As a comet orbits the sun, it produces a long tail stretching millions of kilometers through space. The tail is produced by heated gases leaving the nucleus of the comet.

This image of the head of Comet Tempel-1 was taken by the Hubble Space Telescope on June 30, 2005. It shows the 'coma' formed by these escaping gases about 5 days before its closest approach to the sun (perihelion). The most interesting of these ingredients is ordinary water.

Problem 1 – The NASA spacecraft Deep Impact flew-by Temple-1 and measured the rate of loss of water from its nucleus. The simple quadratic function below gives the number of tons of water produced every minute, W, as Comet Tempel-1 orbited the sun, where T is the number of days since its closest approach to the sun, called perihelion.

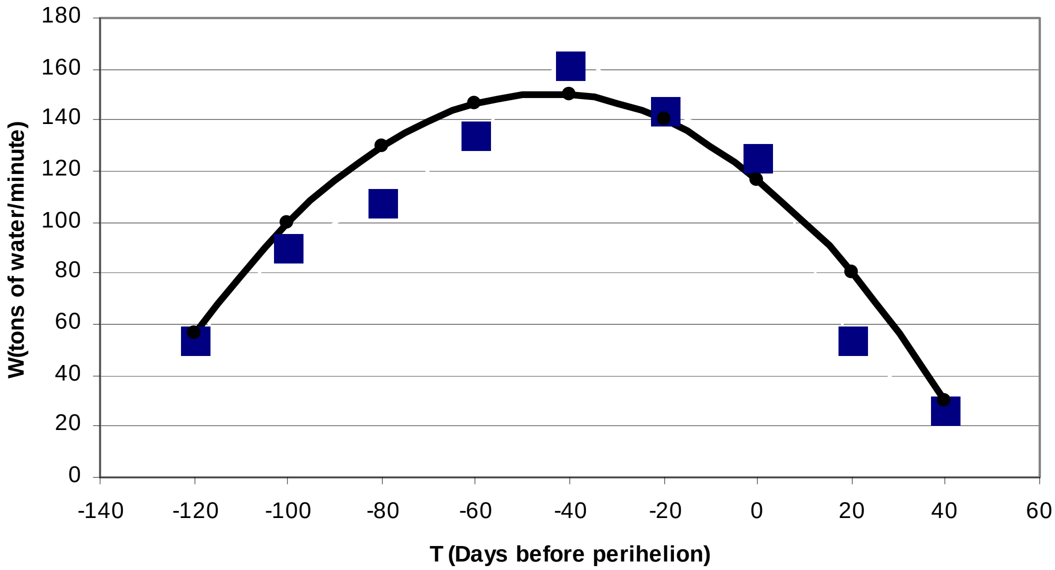
$$W(T) = \frac{(T+140)(60-T)}{60}$$

A) Graph the function W(T). B) For what days, T, will the water loss be zero? C) For what T did the comet eject its maximum amount of water each minute?

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Problem 1 – A) The graph below was created with Excel. The squares represent the actual measured data and are shown as an indicator of the quality of the quadratic model fit to the actual data.



Answer: B) The roots of the quadratic equation, where W(T)=0 are for **T=-140** days and **T=+60** days after perihelion. C) The maximum (vertex of the parabola) occurs half-way between the two intercepts at T = (-140+60)/2 or **T = -40** which indicates 40 days before perihelion.

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Answer: W(-130) = (-130+140)(60+130)/60 = **32 tons/minute**

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Answer: The fitting function W(T) predicts that W(+70) = (70+140)(60-70)/60 = -35 tons per minute. Although this value smoothly follows the prediction curve, it implies that instead of ejecting water (positive answer means a positive rate of change) the comet is absorbing water (negative answer means a negative rate of change), so the prediction is not realistic.

Water on Mars!

NASA/JPL/Malin Space Science Systems

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Question 2: How wide, in meters, are the streams half-way down their flow channels?
Question 3: What is the smallest feature you can see in the image?
Question 4: How wide is the top of the crater wall at its sharpest edge?

Answer Key:

This NASA, Mars Orbiter image was taken of a crater wall in the southern hemisphere of mars from an altitude of 450 kilometers. It shows the exciting evidence of water gullies flowing downhill from the top left to the lower right.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the length of the dark bar is a distance of 300 meters.

Step 1: Measure the length of the bar with a metric ruler. How many millimeters long is the bar?
Answer: 13 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert this to meters.
Answer: 300 meters

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 300 meters / 13 mm = 23 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in kilometers, of this image?
Answer: 134 mm x 120 mm = 3.1 km x 2.8 km

Question 2: How wide, in meters, are the streams half-way down their flow channels?
Answer: 0.5 millimeters = 12 meters.

Question 3: What is the smallest feature you can see in the image?
Answer: Sand dunes in upper left of image = 0.3 millimeters or 7 meters wide.

Question 4: How wide is the top of the crater wall at its sharpest edge?
Answer: 0.2 millimeters or 4 meters wide.

Avalanche on Mars!

This image was taken by NASA's Mars Reconnaissance Orbiter on February 19, 2008. It shows an avalanche photographed as it happened on a cliff on the edge of the dome of layered deposits centered on Mars' North Pole. From top to bottom this impressive cliff is over 700 meters (2300 feet) tall and reaches slopes over 60 degrees. The top part of the scarp, to the left of the image, is still covered with bright (white) carbon dioxide frost which is disappearing from the polar regions as spring progresses. The upper mid-toned (pinkish-brownish) section is composed of layers that are mostly ice with varying amounts of dust. The dust cloud extends 190 meters from the base of the cliff.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?

Step 2: Use clues in the image description to determine a physical distance or length.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Question 1: What are the dimensions, in meters, of this image?
Question 2: What is the smallest detail you can see in the ice shelf?
Question 3: What is the average thickness of the red layers on the cliff?
Question 4: What is the total width of the reddish rock cliff?

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?

Answer Key:

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the cloud extends 190 meters from the base of the cliff.

Step 1: Measure the length of the dust cloud with a metric ruler. How many millimeters long is the cloud?
Answer: 60 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length.
Answer: 190 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 190 meters / 60 mm = 3.2 meters / mm

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

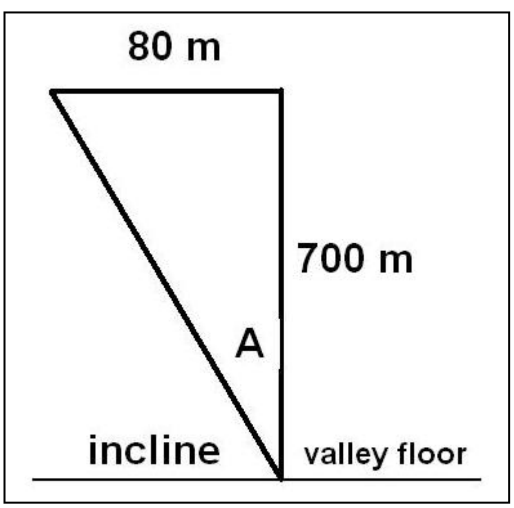
Question 1: What are the dimensions, in meters, of this image?
Answer: 140 mm x 86 mm = 448.0 meters x 275.2 meters, but to two significant figures this becomes 450 meters x 280 meters.

Question 2: What is the smallest detail you can see in the ice shelf?
Answer: 0.2 mm = 0.6 meters

Question 3: What is the average thickness of the red layers on the cliff?
Answer: 1.0 millimeter = 3.2 meters.

Question 4: What is the total width of the reddish rock cliff?
Answer: 25 millimeters = 80 meters.

For experts: Two sides of the right triangle measure 700 meters, and your answer to Question 4. What is the angle of the cliff at the valley floor?
Answer: Draw the triangle with the 700-meter side being vertical and the 80 meter side being horizontal. The tangent of the angle is (80 / 700) = 0.11 so the angle is 6.5 degrees. This is the angle from the vertical, so the incline angle from the floor of the valley is 90 - 6.5 = 84 degrees. This is a nearly vertical wall!



Kelvin Temperatures and Very Cold Things!

183 K	Vostok, Antarctica
160 K	Phobos
134 K	Superconductors
128 K	Europa summer
120 K	Moon at night
95 K	Titan
90 K	Liquid oxygen
88 K	Miranda
81 K	Enceladus summer
77 K	Liquid nitrogen
70 K	Mercury at night
63 K	Solid nitrogen
55 K	Pluto summer
54 K	Solid oxygen
50 K	Quaoar
45 K	Moon - shadowed crater
40 K	Star-forming region
33 K	Pluto wInter
20 K	Liquid hydrogen
19 K	Bose-Einstein Condensates
4 K	Liquid helium
3 K	Cosmic Background Radiation
2 K	Liquid helium
1 K	Boomerang Nebula
0 K	ABSOLUTE ZERO

To keep track of some of the coldest things in the universe, scientists use the Kelvin temperature scale which begins at 0 Kelvin, which is also called Absolute Zero. Nothing can ever be colder than Absolute Zero because at this temperature, all motion stops. The table to the left shows some typical temperatures of different systems in the universe.

You are probably already familiar with the Centigrade (C) and Fahrenheit (F) temperature scales. The two formulas below show how to switch from degrees-C to degrees-F.

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32$$

Because the Kelvin scale is related to the Centigrade scale, we can also convert from Centigrade to Kelvin (K) using the equation:

$$K = 273 + C$$

Use these three equations to convert between the three temperature scales:

Problem 1: 212 F converted to K

Problem 2: 0 K converted to F

Problem 3: 100 C converted to K

Problem 4: -150 F converted to K

Problem 5: -150 C converted to K

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F. A) What are the equivalent temperatures on the Kelvin scale; B) What is the average daytime temperature on the Kelvin scale?

Answer Key

$$C = \frac{5}{9}(F - 32) \qquad F = \frac{9}{5}C + 32 \qquad K = 273 + C$$

Problem 1: 212 F converted to K :
First convert to C: C = 5/9 (212 - 32) = +100 C. Then convert from C to K:
K = 273 + 100 = **373 Kelvin**

Problem 2: 0 K converted to F: First convert to Centigrade:
0 = 273 + C so C = -273 degrees. Then convert from C to F:
F = 9/5 (-273) + 32 = **-459 Fahrenheit**.

Problem 3: 100 C converted to K : K = 273 - 100 = **373 Kelvin**.

Problem 4: -150 F converted to K : Convert to Centigrade
C = 5/9 (-150 - 32) = -101 C. Then convert from Centigrade to Kelvin: K = 273 - 101 = **172 Kelvin**.

Problem 5: -150 C converted to K : K = 273 + (-150) = **123 Kelvin**

Problem 6: Two scientists measure the daytime temperature of the moon using two different instruments. The first instrument gives a reading of + 107 C while the second instrument gives + 221 F.

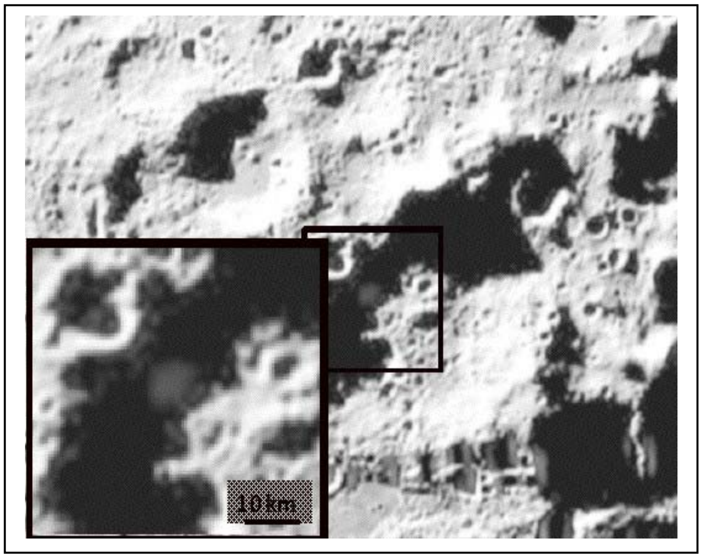
A) What are the equivalent temperatures on the Kelvin scale?;
107 C becomes K = 273 + 107 = **380 Kelvins.**
221 F becomes C = 5/9 (221 - 32) = 105 C, and so K = 273 + 105 = **378 Kelvins**.

B) What is the average daytime temperature on the Kelvin scale?
Answer: (380 + 378)/2 = **379 Kelvins**.

C) Explain why the Kelvin scale is useful for calculating averages of different temperatures. Answer: **Because the degrees are in the same units in the same measuring scale so that the numbers can be averaged.**

Note: Students may recognize that in order to average +107 C and +221 F they could just as easily have converted both temperatures to the Centigrade scale or the Fahrenheit scale and then averaged those temperatures. You may challenge them to do this, and then compare the averaged values in the Centigrade, Fahrenheit and Kelvin scales. They should note that the final answer will be the same as 379 Kelvins converted to F and C scales using the above formulas.

LCROSS 'Sees' Water on the Moon!



On October 9, 2009 the LCROSS spacecraft and its companion Centaur upper stage, impacted the lunar surface within the shadowed crater Cabeus located near the moon's South Pole. The Centaur impact speed was 9,000 km/hr with a mass of 2.2 tons.

The impact created a crater about 20 meters across and about 3 meters deep. Some of the excavated material formed a plume of debris visible to the LCROSS satellite as it flew by. Instruments on LCROSS detected about 25 gallons of water.

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?
Answer: V = (3.14) x (10 meters)^2 x 3 = **942 cubic meters**.

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?
Answer: 3000 kg/m^3 x (942 meters^3) = 2,800,000 kilograms. Since 1000 kg = 1 ton, there were **2,800 tons of regolith excavated**.

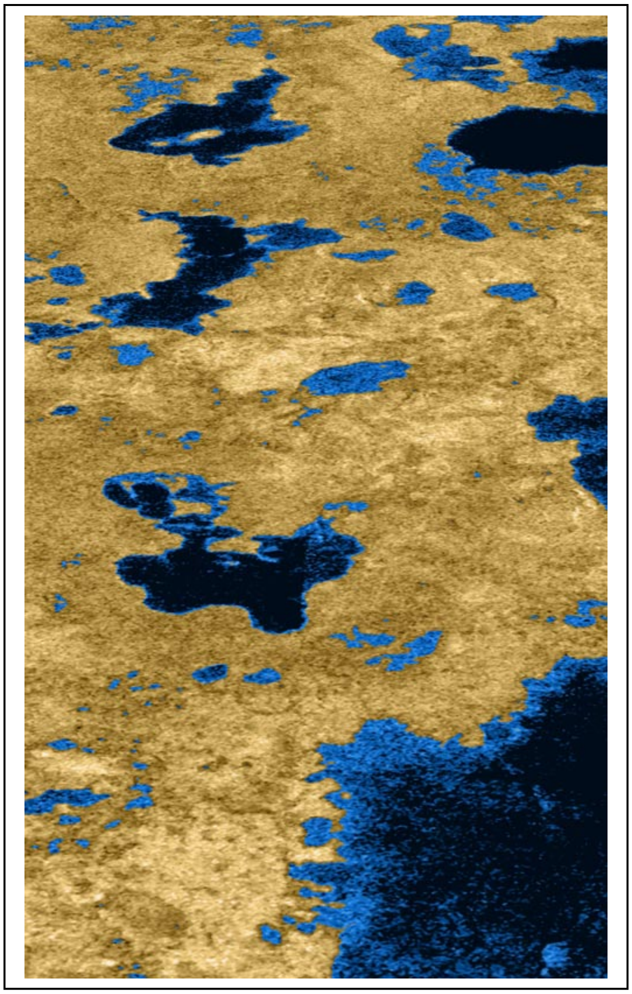
Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?
Answer: The area of the ejecta blanket is given by A = π(35 meters)^2 - π(10 meters)^2 = 3,846 - 314 = 3500 meters^2. The volume is A x h = (3500 meters^2) x 0.2 meters = 700 meters^3 . Then the mass is just M = (700 meters^3) x (3,000 kilograms/meter^3) = 2,100,000 kilograms or **2,100 tons in the ejecta blanket**.

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Answer: The amount of ejected regolith was 2,800 tons - 2,100 tons or 700 tons. The detected water amounted to 25 gallons or 25 gallons x (3.78 liters/ 1 gallon) = 95 liters. So the concentration was about C = 700 tons/95 liters = **7 tons/liter**.

Note to teacher: The estimated concentration, C, in Problem 4 is based on an approximated geometry for the crater (cylinder), an average thickness for the ejecta blanket (about 0.2 meters) and whether all of the remaining material (700 tons) was actually involved in the plume measured by LCROSS. Students may select, by scaled drawing, other geometries for the crater, and thickness for the ejecta blanket to obtain other estimates for the concentration, C. The scientific analysis of the LCROSS data may eventually lead to better estimates for C.

Lakes of Methane on Titan



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Answer: 150 km / 77 millimeters = **1.9 km/mm**.

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Answer: Combining the areas over the rectangular field of view gives about 1/4 of the area covered. The field of view measures 77 mm x 130mm or 150 km x 247 km or an area of 37,000 km^2. The dark areas therefore cover about 1/4 x 37,000 km^2 or **9,300 km^2.**

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Answer: Volume = area x height = 9,300 km^2 x (0.02 km) = **190 km^3**.

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?
.
Answer: 190 km^3 / 150 km^3 = **1.3 Lake Tahoes**.

How Hot is that Planet?

Planets have been spotted orbiting hundreds of nearby stars, but this makes for a variety of temperatures depending on how far the planet is from its star and the stars luminosity.

The temperature of the planet will be about

$$T=273\left(\frac{(1-A)L}{D^2}\right)^{1/4}$$

where A is the reflectivity (albedo) of the planet, L is the luminosity of its star in multiples of the sun's power, and D is the distance between the planet and the star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). The resulting temperature will be in units of Kelvins. (i.e. 0^o Celsius = +273 K, and Absolute Zero is defined as 0 K).

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4?

Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of our sun were increased 1000 times and all other quantities remained the same?

Problem 3 - The recently discovered planet CoRoT-7b (see artist's impression above, from ESA press release), orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	
Water+Land	Earth	0.40	
Gas	Jupiter	0.70	

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4? **Answer: $T = 273\ (0.6)^{1/4} = 240$ K**

Note: The equilibrium temperature of Earth is much lower than the freezing point of water. Were it not for the trace gases of carbon dioxide and to a lesser extent water vapor and methane providing 'greenhouse heating' our planet would be unlivable even with an atmosphere!

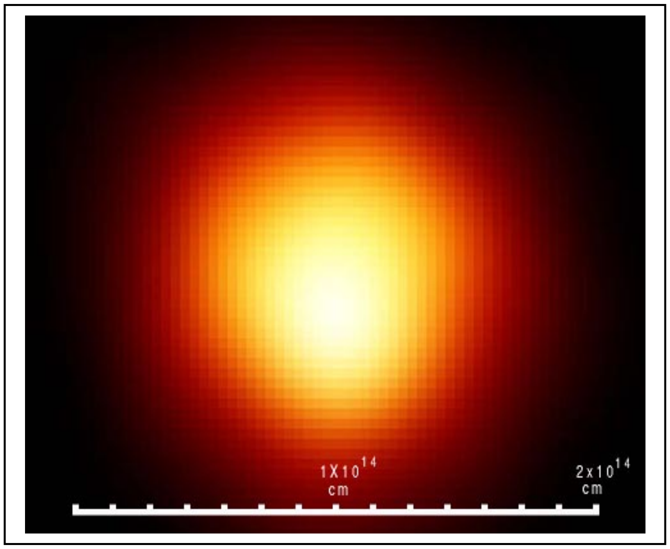
Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of our sun were increased 1000 times and all other quantities remained the same? Answer: From the formula, T = 240 and L = 1000 so
$240 = 273(0.6 \times 1000/D^2)^{1/4}$ and so **D = 5.6 AU**. This is about near the orbit of Jupiter.

Problem 3 - The recently discovered planet CoRoT-7b orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	1840
Water+Land	Earth	0.40	1699
Gas	Jupiter	0.70	1422

Example: For an albedo similar to that of our Moon:
$T = 273 * ((1-0.06)*0.71/(0.017)^2)^{.25}$
= **1,892 Kelvin**

Note: To demonstrate the concept of Significant Figures, the values for L, D and A are given to 2 significant figures, so the answers should be rounded to 1900, 1800, 1700 and 1400 respectively.



The amount of power that a star produces in light is related to the temperature of its surface and the area of the star. The hotter a surface is, the more light it produces. The bigger a star is, the more surface it has. When these relationships are combined, two stars at the same temperature can be vastly different in brightness because of their sizes.

Image: Betelgeuse (Hubble Space Telescope.) It is 950 times bigger than the sun!

The basic formula that relates stellar light output (called luminosity) with the surface area of a star, and the temperature of the star, is L = A x F where the star is assumed to be spherical with a surface area of $A = 4\pi R^2$, and the radiation emitted by a unit area of its surface (called the flux) is given by $F = \sigma T^4$. The constant, σ, is the Stefan-Boltzman radiation constant and it has a value of $\sigma = 5.67 \times 10^{-5}$ ergs/ (cm^2 sec deg^4). The luminosity, L, will be expressed in power units of ergs/sec if the radius, R, is expressed in centimeters, and the temperature, T, is expressed in Kelvins. The formula then becomes,

$$L = 4\pi R^2 \sigma T^4$$

Problem 1 - The Sun has a temperature of 5700 Kelvins and a radius of 6.96×10^5 kilometers, what is its luminosity in A) ergs/sec? B) Watts? (Note: 1 watt = 10^7 ergs/sec).

Problem 2 - The red supergiant Antares in the constellation Scorpius, has a temperature of 3,500 K and a radius of 700 times the radius of the sun. What is its luminosity in A) ergs/sec? B) multiples of the solar luminosity?

Problem 3 - The nearby star, Sirius, has a temperature of 9,200 K and a radius of 1.76 times our Sun, while its white dwarf companion has a temperature of 27,400 K and a radius of 4,900 kilometers. What are the luminosities of Sirius-A and Sirius-B compared to our Sun?

Calculus:

Problem 4 - Compute the total derivative of L(R,T). If a star's radius increases by 10% and its temperature increases by 5%, by how much will the luminosity of the star change if its original state is similar to that of the star Antares? From your answer, can you explain how a star's temperature could change without altering the luminosity of the star. Give an example of this relationship using the star Antares!

Answer Key

Problem 1 - We use L = 4 (3.141) R^2 (5.67 x 10^{-5}) T^4 to get L (ergs/sec) = 0.00071 $R(cm)^2$ $T(K)^4$ then,

A) L(ergs/sec) = 0.00071 x (696,000 km x 10^5 cm/km)^2 (5700)^4 = **3.6 x 10^{33} ergs/sec**

B) L(watts) = 3.6 x 10^{33} (ergs/sec)/ 10^7 (ergs/watt) = **3.6 x 10^{25} watts**.

Problem 2 - A) The radius of Antares is 700 x 696,000 km = 4.9 x 10^8 km.
L(ergs/sec) = 0.00071 x (4.9 x 10^8 km x 10^5 cm/km)^2 (3500)^4 = **2.5 x 10^{38} ergs/sec**
B) L(Antares) = (2.5 x 10^{38} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **71,000 L(sun).**

Problem 3 - Sirius-A radius = 1.76 x 696,000 km = 1.2 x 10^6 km
L(Sirius-A) = 0.00071 x (1.2 x 10^6 km x 10^5 cm/km)^2 (9200)^4 = **7.3 x 10^{34} ergs/sec**
L = (7.3 x 10^{34} ergs/sec) / (3.6 x 10^{33} ergs/sec) = **20.3 L(sun).**
L(Sirius-B) = 0.00071 x (4900 km x 10^5 cm/km)^2 (27,400)^4 = **9.5 x 10^{31} ergs/sec**
L(Sirius-B) = 9.5 x 10^{31} ergs/sec / 3.6 x 10^{33} ergs/sec = **0.026 L(sun).**

Advanced Math:

Problem 4 *(*Note: In the discussion below, the symbol *d* represents a partial derivative)

$$dL(R,T) = \frac{dL(R,T)}{dR}\, dR + \frac{dL(R,T)}{dT}\, dT$$

$$dL = [\, 4\pi\ (2)\ R\sigma T^4\,]\ dR + [\, 4\pi\ (4)\ R2\,\sigma T^3\,]\ dT$$

$$dL = 8\pi\ R\sigma T^4\ dR + 16\pi\ R^2\sigma T^3\ dT$$

To get percentage changes, divide both sides by $L = 4\pi\ R^2 \sigma T^4$

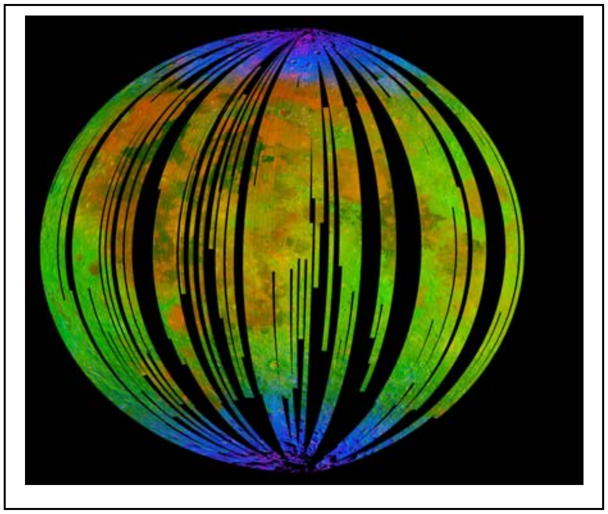
$$\frac{dL}{L} = \frac{8\pi\ R\sigma T^4}{4\pi\ R^2\sigma T^4}\, dR + \frac{16\pi\ R^2\sigma T^3}{4\pi\ R^2\sigma T^4}\, dT$$

Then dL/L = 2 dR/R + 4 dT/T so for the values given, dL/L = 2 (0.10) + 4 (0.05) = **0.40 The star's luminosity will increase by 40%.**

Since dL/L = 2 dR/R + 4 dT/T, we can obtain no change in L if 2 dR/R + 4 dT/T = 0. This means that 2 dR/R = - 4 dT/T and so, - 0.5 dR/R = dT/T. **The luminosity of a star will remain constant if, as the temperature decreases, its radius increases.**

Example. For Antares, its original luminosity is 71,000 L(sun) or 2.5 x 10^{38} ergs/sec. If I increase its radius by 10% from 4.9 x 10^8 km to 5.4 x 10^8 km, its luminosity will remain the same if its temperature is decreased by dT/T = 0.5 x 0.10 = 0.05 which will be 3500 x 0.95 = 3,325 K so L(ergs/sec) = 0.00071 x (5.4 x 10^8 km x 10^5 cm/km)^2 (3325)^4 = **2.5 x 10^{38} ergs/sec**

Water on the Moon !



The debate has gone back and forth over the last 10 years as new data are found, but measurements by Deep Impact/EPOXI, Cassini and most recently the Lunar Reconnaissance Orbiter and Chandrayaan-1 are now considered conclusive. Beneath the shadows of polar craters, billions of gallons of water may be available for harvesting by future astronauts.

The image to the left created by the Moon Minerology Mapper (M3) instrument onboard Chandrayaan-1, shows deposits and sources of hydroxyl molecules. The data has been colored blue and superimposed on a lunar photo.

Complimentary data from the Deep Impact/EPOXI and Cassini missions of the rest of the lunar surface also detected hydroxyl molecules covering about 25% of the surveyed lunar surface. The hydroxyl molecule (symbol OH) consists of one atom of oxygen and one of hydrogen, and because water is basically a hydroxyl molecule with a second hydrogen atom added, detecting hydroxyl on the moon is an indication that water molecules are also present.

How much water might be present? The M3 instrument can only detect hydroxyl molecules if they are in the top 1-millimeter of the lunar surface. The measurements also suggest that about 1 metric ton of lunar surface has to be processed to extract 1 liter (0.26 gallons) of water.

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1?

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the lunar surface contains water?

Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters).

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Answer: The surface area of a sphere is given by $S = 4\pi r^2$ and so the volume of a layer with a thickness of L is $V = 4\pi r^2 L$ provided that L is much smaller than r.
$V = 4 \times (3.141) \times (1731000)^2 \times 0.001 =$ **3.76×10^{10} m^3**

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1? Answer: 3.76×10^{10} m^3 x (3000 kg/m^3) x (1 ton/1000 kg) = **1.13×10^{11} metric tons.**

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the surface contains water? Answer: 1.13×10^{11} tons x (1 liter water/1 ton regolith) x 1/4 = **2.8×10^{10} liters of water.**

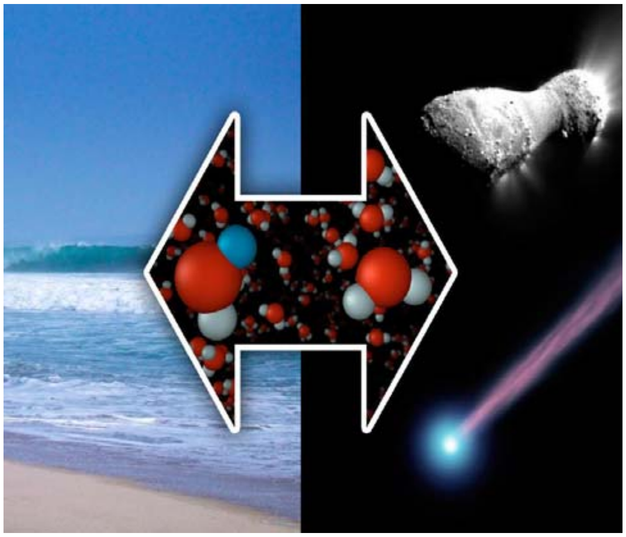
Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters). Answer: 2.8×10^{10} liters x (1 gallon / 3.78 liters) = 7.5×10^9 gallons of water or about **8 billion gallons of water.**

Note: This is similar to the roughly '7 billion gallon' estimate made by the M3 scientists as described in the NASA Press Release for this discovery in September 2009.

For more information, visit:

Moon Minerology Mapper News - http://moonmineralogymapper.jpl.nasa.gov/

<u>The front picture</u> of the moon is from NASA's Moon Mineralogy Mapper on the Indian Space Research Organization's Chandrayaan-1 mission. It is a three-color composite of reflected near-infrared radiation from the sun, and illustrates the extent to which different materials are mapped across the side of the moon that faces Earth. Small amounts of water and hydroxyl (blue) were detected on the surface of the moon at various locations. This image illustrates their distribution at high latitudes toward the poles. **Blue** shows the signature of water and hydroxyl molecules as seen by a highly diagnostic absorption of infrared light with a wavelength of three micrometers. **Green** shows the brightness of the surface as measured by reflected infrared radiation from the sun with a wavelength of 2.4 micrometers. **Red** shows an iron-bearing mineral called pyroxene, detected by absorption of 2.0-micrometer infrared light.



The abundance of heavy-water in Earth's oceans is about 0.015%. The abundance of heavy-water in Hartley-2 is about 0.016%, so comets like Hartley-2 could have impacted Earth and deposited over time Earth's ocean water. (Image courtesy NASA/JPL-Caltech)

New measurements from the Herschel Space Observatory show that comet Hartley 2, which comes from the distant Kuiper Belt, contains water with the same chemical signature as Earth's oceans. This remote region of the solar system, some 30 to 50 times as far away as the distance between Earth and the sun, is home to icy, rocky bodies including Pluto, other dwarf planets and innumerable comets.

Herschel detected the signature of vaporized water in this coma and, to the surprise of the scientists, Hartley 2 possessed half as much "heavy water" as other comets analyzed to date. In heavy water, one of the two normal hydrogen atoms has been replaced by the heavy hydrogen isotope known as deuterium. The amount of deuterium is similar to the abundance of this isotope in Earth's ocean water.

The deposition of Earth's oceans probably occurred between 4.2 and 3.8 billion years ago. Suppose that the comet nuclei consisted of three major types, each spherical in shape and made of pure water-ice: Type 1 consisting of 2km in diameter bodies arriving once every 6 months, Type-2 consisting of 20 km diameter bodies arriving once every 600 years and Type-3 consisting of 200 km diameter bodies arriving every one million years.

Problem 1 - What are the volumes of the three types of comet nuclei in km^3?

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Space Observatory Provides Clues to Creation of Earth's Oceans

http://www.nasa.gov/mission_pages/herschel/news/herschel20111005.html

Problem 1 - What are the volumes of the three types of comet nuclei in km^3?

Answer: $V = 4/3\ \pi\ R^3$ so

Type 1 Volume = **4.2 km^3**

Type 2 Volume = **4,200 km^3**

Type 3 Volume = **4.2×10^6 km^3**

Problem 2 - The volume of Earth's liquid water oceans is 1.33×10^9 cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Answer: 1.33×10^9 km^3 of water requires
$6 \times (1.33 \times 10^9\ km^3) = 8.0 \times 10^9\ km^3$ of ice.

The average delivery rate would be about
$R = 8.0 \times 10^9\ km^3$ of ice / 400 million years
= **20 km^3 of ice per year**.

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Type 1: $R1 = 4.2\ km^3/0.5$ yrs = **8.4 km^3/yr**

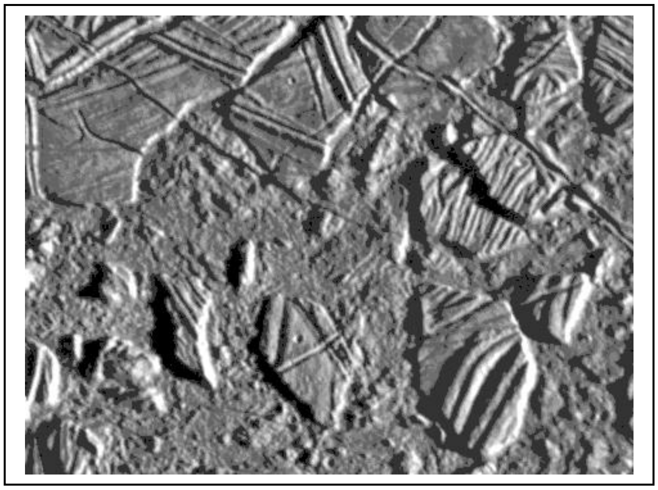
Type 2: $R2 = 4200\ km^3/600$ yrs = **7.0 km^3/yr**

Type 3: $R3 = 4.2 \times 10^6\ km^3$ / 1000000 yrs = **4.2 km^3/yr**

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Answer: The total deposition rate is $R1+R2+R3 = 20\ km^3$/yr, so it would take
$T = 8.0 \times 10^9\ km^3$ of ice / ($20\ km^3$/yr) = **400 million years**.

Water on Planetary Surfaces



Space is very cold! Without a source of energy, like a nearby star, water will exist at a temperature at nearly -270 C below zero and frozen solid. To create a permanent body of liquid water in which pre-biotic chemistry can occur, a steady source of energy must flow into the ice to keep it melted and in liquid form. Common sources of energy on Earth are volcanic activity, oceanic vents and fumaroles, and sunlight.

The picture above was taken by NASA's Galileo spacecraft of the surface of Jupiter's moon Europa. Its icy crust is believed to hide a liquid-water ocean beneath. The energy for keeping the water in a liquid state is probably generated by the gravity of Jupiter, which distorts Europa's shape through tidal action. The tidal energy may be enough to keep the oceans liquid for billions of years.

A common measure of energy flow or usage is the Watt. One Watt equals one Joule of energy emitted or consumed in one second.

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Water ice at 0 C requires 330,000 Joules of energy to become liquid for each kilogram of ice. Suppose the ice absorbed all the energy that fell on it. Ice doesn't really work that way, but let's suppose that it does just to make a simple mathematical model!

Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Answer Key

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Answer: 100 watts is the same as 100 Joules/sec, so if 1 hour = 3600 seconds, the energy consumed is 100 Joules/sec x 3600 seconds = **360,000 Joules.**

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Answer: 3,000 kilowatts x (1,000 watts/1 kilowatt) = 3,000,000 watts. Since this equals 3,000,000 Joules/sec, in 1 hour (3600 seconds) the consumption is 3,000,000 watts x 3,600 seconds = **10,800,000,000 Joules or 10.8 billion Joules.**

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Answer: 3,600,000 Joules in 1 hour is an average rate of 3,600,000 Joules/3,600 seconds = 1,000 Joules/sec or **1,000 Watts**. This is the maximum rate at which the appliances can consume before exceeding the capacity of the solar system.

Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Answer: From the information provided, it takes 330,000 Joules to melt 1 kilogram of ice. So since the mass of the ice block is 10 kilograms, it will take 330,000 x 10 = **3,300,000 Joules**

If the ice stores the energy falling on it from the hair dryer, all we need to do is to calculate how long a 2,000-watt hair dryer needs to be run in order to equal 3,300,000 Joules. This will be Time = 3,300,000 Joules/2000 watts = **1650 seconds or about 27.5 minutes.**

Proving that the unit will automatically be in seconds is a good exercise in unit conversions and using the associative law and reciprocals:

1 Joule / (1 watt)	= 1 Joule / (1 Joule/sec)	
	= 1 Joule x (1 sec/1 joule)	: Multiply be the reciprocal
	= (1 Joule x 1 sec)/ 1 joule	: Re-write
	= 1 sec x (1 joule / 1 joule)	: Re-group common terms
	= 1 sec.	:after cancling the 'joules' unit.