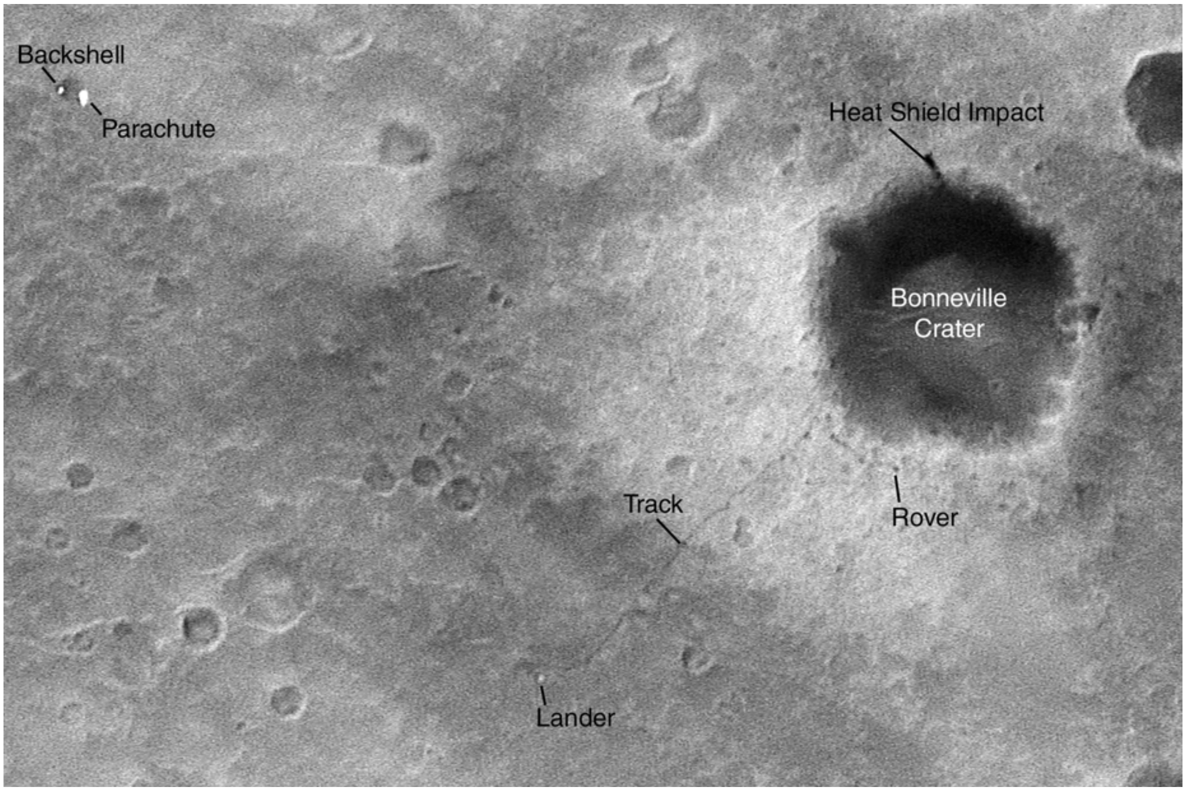
Mars Rover Landing Site



This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the image is exactly 895 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?
Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.
Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters to two significant figures.

Problem 1: About what is the diameter of Bonneville Crater rounded to the nearest ten meters?

Problem 2: How wide, in meters, is the track of the Rover?

Problem 3: How big is the Rover?

Problem 4: How small is the smallest well-defined crater to the nearest meter in size?

Problem 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?

Answer Key:

This NASA, Mars Orbiter image of the Mars Rover, Spirit, landing area near Bonneville Crater. The width of the crater is 200 meters. (Credit: NASA/JPL/MSSS). It shows the various debris left over from the landing, and the track of the Rover leaving the landing site.

The scale of an image is found by measuring with a ruler the distance between two points on the image whose separation in physical units you know. In this case, we are told the width of the image is 895 meters.

Step 1: Measure the width of the image with a metric ruler. How many millimeters wide is it?
Answer: 157 millimeters.

Step 2: Use clues in the image description to determine a physical distance or length. Convert to meters.
Answer: 895 meters.

Step 3: Divide your answer to Step 2 by your answer to Step 1 to get the image scale in meters per millimeter to two significant figures.
Answer: 895 m/157 mm = 5.7 meters / millimeter.

Once you know the image scale, you can measure the size of any feature in the image in units of millimeters. Then multiply it by the image scale from Step 3 to get the actual size of the feature in meters.

Problem 1: About what is the diameter of Bonneville Crater rounded to the nearest 10 meters?
Answer: Students answers for the diameter of the crater in millimeters may vary, but answers in the range from 30-40 mm are acceptable. Then this equals 30x5.7 = 170 meters to 40x5.7=230 meters. Students may average these two measurements to get (170+230)/2 = 200 meters.

Problem 2: How wide, in meters, is the track of the Rover?
Answer: 0.2 millimeters = 1 meter.

Problem 3: How big is the Rover?
Answer: 0.3 millimeters = 1.7 meters but since the measurement is only 1 significant figure, the answer should be 2 meters.

Problem 4: How small is the smallest well-defined crater in meters?
Answer: 2 millimeters x 5.7 = 11.4 meters, which to the nearest meter is 11 meters.

Problem 5: A boulder is typically 5 meters across or larger. Are there any boulders in this picture?
Answer: Students answers may vary and lead to interesting discussions about what features are real, and which ones are flaws in the printing of the picture. This is an important discussion because 'image artifacts' are very common in space-related photographs. 5 meters is about 1 millimeter, and there are no obvious rounded objects this large or larger in this image.

Mercury and the Moon - Similar but different!

The image of Mercury's surface on the left was taken by the MESSENGER spacecraft on March 30, 2011 of the region near crater Camoes near Mercury's south pole. In an historic event, the spacecraft became the first artificial satellite of Mercury on March 17, 2011. The image on the right is a similar-sized area of our own Moon near the crater King, photographed by Apollo 16 astronauts.

The Mercury image is 100 km wide and the lunar image is 115 km wide.

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Answer: Mercury image width is 80 mm, scale is 100 km/80mm = **1.3 km/mm**
Moon image width = 68 mm, scale is 115 km/68mm = **1.7 km/mm**

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Answer: Mercury = 0.5 mm x (1.3 km/mm) = **700 meters**.
Moon = 1.0 mm x (1.7 km/mm) = **1,700 meters**.

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Answer: **On Mercury, less of the material ejected by the impact gets away, and so more of it falls back to the surface near the crater. For the Moon, the escape velocity is so low that ejected material can travel great distances, or even into orbit and beyond, so less of it falls back to the surface to make additional craters**.

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

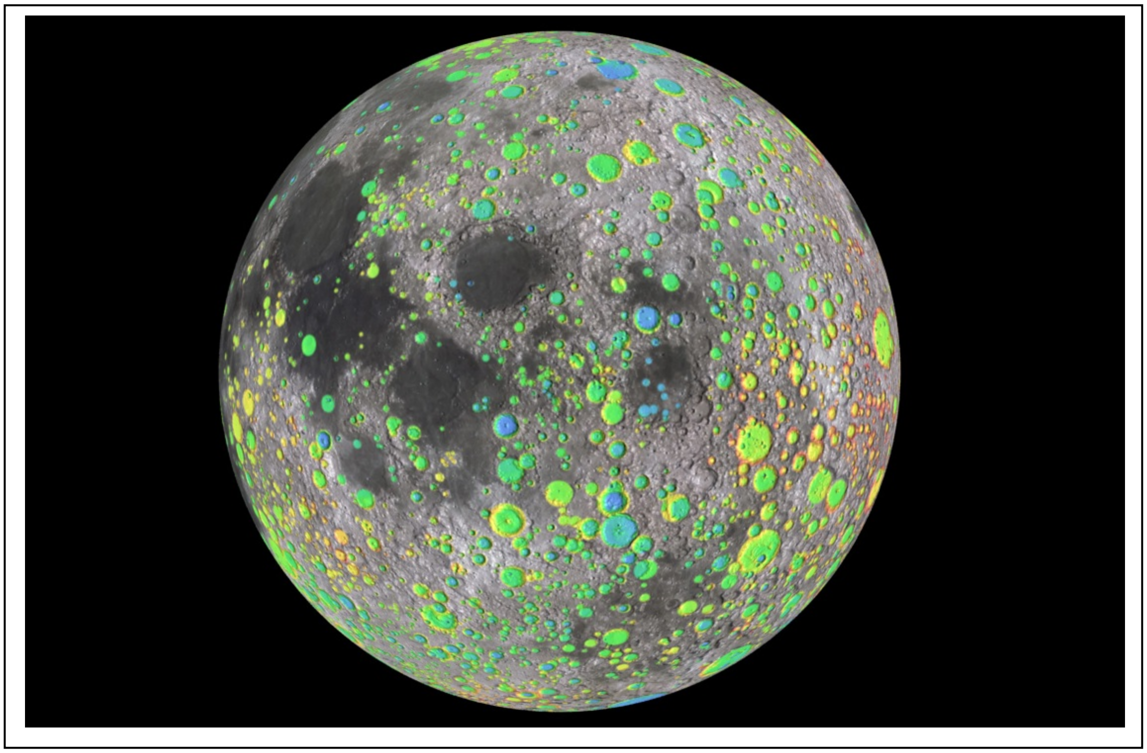
Answer: The volume of a sphere is given by $4/3\ \pi\ R^3$, so if you increase the radius of a sphere by a factor of 1.4, you will increase its volume by a factor of 1.4^3 = 2.7 times, so the volume of Mercury is **2.7 times** larger than the volume of the Moon.

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Answer: The density of Mercury is a factor of 5400/3400 = 1.6 times that of the Moon. Since the volume of Mercury is 2.7 times larger than the Moon, the mass of Mercury will be density x volume or 1.6 x 2.7 = **4.3 times than of the Moon**.

Note: Actual masses for Mercury and the Moon are 3.3×10^{23} kg and 7.3×10^{22} kg respectively, so that numerically, Mercury is 4.5 times the Moon's mass...which is close to our average estimate.

LRO Determines Lunar Cratering History



The Lunar Reconnaissance Orbiter used millions of measurements of the lunar surface to establish the history of cratering on the surface.

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.
Answer: The image diameter is 90 millimeters so the scale is 3400 km/90 mm = **38 km/mm.**

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?
Answer: 70 km equals 2 millimeters at this image scale. There are about **56 craters larger than 2 mm on the image. Students answers may vary from 40 to 60.**

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?
Answer: You would expect to find about 0.2 x 56 = **11 craters larger than 70 km.**

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Answer: **The lunar highlands were present first and were impacted by asteroids until just before the maria formed. There are few/no craters in the maria regions larger than 70 km, so the maria formed after the episode of large impactors ended.**

For more information, see the LRO press release at:

"LRO Exposes Moon's Complex, Turbulent Youth"

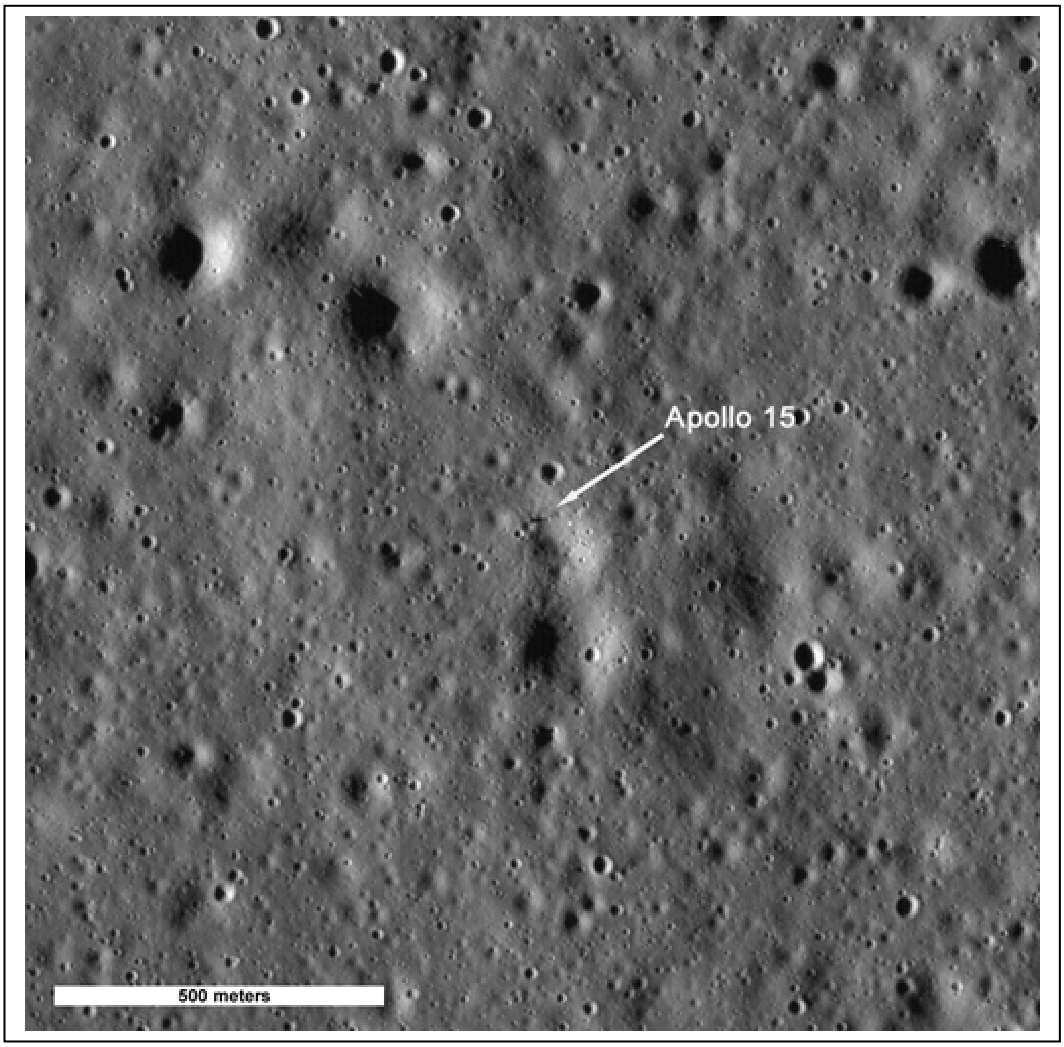
http://www.nasa.gov/mission_pages/LRO/news/turbulent-youth.html

LRO - The Relative Ages of Lunar Surfaces

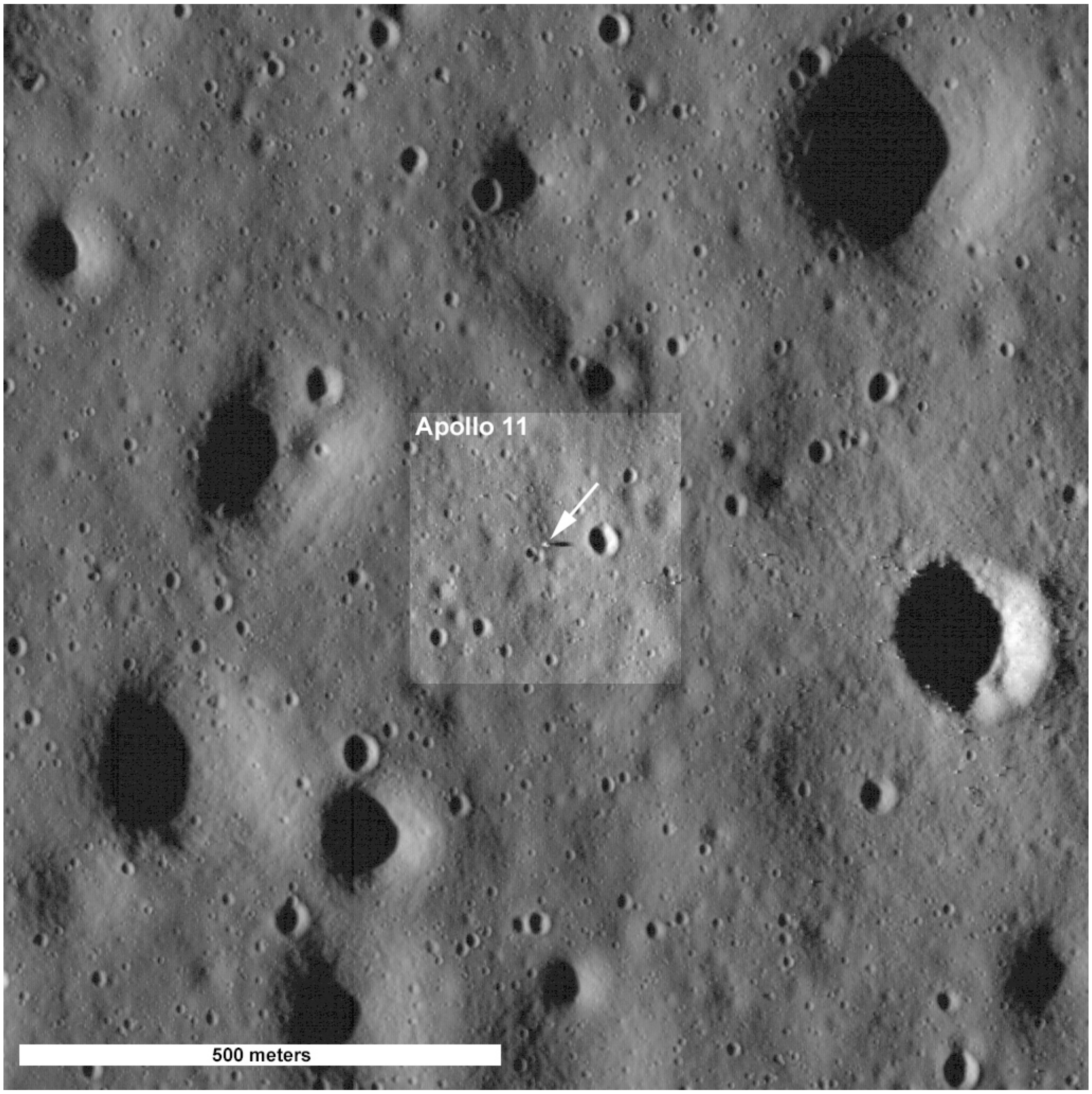
We have all sees pictures of craters on the moon. The images on the next two pages show close-up views of the cratered lunar surface near the Apollo 15 and Apollo 11 landing areas. They were taken by NASA's Lunar Reconnaissance Orbiter (LRO) from an orbit of only 25 kilometers!

Meteors do not arrive on the moon at the same rates. Very large meteors that produce the largest craters are much less common than the smaller bodies producing the smallest craters. That's because there are far more small bodies in space than large ones. Astronomers can use this fact to estimate the ages of various surfaces in the solar system by just comparing the number of large craters and small craters that they find in a given area.

Let's have a look the images below, and figure out whether Apollo-11 landed in a relatively younger or older region than Apollo 15!



This is the Apollo-15 landing area near the foot of the Apennine Mountain range. Note the bar indicating the 'scale' of the image. The arrow points to the location of the Lunar Descent Module.



This image taken by LRO is of the Apollo-11 landing area in Mare Tranquilitatus, with the arrow pointing to the Lunar Descent Module. The LDM was the launching platform for the Apollo-11 Lunar Excursion Module, which carried astronauts Neil Armstrong and Buzz Aldrin back to the orbiting Command Module for the trip back to Earth. Note the length of the '500 meter' bar, which gives an indication of the physical scale of the image. How long would it take you to walk 500 meters?

Astronomers assume that during the last 3 billion years following the so-called 'Late Heavy Bombardment Era' the average time between impacts that created craters has been constant. That means that the more time that passes, the more craters you will find, and that they are produced at a more or less steady number for each million years that passes. Also, by the Law of Superposition, younger craters lie on top of older craters.

Dating a cratered surface.

For each of the above images, perform these steps.

Step 1 - With the help of a millimeter ruler, and the '500 meter' line in the image, calculate the scale factor for the image in terms of meters per millimeter.

Step 2 - Calculate the total area of the image in square kilometers.

Step 3 - Identify and count all craters that are bigger than 20 meters in diameter.

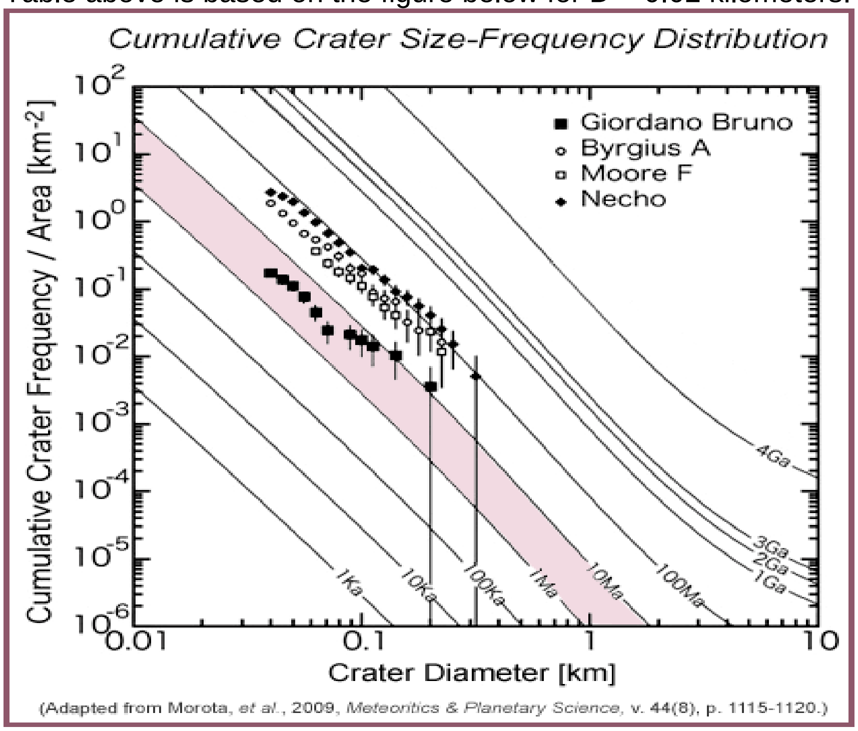
Step 4 - Divide your answer in Step 3 by the area in square kilometers in Step 2.

Step 5 - Look at the table below and estimate the average age of the surface.

Problem 1 - From your answer for each Apollo landing area in Step 5, which region of the moon is probably the youngest?

Estimated Age	Total number of craters per square kilometer
1000 years	0.0008
10000 years	0.008
100,000 years	0.08
1 million years	0.8
10 million years	8.0
100 million years	80.0
1 billion years	800.0

Table above is based on the figure below for D = 0.02 kilometers.



Another thing you might consider doing is to measure the diameters of as many craters as you can, and then plot a histogram (bar graph) of the number of craters you counted in a range of size intervals such as 5 - 10 meters, 11 to 20, 21 to 30 and so on. Because erosion (even on the moon!) tends to eliminate the smallest craters first, you can compare two regions on the moon in terms of how much erosion has occurred.

Problem 2 - Select the same-sized area on each of the Apollo images and count all the craters you can find within the size intervals you selected. How do the two landing areas compare to one another in terms of their crater frequency histograms?

Problem 3 - Which surface do you think has experienced the most re-surfacing or erosion?

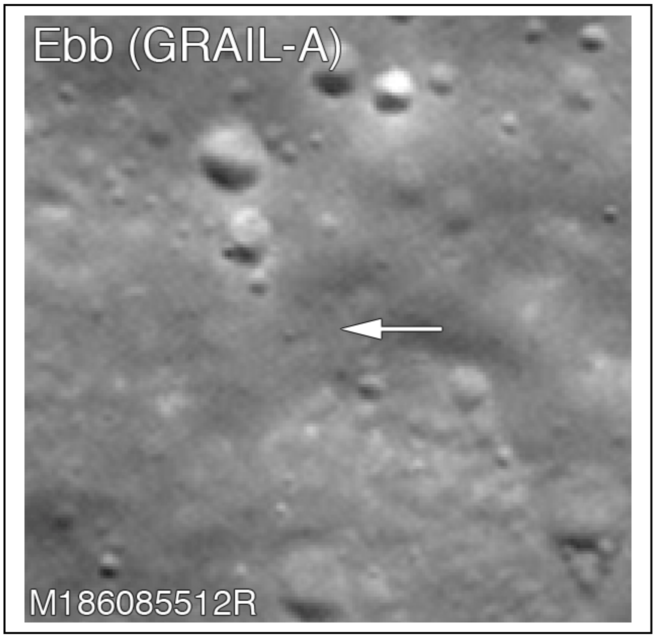
Problem 4 - Without an atmosphere, winds or running water, what do you think could have caused changes in the lunar surface after the craters were formed?

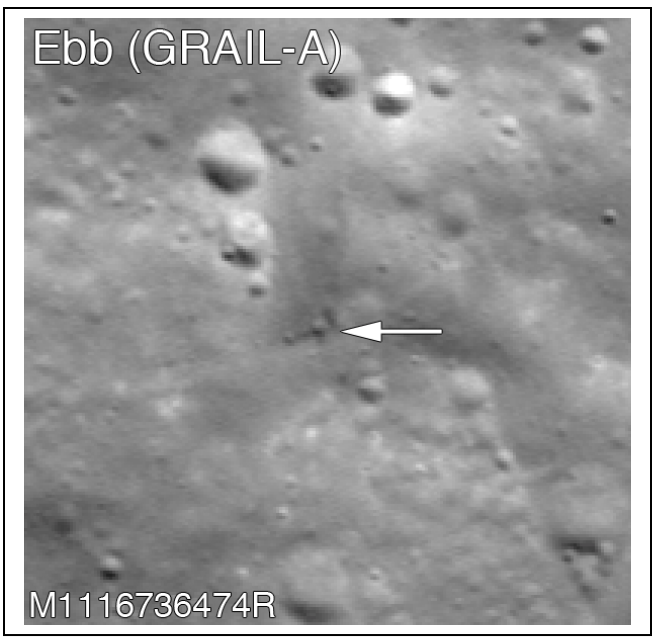
Note to Teachers: More technical information on crater dating can be found at

"How young is the Crater Giordano Bruno"

http://www.psrd.hawaii.edu/Feb10/GiordanoBrunoCrater.html

Grail Creates a New Crater on the Moon





On December 17, 2012 each of the twin Grail spacecraft ended their lunar mapping missions by crashing into the lunar surface. The two images to the left were taken by the Lunar Reconnaissance Orbiter as it flew over the impact site for the Grail-A spacecraft.

The width of each image is 213 meters.

At the moment of impact, each Grail spacecraft was traveling at a speed of 6,070 km/h and carried a mass of 200 kg.

Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater?

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

NASA's LRO Sees GRAIL's Explosive Farewell
http://www.nasa.gov/mission_pages/LRO/news/grail-results.html

Problem 1 – What was the diameter, in meters and feet, of the crater left behind when Ebb impacted the surface?

Answer: Use a millimeter ruler to measure the width of the image. You should get about 80 mm .The scale is then 213 meters/80 mm = 2.7 meters/mm. The crater is about 1mm across so this is just **2.7 meters**. Since 3 feet = 1 meter, this is about **8 feet across.**

Problem 2 – The diameter of the Grail spacecraft was about 1-meter. How many times larger was the crater then the spacecraft?

Answer: About **2.7 times larger**.

Problem 3 – For the largest crater in the image, how large would the meteorite have to be to make the crater?

Answer: The largest crater is about 9 mm across or 24 meters. Using Grail, we get 24 meters/2.7 = **8.9 meters across**.

Problem 4 – Assume that the crater was a cylinder with a depth ¼ its diameter. If the density of the lunar soil is 2700 kg/m^3, how many kilograms of lunar soil were excavated by the impact?

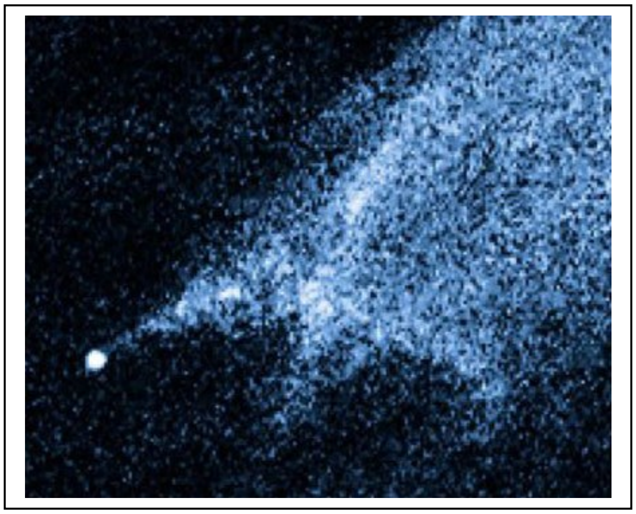
Answer: Volume = $\pi R^2 h$ so V = 3.14 x $(2.7/2)^2$ x (2.7/4) = 3.8 meters^3.
Mass = density x volume so Mass = 2700 x 3.8 = **10,000 kilograms**.

Problem 5 – What is the ratio of the excavated mass to the spacecraft mass?

Answer: 10,000 kilograms/200 kg = **50**.

So the amount of excavated mass is 50 times the mass of the impacting spacecraft.

Hubble Spies Colliding Asteroids!



It doesn't look like much, but this picture taken by the Hubble Space Telescope on January 25, 2010 shows all that remains of two asteroids that collided! The object, called P/2010 A2, was discovered in the asteroid belt 290 million kilometers from the sun, by the Lincoln Near-Earth Asteroid Research sky survey on January 6, 2010.

Hubble shows the main nucleus of P/2010 A2, about 150 meters in diameter, lies outside its own halo of dust. This led scientists to the interpretation that it is the result of a collision.

How often do asteroids collide in the asteroid belt? The collision time can be estimated by using the formula:

$$T = \frac{1}{NAV}$$

Where N is the number of bodies per cubic kilometer, v is the speed of the bodies relative to each other in kilometers/sec and A is the cross-sectional area of the body in square-kilometers. The answer, T, will be in the average number of seconds between collisions.

Estimating A: Assume that the bodies are spherical and 100 meters in diameter .What will be A, the area of a cross-section through the body?

Estimating the asteroid speed V: At the orbit of the asteroids, they travel once around the sun in about 3 years. What is the average speed of the asteroid in kilometers/sec at a distance of 290 million kilometers?

Estimating the density of asteroids N: - This quantity is the number of asteroids in the asteroid belt, divided by the volume of the belt in cubic kilometers. A) Assume that the asteroid belt is a thin disk 1 million kilometers thick, with an inner radius of 1.6 AU and an outer radius of 2.5 AU. If 1 AU = 150 million kilometers, want is the volume? B) Based on telescopic observations, an estimate for the number of asteroids in the belt that are larger than 100 meters across is about 30 billion. From this information, and your volume estimate, what is the average density of asteroids in the asteroid belt?

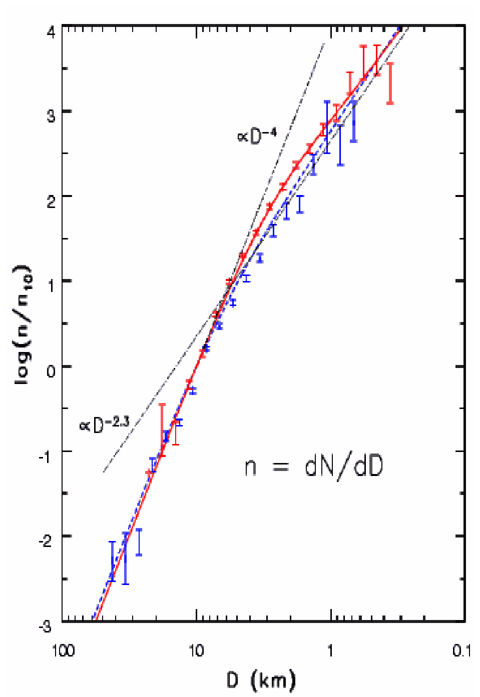
Estimating the collision time T: From the formula, A) what do your estimates for N, V and A imply for the average time between collisions in years? B) What are the uncertainties in your estimate?

Estimating A: Answer: The cross-section of a sphere is a circle, so $A = \pi R^2$ or for R = 100 meters, we have $A = 3.14 (0.05)^2 = 8x10^{-3}$ km^2.

Estimating the asteroid speed V: Answer: The circumference of the orbit is C = 2 π (290 million km) = 1.8 x 10^9 kilometers, and the time in seconds is 3.0 years x (3.1 x 10^7 seconds/1 year) = 9.3 x 10^7 seconds, so the average speed V= C/T = 19 kilometers/sec. But what we want is the <u>relative</u> speed. If all the asteroids were going around in their orbits at a speed of 19 km/sec, their <u>relative</u> speeds would be zero. Example, although two cars are on the freeway traveling at 65 mph, their relative speeds are zero since they are not passing or falling behind each other.

Estimating the density of asteroids N: - Answer: R(inner) = 2.4 x 10^8 km .R(outer)= 3.4 x 10^8 km, so the volume of the disk is V = disk area x thickness = [$\pi(3.4x10^8)^2$ - $\pi(2.4 \times 10^8)^2$] x 10^6 so the volume is **1.8 x 10^{23} km^3**. Then the average asteroid density is N= 3x10^{10} asteroids/1.8 x 10^{23} km^3 = **1.7 x 10^{-13} asteroids/km^3.**

Estimating the collision time T: Answer: A) T = 1/NAV so T = 1/(1.7 x 10^{-13} x 8x10^{-3} x 19) = 3.8 x 10^{13} seconds. Since 1 year = 3.1 x 10^7 seconds, we have about **1,200,000 years between collisions**. B) Students can explore a number of places where large uncertainties might occur such as: 1) the thickness of the asteroid belt; 2) the number of asteroids; 3) their actual relative speeds. 4) The non-uniform distribution of the asteroids…not smoothly distributed all over the volume of the asteroid belt, but may be in clumps or rings that occupy a smaller actual volume. Encourage them to come up with their own estimates and see how that affects the average time between collisions. For advanced students see the problem below.



Extra Credit with Calculus: Data from the Sloan Digital Sky Survey suggests that the number of asteroids in specific size ranges follow a piecewise power-law distribution:

$$N(D) = \begin{cases} 1.8x10^9 D^{-2.3} & \text{for D <70 km} \\ 2.4x10^{12} D^{-4.0} & \text{for D>70 km} \end{cases}$$

Integrate N(D) from 100 meters to infinity to determine the number of asteroids larger than 100 meters.

Answer: $n = \int_{0.1}^{70} N(D)dD + \int_{70}^{\infty} N(D)dD$

= 2.7x10^{10} + 2.32x10^6 = 2.7x10^{10} asteroids.

Note: the figure is scaled to the number of 10-km asteroids (n_{10}) which we take to be 10,000.

The Frequency of Large Meteor Impacts

On February 14, 2013 a 10,000 ton meteor about 17-meters in diameter entered Earth's atmosphere over Russia traveling at 40,000 mph (18 km/s). It detonated in the air over the town of Chelyabinsk and the explosion caused major damage to the town injuring 1,000 people. The people were hurt by flying glass when the windows of over 3000 buildings blew out over an area of about 1000 km^2. Unlike the famous Tunguska Event of 1908 which blew down 80 million trees and was nit 'discovered' for many decades afterwards, the Chelyabinsk Meteor was extensively videoed by hundreds of dashcams and cell phones as it happened.

Studies of thousands of meteor sightings by scientists can now tell us just how often asteroids of 4-meters or larger enter Earth's atmosphere. About two of these events happens each year over the entire surface area of Earth, which is 500 million km^2!

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Problem 3 - The kinetic energy in Joules of a large meteor is given by $KE = 1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded if 1 ton = 1000 kg?

Problem 1 – The surface area of Earth is consists of 72% oceans and 28% land. Of the land area, only 3% is inhabited. How many years would you have to wait to hear about one of these large meteor events in the News?

Answer: The inhabited area of Earth is 3% of 28% of the total surface area or just 0.8% of Earth's total surface area. This is 1/125 of the full area. If you had one event per year, you would have to wait 125 years for the next one. If you had 2 events per year, you would have to wait half this time or about **62 years**. So, in a typical 70-year human lifetime, you will hear about one or two of these major impact events in the News!

Problem 2 – Fireballs are very bright meteors that streak across the sky. They are caused by pieces of meteors that can be 500 grams or more in mass. Astronomers estimate that 50,000 of these 'Bolides' can be seen every year over the entire surface area of Earth. From an inhabited spot on Earth, about how many Bolides should you be able to see in your lifetime if you paid attention to the sky if you could see any bolide entering over an area about 100 km^2?

Answer: The 50,000 bolides arrive somewhere over Earth each year. The chance that that this area is over an inhabited region of Earth is 0.8% or 1/125. So 1/125 of the bolides arrive over an inhabited area which is 50000/125 = 400 each year. For you to personally see the event, it has to happen within 100 km^2 of where you are standing. The inhabited area of Earth has an area of 1/125 x 500 million km^2 = 4 million km^2. Your 100 km^2 is only 1/40000 of this area. So you will see 400 bolides x 1/40000 = 1/100 bolides each year, or will **have to wait about 100 years to see just one**! If you watched the sky every night for your entire life, you might see one of these events!

BUT, because of our world-wide news and internet coverage, you could hear about any bolide that flashed over the inhabited area of Earth or 400 bolides each year! We only hear about a few of these because most of them are too unimpressive to get the attention of the news system. After the Chelyabinsk Meteor event on February 14, 2013, there were many announcements of fireballs or bolides over Los Angeles, San Francisco and other cities as this news topic became popular for a few weeks.

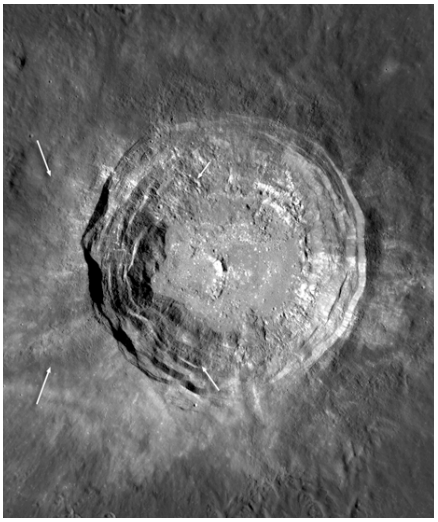
Problem 3 - The kinetic energy in Joules of a large meteor is given by KE = $1/2mV^2$ where m is its mass in kilograms and V is its speed in meters/sec. One ton of TNT explodes with an energy of 4.2×10^9 Joules. How many tons of TNT did the Chelyabinsk Meteor yield as it exploded?

Answer: The mass was 10,000 tons or 10,000 tons x 1000 kg/1 ton = 10 million kg.
The speed was 18 km/s x 1000 m/km = 18,000 m/s, so the energy was

$KE = \frac{1}{2} (1.0 \times 10^7) \times (1.8 \times 10^4)^2 = 1.62 \times 10^{15}$ Joules.

This is equal to 1.62×10^{15} Joules x (1 ton/4.2×10^9Joules) = **386,000 tons of TNT** or about 10 times the energy of a small atom bomb. This is similar to the estimates found in the news reports of this event, and explains why it did so much damage!

The Volume of a Lunar Impact Crater



Lunar craters have been excavated by asteroid impacts for billions of years. This has caused major remodeling of the lunar surface as the material that once filled the crater is ejected. Some of this returns to the lunar surface hundreds of kilometers from the impact site.

An example of a lunar crater is shown in the image to the left taken of the crater Aristarchus by the NASA Lunar Reconnaissance Orbiter (LRO).

Astronomers create models of the rock that was displaced that try to match the overall shape of the crater. The shape of a crater can reveal information about the density of the rock, and even the way that it was layered below the impact area.

One such mathematical model was created for a 17-kilometer crater located at lunar coordinates 38.4 °North, and 194.9 °West. For this particular crater, its depth, D, at a distance of x from its center can be approximated by the following 4^{th}-order polynomial:

$$\mathbf{D = 0.0001x^4 - 0.0055x^3 + 0.0729x^2 - 0.2252x - 0.493}$$

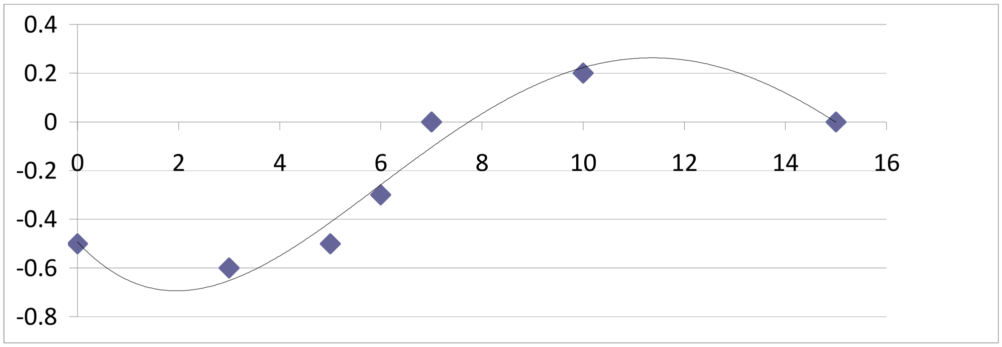
where D and x are in kilometers. The crater is symmetric around the axis x=0.

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.

Problem 2 - To 2 significant figures, what is the approximate excavated volume of this symmetric crater using the method of inscribed and circumscribed disks?

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141)

Problem 1 – Graph this function in the interval (0, +15.0) for which it provides a suitable model.



Problem 2 - To 2 significant figures, what is the approximate excavated volume of the symmetric crater using the method of inscribed and circumscribed disks? Answer; The volume of a disk is V = π R^2 h. For the crater, the inscribed disk has a radius of 5 kilometers and a height of 0.7 km, so its volume is Vi = 3.141 $(5)^2$ (0.7) = 55 km^3. The circumscribed disk has a radius of 10 km and a height of 0.8 km, so Vc = 3.141 $(10)^2$ (0.8) = 251 km^3. The estimated volume is then the average of these two or V = (Vc + Vi)/2 = 153 km^3. To 2 Significant Figure, the correct answer would be V = 150 km^3.

Problem 3 – To 2 significant figures, what is the volume of this crater bounded by the function h(R) and the line y = +0.2 km, and rotated about the y-axis? (Use π = 3.141) Answer: Using the method of shells, the volume differential for this problem is dV = 2π x [0.2 - h(x)] dx

The definite integral to evaluate is then $V = \int_0^{10} 2\pi x[0.2 - h(x)]dx$ Then :

$$V = 2\pi(0.2)\int_0^{10} x dx - 2\pi \int_0^{10} 0.0001x^5 - 0.0055x^4 + 0.0729x^3 - 0.2252x^2 - 0.493x \, dx$$

$$V = 2\pi(0.2)\frac{x^2}{2}\Bigg]_0^{10} - 2\pi\left(0.0001\frac{x^6}{6} - 0.0055\frac{x^5}{5} + 0.0729\frac{x^4}{4} - 0.2252\frac{x^3}{3} - 0.493\frac{x^2}{2}\right)_0^{10}$$

$$V = 2\pi(0.2)\frac{10^2}{2} - 2\pi\left(0.0001\frac{10^6}{6} - 0.0055\frac{10^5}{5} + 0.0729\frac{10^4}{4} - 0.2252\frac{10^3}{3} - 0.493\frac{10^2}{2}\right)$$

V = 2(3.141) [10 - 16.7 + 110 - 182.3 + 75.1 + 24.7] V = 6.24 [20.8]

V = 129.8 km^3 To 2 significant figures this becomes **V = 130 km^3**. So to check that Vi < V < Vo we have 55 km^3 < 130 km^3 < 251 km^3.