer: At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about 40 AU. More accurate estimates using more realistic emission properties for Pluto suggest 90 AUs as an actual limit.



$$R = 1.22 \frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (1.0 meters/pixel at a 50km orbit elevation: aperture = 0.8 meter). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer Key

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = **0.14 degrees.** And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = **8.4 arcminutes.**

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?
Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is **0.069 microradians.** B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = **14 milliarcseconds.**

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/(0.0000048 radians) = **5.1 meters.**