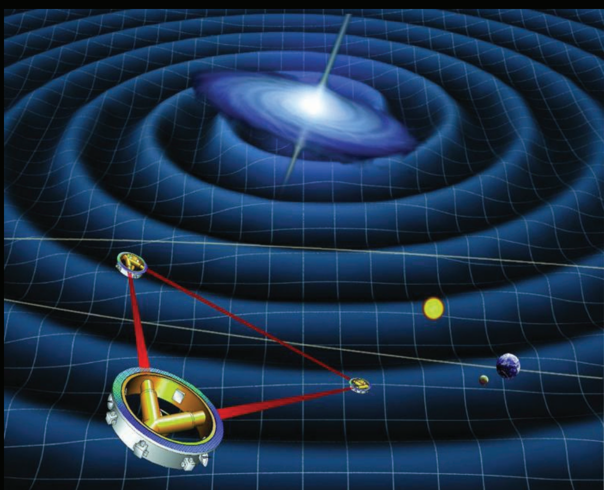
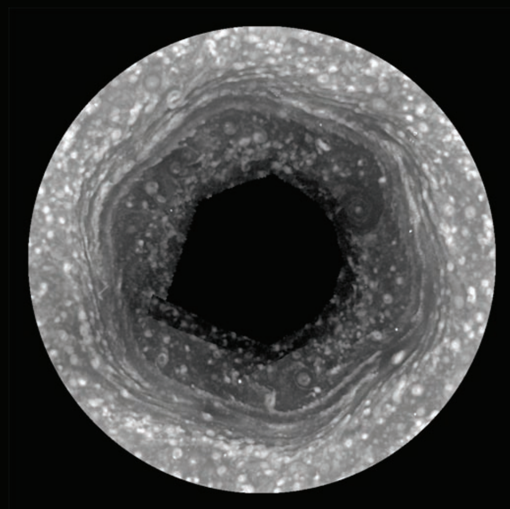
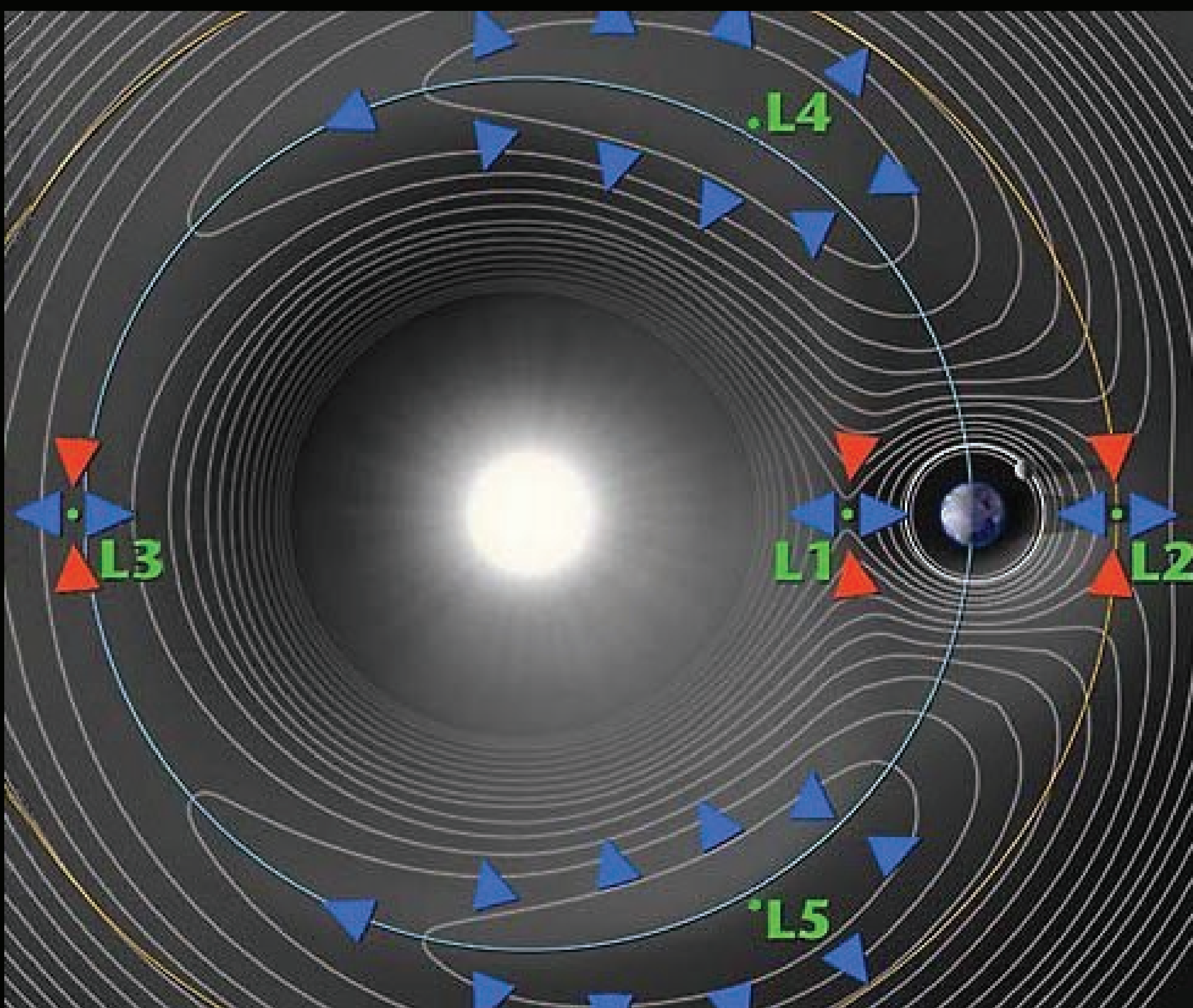
National Aeronautics and Space Administration

Space Math VI







This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2009-2010 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

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This booklet was created through an education grant NNH06ZDA001N-EPO from NASA's Science Mission Directorate.

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Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry																															
Technology, rulers						X	X					X	X			X	X				X			X	X						
Numbers, patterns, percentages	X	X	X	X	X			X		X					X			X		X		X		X			X				
Averages										X					X																
Time, distance,speed						X	X										X				X							X			
Areas and volumes															X									X							
Scale drawings						X	X					X	X			X	X	X			X				X	X					
Geometry																		X		X		X				X		X			
Scientific Notation						X										X					X										
Unit Conversions									X					X																	
Fractions																														X	
Graph or Table Analysis										X	X								X				X								
Solving for X																														X	X
Evaluating Fns																													X		
Modeling																			X								X	X		X	
Probability					X																	X									
Rates/Slopes																			X												
Logarithmic Fns																															X
Polynomials																															
Power Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																										X					
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem													Numbers																	
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry																															
Technology, rulers			X								X																				
Numbers, patterns, percentages																															
Averages																															
Time, distance,speed																															
Areas and volumes				X	X			X			X		X											X		X	X	X			
Scale drawings			X								X																				
Geometry	X		X	X	X			X			X	X											X	X		X	X	X			X
Scientific Notation							X	X					X		X	X	X	X			X					X	X	X	X	X	
Unit Conversions				X	X		X						X	X	X	X	X									X	X	X			
Fractions																															
Graph or Table Analysis		X				X				X												X									
Solving for X	X						X												X	X	X	X			X	X	X	X	X	X	
Evaluating Fns						X			X									X		X	X	X			X	X	X	X	X	X	X
Modeling						X				X									X		X		X	X	X	X	X	X			
Probability																															
Rates/Slopes										X															X						
Logarithmic Fns																															
Polynomials																				X									X	X	X
Power Fns																					X										
Exponential Fns																															
Conics																							X								
Piecewise Fns																															
Trigonometry	X											X																			
Integration																										X	X	X			
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93
Inquiry																															
Technology, rulers								X	X	X	X	X		X		X	X	X	X		X	X	X								
Numbers, patterns, percentages													X		X																
Averages																															
Time, distance,speed														X			X				X						X	X	X		
Areas and volumes							X					X				X		X	X		X	X	X			X					
Scale drawings								X	X	X	X	X	X	X		X	X	X	X		X	X	X			X					
Geometry					X	X	X		X							X	X	X	X		X	X	X			X					
Scientific Notation	X														X				X		X		X	X	X	X					X
Unit Conversions								X	X	X	X	X				X	X	X	X	X	X	X	X	X	X					X	
Fractions																															
Graph or Table Analysis																										X					
Solving for X	X	X	X	X		X																		X			X	X	X	X	X
Evaluating Fns	X	X	X	X		X																		X			X	X	X	X	X
Modeling	X	X	X	X		X	X					X	X									X	X			X	X	X	X	X	X
Probability																															
Rates/Slopes																															
Logarithmic Fns		X																													
Polynomials	X																														
Power Fns			X																												
Exponential Fns																															
Conics																															
Piecewise Fns																															
Trigonometry					X																										
Integration			X		X																										
Differentiation				X	X	X																									
Limits				X																											

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110													
Inquiry																														
Technology, rulers						X			X								X													
Numbers, patterns, percentages										X																				
Averages																														
Time, distance,speed						X																								
Areas and volumes		X	X	X					X		X	X	X			X	X													
Scale drawings					X	X			X								X													
Geometry			X	X	X				X		X						X													
Scientific Notation	X	X	X	X		X	X		X	X	X	X		X		X														
Unit Conversions	X	X	X	X		X	X		X																					
Fractions																														
Graph or Table Analysis													X		X															
Solving for X	X																													
Evaluating Fns	X					X	X	X				X			X															
Modeling	X	X	X	X		X	X	X							X	X	X													
Probability																														
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Differentiation																														
Limits																														

How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Space Math VI**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the Space Math VI book. She copied each problem for student teams to work on. She decided to have the students develop a factious space craft. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the Space Math VI book and images taken from the Space Weather Media Viewer, http://sunearth.gsfc.nasa.gov/spaceweather/FlexApp/bin-debug/index.html#

Space Math VI can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Alignment with Standards

x

AAAS: Project:2061 Benchmarks

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM: Principles and Standards for School Mathematics

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Teacher Comments

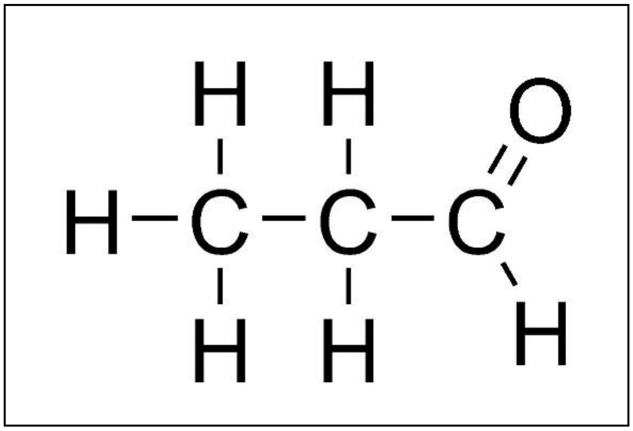
"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I'm not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than “Consider a particle of mass ‘m’ and speed ‘v’ that…” (Associate Professor of Physics)

Counting Atoms in a Molecule



The complex molecule Propanal was discovered in a dense interstellar cloud called Sagittarius B2(North) located near the center of the Milky Way galaxy about 26,000 light years from Earth. Astronomers used the giant radio telescope in Greenbank, West Virginia to detect the faint signals from a massive cloud containing this molecule. It is one of the most complex molecules detected in the 35 years that astronomers have searched for molecules in space. Over 140 different chemicals are now known.

Problem 1 - How many atoms of hydrogen (H), carbon (C) and oxygen (O) are contained in this molecule?

Problem 2 - What percentage of all atoms are hydrogen? Carbon? Oxygen?

Problem 3 - What is the ratio of carbon atoms to hydrogen atoms in propanal?

Problem 4 - If the mass of a hydrogen atom is defined as 1 AMU, and carbon and oxygen have masses of 12.0 and 16.0 AMUs, what is the total mass of a propanal molecule in AMUs?

Problem 5 - What is the complete chemical formula for propanal?

C_3 H __ O__

Problem 6 - If this molecule could be broken up, how many water molecules could it make if the formula for water is H_2O?

Answer Key

Problem 1 - How many atoms of hydrogen (H), carbon (C) and oxygen (O) are contained in this molecule?

Answer; **There are 6 atoms of hydrogen, 3 atoms of carbon and 1 atom of oxygen.**

Problem 2 - What percentage of all atoms are hydrogen? Carbon? Oxygen?

Answer: There are a total of 10 atoms in propanal so it contains 100% x (6 atoms /10 atoms) = 60% hydrogen; 100%x(3/10) = 30% carbon and 100% x (1/10) = 10 % oxygen.

Problem 3 - What is the ratio of carbon atoms to hydrogen atoms in propanal?

Answer: 3 carbon atoms / 6 hydrogen atoms = 1/2.

Problem 4 - If the mass of a hydrogen atom is defined as 1 AMU, and carbon and oxygen have masses of 12.0 and 16.0 AMUs, what is the total mass of a propanal molecule in AMUs?

Answer: 1 AMU x 6 hydrogen + 12 AMU x 3 carbon + 16 AMU x 1 oxygen = 6 AMU + 36 AMU + 16 AMU = 58 AMU for the full molecule mass.

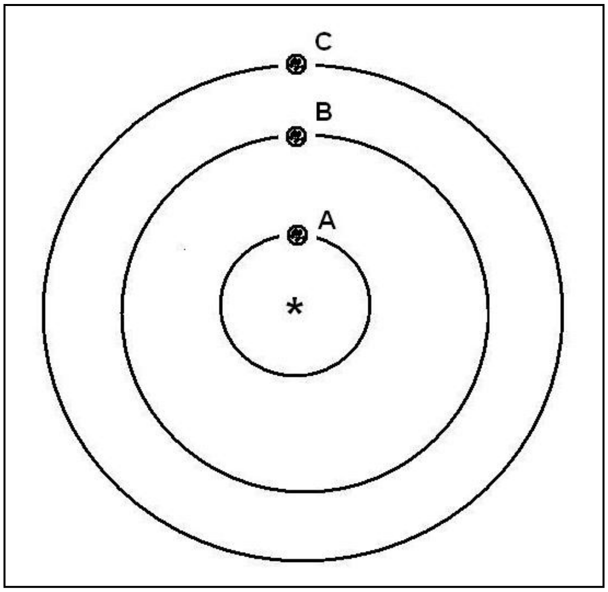
Problem 5 - What is the complete chemical formula for propanal?

$$C_3 H_6 O$$

Problem 6 - If this molecule could be broken up, how many water molecules could it make if the formula for water is H_2O?

Answer: A single water molecule requires exactly 1 oxygen atom ,and since propanal only has 1 atom of oxygen per molecule, you can only get 1 molecule of water by dissolving a single propanal molecule.

Planetary Conjunctions



Since 1995, astronomers have detected over 350 planets orbiting distant stars. Our solar system has 8 planets, and for thousands of years astronomers have studied their motions. The most interesting events happen when planets are seen close together in the sky. These are called conjunctions, or less accurately, alignments.

The figure shows a simple 3-planet solar system with the planets starting out 'lined up' with their star. Each planet revolves around the star at a different pace, so it is a challenge to predict when they will all line up again.

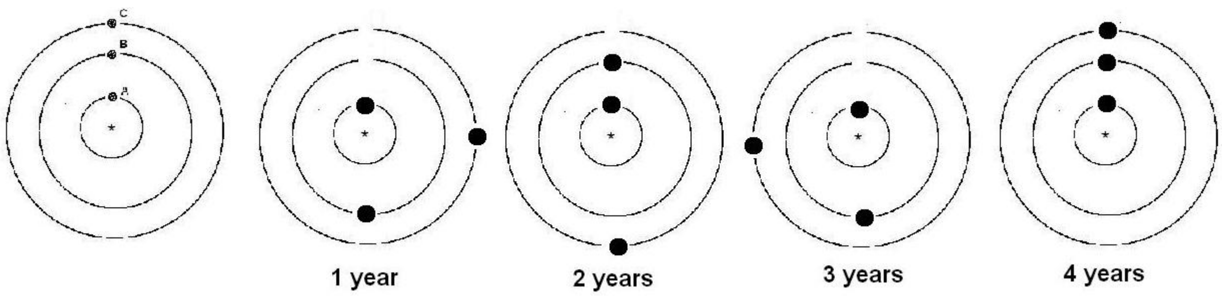
Problem 1 - An astronomer detects three planets, A, B, C, that orbit their star once every 1, 2 and 4 years in a clockwise direction. Using the diagram above, draw a series of new diagrams that show where will the planets be in their orbits after: A) 1 year? B) 2 years? C) 3 years? D) 4 years?

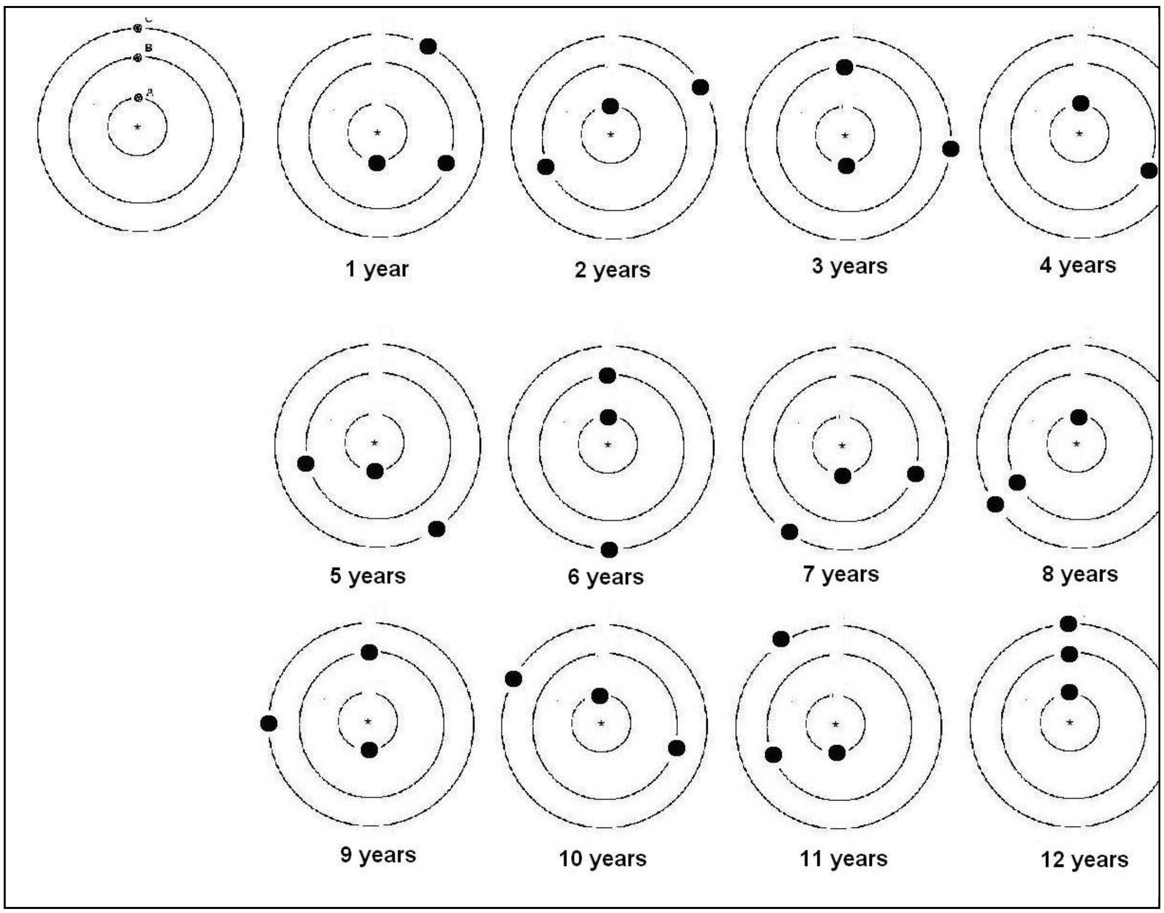
Problem 2 - Suppose the three planets, A, B and C, orbit their star once every 2 years, 3 years and 12 years. A) How long would it take for all three planets to line up again? B) Where would the planets be after 6 years?

Answer Key

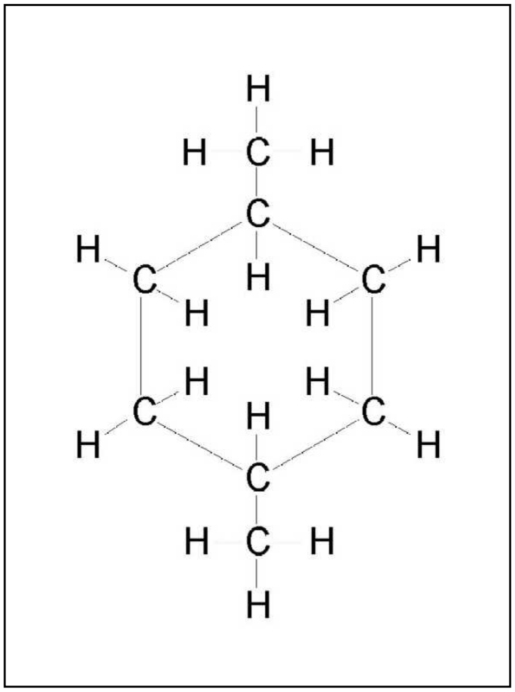
Problem 1 - Students will draw dots located as follows in the top diagram series:

Problem 2 - The bottom series of 12 possibilities indicates it will take 12 years to return to the original line-up. The pattern after 6 years is also shown.





Parts Per Hundred (pph)



A common way of describing the various components of a population of objects is by the number of parts that they represent for every 100, 1000 or 1 million items that are sampled.

For instance, if there was a bag of 100 balls: 5 were red and 95 were white, you would say that the red balls represented 5 parts per hundred (5 pph) of the sample.

This also means that if you had a bag with 300 balls in the same proportions of red and white balls, the red balls would still be '5 parts per hundred' even though there are now 15 red balls in the sample (15/300 = 5/100 = 5 pph).

For each of the situations below, calculate the parts-per-hundred (pph) for each sample.

Problem 1 - Your current age compared to 1 century

Problem 2 - 10 cubic centimeter (10 cc) of food coloring blended into 1 liter (1000 cc) of water.

Problem 3 - The 4 brightest stars in the Pleiades star cluster, compared to the total population of the cluster consisting of 200 stars.

Problem 4 - One day compared to one month (30 days)

Problem 5 - Five percent of anything.

Problem 6 - The figure above shows the atoms of hydrogen (H) and carbon (C) in the molecule of dimethylcyclohexane. What is the pph of the carbon atoms in this molecule?

Answer Key

Problem 1 - Your current age compared to 1 century

Answer: If your current age is 14 years, then compared to 100 years, your lifetime is 14 pph of a century.

Problem 2 - 1 cubic centimeter (10 cc) of food coloring blended into 1 liter (1000 cc) of water.

Answer: 10 cc / 1000 cc is the same as 1 /100 so the food coloring is 1 pph of a liter.

Problem 3 - The 4 brightest stars in the Pleiades star cluster, compared to the total population of the cluster consisting of 200 stars.

Answer: 4 stars / 200 members = 2 / 100 = 2 pph of the cluster stars.

Problem 4 - One day compared to one month (30 days)

Answer: 1 day / 30 days = 0.0333 so 0.033 x 100 = 3.3 pph of a month.

Problem 5 - Five percent of anything.

Answer: 5 % = 5 / 100 = 5 pph of anything.

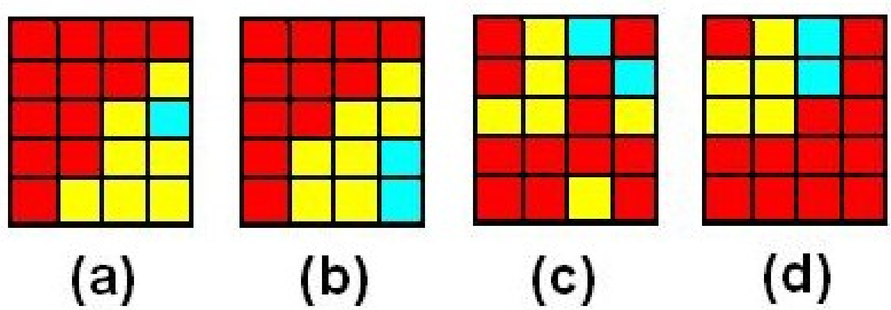
Problem 6 - The figure above shows the atoms of hydrogen (H) and carbon (C) in the molecule of dimethylcyclohexane. What is the pph of the carbon atoms in this molecule?

Answer: There area total of 24 atoms in this molecule. There are a total of 8 carbon atoms, so the fraction is 8/24 = 1/3 which equals 0.333 x 100 = 33 pph of the total atoms.

Star Cluster Math

Astronomers classify stars so that they can study their similarities and differences. A very common way to classify stars is by their temperature. This scale assigns a letter from the set [O, B, A, F, G, K, M] to represent stars with temperatures from 30,000 C (O-type) and 6,000 C (G-type), to 3,000 C (M-type).

Problem 1 – An astronomer studies a sample of stars in a cluster and identifies 6 as G-type like our Sun, 12 as M-type like Antares, and 2 stars as O-type like Rigel. Circle the pattern below, a, b, c or d, that graphically represents this information.



Problem 2 – What fraction of the stars in the sample are G-type?

A) 6/9 B) 20/6 C) 6/20 D) 6/8

Problem 3 – What fraction of the G and M-type stars in the cluster are G-type?

A) 12/18 B) 6/12 C) 12/6 D) 6/18

Problem 4 – If you selected 2 stars randomly from this cluster, which calculation would give the probability that these would both be O-type stars?

A) 1/20 x 1/20 B) 2/20 x 1/20 C) 1/20 x 1/19 D) 2/20 x 1/19

Problem 5 – A second star cluster has a total of 2,040 stars. If the proportion of O, G and M-types stars is the same as in the first cluster, how many G-type stars would be present?

A) 612 B) 340 C) 1428 D) 680

Answer Key

1 – The boxes are colored red for M-type stars, yellow for G-type stars and blue for O-type stars. Count the boxes carefully. Only C) has the correct number of star boxes colored. **Answer: C)**

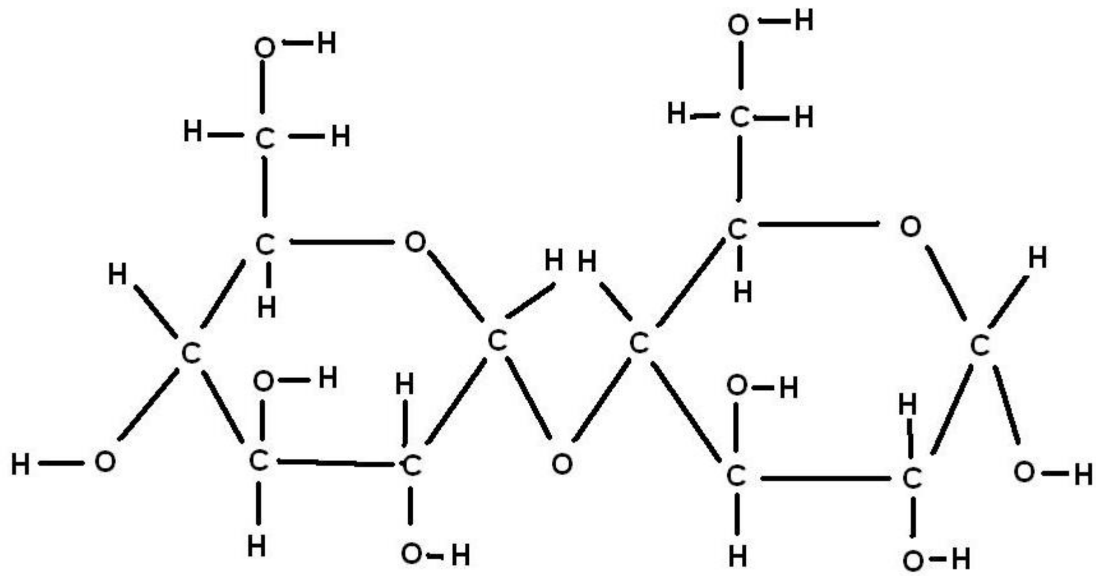
2 – There are 6 G-type stars in the cluster, which contains 20 stars, so the answer is C) 6/20. **Answer: C)**

3 – There are a total of 18 G and M-type stars in the sample, and since only 6 are G-type, the correct fraction is D) 6/18. **Answer: D)**

4 – There are 2 O-type stars in a sample of 20 stars. On the first draw, the probability is 2/20 that an O-type star will be selected. Now there are only 19 stars left and only 1 O-type, so the probability that the next star selected is an O-type star is now 1/19. The probability that both O-type stars are drawn in the first two draws is then D) 2/20 x 1/19. **Answer: D)**

5 – The correct answer is A) given by 2040 x 6/20 = 612 stars. The fraction 6/20 represents the proportion of 6 G-type stars out of the 20 stars in the first sample, and we are assuming that this proportion is the same for the second cluster. The incorrect answers come about by B) dividing 2040 by the number of G-type stars; C) multiplying 2040 by the fraction of stars that are O and M-class and; D) dividing 2040 by the number of star classes, which incorrectly assumes an equal number in each class. **Answer: A)**

Atoms - How sweet they are!



Glucose is a very important sugar used by all plants and animals as a source of energy. Maltose is the next most complicated sugar, and is formed from two glucose molecules. The atomic ingredients of the maltose molecule is shown in the diagram above, which is called the structural formula for maltose. As an organic compound, it consists of three types of atoms: hydrogen (H), carbon (C), and oxygen (O).

Problem 1 - How many molecules does maltose contain of A) hydrogen? B) oxygen? C) carbon?

Problem 2 - What is the ratio of the number of hydrogen atoms to oxygen atoms?

Problem 3 - What fraction of all the maltose atoms are carbon?

Problem 4 - If the mass of 1 hydrogen atoms is 1 AMU, and 1 carbon atom is 12 AMU and 1 oxygen atom is 16 AMU, what is the total mass of one maltose molecule in AMUs?

Problem 5 - Write the chemical formula of maltose by filling in the missing blanks:

C__ H__ O_{11}

Answer Key

Problem 1 - How many molecules does maltose contain of A) hydrogen? B) oxygen? C) carbon? Answer: A) There are 22 hydrogen atoms; B) There are 11 oxygen atoms; C) there are 12 carbon atoms.

Problem 2 - What is the ratio of the number of hydrogen atoms to oxygen atoms? Answer: 22 hydrogen atoms / 11 oxygen atoms so the ratio is 2/1

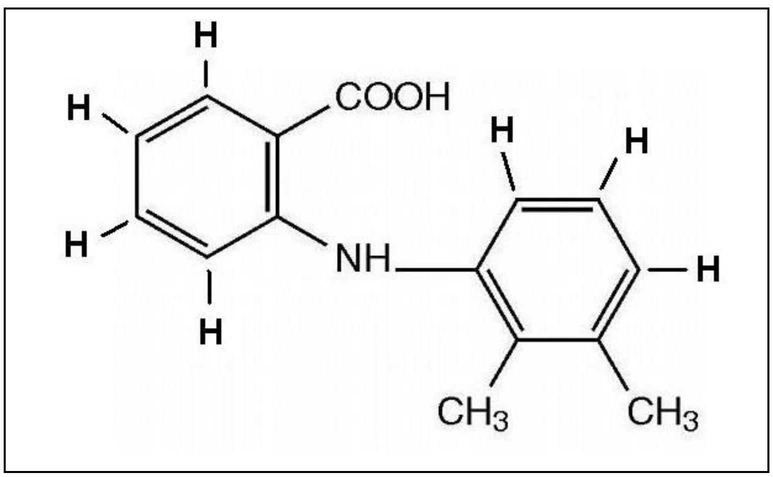
Problem 3 - What fraction of all the maltose atoms are carbon? Answer: The total number of atoms is 12 + 22 + 11 = 45, so carbon atoms are 11/45 of the total.

Problem 4 - If the mass of 1 hydrogen atoms is 1 AMU, and 1 carbon atom is 12 AMU and 1 oxygen atom is 16 AMU, what is the total mass of one maltose molecule in AMUs? Answer: 1 AMU x 22 atoms hydrogen + 12 AMU x 12 atoms carbon + 16 AMU x 11 atoms oxygen = 342 AMU.

Problem 5 - Write the chemical formula of maltose by filling in the missing blanks:

$C_{12} H_{22} O_{11}$

More Challenging Extra Problem:
Below is the structural formula for Mefenamic Acid. Students can determine its chemical formula as $C_{15}H_{15} N O_2$ and its mass as 241 AMU. In this kind of diagram, carbon atoms in the hexagonal rings are located at each vertex. Hydrogen atoms at the vertices are not labeled as well. Challenge your students to GOOGLE the term 'structural diagram' in the 'images' area and try to decipher other more complex molecules such as Lorazepam or Vancocin !



The Hand of Chandra!

A small, dense object only twelve miles in diameter is responsible for this beautiful X-ray nebula that spans 150 light years and resembles a human hand!

At the center of this image made by NASA's Chandra X-ray Observatory is a very young and powerful pulsar known as PSR B1509-58.

The pulsar is a rapidly spinning neutron star which is spewing energy out into the space around it to create complex and intriguing structures, including one that resembles a large cosmic hand.

Astronomers think that the pulsar and its nebula is about 1,700 years old, and is located about 17,000 light years away. Finger-like structures extend to the upper right in the image, apparently energizing knots of material in a neighboring gas cloud known as RCW 89. The transfer of energy from the wind to these knots makes them glow brightly in X-rays (orange and red features to the upper right).

Problem 1 - This field of view is 19 arcminutes across, where one arcminute is exactly 1/60 of a degree. Using similar triangles and proportions, if 1 arcminute at a distance of 3,260 light years equals a length of 1 light year, how wide is the image in light years?

Problem 2 - Measure the width of this image with a millimeter ruler. What is the scale of this image in parsecs per millimeter? How far, in light years, is the bright spot in the 'palm' where the pulsar is located, from the center of the ring-like knots in RCW 89? (1 parsec = 3.26 light years). Round your answer to the nearest light year.

Problem 3 - If the speed of the interstellar gas is 10,000 km/sec, how many years did it take for the gas to reach RCW-89 if 1 light year = 9.5×10^{12} kilometers, and there are 3.1×10^{7} seconds in 1 year?

Answer Key

Problem 1 - This field of view is 19 arcminutes across. Using similar triangles and proportions, if 1 arcminute at a distance of 3,260 light years equals a length of 1 light year, how wide is the image in parsecs?

Answer: If 1 light year seen from a distance of 3,260 light years equals an angle of 1 arcminute, then 1 arcminute at a distance of 5,200 parsecs will subtend

$$\frac{X}{\text{1 light year}} = \frac{\text{17,000 light years}}{\text{3,260 light years}}$$

so that X = 1.0 x (17000/3260) = 5.2 light years. Then 19 arcminutes will subtend

$$\frac{X}{\text{5.2 light years}} = \frac{\text{19 arcminutes}}{\text{1 arcminute}}$$

so that X = 5.2 x (19/1) = **98.8 light years.**

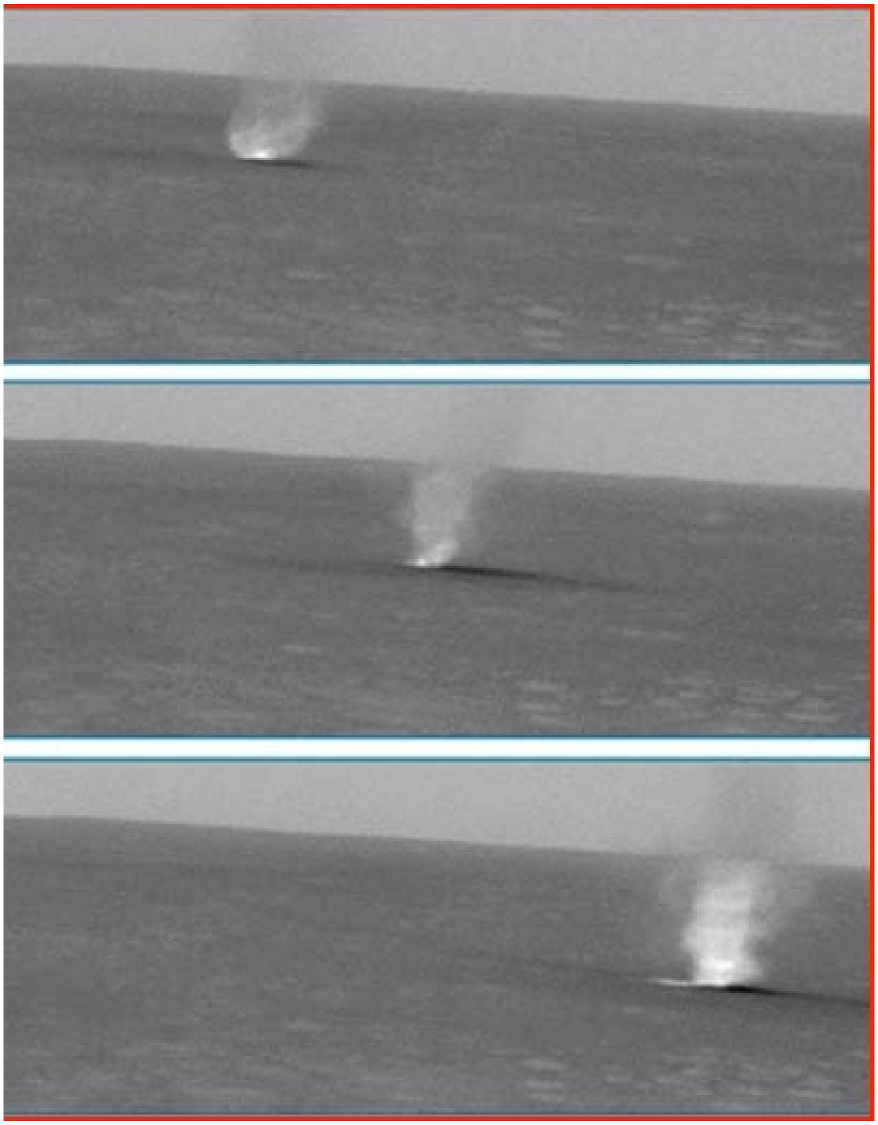
Problem 2 - Measure the width of this image with a millimeter ruler. What is the scale of this image in light years per millimeter? How far, in light years, is the center of the 'palm' where the pulsar is located, from the ring-like knots in RCW 89?

Answer: The width is about 96 millimeters. The scale is then 98.8 light years/96 millimeters or **1.0 light years/millimeter.** The distance from the bright spot to the ring of knots is about 40 millimeters. At the scale of the image, this equals 40 mm x 1.0 ly/mm = **40 light years**.

Problem 3 - If the speed of the interstellar gas is 10,000 km/sec, how many years did it take for the gas to reach RCW-89?

Answer: Time = distance/speed. First convert light years to kilometers: 40 light years x (9.5 x 10^{12} kilometers/light year) = 3.8 x 10^{14} kilometers. Then divide this by the speed to get Time = 3.8 x 10^{14} kilometers / (10,000 km/s) = 3.8 x 10^{10} seconds. Converting this to years: 3.8 x 10^{10} seconds x (1 year/3.1 x 10^{7} seconds) = **1,220 years.**

The Martian Dust Devils



A dust devil spins across the surface of Gusev Crater just before noon on Mars. NASA's Spirit rover took the series of images to the left with its navigation camera on March 15, 2005.

The images were taken at:

11:48:00 (T=top)
11:49:00 (M=middle)
11:49:40 (B=bottom)

based upon local Mars time.

The dust devil was about 1.0 kilometer from the rover at the start of the sequence of images on the slopes of the "Columbia Hills."

A simple application of the rate formula

$$speed = \frac{\text{distance}}{\text{time}}$$

lets us estimate how fast the dust devil was moving.

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top (T) and bottom (B) frames?

Problem 2 - What was the time difference, in seconds, between the images T-M, M-B and T-B?

Problem 3 - What was the distance, in meters, traveled between the images T-M and M-B?

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B. If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B?

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B?

Answer Key

Problem 1 - At the distance of the dust devil, the scale of the image is 7.4 meters/millimeter. How far did the dust devil travel between the top and bottom frames? Answer: The location of the dust devil in frame B when placed in image T is a shift of about 65 millimeters, which at a scale of 7.4 meters/mm equals about 480 meters.

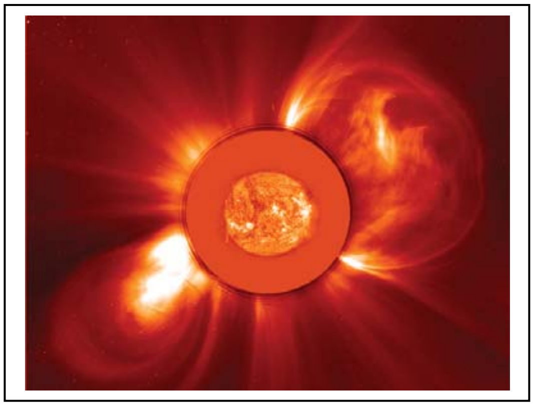
Problem 2 - What was the time difference between the images T-M, M-B and T-B? Answer: T-M = 11:49:00 - 11:48:00 - 1 minute or 60 seconds. For M-B the time interval is 40 seconds. For T-B the time interval is 100 seconds.

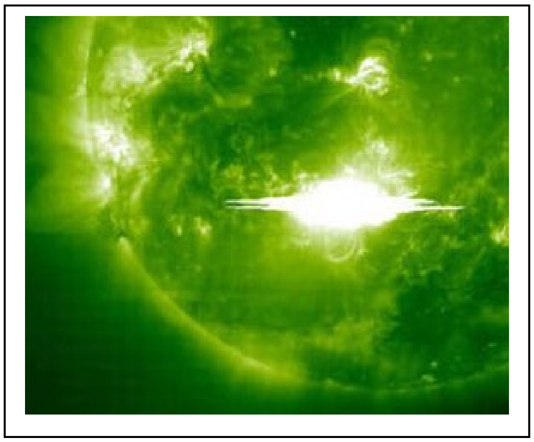
Problem 3 - What was the distance traveled between the images T-M and M-B? Answer: T-M = about 30 mm or 222 meters; M-B = about 35 mm or 259 meters.

Problem 4 - What was the average speed, in meters/sec, of the dust devil between T-B? If an astronaut can briskly walk at a speed of 120 meters/minute, can she out-run a martian dust devil?
Answer: Speed = distance/time so 480 meters/100 seconds = 4.8 meters/sec. This is about twice as fast as an astronaut can walk, so running would be a better option.

Problem 5 - What were the speeds during the interval from T-M, and the interval M-B? Answer: Speed(T-M) = 222 meters/60 seconds = 3.7 meters/sec. Speed(M-B) = 259 meters/40 seconds = 6.5 meters/sec.

Problem 6 - Was the dust devil accelerating or decelerating between the times represented by T-B? Answer: because the speed increased from 4.8 meters/sec to 6.5 meters/sec, the dust



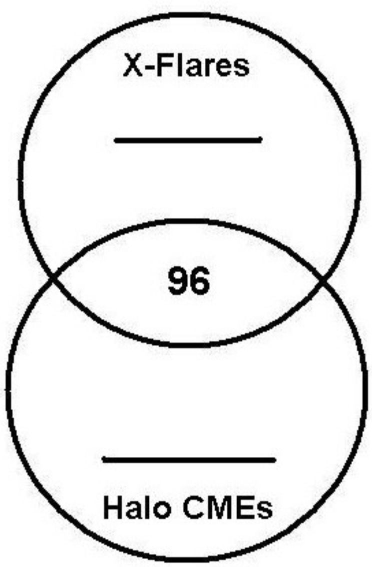


Solar storms come in two varieties:

Coronal Mass Ejections (CMEs) are clouds of gas ejected from the sun that can reach Earth and cause the Aurora Borealis (Northern Lights). These clouds can travel at over 2 million kilometers/hr, and carry billions of tons of matter in the form of charged particles (called a plasma). The picture to the left shows one of many CMEs witnessed by the SOHO satellite.

Solar Flares are intense bursts of X-ray energy that can cause short-wave radio interference on Earth. The picture to the left shows a powerful X-ray flare seen by the SOHO satellite on October 28, 2003.

Between 1996 and 2006, astronomers detected 11,031 coronal mass ejections (CMEs), and of these, 593 were directed towards Earth. These are called 'Halo CMEs' because the ejected gas surrounds the sun's disk on all sides and looks, like a halo around the sun. During these same years, astronomers also witnessed 122 solar flares that were extremely intense X-flares. Of these X-flares, 96 happened at the same time as the Halo CMEs.



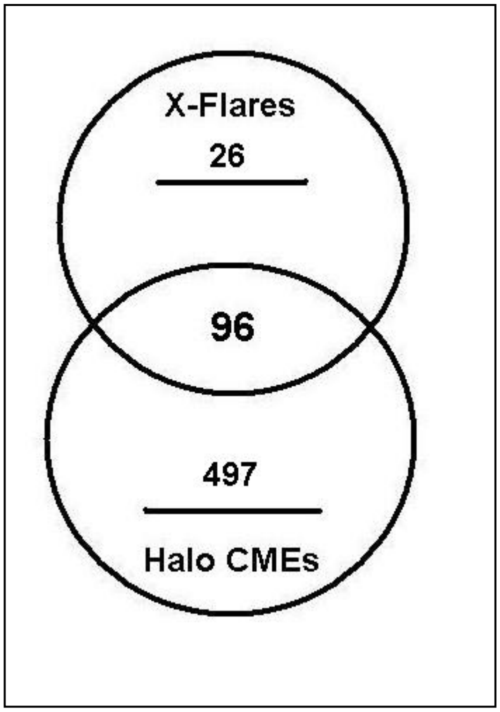
Problem 1 - From this statistical information, fill-in the missing numbers in the circular Venn Diagram to the left.

Problem 2 – What percentage of X-Flares also happened at the same time as a Halo CME?

Problem 3 – What percentage of Halo CMEs happened at the same time as an X-Flare?

Problem 4 – What percentage of all CMEs detected between 1996 and 2006 produced X-Flares?

Answer Key:



Answer 1 - The total number of Halo CMEs is 593 and the total number of X-Flares is 122. The intersecting area of the two circles in the Venn Diagram shows the 96 events in which a Halo CME and X-Flare are BOTH seen together. The areas of the circles not in the intersection represent all of the X-flares that are not spotted with Halo CMEs (top ring) and all of the Halo CMEs that are not spotted with X-Flares (bottom ring).

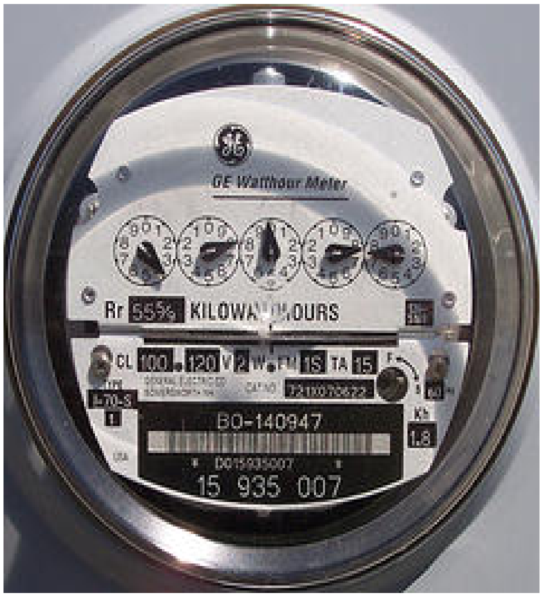
The missing number in the X-Flare ring is just 122 – 96 = 26, and for the Halo CMes we have 593 – 96 = 497.

Answer 2 – The total number of X-Flares is 122 and of these only 96 occurred with a Halo CME, so the fraction of X-Flares is just 96/122 = 0.79. In terms of percentage, this represents 79 %.

Answer 3 – The total number of Halo CMEs is 593 and of these only 96 occurred with an X-Flare, so the fraction of Halo CMEs is just 96/593 = 0.16. In terms of percentage, this represents 16 %.

Answer 4 – There were 11,031 CMEs detected, and of these only 96 coincided with X-Flares, so the fraction is 96/11031 = 0.0087. In terms of percentage, this represents 0.87 % or less than 1 % of all CMEs.

Energy in the Home



Every month, we get the Bad News from our local electrical company. A bill comes in the mail saying that you used 900 Kilowatt Hours (kWh) of electricity last month, and that will cost you $100.00! *What is this all about?*

Definition: 1 kiloWatt hour is a unit of energy determined by multiplying the electrical power, in kilowatts, by the number of hours of use.

Example: A 100-watt lamp is left on all day. E = 0.1 kilowatts x 24-hours = 2.4 kWh. Note: At 11-cents per kWh, this costs you 2.4 x 11 = 26 cents!

Problem 1 – You and your sister fired-up your two computers at 3:00 PM, and finished your homework at 9:00 PM, but you forgot to turn them off before going to bed. At 7:00 AM, they were finally shut off after being on all night. If this happened each school day in the month (25 days):

A) How many kilowatt hours did it cost to run the computers this way for 25 days?

B) How many kilowatt hours were wasted?

C) If each computer runs at 350 watts, and if electricity costs 11-cents per kilowatt hour, how much did this waste cost each month?

D) How many additional songs can you buy with iTunes for the wasted money each month?

Problem 2 – The Tevatron 'atom smasher' at Fermilab in Batavia, Illinois collides particles together at nearly the speed of light to explore the innermost structure of matter. When operating, the accelerator requires 70 megaWatts of electricity – about the same as the power consumption of the entire town of Batavia (population: 27,000). If an experiment, from start to finish, lasts 24 hours:

A) What is the Tevatron's electricity consumption in kilowatt hours?

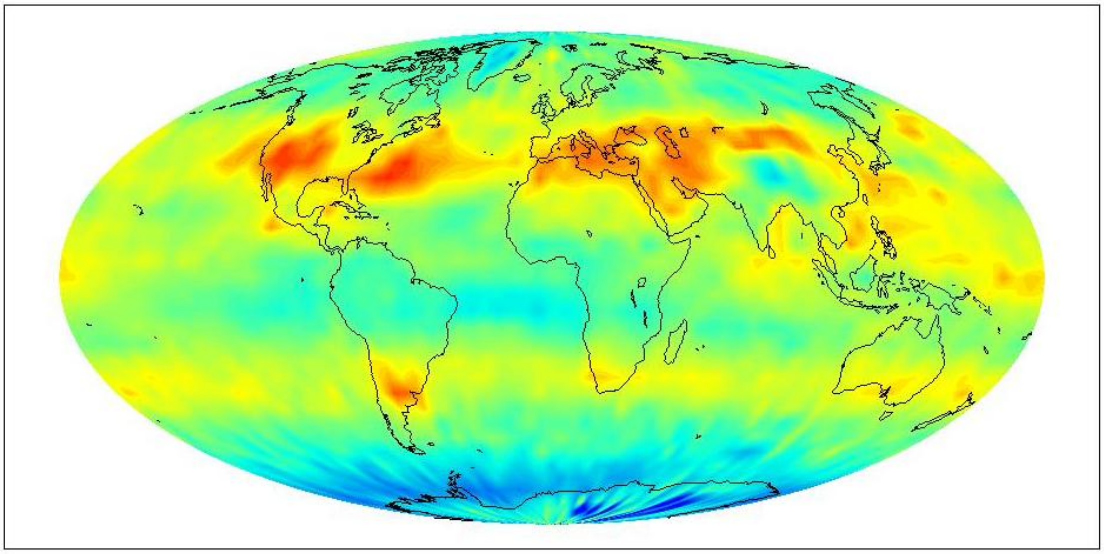
B) At $0.11 per kilowatt hour, how much does one experiment cost to run?

Answer Key

Problem 1 – You and your sister fired-up your two computers at 3:00 PM, and finished your homework at 9:00 PM, but you forgot to turn them off before going to bed. At 7:00 AM, they were finally shut off after being on all night. If this happened each school day in the month (25 days) A) How many kilowatt hours did it cost to run the computers this way for 25 days? Answer: The computers were turned off between 7:00 AM and 3:00 PM which is 8-hours, so the total time they stayed on each day is 24 – 8 = 16 hours. Over 25 days, this comes to 25 x 16 = 400 hours for each computer, or 800 hours for both. B) How many kilowatt hours were wasted? Answer: The computers were used between 3:00 PM and 9:00 PM so this is 6-hours out of the 16, so the wasted time was 10 hours each day per computer, or 2 x 10 x 25 = 500-hours of wasted 'ON' time. C) If each computer runs at 350 watts, and if electricity costs 11-cents per kilowatt hour, how much did this waste cost each month? Answer: The wasted time = 500 hours, and the total wattage of the two computers is 700 watts, so the number of kilowatt hours is 0.7 kilowatts x 500 hours = 350 kilowatt hours. At $.11 per kilowatt hours, this becomes 350 x 0.11 = $38.50. D) How many additional songs can you buy with iTunes for the wasted money each month? Answer: At $0.99 per song, you can buy $38.50/.99 = 38 songs!

Problem 2 – The Tevatron 'Atom Smasher' at Fermilab in Batavia, Illinois collides particles together at nearly the speed of light to explore the innermost structure of matter. When operating, the accelerator requires 70 megaWatts of electricity – about the same as the power consumption of the entire town of Batavia (population: 27,000). If an experiment, from start-up to finish, lasts 24 hours, what is the Tevatron's electricity consumption in kilowatt hours? Answer: 70 megaWatts x (1,000 kiloWatts/1 megaWatt) = 70,000 kiloWatts. Then 70,000 kiloWatts x 24 hours = 1,680,000 kiloWatt hours. B) At $0.11 per kilowatt, how much does one experiment cost to run? Answer: 1,680,000 kiloWatts x $0.11 / kilowatt = $184,800.00

Below is a picture of Fermilab's Tevatron accelerator. The ring has a diameter of 2 kilometers.



The Atmospheric Infrared Sounder (AIRS) instrument on NASA's Aqua spacecraft has been used by scientists to observe atmospheric carbon dioxide. The above map shows the concentrations of atmospheric carbon dioxide in units of 'parts per million', and range from 363 ppm (dark blue) to 380 ppm (red). The data was obtained in July 2003, and the gas is at an altitude of 8 kilometers. The map shows that carbon dioxide is not evenly mixed in the atmosphere, but there are regional differences that change in time. For example, the red 'clouds' move in time and change size and shape.

Problem 1 - From the color bar, about what is the average concentration of carbon dioxide across the globe, in ppm, not including the orange or red areas?

Problem 2 - What is the difference in ppm between your answer to Problem 1, and the highest levels of concentration?

Problem 3 - At these altitudes, atmospheric winds generally blow from west to east (left to right on the map). What geographic regions are nearest the highest concentrations of carbon dioxide in this map?

Problem 4 - The average mass of carbon dioxide in the atmosphere, at a concentration of 1 ppm equals 15 tons per square kilometer. How many tons/km^2 are represented by the: A) Red color? B) Yellow color? and C) The difference between red and yellow?

Answer Key

Problem 1 - From the color bar, about what is the average concentration of carbon dioxide across the globe, in ppm, not including the orange or red areas?
Answer: Most of the areas are yellowish, but the rest is light blue, so according to the color bar, the concentrations is about 375 ppm…very roughly.

Problem 2 - What is the difference in ppm between your answer to Problem 1, and the highest levels of concentration? Answer: The darkest reds are near 382 ppm, so the difference is about 7 ppm.

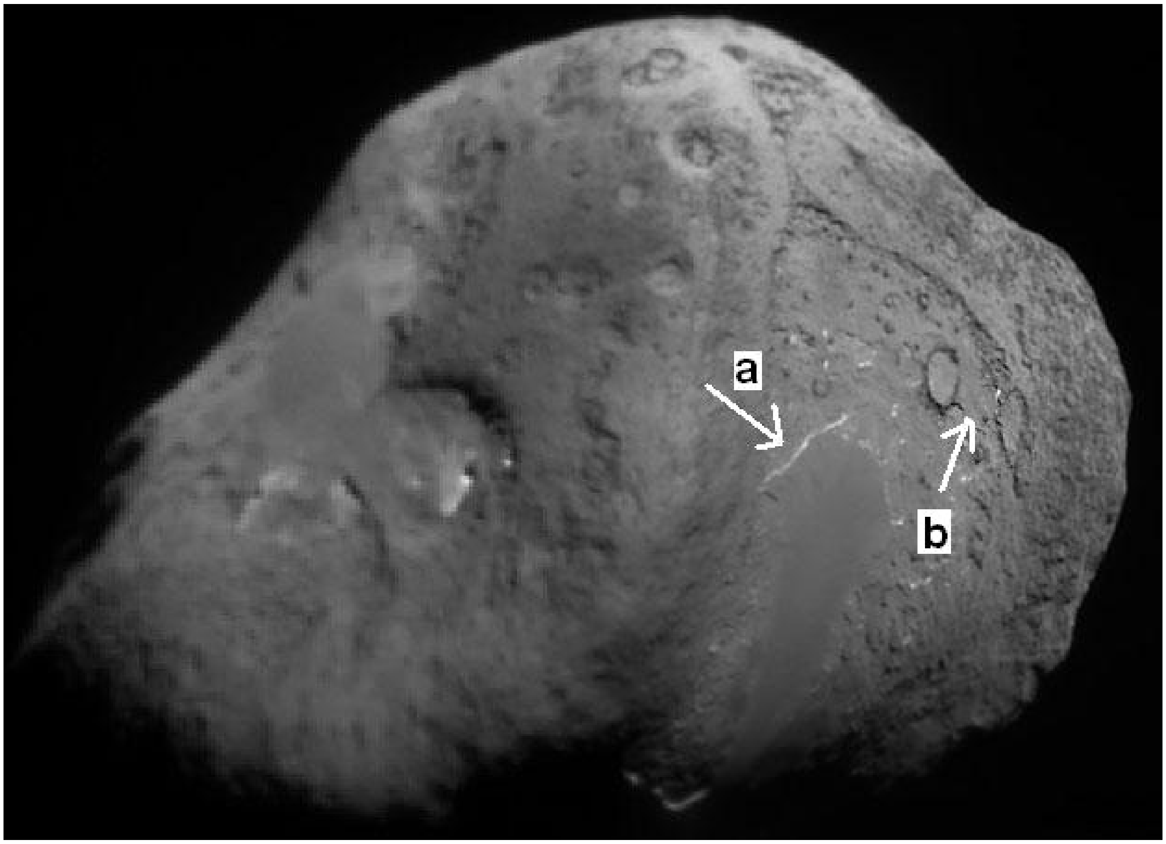
Problem 3 - At these altitudes, atmospheric winds generally blow from west to east (left to right on the map) in the Northern Hemisphere. What geographic regions are nearest the highest concentrations of carbon dioxide in this map? Answer: Western United States, the East Coast of the US, and regions in the Mediterranian and mid-East, some of which are 'downwind' from Europe.

Problem 4 - The average mass of carbon dioxide in the atmosphere, at a concentration of 1 ppm equals 15 tons per square kilometer. How many tons/km^2 are represented by the: A) Red color? B) Yellow color? and C) The difference between red and yellow?

Answer: A) The color bar says that red = 382 ppm so the concentration equals 382 x 15 = 5730 tons/km^2. B) yellow = 378 ppm so the concentration equals 378 x 15 = 5670 tons/km^2. C) the difference is 5730 - 5670 = 60 tons/km^2 of additional CO2.

Teacher Note: Most studies suggest that human activity add about 25 gigatons per year. Since the NASA map represents an average over a month, we see that our very crude estimate of 2.3 gigatons/month x 12 months = 28 gigatons/year which is close to more detailed estimates. This similarity may, however, be accidental since certain approximations had to be used in deriving our estimate that would not be made in the more careful studies. Also, the satellite only observed the carbon dioxide at an altitude of 8 kilometers, not all of the additional carbon dioxide down to sea-level.

Tempel 1 - Close-up of a Comet!



On July 4, 2005, the Deep Impact spacecraft flew within 500 km of the nucleus of comet Tempel 1. This composite image of the surface of the nucleus was put together from images taken by the Impactor probe as it plummeted towards the comet before finally hitting it and excavating its own crater. The width of this picture is 8.0 kilometers.

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus by the arrow? D) What is the size of some of the smallest details you can see in the picture?

Problem 2 - The white streak identified by Arrow A is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of Arrow B. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus? D) What is the size of some of the smallest details you can see in the picture?

Answer: By using a millimeter ruler, what is the scale of this image in meters per millimeter? Answer: A) Width = 153 millimeters, so the scale is 8000 meters/153 mm = **52 meters/mm**. B) Width = 147 mm x 110 mm or **7.6 km x 5.7 km**. C) Although the craters are foreshortened, the maximum size gives a better indication of their 'round' diameters of about 7mm or **360 meters**. D) **Students may find features about 1 millimeter across or 50 meters.**

Note to Teacher: Depending on the quality of your printer, the linear scale of the image in millimeters may differ slightly from the 153 mm stated in the answer to Problem 1. Students may use their measured value as a replacement for the ‘153 mm’ stated in the problem.

Problem 2 - The white streak near the center of the picture is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Answer: The width of the irregular white feature is about 0.5 millimeters or 26 meters. The length is about 15 millimeters or 15x52 = **780 meters**.

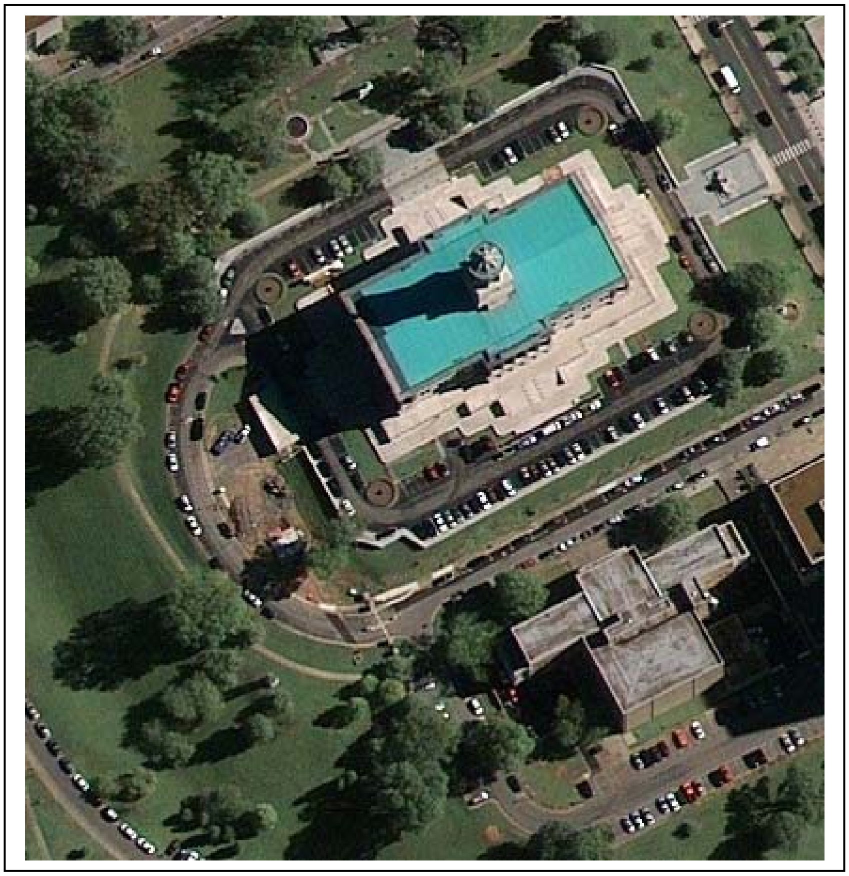
Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of the arrow. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Answer: The picture shows that the shortest distance to the edge of the nucleus is about 20 millimeters to the right, so this is a distance of about 20 x 52 = **1 kilometer**!

Note to Teacher:
Since the distance to the Earth was about 100 million kilometers, the spacecraft orbit had to be calculated to better than 1 part in 100 million over this distance in order for the probe to hit Tempel-1 as planned.

A High-Resolution Satellite Photo

The Lunar Reconnaissance Orbiter (LRO) will take photographs of the lunar surface at a resolution of 0.5 meters per pixel. The 425x425 pixel image below (Copyright © 2009 GeoEye) was taken of the Tennessee Court House from the GeoEye-1 satellite with a width of about 212 meters.



Problem 1 - What is the scale of the image in: A) meters per millimeter? B) meters per pixel?

Problem 2 - How does the resolution of the expected LRO images compare with the resolution of the above satellite photo?

Problem 3 - What are the smallest features you can easily identify in the above photo?

Problem 4 - From the length of the shadows, what would you estimate as the elevation of the sun above the horizon?

Answer Key

Problem 1 - What is the scale of the image in: A) meters per millimeter? B) meters per pixel? Answer: A millimeter ruler would indicate a width of 105 millimeters, so the scale is A) 212 meters/105 mm = 2 meters/millimeter and B) 212 meters/425 pixels = 0.5 meters/pixel.

Problem 2 - How does the resolution of the expected LRO images compare with the resolution of the above satellite photo? Answer: The resolutions are identical and both equal to 0.5 meters/pixel, so we should be able to see about the same kinds of details at the lunar surface with LRO.

Problem 3 - What are the smallest features you can easily identify in the above photo?
Answer: With a millimeter ruler you can determine that the smallest features are comfortably about 0.5 millimeters across or 1 meter. Examples would include the parked cars, the widths of the various footpaths, and the lane and street markings stripes. Some of the smaller spots may be the shadows of people!

Problem 4 - From the length of the shadows, what would you estimate as the elevation of the sun above the horizon?

Answer: This is a challenging problem for students because they first need to estimate what the height of the object is that is casting the shadow. From this they can construct a triangle and determine the sun angle, or use trigonometry.

For example, suppose that the large tree in the lower left corner of the picture has a height of 50-feet (17 meters). We can measure the shadow length with a millimeter ruler to get 20 millimeters or 20 mm x 2 M/mm = 40 meters. Then tan(theta) = 17 meters/40 eters = 0.45, and so theta = 24 degrees above the horizon.

This problem shows students one of the problems encountered when studying photos like this. We can easily measure the widths of objects, but measuring their heights can be challenging, especially when the objects in question are unfamiliar.

Some Famous Unit Conversion Errors!

Story 1: On September 23, 1999 NASA lost the $125 million Mars Climate Orbiter spacecraft after a 286-day journey to Mars. Miscalculations due to the use of English units instead of metric units apparently sent the craft slowly off course -- 60 miles in all. Thrusters used to help point the spacecraft had, over the course of months, been fired incorrectly because data used to control the wheels were calculated in incorrect units. Lockheed Martin, which was performing the calculations, was sending thruster data in English units (pounds) to NASA, while NASA's navigation team was expecting metric units (Newtons).

Problem 1 - A solid rocket booster is ordered with the specification that it is to produce a total of 10 million pounds of thrust. If this number is mistaken for the thrust in Newtons, by how much, in pounds, will the thrust be in error? (1 pound = 4.5 Newtons)

Story 2: On January 26, 2004 at Tokyo Disneyland's Space Mountain, an axle broke on a roller coaster train mid-ride, causing it to derail. The cause was a part being the wrong size due to a conversion of the master plans in 1995 from English units to Metric units. In 2002, new axles were mistakenly ordered using the pre-1995 English specifications instead of the current Metric specifications.

Problem 2 - A bolt is ordered with a thread diameter of 1.25 inches. What is this diameter in millimeters? If the order was mistaken for 1.25 centimeters, by how many millimeters would the bolt be in error?

Story 3: On 23 July 1983, Air Canada Flight 143 ran completely out of fuel about halfway through its flight from Montreal to Edmonton. Fuel loading was miscalculated through misunderstanding of the recently adopted metric system. For the trip, the pilot calculated a fuel requirement of 22,300 kilograms. There were 7,682 liters already in the tanks.

Problem 3 - If a liter of jet fuel has a mass of 0.803 kilograms, how much fuel needed to be added for the trip?

Problem 1 - A solid rocket booster is ordered with the specification that it is to produce a total of 10 million pounds of thrust. If this number is mistaken for the thrust in Newtons, by how much, in pounds, will the thrust be in error? (1 pound = 4.5 Newtons)

Answer: 10,000,000 'Newtons' x (1 pound / 4.448 Newtons) = 2,200,000 pounds instead of 10 million pounds so the error is a 'missing' 7,800,000 pounds of thrust…an error that would definitely be noticed at launch!!!

Problem 2 - A bolt is ordered with a thread diameter of 1.25 inches .What is this diameter in millimeters? If the order was mistaken for 1.25 centimeters, by how many millimeters would the bolt be in error? Answer: 1- inch = 25.4 millimeters so 1.25 inches x (25.4 mm/ 1 inch) = 31.75 millimeters. Since 1.25 centimeters = 12.5 millimeters, the bolt would delivered 31.75 - 12.5 = 19.25 millimeters too small!

Problem 3 - In order to calculate how much more fuel had to be added, the crew needed to convert the quantity in the tanks, 7,682 liters, to a weight, subtract that figure from 22,300 kilograms, and convert the result back into a volume (liters).

7,682 liters x (0.803 kilograms/ 1 liter)= 6,169 kg
22,300 kg – 6,169 kg = 16,131 kg
16,131 kg x (1 liter / 0.803 kilograms) = 20,088 liters of jet fuel.

Between the ground crew and flight crew, however, they arrived at an incorrect conversion factor of 1.77, the weight of a liter of jet fuel in pounds. This was the conversion factor provided on the refueller's paperwork and which had always been used for the rest of the airline's fleet. Their calculation produced:

7,682 liters x (1.77 pounds/liter) = 13,597 which they interpreted as kilograms but was actually the fuel mass in pounds! Then they continued the calculation:

22,300 kg – 13,597 'kg' = 8,703 kg
8,703 kg ÷ 1.77 = 4,916 liters ….so they were actually 15,172 liters short of fuel!

Identifying Materials by their Reflectivity

50%	78%
3%	30%

Material	Reflectivity
Snow	80%
White Concrete	78%
Bare Aluminum	74%
Vegetation	50%
Bare Soil	30%
Wood Shingle	17%
Water	5%
Black Asphalt	3%

When light falls on a material, some of the light energy is absorbed while the rest is reflected. The absorbed energy usually contributed to heating the body. The reflected energy is what we use to actually see the material! Scientists measure reflectivity and absorption in terms of the percentage of energy that falls on the body, which is called its albedo. The combination must add up to 100%.

The table above shows the reflectivity of various common materials. For example, snow reflects 80% of the light that falls on it, which means that it absorbs 20% and so 80% + 20% = 100%. This also means that if I have 100 watts of light energy falling on the snow, 80 watts will be reflected and 20 watts will be absorbed.

Problem 1 - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Problem 2 - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Problem 3 - What is the average reflectivity of these four equal-area regions combined?

Problem 4 - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

Answer Key

Problem 1 - If 1000 watts falls on a body, and you measure 300 watts reflected, what is the reflectivity of the body, and from the Table, what might be its composition?

Answer: The reflectivity is 100% x (300 watts/1000 watts) = 30%. From the table, Bare Soil has this same reflectivity and so is a likely composition.

Problem 2 - You are given the reflectivity map at the top of this page. What are the likely compositions of the areas in the map?

Answer: 50% = Vegetation
78% = White Concrete
30% = Bare Soil
3% = Black Asphalt

Problem 3 - What is the average reflectivity of these four equal-area regions combined? Answer: Because each of the four materials cover the same area, we just add up their reflectivities and divide by 4 to get (50% + 78% + 30% + 3%)/4 = 40%.

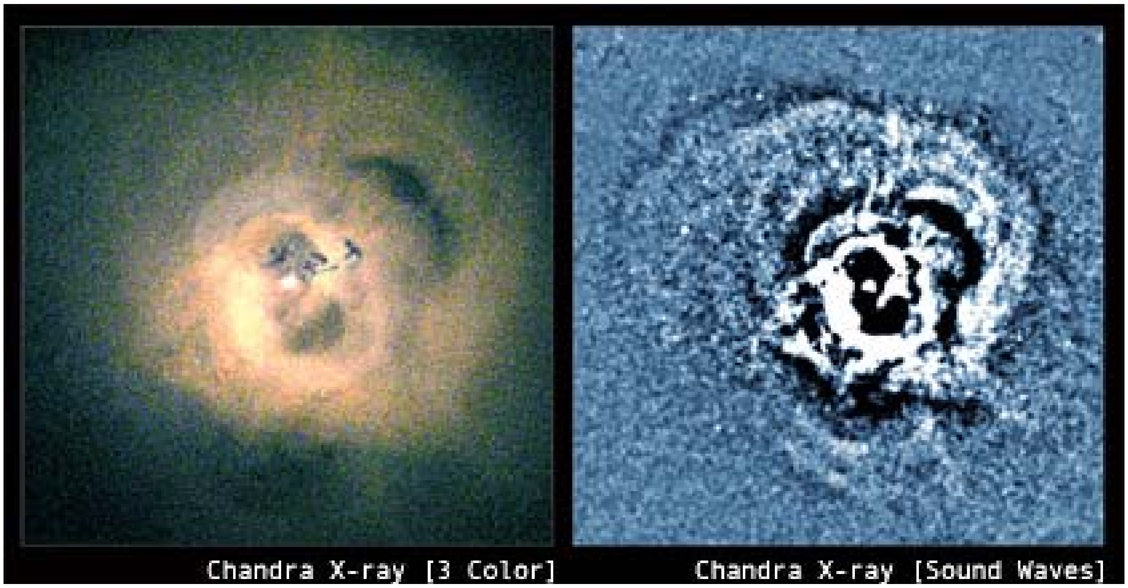
Problem 4 - Solar radiation delivers 1300 watts per square meter to the surface of Earth. If the area in the map is 20 meters on a side; A) how much solar radiation, in watts, is reflected by each of the four materials covering this area? B) What is the total solar energy, in watts, reflected by this mapped area? C) What is the total solar energy, in watts, absorbed by this area?

Answer: Each material covers 10 meters x 10 meters = 100 square meters:

A) Vegetation: 0.5 x 1300 watts/m^2 x 100 m^2 = 65,000 watts.
Concrete: 0.78 x 1300 watts/m^2 x 100 m^2 = 101,400 watts.
Bare Soil: 0.30 x 1300 watts/m^2 x 100 m^2 = 39,000 watts.
Black Asphalt: 0.03 x 1300 watts/m^2 x 100 m^2 = 3,900 watts.

B) 65,000 + 100,000 + 39,000 + 3,900 = 209,300 watts.

C) The total wattage entering this area is 1,300 watts/m^2 x 100 m^2 x 4 = 520,000 watts. Since 209,300 watts are reflected, that means that 520,000 watts - 209,300 watts = 310,700 watts are being absorbed.



In September 2003, the Chandra Observatory took an x-ray image of a massive black hole in the Perseus Galaxy Cluster located 250 million light years from Earth. Although it could not see the black hole, it did detect the x-ray light from the million-degree gas in the core of the cluster. Instead of a featureless blob, the scientists detected a series of partial concentric rings which they interpreted as sound waves rushing out from the vicinity of the black hole as it swallowed gas in a series of explosions. The image above left shows the x-ray image, and to the right, an enhanced version that reveals the details more clearly.

Problem 1 - The image has a physical width of 350,000 light years. Using a millimeter ruler, what is the scale of the image in light years/millimeter?

Problem 2 - Examine the image on the right very carefully and estimate how far apart the consecutive crests of the sound wave are in millimeters. What is the wave length of the sound wave in light years?

Problem 3 - The wavelength of middle-C on a piano is 1.3 meters. If 1 light year = 9.5×10^{15} meters, and if 1 octave represents a change by a factor of 1/2 change in wavelength, how many octaves below middle-C is the sound wave detected by Chandra?

Problem 1 - The image has a physical width of 350,000 light years. Using a millimeter ruler, what is the scale of the image in light years/millimeter?

Answer: The width is approximately 70 millimeters wide, so the scale is 350,000 light years / 70 millimeters = **5,000 light years/millimeter**.

Problem 2 - Examine the image on the right very carefully and estimate how far apart the consecutive crests of the sound wave are in millimeters. What is the wave length of the sound wave in light years?

Answer: Depending on where the student makes the measurement, such as the set of two bright parallel features on the lower part (7-o'clock position) of the image, the separations will be about 6 millimeters, so the wavelength is 6 mm x (5,000 light years/ 1 mm) = **30,000 light years!**

Problem 3 - The wavelength of middle-C on a piano is 1.3 meters. If 1 light year = 9.5×10^{15} meters, and if 1 octave represents a change by a factor of 1/2 change in wavelength ,how many octaves below middle-C is the sound wave detected by Chandra?

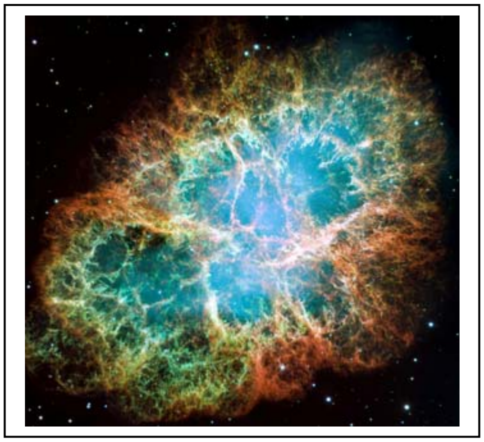
Answer: The sound wave has a wavelength of 30,000 light years x (9.5×10^{15} meters / 1 light year) = 2.9×10^{20} meters. Octaves are determined in terms of powers of two changes in sound waves so that a wavelength change from 32 meters to 16 meters = 1/2, from 32 meters to 8 meters = 1/2 x 1/2 = 1/4 or 2^{-2}, and so on. Middle-C has a wavelength of 1.3 meters, so the Perseus Cluster sound wave differs from middle-C by a factor of 2.9×10^{20} meters /1.3 meters = 2.2×10^{20} times. We have to find N such that $2^N = 2.2 \times 10^{20}$. Using a calculator and repetitive multiplications (or visit the power-of-two table at http://web.njit.edu/~walsh/powers/) we get the table below:

N	factor
10	1,024
30	1.07×10^{9}
40	1.1×10^{12}
57	1.4×10^{17}
68	2.9×10^{20}

So the answer is approximately **68 octaves below middle-C**.

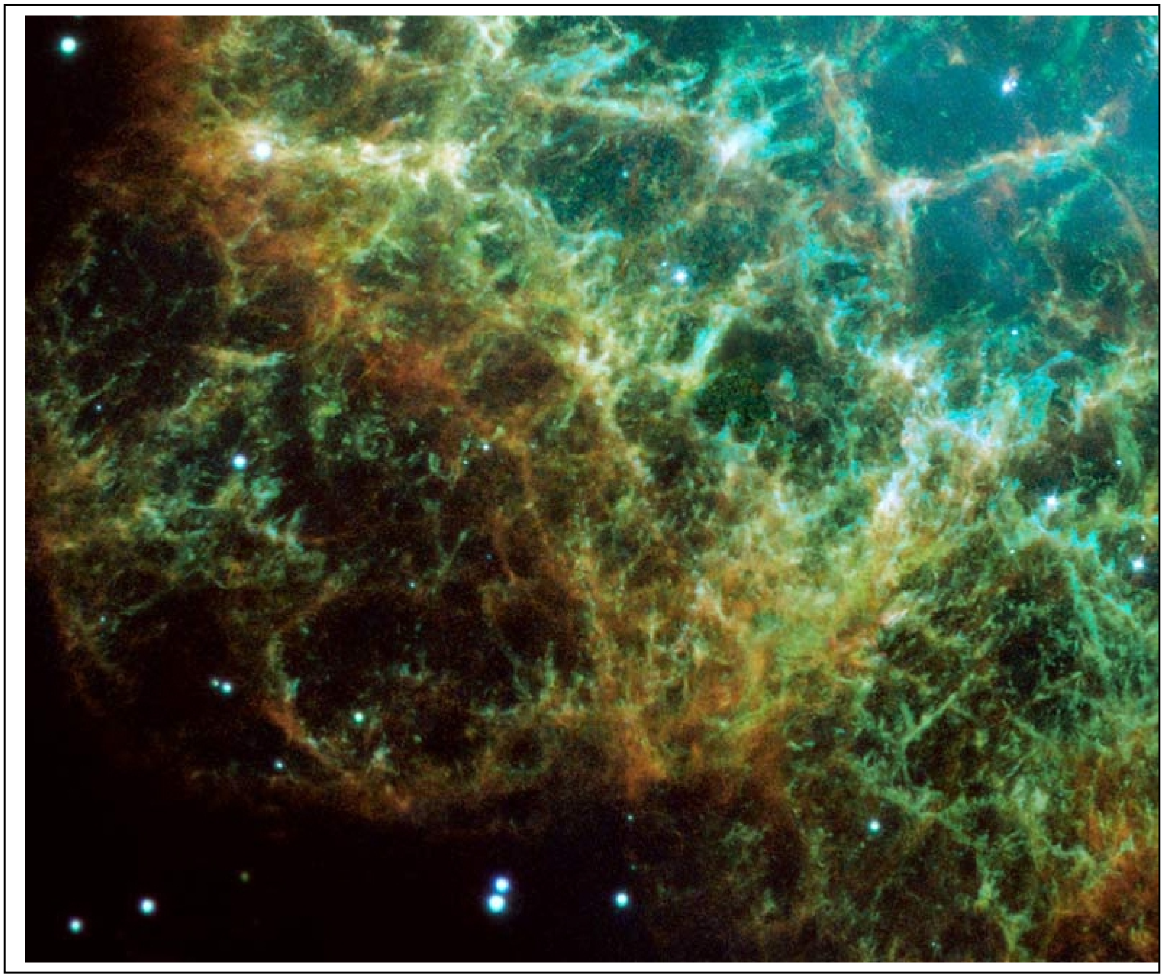
Note to Teacher: A tremendous amount of energy is needed to generate the cavities, as much as the combined energy from 100 million supernovae. For more information, visit the Chandra web page at http://chandra.harvard.edu/press/03_releases/press_090903.html

Details from an exploding star



These dramatic images of the Crab Nebula were taken in 2005 by the Hubble Space Telescope. The image on the left has a scale of 0.2 light years/millimeter. The enlargement below has a scale of 0.025 light years/millimeter.

The star that produced this nebula exploded as a supernova in the year 1054 AD, and the expanding gas has been traveling outwards ever since.



Problem 1 – From the information given, what is the average speed of the expanding gas cloud in kilometers/hour? (Note that 1 light year = 62,000 Astronomical Units, and 1 Astronomical Unit = 150 million kilometers, also 1 year = 8760 hours).

Problem 2 – How large are the smallest clumps of the gas in the expanding cloud?

Problem 3 – Draw a sketch, to scale, of the diameter of the solar system (80 Astronomical Units) compared to the size of two or three of the smallest gas clumps.

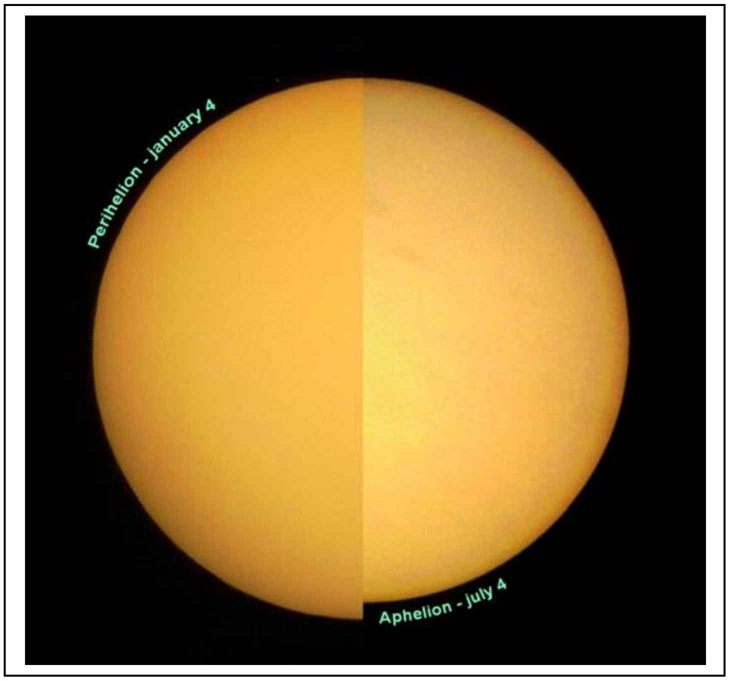
Answer Key

Problem 1 – From the information given, what is the average speed of the expanding gas cloud in kilometers/hour? (Note that 1 light year = 62,000 Astronomical Units, and 1 Astronomical Unit = 150 million kilometers, also 1 year = 8760 hours).

Answer: The fastest gas will have traveled the farthest distance in the picture, so we will measure the longest dimension of the nebula. From the upper image, the largest diameter of the nebula is about 66 millimeters, so the radius is 33 millimeters. At a scale of 0.2 light years/mm, this equals 33 x 0.2 = 6.5 light years. From the conversion information, this equals 6.5 x (62,000 AU/ly) x (150 million km/AU) = 60 trillion kilometers. The time taken to travel this distance is the number of years between 1054 and 2005, which is 951 years. This equals 951 years x (8760 hours/yr) = 8,331,000 hours. The speed average is then 60 trillion km/8,331,000 hours = 7,300,000 kilometers/hour.

Problem 2 – How large are the smallest clumps of the gas in the expanding cloud?
Answer: In the larger image, students will find filaments and condensations that are about 0.2 millimeters across, which corresponds to 0.2 x 0.025 light years/mm = 0.005 light years across. In terms of Astronomical Units, this equals 0.005 Light years x 62,000 AU/ly) = 310 AU.

Problem 3 – Draw a sketch, to scale, of the diameter of the solar system (80 Astronomical Units) compared to the size of two or three of the smallest gas clumps.
Answer: Creating a scaled image is a bit of a challenge. You want the scale to accommodate 1) enough resolution that you can comfortably draw the smallest object you want to represent, and 2) include a full rendition of the largest object you want to represent. For common 8.5 x 11-inch paper, a scale of 5 AU per millimeter would cover the entire solar system (Diameter of 16 millimeters) and a Crab nebula globule (Diameter of 310 AU = 62 millimeters). Students may color the image with a range of colors suggested by the Hubble photo and include several globules and filaments of the proper scale in the field.



Earth's orbit is not a perfect circle centered on the sun, but an ellipse! Because of this, in January, Earth is slightly closer to the sun than in June. This means that the sun will actually appear to have a bigger disk in the sky in June than in January…but the difference is impossible to see with the eye, even if you could do so safely!

The figure above shows the sun's disk taken by the SOHO satellite. The left side shows the disk on January 4 and the right side shows the disk on June 4, 2009. As you can see, the diameter of the sun appears to change slightly between these two months.

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure?

Problem 2 - By what percentage did the diameter of the Sun change between January and June compared to its average diameter?

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in June compared to January?

Problem 1 - What is the average diameter of the Sun, in millimeters, in this figure?
Answer: Using a millimeter ruler, and measuring vertically along the join between the two images, the left-hand, January, image is 72 millimeters in diameter, while the right-hand image is 69 millimeters in diameter. The average of these two is (72 + 69)/2 = 70.5 millimeters.

Problem 2 - By what percentage did the diameter of the Sun change between January and June compared to its average diameter?
Answer: In January the moon was larger then the average diameter by 100% x (72 - 70.5)/70.5 = 2.1 %. In June it was smaller then the average diameter by 100% x (70.5 - 69)/70.5 = 2.1%.

Problem 3 - If the average distance to the Sun from Earth is 149,600,000 kilometers, how much closer is Earth to the Sun in June compared to January?

Answer: The diameter of the sun appeared to change by 2.1% + 2.1% = 4.2% between January and June. Because the apparent size of an object is inversely related to its distance (i.e. the farther away it is the smaller it appears), this 4.2% change in apparent size occurred because of a 4.2% change in the distance between Earth and the Sun, so since 0.042 x 149,600,000 km = 6,280,000 kilometers, the change in the Sun's apparent diameter reflects the 6,280,000 kilometer change in earth's distance between January and June. The Earth is 6,280,000 kilometers closer to the Sun in June than in January.

Period	Age (years)	Days per year	Hours per day
Current	0	365	
Upper Cretaceous	70 million	370	
Upper Triassic	220 million	372	
Pennsylvanian	290 million	383	
Mississippian	340 million	398	
Upper Devonian	380 million	399	
Middle Devonian	395 million	405	
Lower Devonian	410 million	410	
Upper Silurian	420 million	400	
Middle Silurian	430 million	413	
Lower Silurian	440 million	421	
Upper Ordovician	450 million	414	
Middle Cambrian	510 million	424	
Ediacarin	600 million	417	
Cryogenian	900 million	486	

We learn that an 'Earth Day' is 24 hours long, and that more precisely it is 23 hours 56 minutes and 4 seconds long. But this hasn't always been the case. Detailed studies of fossil shells, and the banded deposits in certain sandstones, reveal a much different length of day in past eras! These bands in sedimentation and shell-growth follow the lunar month and have individual bands representing the number of days in a lunar month. By counting the number of bands, geologists can work out the number of days in a year, and from this the number of hours in a day when the shell was grown, or the deposits put down. The table above shows the results of one of these studies.

Problem 1 - Complete the table by calculating the number of hours in a day during the various geological eras. It is assumed that Earth orbits the sun at a fixed orbital period, based on astronomical models that support this assumption.

Problem 2 - Plot the number of hours lost compared to the modern '24 hours' value, versus the number of years before the current era.

Problem 3 - By finding the slope of a straight line through the points can you estimate by how much the length of the day has increased in seconds per century?

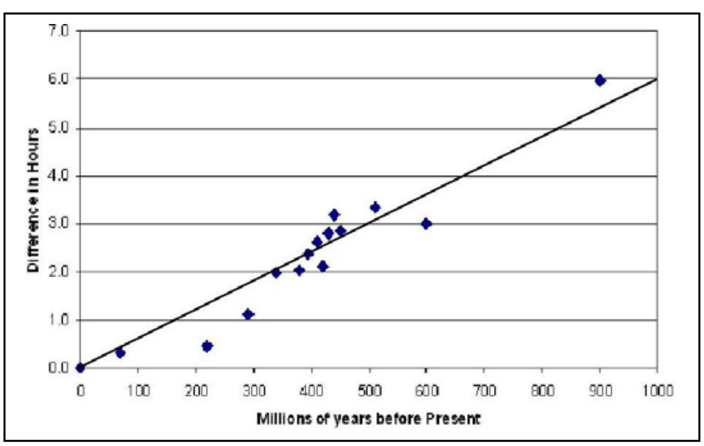
Answer Key

Period	Age (years)	Days per year	Hours per day
Current	0	365	**24.0**
Upper Cretaceous	70 million	370	**23.7**
Upper Triassic	220 million	372	**23.5**
Pennsylvanian	290 million	383	**22.9**
Mississippian	340 million	398	**22.0**
Upper Devonian	380 million	399	**22.0**
Middle Devonian	395 million	405	**21.6**
Lower Devonian	410 million	410	**21.4**
Upper Silurian	420 million	400	**21.9**
Middle Silurian	430 million	413	**21.2**
Lower Silurian	440 million	421	**20.8**
Upper Ordovician	450 million	414	**21.2**
Middle Cambrian	510 million	424	**20.7**
Ediacarin	600 million	417	**21.0**
Cryogenian	900 million	486	**18.0**

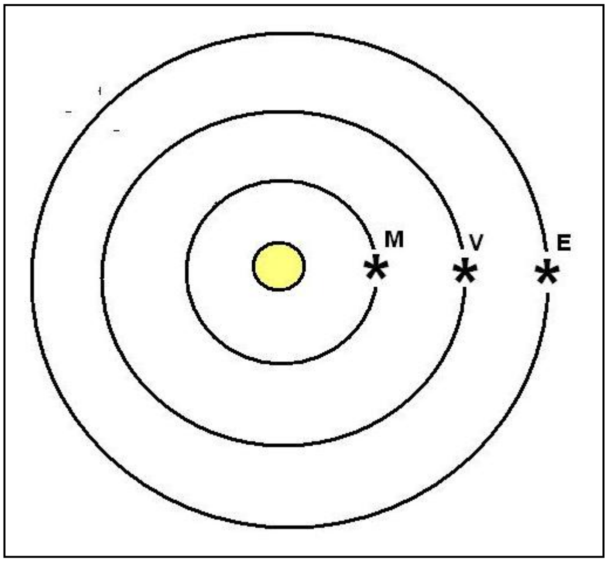
Problem 1 - Answer; **See table above. Example for last entry: 486 days implies 24 hours x (365/486) = 18.0 hours in a day.**

Problem 2 - Answer; **See figure below**

Problem 3 - Answer: From the line indicated in the figure below, the slope of this line is m = (y2 - y1)/ (x2 - x1) = 6 hours / 900 million years or 0.0067 hours/million years. Since there are 3,600 seconds/ hour and 10,000 centuries in 1 million years (Myr), this unit conversion yields 0.0067 hr/Myr x (3600 sec/hr) x (1 Myr / 10,000 centuries) = **0.0024 seconds/century.** This is normally cited as 2.4 milliseconds per century.



Planetary Alignments



One of the most interesting things to see in the night sky is two or more planets coming close together in the sky. Astronomers call this a conjunction. As seen from their orbits, another kind of conjunction is sometimes called an 'alignment' which is shown in the figure to the left and involves Mercury, M, Venus, V, and Earth, E. As viewed from Earth's sky, Venus and Mercury would be very close to the sun, and may even be seen as black disks 'transiting' the disk of the sun at the same time, if this alignment were exact. How often do alignments happen?

Earth takes 365 days to travel one complete orbit, while Mercury takes 88 days and Venus takes 224 days, so the time between alignments will require each planet to make a whole number of orbits around the sun and return to the pattern you see in the figure above. Let's look at a simpler problem. Suppose Mercury takes 1/4 Earth-year and Venus takes 2/3 of an Earth-year to make their complete orbits around the sun. You can find the next line-up from one of these two methods:

Method 1: Work out the three number series like this:

Earth = 0, 1, **2**, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,…
Mercury = 0, 1/4, 2/4, 3/4 ,4/4, 5/4, 6/4, 7/4, **8/4**, 9/4, 10/4, 11/4, 12/4,…
Venus = 0, 2/3, 4/3, **6/3**, 8/3, 10/3, 12/3, 14/3, 16/3, 18/3, 20/3, …

Notice that the first time they all coincide with the same number is at **2 years**. So Mercury has to go around the Sun 8 times, Venus 3 times and Earth 2 times for them to line up again in their orbits.

Method 2: We need to find the Least Common Multiple (LCM) of 1/4, 2/3 and 1. First render the periods in multiples of a common time unit of 1/12, then the sequences are:

Mercury = 0, 3, 6, 9, 12, 15, 18, 21, **24**,
Venus = 0, 8, 16, **24**, 32, 40, …
Earth, 0, 12, **24**, 36, 48, 60, …

The LCM is 24 which can be found from prime factorization:

Mercury: 3 = 3
Venus: 8 = 2 x 2 x 2
Earth: 12 = 2 x 2 x 3

The LCM the product of the highest powers of each prime number or 3 x 2 x 2 x 2 = 24. and so it will take 24/12 = **2 years.**

Problem 1 - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 Earth-years and Venus takes 26/42 Earth-years. After how many Earth-years will the alignment shown in the figure above reoccur?

Problem 1 - Suppose a more accurate estimate of their orbit periods is that Mercury takes 7/30 Earth-years and Venus takes 26/42 Earth-years. After how many Earth-years will the alignment reoccur?

Mercury = 7/30 x 365 = 85 days vs actual 88 days
Venus = 26/42 x 365 = 226 days vs actual 224 days
Earth = 1

The common denominator is 42 x 30 = 1,260 so the series periods are
Mercury = 7 x 42 = 294 so 7/30 = 294/1260
Venus = 26 x 30 = 780 so 26/42 = 780/1260
Earth = 1260 so 1 = 1260/1260

The prime factorizations of these three numbers are

294 = 2 x 2 x 3 x 7 x 7
780 = 2 x 2x 5 x 3 x 13
1260 = 2 x 2 x 3 x 3 x 5 x 7

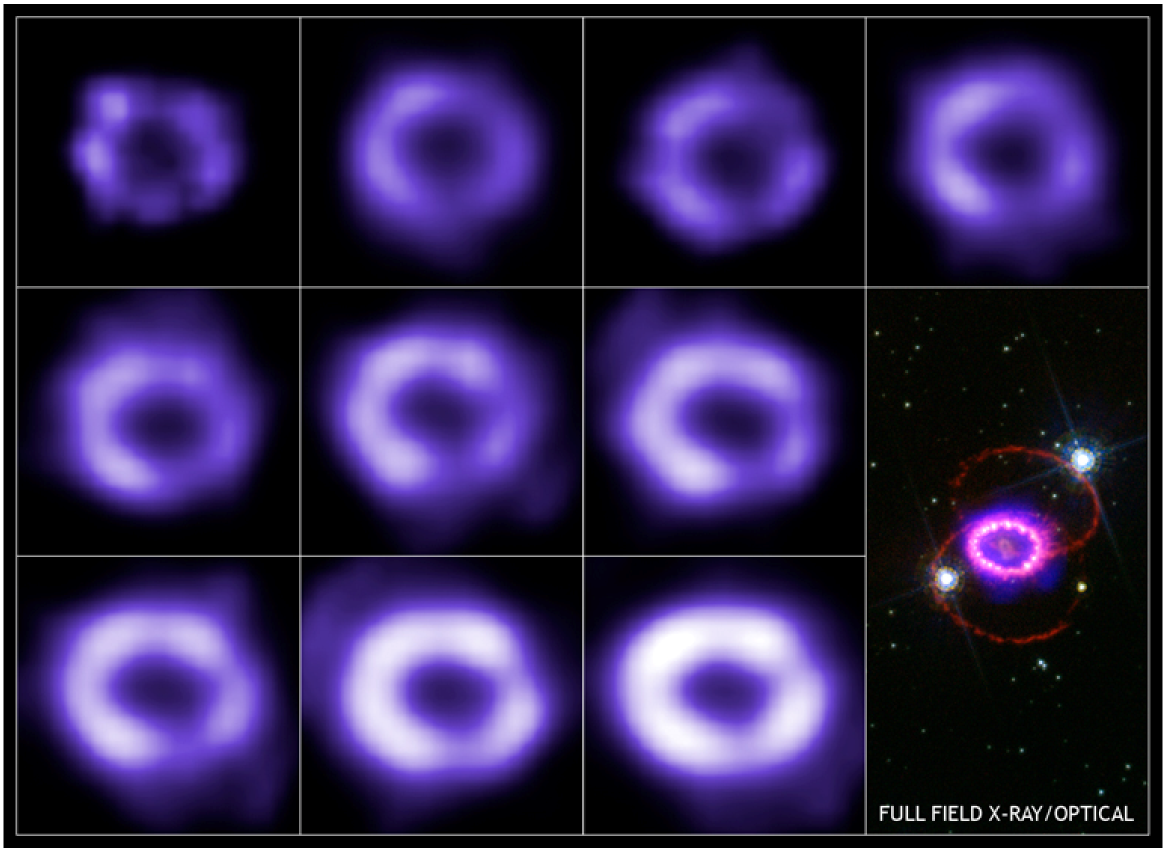
LCM = 2 x 2 x 3 x 3 x 5 x 7 x 7 x 13 = 114,660

So the time will be 114,660 / 1260 = 91 years! In this time, Mercury will have made exactly 114,660/294 = 390 orbits and Venus will have made 114,660/780 = 147 orbits

Note to Teacher: Why did the example problem give only 2 years while this problem gave 91 years for the 'same' alignment? Because we used a more accurate approximation for the orbit periods of the three planets. Mercury actual period = 88 days but 1/4 Earth-year = 91.25 days compared to 7/30 Earth-year = 85 days. Venus actual period = 224 days but 2/3 Earth-year = 243 days and 26/42 Earth-year = 226 days.

This means that after 2 years and exactly 8 orbits (8 x 91.25 = 730 days), Mercury will be at 8/4 x 365 = 730 days while the actual 88-day orbit will be at 88 x 8 = 704 days or a timing error of 26 days. Mercury still has to travel another 26 days in its orbit to reach the alignment position. For Venus, its predicted orbit period is 2/3 x 365 = 243.3 days so its 3 orbits in the two years would equal 3 x 243.3 days = 730 days, however its actual period is 224 days so in 3 orbits it accumulates 3 x 224 = 672 days and the difference is 730-672 = 58 days so it has to travel another 58 days to reach the alignment. In other words, the actual positions of Mercury and Venus in their orbits is far from the 'straight line' we were hoping to see after exactly 2 years, using the approximate periods of 1/4 and 2/3 earth-years!

With the more accurate period estimate of 7/30 Earth-years (85 days) for Mercury and 26/42 Earth-years (226 days) for Venus, after 91 years, Mercury will have orbited exactly 91 x 365 days/88 days = 377.44 times, and Venus will have orbited 91x365/224 = 148.28 times. This means that Mercury will be 0.44 x 88d = 38.7 days ahead of its predicted alignment location, and Venus will be 0.28 x 224 = 62.7 days behind its expected alignment location. Comparing the two predictions, Prediction 1: Mercury= - 26 days, Venus= - 58 days; Prediction 2: Mercury = +26 days and Venus = - 22 days. Our prediction for Venus has significantly improved while for Mercury our error has remained about the same in absolute magnitude. In the sky, the two planets will appear closer together for Prediction 2 in 1911 years than for Prediction 1 in 2 years. If we want an even 'tighter' alignment, we have to make the fractions for the orbit periods much closer to the actual periods of 88 and 224 days.



In March, 1987 a supernova occurred in the Large Magellanic Cloud; a nearby galaxy to the Milky Way about 160,000 light years away from Earth. The site of the explosion was traced to the location of a blue supergiant star called Sanduleak -69º 202 (SK -69 for short) that had a mass estimated at approximately 20 times our own sun.The series of image above, taken by the Chandra X-ray Observatory, shows the expansion of the million-degree gas ejected by the supernova between January, 2000 (top left image) to January, 2005 (lower right image). The width of each image is 1.9 light years.

Problem 1 - Using a millimeter ruler, what is the scale of each image in light years/millimeter?

Problem 2 - If 1 light year = 9.5×10^{12} kilometers, and 1 year = 3.1×10^{7} seconds, what was the average speed of the supernova gas shell between 2000 and 2005?

Problem 1 - Using a millimeter ruler, what is the scale of each image in light years/millimeter?

Answer: The width of each image is about 38 millimeters, so the scale is 1.9 light years / 38 mm = **0.05 light years/mm**.

Problem 2 - If 1 light year = 9.5×10^{12} kilometers, and 1 year = 3.1×10^{7} seconds, what was the average speed of the supernova gas shell between 2000 and 2005?

Answer: First convert the scale to kilometers/mm to get 0.05 LY/mm x (9.5×10^{12} kilometers / 1 LY) = 4.8×10^{11} kilometers/mm.

Next, students will have to measure the outer diameter of an irregular shape. They may do this by making several measurements across different chords through the center of the shell, and averaging the numbers. Because the outer edge is not sharp due to brightness gradients, students will also have to decide at what point the edge of the shell occurs and then use this visual definition consistently in the other edge measurements.

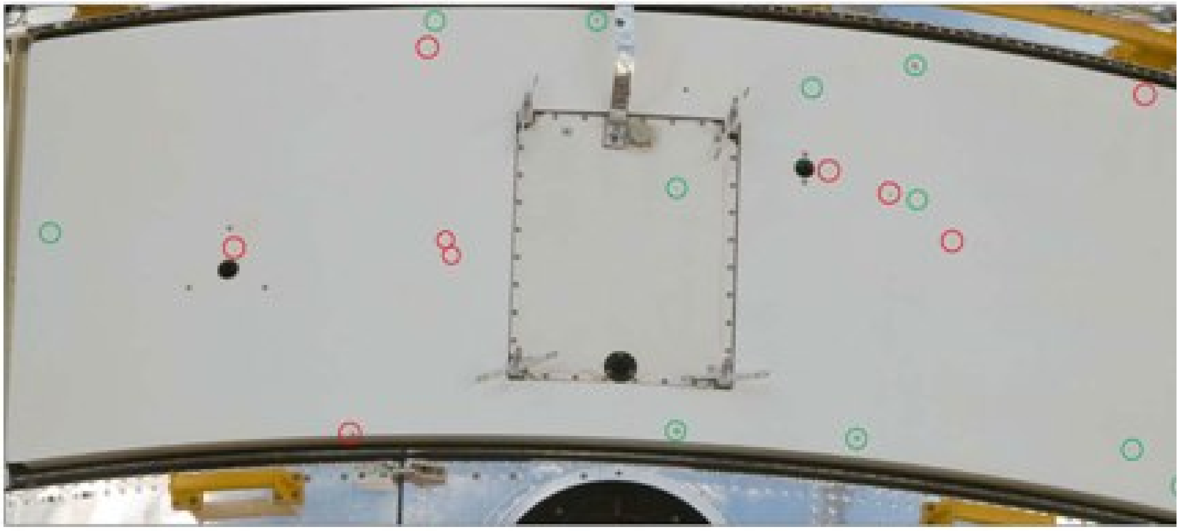
The diameter of the top left supernova ring is about 19 millimeters. The bottom-right ring has a diameter of about 27 millimeters, so the change on radius was (27 - 19)/2 = 4 millimeters. This corresponds to a physical distance of 4 mm x (4.8×10^{11} kilometers/mm) = 1.9×10^{12} kilometers. The elapsed time was (January 2005 - January 2000) = 5 years or 5 years x (3.1×10^{7} seconds / 1 year) = 1.6×10^{8} seconds. The average speed is then V = 1.9×10^{12} kilometers / 1.6×10^{8} seconds = **12,000 kilometers/sec**.

Note to Teacher: Although this is the average speed, students can investigate whether the shell has been moving at a constant speed during this 5-year period, or if the gas shell has been accelerating or deceleration by measuring the speed differences between the consecutive images which can be found at:

http://chandra.harvard.edu/photo/2005/sn87a/more.html ………..for specific image dates

http://chandra.harvard.edu/photo/2005/sn87a/index.html ……….. supernova information

Counting Craters On the Hubble Space Telescope!



The STS-125 Atlantis astronauts retrieved the Hubble Space Telescope Wide-field Planetary Camera 2 (WFPC2) during a very successful and final servicing mission in May 2009. The radiator (above photo) attached to WFPC2 has dimensions of 2.2 meters by 0.8 meters. Its outermost layer is a 4-mm-thick aluminum, curved plate coated with white thermal paint. This radiator has been exposed to space since the deployment of WFPC2 in 1993. During this time, it received numerous impacts by micrometeoroids and man-made particles (flecks of paint, etc). The circles drawn on the radiator plate show the locations of these impacts.

Problem 1 - What is the total surface area of the WFPC2 radiator plate in square meters?

Problem 2 - How many impacts were counted over this area?

Problem 3 - What is the surface density of impacts in units of impacts per square meter?

Problem 4 - How many years was this panel in space?

Problem 5 - What is the impact rate in units of impacts per meter^2 per year?

Problem 6 - The solar panels on the International Space Station have a total surface area of 1,632 meters^2. A) How many impacts per year will these solar panels experience? B) What will be the average time, in hours, between impact?

Problem 1 - What is the total surface area of the WFPC2 radiator plate in square meters?
Answer: 2.2 meter x 0.l8 meter = 1.8 $meters^2$.

Problem 2 - How many impacts were counted over this area?
Answer: There are 20 circular marks, or portions along the edge, on the panel.

Problem 3 - What is the surface density of impacts in units of impacts per square meter?
Answer: 20 impacts / 1.8 $meters^2$ = 11.1 $impacts/meter^2$

Problem 4 - How many years was this panel in space?
Answer: 2009 - 1993 = 16 years.

Problem 5 - What is the impact rate in units of impacts per $meter^2$ per year?
Answer: (11.1 $impacts/meter^2$) / (16 years) = 0.7 $impacts/meter^2/year$

Problem 6 - The solar panels on the International Space Station have a total surface area of 1,632 $meters^2$. A) How many impacts per year will these solar panels experience? B) What will be the average time, in hours, between impact?
Answer: A) 1,632 $meters^2$ x (0.7 $impacts/meter^2/year$) = 1,142 impacts/year.
B) There are 1,142 impacts in a year,
so since there are 365 days/year x 24 hours/day = 8760 hours/year,
there will be about 8,760 hours/year x 1 year/ 1,142 impacts
or **7.7 hours between impacts on the average**.

The images below show some close-up images of impact craters on the Hubble Space Telescope as viewed from the Space Shuttle using a telephoto lens.

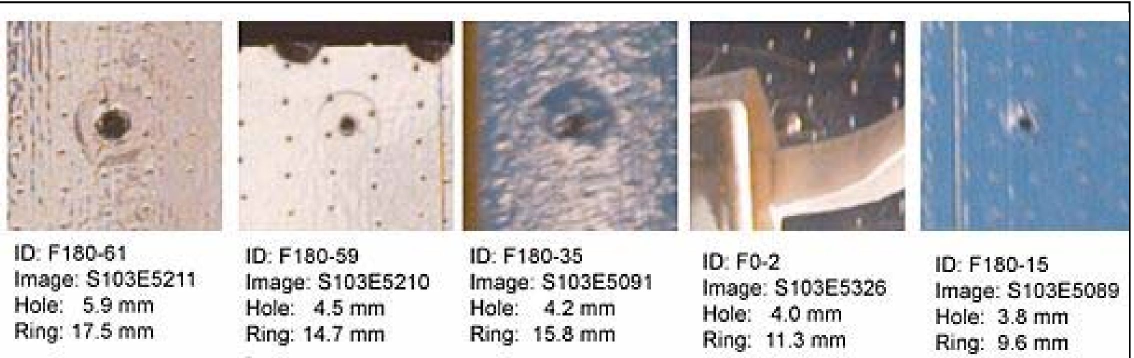
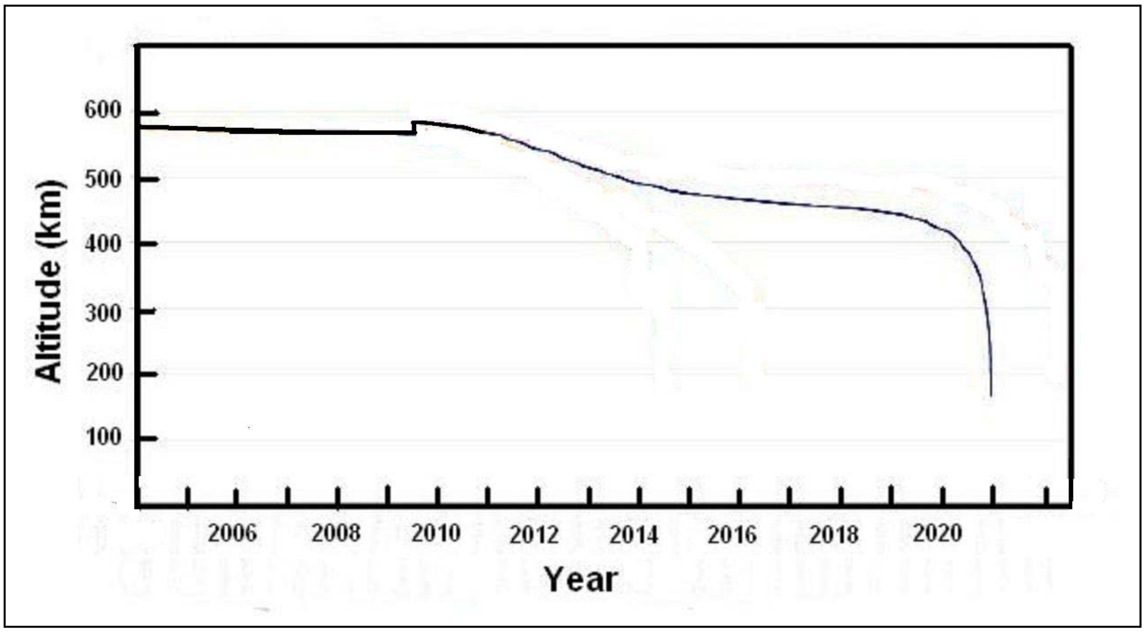


Figure 1. Five HST impact sites photographed with 400 mm lens.

Satellite Drag and the Hubble Space Telescope

The Hubble Space Telescope was never designed to operate forever. What to do with the observatory remains a challenge for NASA once its scientific mission is completed in 2012. Originally, a Space Shuttle was proposed to safely return it to Earth, where it would be given to the National Air and Space Museum in Washington DC. Unfortunately, after the last Servicing Mission, STS-125, in May, 2009, no further Shuttle visits are planned. As solar activity increases, the upper atmosphere heats up and expands, causing greater friction for low-orbiting satellites like HST, and a more rapid re-entry.

The curve below shows the predicted altitude for that last planned re-boost in 2009 of 18-km. NASA plans to use a robotic spacecraft after ca 2015 to allow a controlled re-entry for HST, but if that were not the case, it would re-enter the atmosphere sometime after 2020.



Problem 1 – The last Servicing Mission in 2009 will only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Problem 1 – The last Servicing Mission in 2009 will only extend the science operations by another 5 years. How long after that time will the HST remain in orbit?

Answer: The Servicing Mission will occur in 2009. The upgrades an gyro repairs will extend the satellite's operations by 5 more years, so if it re-enters after 2020 it will have about 6 years to go before uncontrolled re-entry.

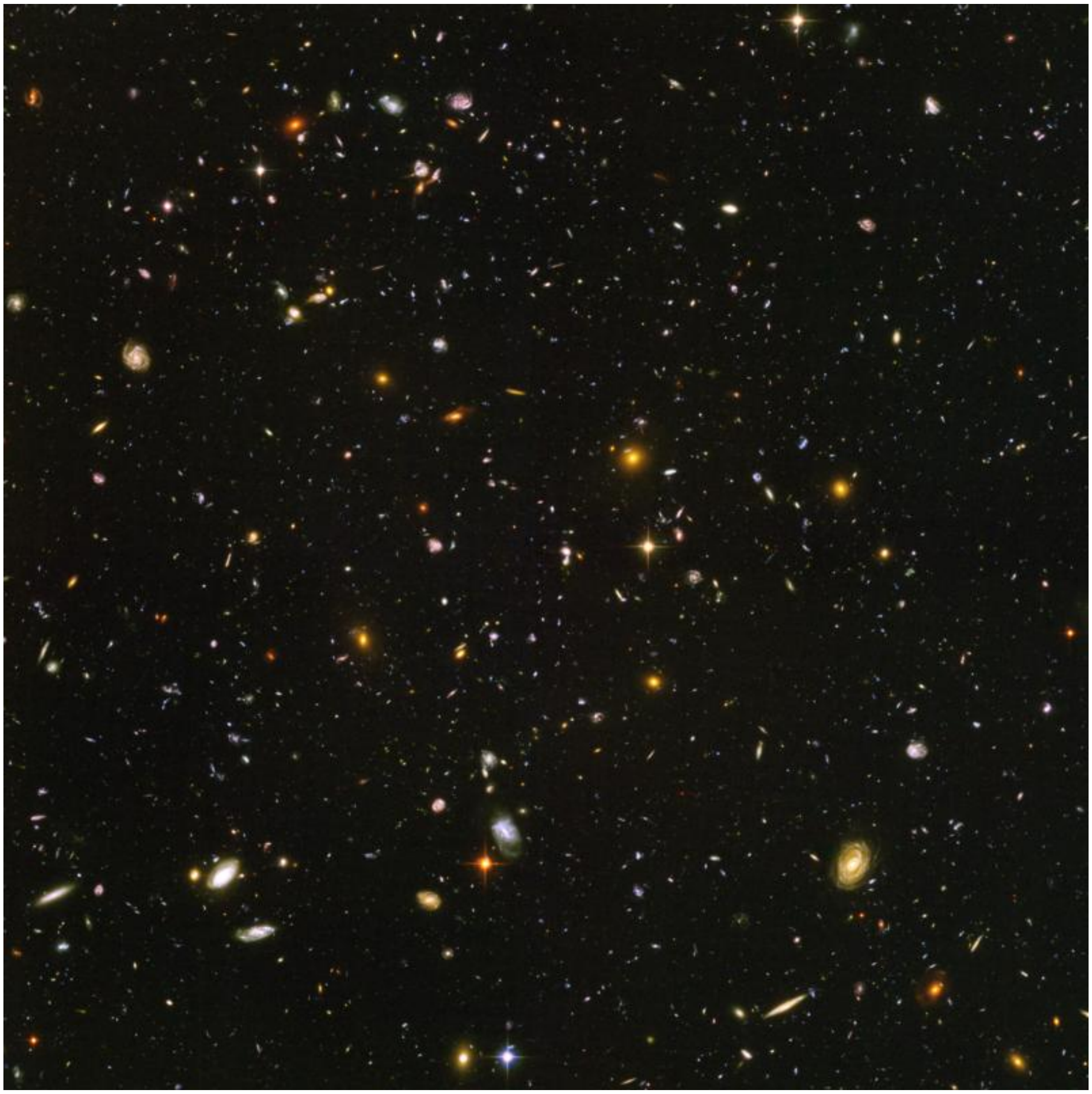
Problem 2 – Once HST reaches an altitude of 400 km, with no re-boosts, about how many weeks will remain before the satellite burns up? (Hint: Use a millimeter ruler.)

Answer: Use a millimeter ruler to determine the scale of the horizontal axis in weeks per millimeter. Mark the point on the curve that corresponds to a vertical value of 400 km. Draw a line to the horizontal axis and measure its distance from 2013 in millimeters. Convert this to weeks using the scale factor you calculated. The answer should be about 50 weeks.

"HST science lifetime could potentially be limited by HST spacecraft orbital decay. Long-term orbit decay predictions are developed based on atmospheric models and solar flux predictions. All contributing combinations of solar flux strength and timing are run in order to bound the orbit decay predictions from a best case atmosphere to a worst case ("unkind") atmosphere. The predictions also consider the effects of Space Shuttle re-boost during HST Servicing Missions. The figure shows the model results for a worst case, 2-sigma high solar cycle (Cycle 24), followed by an early Cycle 25 of average intensity. Figure 3 depicts four curves for various shuttle re-boost scenarios. For the case of no further HST re-boost in any future servicing mission, the prediction is that HST will reenter the Earth's atmosphere in late 2013 or early 2014. The HST science program will cease approximately one year prior to re-entry due to loss of the precise attitude control capability required for science observing, as the atmospheric drag increases. The earliest expected end of the HST science program due to orbital decay is thus late 2012. Further information about this topic is contained in the accompanying Hubble Fact Sheet, entitled "HST Orbit Decay and Shuttle Re-boost." [From 'Expected HST Science Lifetime after SM4", HST Program Office; July 21, 2003]

Counting Galaxies with the Hubble Space Telescope

In 2004, the Hubble Space Telescope took a million-second exposure of a small part of the sky to detect as many galaxies as possible. Here's what they saw!



Problem 1 – Divide the field into 16 equal areas. Label the grids alphabetically from A to P starting from the top left cell. Count the number of 'smudges' in four randomly selected cells. What is the average number of galaxies in of one of the cells in the picture? What uncertainties can you identify in counting these galaxies?

Problem 2 – One square degree equals 3,600 square arcminutes. If Hubble Ultra Deep Field picture is 3 arcminutes on a side, what is the area of one of your cells in square degrees? (Note: 1 arcminute equals 1/60 of a degree of angle measure).

Problem 3 – There are 41,250 square degrees in the sky, about how many galaxies are in the full sky as faint as the faintest galaxy that Hubble detected in the Deep Field image?

Answer Key

Problem 1 – Divide the field into 16 equal areas. Count the number of 'smudges' in four randomly selected cells. What is the average number of galaxies in an area of the sky equal to the size of one of the cells in the picture? What uncertainties can you identify in counting these galaxies?

Answer: Students counting will vary depending on how many 'spots' they can easily see. There should be vigorous discussions about which smudges are galaxies and which ones are photocopying artifacts. Note that, depending on the quality of the laser printer used, the number of galaxies will vary considerably.

The full image is about 145 millimeters wide, so the cells should measure about 145/4 = 36 millimeters wide. For a typical photocopy quality, here are some typical counts in 4 cells:
D=160, F=170; K=112; M=164. The average is (160+170+112+164)/4 = 151 galaxies/cell. Students estimates may vary depending on the number of galaxies they could discern in the photocopy of the Hubble Ultra-Deep Field image. A reasonable range is from 50 – 200 galaxies per cell on average.

Problem 2 – One square degree equals 3,600 square arcminutes. If the Hubble Ultra Deep Field picture is 3 arcminutes on a side, what is the area of one of your cells in square degrees?

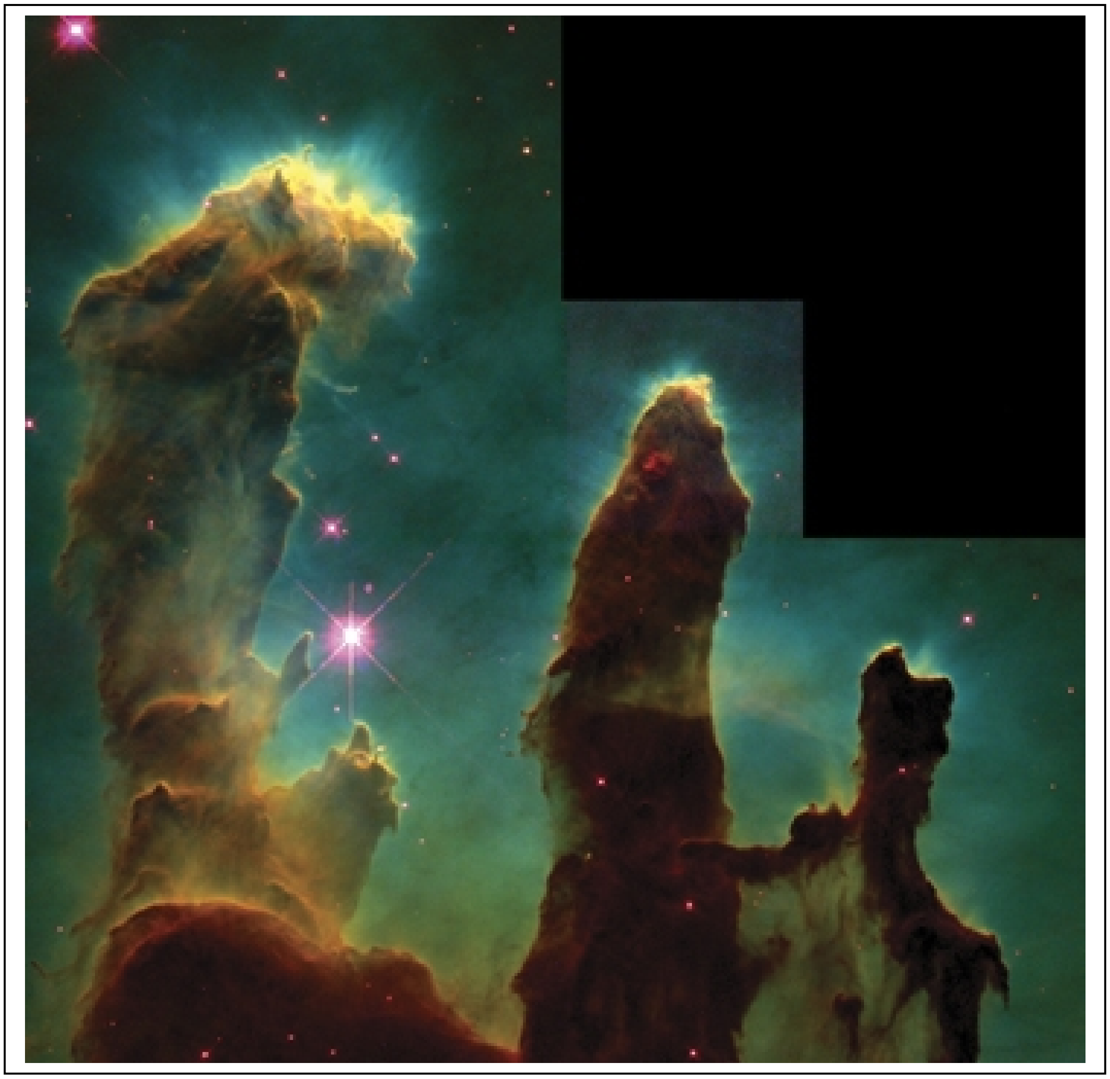
Answer: The field is 3 arcminutes wide, so one cell is 3 / 4 = 0.75 arcminutes on a side. Since there are 60 arcminutes in one degree, this equals 0.75 x 1/60 = 0.0125 degrees. The area of the cell is 0.0125 x 0.0125 = 0.000156 square degrees.

Problem 3 – There are 41,250 square degrees in the sky, about how many galaxies are in the full sky as faint as the faintest galaxy that Hubble detected in the Deep Field image?

Answer: The number of these cells in the full sky is 41,250 square degrees/0.000156 square degrees = 264 million. Since there are on average 151 galaxies per cell, the total number of galaxies in the sky is 151 galaxies/cell x 264 million cells/full sky = 39.4 billion galaxies.

Note, astronomers using the original image data counted an average of 625 galaxies in each cell, for an estimated total of 165 billion galaxies in the full sky.

Eagle Nebula - Closeup



The Hubble Space Telescope took the image of the Eagle Nebula (M-16). This star-forming region is in the constellation Serpens, and located 6,500 light years from Earth. It is only about 6 million years old. The dense clouds of interstellar gas are still collapsing to form new stars. This image is 2.5 arcminutes across. (Note: There are 60 arcminutes in one degree)

Problem 1 – If an angular size of 200 arcseconds corresponds to 1 light year at a distance of 1000 light years. What is the size of this field at the distance of the nebula? (Note: There are 60 arcseconds in one arcminjute.)

Problem 2 – What is the scale of this image in light years/millimeter?

Problem 3 – Our Solar System is about 1/400 of a light year across. How big is it, in millimeters, at the scale of this photo?

Problem 4 - How many times the size of our solar system is the smallest nebula feature you can see in the photo?

Answer Key

Problem 1 – If an angular size of 200 arcseconds corresponds to 1 light years at a distance of 1000 light years. What is the size of this field at the distance of the nebula?

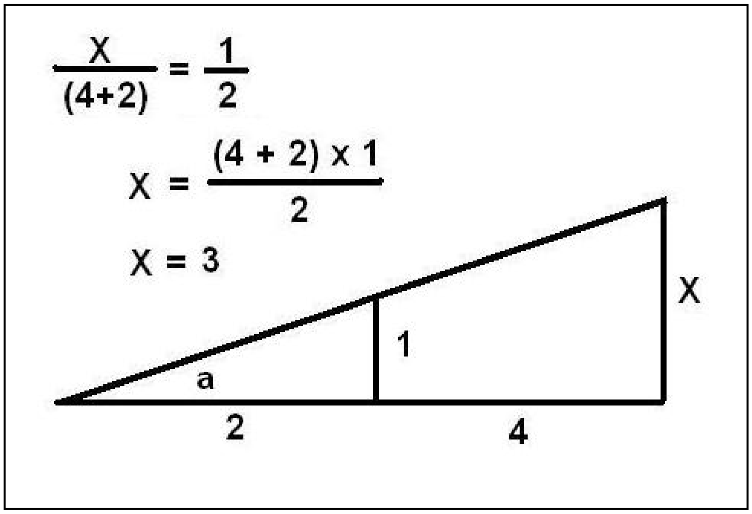
Answer: First we have to find out the scale of the image at the distance of the nebula. The field is stated to be 2.5 arcminutes across. At the distance of the nebula, 6,500 light years, the scale would be 200 arcseconds = 6,500/1000 x 1 light year = 6.5 light years.

Since 1 arcminute = 60 arcseconds, by converting units we have 2.5 arcminutes x (60 arcseconds/arcminutes) x (6.5 ly/200 arcseconds) = 4.9 light years. The field is 4.9 x 4.9 light years in size.

Problem 2 – What is the scale of this image in light years/millimeter? Answer: The Hubble image is 140 millimeters across. Since this equals 4.9 light years, the scale is 4.9 ly/140mm = 0.035 light years/millimeter.

Problem 3 – Our Solar System is about 1/400 of a light year across. How big is it, in millimeters, at the scale of this photo? Answer: At the scale of the photo ,1/400 of a light years = 0.0025 light years = 0.0025/.035 = **0.07 millimeters**. This is about the same size as the 'period' at the end of this sentence .

Problem 4 - How many times the size of our solar system is the smallest nebula feature you can see in the photo? Answer: Some features are about 0.2 millimeters in size, which equals 0.2 mm/0.07 = **2.8 times the diameter of our solar system!**

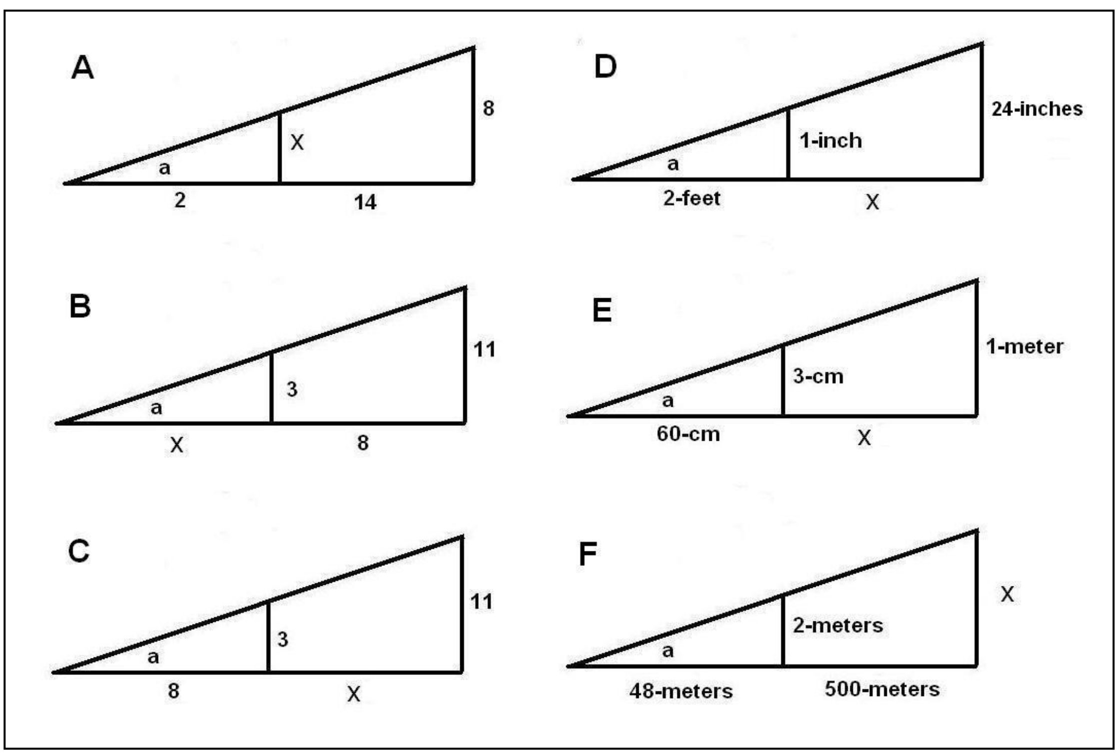


The corresponding sides of similar triangles are proportional to one another as the illustration to the left shows. Because the vertex angle of the triangles are identical in measure, two objects at different distances from the vertex will subtend the same angle, **a** . The corresponding side to 'X' is '1' and the corresponding side to '2' is the combined length of '2+4'.

Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

Problem 2: Which triangles must have the same measure for the indicated angle **a** ?

Problem 3: The sun is 400 times the diameter of the moon. Explain why they appear to have about the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?



Answer Key

Problem 1: Use the properties of similar triangles and the ratios of their sides to solve for 'X' in each of the diagrams below.

A) X / 2 = 8 / 16 so **X = 1**

B) 3 / X = 11 / (X+8) so 3 (X + 8) = 11 X ; 3X + 24 = 11 X ; 24 = 8X and so **X = 3.**

C) 3 / 8 = 11 / (x + 8) so 3 (x + 8) = 88 ; 3X + 24 = 88 ; 3X = 64 and so **X = 21 1/3**

D) 1-inch / 2-feet = 24 inches / (D + 2 feet); First convert all units to inches;
1 / 24 = 24 / (D + 24); then solve (D + 24) = 24x24 so D = 576 - 24;
D = 552 inches or 46 feet.

E) 3 cm / 60 cm = 1 meter / (X + 60 cm). 3/60 = 1 meter / (X + 0.6 m) then
3(X + 0.60) = 60 ; 3X + 1.8 = 60 ; 3X = 58.2 meters so **X = 19.4 meters.**

F) 2 meters / 48 meters = X / 548 meters ; 1/24 = X/548 ; X = 548 / 24 ; so **X = 22.8.**

Problem 2: Which triangles must have the same measure for the indicated angle **a** ?

Answer: Because the triangle (D) has the side proportion 1-inch /24-inches = 1/24 and triangle (F) has the side proportion 2 meters / 48 meters = 1/24 these two triangles, **D and F, have the same angle measurement for angle a**

Problem 3: The Sun is 400 times the diameter of the Moon. Explain why they appear to have the same angular size if the moon is at a distance of 384,000 kilometers, and the sun is 150 million kilometers from Earth?

Answer: From one of our similar triangles, the long vertical side would represent the diameter of the sun; the short vertical side would represent the diameter of the moon; the angle **a** is the same for both the sun and moon if the distance to the sun from Earth were 400x farther than the distance of the moon from Earth. Since the lunar distance is 384,000 kilometers, the sun must be at a distance of 154 million kilometers, which is close to the number given.

ISS - Orbit Altitude Changes

The International Space Station is a 400-ton, $160 billion platform that supports an international team of 3-5 astronauts for tours of duty lasting up to 6 months at a time. Like all satellites that orbit close to Earth, the atmosphere causes the ISS orbit to decay steadily every day, so the ISS has to be 're-boosted' every few months to prevent it from burning up in the atmosphere.

Problem 1 - Based on the following information, what is the altitude of the ISS by April 2009?

"In January, the altitude was 340 kilometers. By March it has lost 8 kilometers before the Progress-59 supply ship raised its altitude by 5 kilometers. In May, the ISS lost 4 1/2 kilometers and was re-boosted by the Progess-60 supply ship by 5 1/2 kilometers. Again the ISS continued to lose altitude by 5 1/2 kilometers by July when the Progress-61 supply ship raised its orbit by 9 1/2 kilometers. The ISS altitude then fell by 3 kilometers by October when the Soyuz TMA-11 mission re-boosted the station by 5 kilometers. The ISS continued to lose altitude until late December, 2007 when it had lost a total of 8 1/2 kilometers since its last re-boost by Soyuz. Since December 2007, the total of all the declines and re-boosts added up to a net change of + 11 1/2 kilometers by April 2009. "

Answer Key

Problem 1 - Based on the following information, what is the altitude of the ISS by April 2009?

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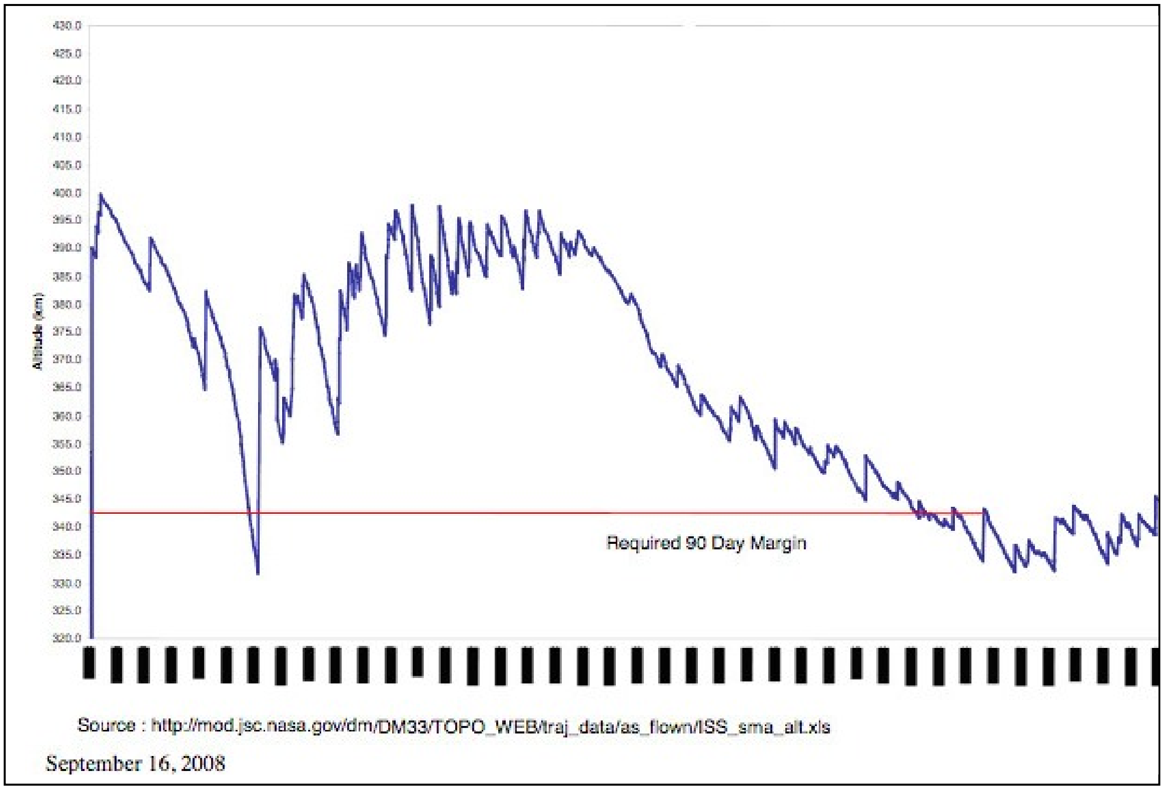
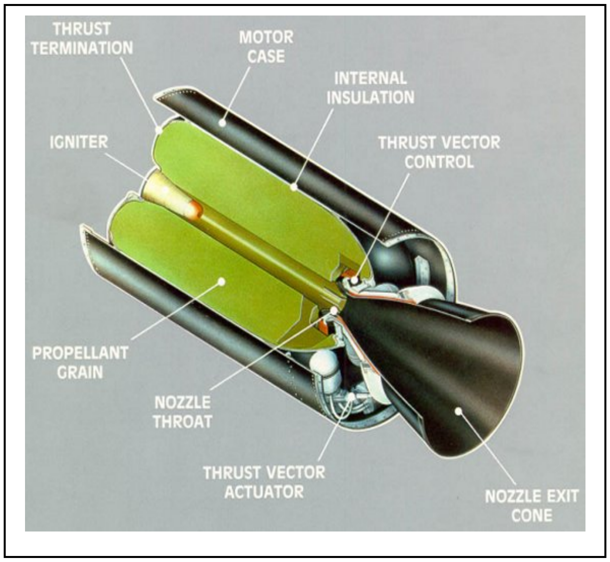
Answer: 340 kilometers - 8 kilometers + 5 kilometers - 4 1/2 kilometers + 5 1/2 kilometers - 5 1/2 kilometers + 9 1/2 kilometers - 3 kilometers + 5 kilometers - 8 1/2 kilometers + 11 1/2 kilometers = 347 kilometers.

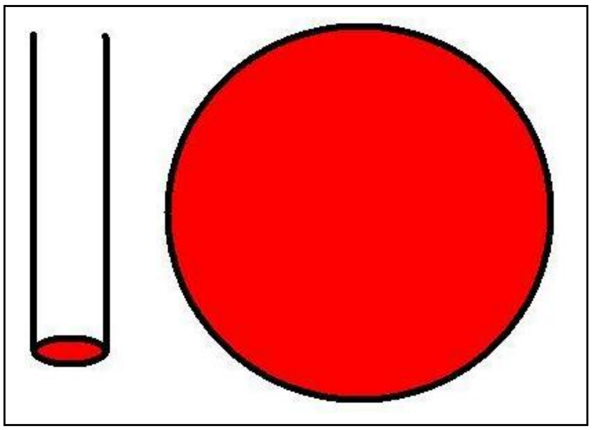
Note to Teacher: The figure below shows the altitude changes between November 1998 and July 2008.



Solid Rocket Boosters



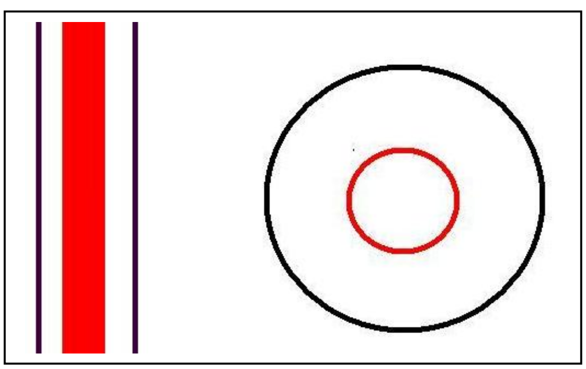
The two solid rocket boosters (SRBs) for the Ares-V launch vehicle will each generate a total thrust of 3.8 million pounds (16.9 megaNewtons). The SRBs from tip to ground are 193 feet long and have a diameter of 12.2 feet. The actual fuel occupies a cylindrical volume about 180 feet (60 meters) long and 11.5 feet (3.7 meters) in diameter. They will 'burn' for a total of 126 seconds. You might think that the fuel burns from the bottom of the cylinder to the top, the way that a candle consumes its wax. The fuel in an SRB is actually designed to burn from the central axis of the cylinder to the outside casing! Let's see why this is a much better way to launch a rocket!



Problem 1 - Thrust is created by burning the exposed surface area of the fuel in the SRB. To launch the 3.4 million pound Ares-V rocket, they have a combined burn rate of 8,740 kg of fuel each second. The density of the fuel is 1770 kg/m^3. If the exposed fuel area is just the circular cross section of the cylinder (see red area in the figure to left), and the burn depth is 0.025 meters each second;

A) What is the total burn rate?

B) Is this enough to launch Ares-V?



Problem 2 - Suppose, instead, that a cylindrical core (red circle) with a diameter of 0.6 meters is cut out along the axis of the booster from top to bottom. The figure to the left shows the red areas where the fuel is burning.

A) What is the surface area of the exposed fuel in the core region?

B) If the burn depth is 0.025 meters each second, what is the mass rate in kg/sec?

C) Is this enough to launch Ares-V?

Answer Key

Problem 1 - Thrust is created by burning the exposed surface area of the fuel in the SRB. To launch the 3.4 million pound Ares-V rocket, they have to have a combined burn rate of 8,740 kg of fuel each second. The density of the fuel is 1770 kg/m^3. If the exposed fuel area is just the circular cross section of the cylinder (see red area in the figure to left), and the burn depth is 0.025 meters each second,

A) What is the total burn rate? Answer: A) Area = 3.14 x $(3.7m/2)^2$ = 10.7 m^2. Volume change of disk = 10.7 m^2 x 0.025 m/sec = 0.27 m^3/sec. Mass = 1770 x 0.27 = 480 kg/sec to two significant figures. For both SRBs this yields a total of **960 kg/sec**.

B) Is this enough to launch Ares-V? Answer: **This is not a fast enough** fuel burn rate to ensure the proper thrust, so an SRB cannot provide enough thrust by burning only the fuel exposed by the cylindrical cross-section at the tail-end of the booster.

Problem 2 - Suppose, instead, that a cylindrical core with a diameter of 0.6 meters is cut out along the axis of the booster from top to bottom.

A) What is the surface area of the exposed fuel in the core region? Answer: Burn Area = $2\pi Rh$ = 2 x 3.14 x (0.6 meters/2) x 60 meters = **113 m^2**.

B) If the burn rate is 0.025 meters each second, what is the mass rate in kg/sec? Answer: 113 m^2 x 0.025 m/sec x 1770 kg/m^3 = 5,000 kg/sec. For both SRBs this equals **10,000 kg/sec.**

C) Ares-V needs 8,740 kg/sec to launch so the 10,000 kg/sec produced by burning along the cylindrical core surface **provides more than enough thrust!**

Note to Teacher: Problem 2 used the 'tubular' method. Sketch this cross section on the board and ask your students to predict what the thrust profile will look like as the fuel burns outward from the core to the outer layer of the cylinder. Repeat this exercise for other kinds of fuel shapes and cross-sections.

Evaluating Secondary Physical Constants

Symbol	Name	Value
c	Speed of light	2.9979×10^{10} cm/sec
h	Planck's constant	6.6262×10^{-27} erg sec
m	Electron mass	9.1095×10^{-28} gms
e	Electron charge	4.80325×10^{-10} esu
G	Gravitation constant	6.6732×10^{-8} dyn cm^{2} gm^{-2}
M	Proton mass	1.6726×10^{-24} gms

Also use $\pi = 3.1415926$

Although there are only a dozen fundamental physical constants of Nature, they can be combined to define many additional basic constants in physics, chemistry and astronomy.

In this exercise, you will evaluate a few of these 'secondary' constants to three significant figure accuracy using a calculator and the defined values in the table.

Problem 1 - Bremstrahlung Radiation Constant: $\frac{32\pi^2 e^6}{3(2\pi)^{1/2} m^3 c}$

Problem 2 - Photoionization Constant: $\frac{32\pi^2 e^6 (2\pi^2 e^4 m)}{3^{3/2} h^3}$

Problem 3 - Stark Line Limit: $\frac{16\pi^4 m^2 e^4}{h^4 M^5}$

Problem 4 - Thompson Scattering Cross-section: $\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$

Problem 5 - Gravitational Radiation Constant: $\frac{32}{5}\frac{G^5}{c^{10}}$

Problem 6 - Thomas-Fermi Constant: $\frac{324}{175}\left(\frac{4}{9\pi}\right)^{2/3}$

Problem 7 - Black Hole Entropy Constant: $\frac{c^3}{2hG}$

Answer Key

Method 1: Key-in to a calculator all the constants with their values as given to all indicated significant figures, write down final calculator answer, and round to three significant figures.

Method 2: Round all physical constants to 4 significant figures, key-in these values on the calculator, then round final calculator answer to 3 significant figures.

Note: When you work with numbers in scientific notation, Ex 1.23×10^{5}, the leading number '1.23' has 3 significant figures, but 1.23000 has 6 significant figures if the '000' are actually measured to be '000', otherwise they are just non-significant placeholders.

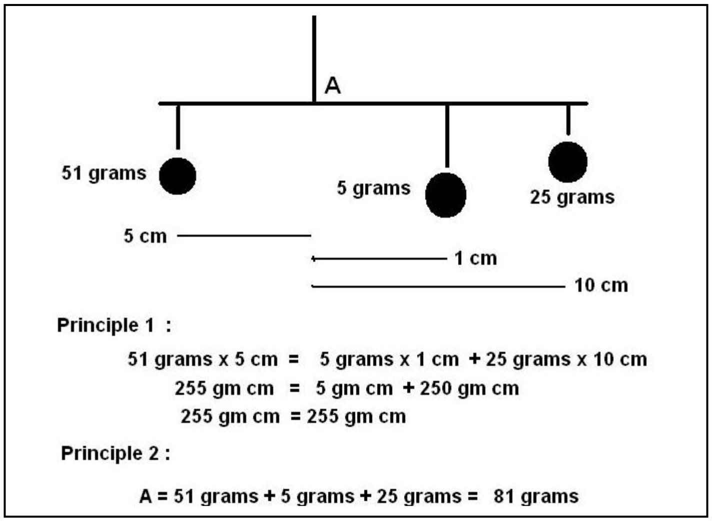
Also, you cannot have a final answer in a calculation that has more significant figures than the smallest significant figure number in the set. For example, 6.25*5.1 which a calculator would render as 31.875 is 'only good' to 2 significant figures (determined from the number 5.1) so the correct, rounded, answer is 32.

Problem	Method 1	Method 2
1	2.28×10^{16}	2.27×10^{16}
2	2.46×10^{-39}	2.46×10^{-39}
3	2.73×10^{135}	2.73×10^{135}
4	6.65×10^{-25}	6.64×10^{-25}
5	1.44×10^{-140}	1.44×10^{-140}
6	5.03×10^{-1}	5.03×10^{-1}
7	3.05×10^{64}	3.05×10^{64}

Note Problem 1 and 4 give slightly different results.

Problem 1: Method 1 answer 3.8784/1.7042 = 2.27578 or 2.28
Method 2 answer 3.8782/1.7052 = 2.2743 = 2.27

Problem 4: Method 1 answer 1.3378/2.0108 = 0.6653 = 0.665
Method 2 answer 1.3376/2.0140 = 0.6642 = 0.664

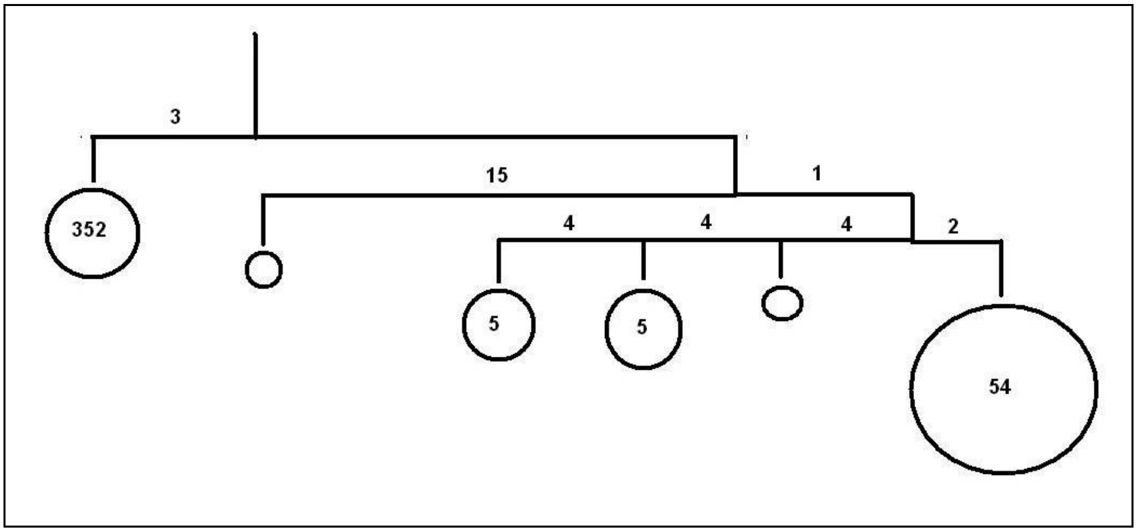


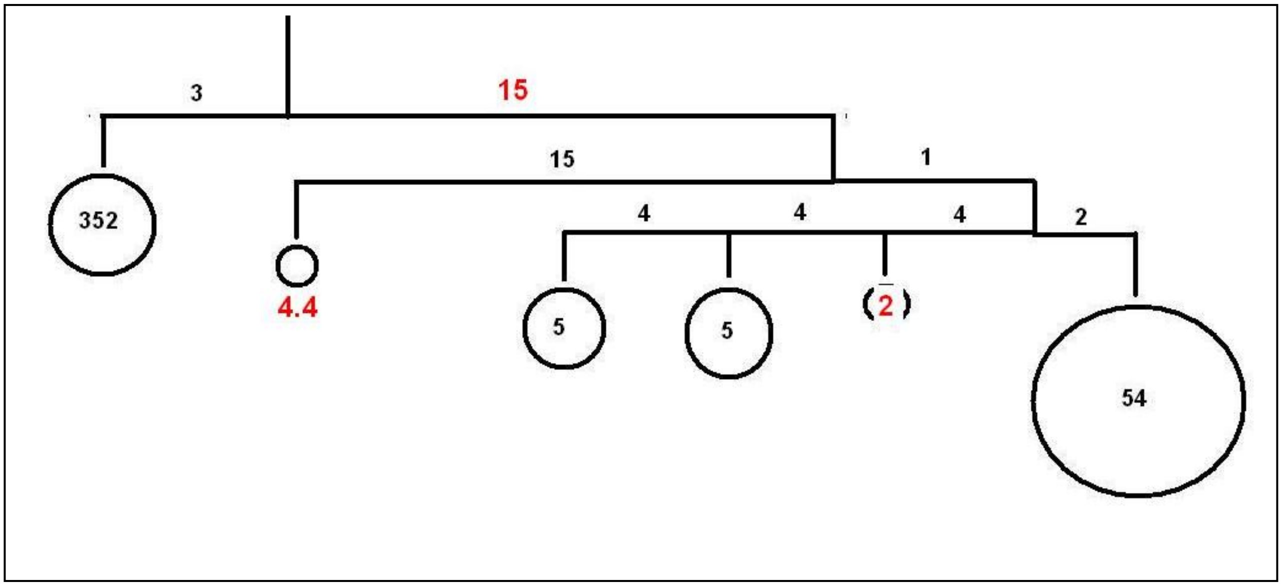
Mobiles are fun to build, but are an example of several important physical principles. The example to the left displays the two basic ones:

1 - The product of the mass x distance from the suspension point of each body must equal zero at the balance point.

2 - The mass at the balance point equals the sum of the masses on the suspended bar.

Using the above two principles, fill-in the missing numbers in the mobile below. The masses of each ball are shown by the numbers inside each circle. The lengths of each cross-bar are indicated by the corresponding numbers. Can you design more elaborate mobiles using these two principles?





Lower Crossbar:
Principle 2:
54 x 2 = M x 4 + 5 x (4 + 4) + 5 x (4 + 4 + 4)
108 = 4m + 100
8 = 4 M
2 = M answer

Middle Crossbar:
Principle 1: Total mass = 54 + 2 + 5 + 5 = 66
Principle 2: 66 x 1 = 15 x M
66/15 = M
4.4 = M answer

Top Crossbar
Principle 1: Total mass = 66 + 4.4 = 70.4

Principle 2: 352 x 3 = L x 70.4
1056/70.4 = L
15 = L answer

Spotting an Approaching Asteroid or Comet

An asteroid, or comet, viewed from Earth will be either bright or faint depending on many quantifiable factors. Of course the size of the body and its reflectivity make a big difference. So does its distance from the sun and earth at the time you see it. The brightness also depends on whether, from Earth, it is fully-illuminated like the full moon, or only partly-illuminated like the crescent moon.

Astronomers can put all of these variables together into one single equation which works pretty well to predict a body's brightness just about anywhere inside the solar system!

The streak in the photo above is the asteroid 1999AN10 (Courtesy Palomar Digital Sky Survey). Orbit data suggest that on August 7, 2027 it will pass within 37,000 kilometers of Earth. The formula for the brightness of the asteroid is given by:

$$R = 0.011\, d\; 10^{-\frac{1}{5}(m)}$$

where: R is the asteroid radius in meters, d is the distance to Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The photograph above shows stars as faint as m = +20. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 37,000 kilometers, what is the formula R(m) for the asteroid?

Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses?

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be d = 37,000 kilometers, what is the formula R(m) for the asteroid? Answer: Substitute the given values into the equation and simplify. The formula will give the radius of the asteroid in meters as a function of its apparent brightness (called apparent magnitude by astronomers) given by m.

$R(m) = 0.011\ (37000)10^{-0.2m}$

$\mathbf{R(m) = 407\ 10^{-0.2m}}$

Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses? Answer: Solve the formula for R(m) for m(R) and evaluate for R = 200 meters and R = 1000 meters to obtain the range of the function.

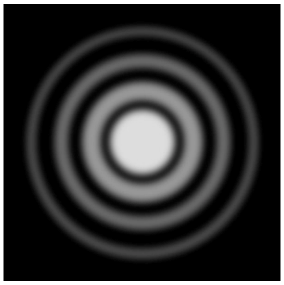
$m(R) = -5 \log_{10}(R/4224)$

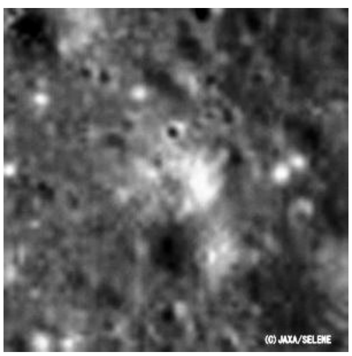
so m(R) for R = 200 yields m(200) = -5 (-11.3) so m(200) = +1.5
and m(1000) = -5(0.39) so m(1000) = -2.0

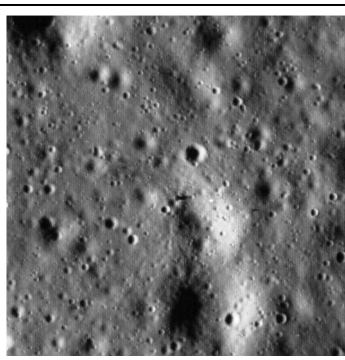
so Domain R: [200,1000]
and Range m: [+1.5, -2.0]

Note: The planet Venus can be as bright as m = -2.5 so this asteroid should be easily visible if it is in this size domain.

$$R = 1.22 \frac{L}{D}$$







There are many equations that astronomers use to describe the physical world, but none is more important and fundamental to the research that we conduct than the one to the left! You cannot design a telescope, or a satellite sensor, without paying attention to the relationship that it describes.

In optics, the best focused spot of light that a perfect lens with a circular aperture can make, limited by the diffraction of light. The diffraction pattern has a bright region in the center called the Airy Disk. The diameter of the Airy Disk is related to the wavelength of the illuminating light, L, and the size of the circular aperture (mirror, lens), given by D. When L and D are expressed in the same units (e.g. centimeters, meters), R will be in units of angular measure called radians (1 radian = 57.3 degrees).

You cannot see details with your eye, with a camera, or with a telescope, that are smaller than the Airy Disk size for your particular optical system. The formula also says that larger telescopes (making D bigger) allow you to see much finer details. For example, compare the top image of the Apollo-15 landing area taken by the Japanese Kaguya Satellite (10 meters/pixel at 100 km orbit elevation: aperture = about 15cm) with the lower image taken by the LRO satellite (1.0 meters/pixel at a 50km orbit elevation: aperture = 0.8 meter). The Apollo-15 Lunar Module (LM) can be seen by its 'horizontal shadow' near the center of the image.

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?

Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer Key

Problem 1 - The Senator Byrd Radio Telescope in Green Bank West Virginia with a dish diameter of D=100 meters is designed to detect radio waves with a wavelength of L= 21-centimeters. What is the angular resolution, R, for this telescope in A) degrees? B) Arc minutes?

Answer: First convert all numbers to centimeters, then use the formula to calculate the resolution in radian units: L = 21 centimeters, D = 100 meters = 10,000 centimeters, then R = 1.22 x 21 cm /10000 cm so R = 0.0026 radians. There are 57.3 degrees to 1 radian, so A) 0.0026 radians x (57.3 degrees/ 1 radian) = **0.14 degrees.** And B) There are 60 arc minutes to 1 degrees, so 0.14 degrees x (60 minutes/1 degrees) = **8.4 arcminutes.**

Problem 2 - The largest, ground-based optical telescope is the D = 10.4-meter Gran Telescopio Canaris. If this telescope operates at optical wavelengths (L = 0.00006 centimeters wavelength), what is the maximum resolution of this telescope in A) microradians? B) milliarcseconds?
Answer: R = 1.22 x (0.00006 cm/10400 cm) = 0.000000069 radians. A) Since 1 microradian = 0.000001 radians, the resolution of this telescope is **0.069 microradians.** B) Since 1 radian = 57.3 degrees, and 1 degree = 3600 arcseconds, the resolution is 0.000000069 radians x (57.3 degrees/radian) x (3600 arcseconds/1 degree) = 0.014 arcseconds. One thousand milliarcsecond = 1 arcseconds, so the resolution is 0.014 arcsecond x (1000 milliarcsecond / arcsecond) = **14 milliarcseconds.**

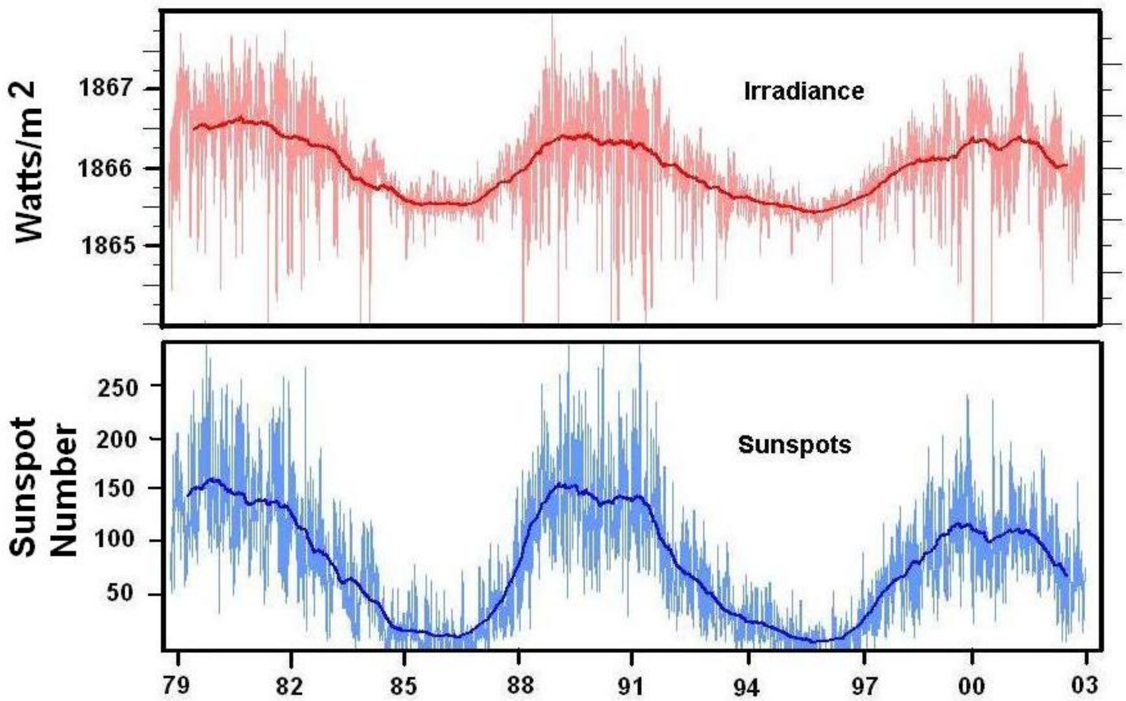
Problem 3 - An astronomer wants to design an infrared telescope with a resolution of 1 arcsecond at a wavelength of 20 micrometers. What would be the diameter of the mirror?

Answer: From R = 1.22 L/D we have R = 1 arcsecond and L = 20 micrometers and need to calculate D, so with algebra we re-write the equation as D = 1.22 L/R.
Convert R to radians:

R = 1 arcsecond x (1 degree/3600 arcsecond) x (1 radian / 57.3 degrees) = 0.0000048 radians.

L = 20 micrometers x (1 meter/1,000,000 micrometers) = 0.00002 meters.

Then D = 1.22 (0.00003 meters)/(0.0000048 radians) = **5.1 meters.**



Irradiance is a measure of the amount of energy in sunlight that is hitting an exposed surface each second, and is commonly expressed as watts per square meter. The figure above is a plot of the solar irradiance and sunspot number since January 1979 according to NOAA's National Geophysical Data Center (NGDC). The thin lines indicate the daily irradiance (red) and sunspot number (blue), while the thick lines indicate the running annual average for these two parameters. The total variation in solar irradiance is about 1.3 watts per square meter during one sunspot cycle. The solar irradiance data obtained by the ACRIM satellite, measures the total number of watts of sunlight that strike Earth's upper atmosphere before being absorbed by the atmosphere and ground.

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003?

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance?

Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989?

Answer Key

Problem 1 - About what is the average value of the solar irradiance between 1978 and 2003? Answer: Draw a horizontal line across the upper graph that is mid-way between the highest and lowest points on the curve. An approximate answer would be **1366.3 watts per square meter.**

Problem 2 - What appears to be the relationship between sunspot number and solar irradiance? Answer: What there are a lot of sunspots on the sun (called sunspot maximum) the amount of solar radiation is higher than when there are fewer sunspots, and the solar irradiance changes follow the 11-year sunspot cycle.

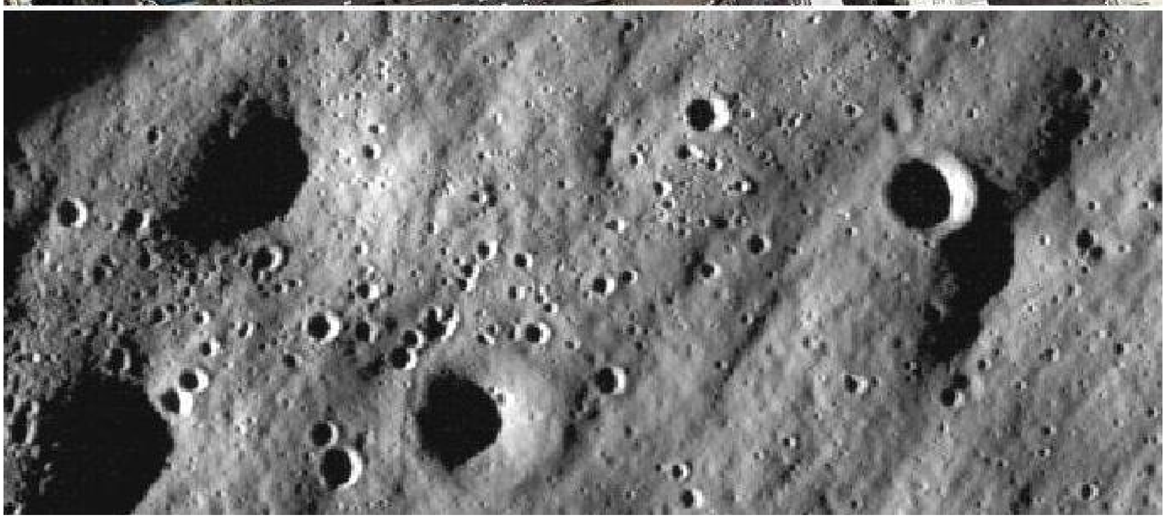
Problem 3 - A homeowner built a solar electricity (photovoltaic) system on his roof in 1985 that produced 3,000 kilowatts-hours of electricity that year. Assuming that the amount of ground-level solar power is similar to the ACRIM measurements, about how much power did his system generate in 1989? Answer: In 1985, the amount of insolation was about 1365.5 watts per square meter when the photovoltaic system was built. In 1989 the insolation increased to about 1366.5 watts per square meter. This insolation change was a factor of 1366.5/1365.5 = 1.0007 .That means that by scaling, if the system was generating 3,000 kilowatt-hours of electricity in 1985, it will have generated 1.0007 x 3,000 kWh = **2 kWh more** in 1989 during sunspot maximum! That is equal to running one 60-watt bulb for about 1 day (actually 33 hours).

Note: Physicists usually distinguish between ‘Irradiance’ and ‘Flux’.

Flux is the quantity of energy or particles flowing through a given area per unit time (example Joules/meter^2/sec or particles/meter^2/sec). It is often called **irradiance** when referring to solar energy reaching Earth’s surface, and is measured in watts/meter^2.

Spectral Irradiance is a measure of the amount of radiant energy (light) of a specific wavelength that is flowing through a given area per unit time, and has units such as Joules/meter^2/sec/nanometer.

Mare Nubium and Las Vegas



The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above 700-meter wide image shows downtown Las Vegas, Nevada (Top - Courtesy of Digital Globe, Inc.), and Mare Nubium (bottom - LRO) at this same resolution.

Problem 1 - About how big, in meters, are the large, medium and small-sized craters in the LRO image?

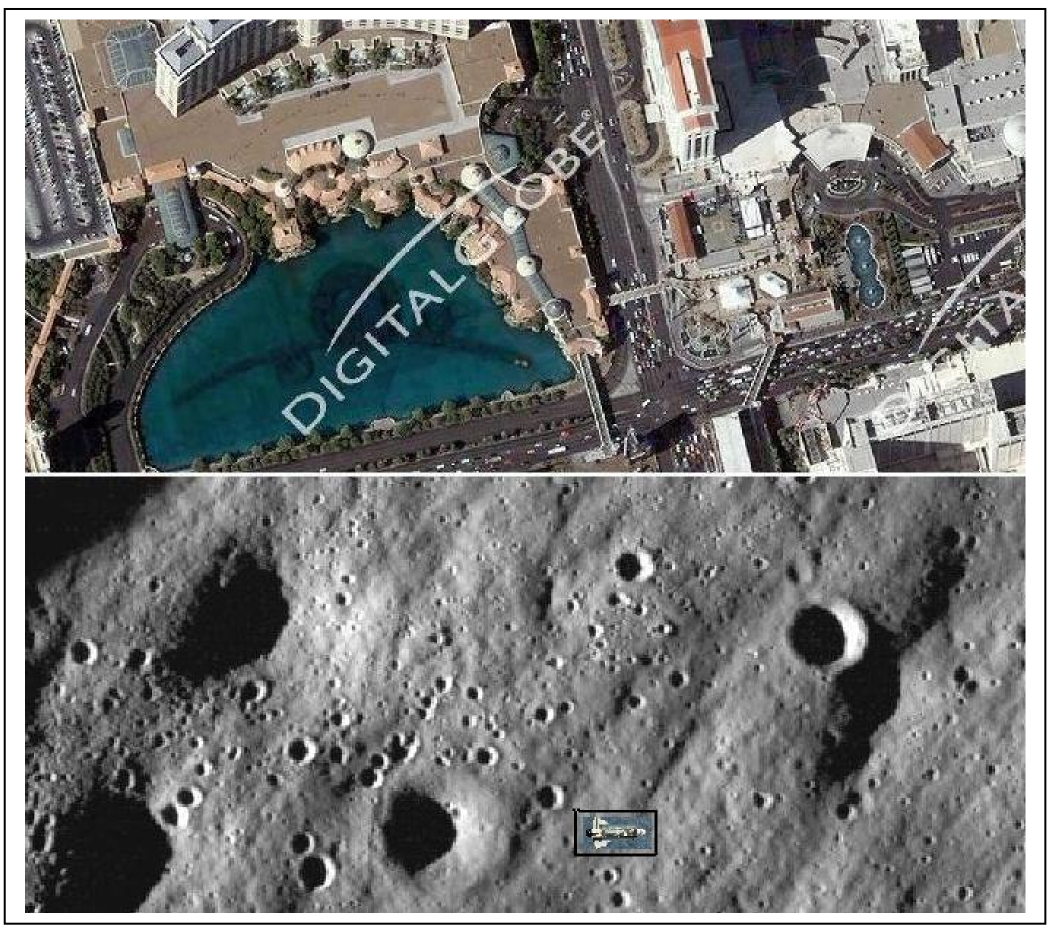
Problem 2 - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood?

Problem 3 - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!).

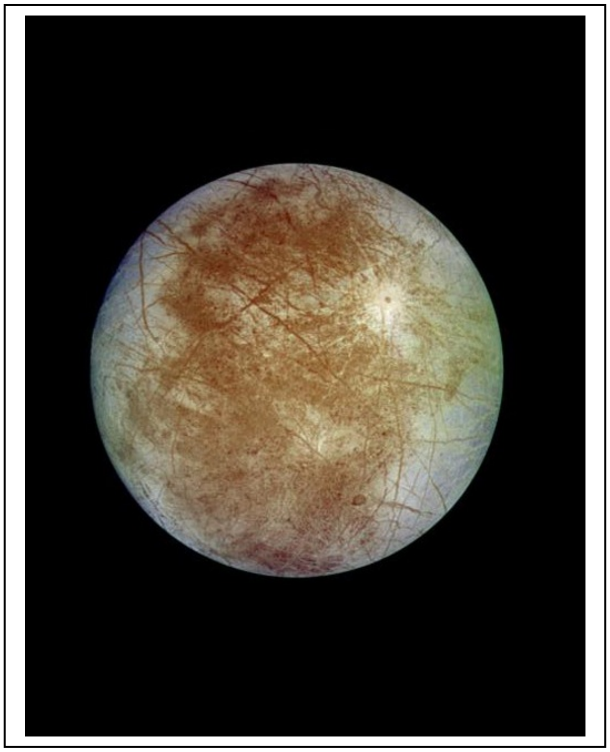
Problem 1 - About how big, in meters, are the large, medium and small-sized craters in the LRO image? Answer: the image is 153 millimeters wide so the scale is 700 meters/153 mm = **4.6 meters/mm.** Small craters are about 4-5 meters across; medium craters are about 10 to15 meters across, and the few large craters are about 30 to 100 meters across.

Problem 2 - How do the large, medium and small-sized craters compare to familiar objects in Downtown Las Vegas, or in your neighborhood? Answer: The small craters are about as wide as your car, mini-van or street. The medium craters are about as wide as large as your house. The big craters are as big as your entire yard or a large Boulevard.

Problem 3 - The Space Shuttle measures 37 meters long and has a wingspan of 24 meters. Draw a sketch of the Shuttle in the LRO image. Would you be able to see the Space Shuttle on the moon's surface at this resolution scale? (Note that the Space Shuttle is not equipped to travel to the moon and land!). Answer: **The shuttle would be 37 meters/4.6 M/mm = 8 millimeters long by 24/4.6 = 5.2 millimeters wide**. The figure below shows the Shuttle to the same scale as the LRO image.



Ice or Water?



There are no known terrestrial organisms that can exist at a temperature lower than the freezing temperature of water. It is also believed that liquid water is a crucial ingredient to the chemistry that leads to the origin of life. To change water-ice to liquid water requires energy.

First, you need energy to raise the ice from wherever temperature it is, to 0 Celsius. This is called the Specific Heat and is 2.04 kiloJoules/kilogram C

Then you need enough energy added to the ice near 0 C to actually melt the ice by increasing the kinetic energy of the water molecules so that their hydrogen bonds weaken, and the water stops acting like a solid. This is called the Latent Heat of Fusion and is 333 kiloJoules/kilogram.

Let's see how this works!

Example 1: You have a 3 kilogram block of ice at a temperature of -20 C. The energy needed to raise it by 20 C to a new temperature of 0 C is Eh = 2.04 kiloJoules/kg C) x 3 kilograms x (20 C) = 2.04 x 3 x 20 = 122 kiloJoules.

Example 2: You have a 3 kilogram block of ice at 0 C and you want to melt it completely into liquid water. This requires Em = 333 kiloJoules/kg x 3 kilograms = 999 kiloJoules.

Example 3: The total energy needed to melt a 3 kilogram block of ice from -20 C to 0C is E = Eh + Em = 122 kiloJoules + 999 kiloJoules = 1,121 kiloJoules.

Problem 1 - On the surface of the satellite Europa (see NASA's Galileo photo above), the temperature of ice is -220 C. What total energy in kiloJoules is required to melt a 100 kilogram block of water ice on its surface? (Note: Calculate Eh and Em separately then combine them to get the total energy.)

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8×10^{16} kilograms. How many Joules would be required to melt the entire surface of Europa to this depth? (Note: Calculate Eh and Em separately then combine them to get the total energy. Then convert kiloJoules to Joules)

Problem 3 - The sun produces 4.0×10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the sun's energy could be used? (Note: The numbers are BIG, but don't panic!)

Problem 1 - On the surface of the satellite Europa, the temperature of ice is -220 C. What total energy is required to melt a 100 kilogram block of water ice on its surface?

Answer: You have to raise the temperature by 220 C, then

E = 2.04 x 220 x 100 + 333 x 100

= 44,880 + 33,300

= **78,180 kiloJoules.**

Problem 2 - To a depth of 1 meter, the total mass of ice on the surface of Europa is 2.8×10^{16} kilograms. How many joules would be required to melt the entire surface of Europa to this depth?

Note: The radius of Europa is 1,565 km. The surface area is $4 \times \pi \times (1{,}565{,}000\ m)^2 = 3.1 \times 10^{13}$ meters^2. A 1 meter thick shell at this radius has a volume of 3.1×10^{13} meters^2 x 1 meter = 3.1×10^{13} meters^3. The density of water ice is 917 kilograms/m^3 ,so this ice layer on Europa has a mass of $3.1 \times 10^{13} \times 917 = 2.8 \times 10^{16}$ kilograms.

Energy = $(2.04 \times 220 + 333) \times 2.8 \times 10^{16}$ kg

= 2.2×10^{19} kiloJoules

= **2.2×10^{22} Joules.**

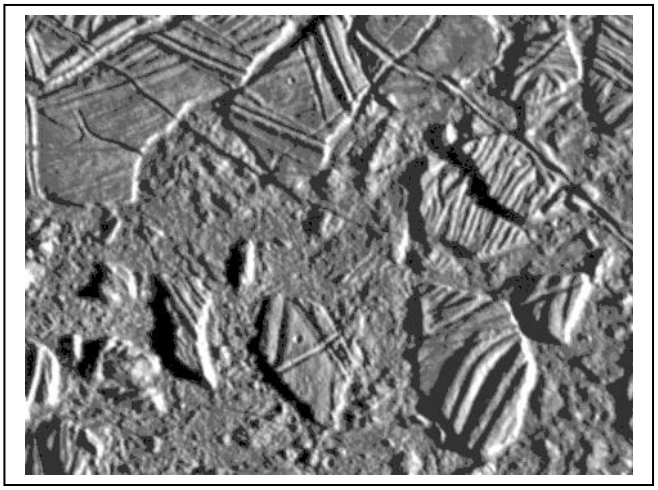
Problem 3 - The sun produces 4.0×10^{26} Joules every second of heat energy. How long would it take to melt Europa to a depth of 1 meter if all of the sun's energy could be used?

Answer: Time = Amount / Rate

= 2.2×10^{22} Joules / 4.0×10^{26} Joules

= **0.000055 seconds.**

Water on Planetary Surfaces



Space is very cold! Without a source of energy, like a nearby star, water will exist at a temperature at nearly -270 C below zero and frozen solid. To create a permanent body of liquid water in which pre-biotic chemistry can occur, a steady source of energy must flow into the ice to keep it melted and in liquid form. Common sources of energy on Earth are volcanic activity, oceanic vents and fumaroles, and sunlight.

The picture above was taken by NASA's Galileo spacecraft of the surface of Jupiter's moon Europa. Its icy crust is believed to hide a liquid-water ocean beneath. The energy for keeping the water in a liquid state is probably generated by the gravity of Jupiter, which distorts Europa's shape through tidal action. The tidal energy may be enough to keep the oceans liquid for billions of years.

A common measure of energy flow or usage is the Watt. One Watt equals one Joule of energy emitted or consumed in one second.

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Water ice at 0 C requires 330,000 Joules of energy to become liquid for each kilogram of ice. Suppose the ice absorbed all the energy that fell on it. Ice doesn't really work that way, but let's suppose that it does just to make a simple mathematical model!

Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Answer Key

Problem 1: How much energy, in Joules, does a 100 watt incandescent bulb consume if left on for 1 hour?

Answer: 100 watts is the same as 100 Joules/sec, so if 1 hour = 3600 seconds, the energy consumed is 100 Joules/sec x 3600 seconds = **360,000 Joules.**

Problem 2: A house consumes about 3,000 kilowatts in one hour. How many Joules is this?

Answer: 3,000 kilowatts x (1,000 watts/1 kilowatt) = 3,000,000 watts. Since this equals 3,000,000 Joules/sec, in 1 hour (3600 seconds) the consumption is 3,000,000 watts x 3,600 seconds = **10,800,000,000 Joules or 10.8 billion Joules.**

Problem 3: A homeowner has a solar panel system that produces 3,600,000 Joules every hour. How many watts of electrical appliances can be run by this system?

Answer: 3,600,000 Joules in 1 hour is an average rate of 3,600,000 Joules/3,600 seconds = 1,000 Joules/sec or **1,000 Watts**. This is the maximum rate at which the appliances can consume before exceeding the capacity of the solar system.

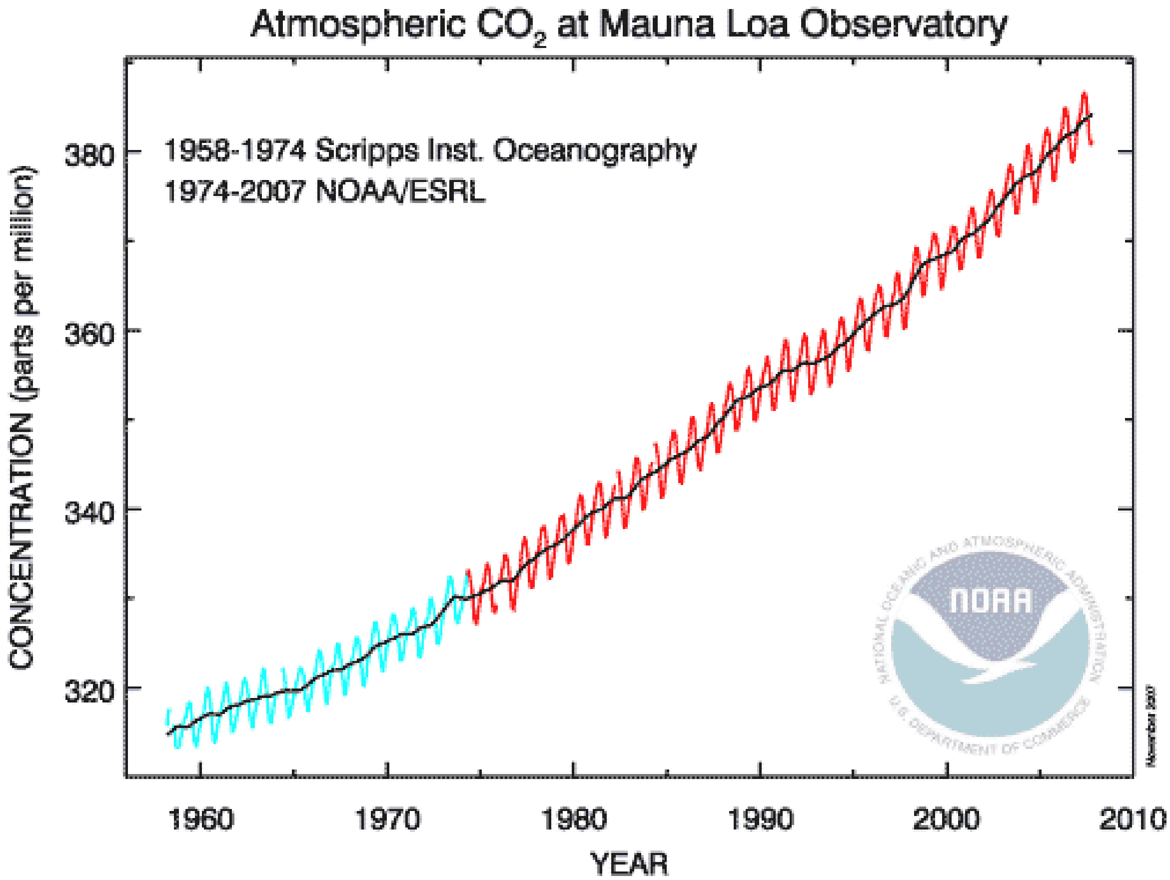
Problem 4: A student wants to melt a 10 kilogram block of ice with a 2,000-watt hair dryer. How many seconds will it take to melt the ice block completely? How many minutes?

Answer: From the information provided, it takes 330,000 Joules to melt 1 kilogram of ice. So since the mass of the ice block is 10 kilograms, it will take 330,000 x 10 = **3,300,000 Joules**

If the ice stores the energy falling on it from the hair dryer, all we need to do is to calculate how long a 2,000-watt hair dryer needs to be run in order to equal 3,300,000 Joules. This will be Time = 3,300,000 Joules/2000 watts = **1650 seconds or about 27.5 minutes.**

Proving that the unit will automatically be in seconds is a good exercise in unit conversions and using the associative law and reciprocals:

1 Joule / (1 watt)	= 1 Joule / (1 Joule/sec)	
	= 1 Joule x (1 sec/1 joule)	: Multiply be the reciprocal
	= (1 Joule x 1 sec)/ 1 joule	: Re-write
	= 1 sec x (1 joule / 1 joule)	: Re-group common terms
	= 1 sec.	:after cancling the 'joules' unit.



This is the Keeling Curve, derived by researchers at the Mauna Kea observatory from atmospheric carbon dioxide measurements made between 1958 - 2005. The accompanying data in Excel spreadsheet form for the period between 1982 and 2008 is provided at

http://spacemath.gsfc.nasa.gov/data/KeelingData.xls

Problem 1 - Based on the tabulated data, create a single mathematical model that accounts for, both the periodic seasonal changes, and the long-term trend.

Problem 2 - Convert your function, which describes the carbon dioxide volume concentration in parts per million (ppm), into an equivalent function that predicts the mass of atmospheric carbon dioxide if 383 ppm (by volume) of carbon dioxide corresponds to 3,000 gigatons.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm) and mass for the years: A) 2020? B)2050, C)2100?

Answer Key

Data from: C. D. Keeling, S. C. Piper, R. B. Bacastow, M. Wahlen, T. P. Whorf, M. Heimann, and H. A. Meijer, Exchanges of atmospheric CO_2 and $^{13}CO_2$ with the terrestrial biosphere and oceans from 1978 to 2000. I. Global aspects, SIO Reference Series, No. 01-06, Scripps Institution of Oceanography, San Diego, 88 pages, 2001. Excel data obtained from the Scripps CO2 Program website at http://scrippsco2.ucsd.edu/data/atmospheric_co2.html

Problem 1 - Answer: The general shape of the curve suggests a polynomial function of low-order, whose amplitude is modulated by the addition of a sinusoid. The two simplest functions that satisfy this constraint are a 'quadratic' and a 'cubic'… where 't' is the elapsed time in years since 1982

F1 = A sin (Bt + C) + (Dt2 + Et + F) and F2 = A sin(Bt + C) + (Dt3 + Et2 + Ft + G)

We have to solve for the two sets of constants A, B, C, D , E, F and for A, B, C, D, E, F, G. Using *Excel* and some iterations, as an example, the constants that produce the best fits appear to be: F1: (3.5, 6.24, -0.5, +0.0158, +1.27, 342.0) and F2: (3.5, 6.24, -0.5, +0.0012, -0.031, +1.75, +341.0). Hint: Compute the yearly averages and fit these, then subtract this polynomial from the actual data and fit what is left over (the residual) with a sin function.) The plots of these two fits are virtually identical. We will choose Fppm = F1 as the best candidate model because it is of lowest-order. The comparison with the data is shown in the graph below: red=model, black=monthly data. Students should be encouraged to obtain better fits.

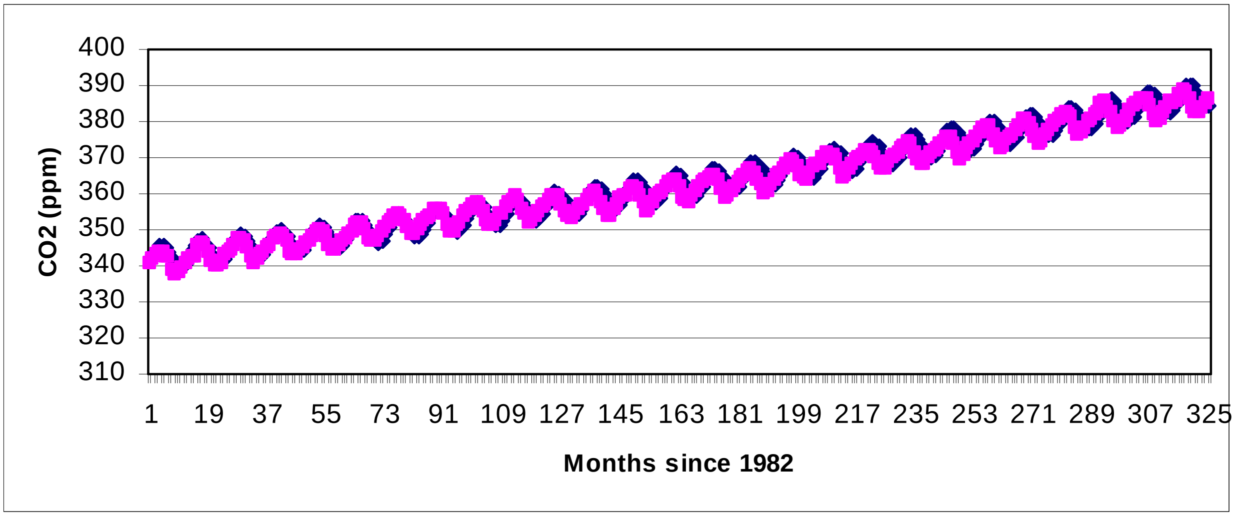
Problem 2 - Answer: The model function gives the atmospheric carbon dioxide in ppm by volume. So take Fppm and multiply it by the conversion factor (3,000/383) = 7.83 gigatons/ppm to get the desired function, Fco2 for the carbon mass.

Problem 3 - What would you predict as the carbon dioxide concentration (ppm), and mass for the years: A) 2020? B)2050, C)2100? Answer:

A) t = 2020-1982 = 38, so Fco2(38) = 7.83 x 410 ppm = **3,200 gigatons**

B) t = 2050-1982 = 68, so Fco2(68) = 7.83 x 502 ppm = **3,900 gigatons**

C) t = 2100-1982 = 118, so Fco2(118) = 7.83 x 718 ppm = **5,600 gigatons**



On July 4, 2005 at 5:45 UT the 362-kilogram Impactor from NASA's Deep Impact mission, collided with the nucleus of the comet Tempel 1, causing a bright flash of light and a plume of ejected gas (see photo).

Traveling at 10.3 km/sec, the Impactor created a crater on the nucleus and ejected about 10,000 tons of material.

The average density of the comet nucleus is 400 kg/m^3 and its size can be approximated as a sphere with a radius of 3 kilometers.

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Problem 2 - If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Problem 3 – How far, in kilometers, will the comet nucleus have drifted 'sideways' to its orbit after 1 million years?

Problem 4 – Suppose that the comet had been headed toward Earth, and it was predicted that in 50 years it would collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5×10^8 kilogram kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Answer: The spherical volume was $V = 4/3\ \pi\ (3000\ \text{meters})^3 = 1.1 \times 10^{11}\ \text{meters}^3$. The density was 400 kg/m^3, so Mass = Density x Volume = $400 \times 1.1 \times 10^{11}$ = **45 trillion kilograms.**

Problem 2 -- If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Answer: A) Vc = mi Vi/Mc = (362 kg) x (10.3 km/sec)/45 trillion kg = **8×10^{-11} kilometers/sec.**

B) 8×10^{-11} km/s x (1000 m/km) x (3600 s/hr) x (24 hr/day) x (365 d/yr) = **2.5 meters/year.**

Problem 3 – How far, in kilometers, will the comet nucleus have drifted sideways to its orbit after 1 million years?

Answer: From Problem 2, the drift is 2.5 meters/year, so after 1 million years the nucleus will have drifted about 2,500,000 meters or **2,500 kilometers.**

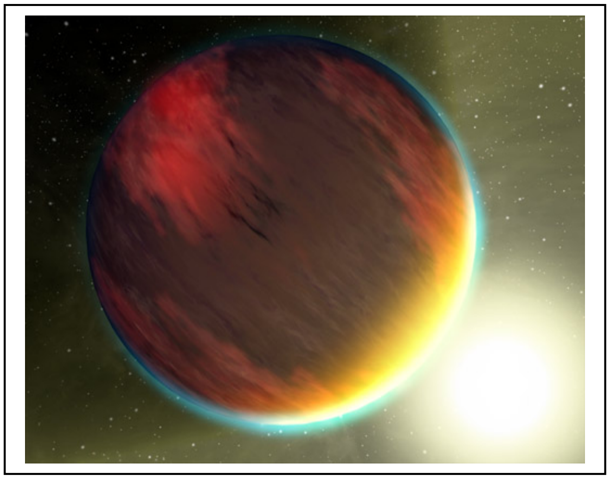
Problem 4 – Suppose that the comet had been headed towards Earth, and it is predicted that in 50 years it will collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5×10^8 kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Answer: Using miVi = mcVc, we get

$$Vc = (7.5 \times 10^8 \text{ kilograms}) \times (10.3\text{km/sec})/45 \text{ trillion kg}$$
$$= 0.00017 \text{ kilometers/sec.}$$

A) In 20 years ($20 \times 3.1 \times 10^7$ seconds) it travels **100,000 kilometers.**
B) **Yes,** since it only needs to travel 12,000 kilometers sideways to avoid hitting Earth, the detonation did help to avoid the collision…assuming the comet wasn't fragmented into a large cloud of debris!

Organic Molecules Detected on Distant Planet!



The basic chemistry for life has been detected in a second hot gas planet, HD 209458b, depicted in this artist's concept. Two of NASA's Great Observatories – the Hubble Space Telescope and Spitzer Space Telescope, yielded spectral observations that revealed molecules of carbon dioxide, methane and water vapor in the planet's atmosphere. HD 209458b, bigger than Jupiter, occupies a tight, 3.5-day orbit around a sun-like star about 150 light years away in the constellation Pegasus. (NASA Press release October 20, 2009)

Some Interesting Facts: The distance of the planet from the star HD209458 is 7 million kilometers, and its orbit period (year) is only 3.5 days long. At this distance, the temperature of the outer atmosphere is about 1,000 C (1,800 F). At these temperatures, water, methane and carbon dioxide are all in gaseous form. It is also known to be losing hydrogen gas at a ferocious rate, which makes the planet resemble a comet! The planet itself has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter. The unofficial name for this planet is Osiris.

Problem 1 - The mass of Jupiter is 1.9×10^{30} grams. The radius of Jupiter is 71,500 kilometers. A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere? B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?

Problem 2 - From the information provided; A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere? B) What is the mass of Osiris in grams? C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock ……………………………….	3.0 grams/cc
Iron ……………………………….	9.0 grams/cc
Water ………………………………	5.0 grams/cc
Ice ………………………………….	1.0 gram/cc
Mixture of hydrogen + helium ……	0.7 grams/cc

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Problem 1 - The mass of Jupiter is 1.9 x 10^{30} grams. The radius of Jupiter is 71,500 kilometers.
A) What is the volume of Jupiter in cubic centimeters, assuming it is a perfect sphere?
Answer: The radius of Jupiter, in centimeters, is
R = 71,500 km x (100,000 cm/1 km)
= 7.15 x 10^{9} cm.

For a sphere, V = 4/3 π R^{3} so the volume of Jupiter is
V = 1.33 x (3.141) x (7.15 x $10^{9})^{3}$
V = 1.53 x 10^{30} cm ^{3}

B) What is the density of Jupiter in grams per cubic centimeter (cc), based on its mass and your calculated volume?
Answer: Density = Mass/Volume so the density of Jupiter is D = (1.9 x 10^{30} grams)/(1.53 x 10^{30} cm ^{3}) = **1.2 gm/cc**

Problem 2 - From the information provided;
A) What is the volume of Osiris in cubic centimeters, if it is in the shape of a perfect sphere?
Answer: The information says that the volume is 146% greater than Jupiter so it will be V = V=1.46 x (1.53 x 10^{30} cm ^{3})
= **2.23 x 10^{30} cm ^{3}**

B) What is the mass of Osiris in grams?
Answer: the information says that it is 69% of Jupiter so
M = 0.69 x (1.9 x 10^{30} grams)
= **1.3 x 10^{30} grams**
C) What is the density of Osiris in grams/cc, and how does this compare to the density of Jupiter?
Answer: D = Mass/volume
= 1.3 x 10^{30} grams / 2.23 x 10^{30} cm ^{3}
= **0.58 grams/cc**

Problem 3 - The densities of some common ingredients for planets are as follows:

Rock	3 grams/cc	Ice	1 gram/cc
Iron	9 grams/cc	Mixture of hydrogen + helium	0.7 grams/cc
Water	5 grams/cc		

Based on the average density of Osiris, from what substances do you think the planet is mostly composed?

Answer: Because the density of Osiris is only about 0.6 grams/cc, the closest match would be **a mixture of hydrogen and helium.** This means that, rather than a solid planet like earth, which is a mixture of higher-density materials such as iron, rock and water, Osiris has much in common with Jupiter which is classified by astronomers as a Gas Giant!

How Hot is that Planet?

Planets have been spotted orbiting hundreds of nearby stars, but this makes for a variety of temperatures depending on how far the planet is from its star and the stars luminosity.

The temperature of the planet will be about

$$T=273\left(\frac{(1\text{-}A)L}{D^2}\right)^{1/4}$$

where A is the reflectivity (albedo) of the planet, L is the luminosity of its star in multiples of the sun's power, and D is the distance between the planet and the star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). The resulting temperature will be in units of Kelvins. (i.e. 0^{o} Celsius = +273 K, and Absolute Zero is defined as 0 K).

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4?

Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of the star were increased 1000 times and all other quantities remained the same?

Problem 3 - The recently discovered planet CoRoT-7b (see artist's impression above, from ESA press release), orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	
Water+Land	Earth	0.40	
Gas	Jupiter	0.70	

Problem 1 - Earth is located 1.0 AU from the sun, for which L = 1.0. What is the surface temperature of Earth if its albedo is 0.4? **Answer: $T = 273\ (0.6)^{1/4} = 240$ K**

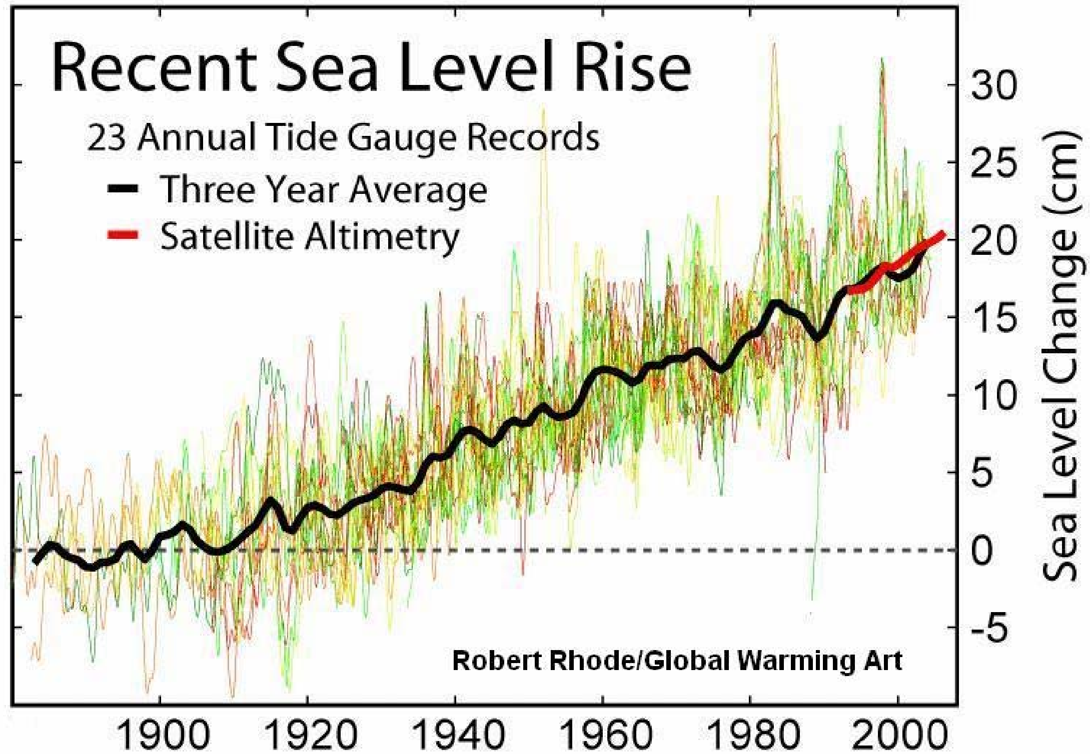
Problem 2 - At what distance would Earth have the same temperature as in Problem 1 if the luminosity of the star were increased 1000 times and all other quantities remained the same? Answer: From the formula, T = 240 and L = 1000 so
$240 = 273(0.6 \times 1000/D^2)^{1/4}$ and so **D = 5.6 AU**. This is about near the orbit of Jupiter.

Problem 3 - The recently discovered planet CoRoT-7b orbits the star CoRoT-7 which is a sun-like star located about 490 light years from Earth in the direction of the constellation Monoceros. If the luminosity of the star is 71% of the sun's luminosity (L = 0.71) and the planet is located 2.6 million kilometers from its star (D= 0.017 AU) what are the predicted surface temperatures of the day-side of CoRoT-7b for the range of albedos shown in the table below?

Surface Material	Example	Albedo (A)	Surface Temperature (K)
Basalt	Moon	0.06	1892
Iron Oxide	Mars	0.16	1840
Water+Land	Earth	0.40	1699
Gas	Jupiter	0.70	1422

Example: For an albedo similar to that of our Moon:
$T = 273 * ((1-0.06)*0.71/(0.017)^2)^{.25}$
= **1,892 Kelvin**

Note: To demonstrate the concept of Significant Figures, the values for L, D and A are given to 2 significant figures, so the answers should be rounded to 1900, 1800, 1700 and 1400 respectively



The graph, produced by scientists at the University of Colorado and published in the IPCC Report-2001, shows the most recent global change in sea level since 1880 based on a variety of tide records and satellite data. The many colored curves show the individual tide gauge trends. The black line represents an average of the data in each year.

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line?

Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form?

Problem 3 - If the causes for the rise remained the same, what would you predict for the sea level rise in A) 2050? B) 2100? C) 2150?

Problem 1 - If you were to draw a straight line through the curve between 1920 and 2000 representing the average of the data, what would be the slope of that line? Answer; See figure below. First, selecting any two convenient points on this line, for example X= 1910 and Y = 0 cm (1910, +0) and X = 1980 Y= +15 cm (1980, +15). The slope is given by m = (y2-y1)/(x2-x1) = 15 cm / 70 years = **0.21 cm/year.**

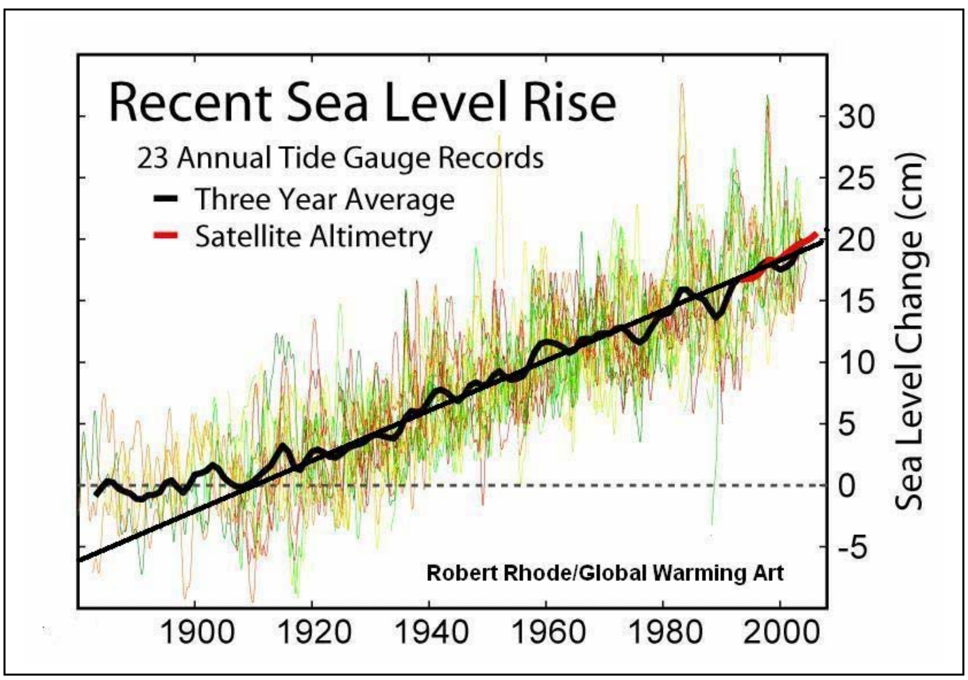
Problem 2 - What would be the equation of the straight line in A) Two-Point Form? B) Point-Slope Form? C) Slope-Intercept Form? Answer:

A) $y - y1 = \frac{(y2 - y1)}{(x2 - x1)} (x - x1)$ so $\mathbf{y - 0 = \frac{(15 - 0)}{(1980 - 1910)} (x - 1910)}$

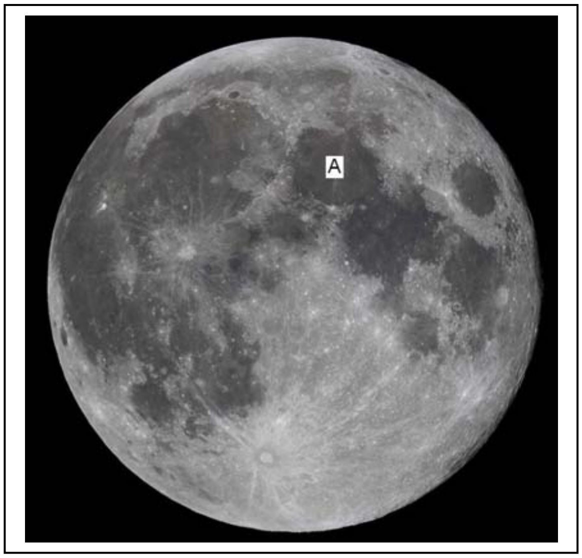
B) y - y1 = m (x - x1) so **y - 0 = 0.21 (x - 1910)**

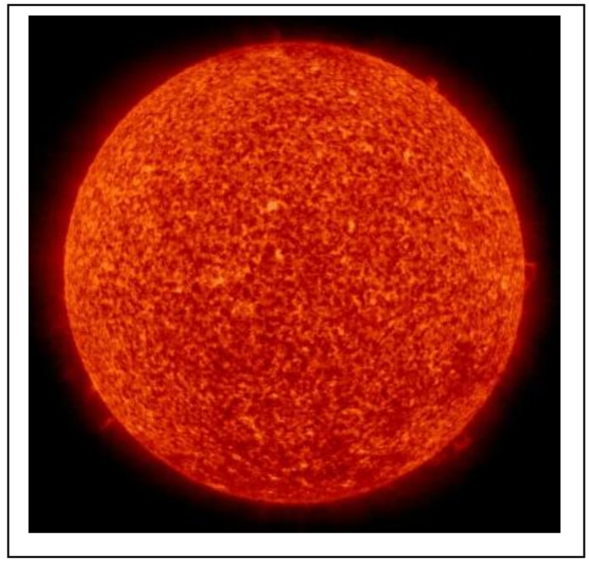
C) y = 0.21 x - 0.21(1910) so **y = 0.21x - 401.1**

Problem 3 - If the causes for the rise remained the same, what would you predict for the sea level rise in A) 2050? B) 2100? C) 2150? Answer:
A) y = 0.21 (2050) - 401.1 = **29.4 centimeters**. (Note; this equals 12 inches)
B) y = 0.21 (2100) - 401.1 = **39.9 centimeters** (Note: this equals 16 inches)
C) y = 0.21 (2150) - 401.1 = **50.4 centimeters.** (Note: this equals 20 inches)



Getting an Angle on the Sun and Moon





The Sun (Diameter = 1,400,000 km) and Moon (Diameter = 3,476 km) have very different physical diameters in kilometers, but in the sky they can appear to be nearly the same size. Astronomers use the angular measure of arcseconds (asec) to measure the apparent sizes of most astronomical objects. (1 degree equals 60 arcminutes, and 1 arcminute equals 60 arcseconds). The photos above show the Sun and Moon at a time when their angular diameters were both about 1,865 arcseconds.

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter?

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon?

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon?

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle?

Problem 5 - What is the area of Mare Serenitatis in square kilometers?

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun?

Answer Key

Problem 1 - Using a metric ruler, what is the angular scale of each image in arcseconds per millimeter? Answer: Moon diameter = 65 mm and sun diameter = 61 mm so the lunar image scale is 1,865 asec/65mm = **28.7 asec/mm** and the solar scale is 1865 asec/61 mm = **30.6 asec/mm.**

Problem 2 - In arcseconds, what is the size of the smallest feature you can see in the images of the Sun and Moon? Answer: the smallest feature is about 0.5 mm or 0.5 x 28.7 asec/mm = **14.4 asec for the Moon** and 0.5 x 30.6 asec/mm = **15.3 asec for the Sun.**

Problem 3 - About what is the area, in square arcseconds ($asec^2$) of the circular Mare Serenitatis (A) region in the photo of the Moon? Answer: The diameter of the mare is 1 centimeter, so the radius is 5 mm or 5 mm x 28.7 asec/mm = 143.5 asec. Assuming a circle, the area is A = π x (143.5 asec)^2 = **64,700 $asec^2$**.

Problem 4 - At the distance of the Moon, 1 arcsecond of angular measure equals 1.9 kilometers. The Sun is exactly 400 times farther away than the Moon. On the photograph of the Sun, how many kilometers equals 1 arcsecond of angle? Answer: The angular scale at the sun would correspond to 400 x 1.9 km = **760 kilometers per arcsecond**.

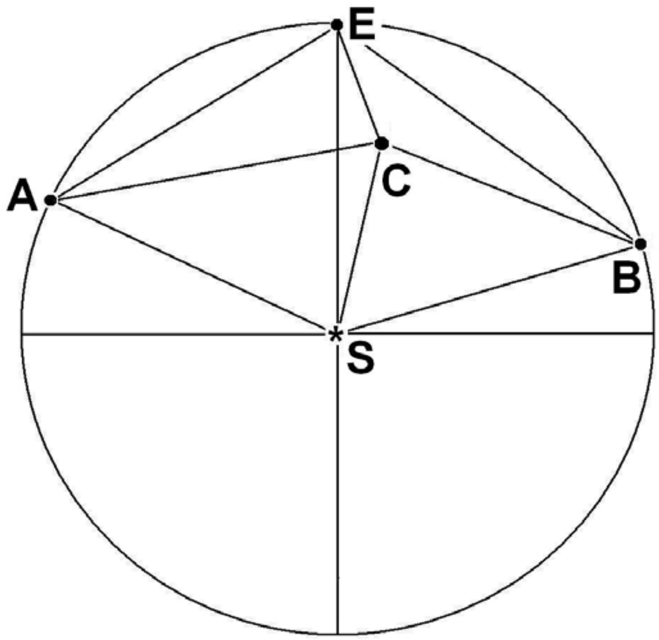
Problem 5 - What is the area of Mare Serenitatis in square kilometers? Answer: We have to convert from square arcseconds to square kilometers using a two-step unit conversion 'ladder'.

64,700 $asec^2$ x (1.9 km/asec) x (1.9 km/asec) = **233,600 km^2.**

Problem 6 - What would be the physical area, in square-kilometers, of an identical angular area to Mare Serenitatis if it were located on the surface of the sun? Answer: The angular area is 400-times further away, so we have to use the scaling of 760 kilometers/asec deduced in Problem 4. The unit conversion for the solar area becomes:

64,700 $asec^2$ x (760 km/asec) x (760 km/asec) = **37,400,000,000 km^2.**

Seeing Solar Storms in STEREO - II



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical unit (150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. In the previous math problem the astronomers knew the ejection angle of the CME, mESC, but in fact they didn't need to know this in order to solve the problem below!

Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:

SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Answer Key

Givens from satellite orbits:

SB = SA = SE = 150 million km   AE = 136 million km   BE = 122 million km
mASE = 54 degrees   mBSE = 48 degrees
mEAS = 63 degrees   mEBS = 66 degrees   mAEB = 129 degrees
use units of megakilometers i.e. 150 million km = 150 Mkm.

Method 1: Students construct a scaled model of the diagram based on the angles and measures, then use a protractor to measure the missing angles, and from the scale of the figure (in millions of kilometers per millimeter) they can measure the required segments. **Segment EC is about 49 Mkm at an angle, mSEB of 28 degrees.**

Method 2 use the Law of Cosines and the Law of Sines to solve for the angles and segment lengths exactly.

mASB = mASE+mBSE = 102 degrees
mASC = theta
mACS = 360 - mCAS - mASC = 315 - theta
mBSC = mASB - theta = 102 - theta
mBCS = 360 - mCBS - mBSC = 208 + theta

Use the Law of Sines to get
sin(mCAS)/L = sin(mACS)/150 Mkm   and   sin(mBCS)/L = sin(mBCS)/150 Mkm.

Eliminate L :   150sin(45)/sin(315-Theta) = 150 sin(50)/sin(208+theta)

Re-write using angle-addition and angle-subtraction:
sin50 [sin(315)cos(theta)-cos(315)sin(theta)] = sin(45) [sin(208)cos(theta) + cos(208)sin(theta)]

Compute numerical factors by taking indicated sines and cosines:
-0.541cos(theta) - 0.541sin(theta) = -0.332cos(theta) - 0.624sin(theta)

Simplify: cos(theta) = 0.397sin(theta)

Use definition of sine: $\cos(\theta)^2 = 0.158 (1 - \cos(\theta)^2)$

Solve for cosine: $\cos(\theta) = (0.158/1.158)^{1/2}$ so theta = 68 degrees. And so **mASC=68**

Now compute segment CS = 150 sin(45)/sin(315-68) = **115 Mkm.**
BC = 115 sin(102-68)/sin(50) = **84 Mkm**.
Then $EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(mEBS-mCBS)$
$EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(66-50)$ So **EC = 47 Mkm**.

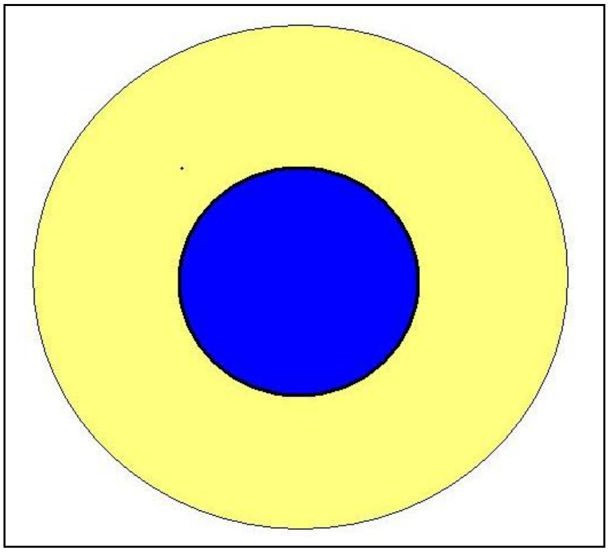
mCEB from Law of Cosines: $84^2 = 122^2 + 47^2 - 2(122)(47)\cos(mCEB)$ so **mCEB = 29 degrees**

And since   mAES = 180 - mASE - mEAS = 180 - 54 - 63 = 63 degrees
so mSEC = mAEB - mAES - mCEB = 129 - 63 - 29 = **37 degrees**

So, the two satellites are able to determine that the CME is 49 million kilometers from Earth and approaching at an angle of 37 degrees from the sun.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?
Answer: 115 million kilometers / 2 million km/hr = **58 hours or 2.4 days**.

How to Build a Planet from the Inside Out!



The planet Osiris orbits 7 million kilometers from the star HD209458 located 150 light years away in the constellation Pegasus. The Spitzer Space Telescope has recently detected water, methane and carbon dioxide in the atmosphere of this planet. The planet has a mass that is 69% that of Jupiter, and a volume that is 146% greater than that of Jupiter.

By knowing the mass, radius and density of a planet, astronomers can create plausible models of the composition of the planet's interior. Here's how they do it!

Among the first types of planets being detected in orbit around other stars are enormous Jupiter-sized planets, but as our technology improves, astronomers will be discovering more 'super-Earth' planets that are many times larger than Earth, but not nearly as enormous as Jupiter. To determine whether these new worlds are Earth-like, they will be intensely investigated to determine the kinds of compounds in their atmospheres, and their interior structure. Are these super-Earths merely small gas giants like Jupiter, icy worlds like Uranus and Neptune, or are they more similar to rocky planets like Venus, Earth and Mars?

Problem 1 - A hypothetical planet is modeled as a sphere. The interior has a dense rocky core, and on top of this core is a mantle consisting of a thick layer of ice. If the core volume is 4.18×10^{12} cubic kilometers and the shell volume is 2.92×10^{13} cubic kilometers, what is the radius of this planet in kilometers?

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet? B) How many Earths could fit inside the mantle of this hypothetical planet?

Problem 3 - Suppose the astronomer who discovered this super-Earth was able to determine that the mass of this new planet is 8.3 times the mass of Earth. The mass of Earth is 6.0×10^{24} kilograms. What is A) the mass of this planet in kilograms? B) The average density of the planet in kilograms/cubic meter?

Problem 4 - Due to the planet's distance from its star, the astronomer proposes that the outer layer of the planet is a thick shell of solid ice with a density of 1000 kilograms/cubic meter. What is the average density of the core of the planet?

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?

Problem 1 - The planet is a sphere whose total volume is given by $V = 4/3\ \pi\ R^3$. The total volume is found by adding the volumes of the core and shell to get $V = 4.18 \times 10^{12} + 2.92 \times 10^{13} = 3.34 \times 10^{13}$ cubic kilometers. Then solving the equation for R we get $R = (3.34 \times 10^{13} / (1.33*3.14))^{1/3} =$ 19,978 kilometers. Since the data is only provided to 3 place accuracy, the final answer can only have three significant figures, and with rounding this equals a radius of R = **20,000 kilometers.**

Problem 2 - If the volume of Earth is 1.1×10^{12} cubic kilometers, to the nearest whole number, A) how many Earths could fit inside the core of this hypothetical planet?
Answer: V = 4.18×10^{12} cubic kilometers / 1.1×10^{12} cubic kilometers = **4 Earths.**

B) How many Earths could fit inside the mantle of this hypothetical planet?
Answer: V = 2.92×10^{13} cubic kilometers / 1.1×10^{12} cubic kilometers = **27 Earths.**

Problem 3 - What is A) the mass of this planet in kilograms? Answer: $8.3 \times 6.0 \times 10^{24}$ kilograms = **5.0×10^{25} kilograms.**
B) The average density of the planet in kilograms/cubic meter?
Answer: Density = total mass/ total volume
= 5.0×10^{25} kilograms/3.34×10^{13} cubic kilometers
= 1.5×10^{12} kilograms/cubic kilometers.
Since 1 cubic kilometer = 10^9 cubic meters,
= 1.5×10^{12} kilograms/cubic kilometers x (1 cubic km/10^9 cubic meters)
= **1,500 kilograms/cubic meter.**

Problem 4 - We have to subtract the total mass of the ice shell from the mass of the planet to get the mass of the core, then divide this by the volume of the core to get its density. Mass = Density x Volume, so the shell mass is $1,000\ kg/m^3 \times 2.92 \times 10^{13}\ km^3 \times (10^9\ m^3/km^3) = 2.9 \times 10^{25}$ kg. Then the core mass = 5.0×10^{25} kilograms - 2.9×10^{25} kg = 2.1×10^{25} kg. The core volume is $4.18 \times 10^{12}\ km^3 \times (10^9\ m^3/km^3) = 4.2 \times 10^{21}\ m^3$, so the density is $D = 2.1 \times 10^{25}\ kg / 4.2 \times 10^{21}\ m^3$= **5,000 kg/m^3.**

Problem 5 - The densities of some common ingredients for planets are as follows:

Granite 3,000 kg/m^3 ; Basalt 5,000 kg/m^3 ; Iron 9,000 kg/m^3

From your answer to Problem 4, what is the likely composition of the core of this planet?
Answer: **Basalt.**

Note that, although the average density of the planet (1,500 kg/m^3) is not much more than solid ice (1,000 kg/m^3), the planet has a sizable rocky core of higher density material. Once astronomers determine the size and mass of a planet, these kinds of 'shell-core' models can give valuable insight to the composition of the interiors of planets that cannot even be directly imaged and resolved! In more sophisticated models, the interior chemistry is linked to the temperature and location of the planet around its star, and proper account is made for the changes in density inside a planet due to compression and heating. The surface temperature, which can be measured from Earth by measuring the infrared 'heat' from the planet, also tells which compounds such as water can be in liquid form.

Energy is measured in a number of ways depending on what property is being represented.

Total Energy - Joules and ergs - The total amount of energy in various forms (kinetic, potential, magnetic, thermal, gravitational)

Power - Watts, Joules/second or ergs/second – the rate at which energy is produced or consumed in time. Power = Energy/Time

Flux - Watts/meter^2, Joules/sec/meter^2 or ergs/sec/meter^2 – the rate with which energy flows through a given area in given amount of time: Flux=Power/Area

1 Joule = 10 million ergs	1 kilowatt = 1,000 watts
1 Watt = 1 Joule/1 second	1 megaJoule = 1,000,000 Joules
1 hour = 3600 seconds	3 feet = 1.0 meters

Example: A 5-watt flashlight is left on for 1 hour: Convert its energy consumption of 5 watt-hours to Joules.

$$5 \text{ Watt-hours} \times \frac{1 \text{ Joule}}{1 \text{ sec } 1 \text{ watt}} \times \frac{3{,}600 \text{ sec}}{1 \text{ hour}} = 18{,}000 \text{ Joules}$$

Notice how the compound unit 'watt' is handled so that the appropriate units in the unit conversion ladder cancel.

Problem 1 – The flux of sunlight at Earth's surface is 1300 Watts/meter^2. Convert this flux to ergs/sec/cm^2.

Problem 2 – A 100-watt bulb shines light over a wall with a surface area of 25 meters^2. What is the flux of light energy in Joules/sec/meter^2?

Problem 3 – The common energy unit for electricity is the watt-hour (Wh), which can be written as 1 watt x 1 hour. How many megajoules equal 1 kilowatt-hour (1 kWh)?

Problem 4 – How many ergs of energy are collected from a solar panel on a roof, if the sunlight provides a flux of 300 Joules/sec/meter^2, the solar panels have an area of 27 square feet, and are operating for 8 hours during the day?

Problem 1 –

Answer: $1300 \frac{\text{Watts}}{\text{meter}^2} \times \frac{\text{10 million ergs}}{\text{sec Watt}} \times \frac{\text{1 meter}}{\text{100 cm}} \times \frac{\text{1 meter}}{\text{100 cm}} = \mathbf{1.3 \times 10^6 \frac{ergs}{sec\ cm^2}}$

Problem 2 –

Answer: $100 \text{ watts} \times \left(\frac{\text{1 Joule}}{\text{1 sec watt}}\right) \times \frac{1}{\text{25 meters}^2} = \mathbf{4.0 \frac{Joules}{sec\ meters^2}}$

Problem 3 –

Answer: $1 \text{ kilowatt-hour} \times \frac{\text{1 Joule}}{\text{1 sec 1 Watt}} = 1{,}000 \frac{\text{Joules}}{\text{sec}} \times 1 \text{ hour}$

$$1{,}000 \frac{\text{Joules}}{\text{sec}} \times 1 \text{ hour} \times \left(\frac{\text{3,600 sec}}{\text{1 hour}}\right)$$

$$1{,}000 \frac{\text{Joules}}{\text{sec}} \times 3{,}600 \text{ seconds}$$

$$3{,}600{,}000 \text{ Joules} \times \frac{\text{1 megaJoule}}{\text{1,000,000 Joules}} = \mathbf{3.6\ megaJoules}$$

Problem 4 -

Answer: Area of roof = $27 \text{ feet}^2 \times (1 \text{ meter}/ 3 \text{ feet}) \times (1 \text{ meter}/ 3 \text{ feet}) = 3 \text{ meter}^2$

Flux = Power/Area so Power = Flux x Area:

$$\text{Power} = 300 \frac{\text{Joules}}{\text{sec meter}^2} \times 3 \text{ meters}^2 = 900 \frac{\text{Joules}}{\text{sec}}$$

Energy = Power x Time

$$= 900 \frac{\text{Joules}}{\text{sec}} \times \left(\frac{\text{3,600 sec}}{\text{1 hour}}\right) \times 8 \text{ hours} = 25{,}920{,}000 \text{ Joules}$$

$$= 25{,}920{,}000 \text{ Joules} \times \left(\frac{\text{10 million ergs}}{\text{1 Joule}}\right) = \mathbf{258.2\ trillion\ ergs}$$

The Elementary Particle Masses

Particle	Mass	Status
Higgs Boson	>110 GeV	Predicted
Top Quark	170 GeV	Found
Z Boson	91 GeV	Found
W Boson	80 GeV	Found
Bottom Quark	4.2 GeV	Found
Tauon	1.8 GeV	Found
Charm Quark	1.2 GeV	Found
Strange Quark	120 MeV	Found
Muon	105 MeV	Found
Down Quark	4.0 MeV	Found
Up Quark	2.0 MeV	Found
Tau Neutrino	< 16 MeV	Found
Electron	0.5 MeV	Found
Muon Neutrino	< 0.2 MeV	Found
Electron Neutrino	< 2 eV	Found
Axion	< 1 eV	Predicted
Graviton	0	Predicted
Gluon	0	Found
Photon	0	Found

Most people are familiar with the Periodic Table of the Elements, which summarizes the properties of the 110 known elements starting from Hydrogen. These elements are composed of elementary particles called electrons, protons and neutrons, which combine in various numbers to build up all of the elements.

Since the 1950's, physicists have discovered an even more fundamental collection of particles that seem to be truly elementary, and which combine to produce not only all of the elements, but the very forces that hold them together. The table to the left shows the names of these particles and their mass. Some particles, such as the Higgs Boson are being proposed to exist because some theories require them in order to complete our understanding of how forces and particles interact.

A basic property of a particle is its mass. Because the masses of the fundamental particles are vastly smaller than a gram or a kilogram, physicists use another unit called the electron Volt. This is actually a unit of energy, but because Albert Einstein demonstrated that energy, E, and mass, m, are basically the same things ($E = mc^2$), physicists conveniently state mass as $M = E/c^2$, and then drop the speed-of-light factor c^2 because it is understood to be part of the definition of mass when energy units are used as in the table above. For example, 1 billion electron Volts (1 GeV) are equal to 1.8×10^{-27} kilograms, which is about equal to the mass of a single proton, which is 1.7×10^{-27} kilograms (938 MeV). (Note also that 1 MeV = 1 million electron Volts.)

Problem 1 - What is the mass, in kilograms, of A) An electron? B) A Strange Quark? C) A Top Quark? and D) A Muon?

Problem 2 - A proton consists of two Up Quarks and one Down Quark held tightly together by the strong nuclear force. What is the total quark mass of a proton in A) MeV? B) In kilograms?

Problem 3 - From your answer to Problem 2, what is the mass difference between 1 proton and the combination of the two Up Quarks and one Down Quark in units of A) MeV? B) kilograms?

Problem 4 - An oxygen atom contains a total of 16 protons and neutrons and has a mass of 2.7×10^{-26} kilograms. What is the mass of a Top Quark in terms of the number of oxygen atoms?

Problem 1 - What is the mass, in kilograms, of A) An electron? B) A Strange Quark? C) A Top Quark? and D) A Muon?
Answer:
A) 0.5 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **9.0 x 10^{-31} kilograms.**
B) 120 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **2.1 x 10^{-28} kilograms.**
C) 170 GeV x (1.8 x 10^{-27} kilograms / 1 GeV) = **3.1 x 10^{-25} kilograms.**
D) 105 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **1.9 x 10^{-28} kilograms.**

Problem 2 - A proton consists of two Up Quarks and one Down Quark held tightly together by the strong nuclear force. What is the total quark mass of a proton in A) MeV? B) In kilograms?

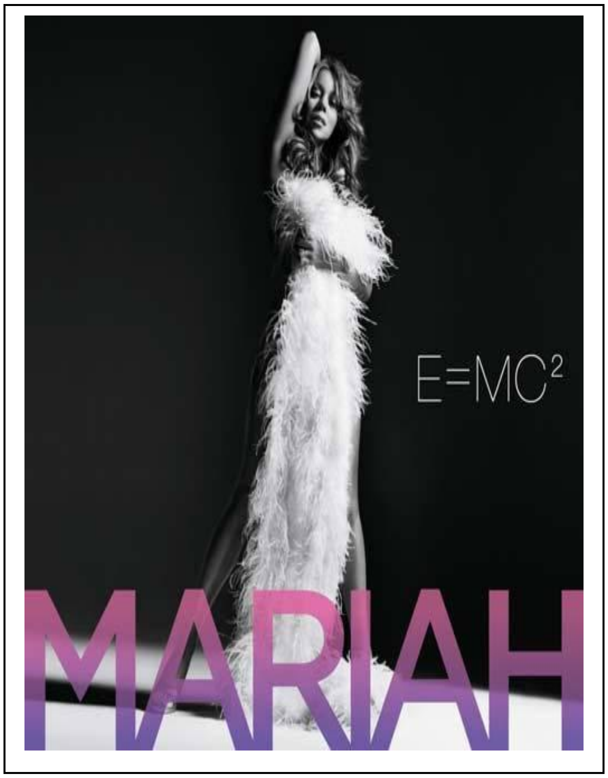
Answer: A) 2 Up + 1 Down = 2 x (2 MeV) + 1 x (4 MeV) = **8 MeV**. B) Mass = 8 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **1.4 x 10^{-29} kilograms.**

Problem 3 - From your answer to Problem 2, what is the mass difference between 1 proton and the combination of the two Up Quarks and one Down Quark in units of A) MeV? B) kilograms?

Answer:
A) 938 MeV - 8 MeV = **930 MeV.**
B) Mass = 930 MeV x (1 GeV/1,000 MeV) x (1.8 x 10^{-27} kilograms / 1 GeV) = **1.7 x 10^{-27} kilograms.**

Problem 4 - An oxygen atom contains a total of 16 protons and neutrons and has a mass of 2.7x10^{-26} kilograms. What is the mass of a Top Quark in terms of the number of oxygen atoms?

Answer: From Problem 1C, the Top Quark mass is 3.1 x 10^{-25} kilograms, and so the number of oxygen atoms needed to equal the mass of one Top Quark is 2.7x10^{-26} kilograms / 3.1 x 10^{-25} kilograms is 8.7 or about **9 oxygen atoms**.



Einstein's formula $E = mc^2$ is almost legendary, and sometimes appears even on T-shirts or even Mariah Carey's pop music album cover! But what does the formula really mean?

It means that, what we consider to be energy in its many forms (like light and heat), which we measure in Joules, can also be considered equivalent to mass in its many forms (like grains of sand or mountains), which we measure in units of kilograms!

The problem is that it takes a LOT of energy to make a kilogram of mass! That's why we never see matter suddenly just appearing out of nowhere in our daily lives. That little formula says that 1 kilogram of mass, m, is exactly equal to 9.0×10^{16} Joules of energy, E. That's the energy released by just one, 20 megaton hydrogen bomb!

The formula depends on the speed of light-squared, c^2, to do the conversion from energy units (E in Joules) to matter units (m in kilograms). If c = 300 million meters/sec, we have the simpler formula: E(Joules) = 9×10^{16} x (mass in kilograms). Use this formula to perform the following conversions using a calculator, and providing answers to 2 significant figures in scientific notation:

Problem 1 1-proton mass = 1.6726×10^{-27} kg equals --------------- Joules

Problem 2 1 Joule of Energy = ------------------- kilograms

Problem 3 1 electron Volt = 1.6×10^{-19} Joules = ------------------ kilograms

Problem 4 1 neutron = 1.6749×10^{-27} kg equals --------------- Joules

Problem 5 1 deuterium nucleus (1 proton+1 neutron) = 3.3444×10^{-27} kg equals --------------- Joules

Problem 6 What is: A) The difference in mass between a single proton plus a single neutron, and the deuterium nucleus? B) The difference in energy? C) The difference in Volts?

Answer Key

Problem 1 1-proton mass = 1.6726×10^{-27} kg

$9 \times 10^{16} \times 1.6726 \times 10^{-27}$ kg = **1.505×10^{-10} Joules**

Problem 2 1 Joule of Energy = $1 / (9 \times 10^{16})$ = **1.1×10^{-17} kilograms**

Problem 3 1 electron Volt = 1.6×10^{-19} Joules = **1.8×10^{-36} kilograms**

Problem 4 1 neutron = 1.6749×10^{-27} kg equals **1.507×10^{-10} Joules**

Problem 5 1 deuterium nucleus (1 proton+1 neutron) = 3.3444×10^{-27} kg = **3.010×10^{-10} Joules**

Problem 6 What is:

A) The difference in mass between a single proton plus a single neutron, and the deuterium nucleus?

$(1.6726 \times 10^{-27}$ kg + 1.6749×10^{-27} kg) - 3.3444×10^{-27} kg = **3.1×10^{-30} kg**

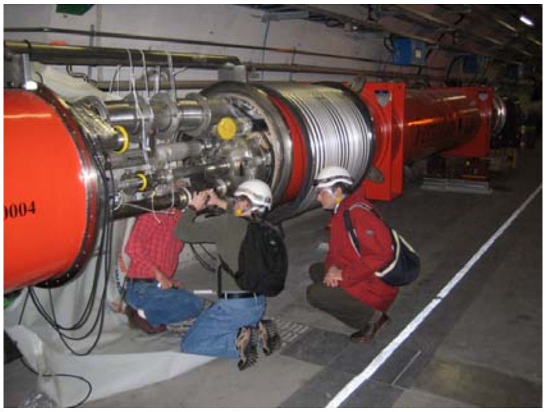
B) The difference in energy?

$(1.505 \times 10^{-10}$ Joules + 1.507×10^{-10} Joules) - 3.010×10^{-10} Joules = **2.0×10^{-13} Joules**

C) The difference in Volts?
Answer: Since 1 volt = 1.8×10^{-36} kilograms

3.1×10^{-30} kg x (1 Volt / 1.8×10^{-36} kg) = **1.7 million Volts or 1.7 MeV.**

A more accurate calculation yields 2.2 MeV



A small section of the LHC (Photo: Peter Limon)

During November, 2009 the Large Hadron Collider experiment at CERN began a slow, step-by-step process of being turned on. Its goal is to accelerate two beams of protons and anti-protons to very high energies, and collide them to form new sub-atomic particles for physicists to discover and study.

Although it is not a NASA research program, it will directly impact the findings of many NASA research satellites such as Chandra and WMAP in searching for Dark Matter and Dark Energy. To understand the operation of LHC, we must first learn how to measure and discuss very small amounts of energy!

The 27-kilometer diameter LHC ring, buried deep underground, uses thousands of magnets to steer two beams of protons so that they collide at specific points along the ring. The beams are no bigger than a human hair in diameter, but contain millions of protons and anti-protons traveling at nearly the speed of light and circulating in the 27-kilometer ring in opposite directions. When these particles collide, their energies combine to create a momentary explosion out of which is created hundreds of particles including electrons, quarks and even more massive particles. Thanks to Albert Einstein's $E=mc^2$, the energy of the two protons can create a burst of new particles, m, literally created out of the raw energy, E, of the collisions.

Physicists use the 'electron volt' (eV) as a convenient unit of energy. One 'eV' is the amount of energy that an electron gains as it falls through a voltage difference of exactly 1 volt. This energy is equal to 1.6×10^{-19} Joules. Physicists find it far easier to remember and write energy measured in units of eV than in Joules! The following problems exercise your ability to translate between eV units and Joules.

Problem 1 - The mass of an electron is equivalent to 511,000 electron-Volts of energy (also written as 511 keV). How many Joules is this?

Problem 2 - The mass of a Top Quark is 175 billion eV (or 175 Giga-electron Volts abbreviated as 175 GeV). How many Joules is this?

Problem 3 - A proton has an energy equivalent to 1.5×10^{-10} Joules. How many electron volts is this?

Problem 4 - A small gnat in flight carries about 1.6×10^{-7} Joules of energy. About how many electron Volts is this equivalent to?

Problem 5 - When the LHC began operation, on November 30, 2009 it achieved an energy of 1.2 trillion electron Volts (1.2 Terra eV or 1.2 TeV). How many Joules of energy is this?

Problem 6 - When fully operational, the LHC will carry 100 trillion protons along each beam, with each proton carrying an energy of 15 TeV. What is the total energy of each proton beam in the LHC compared to: A) A baseball in flight (120 Joules)? B) A small car traveling at 60 mph (300,000 Joules)? C) A Boeing 767 in flight (4 billion Joules)?

Problem 1 - The mass of an electron is equivalent to 511,000 eV of energy (also written as 511 keV). How many Joules is this?
Answer: 511,000 eV x (1.6 x 10^{-19} Joules/1 eV) = **8.2 x 10^{-14} Joules.**

Problem 2 - The mass of a Top Quark is 175 billion eV (or 175 Giga-electron Volts abbreviated as 175 GeV). How many Joules is this?
Answer: 175,000,000,000 eV x (1.6 x 10^{-19} Joules/1 eV) = **2.8 x 10^{-6} Joules.**

Problem 3 - A proton has an energy equivalent to 1.5 x 10^{-10} Joules. How many electron volts is this?
Answer; 1.5 x 10^{-10} Joules x (1 eV/ 1.6 x 10^{-19} Joules) = **938,000,000 eV**
Also written as 938 MeV.

Problem 4 - A small gnat in flight carries about 1.6 x 10^{-7} Joules of energy. About how many electron Volts is this equivalent to?
Answer; 1.6 x 10^{-7} Joules x (1 eV/ 1.6 x 10^{-19} Joules) = **1,000,000,000,000 eV**
Also written as 1 terra eV or 1 TeV.

Problem 5 - When the LHC began operation, on November 30, 2009 it achieved an energy of 1.2 trillion electron Volts (1.2 Terra eV or 1.2 TeV). How many Joules of energy is this?
Answer: 1,200,000,000,000 eV x (1.6 x 10^{-19} Joules/1 eV) = **1.9 x 10^{-7} Joules.**

Problem 6 - When fully operational, the LHC will carry 100 trillion protons along each beam, with each proton carrying an energy of 15 TeV. What is the total energy of each proton beam in the LHC compared to: A) A baseball in flight (120 Joules)? B) A small car traveling at 60 mph (300,000 Joules)? C) A Boeing 767 in flight (4 billion Joules)?

Answer: Each proton will carry 15,000,000,000,000 eV x (1.6 x 10^{-19} Joules/1 eV) = 2.4 x 10^{-6} Joules. So the total energy of each beam will be 100 trillion times this or **2.4 x 10^{8} Joules!**

A) 2.4 x 10^{8} Joules / 120 Joules = **2 million times a baseball in flight**.

B) 2.4 x 10^{8} Joules / 300,000 Joules = **800 times a car in motion.**

C) 2.4 x 10^{8} Joules / 4,000,000,000 Joules = **1/17 of a passenger jet.**

Note to Teacher: An electron-Volt is a unit of energy and is not directly a unit of mass but because E = mc^2, what we are actually talking about is the mass of a particle in terms of eV divided by c^2 .Physicists all understand that the 'c^2' is implicit in using eV as a mass unit but set c = 1 for convenience.

$$T = \frac{10^{10}}{\sqrt{t}} \text{ Kelvin}$$

$$E = \frac{860{,}000}{\sqrt{t}} \text{ eVolts}$$

T is the temperature in degrees Kelvin that was reached **t** seconds after the Big Bang.

E is the energy in electron Volts that was reached **t** seconds after the Big Bang.

Moments after the Big Bang, the universe was brilliantly hot, but steadily cooled with every passing second. All of the matter we see around us today was once fragmented into its individual parts, and we can re-create many of the original Big Bang conditions in our laboratories today. All we have to do is heat up matter so that the parts collide with the same energy as they would have during the Big Bang.

Physicists do this by using 'atom smashers' that accelerate individual particles such as electrons and protons, and then collide them. Thanks to Relativity, and Einstein's famous equation $E = mc^2$, all of the energy of the collision is then available for creating new kinds of particles…if they exist.

Astronomers can also use the Big Bang model to calculate at exactly what time after the Big Bang particles of matter typically had the energies being explored under laboratory conditions. There are two different formula, depending on whether you want to predict the temperature of the gas, or the average energy of the particles in the gas, as shown on the left.

Problem 1 - At 100 seconds after the Big Bang, what was the temperature of the universe, and what was the average collision energy, in kilovolts, of the particles at that time?

Problem 2 - Collisions between particles with a combined energy of 2 billion Volts (2 GeV) can produce a pair of particles, one proton and one anti-proton, out of pure energy. How many seconds after the Big Bang were particles colliding with these energies?

Problem 3 - How hot did the Big Bang have to be in order for it to create particles as massive as a pair of Top Quarks (E = 175 GeV for one top quark), and how long after the Big Bang was this temperature achieved?

Problem 4 - The Large Hadron Collider at CERN in Switzerland was recently 'powered-up' and achieved collision energies of 1.2 TeV, with an ultimate goal of about 15 TeV when it is fully operational in 2010. If 1 TeV = 1 trillion electron Volts (1 Terra eV), A) how many seconds after the Big Bang will the LHC be able to explore the state of matter at the lower and upper energy limits achieved in 2009 and 2010? B) What will be the temperature of matter at these two times?

Problem 1 - At 100 seconds after the Big Bang, what was the temperature of the universe, and what was the average collision energy, in kilovolts, of the particles at that time?
Answer;
T = 10 billion / $(100)^{1/2}$ = 10 billion/10 = **1 billion K**.
E = 860,000 eV / $(100)^{1/2}$ = 860,000/10 = 86,000 eV or **86 kiloVolts**. (abbrev. 86 keV)

Problem 2 - Collisions between particles with a combined energy of 2 billion Volts (2 GeV) can produce a pair of particles, one proton and one anti-proton, out of pure energy. How many seconds after the Big Bang were particles colliding with these energies?

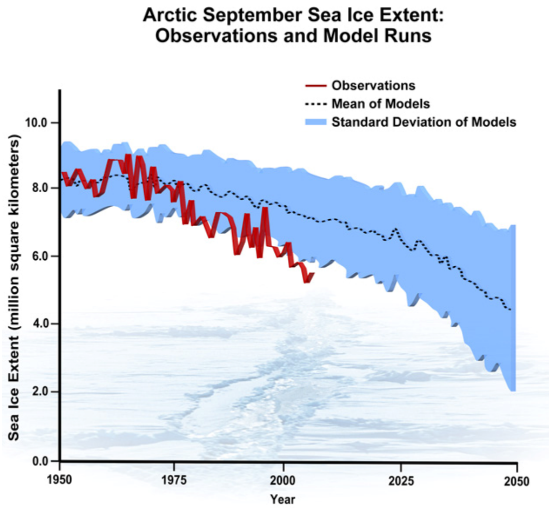
Answer: From the equation for E, we can solve it for t to get t = $(860{,}000/E)^2$ .Since E = 2,000,000,000 eV, we get t = $(860{,}000/2{,}000{,}000{,}000)^2$ = 0.00000018 seconds or **0.18 microseconds after the Big Bang**.

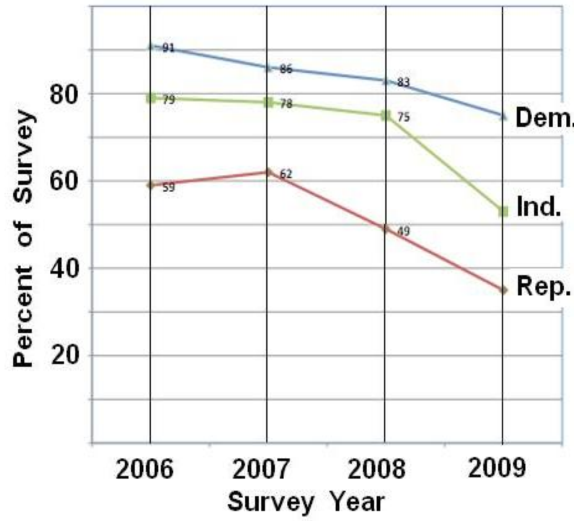
Problem 3 - How hot did the Big Bang have to be in order for it to create particles as massive as a pair of Top Quarks (E = 175 GeV for one top quark), and how long after the Big Bang was this temperature achieved?

Answer: E = 2 x 175 GeV = 350,000,000,000 eV. Then since t = $(860{,}000/E)^2$ we have t = $(860{,}000 / 350{,}000{,}000{,}000)^2$ = 0.000000000006 seconds or **6.0 trillionths of a second** after the Big Bang (also written as 6 picoseconds). At this time, the temperature was T = 10^{10} K/$(6 \times 10^{-12}$ sec$)^{1/2}$ = **4.1×10^{15} Kelvin or 4,100 trillion Kelvin**.

Problem 4 - The Large Hadron Collider at CERN in Switzerland was recently 'powered-up' and achieved collision energies of 1.2 TeV, with an ultimate goal of about 15 TeV when it is fully operational in 2010. If 1 TeV = 1 trillion electron Volts (1 Terra EV), A) how many seconds after the Big Bang will the LHC be able to explore the state of matter at the lower and upper energy limits achieved in 2009 and 2010? B) What will be the temperature of matter at these two times?

Answer: A) E = 1.2 TeV so t = $(860{,}000/1{,}200{,}000{,}000{,}000)^2$ = **5.1×10^{-13} seconds**
And for E = 15 TeV; $(860{,}000/15{,}000{,}000{,}000{,}000)^2$ = **3.3×10^{-15} seconds**
The temperature will be T = $10^{10}/(5.1 \times 10^{-13})^{1/2}$ = **1.4×10^{16} Kelvin**
And T = $10^{10}/(3.3 \times 10^{-15})^{1/2}$ = **1.7×10^{17} Kelvin**





The graph above, based upon research by the National Sea Ice Data Center (Courtesy Steve Deyo, UCAR), shows the amount of Arctic sea ice in September (coldest Arctic month) for the years 1950-2006, based on satellite data (since 1979) and a variety of direct submarine measurements (1950 - 1978). The blue region indicates model forecasts based on climate models.

The figure on the right shows the results of polls conducted between 2006 and 2009 of 1,500 adults by the Pew Research Center for the People & the Press. The graph indicates the number of people, in all three major political parties, believing there is strong scientific evidence that the Earth has gotten warmer over the past few decades.

Problem 1 - Based on the curve in the sea ice graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006?

Problem 2 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Problem 1 - Based on the red curve in the graph, which gives the number of millions of square kilometers of Arctic sea ice identified between 1950 and 2006, what is a linear equation that models the average trend in the data between 1950-2006?

Answer: The linear equation will be of the form y = mx + b. From the graph, the y-intercept for the actual data is 8.5 million km^2 for 1950. The value for 2006 is 5.5 million km^2. The slope is m = (5.5 - 8.5) / (2006 - 1950) = -0.053, so the equation is given by **Y = -0.053(x-1950) + 8.5** in millions of km^2.

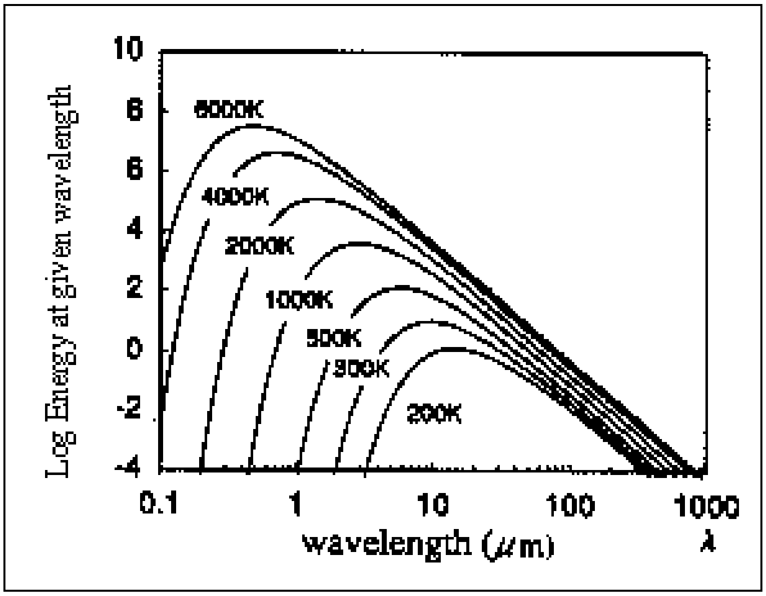
Problem 2 - From your linear model for Arctic ice cover, about what year will the Arctic Ice Cap have lost half the sea ice that it had in 1950-1975?

Answer: In 1950-1975 there were about 8.5 million km^2 of sea ice in September. Half of this is 4.3 million km^2. Set y = 4.3 and solve for x:

Solve 4.3 = -0.053(x-1950)+ 8.5 to get
-4.2 = -0.053(x-1950)
4.2 = 0.053(x-1950)
4.2/0.053 = x-1950
79 = x - 1950
And so x = 2029. So, during the year **2029 AD** there will only be half as much sea ice in the Arctic in September.

Note: If we use only the slope data since 1975 when the ice cover was 8.0 million km^2, the slope would be m = (5.5 - 8.0)/(2006-1975) = -0.083, and linear equation is y = - 0.083(x-1975) + 8.0. The year when half the ice is present would then be about 2023 AD, because the slope is steeper during the most resent 30 years. If the slope continues to steepen with time, the year when only half the ice is present will move closer to the current year.

Star Light…Star Bright



Astronomers can 'take the temperature' of a star by measuring the star's brightness through two filters that pass radiation in the 'blue' and 'visual' regions of the visible spectrum.

From the ratio of these brightnesses, a simple cubic relationship yields the temperature of the star, in Kelvins as follows:

$$T(x) = 9391 - 8350\, x + 5300\, x^2 - 1541\, x^3$$

This formula works for star temperatures between 9,000 and 3,500 Kelvin.

Problem 1 - From the indicated temperature range, what is the domain of this function?

Problem 2 – The sun has a temperature of 5770 K. What is the corresponding value for x?

Problem 3 – To save computation time, an astronomer uses the approximation for T(x) based on a quadratic formula given by $\mathbf{F(x) = 1844x^2 - 6410x + 9175}$. What is the formula that gives the percentage difference, P(x), between F(x) and T(x)?

Problem 1 - From the indicated temperature range, what is the domain of this function?
Answer: We have to solve T(x) = 9,000 and T(x) = 3,500. This can be done using a graphing calculator by programming T(X) into an Excel spreadsheet. Acceptable answers for [9000,3500] should be near **[0.06, 1.50]**

Problem 2 – The sun has a temperature of 5770 K. What is the corresponding value for x? Answer: T(x) = 5770, so **x = 0.67**

Problem 3 – To save computation time, an astronomer uses the approximation for T(x) based on a quadratic formula given by $F(x) = 1844x^2 - 6410x + 9175$. What is the formula that gives the percentage error, P(x) between F(x) and T(x)?

Answer: P(x) = 100% x (estimate – actual) / actual

$$= 100\,(F(x)-T(x))\,/\,T(x)$$

$$= \frac{100(1844x^2 - 6410x + 9175 - (9391 - 8350\,x + 5300\,x^2 - 1541\,x^3))}{9391 - 8350\,x + 5300\,x^2 - 1541\,x^3}$$

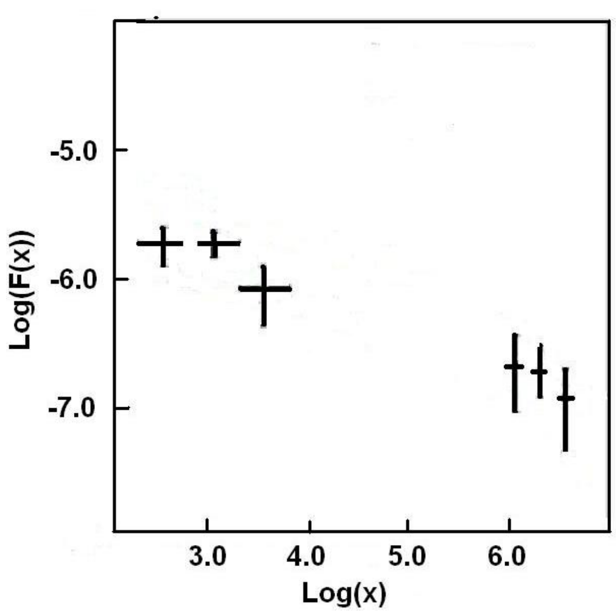
$$= \frac{100\,[1541\,x^3 - 3456\,x^2 + 1940\,x - 216]}{9391 - 8350\,x + 5300\,x^2 - 1541\,x^3}$$

Problem 4 – A what temperature, T(x), does P(x) defined in Problem 3 have its maximum absolute percentage error in the domain of x: [0,1.4], and what is this value?

Answer: Students can use a graphing calculator to display this function, or use Excel to program and plot it, to manually determine the maximum. The graph shows a local minimum of -4.6% at x=1.1, and a local maximum of +1.55% at x=0.4. The absolute maximum error is then 4.6% at x=1.1. The function T(X) evaluated at x=1.1 yields **T = 4,600 K**.

M82, or the Cigar Galaxy, is a starburst galaxy about 12 million light-years away from Earth. In the galaxy's center, stars are being born 10 times faster than they are inside the entire Milky Way galaxy.

In this false-color image, X-ray data recorded by the Chandra X-ray observatory is blue; infrared light recorded by the Spitzer infrared telescope is red; Hubble space telescope observations of hydrogen line emission is orange, and the bluest visible light is yellow-green. (Credit: NASA/JPL-Caltech, STScI, CXC, Uof A, ESA, AURA, JHU)



The Fermi Gamma-Ray Space Telescope has recently confirmed that the nearby galaxy, Messier 82, is the major source of high-energy gamma-rays seen at Earth: over 12 million light years away!

This galaxy has an active core in which a massive black hole is absorbing matter and turning it into energy at a ferocious rate.

The gamma-rays arrive at Earth, at a rate of about one or two per hour, and span a range of energies (x given in MeV) that are shown in the Log-Log plot to the left. F(x) is related to the number of gamma-rays detected per second over an area of 1 square centimeter.

Power-Laws

A surprising number of physical phenomena can be mathematically represented, at least over a part of their range, in terms of a power-law function $\mathbf{F(x) = ax^n}$. We are going to explore some interesting, and convenient, features of power-law functions in analyzing data.

Problem 1 - Show that the graph of Log(F(x)) vs Log(x) is a straight line.

Problem 2 - The graph to the left shows the gamma-ray energy spectrum measured by Fermi. The data points are presented as crosses (called error bars), with the measured value being at the center of the cross. The size of each error bar shows the acceptable range of the measurement. Using a ruler, what linear equation passes through the crosses for the entire collection of data?

Problem 3 - From your answer to Problem 2, what is the 'best fit' power-law, F(x), defined by the linear equation you derived from the data?

Problem 4 - Evaluate F(4.0) and F(5.0) to determine the number of gamma rays/sec/cm^2 at energies of 10,000 and 100,000 MeV.

Problem 1 - Show that the graph of Log(F(x)) vs Log(x) is a straight line.
Answer: $F(x) = ax^n$ so Log(F) = Log(a) + n Log(x) . This is of the form y(x) = b + mx which is the equation for a line.

Problem 2 - Using a ruler, what linear equation passes through the crosses for the entire collection of data? Answer: Students may 'fit' several different lines to the data. One possibility is shown below and has the form Y = mX + b. Using the 'Point-Slope Form' of a linear equation where Y = Log(F(x)) and X = Log(x):

$$Y - Y1 = \frac{Y2 - Y1}{X2 - X1}(X - X1)$$ where X1 = 2.0, Y1 = -5.6, X2 = 7.0 and Y2 = -7.0 we have

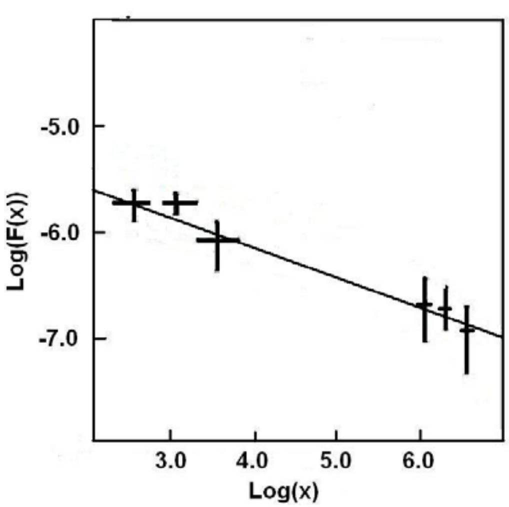
Y = -0.28 X - 5.0

Problem 3 - From your answer to Problem 2, what is the 'best fit' power-law, F(x), defined by the linear equation you derived from the data? Answer: From the above linear example, Y = Log(F(x)) and X = Log(x), so Log(F(x)) = -0.28Log(x) - 5.0 and

$10^{Log(F(X))} = 10^{-0.28 Log(x) - 5.0}$ becomes $F(x)=10^{-5}\, x^{-0.28}$ or $\mathbf{F(x)=0.00001x^{-0.28}}$

Problem 4 - Evaluate F(4.0) and F(5.0) to determine the number of gamma rays/sec/cm^2 at energies of 10,000 and 100,000 MeV. Answer:
$F(10,000) = 0.00001(10,000)^{-0.28} = 7.58\times10^{-7}$ gamma rays/sec/cm^2 (Log(F) = -6.1)
$F(100,000) = 0.00001(100,000)^{-0.28} = 3.98\times10^{-7}$ gamma rays/sec/cm^2 (Log(F) = -6.4)

Note: that these two points, plotted on the Log-Log graph are (+4.0, -6.1) and (+5.0, -6.4) and 'fill-in' the gap between the data points. This is an example of interpolation.



Graph adapted from Abdo, A. A. et al, 2009, 'Detection of Gamma-Ray Emission from the Starburst Galaxies M-82 and NGC-253 with the Large Area Telescope on Fermi', (http://arxiv.org/PS_cache/arxiv/pdf/0911/0911.5327v1.pdf)

Note x is the gamma-ray energy in millions of electron volts (MeV), so '3.0' is 1,000 MeV, and F(x) is in units of MeV/cm^2/sec.


Life-size model of the Webb Space Telescope.

In 2014, the new Webb Space Telescope will be launched. This telescope, designed to detect distant sources of infrared 'heat' radiation, will be a powerful new instrument for discovering distant dwarf planets far beyond the orbit of Neptune and Pluto.

Scientists are already predicting just how sensitive this new infrared telescope will be, and the kinds of distant bodies it should be able to detect in each of its many infrared channels. This problem shows how this forecasting is done.

Problem 1 - The angular diameter of an object is given by the formula:

$$\theta(R) = 0.0014\frac{L}{R} \text{ arcseconds}$$

Create a single graph that shows the angular diameter, $\theta(R)$, for an object the size of dwarf planet Pluto (L=2,300 km) spanning a distance range, R, from 30 AU to 100 AU, where 1 AU (Astronomical Unit) is the distance from Earth to the sun (150 million km). How big will Pluto appear to the telescope at a distance of 90 AU (about 3 times its distance of Pluto from the sun)?

Problem 2 - The temperature of a body that absorbs 40% of the solar energy falling on it is given by

$$T(R) = \frac{250}{\sqrt{R}}$$

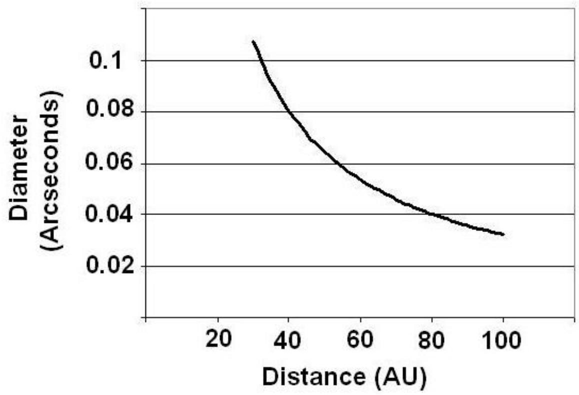
where R is the distance from the sun in AU. Create a graph that shows $\theta(R)$ vs R for objects located in the distance range from 30 to 100 AU. What will be the predicted temperature of a Pluto-like object at 90 AU?

Problem 3 - A body with an angular size $\theta(R)$ given in arcseconds emits 40% of its light energy in the infrared and has a temperature given by T(R) in Kelvin degrees. Its brightness in units of Janskys, F, at a wavelength of 25 microns (2500 nanometers) will be given by:

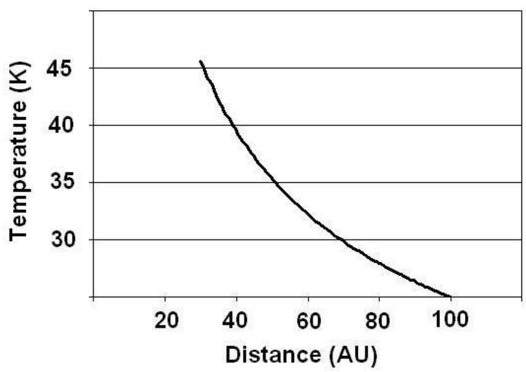
$$F(T) = \frac{11500}{(e^x - 1)}\theta(R)^2 \text{ Janskys} \qquad \text{where} \qquad x = \frac{580}{T(R)}$$

From the formula for $\theta(R)$ and T(R), create a curve F(R) for a Pluto-like object. If the Webb Space Telescope cannot detect objects fainter than 4 nanoJanskys, what will be the most distant location for a Pluto-like body that this telescope can detect? (Hint: Plot the curve with a linear scale in R and a log10 scale in F.)

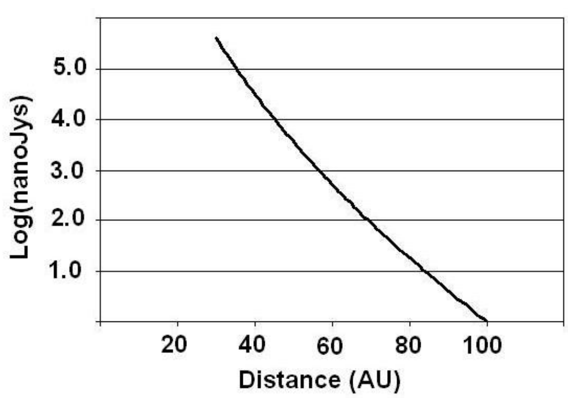
Problem 1 - Answer: At 90 AU, the disk of a Pluto-sized body will be 0.035 arcseconds in diameter.



Problem 2 Answer; At 90 AU, the predicted temperature will be about 27 K.



Problem 3 - Answer; At 4 nanoJanskies, Log(4 nanoJy) = 0.60 which occurs at a distance of about 90 AU.



Computing the Orbit of a Comet

This historic image of the nucleus of Halley's Comet by the spacecraft Giotto in 1986 reveals the gases leaving the icy body to form the tail of the comet.

Once astronomers discover a new comet, a series of measurements of its location allows them to calculate the orbit of the comet and predict when it will be closest to the Sun and Earth.

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which is a unit equal to the distance from Earth to the sun (150 million km). The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law if $P^2 = a^3$ where a is in Astronomical Units and P is in years?

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

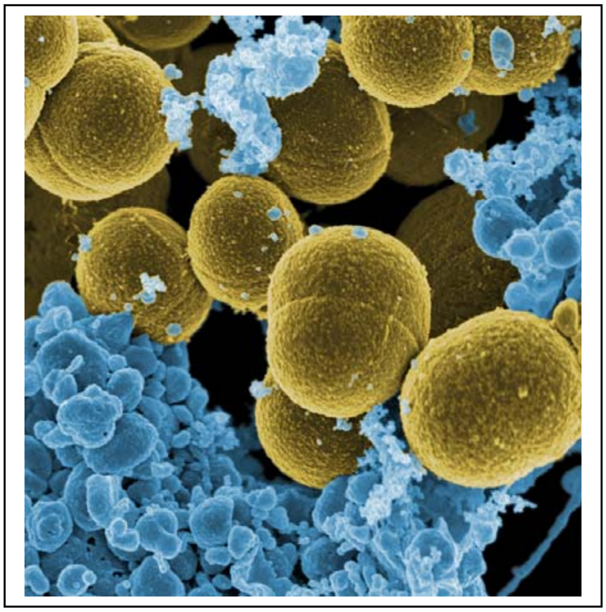
Answer: The standard formula for an ellipse is $x^2/a^2 + y^2/b^2 = 1$ so we can re-write this as $b^2x^2 + a^2y^2 = a^2b^2$.

Then for Point 1 we have
$10^2b^2 + 4^2a^2 = (ab)^2$ so $\mathbf{100b^2 + 16a^2 = (ab)^2}$. Similarly for Point 2 we have
$14^2b^2 + 3^2a^2 = (ab)^2$ so $\mathbf{196b^2 + 9a^2 = (ab)^2}$

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.
Answer:
$100b^2 + 16a^2 = (ab)^2$
$196b^2 + 9a^2 = (ab)^2$
Difference the pair to get $7a^2 = 96b^2$ so $a^2 = (96/7)b^2$.
Substitute this into the first equation to eliminate b^2 to get
$(700/96) + 16 = (7/96)a^2$ or $a^2 = 2236/7$ and so **a = 17.8 AU**.
Then substitute this value for a into the first equation to get
$5069 = 217\ b^2$ and so **b = 4.8 AU**.

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years? Answer: $P = a^{3/2}$ so for a = 17.8 AU we have **P = 75.1 years**.

Problem 4 – The perihelion of the comet is defined as d = a – c where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the Sun in this orbit in kilometers?
Answer: From the definition for c as $c = (a^2 - b^2)^{1/2}$ we have c = 17.1 AU and so the perihelion distance is just d = 17.8 – 17.1 = 0.7 AU. Since 1 AU = 150 million km, it comes to within **105 million km** of the Sun. This is near the orbit of Venus.



NASA's astrobiology program is exploring the basic ingredients of living systems. One of these basic elements is the cell.

A simple living cell generates wastes from the volume inside its cell wall, and passes the wastes outside its cell wall by a process called passive diffusion.

Photo: *S. aureus* bacteria escaping destruction by human white blood cells. (Credit: NIAID / RML)

If a cell cannot remove the waste fast enough, toxins will build up that eventually kill the cell. The balance between waste generation and diffusion, therefore, determines how much volume a cell may have and therefore its typical size.

Suppose the cell has a spherical volume, and that it generates waste at a rate of **a** molecules per cubic micron per second. Suppose it removes the waste through its surface at a rate of b molecules per square micron per second, where 1 micron is 0.000001 meters.

Problem 1 - What is the equation that defines the rate, R, at which the organism changes the amount of its net waste products?

Problem 2 - For what value for the cell's radius will the net change be zero ,which means the cell is in equilibrium?

Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Problem 4 - Two organisms are discovered that have a size of 1 micron and 10 microns. A) How does the ratio of their diffusion rates compare? B) If the surface waste diffusion rates are the same, how do their metabolic rates of waste production compare?

Problem 1 - What is the equation that defines the rate, R, at which the organism changes its net waste products?

Answer:

$$R = \frac{4}{3}\pi r^3 a - 4\pi r^2 b$$

Problem 2 - At what value for the cell's radius will the net change be zero?
Answer: Set R = 0, and then solve for r in terms of *a* and *b* to get:

$$\frac{4}{3}\pi r^3 a = 4\pi r^2 b$$

$$\frac{ra}{3} = b$$

$$r = \frac{3b}{a}$$

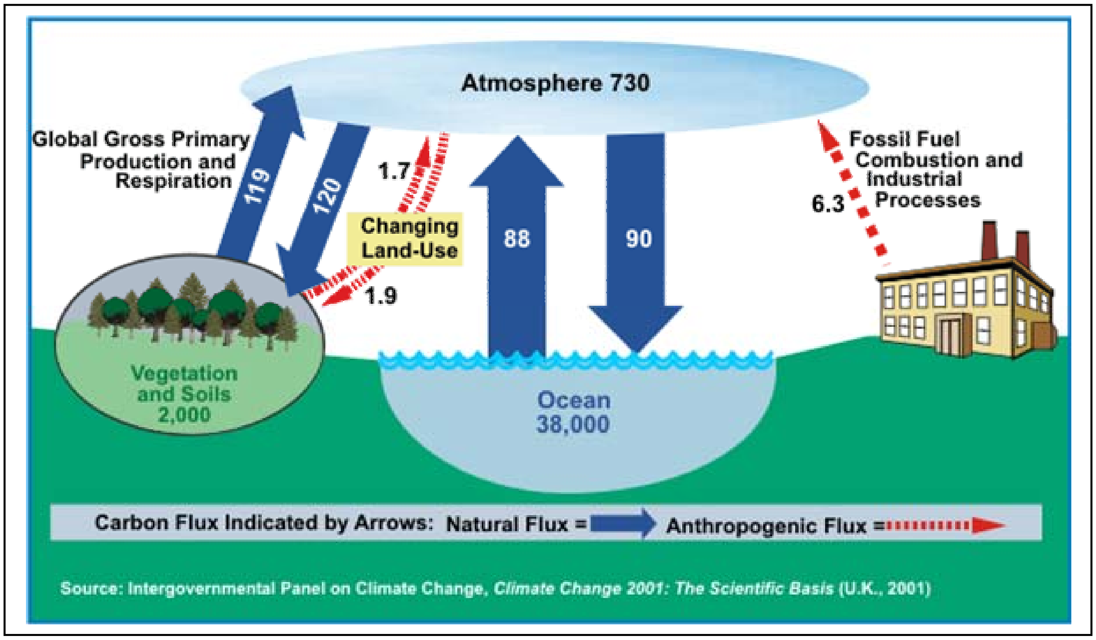
Problem 3 - A hypothetical cell metabolism is measured to be a = 800 molecules/μm^3/sec and b = 2000 molecules/μm^2/sec , about how large might such a cell be if it removed waste products only by passive diffusion?

Answer: r = 3 (2000/800) = **7.5 microns in radius.**

Problem 4 - Two organisms are discovered that have a size of 1 micron and 10 microns.

A) How does the ratio of their diffusion rates compare? Answer: **The smaller organism has 1/10 the ratio of the diffusion rates, b / a, as the larger organism.**

B) If the surface diffusion rates are the same, how do their metabolic rates of waste production compare? Answer: If the surface waste diffusion rate, b, is the same, and the smaller organism has the lower ratio, then **the smaller organism must have 10 times the waste production rate, a,** if only passive diffusion is involved.



The figure above shows a simplified view of the sources and sinks of the element carbon on Earth. Note that, for every 44 gigatons of the carbon dioxide molecule, there are 12 gigatons of the element carbon.

Problem 1 - What are the sources of carbon increases to the atmosphere in the above diagram? What are the sinks of carbon?

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon in gigatons.

Problem 3 - Integrate the equation in Problem 2, assuming that C(2005) = 730 gigatons, and derive the simple equation describing the total amount of carbon in the atmosphere as a function of time.

Problem 4 - What does your model predict for the amount of carbon in the atmosphere in 2050 if the above source and sink rates remain the same?

Answer Key

Problem 1 - Answer: The sources of the carbon (arrow pointed into the atmosphere in the figure) are Vegetation (+119.6 gigatons/yr), oceans (+88 gigatons/yr), human activity (+6.3 gigatons/yr) and changing land use (+1.7 gigatons/yr). The sinks remove carbon (the arrows pointed down in the figure) and include vegetation (-120 gigatons/yr), oceans (-90 gigatons/yr), and changing land use (-1.9 gigatons/yr).

Problem 2 - From the values of the sources and sinks, and assuming they are constant in time, create a simple differential equation that gives the rate-of-change of atmospheric carbon dioxide, C(t), in gigatons.

Answer:

$$\frac{dC(t)}{dt} = +119 + 88 + 6.3 + 1.7 - 120 - 90 - 1.9$$

so

$$\frac{d\,C(t)}{dt} = +3.1$$

Problem 3 - Answer: $C(t) = 3.1\,t + a$ where a is the constant of integration.

Since C(2005) = 730

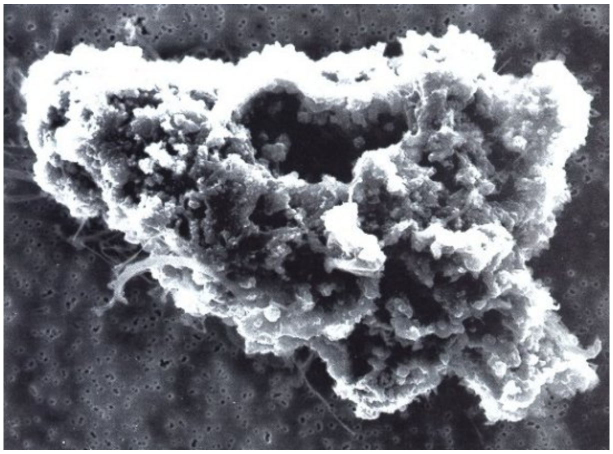
$730 = 3.1\,(2005) + a$ and so $a = -5500$

$C(t) = 3.1\,t - 5500$. for the total element carbon.

Problem 4 - What does your model predict for the amount of carbon dioxide in the atmosphere in 2050 if the above source and sink rates remain the same?

Answer: $C(2050) = 3.1\,(2050) - 5500$

C(2050) = 860 gigatons of carbon.



A cosmic dust grain about 0.1mm across captured by a NASA high-altitude aircraft. Probably debris from a passing comet

Planets are built in several stages. The first of these involves small, micron-sized interstellar dust grains that collide and stick together to eventually form centimeter-sized bodies. A simple model of this process can tell us about how long it takes to 'grow' a rock-sized body starting from microscopic dust. This process occurs in dense interstellar clouds, which are known to be the birth places for stars and planets.

Problem 1 – Assume that the forming rock is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the rock as a function of time, M(t)?

Problem 2 – The rock grows by absorbing incoming dust grains that have an average mass of m grams and a density of N particles per cubic centimeter in the dust cloud. The particles collide with the surface of the rock at a speed of V cm/sec, what is the equation that gives the rate of growth of the rock's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the rock when it reaches a diameter of 1 centimeter if its density is 3 grams/cc?

Problem 6 – The rock begins at t=0 with a mass of 1 dust grain, m = 8×10^{-12} grams. The cloud density N = 3.0×10^{-5} dust grains/cc and the speed of the dust grains striking the rock, without destroying the rock, is V=10 cm/sec. How many years will the growth phase have to last for the rock to reach a diameter of 1 centimeter?

Problem 1 – Answer: Because mass = density x volume, we have
M = 4/3 π R^3 ρ and so **M(t) = 4/3 π ρ $R(t)^3$**

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of dust grains land on the surface area of the rock per unit time, dt. The amount is proportional to the surface area of the rock, since the more surface area the rock has, the more dust particles will be absorbed. Also, the rate at which dust grain mass is brought to the surface is proportional to the product of the dust grain density in the interstellar gas, times the speed of the grains landing on the surface. This leads to m x N x V where m is in grams per dust grain, N is in dust grains per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the rock, will then have the units of grams/sec representing the rate at which the rock mass is growing. The full formula for the growth of the rock mass is then
dM/dt = 4 p R^2 m N V

Problem 3 – Answer: From Problem 1 we see that R(t) = $(3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have dM/dt = 4 π m N V $(3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}$ dM = 4 π m N V $(3/4\ \pi\ \rho)^{2/3}$ dt Then integrate both sides to get:
3 $M(t)^{1/3}$ = 4 π m N V $(3/4\ \pi\ \rho)^{2/3}$ t + c. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 –Answer: The radius will be 0.5 centimeters so, m = 4/3 π $(0.5\ cm)^3$ x 3.0 gm/cc = **1.6 grams.**

Problem 6 – Answer: For t = 0, M(0) = m so the constant of integration is c = $m^{1/3}$ so c = $(8\times10^{-12})^{1/3}$ = 2 x 10^{-4}.
Then M(t) = (4/3 (3.14) (8 x 10^{-12})(3.0 x 10^{-5})(10.0)$(3/(4(3.14)\ (3.0)))^{2/3}$ t + 2 x 10^{-4} $)^3$
M(t) = $(1.9 \times 10^{-15}\ t + 2 \times 10^{-4})^3$

To reach M(t) = 1.6 grams, t = 6.1 x 10^{14} seconds or about **19 million years!**

Asteroid Gaspra about 15 km across. Image taken by the NASA Galileo spacecraft.

Planets are built in several stages. The first of these involves small, interstellar dust grains that collide and stick together to form centimeter-sized bodies. This can take millions of years. The second stage involves the formation of kilometer-sized asteroids from the centimeter-sized rocks. A simple model of this process can tell us about how long it takes to 'grow' an asteroid from rock-sized bodies.

Problem 1 – Assume that the forming asteroid is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the asteroid as a function of time, M(t)?

Problem 2 – The asteroid grows by absorbing incoming rocks that have an average mass of 5.0 grams and a density of N rocks per cubic centimeter in the cloud. The rocks collide with the surface of the forming asteroid at a speed of V cm/sec, what is the equation that gives the rate of growth of the asteroid's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc?

Problem 6 – The asteroid begins at t=0 with a mass of m=5 grams. The cloud density N = 1.0×10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer?

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so $\mathbf{M(t) = 4/3\ \pi\ \rho\ R(t)^3}$

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of rocks land on the surface area of the forming asteroid per unit time, dt. The amount is proportional to the surface area of the asteroid, since the more surface area the asteroid has, the more rocks will be absorbed. Also, the rate at which rock mass is brought to the surface of the asteroid is proportional to the product of the rock density in the solar nebula, times the speed of the rocks landing on the surface of the asteroid. This leads to m x N x V where m is in grams per rock, N is in rocks per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the asteroid, will then have the units of grams/sec representing the rate at which the asteroid mass is growing. The full formula for the growth of the asteroid mass is then
$\mathbf{dM/dt = 4\ p\ R^2\ m\ N\ V}$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}\ dM = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ dt$ Then integrate both sides to get:
$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t + c$. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV\left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

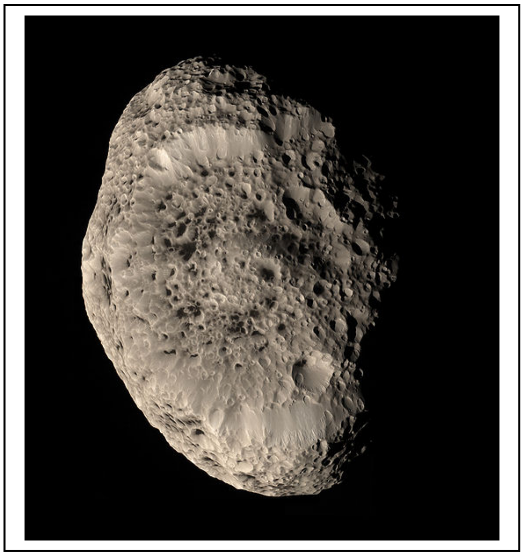
Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc? Answer $M = 4/3\ \pi\ (50{,}000\ \text{cm})^3 \times 3.0$ gm/cc = $\mathbf{1.6 \times 10^{15}}$ **grams.**

Problem 6 – The rock begins at t=0 with a mass of 1 rock, m = 5 grams. The cloud density N = 1.0×10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer? Answer: For t = 0, M(0) = m so the constant of integration is $c = m^{1/3}$ so c = 1.7.

Then $M(t) = (4/3\ (3.14)\ (5)(1.0 \times 10^{-8})(100{,}000)(3/(4(3.14)\ (3.0)))^{2/3}\ t + 1.7\ )^3$

$M(t) = (0.0039\ t + 1.7)^3$

So to get $M(t) = 1.6 \times 10^{15}$ grams, solve for t to get t = 29,600,000 seconds or about **342 days!**



Planets are built in several stages. Dust grains grow to large rocks in a million years, then rocks accumulate to form asteroids in a few years or so. The third stage combines kilometer-wide asteroids to make rocky planets. A simple model of this process can tell us about how long it takes to 'grow' a planet by accumulating asteroid-sized bodies through collisions. Saturn's moon Hyperion (see image) is 300 km across and is an example of a 'small' planet-sized body called a planetoid.

Problem 1 – Assume that the forming planet is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the planet as a function of time, M(t)?

Problem 2 – The planet grows by absorbing incoming asteroids that have an average mass of 10^{15} grams and a density of N asteroids per cubic centimeter in the cloud. The asteroids collide with the surface of the forming planet at a speed of V cm/sec, what is the equation that gives the rate of growth of the planet's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to Problem 3 so determine M(t).

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc?

Problem 6 – The planetoid begins at t=0 with a mass of m= 2 x 10^{15} grams. The cloud density N = 1.0 x 10^{-24} asteroids/cc (1 asteroid per 1000 cubic kilometers), and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers?

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3\ \pi\ R^3\ \rho$ and so $\mathbf{M(t) = 4/3\ \pi\ \rho\ R(t)^3}$

Problem 2 –**Answer:** The change in the mass, dM, occurs as a quantity of asteroids land on the surface area of the planet per unit time, dt. The amount is proportional to the surface area of the planet, since the more surface area the planet has, the more asteroids will be absorbed. Also, the rate at which asteroid mass is brought to the surface of the forming planet is proportional to the product of the asteroid density in the planetary nebula, times the speed of the asteroids landing on the surface of the planet. This leads to m x N x V where m is in grams per dust grain, N is in asteroids per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the planet, will then have the units of grams/sec representing the rate at which the planet mass is growing. The full formula for the growth of the planet mass is then
$\mathbf{dM/dt = 4\ p\ R^2\ m\ N\ V}$

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3\ M(t)/\ 4\ \pi\ \rho)^{1/3}$. Then substituting into dM/dt we have $dM/dt = 4\ \pi\ m\ N\ V\ (3\ M(t)/4\ \pi\ \rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV \left(\frac{3}{4\pi\rho} \right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand $M(t)^{-2/3}\ dM = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ dt$ Then integrate both sides to get:
$3\ M(t)^{1/3} = 4\ \pi\ m\ N\ V\ (3/4\ \pi\ \rho)^{2/3}\ t + c$. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3} \pi mNV \left(\frac{3}{4\pi\rho} \right)^{\frac{2}{3}} t + c \right]^3$$

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc? Answer $M = 4/3\ \pi\ (2.5 \times 10^8\ cm)^3 \times 3.0\ gm/cc =$ $\mathbf{2.0 \times 10^{26}}$ **grams.**

Problem 6 – The planetoid begins at t=0 with a mass of 1 asteroid, $m = 2.0 \times 10^{15}$ grams. The cloud density $N = 1.0 \times 10^{-24}$ asteroids/cc (This equals 1 asteroid per 1000 cubic kilometers) and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers? Answer: For t = 0, M(0) = m so the constant of integration is $c = m^{1/3}$ so $c = (2.0 \times 10^{15}\ g)^{1/3} = 1.3 \times 10^5$.
Then $M(t) = (4/3\ (3.14)\ (2 \times 10^{15})(1.0 \times 10^{-24})(100{,}000)(3/(4(3.14)\ (3.0)))^{2/3}\ t + 1.3 \times 10^5)^3$
$M(t) = (0.00015\ t + 1.3 \times 10^5)^3$

To get $M(t) = 2.0 \times 10^{26}$ grams will take $t = 3.9 \times 10^{12}$ seconds or about **126,000 years!**

The Higgs Boson and the Mystery of Mass

The search is on for the Higgs Boson at the Large Hadron Collider, which began operation on November 23, 2009. This sub-atomic particle plays a major role in explaining why some particles have a measurable mass (electrons and quarks) while others do not (photons, gluons). This remarkable ability to cause particles to have mass can be described by the properties of the following equation:

$$V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}$$

This equation describes the energy, V, stored in what physicists call the Higgs field. The variable x is the mass of the Higgs Boson, and T is the energy that particles such as electrons and protons carry when they are colliding with each other. The value of V determines how much mass a colliding particle will have, and this depends on the mass of the Higgs Boson.

Problem 1 - What is the shape of the function V(x) over the domain [0,+1] for a collision energy of; A) T=0? B) T=0.5? C) T = 0.8 and D) T=1.0?

Problem 2 - The observed mass of the Higgs Boson is defined by the location of the minimum of V(x) over the domain [0, +1]. How does the predicted mass of the Higgs Boson change as the value of T increases from 0 to 1?

Problem 3 - The measured mass of an electron is M_0=9.1 x 10^{-31} kilograms. According to the current models of the Higgs field, the electron's mass is determined by the simple equation M = M_0 x, where x is the mass of the Higgs Boson. What would be the predicted electron masses for T = 0.5?

Problem 4 - The LHC will achieve collision energies of about 5 trillion electron-Volts (5 TeV). At this energy, T = 33. What will the function V(x) look like, and what will be the predicted mass of the electron at these energies?

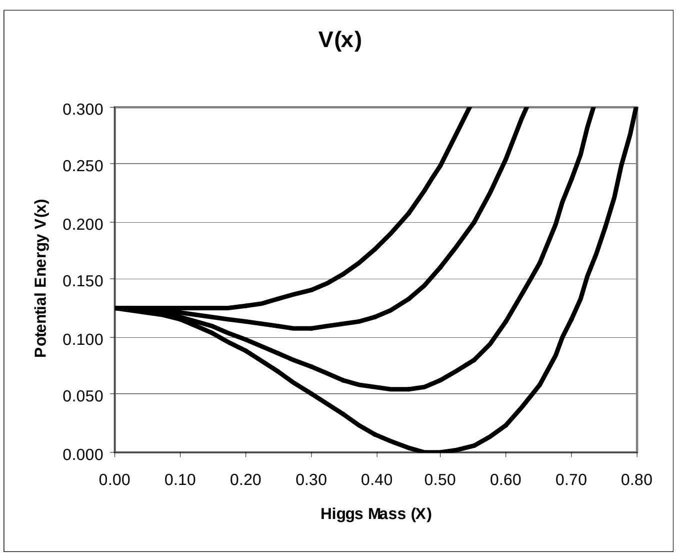
Problem 1 - Answer: The function can be programmed on an Excel spreadsheet or a graphing calculator. Select x intervals of 0.05 and a graphing window of x: [0,1] y:[0,0.3] to obtain the plot below-left. The curves from top to bottom are for T = 1, 0.8, 0.5 and 0 respectively.

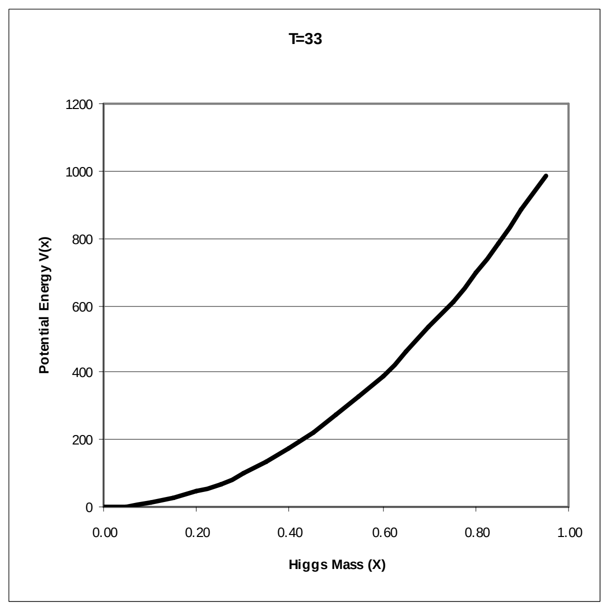
Problem 2 - Answer: The minima of the curves can be found using a graphing calculator display or by interpolating from the spreadsheet calculations. The x values for T = 1.0, 0.8, 0.5 and 0 are approximately 0, 0.3, 0.45 and 0.5. Note that as the collision energy increases, the observed mass of the Higgs Boson will decrease to zero!

Problem 3 - Answer: At T = 0.5, Problem 2 says that the Higgs mass is lowered to x=0.45 so the mass of the electron M = 9.1 x 10^{-31} kilograms x (0.45) = 4.5 x 10^{-31} kilograms. The electrons that are observed in very high energy collisions have lost nearly half their measurable mass.

Problem 4 - The graph is shown to the lower right. The electron's mass will be **near zero** because the curve is nearly a parabola with its vertex at x=0.

Note to Teacher: The function V(x) in this problem is meant to illustrate an important concept related to the way in which the Higgs Boson allows particles such as electrons and quarks to gain mass, rather than to remain massless particles. The answers to Problems 2-4 are not meant to be exact, but only to illustrate the basic mathematics. In actuality, V(X) and its changes with actual collision energies at the Large Hadron Collider are more complex than presented in these problems, and the masses of known particles are not expected to change my more than a few percent over the energy range for T being explored.





The Energy of Empty Space

An important concept in cosmology is that the 'empty space' between stars and galaxies is not really empty at all! Today, the amount of invisible energy hidden in space is just enough to be detected as Dark Energy, as astronomers measure the expansion speed of the universe. Soon after the Big Bang, this Dark Energy caused the universe to expand by huge amounts in less than a second. Cosmologists call this early period of the Big Bang Era, Cosmic Inflation.

Physicists have developed a number of theories to quantify how Cosmic Inflation occurred. The basis for many of these theories is a mathematical equation of the form shown below. This equation connects the energy of empty space, V(x), to the existence of a new field in nature whose strength is determined by the value for x in the equation.

An interesting property of this new field is the way in which it interacts with all other particles in the universe as the temperature of the universe changes. As the universe cools from very high temperatures (T=1) near the Big Bang to very low temperatures (T=0) today, the function V(x) changes its shape. This causes the value of x where the function has its minimum to also change. The consequence of changing the value for x in the universe is that particles such as electrons and quarks will have different masses than what we observe today.

$$V(x) = 2x^4 - (1 - T^2)x^2 + \frac{1}{8}$$

Problem 1 - What are the domain and range of the function V(x)?

Problem 2 - What is the axis of symmetry of V(x)?

Problem 3 - Is V(x) an even or an odd function?

Problem 4 - For T=0, what are the critical points of the function in the domain [-2, +2]?

Problem 5 - Over the domain [0,+2] where are the local minima and maxima located for T=0?

Problem 6 - Using a graphing calculator or an Excel spreadsheet, graph V(x) for the values T=0, 0.5, 0.8 and 1.0 over the domain [0,+1]. Tabulate the x-value of the local minimum as a function of T. In terms of its x location, what do you think happens to the end behavior of the minimum of V(x) in this domain as T increases?

Problem 7 - During the Cosmic Inflation Era, what is the vacuum energy difference defined as V = V(0) - V(1/2)?

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?

Problem 1 - **Answer: Domain [- infinity, + infinity], Range [0,+infinity]**

Problem 2 - **Answer: The y-axis: x=0**

Problem 3 - **Answer: It is an even function.**

Problem 4 - **Answer: X = 0, X = -1/2 and x = +1/2**

Problem 5 - **Answer: The local maximum is at x=0; the local minimum is at x = +1/2**

Problem 6 - **Answer: See the graph below where the curves represent from top to bottom, T = 1.0, 0.8, 0.5 and 0.0. The tabulated minima are as follows:**

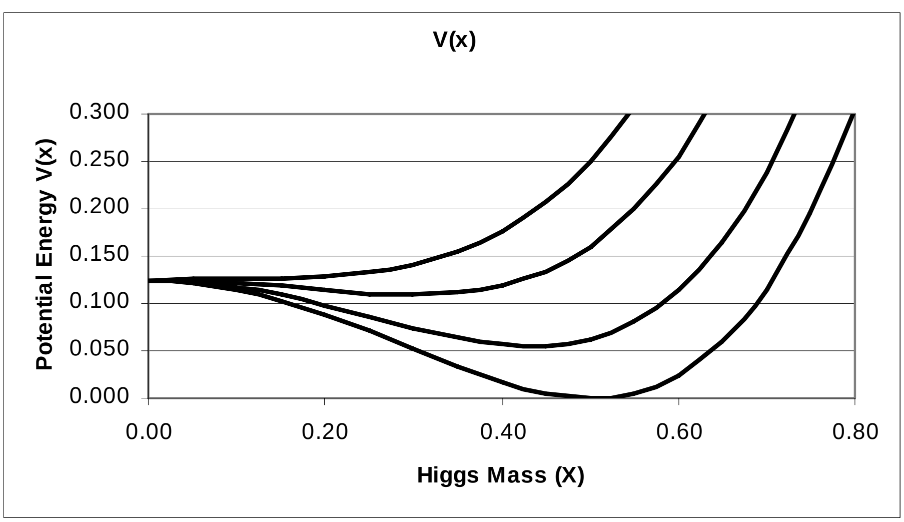
T	X
0.0	0.5
0.5	0.45
0.8	0.30
1.0	0.0

The end behavior, in the limit where T becomes very large, is that V(x) becomes a parabola with a vertex at (0, +1/8)

Problem 7 - What is the vacuum energy difference V = V(0) - V(1/2) during the Cosmic Inflation Era? **Answer: V(0) = 1/8 V(1/2) = 0 so V = 1/8.**

Problem 8 - The actual energy stored in 'empty space' given by V(x) has the physical units of the density of energy in multiples of 10^{35} Joules per cubic meter. What is the available energy density during the Cosmic Inflation Era in these physical units?
Answer: V = 1/8 x 10^{35} Joules/meter^3 = **1.2 x 10^{34} Joules/meter^3**.

Note to Teacher: This enormous energy was available in every cubic meter of space that existed soon after the Big Bang, and the time it took the universe to change from the V(0) to V(1/2) state lasted only about 10^{-35} seconds. This was enough time for the universe to grow by a factor of 10^{35} times in its size during the Cosmic Inflation Era.



The Volume of a Hypersphere

This spherical propellant tank is an important component of testing for the Altair lunar lander, an integral part of NASA's Constellation Program. It will be filled with liquid methane and extensively tested in a simulated lunar thermal environment to determine how liquid methane would react to being stored on the moon.

The volume of a sphere is a mathematical quantity that can be extended to spaces with different numbers of dimensions with some very interesting, and surprising, consequences!

The mathematical formula for the volume of a sphere in a space of N dimensions is given by the recursion relation

$$V(N) = \frac{2\pi R^2}{N} V(N-2)$$

For example, for 3-dimensional space, N = 3 and since from the table to the left, V(N-2) = V(1) = 2R, we have the usual formula

$$V(3) = \frac{4}{3}\pi R^3$$

Dimension	Formula	Volume
0	1	1.00
1	2R	2.00
2	πR^2	3.14
3	$\frac{4}{3}\pi R^3$	4.19
4		
5		
6		
7		
8		
9		
10		

Problem 1 - Calculate the volume formula for 'hyper-spheres' of dimension 4 through 10 and fill-in the second column in the table.

Problem 2 - Evaluate each formula for the volume of a sphere with a radius of 1.00 and enter the answer in column 3.

Problem 3 - Create a graph that shows V(N) versus N. For what dimension of space, N, is the volume of a hypersphere its maximum possible value?

Problem 4 - As N increases without limit, what is the end behavior of the volume of an N-dimensional hypersphere?

Dimension	Formula	Volume
0	1	1.00
1	2R	2.00
2	πR^2	3.14
3	$\frac{4}{3}\pi R^3$	4.19
4	$\frac{\pi^2 R^4}{2}$	4.93
5	$\frac{8\pi^2 R^5}{15}$	5.26
6	$\frac{\pi^3 R^6}{6}$	5.16
7	$\frac{16\pi^3 R^7}{105}$	4.72
8	$\frac{\pi^4 R^8}{24}$	4.06
9	$\frac{32\pi^4 R^9}{945}$	3.30
10	$\frac{\pi^5 R^{10}}{120}$	2.55

Problem 1 - Answer for N=4:

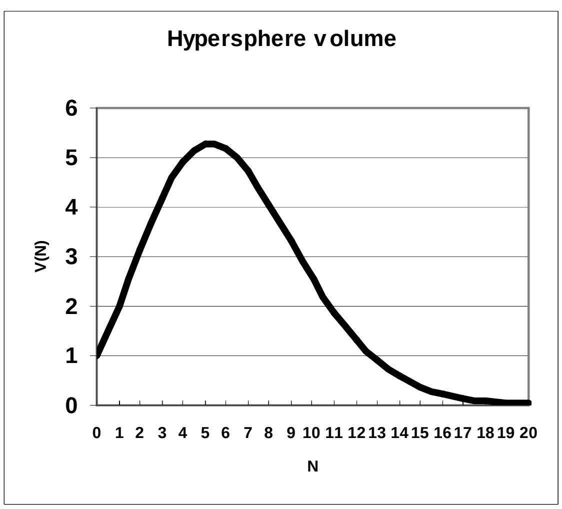
$$V(4) = \frac{2\pi R^2}{4} V(4-2)$$

$$V(4) = \frac{2\pi R^2}{4} V(2)$$

$$V(4) = \frac{2\pi R^2}{4} (\pi R^2)$$

$$V(4) = \frac{\pi^2 R^4}{2}$$

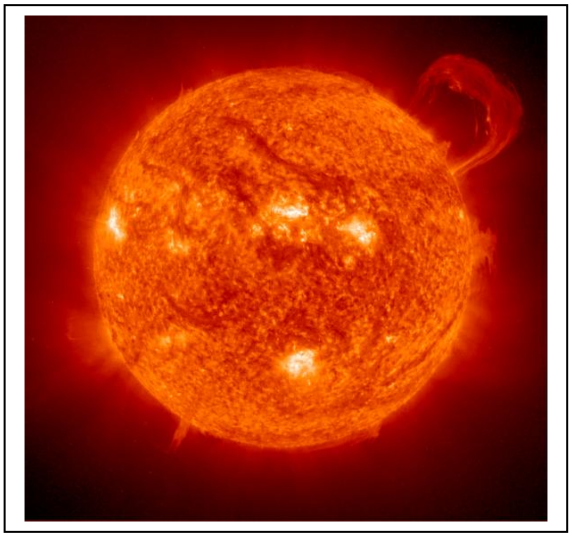
Problem 2 - Answer for N=4:
V(4) = $(0.5)(3.141)^2$ = 4.93.



Problem 3 - The graph to the left shows that the maximum hypersphere volume occurs for spheres in the fifth dimension (N=5). Additional points have been calculated for N=11-20 to better illustrate the trend.

Problem 4 - In the limit for spaces with very large dimensions, the hypersphere volume approaches zero!

The Internal Density and Mass of the Sun



Detailed mathematical models of the interior of the sun are based on astronomical observations and our knowledge of the physics of stars. These models allow us to explore many aspects of how the sun 'works' that are permanently hidden from view.

The Standard Model of the sun, created by astrophysicists during the last 50 years, allows us to investigate many separate properties. One of these is the density of the heated gas throughout the interior. The function below gives a best-fit formula, D(x) for the density (in grams/cm^3) from the core (x=0) to the surface (x=1) and points in-between.

$$D(x) = 519x^4 - 1630x^3 + 1844x^2 - 889x + 155$$

For example, at a radius 30% of the way to the surface, x = 0.3 and so D(x=0.3) = 14.5 grams/cm^3.

Problem 1 - What is the estimated core density of the sun?

Problem 2 - To the nearest 1% of the radius of the sun, at what radius does the density of the sun fall to 50% of its core density at x=0? (Hint: Use a graphing calculator and estimate x to 0.01)

Problem 3 - What is the estimated density of the sun near its surface at x=0.9 using this polynomial approximation?

Problem 4 - Integrate D(x) throughout the volume of the solar interior to estimate the total mass of the sun in grams. (Use the volume element dV = $4\pi x^2$ dx, and use the fact that for x=1, the physical radius of the sun is 6.9×10^{10} centimeters.)

Problem 1 - Answer; At the core, x=0, do D(0) = 155 grams/cm^3.

Problem 2 - Answer: We want D(x) = 155/2 = 77.5 gm/cm3. Use a graphing calculator, or an Excell spreadsheet, to plot D(x) and slide the cursor along the curve until D(x) = 77.5 .Then read out the value of x. The relevant portion of D(x) is shown in the table below:

X	D(x)
0.08	94.87
0.09	88.77
0.1	82.96
0.11	**77.43**
0.12	72.16
0.13	67.16
0.14	62.41

Problem 3 - Answer: At x=0.9 (i.e. a distance of 90% of the radius of the sun from the center).
D(0.9) = $519(0.9)^4 - 1630(0.9)^3 + 1844(0.9)^2 - 889(0.9) + 155$
D(0.9) = 340.516 - 1188.27 + 1493.64 - 800.10 + 155.00
D(0.9) = 0.786 gm/cm^3.

Problem 4 - Integrate D(x) throughout the volume of the solar interior to estimate the total mass of the sun in grams. (Use the volume element dV = $4\pi x^2$ dx and use the fact that for x=1, the physical radius of the sun is 6.9 x 10^{10} centimeters.). Answer:

$$M = \int_0^1 [519x^4 - 1630x^3 + 1844x^2 - 889x + 155] 4\pi x^2 dx$$

$$M(x) = 4\pi[\frac{519}{7}x^7 - \frac{1630}{6}x^6 + \frac{1844}{5}x^5 - \frac{889}{4}x^4 + \frac{155}{3}x^3]$$

$$M(1) = 4(3.14)[74.14 - 271.67 + 368.80 - 222.25 + 51.67]$$

$$M(1) = 4(3.14)[0.69]$$

$$M(1) = 8.71$$

Since x=1 is a physical distance of R = 6.9 x 10^{10} centimeters, we have to multiply M(1) by (6.9 x $10^{10})^3$ to get M = 8.29 x (3.28 x 10^{32} cm^3) or M = 2.7 x 10^{33} grams. The actual astronomical value is 1.98 x 10^{33} grams. The polynomial approximation for the internal density is a close, but not exact, estimate.

How many stars are in the sky?

One of the very first things that astronomers studied was the number of stars in the sky. From this, they hoped to get a mathematical picture of the shape and extent of the entire Milky Way galaxy. This is perhaps why some cartoons of 'astronomers' often have them sitting at a telescope and tallying stars on a sheet of paper! Naked-eye counts usually number a few thousand, but with increasingly powerful telescopes, fainter stars can be seen and counted, too.

Over the decades, 'star count' sophisticated models have been created, and rendered into approximate mathematical functions that let us explore what we see in the sky. One such approximation, which gives the average number of stars in the sky, is shown below:

$$\mathrm{Log}_{10}N(m) = -0.0003\, m^3 + 0.0019\, m^2 + 0.484\, m - 3.82$$

This polynomial is valid over the range [+4.0, +25.0] and gives the Log_{10} of the total number of stars per square degree fainter than an apparent magnitude of m. For example, at an apparent magnitude of +6.0, which is the limit of vision for most people, the function predicts that $\mathrm{Log}_{10}N(6) = -0.912$ so that there are $10^{-0.912} = 0.12$ stars per square degree of the sky. Because the full sky area is 41,253 square degrees, there are about 5,077 stars brighter than, or equal to, this magnitude.

Problem 1 - A small telescope can detect stars as faint as magnitude +10. If the human eye-limit is +6 magnitudes, how many more stars can the telescope see than the human eye?

Problem 2 - The Hubble Space Telescope can see stars as faint as magnitude +25. About how many stars can the telescope see in an area of the sky the size of the full moon (1/4 square degree)?

Problem 3 - A photograph is taken of a faint star cluster that has an area of 1 square degree. If the astronomer counts 5,237 stars in this area of the sky with magnitudes in the range from +11 to +15, how many of these stars are actually related to the star cluster?

Problem 1 - Answer: From the example, there are 0.12 stars per square degree brighter than +6.0

$$\begin{aligned} Log_{10}N(+10) &= -0.0003\ (10)^3 + 0.0019\ (10)^2 + 0.484\ (10) - 3.82 \\ &= -0.3 + 0.19 + 4.84 - 3.82 \\ &= +0.55 \end{aligned}$$

So there are $10^{.55}$ = 3.55 stars per square degree brighter than +10. Converting this to total stars across the sky (area = 41,253 square degrees) we get 5,077 stars brighter than +6 and 146,448 stars brighter than +10. The number of additional stars that the small telescope will see is then 146,448 - 5,077 = **141,371 stars.**

Problem 2 - Answer: $Log_{10}N(25) = -0.0003\ (25)^3 + 0.0019\ (25)^2 + 0.484\ (25) - 3.82$
$= +4.78$

So the number of stars per square degree is $10^{+4.78}$ = 60,256. For an area of the sky equal to 1/4 square degree we get (60,256) x (0.25) = **15,064 stars**.

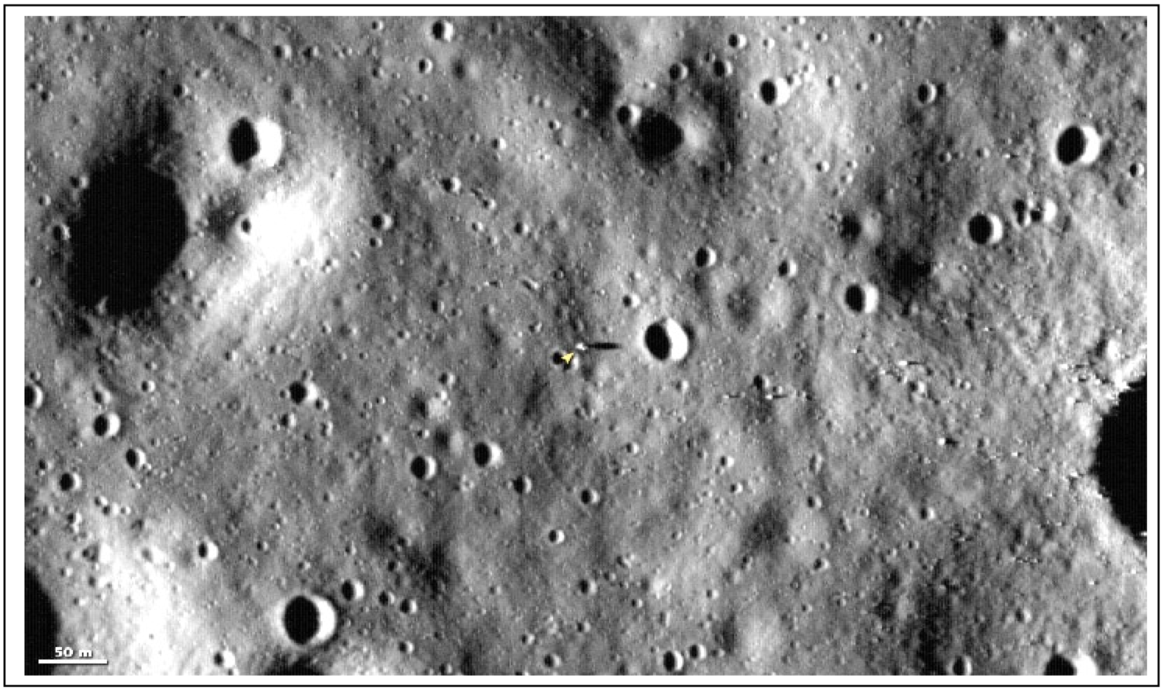
Problem 3 - Answer: $Log_{10}N(15)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+15. $Log_{10}N(10)$ counts up all of the stars in an area of 1 square degree with magnitudes of 0, +1, +2,…+10. The difference between these two will be the number of stars with magnitudes of +11, +12, +13, +15, which is the number of stars in the sky per square degree, in the magnitude range of the star cluster. We then subtract this number from 5,237 to get the number of stars actually in the cluster.

$Log_{10}N(15) = +2.86$ corresponds to $10^{+2.86}$ = 716.1 stars/square degree

$Log_{10}N(11) = +1.33$ corresponds to $10^{+1.33}$ = 21.6 stars/square degree

So the difference between these is 716.1-21.6 = 694.5 stars/deg^2. The star cluster area is 5 square degrees, so we have 694.5 stars/deg^2 x (5 square degrees) = 3,473 stars that we expect to find in the sky in the magnitude range of the star cluster. Since the astronomer counted 5,237 stars in the star cluster field, that means that 3,473 of these stars are probably just part of the general sky population of stars, while only 5,237 - 3,473 = **1,764 stars are actually members of the cluster** itself.

Note to Teacher: Show that the students have to evaluate N first before taking the difference because $Log_{10}N(15) - Log_{10}N(11)$ is not the same as N(15) - N(10).



This image of the 800-meter x 480-meter region near the Apollo-11 landing pad was taken by the Lunar Reconnaissance Orbiter (LRO). It reveals hundreds of craters covering the landing area with sizes as small as 5 meters. The Apollo-11 landing pad is at the center of the image, and is casting a long horizontal shadow to the right of the pad, in the direction of a small crater.

Astronomers use counts of the number of craters per kilometer^2 as a function of crater diameter to determine the age of a given lunar landscape, and the distribution of the sizes of the impactors. Crater counts are also used to determine which areas are safe to land. The power-law function below is based upon the above image from LRO and gives the surface density of craters near the Apollo-11 landing site in terms of craters per kilometer^2 of a given diameter, x, in meters. The range of validity is from 2 meters to 40 meters for this particular lunar area. Apollo-11 astronauts did not find any craters smaller than 2-meters near the landing area.

$$S(x) = 22000\, x^{-2.4} \text{ craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters.

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters.

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? (Assume that the craters do not overlap, which is a good approximation to what the image shows.)

$$S(x) = 22000\, x^{-2.4} \text{ craters/km}^2$$

Problem 1 – Integrate the function S(x) to get the function N(x>m) which gives the number of craters per kilometer^2 with diameters greater than m-meters. Answer: The limits to the definite integral extend from m to infinity:

$$\int_m^{\infty} 22000x^{-2.4}dx \quad = \quad N(x>m) = \frac{22000}{1.4m^{1.4}}$$

Problem 2 - Integrate the function S(x) to get the function N(x<m) which gives the number of craters per kilometer^2 with diameters smaller than m-meters. Answer: The limits to the definite integral extend from 2 to m, because Apollo-11 astronauts did not see any craters smaller than 2 meters (x=2) in this area:

$$\int_2^{m} 22000x^{-2.4}dx \qquad N = \frac{22000}{1.4(2)^{1.4}} - \frac{22000}{1.4m^{1.4}} \qquad \text{so} \qquad N = 5955 - \frac{22000}{1.4m^{1.4}}$$

Problem 3 – The Apollo-11 astronauts surveyed the area shown in the image above in order to find a landing site that was not part of a crater. To two significant figures, what is the maximum fraction of the area in the above image covered by craters larger than 2 meters in diameter? Answer: S(x) gives the number of craters with a diameter of x per km^2. The maximum area occupied by these craters assuming that they are non-overlapping is given by $\pi (x/2)^2$ S(x). The total area covered by non-overlapping craters larger than 2 meters is given by the integral:

$$A = \int_2^{40} \pi \left(\frac{x}{2}\right)^2 22000x^{-2.4}dx$$

so $$A = \frac{22000\pi}{4}\int_2^{40} x^{-0.4}dx$$ then $$A = \frac{22000\pi}{4}\left(\frac{1}{0.6}\right)\left[x^{0.6}\right]_2^{40}$$

$$A = \frac{22000\pi}{4(0.6)}\left[40^{0.6} - 2^{0.6}\right]$$

so that the cratered area is A = 28783 (9.14 - 1.52) = 220,000 square meters. The area in the image is 800 meters x 480 meters = 384,000 square meters, so the cratered area represents 100% x (220,000/384,000) = 57% of the surface area. So, the **maximum area that is covered by craters is 57%.** Note: That means that 43% of the area was safe to land on.

Rotation Velocity of a Galaxy

Spiral galaxy M-101 showing its bright nucleus and spiral arms. The radius of M-101 is about 90,000 light years, which corresponds to x=9 in the formula for V(x). (Hubble image)

Stars orbit the center of a galaxy with speeds that decrease as their orbital distances increase. A simple function, V(x) can model the orbital speeds of stars as a function of their distance, x, from the nucleus of the galaxy:

$$V(x) = \frac{350x}{\left(1+x^2\right)^{\frac{3}{4}}}$$

For example: At a distance of 10,000 light years from the center, x = 1.0 and the rotation speed is V(1.0) = 208 kilometers/sec.

Problem 1 – For small x (i.e. x < 1), what is the limiting form of V(x)?

Problem 2 – For large x, (i.e. x > 1) what is the limiting form of V(x)?

Problem 3 - The radius of M-101 is 90,000 light years. How fast are stars orbiting the center of M-101 according to V(x)? (Hint: At a radius of 90,000 light years, x=9.0. If the units of V(x) are kilometers/sec, what is V(x) at x = 9.0?)

Problem 4 – For what value of x is V(x) maximum?

Problem 5 – For x=1 the physical distance is 10,000 light years. How many years does it take a star to complete one circular orbit at x=1.0 if 1 light year equals 9.5 x 10^{12} km, and there are 3.1 x 10^{7} seconds in a year?

Note: This example of V(x) is for galaxies in which most of the mass is concentrated within their central regions (x < 1), however, astronomers know that this model is not completely accurate. Beyond x = 1, the rotation speeds for some galaxies, including the Milky Way, do not decrease rapidly as suggested by V(x), but actually remain constant. This implies that some galaxies contain substantial amounts of 'Dark Matter' that is not in the form of stars or other known forms of matter.

Problem 1 – For small x, what is the limiting form of V(x)?
Answer: The denominator approaches 1 and so **V(x) = 350x**

Problem 2 – For large x, what is the limiting form of V(x)?
Answer: In the denominator, x^2 dominates over 1 so the denominator approaches $x^{3/2}$ and so $V(x) = 350x/x^{3/2}$ becomes:

$$V(x) = \frac{350}{\sqrt{x}}$$

Problem 3 - The radius of M-101 is 90,000 light years. How fast are stars orbiting the center of M-101 according to V(x)? (Hint: At a radius of 90,000 light years, x=9.0. If the units of V(x) are kilometers/sec, what is V(x) at x = 9.0?)
Answer:

$$V(9) = \frac{350(9)}{(1+9^2)^{\frac{3}{4}}}$$ = **26 kilometers/sec**

Note: X is a pure number. It represents the ratio X = (d /10,000 light years) where d is a physical distance in units of light years. Example: at a physical distance of 40,000 light years from the center of the galaxy, x = 40,000 LY/10,000 LY so x = 4.0. The rotation speed of stars at this distance is just $V(4) = 350(4)/(1+4^2)^{3/4}$ = 167 kilometers/sec.

Problem 4 – For what value of x is V(x) maximum?
Answer: Students can graph this function on a calculator. The maximum should occur near **x = 1.4** with a value V(x) = 217 km/sec.

Advanced students can use differential calculus and solve for x in the equation dV(x)/dx = 0.

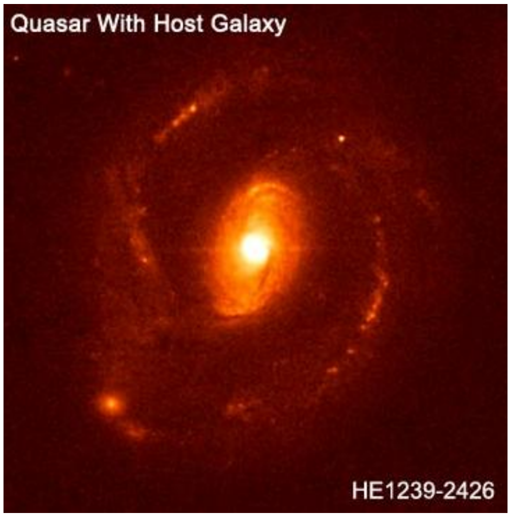
$$\frac{dV(x)}{dx} = \frac{(350x)(3/4)(1+x^2)^{\frac{-1}{4}}(2x) - 350(1+x^2)^{\frac{3}{4}}}{(1+x^2)^{\frac{3}{2}}}$$

so after some algebra:

$$0 = 1 - \frac{3x^2}{2(1+x^2)}$$ so $2 + 2x^2 = 3x^2$ and $x = (2)^{1/2}$ = **1.414**

Problem 5 – For x=1 the physical distance is 10,000 light years. How many years does it take a star to complete one circular orbit at x=1.0 if 1 light year equals 9.5×10^{12} km, and there are 3.1×10^7 seconds in a year? Answer: For x=1 the physical distance is 10,000 light years or 9×10^{16} kilometers. The circumference of the orbit is $2 \pi R = 2 (3.141) (9.5 \times 10^{16}$ km) = 6.0×10^{17} kilometers. The speed is V(1) = 208 km/sec, so the time in seconds is $T = 6 \times 10^{17}$ kilometers / (208 km/sec) = 2.9×10^{15} seconds. Since there are 3.1×10^7 seconds/year, it will take **93 million years for a star to orbit once-around the center of M-101**.

How Many Quasars Are There?



This Hubble Space Telescope image shows the brilliant 'quasar' core of a distant galaxy located 1.5 billion light years from Earth in the direction of the constellation Corvus the Crow.

Quasars are among the most luminous galaxies in the universe, and because of this, astronomers can detect them to great distances exceeding nearly 10 billion light years from Earth. Originally discovered as peculiar faint 'blue stars' in 1964, astronomers have counted up their numbers as a function of their brightness, expressed according to the logarithmic stellar apparent magnitude scale, m. The function that provides a good match to the quasar surface density, N(m), defined over the domain [+13.0, +23.0], is given by:

$$\frac{dN(m)}{dm} = 10^{f(m)}$$

where f(m) is a piecewise function defined as follows:

f(m) = +0.7m -12.0 for $+13.0 \leq m \leq +18.0$
f(m) = +0.5m -9.0 for $+19.0 \leq m \leq +23.0$

Problem 1 - The integral of dN(m)/dm is the total number of quasars present in one square degree of the sky. As a comparison, the full moon subtends an area of 1/4 square degree. How many quasars does the function predict that have magnitudes between +13.0 and +18.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Problem 2 - How many quasars does the function predict that have magnitudes between +19.0 and +23.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Problem 3 - An astronomer wants to study quasars in the brightness interval +13.0 < m < +18.0. What is the minimum sky area, in square degrees, that she needs to photograph in order to have one quasar present in the photograph?

Problem 4 - Combining the quasar sky densities from Problem 1 and 2, and the fact that the sky has a total angular area of 41,253 deg^2, how many quasars are there with magnitudes in the range from +13 to +23, inclusively?

Problem 1 - The integral of dN(m)/dm is the total number of quasars present in one square degree of the sky. As a comparison, the full moon subtends an area of 1/4 square degree. How many quasars does the function predict that have magnitudes between +13.0 and +18.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Answer: $N = \int_{13}^{18} \frac{dN(m)}{dm} dm$ so from the definition of f(m) in the appropriate interval,

we get $N = \int_{13}^{18} 10^{0.7m-12} dm$ With u = (0.7m-12) , du = 0.7 dm, and using $10^u = e^{2.3u}$ the

equivalent integral becomes: $N = \frac{1}{0.7}\int_{-2.9}^{0.6} e^{2.3u} du$ or $N = \frac{1}{0.7(2.3)}\int_{-1.26}^{0.26} e^{x} dx$

so N = 0.62 [$e^{0.26}$ - $e^{-1.26}$] = 0.62[1.29 - 0.28] = 0.63 quasars per deg^2. The full moon has an area of 1/4 square degrees, so there will be 5 x (1/4 deg^2) x (0.63 quasars/deg^2) = **0.78 quasars in this sky area.**

Problem 2 - How many quasars does the function predict that have magnitudes between +19.0 and +23.0 inclusively, over an area of the sky similar to five times the area of the full moon?

Answer; $N = \int_{19}^{23} 10^{0.5m-9} dm$ With u = (0.5m-9) , du = 0.5 dm, and using $10^u = e^{2.3u}$

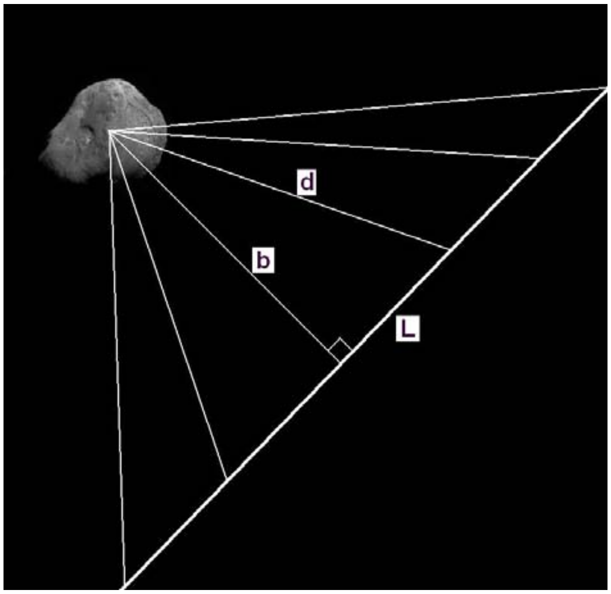
the equivalent integral becomes: $N = \frac{1}{0.5(2.3)}\int_{0.22}^{1.1} e^{x} dx$

so N = 0.87 [$e^{1.1}$ - $e^{0.22}$] = 0.87 [3.00 - 1.24] = 1.53 quasars/deg^2. For an area equal to 5 times the full moon, there are 5 x (1/4 deg^2) x (1.53 quasars/deg^2) = **1.9 quasars in this sky area.**

Problem 3 - An astronomer wants to study quasars in the brightness interval +13.0 < m < +18.0. What is the minimum sky area, in square degrees, that she needs to photograph in order to have one quasar present in the photograph? Answer; From Problem 1, the integral over this magnitude range gives an surface area of 0.63 quasars per square degree. To find one quasar, you need to photograph an area of 1/0.63 square degrees or **1.6 square degrees**. Note: The full moon occupies an area of 0.25 square degrees, so the astronomer will have to photograph an area of the sky about six times the area of the full moon.

Problem 4 - Answer: The combined quasar density is 0.63 + 1.53 = 1.16 quasars/deg^2 , so over the entire sky, there are about 1.16 x 41,253 = **47,853 quasars** in this magnitude range.

Deep Impact - Comet Flyby



On July 4, 2005, the Deep Impact spacecraft flew by the comet Tempel-1 along a path shown in the figure to the left, at a speed of V=10 km/sec. Its closest distance to the comet was b = 500 kilometers at a time, t=0. The distance traveled along the path is given by L = Vt.

The diameter of the comet is D = 8 kilometers, and the distance to the comet in kilometers is d(t), so the angular diameter of the comet in arcminutes is given by

$$\Theta(t) = 3438 \frac{D}{d(t)}$$

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as *d(t)*?

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory, defined by the variables V, b and L? Simplify the formula by defining two constants B = 3438L/b and c = V^2/b^2.

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys by?

Problem 4 - What was the angular diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Problem 1 - What is the formula for the distance to the comet from the spacecraft defined as d(t)? Answer: Use the Pythagorean Theorem and the diagram to determine that

$$d(t) = \sqrt{b^2 + V^2 t^2}$$

Problem 2 - What is the formula for the angular diameter of the comet as seen from the spacecraft at any time, t, along the trajectory? Simplify the formula for Θ(t) by defining two constants B = 3438D/b and c = V^2/b^2. Answer:

$$\Theta(t) = \frac{B}{\sqrt{1+ct^2}}$$

Problem 3 - What is the exact numerical formula for the rate-of-change in time of the angular size of the Tempel-1 as viewed by the spacecraft as it flys by? Answer; Evaluate B and c for the specific values of D, b and V for the Tempel-1 flyby. Remember to use consistent units (all units in meters, and meters/sec). B = 3438 x 8,000 meters/(500,000 meters) so B = 55 arcminutes, and c = (10,000 meters/sec)^2/(500,000 meters)^2 so c = 0.0004 sec^{-2}. Then the formula becomes,

$$\Theta(t) = \frac{55}{\sqrt{1+0.0004t^2}}$$

where θ(t) will be the comet angular diameter in arcminutes. The rate-of-change of Θ(t) with respect to time is just its first-derivative, which we then evaluate for B=55 and c=0.0004. The formula will provide answers in units of arcminutes/sec:

$$\frac{d\Theta(t)}{dt} = -\frac{1}{2}B\left(1+ct^2\right)^{-3/2}(2ct)$$

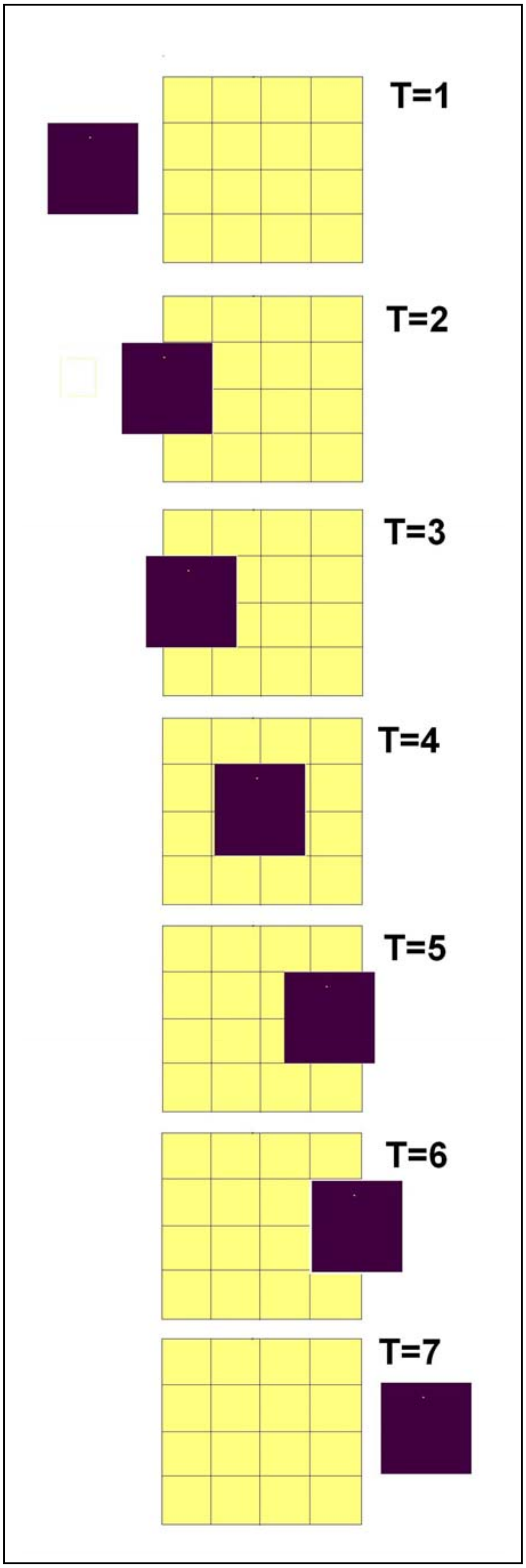
or

$$\frac{d\Theta(t)}{dt} = -\frac{0.022t}{\left(1+0.0004t^2\right)^{3/2}}$$

Problem 4 - What was the angular diameter of Tempel-1 at the closest approach, and how fast will the angular size be decreasing when Tempel-1 reaches one-half its maximum angular size?

Answer: For t=0 we get $\theta(0) = 55$ arcminutes. To decrease by a factor of two its final diameter has to be 27.5 arcminutes, so θ(t)=27.5, and solve for t to get **t= 86.6 seconds.**

From the derivative formula, and evaluating it at t=86.6 seconds, we find that
dθ/dt = **- 0.24 arcminutes/sec**.

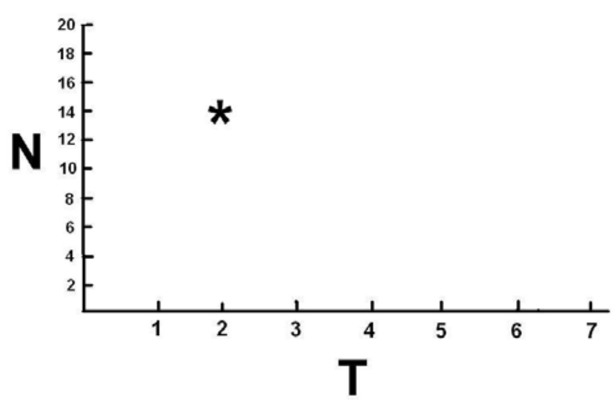


NASA's Kepler spacecraft recently announced the discovery of five new planets orbiting distant stars. The satellite measures the dimming of the light from these stars as planets pass across the face of the star as viewed from Earth. To see how this works, lets look at a simple model.

In the Bizarro Universe, stars and planets are cubical, hot spherical. Bizarro astronomers search for distant planets around other stars by watching planets pass across the face of the stars and cause the light to dim.

Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?



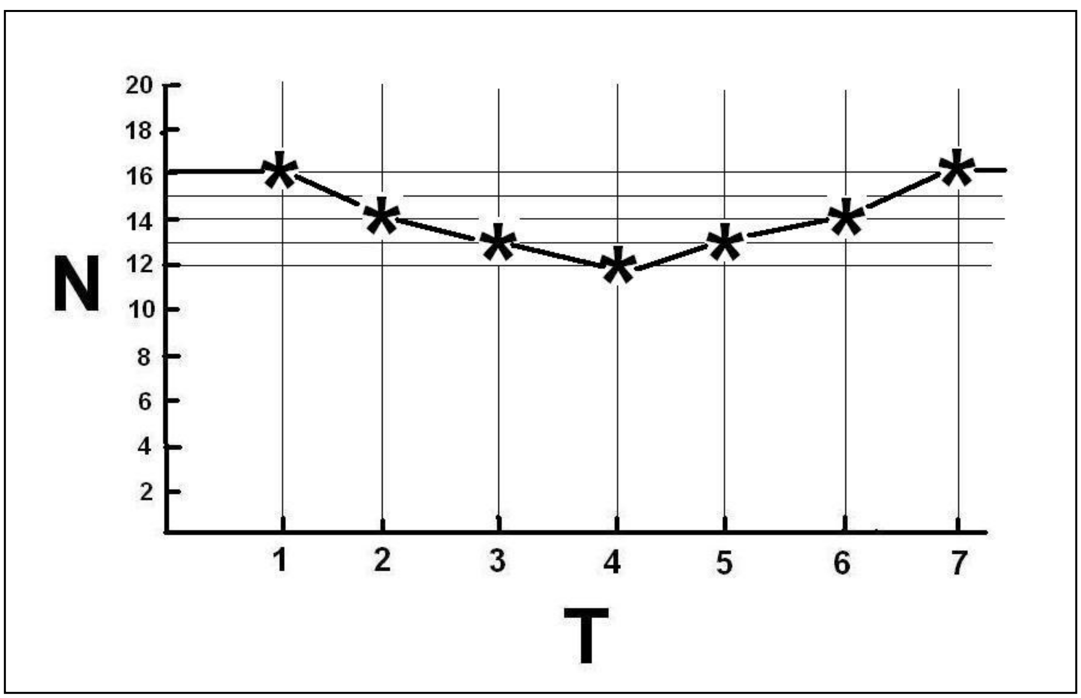
Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Answer: **Count the number of yellow squares in the star and plot these for each value of T in the graph as shown below. Note, for T= 3 and 5, the black square of the planet occupies 2 full squares and 2 half squares for a total of 2 + 1/2 + 1/2 = 3 squares covered, so there are 16 - 3 = 13 squares remaining that are yellow.**

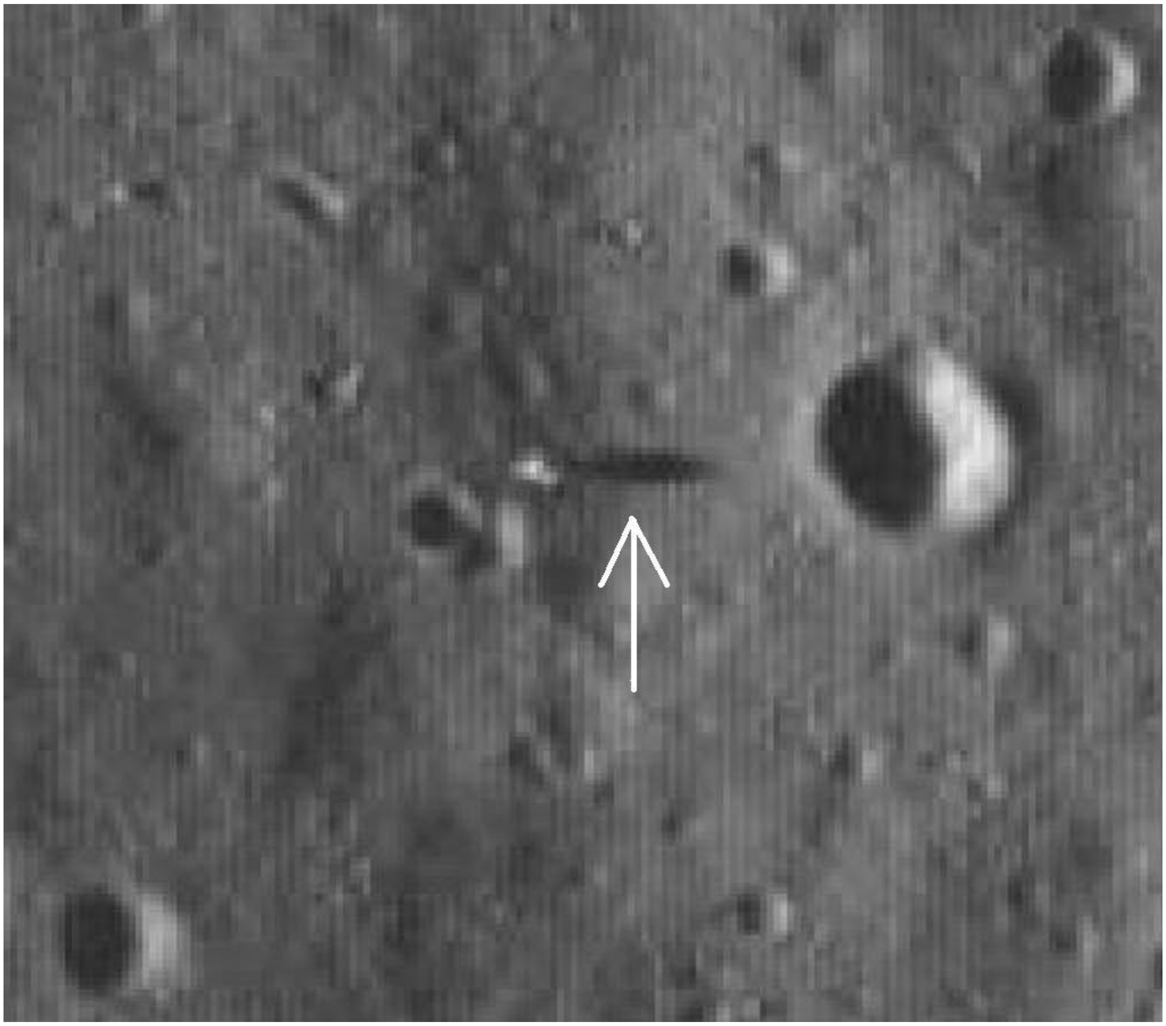
Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?

Answer: The light curve shows that the planet caused the light from the star to decrease from 16 units to 12 units because the planet blocked 16-12 = 4 units of the stars surface area. That means that the planet squares occupy 4/16 of the stars area as seen by the astronomers. The area of the star is just the area of a square, so the area of the square planet is 4/16 of the stars area or Ap = 4/16 x Astar. Since the star as a width of Wstar = 1 million kilometers, the planet will have a width of $Wp = Wstar\sqrt{\frac{4}{16}}$ or **500,000 kilometers**.

The amount of star light dimming is proportional to the ratio of the area of the planet and the star facing the observer. The Kepler satellite can detect changes by as little as 0.0001 in the light from a star, so the smallest planets it can detect have diameters about 1/100 the size of the stars that they orbit. For a star with a diameter of the sun, 1.4 million kilometers, the smallest planet detectable by the Transit Method has a diameter about equal to 14,000 kilometers or about the size of Earth.



LRO Sees Apollo 11 on the Moon!



The LRO satellite recently imaged the Apollo 11 landing area on the surface of the moon. The above (172 pixels wide x 171 pixels high) image shows this area and is 172 meters wide.

Problem 1 - Determine the scale of the image in meters per millimeter and meters per pixel? What is the diameter, in meters, of A) the largest crater? B) the smallest crater?

Problem 2 - The shadow identified by the arrow was cast by the Lunar Landing Module which is about 3.5 meters tall. Using A) trigonometry, or a B) scaled drawing and a protractor, what was the sun angle at the time of the photograph?

Problem 3 - Are there any individual boulders larger than 1 meter across in this area?

Problem 1 - Determine the scale of the image in meters per millimeter and meters per pixel? What is the diameter, in meters, of A) the largest crater? B) the smallest crater?

Answer: The image is 153 millimeters wide, which corresponds to 172 meters, so the scale is **1.1 meters per millimeter,** and the image is 172 pixels wide so the resolution is 172 pixels/153 meters = **1.1 meters/pixel.**

The largest crater is about 25mm x 30 mm in size, which corresponds to 25mm x 1.1 meters/mm = 28 meters wide, and 30 mm x 1.1 = 33 meters long, for an **average size of about 30 meters across.** B) The smallest discernable features are about 1 to 2 mm wide, which corresponds to an actual size of about 1-2 pixels or 1 to 2 meters. Note, there can be no actual details smaller than the pixel resolution of the image (1.1 meters).

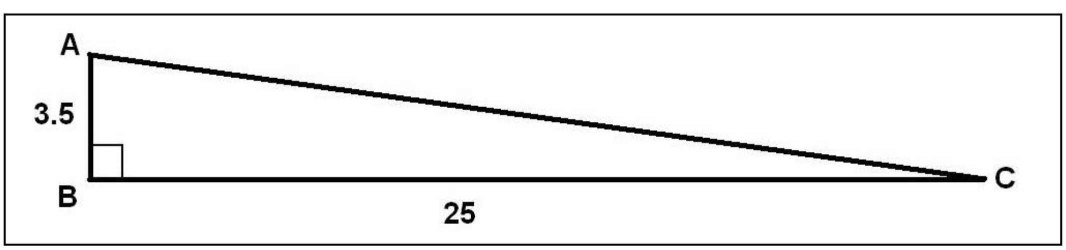
Problem 2 - The shadow near the center of the picture was cast by the Lunar Landing Module which is about 3.5 meters tall. Using A) trigonometry, or a B) scaled drawing and a protractor, what was the sun angle at the time of the photograph?

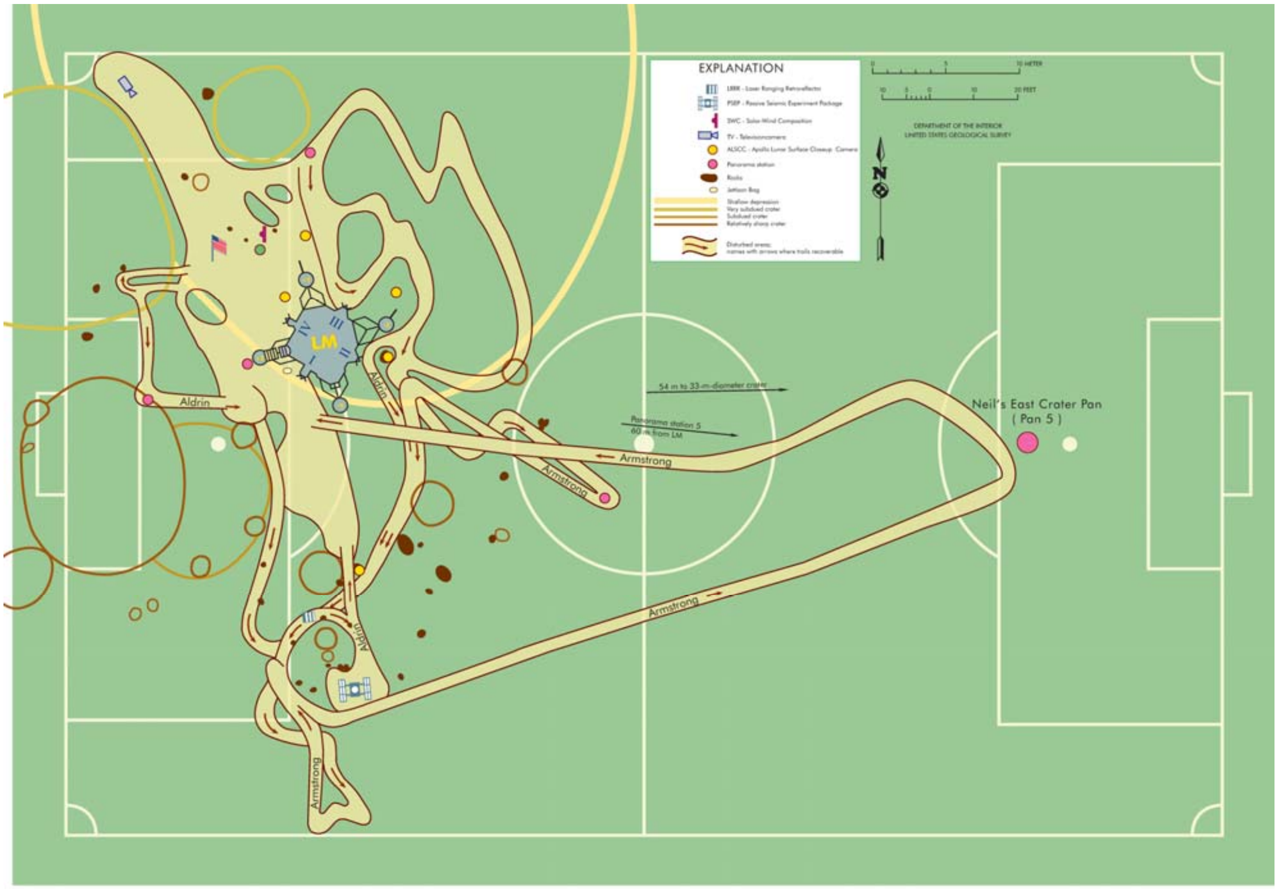
Answer: The length of the shadow from the base of the lander is about 23 millimeters or in actual length, 23 x 1.1 = 25 meters. This makes a right triangle, ABC, with a base length AB= 25 meters and an altitude of AC=3.5 meters and a hypotenuse located along BC, with the right-angle defined as ABC.

Method 1: From trigonometry, Tan(theta) = 3.5 meters/25 meters = 0.14 so the angle whose tangent is 0.14 is **theta=8.0 degrees.**

Method 2: A scaled drawing is shown below, and a protractor may be used to measure the angle directly from the diagram.

Problem 3 - Are there any individual boulders larger than 1 meter across in this area?
Answer: No, because they would have shadows about 7 meters long (1/4 the Apollo 11 module) and there are no such shadows in the image, other than the Apollo-11 Landing Module itself. This area of the moon seems to be boulder-free at a resolution of 1 meter, which is why it was selected by Apollo-11 astronauts for a landing site.





The NASA Lunar Reconnaissance Orbiter (LRO), launched in 2009, will be able to see details on the lunar surface at much higher resolution than any previous lunar mapping mission. The images will have a resolution of about 0.7 meters per pixel. The map shows the area surrounding the Apollo 11 landing site where astronauts deployed experiments and walked around the landing site to gather rock and soil samples. The grid lines show a standard soccer field in comparison, with a distance between the white goal boundaries of 110 meters.

Problem 1 - Use a millimeter ruler and the information provided to determine the scale of the figure in meters per millimeter.

Problem 2 - Draw an overlay of 10 rows and 10 columns on the above figure at a location near the Apollo-11 landing area, with individual cells representing the individual LRO pixels.

Problem 3 - Will LRO be able to see: A) the Lunar Module 'LM' marked on the map? B) The discolorations (shown in yellow) of the lunar soil caused by the paths taken by the astronauts? C) Details on the LM?

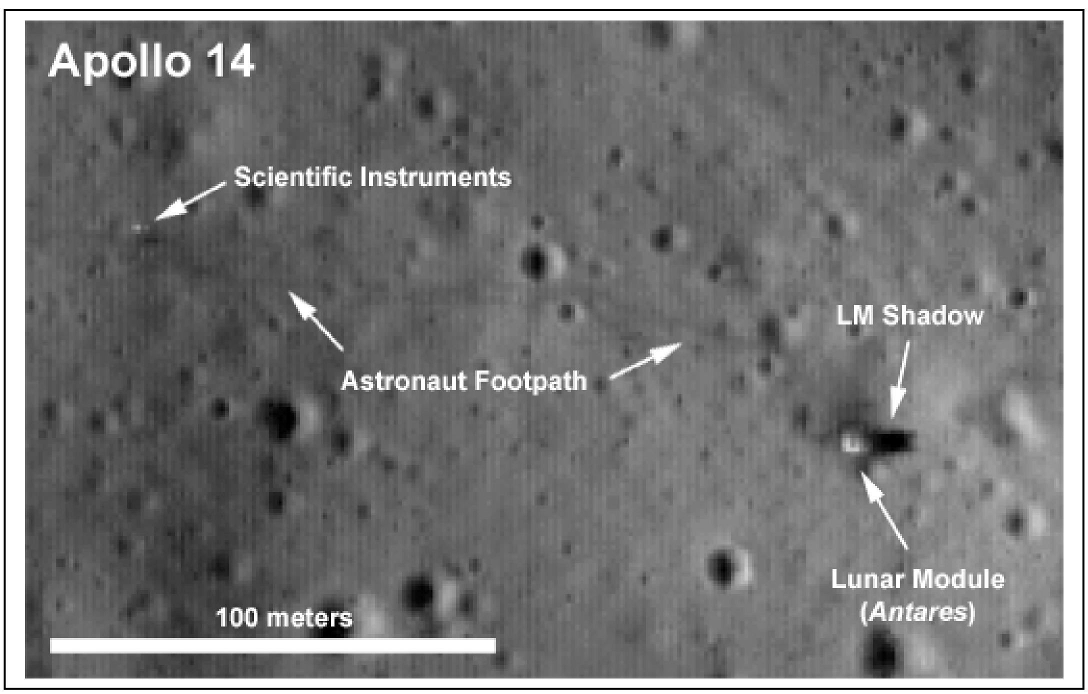
Problem 1 - Use a millimeter ruler and the information provided to determine the scale of the figure in meters per millimeter.

Answer: A millimeter ruler placed along the long-edge of the figure would indicate that the distance between the vertical white goal lines is 153 millimeters .Since this corresponds to 110 meters on the actual field, the scale of this figure is 110 meters/153 mm = *0.7 meters per millimeter.* Alternately, students may measure the width of the green area which is 167 millimeters and corresponds to 120 meters.

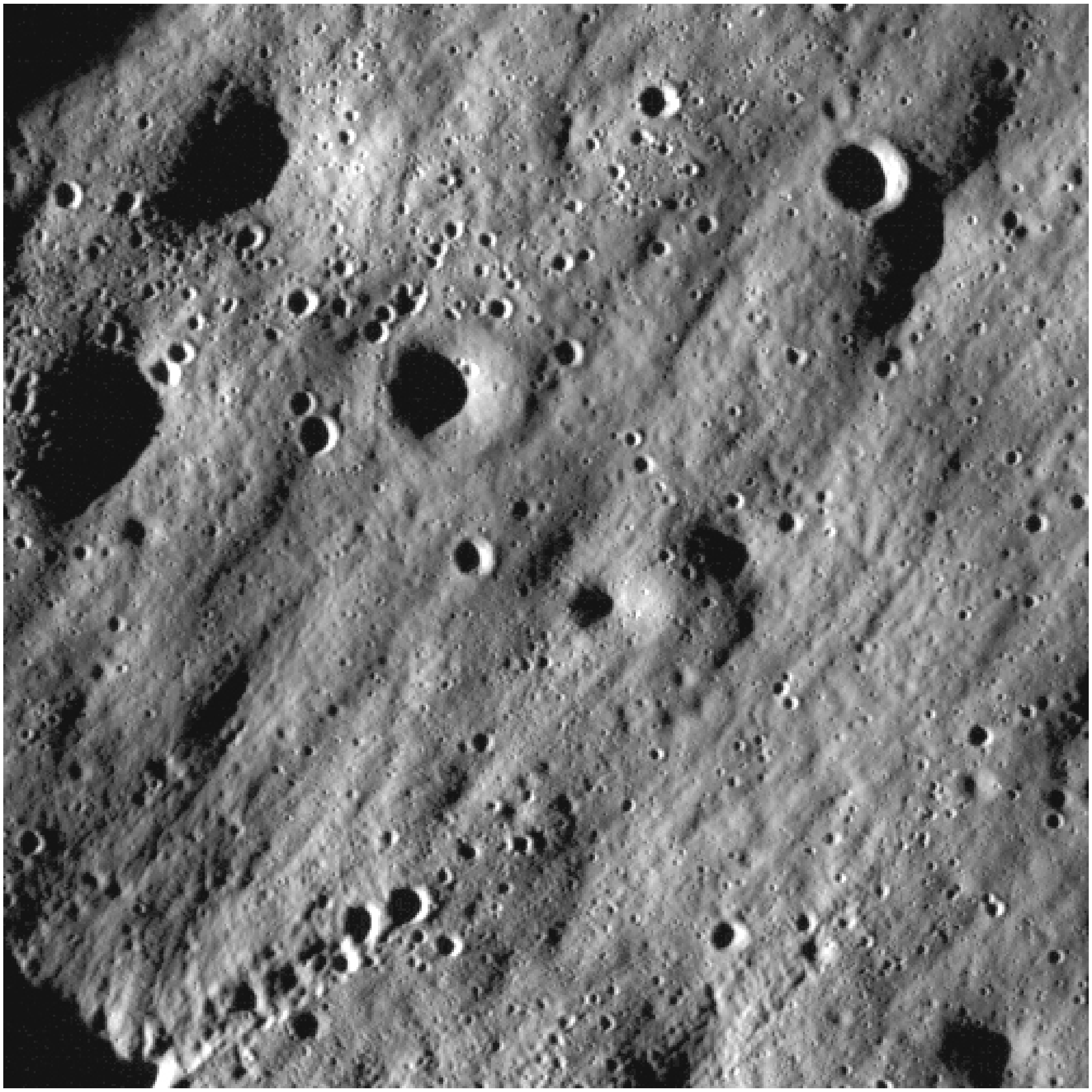
Problem 2 - Draw an overlay of 10 rows and 10 columns on the above figure at a location near the Apollo-11 landing area, with individual cells representing the individual LRO pixels. *Answer: The LRO pixels are 0.7 meters wide, so the students would have to draw a grid with rows and columns 1 millimeter wide.*

Problem 3 - Will LRO be able to see: A) The Lunar Module 'LM' marked on the map? B) The discolorations (shown in yellow) of the lunar soil caused by the paths taken by the astronauts? C) Details on the LM?

Answer: Below is an example of the details seen by the LRO cameras at the Apollo 14 landing site!



LRO's First Image of Mare Nubium



This is one of the first images taken by LRO showing details in Mare Nubium. The width of the image is 700 meters (500 pixels).

Problem 1 - Use a millimeter ruler to determine the scale of the image in meters per millimeter, and meters per pixel.

Problem 2 – What is the diameter, in meters, of the smallest recognizable crater you can find?

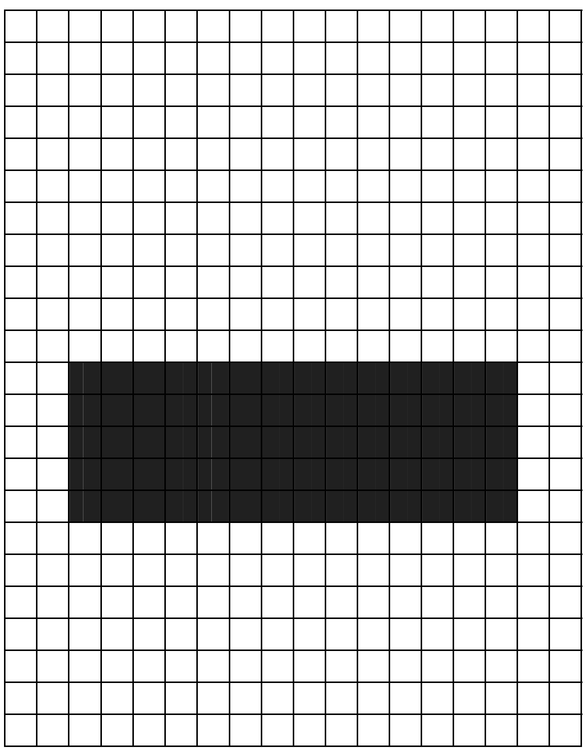
Problem 3 – Suppose your house is 42 feet wide and 60 feet long, and its sits on a property that is 75 feet wide and 96 feet long. Draw two squares at the same pixel scale as the LRO image. (Assume 1 meter = 3 feet)

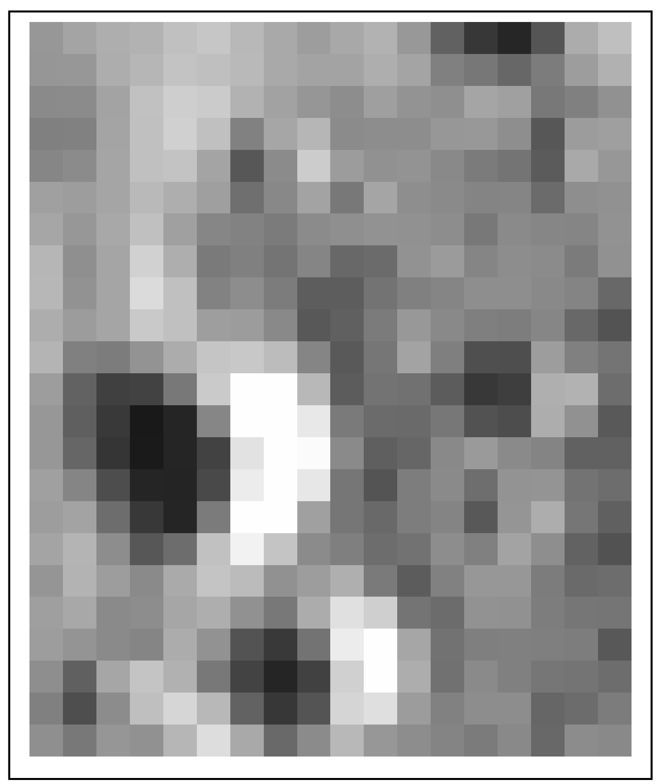
Answer Key

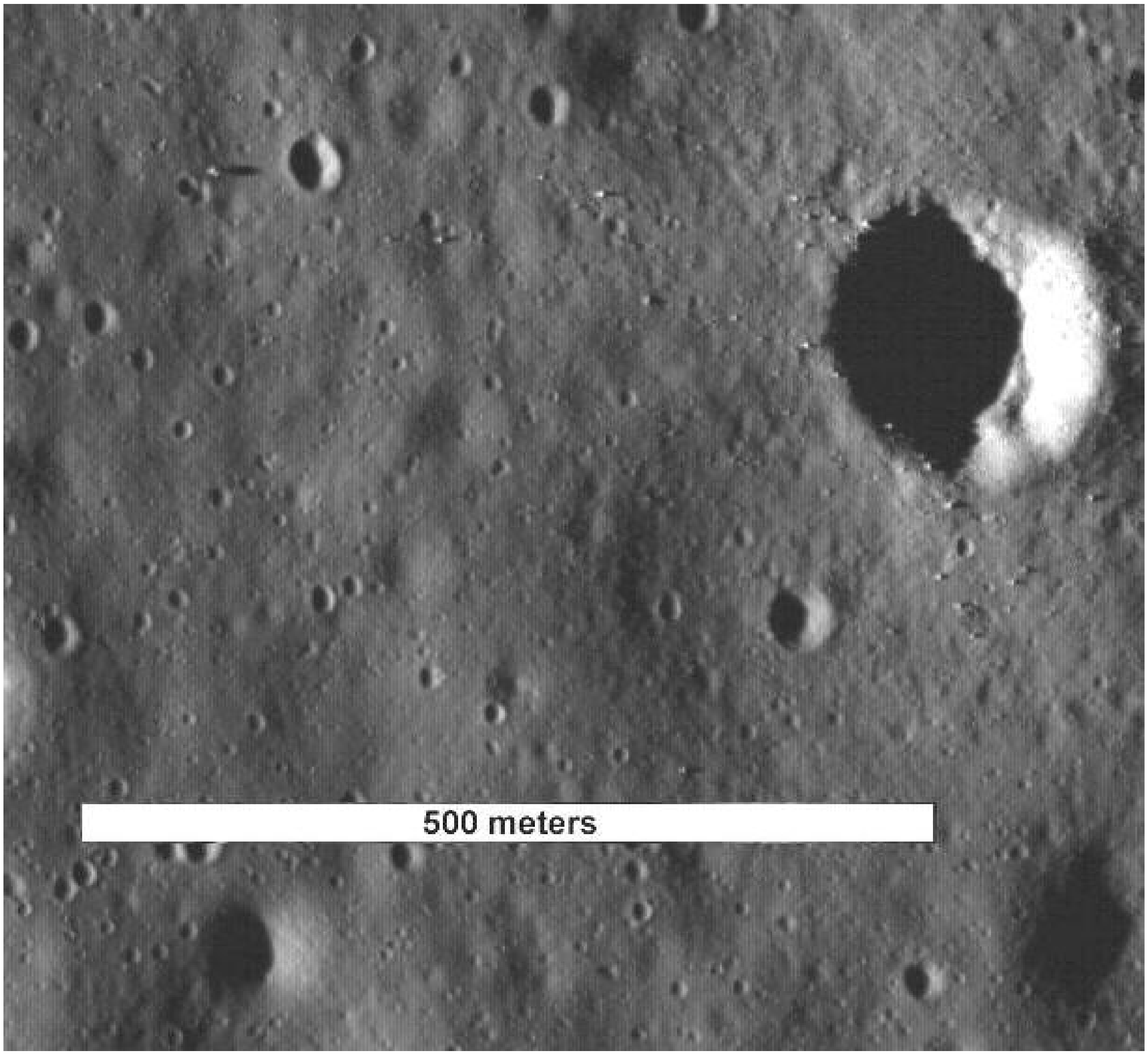
Problem 1 - Use a millimeter ruler to determine the scale of the image in meters per millimeter, and meters per pixel. Answer: Width = 153 millimeters so the scale is 700 meters/153 mm = **4.6 meters/mm**, and 700 meters/500 pixels = **1.4 meters/pixel.**

Problem 2 – What is the diameter, in meters, of the smallest recognizable crater you can find? Answer: Students should see craters as small as 0.5 millimeters or 0.5 mm x 4.6 m/mm = **2.3 meters.**

Problem 3 – Suppose your house is 42 feet wide and 60 feet long, and its sits on a property that is 75 feet wide and 96 feet long. Draw two squares at the same pixel scale as the LRO image. Answer: First convert the feet into metric units. Three feet equals about 1 meter, so the yard measures 75 feet x 96 feet = 25 meters x 32 meters, and the house measures 7 meters x 20 meters. At the scale of the LRO image of 1.4 meters/pixel, the property is **18 pixels x 23 pixels,** and the house measures **5 pixels x 14 pixels.** See sketch below, and the comparison lunar image enlargement.







The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows the region near the Apollo-11 landing area. The Lunar Module (LM) and its shadow are shown to the left of the large crater in the upper right corner. Sunlight comes from the left, and so craters will have their shadow zones on the left-hand side of their depressions. Objects above the surface, like the Apollo LM, will be bright on the left side, and have their right-side in shadow.

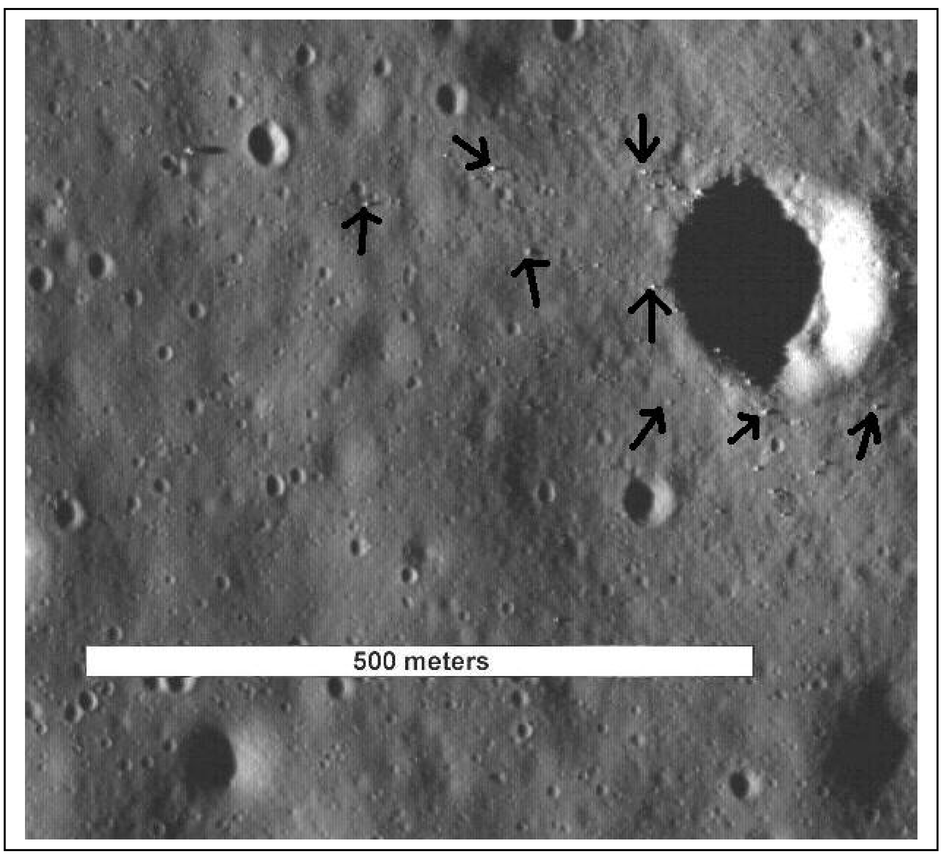
Problem 1 - From the information given, and using a millimeter ruler: A) determine the scale of the image (meters per millimeter); and B)the length of the Apollo LM shadow.

Problem 2 - Find as many boulders as you can, and determine their approximate size using the height of the LM (3.5 meters) and the length of the LM shadow to establish their sizes. Do you think there are smaller boulders that the ones you can easily spot?

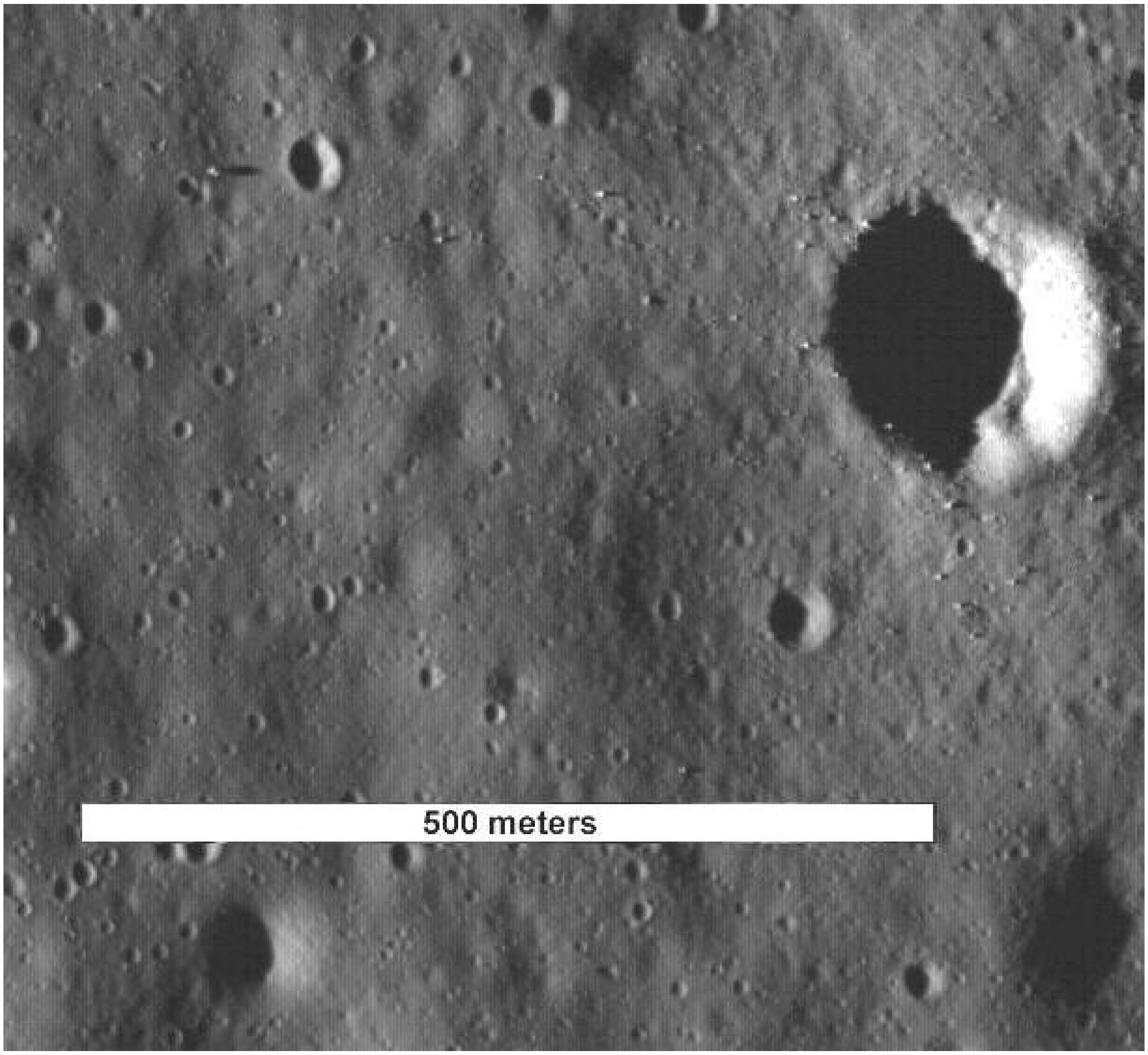
Problem 1 - From the information given, and using a millimeter ruler: A) determine the scale of the image (meters per millimeter); and B) the length of the Apollo LM shadow.

Answer: Measure the length of the white bar on the image, which corresponds to a physical length of 500 meters on the lunar surface. Students should get answers near 111 millimeters. A) so the scale of the image is **4.5 meters/millimeter.** The length of the LM shadow is 6 mm, so its physical length is 6 x 4.5 = **27 meters.**

Problem 2 - Find as many boulders as you can, and determine their approximate size using the height of the LM (3.5 meters) and the length of the LM shadow to establish their sizes. Do you think there are smaller boulders that the ones you can easily spot?
Answer: The image below shows some examples. The LM is 3.5 meters tall and casts a 27-meter shadow, so by using similar triangles and proportions, that means that a 1-meter boulder will cast a shadow that is 1/3.5 x (27 meters) = 7.7 meters long. At the image scale of 4.5 meters/mm, that corresponds to a length on the image that is just under 2-mm long. Students may create tables for actual, numbered, boulders in the image and determine more accurate boulder sizes. Students should realize that, although they cannot directly see boulders smaller than about 1-pixel (1.4 meters) they can easily see the shadows of boulders much smaller than this. For example, a shadow that is 1 millimeter long is from an unobservable boulder about 0.5 meters across!



LRO Explores Lunar Surface Cratering



The LRO satellite recently imaged the surface of the moon at a resolution of 1.4 meters/pixel. The above image shows a region near the Apollo-11 landing site. The Lunar Module (LM) can be seen from its very long shadow near the large crater in the upper left corner of the image.

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Problem 2 - Measure all of the craters larger than or equal to 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_c}$: the Areal Crater Density (craters/km^2).

Problem 3 - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_c}$. About what is the average distance between craters with a diameter close to 5 meters?

Problem 1 - With a millimeter ruler, determine the scale of this image in meters/mm. What is the total area of this image in square-kilometers?

Answer: The 500-meter bar is 111 millimeters long so the scale is 500 M/111mm = **4.5 meters/mm. The image has the dimensions of 149 mm x 136 mm or 670m x 612 m for an area of 0.41 kilometers^2.**

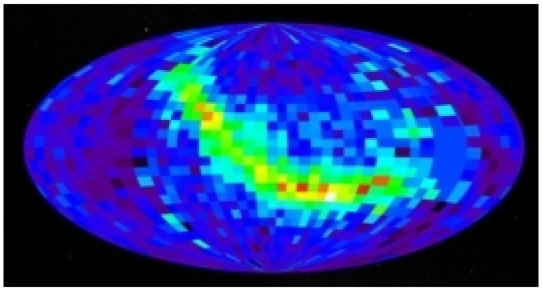
Problem 2 - Measure all of the craters larger than 9 meters and create a histogram of the numbers of the craters. Divide the number of craters in each bin, by the total area of the field, to get $\mathbf{A_C}$: the Areal Crater Density (craters/km^2). Answer: The following table shows an example. Students bin intervals may differ.

Crater diameter (mm)	Crater diameter (meters)	Number of craters close to this size	Areal Density	**Problem 3** Average distance in kilometers
2 mm	9	70	70/0.41 = 171	**0.08**
4 mm	18	6	15	**0.25**
6 mm	27	3	7	**0.38**
8 mm	36	2	5	**0.44**
10 mm	45	1	2	**0.71**

Students may extend this table to include craters of 1-mm diameter and also the single, very large crater that is 35mm in diameter. The number of counted craters, especially in the smallest bins, will vary. Student data may be averaged together to improve accuracy in each bin.

Problem 3 - The average distance between craters of a given size is found by taking the square-root of the reciprocal of $\mathbf{A_C}$. About what is the average distance between craters with a diameter close to 5 meters?

Answer: See above table for tabulated values. Students may also convert the answers to meters. For example, '0.08 km' = 80 meters. Students will need to estimate the Areal Crater Density for craters just below the tabulated threshold of 9 meters. This can be done by estimating the shape of the plotted curve through the points, and extrapolating it to 5 meters. It is also possible to use Excel Spreadsheets by entering the data and plotting the 'scatter plot' with a trendline added. Reasonable values for the Areal Creater Density would range from 171 craters/km^2 to 1000 craters/km^2 ,which lead to distances **between 80 meters and 30 meters, but probably closer to 30 meters given the rapidly decreasing trend of the curve based on the data in the bins for 18-meter and 9-meter crater diameters**



NASA's IBEX satellite recently made headlines by creating a picture of the entire sky, not using light but by using cosmic particles called ENAs (Energetic Neutral Atoms). These fast-moving atoms flow through the solar system. Some of them reach Earth, where they can be captured by the IBEX satellite. By counting how many of these ENAs the satellite sees in different directions in the sky, IBEX can create a unique 'picture' of where ENAs are coming from in space.

The big surprise was that they were not coming from all over the sky as expected. They were also coming from a specific band of directions as we see in the image to the left. This image has the same kind of geometry as the map of the Earth below it! It is called a Mollweide Projection, except that instead of graphing geographic points on Earth, the IBEX image shows points in outer space!

	A	B	C	D	E
1					
2					
3					
4					
5					

Data String:
A5, E2, B2, D4, C3, A1, E4, C3, D4, B2, D4, B3, C4, E5, D5, D4, C2, D3, B1, E5, A2, C3, D5, C5, D4, E4, D3, C4, B4, D2, E3, C1, B5, A3, E1, A4, D1, B3, C2, E3

The IBEX satellite detected a series of particles entering its ENA instrument, and was able to determine the direction that each particle came from in the sky. The grid above shows a portion of the sky as a 5x5 grid with columns labeled by their letter and rows by their number. The data string to the right shows the detections of individual ENA particles with their direction indicated by their cell. 'A5, E2, B2…' means that the first ENA particle came from the direction of cell 'A5', the second from cell 'E2' and the third from cell 'B2' and so on. In some ways this process is like the 'call out' during a Bingo game, except that you keep track of the particle 'tokens' in each square to build a picture! Let's look at an example of constructing an ENA image.

Problem 1 - From the hypothetical data string, tally the number of particles detected in each sky cell in the grid. Select colors to represent the number of ENAs to create an 'image' of the sky in ENAs! How many particles were reported by this data string?

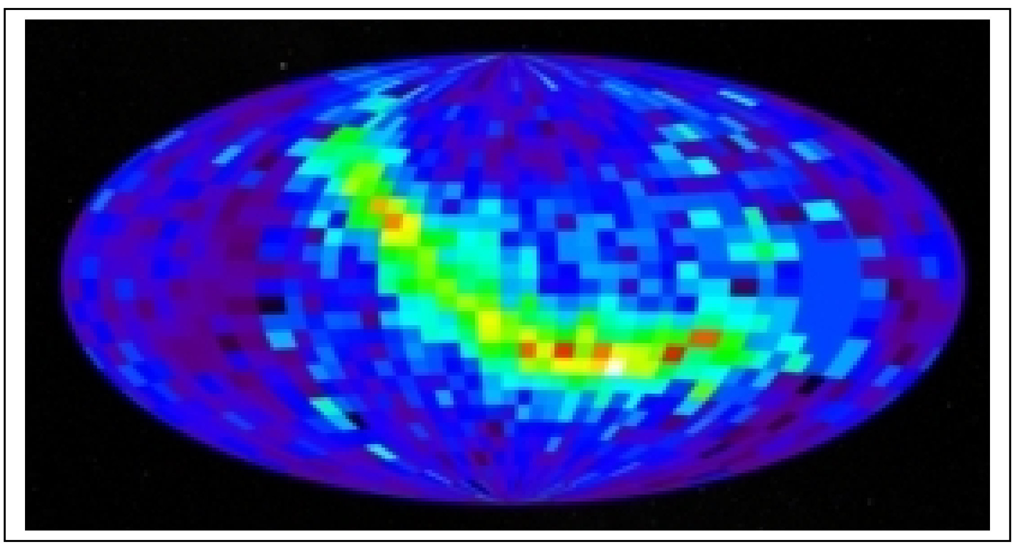
Problem 2 - Suppose that cell B2 is in the direction of the constellation Auriga, cell C3 is towards Taurus and cell D4 is towards Orion, from which constellation in the sky were most of the ENAs detected?

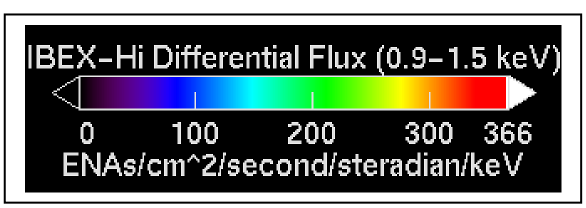
	A	B	C	D	E
1	1	1	1	1	1
2	1	2	2	1	1
3	1	2	3	2	2
4	1	1	2	5	2
5	1	1	1	2	2

Problem 1 - Answer above. Students can select yellow = 5, orange=3, red=2 and blue=1 as an example and colorize the table. This is an example of using 'false color' to highlight data in order to reveal patterns. This technique is used by scientists, but should never be confused with the 'actual' color of an object. If you add up the number of ENAS in the data stream you get **40 ENAs** being reported.

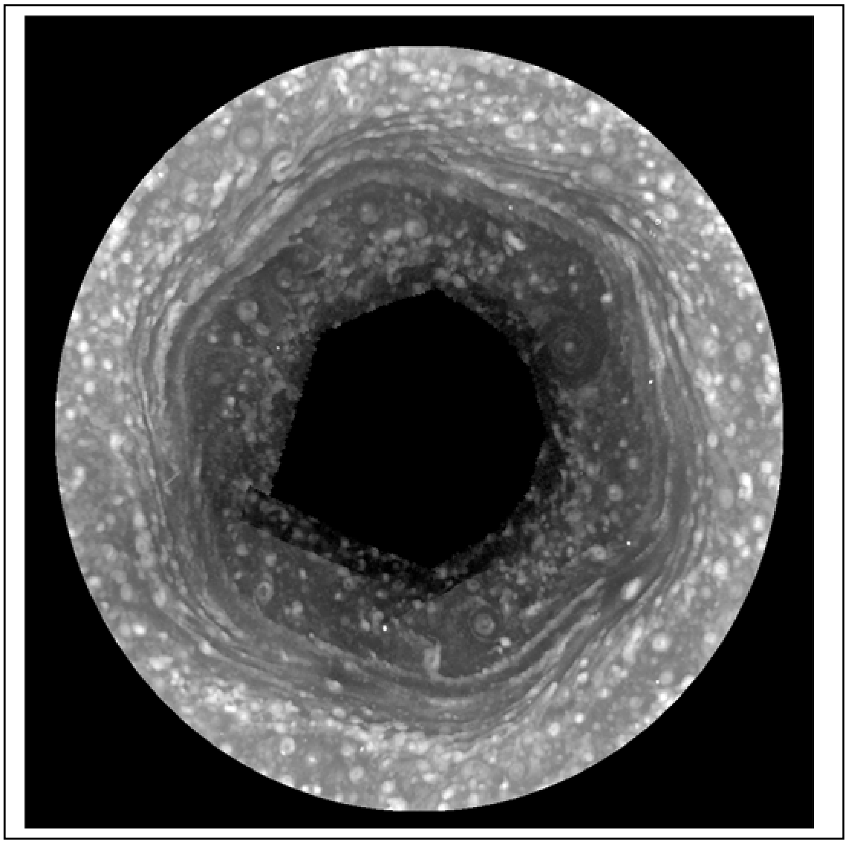
Problem 2 - The most ENAs recorded in any of the cells is '5' from Cell D4, which is in the direction of the constellation Orion.

The image below shows a close-up of actual IBEX data revealing the individual sky cells that make up the image. The color bar below shows the relationship between the color used, and the number of ENAs detected in each sky cell (called the particle flux).





The Mysterious Hexagon on Saturn!



The Cassini spacecraft recently took this high-resolution image of Saturn's north-polar region, which was observed by the Voyager 1 and 2 spacecraft in the early-1980s to have a remarkable hexagonal jet stream! The white spots are individual clouds, much like fair-weather cumulus clouds on Earth. The interior distance between opposite vertices of the hexagon is 25,000 kilometers, and the estimated speed of the winds along the walls of the hexagon is about 100 meters/sec. Use a millimeter ruler, or your knowledge of hexagons, to answer these questions:

Problem 1 - How long does it take the jet stream to make one complete circuit of the hexagon; A) In hours?, B) In days?

Problem 2 - The Earth has a radius of 6,378 kilometers, from a scaled drawing of the hexagon and Earth, how many Earths can fit inside the hexagonal area?

Problem 3 - Acceleration is a measure of the change in the speed and/or direction of motion per unit time interval. A) From the hexagon figure, how much time, T, elapsed in seconds as the velocity of the jet stream completely changed direction as it crossed a vertex region? B) If the total velocity change, V, passing across one vertex is about 173 meters/sec, what was the average acceleration of the jet stream defined as $a = V/T$ in meters/sec^2?

Problem 1 - How long does it take the jet stream to make one complete circuit of the hexagon; A) In hours?, B) In days?

Answer: The interior distance between opposite vertices is 67 mm, which corresponds to 25,000 kilometers, so the scale is 25,000 km/67 mm = 373 km/mm. The length of one side is 35 millimeters (or 13,000 km) so the full perimeter is 6 x 35 mm = 210 mm. Converting this to meters we get 210 mm x (373 km/mm) = 78,300 kilometers or 7.83 x 10^7 meters. The time taken is T = D/V so for D = 7.83 x 10^7 meters and V = 100 meters/sec we get T = 783,000 seconds. A) **217 hours** and B) **9 days**.

Problem 2 - The Earth has a radius of 6,378 kilometers, from a scaled drawing of the hexagon and Earth, how many Earths can fit inside the hexagonal area?
Answer: The earth circle will have a diameter of 2 x 6,378/373 = 34 millimeters. That will be about **2 Earth disks** spanning the inside of the hexagon. The area of a hexagon is given by $A = 3(3)^{1/2}/2\ L^2$ so the Saturn hexagon has an area of A = 3 (1.732)x(0.5) x $(13,000\ km)^2$ = 440,000,000 km^2. The surface area of Earth is $A = 4\ \pi\ R^2$ = 510,000,000 km^2, so the hexagon has nearly the same surface area as all of earth.

Problem 3 - Acceleration is a measure of the change in the speed and/or direction of motion per unit time interval. A) From the hexagon figure, how much time, T, elapsed in seconds as the velocity of the jet stream completely changed direction as it crossed a vertex region? B) If the total velocity change, V, passing across one vertex is about 173 meters/sec, what was the average acceleration of the jet stream defined as a = V/T in meters/sec^2? Answer: A) The shape of the clouds near a vertex suggest that the turn a distance of about 8 millimeters in going from the direction along one face to the other .The physical distance is 8 mm x (373 km/mm) = 3,000 kilometers or 3 million meters. The speed is 100 meters/sec, so the time taken is 3,000,000 m/(100m/s) = **30,000 seconds**. B) The velocity change during this time was 173 meters/sec, so the acceleration A = V/T = (173 meters/sec) / (30,000 sec) = **0.006 meters/sec^2**.
Note, the acceleration of gravity at Earth's surface is 9.8 meters/sec^2, so the winds feel an acceleration of about 0.0006 Gs.

Note: From a study of vectors, the velocity difference between a flow along one face Vf, and the adjacent face is a vector 'turn' of 60 degrees, so the difference vector, Vd, has a magnitude of $Vd^2 = 2(Vf)^2 + 2(Vf)^2\cos(60)$ so for Vf = 100 m/sec, Vd = 173 meters/sec, which is the value used.

Scientific Data - The gift that keeps on giving!

NASA launched its Solar Dynamics Observatory mission in late 2009. The three instruments on board the satellite will take lots and lots of pictures of the Sun. So it will require lots and lots of disc space to store the data.

Imagine you are the engineer responsible for purchasing the computer systems to handle that data. You must figure out how many computer discs and tapes you'll need, and what they will cost, so you can ask NASA for the right amount of money in your mission budget.

8 bits	= 1 byte	
1 kilobyte	= 1 thousand bytes	10^3
1 megabyte	= 1 million bytes	10^6
1 gigabyte	= 1 billion bytes	10^9
1 terabyte	= 1 trillion bytes	10^{12}
1 petabyte	= 1 quadrillion bytes	10^{15}

Data rate:	130 megabits / second
Hours of data/day:	24 hrs
Size of disc drive:	1 terabyte
Cost per disc drive:	$300
Size of backup tape:	800 gigabytes
Cost per backup tape:	$60

Problem 1 - How many bytes will your discs need to hold each day? (Express your answer in terabytes)

Problem 2 - NASA would like you to keep about 60 days of data online (i.e. on disc). Data older than 60 days will be archived and copied to tapes. How many bytes of drive space will you need to hold 60 days of data?

Problem 3 - How many disc drives will you need to purchase, and how much will they cost?

Problem 4 - How many terabytes of tape storage will you need to archive a year's worth of data?

Problem 5 - How many tapes will you need, and what will they cost you?

Problem 6 - There is a rule of thumb (called Moore's Law) that says the costs for electronics halve every 18 months. If you can wait 18 months before you purchase your discs and tapes, estimate how much money you might save.

Answer Key

Problem 1 - Answer: The data rate is 130 megabits per second. We first convert this to a daily rate. There are 60 sec/min and 60 min/hr and 24 hr/day so there are 60 x 60 x 24 = 86,400 sec/day. Since the data rate is 130 x 10^6 bits/sec or 1.3 x 10^8 bits/sec, in one day there will be 1.3 x 10^8 bits/sec x 86,400 sec/day = 1.12 x 10^{13} bits/day. Since there are 8 bits in 1 byte, we have 1.12 x 10^{13} bits/day x (1 bytes/8 bits) = 1.4 x 10^{12} bytes/day. Since 1 terabyte = 10^{12} bytes, we have 1.4 x 10^{12} bytes/day x (1 terabyte/10^{12} bytes) = **1.4 terabytes/day.**

Problem 2 - Answer: For 60 days at 1.4 terabytes/day there will be 60 days x 1.4 terabytes/day = **84 terabytes of data.**

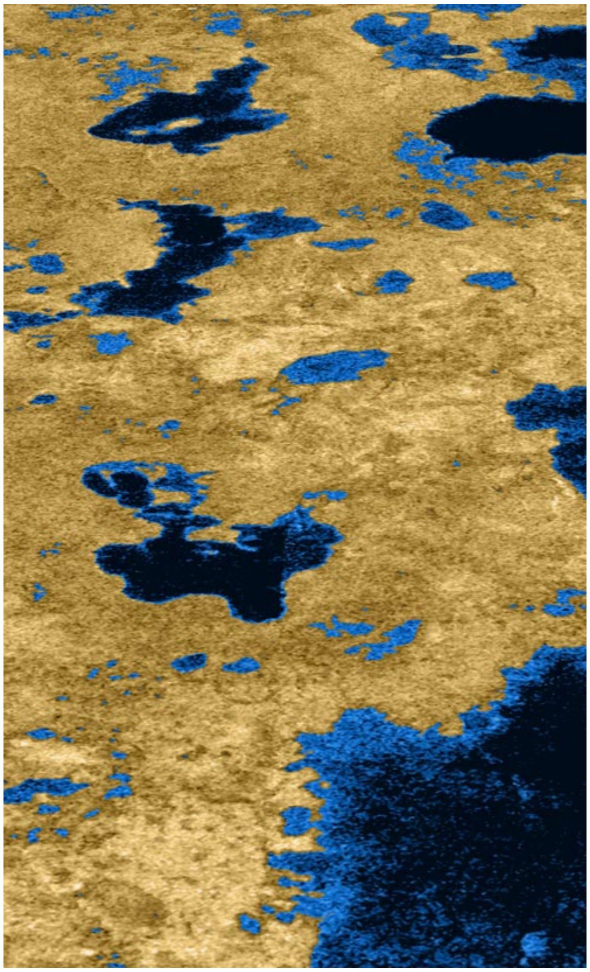
Problem 3 - Answer: 1 disk drive stores 1 terabyte of data so you will need **84 disk drives**. Since the price is $300 per disk drive, the total cost will be 84 drives x $300/drive = **$25,200.**

Problem 4 - Answer: At a rate of 1.4 terabytes/day for 365 days you will have 1.4 terabytes/day x 365 days = **511 terabytes of data to archive.**

Problem 5 - One tape holds 800 gigabytes of data. We first convert this to terabytes. Since 1,000 gigabytes = 1 terabyte, we have 800 gigabytes x (1 terabyte/1000 gigabytes) = 0.8 terabytes per tape. Then for 511 terabytes of data in one year, we need 511 terabytes/year x (1 tape/0.8 terabytes) = 638.75 tapes, but since we cannot buy a fraction of a tape, we round up to **639 tapes.** Each tape costs $60, so the total annual cost for backup tapes will be 639 tapes x ($60/1 tape) = **$38,340**

Problem 6 - Answer: The cost for drives will fall from $25,200 to one-half this cost or $12,600, and the cost for the backup tapes will fall from $38,340 to only $19,170. For the combined purchases, you would save $12,600 + $19,170 = **$31,770**.

Lakes of Methane on Titan



A key goal in the search for life elsewhere in the universe is to detect liquid water, which is generally agreed to be the most essential ingredient for living systems that we know about.

The image to the left is a false-color synthetic radar map of a northern region of Titan taken during a flyby of the cloudy moon by the robotic Cassini spacecraft in July, 2006. On this map, which spans about 150 kilometers across, dark regions reflect relatively little of the broadcast radar signal. Images like this show Titan to be only the second body in the solar system to possess liquids on the surface. In this case, the liquid is not water but methane!

Future observations from Cassini during Titan flybys will further test the methane lake hypothesis, as comparative wind affects on the regions are studied.

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

Problem 1 – From the information provided, what is the scale of this image in kilometers per millimeter?

Answer: 150 km / 77 millimeters = **1.9 km/mm**.

Problem 2 – What is the approximate total surface area of the lakes in this radar image?

Answer: Combining the areas over the rectangular field of view gives about 1/4 of the area covered. The field of view measures 77 mm x 130mm or 150 km x 247 km or an area of 37,000 km^2. The dark areas therefore cover about 1/4 x 37,000 km^2 or **9,300 km^2.**

Problem 3 - Assume that the lakes have an average depth of about 20 meters. How many cubic kilometers of methane are implied by the radar image?

Answer: Volume = area x height = 9,300 km^2 x (0.02 km) = **190 km^3**.

Problem 4 – The volume of Lake Tahoe on Earth is about 150 km^3. How many Lake Tahoes-worth of methane are covered by the Cassini radar image?

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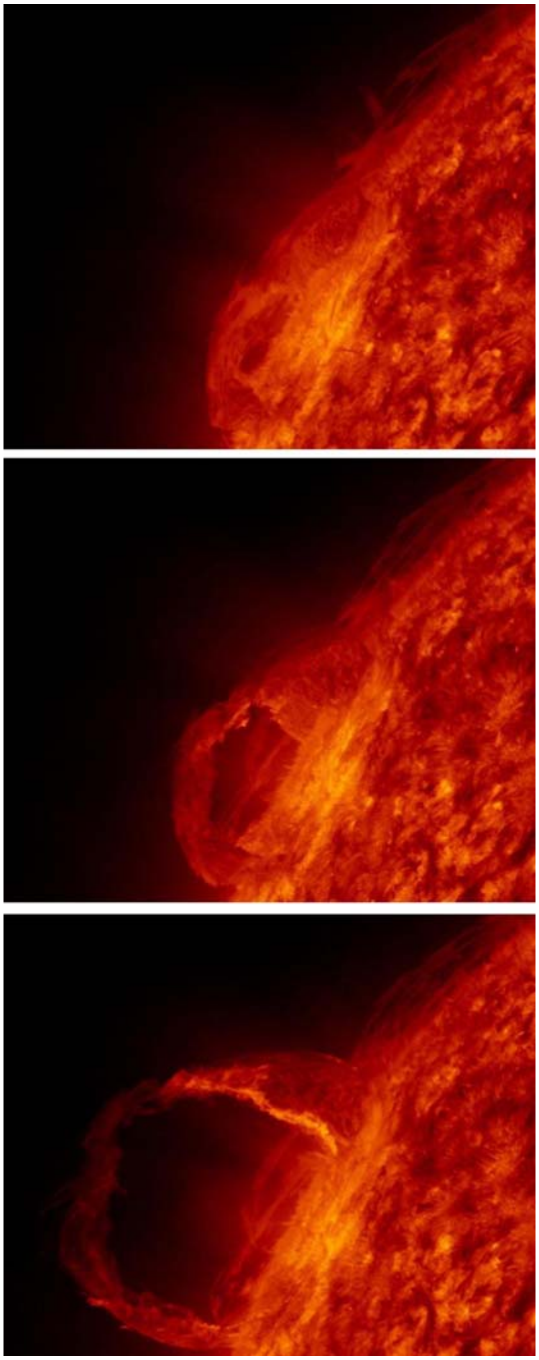
Answer: 190 km^3 / 150 km^3 = **1.3 Lake Tahoes**.



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. Among them was a sequence of images taken on March 30, showing an eruptive prominence ejecting millions of tons of ionized gas (plasma) into space. The three images to the left show selected scenes from the first 'high definition' movie of this event. The top image was taken at 17:50:49, the middle image at 18:02:09 and the bottom image at 18:13:29.

Problem 1 – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Problem 2 – If the Earth were represented by a disk the size of a penny (10 millimeters), on this same scale how big was the loop of the eruptive prominence in the bottom image if the radius of Earth is 6,378 kilometers?

Problem 3 – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

For additional views of this prominence, see the NASA/SDO movies at:

http://svs.gsfc.nasa.gov/vis/a000000/a003600/a003693/index.html

or to read the Press Release:

http://www.nasa.gov/mission_pages/sdo/news/first-light.html

Problem 1 – The width of the image is 300,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The width is 70 millimeters so the scale is 300,000 km/70 mm **= 4,300 km/mm**

Problem 2 – If the Earth were represented by a disk the size of a penny (10 millimeters), on this same scale how big was the loop of the eruptive prominence in the bottom image if the radius of Earth is 6,378 kilometers?

Answer: **The diameter of the loop is about 35 millimeters or 35mm x 4300km/mm = 150,000 km. The diameter of Earth is 13,000 km, so the loop is 12 times the diameter of Earth. At the scale of the penny, 13 penny/Earth's can fit across a scaled drawing of the loop.**

Problem 3 – What was the average speed of the prominence in A) kilometers/second? B) Kilometers/hour? C) Miles/hour?

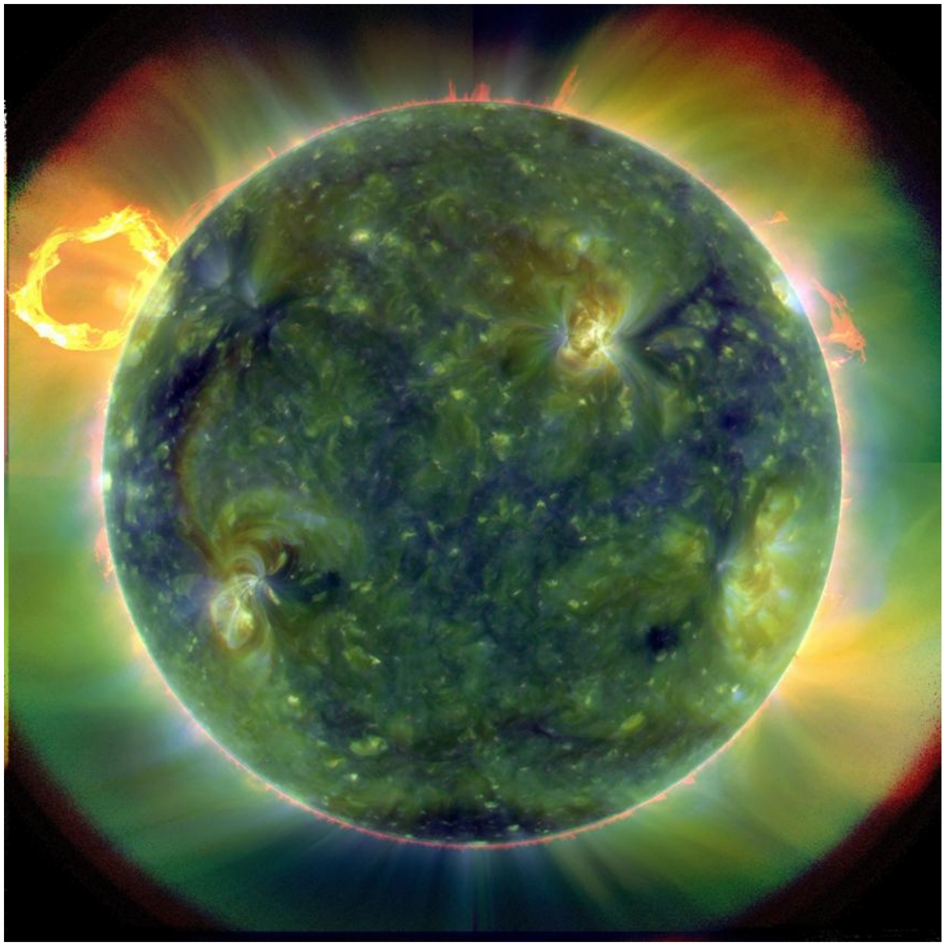
Answer: Speed = distance traveled / time elapsed.
In the bottom image, draw a straight line from the lower right corner THROUGH the peak of the coronal loop. Now draw this same line at the same angle on the other two images. With a millimeter ruler, measure the distance along the line from the lower right corner to the edge of the loop along the line. Example:
Top: 47 mm ;
Middle: 52 mm,
Bottom: 67 mm.
The loop has moved 67mm – 47mm = 20 millimeters. At the scale of the image this equals 20mm x 4,300 km/mm so D = 86,000 km.

The time between the bottom and top images is 18:13:29 – 17:50:49 or 22minutes and 40 seconds or 1360 seconds.

A) The average speed of the loop is then S = 86,000 km/1360 sec = **63 km/sec**.

B) 63 km/sec x 3600 sec/hr = **227,000 km/hour**.

C) 227,000 km/hr x 0.62 miles/km = **140,000 miles/hour**.



On April 21, 2010 NASA's Solar Dynamics Observatory released its much-awaited 'First Light' images of the Sun. The image above shows a full-disk, multi-wavelength, extreme ultraviolet image of the sun taken by SDO on March 30, 2010. False colors trace different gas temperatures. Black indicates very low temperatures near 10,000 K close to the solar surface (photosphere). Reds are relatively cool plasma heated to 60,000 Kelvin (100,000 F); blues, greens and white are hotter plasma with temperatures greater than 1 million Kelvin (2,000,000 F).

Problem 1 – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Problem 2 – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

Problem 3 – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Problem 1 – The radius of the sun is 690,000 kilometers. Using a millimeter ruler, what is the scale of these images in kilometers/millimeter?

Answer: The diameter of the Sun is 98 millimeters, so the scale is 1,380,000 km/98 mm = **14,000 km/mm**.

Problem 2 – What are the smallest features you can find on this image, and how large are they in kilometers, and in comparison to Earth if the radius of Earth is 6378 kilometers?

Answer: **Students should see numerous bright points freckling the surface, the smallest of these are about 0.5 mm across or 7,000 km. This is slightly larger than ½ the diameter of Earth.**

Problem 3 – Where is the coolest gas (coronal holes), and the hottest gas (micro flares), located in this image?

Answer: **There are large irregular blotches all across the disk of the sun that are dark blue-black. These are regions where thee is little of the hot coronal gas and only the 'cold' photosphere can be seen. The hottest gas seems to reside in the corona, and in the very small point-like 'microflare' regions that are generally no larger than the size of Earth**.

Note: Microflares were first observed, clearly, by the Hinode satellite between 2007-2009. Some solar physicists believe that these microflares, which erupt violently, are ejecting hot plasma that eventually ends up in the corona to replenish it. Because the corona never disappears, these microflares happen all the time no matter what part of the sunspot cycle is occurring.

NASA's Terra satellite flew over the Deepwater Horizon rig's oil spill in the Gulf of Mexico on Saturday, May 1 and captured the above natural-color image of the slick from space. The oil slick resulted from an accident at the Deepwater Horizon rig in the Gulf of Mexico. NOAA's estimated release rate of oil spilling into the Gulf is 200,000 gallons per day since April 20 when the accident occurred.

Problem 1 – Using a metric ruler, calculate the scale of this image in kilometers/mm.

Problem 2 – What is the approximate area of this oil leak in A) square kilometers? B) square meters?

Problem 3 - The estimated quantity of oil covering this area is about 2 million gallons. If one gallon of oil has a mass of 3.0 kg, what is the surface density, S, of oil in this patch in A) Gallons/meter^2? B) kg/meter^2?

Problem 4 – The density of crude oil is about D=850 kg/m^3. From your estimate for S, what is the approximate thickness, h, of the oil layer covering the ocean water?

Problem 5 - Suppose that an average 'oil' molecule has a length of about 5 nanometers. About what is the average thickness of this oil layer in molecules if the molecules are lined up end to end?

Problem 1 – Using a metric ruler, calculate the scale of this image in kilometers/cm.

Answer: The '25km' legend mark on the Terra image is 1.7 cm long, so the scale is

25 km / 1.7 cm = **15 km/cm**.

Problem 2 – What is the approximate area of this oil leak in A) square meters? B) square meters?

Answer: The oil spill is about 6 cm in diameter or 6cm x 15 km/cm = 90 km in diameter. A) As a circle, the area is A = π (45 km)^2 = 6,358 km^2 or to 1 significant figures **A = 6,000 km^2**.

B) A = 6,000 km^2 x (1,000 m/1 km)^2 so **A = 6.0 x 10^9 m^2**.

Problem 3 - The estimated quantity of oil covering this area is about 2 million gallons. If one gallon of oil has a mass of 3.0 kg, what is the surface density, S, of oil in this patch in A) Gallons/meter^2? B) kg/meter^2?

Answer: Mass = 2 million gallons x (3 kg/1 gallon) = 6 million kg. Then

A) S = 2 million gallons / 6.0 x 10^9 m^2 so **S = 0.0003 gallons/m^2**.

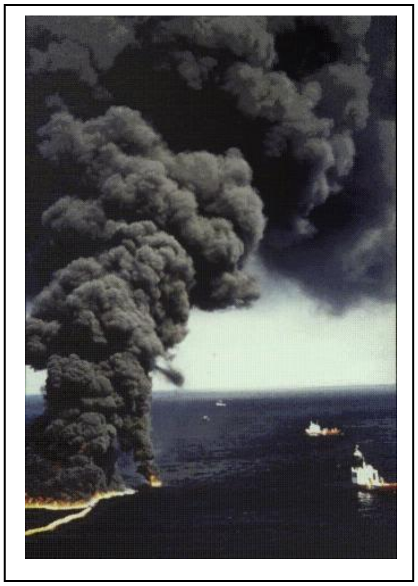
B) S = 6 million kg/ 6.0 x 10^9 m^2 so **S = 0.001 kg/m^2**.

Problem 4 – The density of crude oil is about D=850 kg/m^3. From your estimate for S, what is the approximate thickness, h, of the oil layer covering the ocean water?

Answer: h = S/D so h= (0.001 kg/m^2)/(850 kg/m^3) = **1.0 x 10^{-6} meters** (or 1 micron)

Problem 5 – Suppose that an average 'oil' molecule has a length of about 5 nanometers. About what is the average thickness of this oil layer in molecules if the molecules are lined up end to end?

Answer: Assuming that these cylindrical molecules are stacked up vertically along their maximum length, the layer is about 1.0 x 10^{-6} meters/5.0 x 10^{-9} meters = **200 molecules thick**.



On March 21, 2010 the Eyjafjalla Volcano in Iceland erupted, and the expanding ash cloud grounded over 3,000 flights in Europe. Then on April 20, a major oil spill began in the Gulf of Mexico. Although the preferred method for dealing with the oil spill is to collect it using skimmers, burning it is also a common option (see above left photo). A major concern in burning this oil is the addition of carbon dioxide to the atmosphere during the combustion process.

Problem 1 – The Gulf Oil Spill is predicted to generate 200,000 gallons of crude oil every day. If 50% of this is ultimately burned-off, how many tons/day of carbon dioxide are generated if the combustion of 1 gallon of oil generates 10 kg of carbon dioxide?

Problem 2 – Scientists have estimated that the Iceland volcano generated 15,000 tons of carbon dioxide per day, and this eruption continued for about 28 days. How many days will the Gulf Oil burn-off have to continue before its carbon dioxide contribution equals that of the total carbon dioxide generated by the Eyjafjalla Volcano?

Problem 3 – It has been estimated that the European aviation industry generates 344,000 tons of carbon dioxide each day. If 60% of this industry was shut down by the ash cloud from the Eyjafjalla Volcano, how many tons of carbon dioxide would have been produced by airline flights during the 5-day shut-down of the industry?

Problem 4 – What can you conclude by comparing your answers to Problem 1, 2 and 3?

Problem 1 – The Gulf Oil Spill is predicted to generate 200,000 gallons of crude oil every day. If 50% of this is ultimately burned-off, how many tons/day of carbon dioxide are generated if the combustion of 1 gallon of oil generates 10 kg of carbon dioxide?

Answer: 200,000 gallons/day x (0.50) x (10 kg/ 1 gallon) = 1,000,000 kg/day or **1,000 tons/day**

Problem 2 – Scientists have estimated that the Iceland volcano generated 15,000 tons of carbon dioxide per day, and this eruption continued for about 28 days. How many days will the Gulf Oil burn-off have to continue before its carbon dioxide contribution equals that of the total carbon dioxide generated by the Eyjafjalla Volcano?

Answer: The volcano generated 15,000 tons/day x 28 days = 420,000 tons of CO2. The Gulf Oil burn-off generates 1,000 tons/day, so the Gulf Oil burn-off would have to continue for **420 days** before it equaled the emission of the volcano.

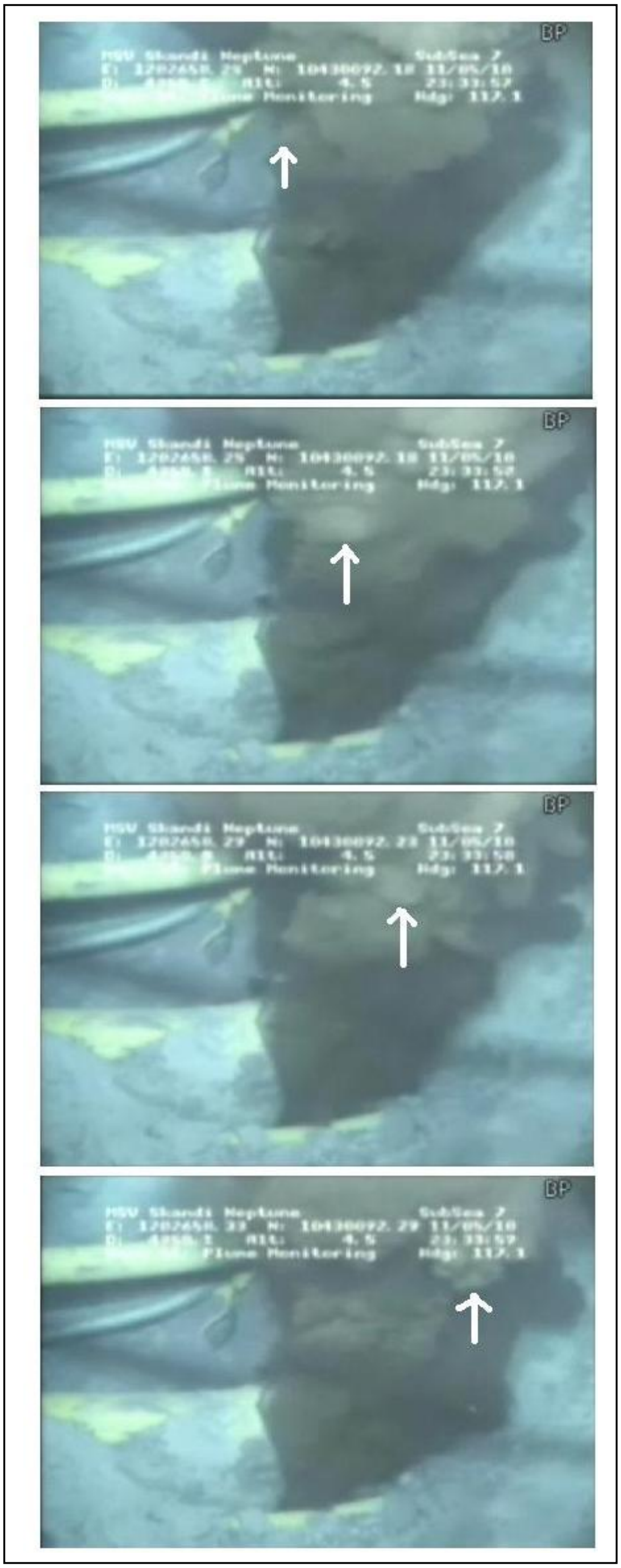
Problem 3 – It has been estimated that the European aviation industry generates 344,000 tons of carbon dioxide each day. If 60% of this industry was shut down by the ash cloud from the Eyjafjalla Volcano, how many tons of carbon dioxide would have been produced by airline flights during the 5-day shut-down of the industry?

Answer: 344,000 tons/day x (0.6) x (5 days) = **1 million tons**.

Problem 4 – What can you conclude by comparing your answers to Problem 1, 2 and 3?

Answer: The total carbon dioxide generated by the volcano is only 40% of what was generated by the European airline industry during the time it was shut down, and the burn-off of the oil spill will only exceed what the volcano generated if the cleanup continues for over one year, which most experts say is very unlikely. The oil spill cleanup is a small part of the carbon dioxide generated by aviation or by the Icelandic volcano.

Note: Since the crude oil was destined to be burned to generate electricity, and combusted in cars, the fact that the clean-up is producing carbon dioxide is almost beside the point since this very same quantity of carbon dioxide would have been generated anyway once the crude oil is used under more controlled conditions.



The April 14, 2010 BP Gulf Oil Leak was in the news for nearly three months, and ranked as one of the most environmentally costly accidents in recent history. Considerable debate continues as to the actual rate at which the leaky British Petrolium (BP) well is leaking oil. Initial estimates from the observed surface oil slick suggested 210,000 gal/day. Following the release of actual videos of the leak, experts estimated a new rate from 3 to 4 million gallons/day.

The images to the left were extracted from the May 12, 2010 video between 23:33:57 and 23:33:58. The arrow shows how far a portion of the billowing oil moved during this time.

The diameter of the pipe fragment shown in the image is 21 inches.

Problem 1 - From the scale of the images, how many inches did the oil spot move in the time between the first and last images?

Problem 2 - What is the area of the open circular pipe in square-feet?

Problem 3 - If the oil is emerging at the same speed as you derived in Problem 1, how many cubic-feet of oil is leaving the pipe each second?

Problem 4 - If 1 cubic foot equals 7.5 gallons, what do you estimate as the rate in gallons/day at which oil is leaving the pipe if A) 100% of the dark material is oil? B) 50% is oil and 50% is gas?

Problem 1 - From the scale of the images, how many inches did the oil spot move in the time between the first and last images?

Answer: Using a metric ruler, the diameter of the 21-inch pipe is 18 millimeters, so the scale of the image is 1.2 inches/mm. The distance between the arrow in the top image and the bottom image is about 25 millimeters or 25 x 1.2 = **30 inches**.

Problem 2 - What is the area of the open circular pipe in square-feet?
Answer: Assuming a circular aperture, and a diameter of 21/12 = 1.8 feet,
$A = \pi (1.8/2)^2$ = **2.5 feet^2**.

Problem 3 - If the oil is emerging at the same speed as you derived in Problem 1, how many cubic-feet of oil is leaving the pipe each second?

Answer: The speed of the flow is 30 inches/1 second or 30 inches/sec. This can be converted to feet/sec to get S = 2.5 feet/sec. Flow = Area x Speed so Flow = 2.5 feet^2 x 2.5 feet/sec so **Flow = 6.3 feet^3/sec**.

Problem 4 - If 1 cubic foot equals 7.5 gallons, what do you estimate as the rate in gallons/day at which oil is leaving the pipe if A) 100% of the dark material is oil? B) 50% is oil and 50% is gas?

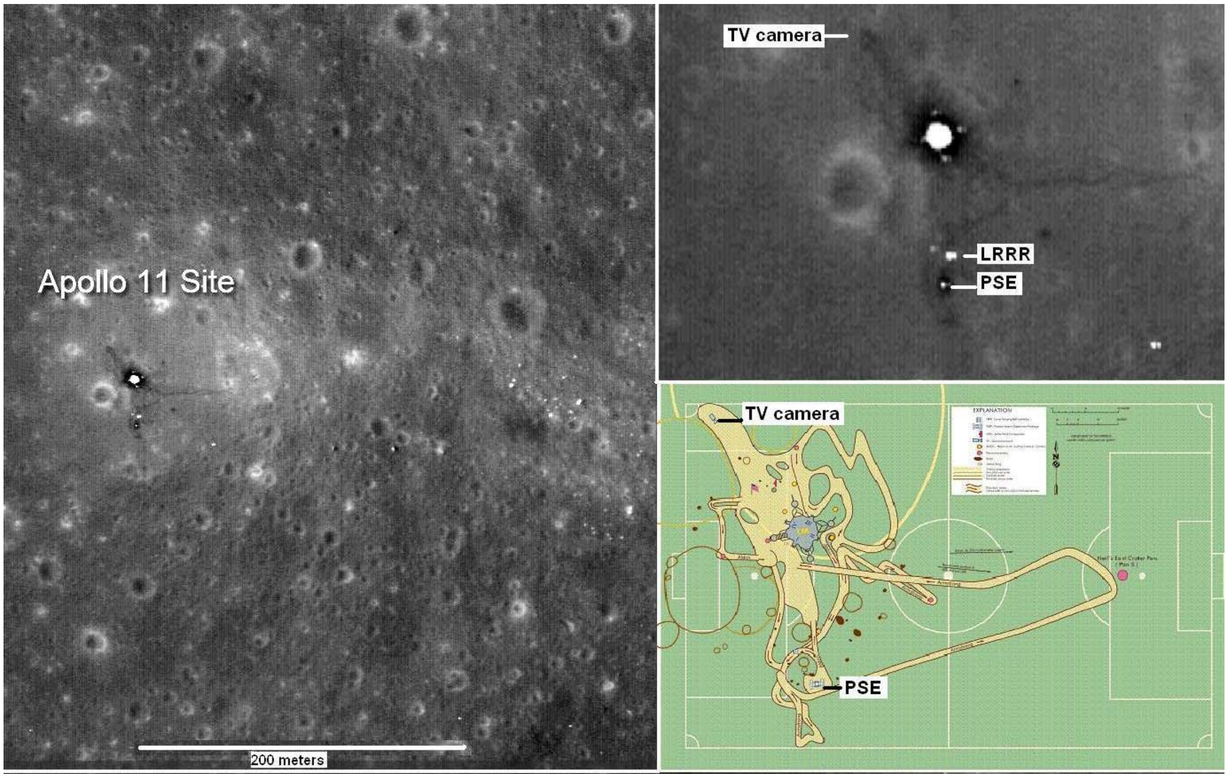
Answer: A) 100% x 6.3 feet^3/sec x (3600 sec/hour) x (24 hour/day) x (7.5 gallons / 1 feet^3) so Rate = **4 million gallons/day.**
B) At a 50% mixture, by volume, of oil and gas, Rate = **2 million gallons/day**.

Note to Teacher: Students may view the actual video at:

http://www.necn.com/05/13/10/Video-Gulf-oil-leak-at-the-source/landing_scitech.html?blockID=234064&feedID=4213

There are many other websites that archive the BP oil leak video. Students may examine other portions of the videos to obtain additional estimates .They may also discuss the issue of the actual concentration of the oil in the outflowing material seen in the videos, and also how to obtain better speed estimates by following 'blobs' in the video. What are some of the problems with using this video? Are there any geometric effects that have to be taken into account because the camera/pipe/cloud are tilted relative to the image?

Comparing this daily flow rate with the rate estimated from the size of the surface oil seen by the Terra satellite, why do you think that scientists believe that there is a significant amount of oil below the surface of the ocean that has not been accounted for yet?



The Lunar Reconnaissance Orbiter (LRO) recently imaged the Apollo-11 landing area at high-resolution and obtained the image above (Top left). An enlargement of the area is shown in the inset (Top right) and a rough map of the area is also shown (bottom right). The landing pad with three of its four foot-pads is clearly seen, together with the Lunar Ranging Retro Reflector experiment (LRRR), the Passive Seismic Experiment (PSE) and the TV camera area. The additional white spots seen in the left image are boulders from the West Crater located just off the right edge of the image.

Problem 1 - Using a millimeter ruler and the '200 meter' metric bar, what is the scale of each of the two images and the map?

Problem 2 - About what is the distance between the TV camera and the PSE?

Problem 3 - From the left-hand image; A) What is the height and width of the field? B) What is the area of the field in square-kilometers?

Problem 4 - In the left-hand image, what is the diameter, in meters, of A) the largest crater, and B) the smallest crater?

Problem 5 - By counting craters in the left-hand image, what is the surface density of cratering in this region of the moon in units of craters per square kilometer?

Problem 1 - Using a millimeter ruler and the '200 meter' metric bar, what is the scale of each of the two images and the map?
Answer: On the main image, the length is 43 millimeters so the scale is 200 meters/43 mm = **4.7 meters/mm for the left-hand image.** The distance between the landing pad and the LRRR on this image is 5 millimeters or 5 x 4.7 = 24 meters. In the upper right image, the landing pad and the LRRR are 16 mm apart, so the scale of this image is 24 meters/16mm = **1.5 meters/mm**. The PSE and landing pad are clearly indicated in the map, which the top image says are 20 mm or 20x1.5 = 30 meters apart. On the map, these points are also 20 mm apart, so the scale is also 1.5 meters/mm on the map.

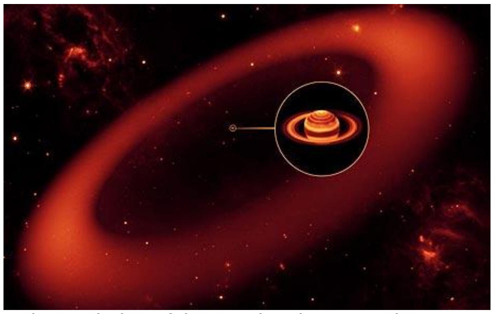
Problem 2 - About what is the distance between the TV camera and the PSE?
Answer: According to the map, the distance is 38 millimeters or 38 mm x (1.5 m/mm) = **57 meters apart**.

Problem 3 - From the left-hand image; A) What is the height and width of the field? B) What is the area of the field in square-kilometers?
Answer; A) Height x Width = 101mm x 86mm and for a scale of 4.7 meters/mm this equals **475m x 404m**. B) The are in square kilometers is 0.475 m x 0.404 m= **0.19 km^2.**

Problem 4 - In the left-hand image, what is the diameter, in meters, of A) the largest crater, and B) the smallest crater? Answer: A) The largest circular feature is about 8mm in diameter or **38 meters across**. B) The smallest feature is about 1 millimeter across or **4.7 meters**.

Problem 5 - By counting craters in the left-hand image, what is the surface density of cratering in this region of the moon in units of craters per square kilometer?
Answer: Depending on the quality of the printed copy, students may count between 20 and 100 craters. Assuming the lower value, the crater density is 20 craters/0.19 km^2 = 105 craters/km^2. If the PDF file is displayed on the computer screen, a much better contrast is obtained and students should be able to count about 225 craters for a density of 1,200 craters/km^2. **Values between 100 and 1000 craters/km^2 are acceptable**.

Spitzer Sees a New Ring Around Saturn



Artist rendering of the new ice ring around Saturn detected by the Spitzer Space Telescope.

"This is one supersized ring," said one of the authors, Professor Anne Verbiscer, an astronomer at the University of Virginia in Charlottesville. Saturn's moon Phoebe orbits within the ring and is believed to be the source of the material.

The thin array of ice and dust particles lies at the far reaches of the Saturnian system. The ring was very diffuse and did not reflect much visible light but the infrared Spitzer telescope was able to detect it. Although the ring dust is very cold -316F it shines with thermal 'heat' radiation. No one had looked at its location with an infrared instrument until now.

"The bulk of the ring material starts about 6.0 million km from the planet, extends outward about another 12 million km, and is 2.6 million km thick. The newly found ring is so huge it would take 1 billion Earths to fill it." (CNN News, October 7, 2009)

Many news reports noted that the ring volume was equal to 1 billion Earths. Is that estimate correct? Let's assume that the ring can be approximated by a washer with an inner radius of r, an outer radius of R and a thickness of h.

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Problem 3 - If $r = 6 \times 10^6$ kilometers, $R = 1.2 \times 10^7$ kilometers and $h = 2.4 \times 10^6$ kilometers, what is the volume of the new ring in cubic kilometers?

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Problem 6 - How does your answer compare to the Press Release information? Why are they different?

Problem 1 - What is the formula for the area of a circle with a radius R from which another concentric circle with a radius r has been subtracted?

Answer: The area of the large circle is given by πR^2 minus area of small circle πr^2 equals $\mathbf{A = \pi (R^2 - r^2)}$

Problem 2 - What is the volume of the region defined by the area calculated in Problem 1 if the height of the volume is h?

Answer: Volume = Area x height so $\mathbf{V = \pi (R^2 - r^2) h}$

Problem 3 - If r = 6×10^6 kilometers, R = 1.2×10^7 kilometers and h = 2.4×10^6 kilometers, what is the volume of the new ring in cubic kilometers?

Answer: $V = \pi (R^2 - r^2) h$

$= (3.141) [(1.2 \times 10^7)^2 - (6.0 \times 10^6)^2] 2.4 \times 10^6$

` $= \mathbf{8.1 \times 10^{20}\ km^3}$

Note that the smallest number of significant figures in the numbers involved is 2, so the answer will be reported to two significant figures.

Problem 4 - The Earth is a sphere with a radius of 6,378 kilometers. What is the volume of Earth in cubic kilometers?

Answer: Volume of a sphere $V = 4/3 \pi R^3$ so for Earth,

$V = 1.33 \times (3.14) \times (6.378 \times 10^3)^3$

$= \mathbf{1.06 \times 10^{12}\ km^3}$

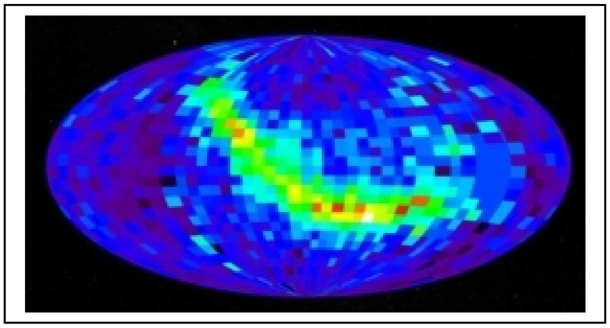
Note that the smallest number of significant figures in the numbers involved is 3, so the answer will be reported to three significant figures.

Problem 5 - About how many Earths can be fit within the volume of Saturn's new ice ring?

Answer: Divide the answer for Problem 3 by Problem 4 to get

$8.1 \times 10^{20}\ km^3 / (1.06 \times 10^{12}\ km^3) = \mathbf{7.6 \times 10^8}$ **times**

Problem 6 - How does your answer compare to the Press release information? Why are they different? Answer: **The Press Releases say 'about 1 billion times' because it is easier for a non-scientist to appreciate this approximate number. If we rounded up 7.6 x 10^8 times to one significant figure accuracy, we would also get an answer of '1 billion times'.**



$$K.E. = \frac{1}{2}mV^2$$

NASA's IBEX satellite has detected fast-moving atoms streaking into the solar system from interstellar space. The energetic neutral atoms (called ENAs) are created in an area of our solar system known as the interstellar boundary region. This region is where charged particles from the sun, called the solar wind, flow outward far beyond the orbits of the planets and collide with material between stars.

The NASA Press release says that the ENAs "…travel inward toward the sun from interstellar space at speeds from 100,000 mph to more than 2.4 million mph."

Question: How do scientists know the speeds of these particles?

Answer: It's all about Kinetic Energy!

The IBEX satellite detects energetic neutral atoms with a kinetic energy (K.E.) of 1,000 electron volts (1 keV), where 1 electron volt (eV) equals 1.6 x 10^{-19} Joules of energy. In the formula above, with KE expressed in Joules and the particle mass, m, expressed in kilograms, the speed of the particle, V, will be in meters/sec.

Problem 1 - What is the formula for the particle speed, V, in terms of the particle's mass and kinetic energy?

Problem 2 - Show that, if the particle is a proton with a mass of 1.7 x 10^{-27} kg and it has a speed of 450 kilometers/sec, to two significant figures, its energy is A) 1.7 x 10^{-16} Joules, or B) 1,100 eV.

Problem 3 - The IBEX satellite measures ENAs with an energy of 1 keV in order to make the image shown above. The most common element in the universe is hydrogen (1 proton). If the detected ENAs are all protons, what is the speed of the protons, in kilometers/sec, detected by IBEX to two significant figures?

Problem 1 - What is the formula for the particle speed, V, in terms of the particle's mass and kinetic energy? Answer:

$$E = \frac{1}{2} m V^2$$

so

$$V = \sqrt{\frac{2E}{m}}$$

Problem 2 - Show that, if the particle is a proton with a mass of 1.7 x 10^{-27} kg and it has a speed of 450 kilometers/sec, to two significant figures its energy is A) 1.7 x 10^{-16} Joules, or B) 1,076 eV.

Answer A) In order to use the formula for K.E., we have to convert km/s to meters/sec, so 450 km/s x (1000 m/km) = 450,000 m/s, then K.E = 1/2 (1.7 x 10^{-27} kg) x (4.5 x 10^{5} m/s)^2 = **1.7 x 10^{-16} Joules**. B) Since 1 eV = 1.6 x 10^{-19} Joules we have
1.7 x 10^{-16} Joules x (1 eV/1.6 x 10^{-19} Joules) = 1,063 eV
which to two significant figures is **1,100 eV.**

Problem 3 - The IBEX satellite measures ENAs with an energy of 1 keV in order to make the image shows above. The most common element in the universe is hydrogen (1 proton). If the detected ENAs are all protons, what is the speed of the protons in kilometers/sec, detected by IBEX to two significant figures?

Answer: 1 keV = 1,000 eV. The equivalent energy in Joules is 1,000 eV x (1.6 x 10^{-19} Joules/eV) = 1.6 x 10^{-16} Joules. The mass of a proton is 1.7 x 10^{-27} kg, so from the formula derived in Problem 1,

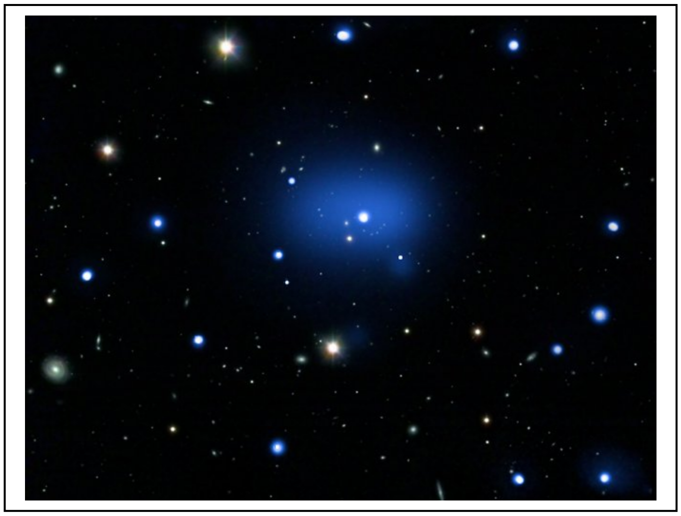
$$V = \sqrt{\frac{2\text{x}(1.6\text{x}10^{-16})}{1.7\text{x}10^{-27}}}$$

$$V = \sqrt{1.88 x 10^{11}}$$

So V = 433,861 m/s by the calculator display, or to 2 significant figures we get 430,000 m/s or **430 kilometers/sec.**

Note: 430 km/sec is equivalent to 960,000 miles/hour or nearly 1 million miles per hour. Also, IBEX measures higher-energy ENAs with energies up to 6 keV, which corresponds to protons with speeds of 2,400,000 million miles/hour as stated in most press releases.

IBEX Press Release: http://www.nasa.gov/mission_pages/ibex/index.html



The most distant galaxy cluster yet has been discovered by combining data from NASA's Chandra X-ray Observatory and optical and infrared telescopes. The cluster is located about 10.2 billion light years away, and is observed as it was when the Universe was only about a quarter of its present age. The galaxy cluster, known as JKCS041, beats the previous record holder by about a billion light years. Galaxy clusters are the largest gravitationally bound objects in the Universe.

JKCS041 was originally detected in 2006 in a survey from the United Kingdom Infrared Telescope (UKIRT). The Chandra data were the final - but crucial - piece of evidence that showed JKCS041 was, indeed, a genuine galaxy cluster. Clusters of galaxies have such strong gravitational fields that they can serve as a bottle for very high temperature gas. These gases often emit x-ray light that can be detected by observatories such as Chandra. The discovery of such a high-temperature gas between the galaxies in JKCS041 supports the original idea that the galaxies seen in that direction are, in fact, members of a cluster. From the X-ray information, astronomers can also measure the total mass of the entire cluster that is responsible for creating the gravitational field holding the gas in place.

Problem 1 - The Chandra satellite detected x-rays coming from the region of the sky containing the galaxy cluster JKS041. The electrons in the gas are emitting the X-rays, and colliding at high speed with the protons in the gas. The energy of the x-rays at the time they were emitted by the hot gas was 21,400 electron Volts (eV). This energy is shared equally between the electrons and protons. The speed of a proton is related to its kinetic energy by $E = 1/2mV^2$ where E is the energy in Joules, V is the proton speed in meters/sec, and m is the mass of a proton ($m = 1.7 \times 10^{-27}$ kg). About how fast are the protons moving? (Note: 1 eV = 1.6×10^{-19} Joules)

Problem 2 -The escape velocity (in km/s) from a body is given by $V = 0.17 (M/R)^{1/2}$ where M is the mass in multiples of the mass of our sun, and R is the average distance, in light years, between the body and the gas particle. Example, for the Milky Way, R = 50,000 light years and M = 300 billion so V = 420 km/sec. Compared to the sun, about how much mass do you need to confine the gas cloud observed by Chandra, if the cluster of galaxies has a radius of about 1 million light years A) in units of the sun's mass? B) In terms of the number of Milky Way galaxies where 1 Milky Way is about 2×10^{12} solar masses?

Problem 1 - The Chandra satellite detected x-rays coming from the region of the sky containing the galaxy cluster JKS041. The electrons in the gas are emitting the X-rays, and colliding at high speed with the protons in the gas. The energy of the x-rays at the time they were emitted by the hot gas was 21,400 electron Volts (eV). This energy is shared equally between the electrons and protons. The speed of a proton is related to its kinetic energy by $E = 1/2mV^2$ where E is the energy in Joules, V is the proton speed in meters/sec, and m is the mass of a proton ($m = 1.7 \times 10^{-27}$ kg). About how fast are the protons moving that are producing the X-ray light seen by Chandra? (Note: 1 eV = 1.6×10^{-19} Joules)

Answer: The information given in the problem is that:

The x-ray energy is 21,400 eV
1 eV = 1.6×10^{-19} Joules of energy
The electrons carry ½ of the x-ray energy
The protons carry ½ of the x-ray energy
The mass of a proton is 1.7×10^{-27} kilograms

The formula requires the energy, E, in units of Joules, so we first have to convert 21,400 eV to Joules. E = 21,400 eV x (1.6×10^{-19} Joules /1 eV)
= 3.4×10^{-15} Joules.
This is the total energy, so protons only carry half of this so that E = **1.7×10^{-15} Joules**

Next, we use the formula $E = 1/2mV^2$ and solve for V to get $V = (2E/m)^{1/2}$ and substitute the known values for m and E to get $V = (2 \times 1.7 \times 10^{-15} \text{ Joules}/ 1.7 \times 10^{-27} \text{ kg})^{1/2}$ = **1,400,000 m/sec or 1,400 km/s.**

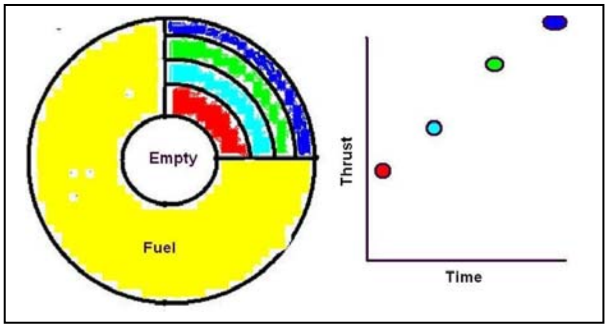
Problem 2 -The escape velocity (in km/s) from a body is given by $V = 0.17 (M/R)^{1/2}$ where M is the mass in multiples of the mass of our sun, and R is the average distance, in light years, between the body and the gas particle. Example, for the Milky Way, R = 50,000 light years and M = 300 billion so V = 420 km/sec. Compared to the sun, about how much mass do you need to confine the gas cloud observed by Chandra, if the cluster of galaxies has a radius of about 1 million light years, A) in units of the sun's mass? B) In terms of the number of Milky Way galaxies where 1 Milky Way is about 2×10^{12} solar masses?

Answer: Solve the stated equation for M to get $M = R (V/0.17)^2$. For R = 1 million light years and V = 1,400 km/sec, then A) **M = 70 trillion suns** and B) N = 70 trillion suns x (1 Milky Way/2 trillion suns) so this is about **35 Milky Ways**. Note, some of this mass is actually in the 16 large galaxies that make up the cluster. Some of it is in the hot cloud of x-ray emitting gas, and some of it may be in Dark Matter.

Note: For more information about this discovery, read the Chandra Press release at: http://www.nasa.gov/mission_pages/chandra/news/09-086.html

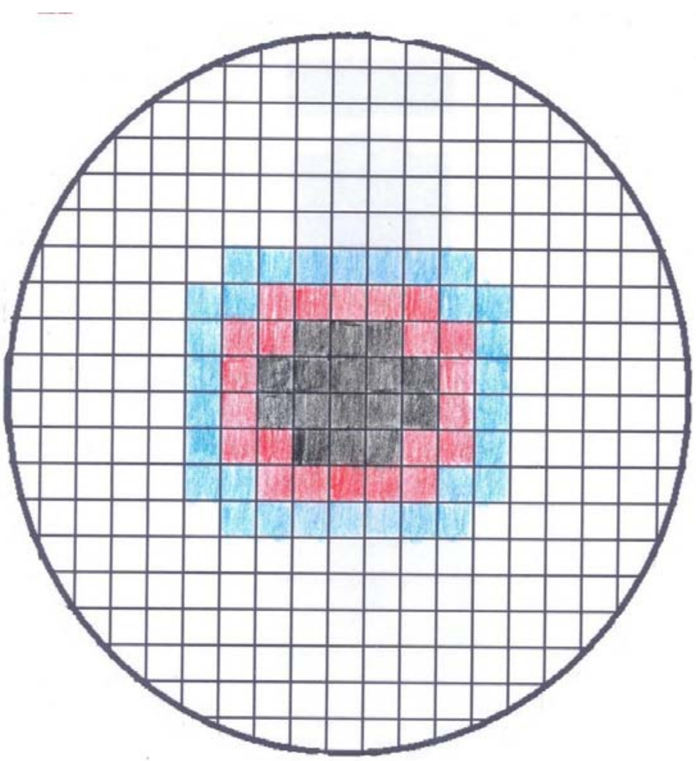
According to the research paper by S. Andreon, B. Maughan, G. Trinchieri, and J. Kirk "JKCS041: a color-detected galaxy cluster at z=1.9 with deep potential well as confirmed by x-ray data" published in the journal Astronomy and Astrophysics, October 2009, the estimated mass based on careful modeling of the data indicated a range between 30 trillion and 670 trillion suns.

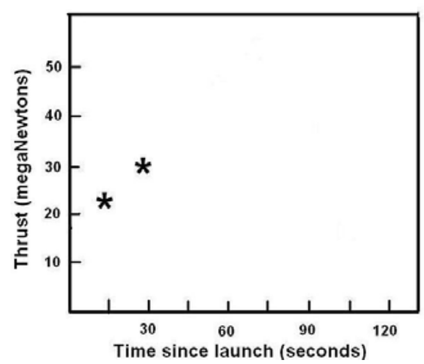
Solid Rocket Boosters - II



The example above shows the thrust curve for fuel burned in shells concentric with a cylindrical empty cavity along the axis of the booster from the inner (red) zone to the outer (blue) zone.

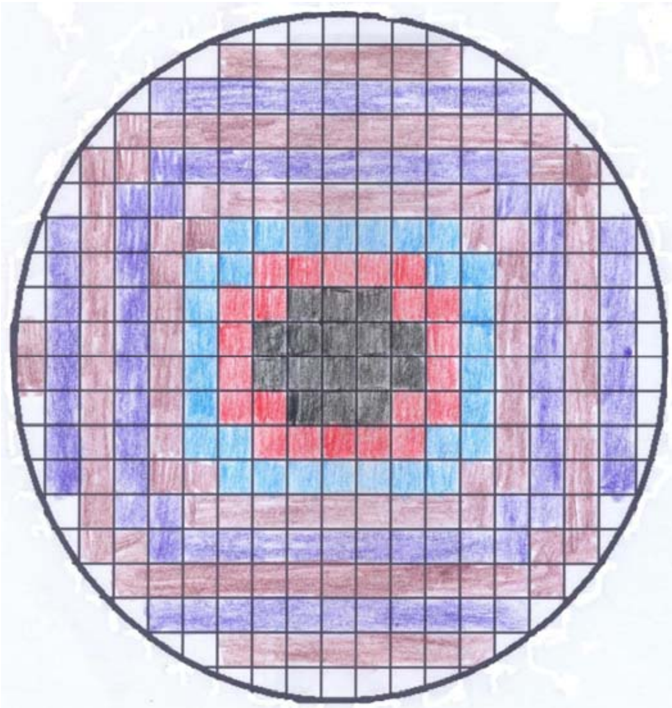
As the fuel in a solid rocket booster burns, it produces gas that exits the nozzle at very high pressure. This produces the thrust needed to launch a rocket. The area under combustion is a hollow core along the long axis of the booster from top to bottom. Depending on the shape of this empty tube, different volumes of gas will be produced from second to second, leading to different patterns of thrust for the rocket during its flight. The curve that describes a rocket engine's 'thrust versus time' is called the **thrust curve**. The more volume of fuel that is burned, the more thrust is produced.

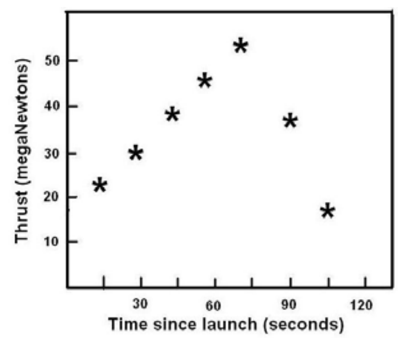




Problem 1 - The grid shows the cross section of a proposed SRB that is a cylinder 80-meters tall. The black squares represent the empty core region. As the fuel burns its way from the core to the outside circle, complete the shading of the rings of combustion at each 15-second time step. Count the number of shaded squares in each ring. The first two rings in red and blue are shown as an example. The graph represents the thrust curve and gives the number of squares shaded (red=22 and blue = 30) at each time step.

Problem 2 - How many seconds after launch does the SRB produce its maximum thrust?





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Answer: see above shaded rings. Note, the rings are only 1-box wide. Students shading may vary. Students may also estimate the areas of the partial boxes near the outer ring as an additional exercise in completing the graph.

Problem 2 - How many seconds after launch does the SRB produce its maximum thrust?

Answer: The thrust curve shows that the booster reaches a maximum thrust of about 54 megaNewtons about 75 seconds after launch.

Note to Teacher: Although the Ares-V rocket boosters are based on a 'star' shaped empty core pattern, it will reach its maximum thrust about 80 seconds after launch. Students may also experiment with other shapes for the empty core region (square, hexagon, triangle) and see what other thrust curves they can produce by the square-counting exercise.

At 11:30 AM EST, NASA successfully launched the Ares 1-X rocket from Cape Canaveral. The top image is an artists illustration of the launch and the bottom photo shows the actual launch from Pad 39B. The flight reached the target sub-orbital altitude of 150,000 feet. The next launch will be in March 2014 of Ares 1-Y. (Images courtesy NASA)

The first half of the flight just before the rocket reached its peak altitude was powered by the first stage. Once the first stage was jettisoned 150 seconds after launch, the capsule traveled under its own inertia under the influence of Earth's gravity. The capsule reached a maximum altitude of 28 miles (45 km) at a point 40 miles (64 km) downrange from the launch pad, traveling at a horizontal speed of 4000 mph (6400 km/hr). We can approximate the capsule's path by a portion of a parabolic arc.

The time that it takes a body to fall to the ground is given by

$$H = \frac{1}{2} g T^2$$

where H is the altitude in meters, T is the time in seconds and g is the acceleration of gravity given by g = 9.8 meters/sec^2.

Problem 1 - To two significant figures, how many seconds did it take the Ares 1-X capsule to fall from an altitude of 45 kilometers?

Problem 2 - At the horizontal speed that the capsule was traveling, and to two significant figures, how far from the launch pad did it come to earth if the capsule reached its maximum altitude 64 kilometers downrange from the launch pad?

Problem 1 - To two significant figures, how many seconds did it take the Ares 1-X capsule to fall from an altitude of 45 kilometers?

Answer: H = 45 kilometers or 45,000 meters
g = 9.8 meters/sec^2

And $$H = \frac{1}{2}gT^2$$

So solving for T we get

$$T = \sqrt{\frac{2H}{g}}$$

$$T = \sqrt{\frac{2x45000}{9.8}}$$

$$T = \sqrt{9184}$$

T = 95.833
T = **96 seconds** to 2 significant figures.

Problem 2 - At the horizontal speed that the capsule was traveling, and to two significant figures, how far from the launch pad did it come to earth if the capsule reached its maximum altitude 64 kilometers downrange from the launch pad?

Answer: The time taken was 96 seconds which is 96 sec x (1 hour/3600 sec) = 0.027 hours. Then V = 6,400 km/hour and the initial distance was 40 kilometers, so in 96 seconds, the capsule lands

X = x0 + VT

X = 64 km + (6400 km/hr) x (0.027 hours)

X = 237 kilometers (or 147 miles) downrange

To 2 significant figures, this becomes **240 kilometers or 150 miles** downrange from the launch pad.

The neat thing about ballistic problems (flying baseballs or rockets) is that their motion in the vertical dimension is independent of their motion in the horizontal dimension. This means we can write one equation that describes the movement in time along the x axis, and a second equation that describes the movement in time along the y axis. In function notation, we write these as y(t) and y(t) where t is the independent variable representing time.

To draw the curve representing the trajectory, we have a choice to make. We can either create a table for X and Y at various instants in time, or we can simply eliminate the independent variable, t, and plot the curve y(x).

Problem 1 - The Ares 1X underwent powered flight while its first stage rocket engines were operating, but after it reached the highest point in its trajectory (apogee) the Ares 1X capsule coasted back to Earth for a water landing. The parametric equations that defined its horizontal downrange location (x) and its altitude (y) in meters are given by

$$x(t) = 64,000 + 1800t$$

$$y(t) = 45,000 - 4.9t^2$$

Using the method of substitution, create the new function y(x) by eliminating the variable t.

Problem 2 - Determine how far downrange from launch pad 39A at Cape Canaveral the capsule landed, (y(x)=0), giving your answer in both meters and kilometers to two significant figures.

Problem 3 - Why is it sometimes easier not to work with the parametric form of the motion of a rocket or projectile?

Problem 1 - Answer: The parametric equations that defined its horizontal downrange location (x) and its altitude (y) in meters are given by

$$x(t) = 64,000 + 1800t$$
$$y(t) = 45,000 - 4.9t^2$$

We want to eliminate the variable, t, from y(t) and we do this by solving the equation x(t) for t and substituting this into the equation for y(t) to get y(t(x)) or just y(x).

$$t = \frac{x - 64,000}{1800}$$

$$y = 45,000 - 4.9\left[\frac{x - 64,000}{1800}\right]^2$$

$$y = 38,800 + 0.19x - 0.0000015x^2$$

Problem 2 - Determine how far downrange from the launch pad the capsule landed, (y(x)=0), giving your answer in both meters and kilometers to two significant figures.
Answer: Using the Quadratic Formula, find the two roots of the equation y(x)=0, and select the root with the largest positive value.

$$x_{1,2} = \frac{-0.19 \pm \sqrt{0.036 - 4(-0.0000015)38800}}{2(-0.0000015)} \text{ meters}$$

so x1 = -109,000 meters or -109 kilometers
x2 = **+237,000 meters or 237 kilometers.**

The second root, x2, is the answer that is physically consistent with the given information. Students may wonder why the mathematical model gives a second answer of -109 kilometers. This is because the parabolic model was only designed to accurately represent the physical circumstances of the coasting phase of the capsule's descent from its apogee at a distance of 64 kilometers from the launch pad. Any extrapolations to times and positions earlier than the moment of apogee are 'unphysical'.

Problem 3 - Why is it sometimes easier not to work with the parametric form of the motion of a rocket or projectile? Answer: In order to determine the trajectory in space, you need to make twice as many calculations for the parametric form than for the functional form y(x) since each point is defined by (x(t), y(t)) vs (x, y(x)).

Potential energy is the energy that a body possesses due to its **location** in space, while kinetic energy is the energy that it has depending on its **speed** through space. For locations within a few hundred kilometers of Earth's surface, and for speeds that are small compared to that of light, we have the two energy formulae:

$$P.E = mgh \qquad K.E = \frac{1}{2}mV^2$$

where g is the acceleration of gravity near Earth's surface and has a value of 9.8 meters/sec^2. If we use units of mass, **m**, in kilograms, height above the ground, **h**, in meters, and the body's speed, **V**, in meters/sec, the units of energy (P.E and K.E.) are Joules.

As a baseball, a coasting rocket, or a stone dropped from a bridge moves along its trajectory back to the ground, it is constantly exchanging, joule by joule, potential energy for kinetic energy. Before it falls, its energy is 100% P.E, while in the instant just before it lands, its energy is 100% K.E.

Problem 1 - A baseball with m = 0.145 kilograms falls from the top of its arc to the ground; a distance of 100 meters. A) What was its K.E., in joules, at the top of its arc? B) What was the baseball's P.E. in joules at the top of the arc?

Problem 2 - The Ares 1-X capsule had a mass of 5,000 kilograms. If the capsule fell 45 kilometers from the top of its trajectory 'arc', how much kinetic energy did it have at the moment of impact with the ground?

Problem 3 - Suppose that the baseball in Problem 1 was dropped from the same height as the Ares 1-X capsule. What would its K.E. be at the moment of impact?

Problem 4 - From the formula for K.E. and your answers to Problems 2 and 3, in meters/sec to two significant figures; A) What was the speed of the baseball when it hit the ground? B) What was the speed of the Ares 1-X capsule when it landed? C) Discuss how your answers do not seem to make 'common sense'.

Problem 5 - Use the formulae for P.E. and K.E. to explain your answers to Problem 4. (Hint: Don't worry about air resistance!!)

Problem 1 - A baseball with m = 0.145 kilograms falls from the top of its arc to the ground; a distance of 100 meters. A) What was its K.E., in joules, at the top of its arc? B) What was the baseball's P.E. in joules at the top of the arc?
Answer: A) **K.E = 0** B) P.E. = mgh = (0.145) x (9.8) x (100) = **142 joules.**

Problem 2 - The Ares 1-X capsule had a mass of 5,000 kilograms. If the capsule fell 45 kilometers from the top of its trajectory 'arc', how much kinetic energy did it have at the moment of impact with the ground?
Answer: At the ground, the capsule has exchanged all of its potential energy for 100% kinetic energy so K.E. = P.E. = mgh. Then K.E. = (5,000 kg) x (9.8) x (45,000 meters) = **2.2 billion joules for the Ares 1-X capsule.**

Problem 3 - Suppose that the baseball in Problem 1 was dropped from the same height as the Ares 1-X capsule. What would its K.E. be at the moment of impact?
Answer: its K.E. would equal 100% of its original P.E. so K.E = mgh = (0.145 kg) x (9.8) x (45,000 meters) = **64,000 joules for the baseball.**

Problem 4 - From the formula for K.E. and your answers to Problems 2 and 3, in meters/sec to two significant figures; A) What was the speed of the baseball when it hit the ground? B) What was the speed of the Ares 1-X capsule when it landed? C) Discuss how your answers do not seem to make 'common sense'.
Answer; A) Baseball: $E = 1/2\ m\ V^2$
$V = (2E/m)^{1/2}$
$= (2(64000)/0.145))^{1/2}$
= **940 meters/sec.**
B) Capsule: $V = (2\ (2{,}200{,}000{,}000)/5{,}000)^{1/2}$
= **940 meters/sec**

C) Our intuition suggests that the much heavier Ares 1-X capsule should have struck the ground at a far-faster speed!

Problem 5 - Use the formulae for P.E. and K.E. to explain your answers to Problem 4.
Answer: The P.E. that it starts with will be equal to the K.E when it lands, so P.E. = K.E. Comparing the formulae, we see that

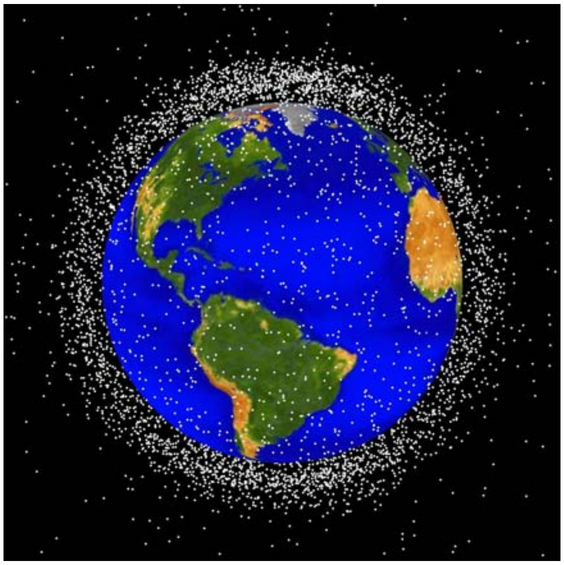
$$mgh = \frac{1}{2} m V^2$$

And since, by algebra, the variables representing mass, m, cancel from both sides, we get

$$gh = \frac{1}{2} V^2$$

so the speed upon impact, V, depends only upon the constant g, and the height from which the body fell, h. Q.E.D.

Note: The above discussion is modified slightly when air resistance is considered.



We live at the bottom of a deep 'gravity well' that takes an enormous amount of energy to climb out of. The figure shows the locations of 12,000 currently in Low Earth Orbit (LEO). To reach any altitude above sea level, we have to work against gravity. The higher we want to climb, the more energy we need to use to reach that altitude.

We can measure this energy in terms of the number of joules per kilogram (J/kg) that is needed to reach an elevation of h meters. The formula is given very simply by

$$E = 9.8\,h \quad J/kg$$

Problem 1 - A mountain climber hikes from sea level to the top of Mt Everest, which has an altitude of 8,848 meters. A) How many Joules/kg will he need for the trip? B) If his total mass is 75 kg, how many Joules of energy will he have to expend to reach the summit?

Problem 2 - The Ares 1-X rocket was launched on Wednesday, October 28 at Cape Canaveral. If the payload reached a maximum altitude of 45 kilometers, A) how many megaJoules/kg were needed for the payload to reach this altitude? B) If the payload mass was 5,000 kg, how many megaJoules were required?

Problem 3 - At what altitudes will the mountain climber and the Ares 1-X need to expend the same number of Joules/kg?

Problem 4 - How many megaJoules/kg will the Ares 1-X need to expend to reach Low Earth Orbit at an altitude of 200 miles? (1 mile = 1.62 kilometers).

Problem 5 - In terms of megaJoules/kg, by what factor is the Ares 1-X energy requirement greater to get into LEO than to reach an altitude of 45 kilometers?

Problem 1 - A mountain climber hikes from sea level to the top of Mt Everest, which has an altitude of 8,848 meters. A) How many Joules/kg will he need for the trip? B) If his total mass is 75 kg, how many Joules of energy will he have to expend to reach the summit?

Answer; A) E = 9.8 x 8848 = **87,000 J/kg.**

B) E = 87,000 J/kg x (75 kg) = **6,500,000 Joules.**

Problem 2 - The Ares 1-X rocket was launched on Wednesday, October 28 at Cape Canaveral. If the payload reached a maximum altitude of 45 kilometers, A) how many megaJoules/kg were needed for the payload to reach this altitude? B) If the payload mass was 5,000 kg, how many megaJoules were required?

Answer: A) E = 9.8 x 45 km x (1000 m/km) = **440,000 J/kg.**
B) E = 440,000 J/kg x (5,000 kg)
= 2,200,000,000 Joules = **2,200 megaJoules.**

Problem 3 - At what altitudes will the mountain climber and the Ares 1-X need to expend the same number of Joules/kg?

Answer: Because E = 9.8 x h and does not depend on the mass of the body, the mountain climber and the Ares 1-X payload will need the SAME number of Joules/kg to reach all elevations that they have in common up to 8,848 meters.

Problem 4 - How many megaJoules/kg will the Ares 1-X need to expend to reach Low Earth Orbit at an altitude of 200 miles? (1 mile = 1.62 kilometers).

Answer: First we have to convert 200 miles to meters:

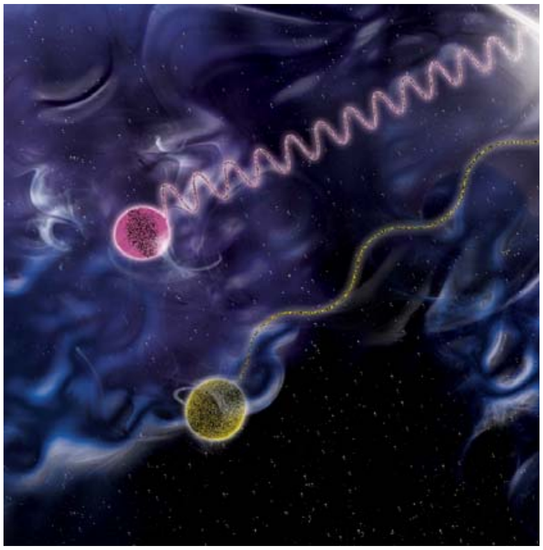
200 miles x (1.61 km/mile) x (1000 meters/1 km) = 322,000 meters.

Then E = 9.8 x (322,000) = 3,200,000 Joules/kg or **3.2 megaJoules/kg**.

Problem 5 - In terms of megaJoules/kg, by what factor is the energy requirement for Ares 1-X greater to get into LEO than to reach an altitude of 45 kilometers?

Answer: To reach 45 km requires 0.44 megaJoules/kg while for LEO it requires 3.2 megaJoules/kg so the energy factor per kilogram of payload mass is 3.2/0.44 = **7.3 times greater.**

Teacher Note: The unit Joules/kg is also called the 'energy density'. Different fuels have different energy densities. A peanut butter and jelly sandwich (500 calories and 0.1 kg) has E = 21,000 Joules/kg, while solid rocket fuel has E = 5 megaJoules/kg!


An artistic impression of two gamma-ray photons traveling through a lumpy space (Courtesy: NASA /Sonoma State University/Aurore Simonnet.)

The most advanced theories of how gravity works have proposed that empty space is not smooth, but may be filled by invisible limps and bumps that distort space into a froth of bubbles that are billions of times smaller than atomic nuclei. That's why we have not detected them in laboratory experiments thus far.

By studying how intense gamma-rays travel through the vast spaces between galaxies in the universe, NASA's Fermi Gamma-ray Observatory may have placed limits on this frothiness that eliminate many of the theories being explored to date.

As the gamma-rays travel through space, the shortest-wavelength gamma-rays take a slightly different path through space than the longer-wavelength gamma-rays. Although all gamma-rays travel at the speed of light, the invisible lumps in space scatter the short-wavelength (high-energy) gamma rays more than the long-wavelength (low-energy) ones so that there is a difference in the travel times of the long and short-wavelength gamma rays. This means that in traveling a distance, L, to Earth, the time should be about t = L/c where c is the speed of light. The theories predict that the lumps in space cause the arrival times for the long and short-wavelength gamma-rays to differ by an amount **$T = (L/c) \times (d/\lambda)$.** In this equation, λ is the wavelength difference between the two gamma-rays (related to their energy-difference), **d** is the length scale corresponding to the lumpiness in space, and **c** is the speed of light, 3×10^{10} cm/sec.

Problem 1 – Suppose two rays of visible light differed in wavelength by 100 nanometers ($\lambda = 10^{-9}$ cm) and traveled from the sun to Earth (L=150 million kilometers) through space with a lumpiness of about the scale of an atomic nucleus ($d=10^{-14}$ cm). What would be the arrival time delay in seconds at Earth between the two visible light photons?

Problem 2 – Suppose there is no indication that arriving visible light photons (10^{-9} cm) experience any delays in arrival time longer than 1 second, from quasars as far away as 5 billion light years. What is the maximum size of the lumps in space that are consistent with this limit?

Problem 3 – The Fermi Telescope measured a gamma-ray pulse from a distant object located 10 billion light years from Earth. The time delay was no more than 0.7 seconds. The wavelength difference in the gamma-rays was 4.0×10^{-12} centimeters (33 billion electron volts of energy). What is the largest size that can be involved in the lumpiness of empty space given this Fermi measurement.

Problem 1 – Suppose two rays of visible light differed by 100 nanometers (10^{-9} cm) in wavelength, and traveled from the sun to earth (150 million kilometers) through space with a lumpiness of about the scale of an atomic nucleus (10^{-14} cm). What would be the arrival time delay in seconds at Earth between the two visible light photons?

Answer: T = (1.5×10^{13} cm) x (1.0×10^{-14}) / ($3 \times 10^{10} \times 1.0 \times 10^{-9}$)
= **0.005 seconds**.

Problem 2 – Suppose there is no indication that arriving visible light photons (10^{-9} cm) experience any delays in arrival longer than one second, from quasars as far away as 5 billion light years. What is the maximum size of the lumps in space that are consistent with this limit?

Answer: L = 5 billion light years; $\lambda = 10^{-9}$ cm ; T = 1 second
Also L/c = 5 billion light years/c
= 5 billion years.
= 5×10^{9} years x 3.1×10^{7} sec/year
= 1.6×10^{17} seconds

So d = T λ c / L
= (1 sec) x (1.0×10^{-9} cm)/ (1.6×10^{17} sec)
= **6.3×10^{-27} centimeters.**

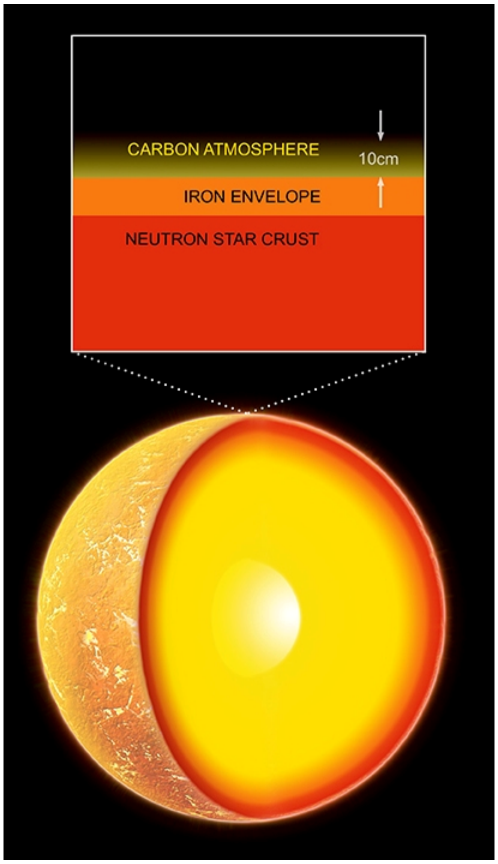
Note: This is 10 trillion times smaller than the size of an atomic nucleus!

Problem 3 – The Fermi Telescope measured a gamma-ray pulse from a distant object located 10 billion light years from Earth. The time delay was no more than 0.7 seconds. The wavelength difference in the gamma-rays was 4.0×10^{-12} centimeters. What is the largest size that can be involved in the lumpiness of empty space given this Fermi measurement.

Answer: L = 10 billion light years; $\lambda = 4.0 \times 10^{-12}$ cm ; T = 0.7 second
Also L/c = 10 billion light years/c
= 10 billion years.
= 1.0×10^{10} years x 3.1×10^{7} sec/year
= 3.1×10^{17} seconds

So d = T λ c / L
= (0.7 sec) x (1.0×10^{-12} cm)/ (3.1×10^{17} sec)
= **2.2×10^{-30} centimeters.**

Note: This is 22,000 trillion times smaller than the size of an atomic nucleus! According to some theories of the structure of space, the fundamental limit is about 1.6×10^{-33} centimeters and is called the Planck Length. The Fermi limit corresponds to about 1,400 Planck Lengths or smaller.



The atmosphere of Earth has a thickness of over 100 kilometers, however the Chandra X-Ray Observatory recently detected an atmosphere on the neutron star Cassiopeia-A that measured only 10 centimeters thick. How can a small planet have a deeper atmosphere than an entire stars-worth of matter?

The size of an atmosphere depends on a balance between the force of gravity pulling it towards the center of the planet, and the pressure of the atmosphere due to its temperature and density, pushing in the opposite direction. A pair of simple equations then defines how the density of the atmosphere has to rearrange itself with height above the surface so that gravity and pressure are always in balance. The equations look like this:

$$n(z) = n_0 e^{-\frac{z}{H}} \quad \text{where} \quad H = \frac{kT}{mg}$$

The exponential equation says that as you get farther from the surface, the density of the gas, N, drops very fast. The quantity, H in meters, is called the 'scale height' and its value is defined by the atmosphere's temperature, T, in Kelvins, and the acceleration of gravity at the surface, g, in multiples of Earth's acceleration (9.8 meters/sec^2). It also depends on the average mass, m, of the particles in the atmosphere. A light atmosphere made from hydrogen (m=1) will produce a value for H that is much larger than one made from pure oxygen (m=16). In this equation, k is Boltzman's Constant and equals 1.38 x 10^{-23} Joules/degree.

Problem 1 – The surface acceleration of the neutron star is 100 billion times that of Earth, the temperature of the gas is 3 million Kelvins compared to Earth's of 300 Kelvins, and the neutron star atmosphere is composed of carbon (A=12) rather than Earth's mixture of nitrogen and oxygen (A=28). From the formula for H, and the way in which it scales with m, T and g, what would you predict as the scale height for the neutron star atmosphere if for Earth, H = 8 kilometers?

Problem 2 – How far from the surface would you have to travel in order for the density of the atmosphere to fall by 1 million times for: A) Earth and B) the neutron star?

Problem 1 – The surface acceleration of the neutron star is 100 billion times that of Earth, the temperature of the gas is 3 million Kelvins compared to Earth's of 300 Kelvins, and the neutron star atmosphere is composed of carbon (A=12) rather than Earth's mixture of nitrogen and oxygen (A=28). From the formula for H, and the way in which it scales with m, T and g, what would you predict as the scale height for the neutron star atmosphere if for Earth, H = 8 kilometers?

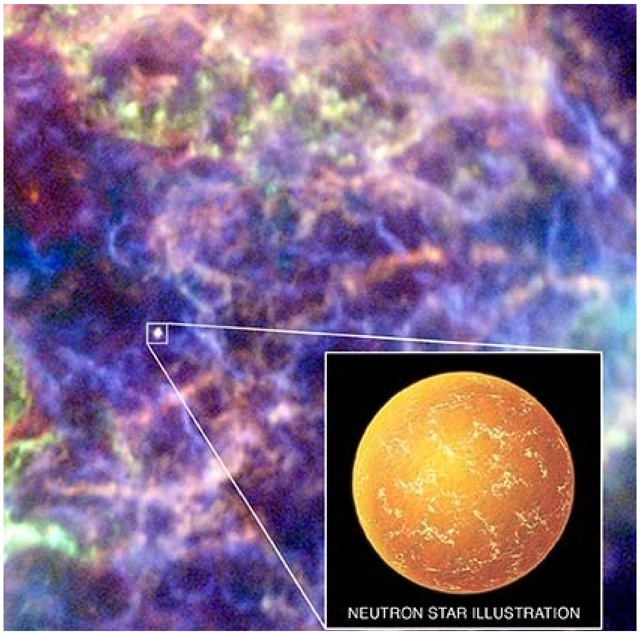
Answer: This is an exercise in scaling and proportionality using equations and their variables. H is proportional to T and inversely proportional to the product of m and g, so that means that if we double only T, H increases by a factor of 2; if we double only g, H decreases by a factor of 2; and if we double only the mass of the atmosphere particles, H decreases by a factor of 2. Combining these proportionalities and starting with the value of H=8 kilometers for Earth, we have that T increases by 3 million/300 = 10,000 times; m decreases by 12/28 = 0.4 and g increases by 100 billion g/1g = 100 billion times. The new scale height is then H = 8 kilometers x (10,000)/(0.4 x 100 billion) **= 0.2 centimeters**.

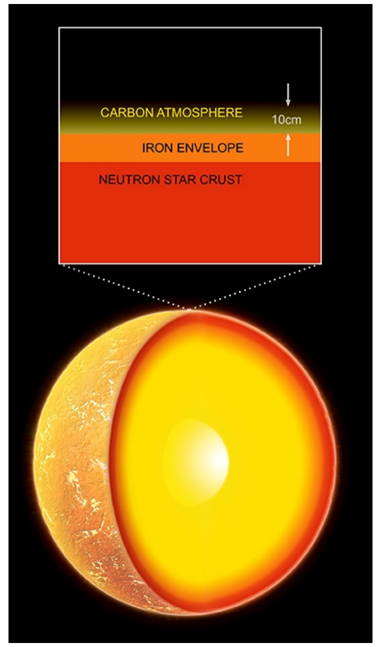
Problem 2 – How far from the surface would you have to travel in order for the density of the atmosphere to fall by 1 million times for: A) the Earth and B) the neutron star?

Answer: We use the equation for the atmosphere density N = n0 e-x/H and determine for what value of the height, x, that $e^{-x/H}$ = 1/1000000. We can solve the equation as follows:

Take reciprocals of both sides:	1,000,000 = e(x/h)
Take the log, base-e, of both sides	13.8 = x/H
Solve for x	13.8 H = x
A) For Earth, H = 8 kilometers, so	13.8 (8 kilometers) = x
	x = **110 kilometers!**
B) For the neutron star, H = 0.13 centimeters so	13.8 (0.2 cm) = x
	x = **2.8 centimeters.**

Note to teacher: At the densities of the neutron star atmosphere (3.5 grams/cc) the carbon is not a gas but acts more like a crystalline solid so that the thickness, H, will not be completely determined by the equilibrium equations we have been using for Earth's atmosphere.





NASA Press Release – “The Chandra X-ray Observatory image to the left shows the central region of the supernova remnant Cassiopeia A. This interstellar cloud 14 light years across, is all that remains of a massive star that exploded 330 years ago. A careful analysis of the X-ray data has revealed that the dense neutron star left behind by the supernova has a thin carbon atmosphere as shown in the figure to the right. The neutron star is only 14 miles (23 kilometers) in diameter, and is as dense as an atomic nucleus (100 trillion gm/cc). The atmosphere is only about four inches (10 cm) thick, has a density similar to diamond (3.5 gm/cc), and a temperature of nearly 2 million Kelvin. The surface gravity on the neutron star is 100 billion times stronger than on Earth, which causes the atmosphere to be incrredibly thing even with such a high temperature.”

How much carbon is there?

Problem 1 – What are the facts that we know about the atmosphere from the news announcement, and what combination of facts will help us estimate the atmosphere’s mass?

Problem 2 – If the volume of a thin spherical shell is $V = 4 \pi R^2 h$ where R is the radius of the sphere and h is the thickness of the shell, what other formula do you need to calculate the atmosphere’s mass?

Problem 3 – What is your estimate for the mass of the carbon atmosphere in A) kilograms? B) metric tons? C) Earth Atmosphere masses (Ae) where 1 Ae = 5.1×10^{18} kg? (Provide answers to two significant figures)

Problem 1 – What are the facts that we know about the atmosphere from the news announcement, and what combination of facts will help us estimate the atmosphere's mass?

Answer: The facts are as follows, with the facts that help estimate the atmosphere's mass indicated in bold face:

1... The interstellar cloud is 14 light years across,

2… The supernova exploded 330 years ago.

3… **The neutron star is only 14 miles (23 kilometers) in diameter**.

4… The neutron star is as dense as an atomic nucleus (100 trillion gm/cc).

5… **The atmosphere is only about four inches (10 cm) thick,**

6… **The atmosphere has a density similar to diamond (3.5 gm/cc),**

7… The atmosphere has a temperature of nearly 2 million Kelvin.

8… The surface gravity on the neutron star is 100 billion times stronger than on Earth.

Problem 2 – If the volume of a thin spherical shell is $V = 4 \pi R^2 h$ where R is the radius of the sphere and h is the thickness of the shell, what other formula do you need to calculate the atmosphere's mass?

Answer: The formula gives the volume occupied by the atmosphere, so you need a relationship that relates volume to mass: **Mass = Density x Volume.**

Problem 3 – What is your estimate for the mass of the carbon atmosphere in A) kilograms? B) metric tons? C) Earth Atmosphere masses (Ae) where 1 Ae = 5.1×10^{18} kg? (Provide answers to two significant figures)

Answer: Convert all measures to centimeters, so the neutron star diameter is 23 kilometers x (100,000 cm/1 km) = 2.3×10^6 cm and its radius is 1.1×10^6 cm. . The volume of the atmosphere is

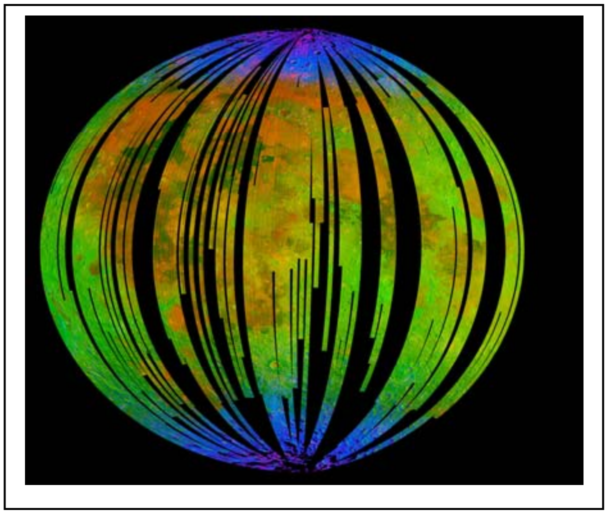
V(shell) = $4 \times 3.14 \times (1.1 \times 10^6 \text{ cm})^2 (10 \text{ cm})$
= $1.5 \times 10^{14} \text{ cm}^3$

A) Mass = $(3.5 \text{ grams/cm}^3) \times 1.5 \times 10^{14} \text{ cm}^3$
= 5.3×10^{14} grams
= 5.3×10^{14} grams x (1 kg/1000 grams) = **5.3×10^{11} kilograms**

B) 5.3×10^{11} kilograms x (1 tons/1000 kilograms) = **5.3×10^8 tons (also 0.53 gigatons)**

C) 5.3×10^{11} kilograms x (1 Ae/ 5.1×10^{18} kilograms) = **1.0×10^{-7} Ae**

Water on the Moon !



The debate has gone back and forth over the last 10 years as new data are found, but measurements by Deep Impact/EPOXI, Cassini and most recently the Lunar Reconnaissance Orbiter and Chandrayaan-1 are now considered conclusive. Beneath the shadows of polar craters, billions of gallons of water may be available for harvesting by future astronauts.

The image to the left created by the Moon Minerology Mapper (M3) instrument onboard Chandrayaan-1, shows deposits and sources of hydroxyl molecules. The data has been colored blue and superimposed on a lunar photo.

Complimentary data from the Deep Impact/EPOXI and Cassini missions of the rest of the lunar surface also detected hydroxyl molecules covering about 25% of the surveyed lunar surface. The hydroxyl molecule (symbol OH) consists of one atom of oxygen and one of hydrogen, and because water is basically a hydroxyl molecule with a second hydrogen atom added, detecting hydroxyl on the moon is an indication that water molecules are also present.

How much water might be present? The M3 instrument can only detect hydroxyl molecules if they are in the top 1-millimeter of the lunar surface. The measurements also suggest that about 1 metric ton of lunar surface has to be processed to extract 1 liter (0.26 gallons) of water.

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1?

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the lunar surface contains water?

Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters).

Problem 1 – The radius of the moon is 1,731 kilometers. How many cubic meters of surface volume is present in a layer that is 1 millimeter thick?

Answer: The surface area of a sphere is given by $S = 4\pi r^2$ and so the volume of a layer with a thickness of L is $V = 4\pi r^2 L$ provided that L is much smaller than r.
$V = 4 \times (3.141) \times (1731000)^2 \times 0.001 =$ **3.76×10^{10} m^3**

Problem 2 – The density of the lunar surface (called the regolith) is about 3000 kilograms/meter^3. How many metric tons of regolith are found in the surface volume calculated in Problem 1? Answer: 3.76×10^{10} m^3 x (3000 kg/m^3) x (1 ton/1000 kg) = **1.13×10^{11} metric tons.**

Problem 3 – The concentration of water is 1 liter per metric ton. How many liters of water could be recovered from the 1 millimeter thick surface layer if 25% of the surface contains water? Answer: 1.13×10^{11} tons x (1 liter water/1 ton regolith) x 1/4 = **2.8×10^{10} liters of water.**

Problem 4 – How many gallons could be recovered if the entire surface layer were mined? (1 Gallon = 3.78 liters). Answer: 2.8×10^{10} liters x (1 gallon / 3.78 liters) = 7.5×10^9 gallons of water or about **8 billion gallons of water.**

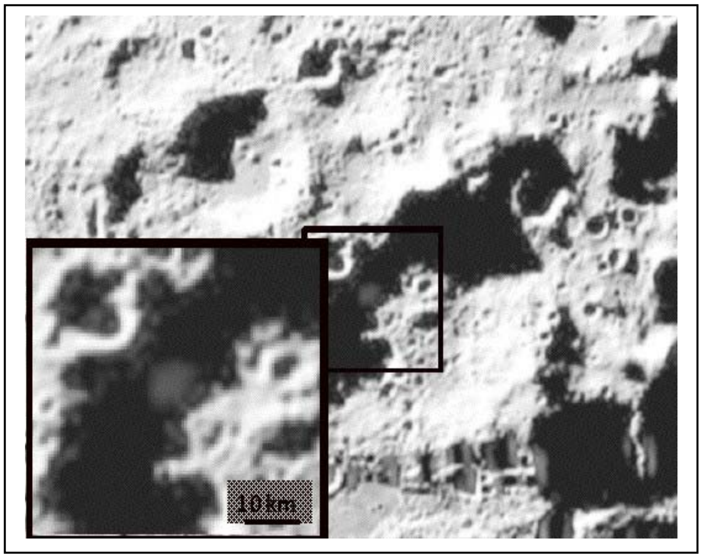
Note: This is similar to the roughly '7 billion gallon' estimate made by the M3 scientists as described in the NASA Press Release for this discovery in September 2009.

For more information, visit:

Moon Minerology Mapper News - http://moonmineralogymapper.jpl.nasa.gov/

The front picture of the moon is from NASA's Moon Mineralogy Mapper on the Indian Space Research Organization's Chandrayaan-1 mission. It is a three-color composite of reflected near-infrared radiation from the sun, and illustrates the extent to which different materials are mapped across the side of the moon that faces Earth. Small amounts of water and hydroxyl (blue) were detected on the surface of the moon at various locations. This image illustrates their distribution at high latitudes toward the poles. **Blue** shows the signature of water and hydroxyl molecules as seen by a highly diagnostic absorption of infrared light with a wavelength of three micrometers. **Green** shows the brightness of the surface as measured by reflected infrared radiation from the sun with a wavelength of 2.4 micrometers. **Red** shows an iron-bearing mineral called pyroxene, detected by absorption of 2.0-micrometer infrared light.

LCROSS 'Sees' Water on the Moon!



On October 9, 2009 the LCROSS spacecraft and its companion Centaur upper stage, impacted the lunar surface within the shadowed crater Cabeus located near the moon's South Pole. The Centaur impact speed was 9,000 km/hr with a mass of 2.2 tons.

The impact created a crater about 20 meters across and about 3 meters deep. Some of the excavated material formed a plume of debris visible to the LCROSS satellite as it flew by. Instruments on LCROSS detected about 25 gallons of water.

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Problem 2 - If the density of the lunar soil (regolith) is about 3000 $kilograms/meter^3$, how many tons of regolith were excavated by the impact?

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Problem 1 - The volume of the crater can be approximated as a cylinder with a diameter of 20 meters and a height of 3 meters. From the formula $V = \pi R^2 h$, what was the volume of the lunar surface excavated by the LCROSS-Centaur impact in cubic meters?

Answer: V = (3.14) x (10 meters)^2 x 3 = **942 cubic meters**.

Problem 2 - If the density of the lunar soil (regolith) is about 3000 kilograms/meter^3, how many tons of regolith were excavated by the impact?

Answer: 3000 kg/m^3 x (942 meters^3) = 2,800,000 kilograms. Since 1000 kg = 1 ton, there were **2,800 tons of regolith excavated**.

Problem 3 - During an impact, most of the excavated material remains as a ring-shaped ejecta blanket around the crater. For the LCROSS crater, the ejecta appeared to be scattered over an area about 70 meters in diameter and perhaps 0.2 meter thick around the crater. How many tons of regolith from the crater remained near the crater?

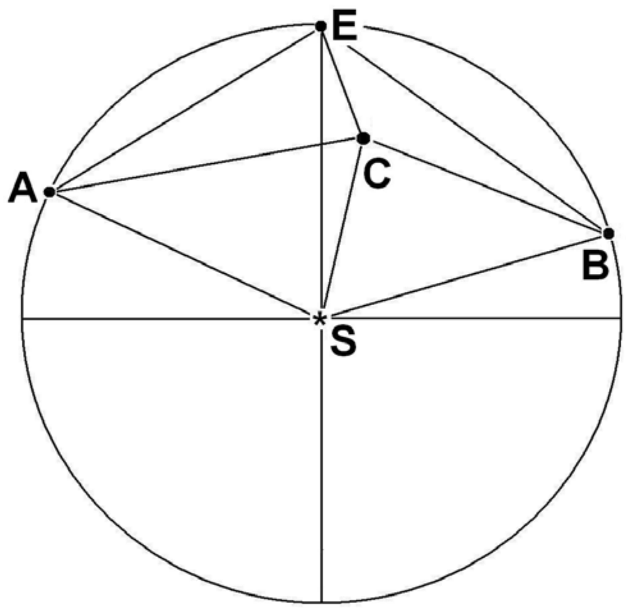
Answer: The area of the ejecta blanket is given by A = π(35 meters)^2 - π(10 meters)^2 = 3,846 - 314 = 3500 meters^2. The volume is A x h = (3500 meters^2) x 0.2 meters = 700 meters^3 . Then the mass is just M = (700 meters^3) x (3,000 kilograms/meter^3) = 2,100,000 kilograms or **2,100 tons in the ejecta blanket**.

Problem 4 - If the detected water came from the regolith ejected in the plume, and not scattered in the ejecta blanket ,what was the concentration of water in the plume in units of tons of regolith per liter of water?

Answer: The amount of ejected regolith was 2,800 tons - 2,100 tons or 700 tons. The detected water amounted to 25 gallons or 25 gallons x (3.78 liters/ 1 gallon) = 95 liters. So the concentration was about C = 700 tons/95 liters = **7 tons/liter**.

Note to teacher: The estimated concentration, C, in Problem 4 is based on an approximated geometry for the crater (cylinder), an average thickness for the ejecta blanket (about 0.2 meters) and whether all of the remaining material (700 tons) was actually involved in the plume measured by LCROSS. Students may select, by scaled drawing, other geometries for the crater, and thickness for the ejecta blanket to obtain other estimates for the concentration, C. The scientific analysis of the LCROSS data may eventually lead to better estimates for C.

Seeing Solar Storms in STEREO - I



The two STEREO spacecraft are located along Earth's orbit and can view gas clouds ejected by the sun as they travel to Earth. From the geometry, astronomers can accurately determine their speeds, distances, shapes and other properties.

By studying the separate 'stereo' images, astronomers can determine the speed and direction of the cloud before it reaches Earth.

Use the diagram, (angles and distances not drawn to the same scale of the 'givens' below) to answer the following question.

The two STEREO satellites are located at points A and B, with Earth located at Point E and the sun located at Point S, which is the center of a circle with a radius ES of 1.0 Astronomical Unit (1 AU is the distance of Earth from the sun or 150 million kilometers). Suppose that the two satellites spot a Coronal Mass Ejection (CME) cloud at Point C. Satellite A measures its angle from the sun mSAC as 45 degrees while Satellite B measures the corresponding angle to be mSBC=50 degrees. The CME is ejected from the sun at the angle mESC=14 degrees.

Problem 1 - The astronomers want to know the distance that the CME is from Earth, which is the length of the segment EC. The also want to know the approach angle, mSEC. Use either a scaled construction (easy: using compass, protractor and millimeter ruler) or geometric calculation (difficult: using trigonometric identities) to determine EC from the available data.

Givens from satellite orbits:

SB = SA = SE = 150 million km   AE = 136 million km   BE = 122 million km
mASE = 54 degrees   mBSE = 48 degrees
mEAS = 63 degrees   mEBS = 66 degrees   mAEB = 129 degrees

Find the measures of all of the angles and segment lengths in thee above diagram rounded to the nearest integer.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?

Answer Key

Givens from satellite orbits:
SB = SA = SE = 150 million km AE = 136 million km BE = 122 million km
mASE = 54 degrees mBSE = 48 degrees
mEAS = 63 degrees mEBS = 66 degrees mAEB = 129 degrees
use units of megakilometers i.e. 150 million km = 150 Mkm.

Method 1: Students construct a scaled model of the diagram based on the angles and measures, then use a protractor to measure the missing angles, and from the scale of the figure (in millions of kilometers per millimeter) they can measure the required segments. **Segment EC is about 49 Mkm at an angle, mSEB of 28 degrees.**

Method 2 use the Law of Cosines and the Law of Sines to solve for the angles and segment lengths exactly.

mEAC = mEAS - mSAC = 63 - 45 = **18 degrees**
mASC = mASE + mESC = 54 + 14 = **68 degrees**
mASB = mASE + mBSE = 54 + 48 = **102 degrees**
mCSB = mASB - mASE - mESC = 102 - 54 - 14 = **34 degrees**

SC from Law of Sines: Sin(45)/SC = Sin(67)/150Mkm so **SC = 115 Mkm.**

CB from Law of Cosines: $CB^2 = 115^2 + 150^2 - 2(115)(150)\cos(34)$ so **CB = 84 Mkm**

mEBC = mEBS - mSBC = 66 - 50 = **16 degrees**

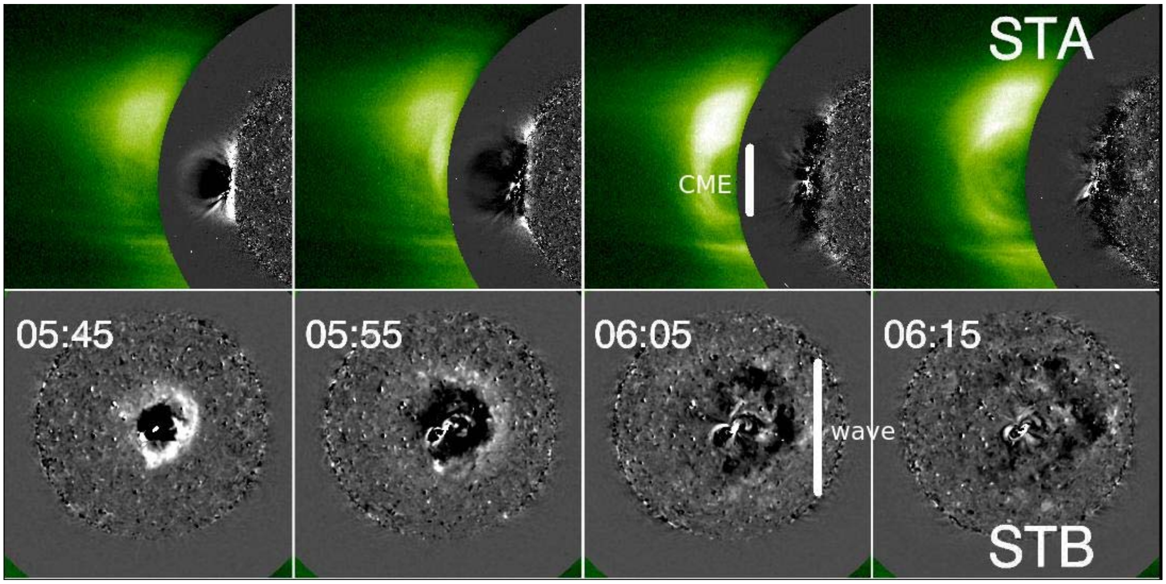
EC from law of Cosines: $EC^2 = 122^2 + 84^2 - 2(84)(122)\cos(16)$
EC = 47 Mkm.

mCEB from Law of Cosines: $84^2 = 122^2 + 47^2 - 2(122)(47)\cos(mCEB)$
mCEB = 29 degrees

And since mAES = 180 - mASE - mEAS = 180 - 54 - 63 = 63 degrees
so mSEC = mAEB - mAES - mCEB = 129 - 63 - 29 = **37 degrees**

So, the two satellites are able to determine that the CME is 49 million kilometers from Earth and approaching at an angle of 37 degrees from the sun.

Problem 2 - If the CME was traveling at 2 million km/hour, how long did it take to reach the distance indicated by the length of segment SC?
Answer: 115 million kilometers / 2 million km/hr = **58 hours or 2.4 days.**



A solar tsunami that occurred in February 13, 2009 has recently been identified in the data from NASA's STEREO satellites. It was spotted rushing across the Sun's surface. STEREO recorded the wave from two positions separated by 90 degrees, giving researchers a spectacular view of the event. Satellite A (STA) provided a side-view of the explosion, called a Coronal Mass Ejection (CME), while Satellite B (STB) viewed the explosion from directly above. The technical name is "fast-mode magnetohydrodynamic wave" – or "MHD wave" for short. The one STEREO saw raced outward at 560,000 mph (250 km/s) packing as much energy as 2,400 megatons of TNT.

Problem 1 - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Problem 2 - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? (Note '05:45' means 5:45 o'clock Universal Time).

Problem 3 - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

Problem 4 - Kinetic Energy is defined by the equation K.E. = $1/2\ m\ V^2$ where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

Problem 5 - If 1 kiloton of TNT has the explosive energy of 4.1×10^{12} Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

Problem 1 - In the lower strip of images, the sun's disk is defined by the mottled circular area, which has a physical radius of 696,000 kilometers. Use a millimeter ruler to determine the scale of these images in kilometers/mm.

Answer: The diameter is 31 millimeters, which corresponds to 2 x 696,000 km or 1,392,000 km. The scale is then 1,392,000 km/31mm = 45,000 km/mm.

Problem 2 - The white circular ring defines the outer edge of the expanding MHD wave. How many kilometers did the ring expand between 05:45 and 06:15? (Note '05:45' means 5:45 o'clock Universal Time).

Answer: From the scale of 45,000 km/mm, the difference in the ring radii is 12mm - 5 mm = 7mm which corresponds to 7 mm x (45,000 km/1 mm) = 315,000 kilometers. Students answers may vary depending on where they defined the outer edge of the ring.

Problem 3 - From your answers to Problem 1 and 2, what was the approximate speed of this MHD wave in kilometers/sec?

Answer: The time difference is 06:15 - 05:45 = 30 minutes. The speed was about 315,000 km / 30 minutes = 11,000 kilometers/minute, which is 11,000 km/minute x (1 minute/60 seconds) = 180 kilometers/sec.

Problem 4 - Kinetic Energy is defined by the equation K.E. = 1/2 m V^2 where m is the mass of the object in kilograms, and V is its speed in meters/sec. Suppose the mass of the CME was about 1 million metric tons, use your answer to Problem 3 to calculate the K.E., which will be in units of Joules.

Answer: The mass of the CME was 1 billion metric tons. There are 1,000 kilograms in 1 metric ton, so the mass was 1.0×10^{12} kilograms. The speed is 180 km/sec which is 180,000 meters/sec. The kinetic energy is then about $0.5 \times 1.0 \times 10^{12} \times (180,000)^2$ = **1.6×10^{22} Joules.**

Problem 5 - If 1 kiloton of TNT has the explosive energy of 4.1×10^{12} Joules, how many megatons of TNT does the kinetic energy of the tsunami represent?

Amswer: 1.6×10^{22} Joules x (1 kiloton TNT/4.1×10^{12} Joules) = 3.9×10^{9} kilotons TNT. Since 1 megaton = 1,000 kilotons, we have an explosive yield of **3,900,000 megatons TNT**. (Note; this answer differs from the STEREO estimate because the speed is approximate, and does not include the curvature of the sun).

Teacher Note: Additional information, and movies of the event, can be found at the STEREO website: **http://stereo.gsfc.nasa.gov/news/SolarTsunami.shtml.** Also published in the Astrophysical Journal Letters (*ApJ* **700** L182-L186)

Calculating Black Hole Power

This image shows the large galaxy, NGC-6872, interacting with a smaller galaxy, IC-4970, located just above the center of NGC-6872. These galaxies are located in the southern constellation Pavo, and about 300 million light years from the Sun. From tip to tip, NGC-6872 measures about 700,000 light years, making it nearly 3 times as big as the Milky Way. The image is a composite made by NASA's Spitzer and Chandra satellites, and a ground-based telescope. Although NGC-6872 is dramatically bigger, IC-4970 is the real 'player' in this collision. It contains a massive black hole that is emitting energy as it absorbs interstellar gas and dust. This material has been gravitationally ripped from the larger galaxy. The X-ray power, alone, is about 450 million times the sun's total light output!

When black holes 'digest' gas, stars and other forms of matter that enter their event horizons, the energy that is produced as the matter falls in can be converted into heat in the orbiting 'accretion disk' that surrounds a black hole. The amount of heat energy that is emitted by the gas in this disk each second (power) can be detected at great distances as a brilliant source of light or other forms of electromagnetic radiation. The formula that approximately relates the rate of in-falling matter into a super-massive black hole (**R** in solar masses per year), to the emitted power (**L** in multiples of the sun's power) is given by $L = 1.1 \times 10^{12}\ R$ solar luminosities. (Note 1 solar mass = 2×10^{33} grams, and 1 solar luminosity = 4×10^{33} ergs/sec).

Problem 1 - What is the minimum accretion rate that is needed to account for the x-ray power of the black hole in the core of IC-4970?

Problem 2 - How much mass would have to be accreted in order for the supermassive black hole to have the same power as an average quasar with a luminosity of about 2 trillion times the luminosity of our sun?

Problem 3 - The supermassive black hole in the center of the Milky Way has an estimated output equal to 2,500 suns. About how fast is it accreting matter?

Problem 1 - What is the minimum accretion rate that is needed to account for the x-ray power of the black hole in the core of IC-4970?
Answer: $L = 1.1 \times 10^{12}$ R solar luminosities, and since Lx = 450 million solar luminosities, solving for R we get $R = 4.5 \times 10^{8} / 1.1 \times 10^{12}$ = **0.00041 solar masses per year**.

Problem 2 - How much mass would have to be accreted in order for the supermassive black hole to have the same power as an average quasar with a luminosity of about 2 trillion times the luminosity of our sun?
Answer: $R = 2.0 \times 10^{12} / 1.1 \times 10^{12}$ = **1.8 solar masses per year**.

Problem 3 - The supermassive black hole in the center of the Milky Way has an estimated output equal to 2,500 suns. About how fast is it accreting matter?
Answer: $R = 2{,}500 / 1.1 \times 10^{12}$ = **2.3×10^{-9} solar masses per year**

Note: The formula is derived by using $E = mc^2$ to convert the in-falling mass into energy, and for non-rotating 'Schwarschild' black holes, the conversion efficiency is 7%, so that only 7% of the available 'rest mass' energy of the in-falling material actually is converted into energy. If R is given in solar masses per year, then the energy liberated is equal to:

L = R (solar mass/year) x (2×10^{33} grams/solar mass) x $(3 \times 10^{10})^2 \times 0.07$
= 1.3 x 1053 R ergs/year

Since 1 solar luminosity =3.8×10^{33} ergs/sec, and 1 year = 3.1×10^{7} seconds we have:

L = 1.3×10^{53} ergs/year x (1 solar luminosity/ 3.8×10^{33}) x (1 year/3.1×10^{7} seconds) R

L = 1.1×10^{12} R solar luminosities.
where R is the accretion rate in solar masses per year.

$$B = B_0 10^{-0.4m}$$

Since the time of the Greek astronomer Hipparchus (190BC), astronomers have used a 'magnitude' scale to indicate the brightness of stars. A star of the First Magnitude (+1.0) is brighter than a star of the Second Magnitude (+2.0) and so on, which means that, the more positive a star's magnitude is, the fainter is the star! Although modern astronomers would have preferred a more conventional scale, we are basically stuck with this ancient convention.

On the stellar magnitude scale, a difference of 5 magnitudes is exactly a brightness difference of 100 times. This magnitude scale, **m**, is related to a physical brightness scale, **B**, using the formula shown to the left.

The star field image is from the Hubble Space Telescope Exoplanet Survey and shows a range of stellar brightnesses spanning a magnitude range from about +13 to +17

Problem 1 - In this formula, to what magnitude does a brightness of B_0 correspond?

Problem 2 - The sun has a magnitude of -26.5, and the faintest star detected by the Hubble Space Telescope has a magnitude of +28.5 A) What is the magnitude difference between these two objects? B) By what factor to they differ in brightness?

Problem 3 - An astronomer has a digital camera that can accommodate a brightness level change of about 10 million from the brightest to the faintest object that can be imaged without 'saturating' the camera. What magnitude difference does this range correspond to?

Problem 4 - NASA's WISE infrared sky survey satellite will detect stars at a wavelength of 4.6 microns (called M-band) to a brightness limit of 160 microJanskys. If B0 is 180 Janskys at this wavelength, and for the visual magnitude scale (called V-band) B0 = 3,781 Janskys. What will be the equivalent magnitude limit of the WISE survey at 4.6 microns, and in the visual band?

Problem 1 - In this formula, to what magnitude does a brightness of B_0 correspond?

Answer: It corresponds to m=0 .This means that B_0 establishes the 'zero-point' for this magnitude scale.

Problem 2 - The sun has a magnitude of -26.5, and the faintest star detected by the Hubble Space Telescope has a magnitude of +28.5 A) What is the magnitude difference between these two objects? B) By what factor do they differ in brightness?

Answer: A) 28.5 - (-26.5) = **55 magnitudes**! B) From the formula, the brightness ratio will be $10^{0.4(55)}$ = **10^{22} times** (or alternately 10^{-22} times).

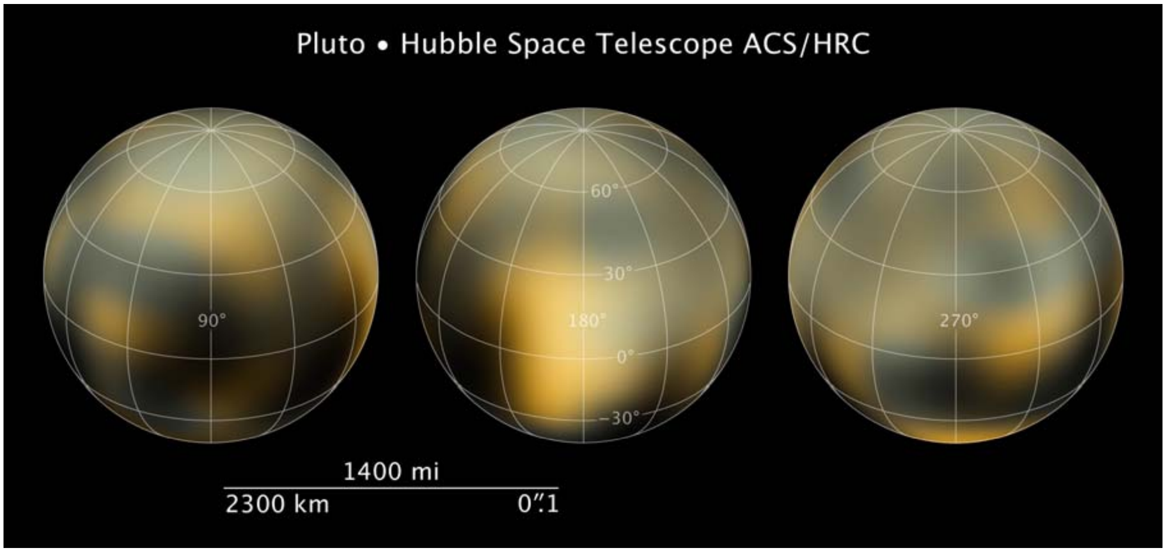
Problem 3 - An astronomer has a digital camera that can accommodates a brightness level change of about 10 million from the brightest to the faintest object that can be imaged without 'saturating' the camera. What magnitude difference does this range correspond?

Answer $10^7 = 10^{0.4(m)}$ so m = **17.5 magnitudes**.

Problem 4 - NASA's WISE infrared sky survey satellite will detect stars at a wavelength of 4.6 microns (called M-band) to a brightness limit of 160 microJanskys. If B0 is 180 Janskys at this wavelength, and for the visual magnitude scale (called V-band) B0 = 3,781 Janskys. What will be the equivalent magnitude limit of the WISE survey at 4.6 microns, and in the visual band?

Answer: For M-band, 0.000160 Jy = 180 Jy $10^{-0.4m}$ so m(M-band) = **+15.1**
For V-band, 0.000160 Jy = 3,781 Jy $10^{-0.4m}$ so m(V-band) = **+18.4**. So, the same brightness level in V-band corresponds to a magnitude of +18.4 which is +3.3 magnitudes fainter than in M-band.

Seeing a Dwarf Planet Clearly: Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. The images, created at the very limits of Hubble's ability to see small details (sometimes called a telescope's resolving power), show enigmatic light and dark regions that are probably organic compounds (dark areas) and methane or water-ice deposits (light areas). Since these photos are all that we are likely to get until NASA's New Horizons spacecraft arrives in 2015, let's see what we can learn from the image!

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter?

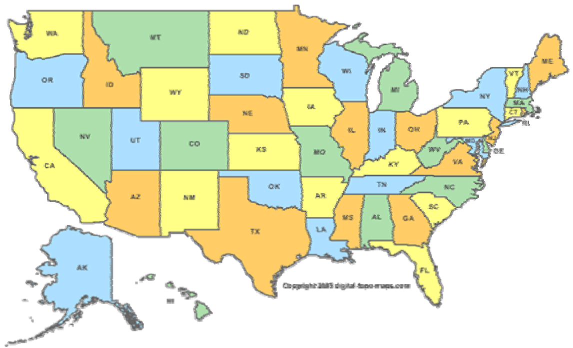
Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?

Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)?

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Problem 1 - Using a millimeter ruler, what is the scale of the Hubble image in kilometers/millimeter? Answer: The Legend bar indicates 2,300 km and is 43 millimeters long so the scale is 2300/43 = **53 km/mm**.

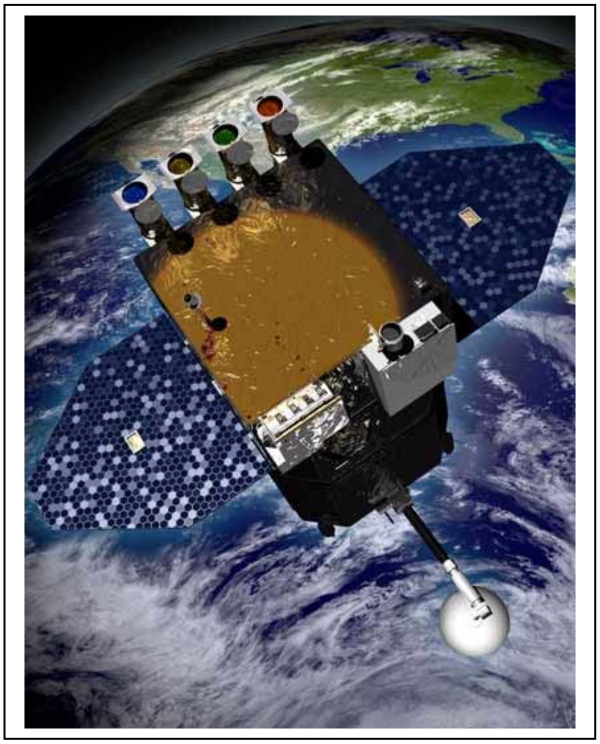
Problem 2 - What is the largest feature you can see on any of the three images, in kilometers, and how large is this compared to a familiar earth feature or landmark such as a state in the United States?
Answer; Student's selection will vary, but on the first image to the lower right a feature measures about 8 mm in diameter which is 8 mm x (53 km/1mm) = **424 kilometers wide**. This is about the same size as the **state of Utah**!



Problem 3 - The satellite of Pluto, called Charon, has been used to determine the total mass of Pluto. The mass determined was about 1.3 x 10^{22} kilograms. From clues in the image, calculate the volume of Pluto and determine the average density of Pluto. How does it compare to solid-rock (3000 kg/m^3), water-ice (917 kg/m^3)? Answer: From the image, Pluto is a sphere with a diameter of 2,300 km, so its volume will be V = 4/3 π $(1,250,000)^3$ = 8.2 x 10^{18} meters^3. Then its density is just D = M/V = (1.3 x 10^{22} kilograms)/ (8.2 x 10^{18} meters^3) so **D = 1,600 kg/m^3**. This would be about the density of **a mixture of rock and water-ice**.

Inquiry: Can you create a model of Pluto that matches its average density and predicts what percentage of rock and ice may be present?

Answer: We want to match the density of Pluto (1,600 km/m^3) by using ice (917 kg/m^3) and rock (2300 kg/m^3). Suppose we made Pluto out of half-rock and half-ice **by mass**. The volume this would occupy would be V = (0.5* 1.3 x 10^{22} kilograms/917 kg/m^3) = 7.1x10^{18} meters^3 for the ice, and V = (0.5* 1.3 x 10^{22} kilograms/3000 kg/m^3) = 2.2 x10^{18} meters^3 for the rock, for a total volume of 9.3 x 10^{18} meters^3 for both. This is a bit larger then the actual volume of Pluto (8.2 x 10^{18} meters^3) so we have to increase the mass occupied by ice, and lower the 50% by mass occupied by the rock component. The result, from student trials and errors should yield after a few iterations **about 40% ice and 60% rock.** This can be done very quickly using an Excel spreadsheet. For advanced students, it can also be solved exactly using a bit of algebra.



The 15 instruments on NASA's latest solar observatory will usher in a new era of solar observation by providing scientists with 'High Definition'-quality viewing of the solar surface in nearly a dozen different wavelength bands.

One of the biggest challenges is how to handle all the data that the satellite will return to Earth, every hour of the day, for years at a time! It is no wonder that the design and construction of this data handling network has taken nearly 10 years to put together! To make sense of the rest of this story, here are some units and prefixes you need to recall (1 byte = 8 bits):

Kilo = 1 thousand
Mega = 1 million
Giga = 1 billion
Tera = 1 trillion
Peta = 1,000 trillion
Exa = 1 million trillion

Problem 1 - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models?

Problem 2 - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute, uncompressed, MPEG-4 song occupies 8 megbytes, about A) How many uncompressed songs can be stored on the HD? B) How many hours of music can be stored on the HD? (Note: music is actually stored in a compressed format so typically several thousand songs can be stored on a large HD)

Problem 3 - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?

Problem 4 - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 5 gigabytes of information, how many DVDs-worth of data will be generated by the SDO: A) Each day? B) Each year?

Problem 5 - How many petabytes of data will SDO generate during its planned 5-year mission?

Problem 6 - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal?

Problem 1 - In 1982 an IBM PC desktop computer came equipped with a 25 megabyte hard drive (HD) and cost $6,000. In 2010, a $500 desktop comes equipped with a 2.5 gigabyte hard drive. By what factor do current hard drives have more storage space than older models? Answer: Take the ratio of the modern HD to the one in 1982 to get 2.5 billion / 25 million = 2,500,000,000/25,000,000 = 2,500/25 = **100 times.**

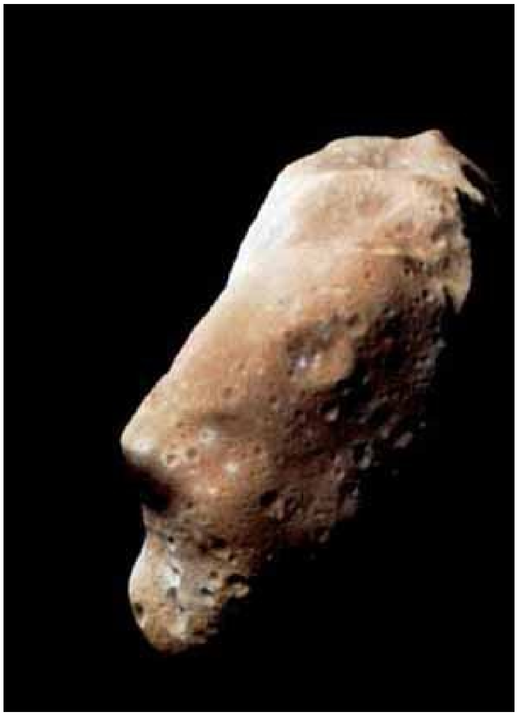
Problem 2 - A 2.5 gigabyte hard drive is used to store music from iTunes. If one typical 4-minute MPEG-4 song occupies 8 megbytes, about A) how many songs can be stored on the HD? B) How many hours of music can be stored on the HD?
Answer: A) Number of songs = 2.5 gigabytes/8 megabytes = 2,500 megabytes/8 megabytes = 312 B) Time = 312 songs x 4 minutes/song = 1,248 minutes = **20.8 hours.**

Problem 3 - How long will it take to download 2 gigabytes of music from the iTunes store A) Using an old-style 1980's telephone modem with a bit rate of 56,000 bits/sec? B)With a modern fiber-optic cable with a bit rate of 16 megabits/sec?
Answer; A) 2 gigabytes x 8 bits/1 byte = 16 gibabits. Then 16,000,000,000 bits x (1 second/56,000 bits) = 285,714 seconds or **79.4 hours**. B) 16,000,000,000 bits x (1 second/16,000,000 bits) = 1,000 seconds or about **17 minutes.**

Problem 4 - The SDO satellite's AIA cameras will generate 67 megabits/sec of data as they take 4096x4096-pixel images every 3/4 of a second. The other two instruments, the HMI and the EVE, will generate 62 megabits/sec of data. The satellite itself will also generate 20 megabits/sec of 'housekeeping' information to report on the health of the satellite. If a single DVD can store 5 gigabytes of information, how many DVDs-worth of data will be generated by the SDO each A) Day? B) Year? Answer: Adding up the data rates for the three instruments plus the satellite housekeeping we get 67+63+20 = 150 megabits/sec. A) In one day this is 150 megabits/sec x (86,400 sec/1day) = 12.96 terabites or since 1 byte = 8 bits we have 1.6 terabytes. This equals 1,600 gigabytes x (1 DVD/8 gigabytes) = **200 DVDs each day**. B) In one year this equals 365 days x 1.6 terabytes/day = 584 terabytes per year or 365 days/1 year x 200 DVDs/1 year = **73,000 DVDs/year.**

Problem 5 - How many petabytes of data will SDO generate during its planned 5-year mission? Answer: In 5 years it will generate 5 years x 584 terabytes/year = 2,920 terabytes. Since 1 petabyte = 1,000 terabytes, this becomes **2.9 petabytes**.

Problem 6 - It has been estimated that the total amount of audio, image and video information generated by all humans during the last million years through 2009 is about 50 exabytes including all spoken words (5 exabytes). How many DVDs does this equal? Answer: 1 exabyte = 1,000 petabytes = 1,000,000 terabytes = 1,000,000,000 gigabytes. So 50 exabytes equals 50 billion gigabytes. One DVD stores 5 gigabytes, so the total human information 'stream' would occupy 50 billion / 5 billon = **10 billion DVDs.**



Asteroid Gaspara

Astronomers studying the asteroid 24-Themis detected water-ice and carbon-based organic compounds on the surface of the asteroid.

NASA detects, tracks and characterizes asteroids and comets passing close to Earth using both ground and space-based telescopes.

NASA is particularly interested in asteroids with water ice because this resource could be used to create fuel for interplanetary spacecraft.

On October 7, 2009, the presence of water ice was confirmed on the surface of this asteroid using NASA's Infrared Telescope Facility. The surface of the asteroid appears completely covered in ice. As this ice layer is sublimated (goes directly from solid to gaseous state) it may be getting replenished by a reservoir of ice under the surface. The orbit of the asteroid varies from 2.7 AU to 3.5 AU (where 1 AU is the 150 million km distance from Earth to the sun) so it is located within the asteroid belt. The asteroid is 200 km in diameter, has a mass of 1.1 x 10^{19} kg, and a density of 2,800 kg/m^3 so it is mostly rocky material similar in density to Earth's.

By measuring the spectrum of infrared sunlight reflected by the object, the NASA researchers found the spectrum consistent with frozen water and determined that 24 Themis is coated with a thin film of ice. The asteroid is estimated to lose about 1 meter of ice each year, so there must be a sub-surface reservoir to constantly replace the evaporating ice.

Problem 1 – Assume that the asteroid has a diameter of 200 km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 km/$meter^3$? (Hint: Volume = Surface area x thickness)

Problem 2 – Suppose that only 1% by volume of the 1-meter-thick 'dirty' surface layer is actually water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

Problem 1 – Assume that the asteroid has a diameter of 200km. How many kilograms of water ice are present in a layer 1-meter thick covering the entire surface, if the density of ice is 1,000 km/meter$^{\mathbf{3}}$? (Hint: Volume = Surface area x thickness)

Answer: Volume = surface area x thickness.

$$SA = 4 \pi r^2$$
$$= 4 (3.14) (100{,}000 \text{ meters})^2$$
$$= 1.3 \times 10^{11} \text{ meters}^2$$
$$\text{Volume} = 1.3 \times 10^{11} \text{ meters}^2 \times 1 \text{ meter}$$
$$= 1.3 \times 10^{11} \text{ meters}^3$$

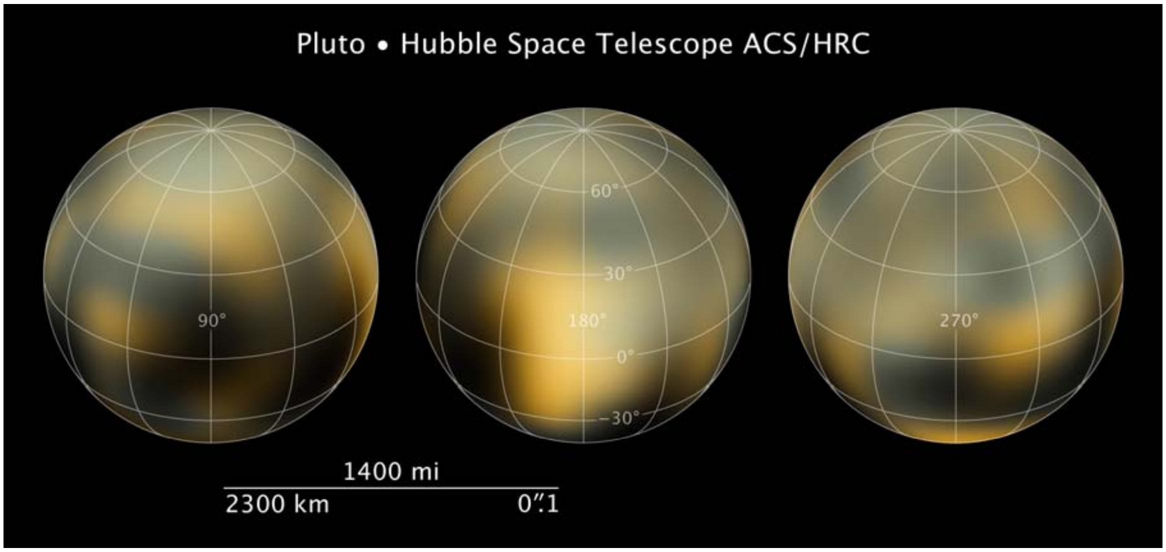
Mass of water = density x volume

$$= 1{,}000 \text{ kg/meter}^3 \times 1.3 \times 10^{11} \text{ meter}^3$$
$$= \mathbf{1.3 \times 10^{14}\ kg} \quad \text{(or 130 billion tons)}$$

Problem 2 – Suppose that only 1% by volume of the 'dirty' 1-meter-thick surface layer is water-ice and that it evaporates 1 meter per year, what is the rate of water loss in kg/sec?

Answer: The mass of water in the outer 1-meter layer is 1% of 1.3×10^{14} kg or 1.3×10^{12} kg. Since 1 year = 365 days x 24h/day x 60m/hr x 60 sec/min = 3.1×10^7 seconds, the mass loss is just 1.3×10^{12} kg / 3.1×10^7 sec = **42,000 kg/sec or 42 tons/sec**.

The Changing Atmosphere of Pluto



Recent Hubble Space Telescope studies of Pluto have confirmed that its atmosphere is undergoing considerable change, despite its frigid temperatures. Let's see how this is possible!

Problem 1 - The equation for the orbit of Pluto can be approximated by the formula $2433600=1521x^2+1600y^2$. Determine from this equation, expressed in Standard Form, A) the semi-major axis, a; B) the semi-minor axis, b; C) the ellipticity of the orbit, e; D) the longest distance from a focus called the aphelion; E) the shortest distance from a focus, called the perihelion. (Note: All units will be in terms of Astronomical Units. 1 AU = distance from the Earth to the Sun = 1.5×10^{11} meters).

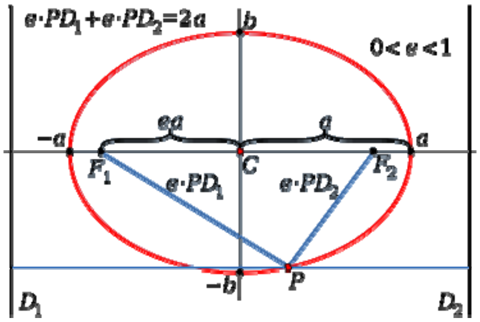
Problem 2 - The temperature of the methane atmosphere of Pluto is given by the formula

$$T(R) = \left(\frac{L(1-A)}{16\pi\sigma R^2}\right)^{\frac{1}{4}} \text{ degrees Kelvin (K)}$$

where L is the luminosity of the sun ($L=4 \times 10^{26}$ watts); σ is a constant with a value of 5.67×10^{-8}, R is the distance from the sun to Pluto in meters; and A is the albedo of Pluto. The albedo of Pluto, the ability of its surface to reflect light, is about A = 0.6. From this information, what is the predicted temperature of Pluto at A) perihelion? B) aphelion?

Problem 3 - If the thickness, H, of the atmosphere in kilometers is given by H(T) = 1.2 T with T being the average temperature in degrees K, can you describe what happens to the atmosphere of Pluto between aphelion and perihelion?

Problem 1 - Answer:



In Standard Form $2433600=1521x^2+1600y^2$ becomes $1=\frac{x^2}{1600}+\frac{y^2}{1521} = \frac{x^2}{a^2}+\frac{y^2}{b^2}$

Then A) **a = 40 AU** and B) **b=39 AU**. C) The ellipticity **$e = (a^2 - b^2)^{1/2}/a = 0.22$**. D) The longest distance from a focus is just a(1 + e) = 40(1+0.22) = **49 AU**. E) The shortest distance is just a(1-e) = (1-0.22)(40) = **31 AU**. Written out in meters we have $a= 6x10^{12}$ meters; $b= 5.8x10^{12}$ meters; aphelion = $7.35x10^{12}$ meters and perihelion = $4.6x10^{12}$ meters.

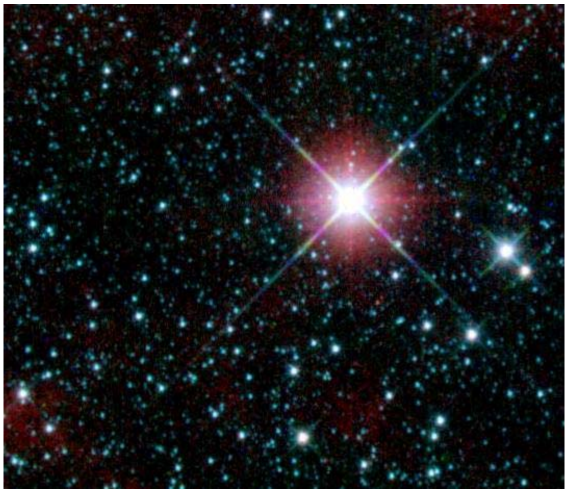
Problem 2 - Answer: For R in terms of AU, the formula simplifies to

$$T(R)=\left(\frac{4x10^{26}(1-0.6)}{16(3.14)(5.67x10^{-8})(1.5x10^{11})^2R^2}\right)^{\frac{1}{4}} \quad \text{so } T(R)=\frac{223}{\sqrt{R}} \text{ degrees K}$$

A) For a perihelion distance of 31 AU we have T = $223/(31)^{1/2}$ = 40 K; B) At an aphelion distance of 49 AU we have T = $223/(49)^{1/2}$ = 32 K. Note: The actual temperatures are about higher than this and average about 50K.

Problem 3 - Answer: At aphelion, the height of the atmosphere is about H=1.2x(32) = 38 kilometers, and at perihelion it is about H=1.2x(40) = 48 kilometers, so as Pluto orbits the sun its atmosphere increases and decreases in thickness.

Note: In fact, because the freezing point of methane is 91K, at aphelion most of the atmosphere freezes onto the surface of the dwarf planet, and at aphelion it returns to a mostly gaseous state. This indicates that the simple physical model used to derive H(T) was incomplete and did not account for the freezing-out of an atmospheric constituent.



There are many situations in astrophysics when two distinct functions are multiplied together to form a new function.

If there are N light bulbs, each with a brightness of W watts, then the total brightness, T of all these bulbs is just N x W. For N=3 bulbs and W = 100 watts we have T = 300 watts.

Suppose N(m) tells us the number of stars in an area of the sky with a brightness of m. Let a second function, S(m), represent the number of watts per square meter at the Earth that a star with a brightness of m produces. Then N(m)S(m) will be the total number of watts/meter^2 produced by the stars in the sample that have a brightness of m.

NASA's, Wide-Field Infrared Survey Explorer (WISE) satellite is surveying the sky to catalog stars visible at a wavelength of 3.5 microns in the infrared spectrum. If the differential star count function $N(m)=0.000005\ m^{+7.0}$ stars, and the star brightness function is defined by $S(m)= 350\ 10^{-0.4m}$ Janskys. Use this information to answer the following problems:

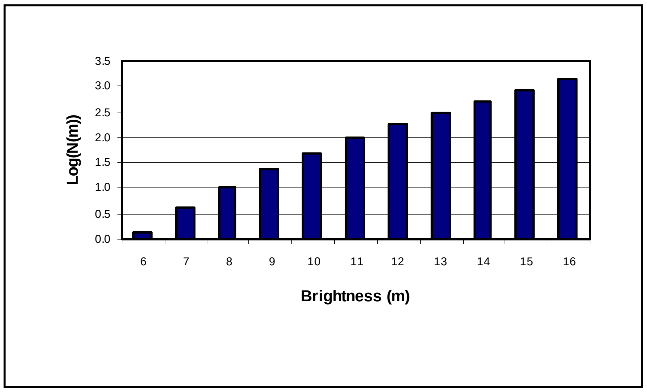
Problem 1 - Graph the functions Log(N(m)) and Log(S(m)) as individual histograms over the domain m:[+6, +16] for integer values of m.

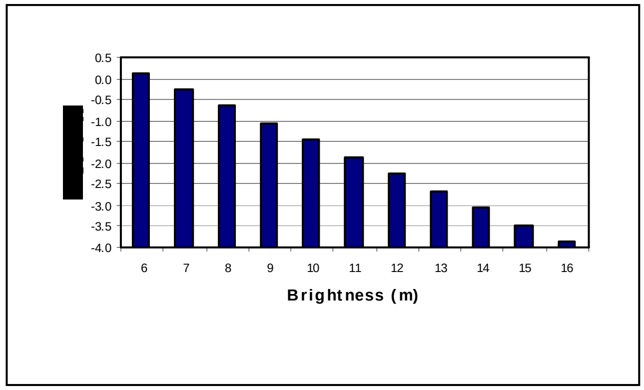
Problem 2 - Graph the product of these functions N(m)S(m) over the domain m:[+6, +16] for integer values of m.

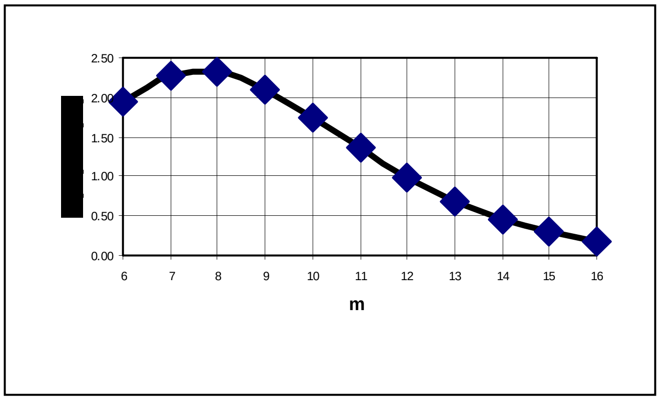
Problem 3 - What is the sum, T, of N(m)S(m) for each integer value of m in the domain m:[+6, +16], and how does this sum relate to the area under the curve for N(m)S(m)?

Problem 4 - What is the integral of N(m)S(m) from m=+6 to m= +16? You do not need to evaluate it!

Problem 1 and 2 - Answer: **See below.**







Problem 3 - Answer: The sum is T = N(6)S(6) + N(7)S(7) + … N(16)S(16)
T = 1.95 + 2.28 + 2.32 + 2.10 + 1.75 + 1.36 + 0.99 + 0.69 + 0.46 + 0.30 + 0.19
T = 14.39 Janskys. **This sum represents the approximate area under the curve N(m)S(m) vs m using vertical rectangles with a width of m= +1.0 and a height of N(m)S(m).**

Problem 4 - Answer:

$$T = \int_{6}^{16} (350)10^{-0.4m}(5x10^{-6})m^{+7}dm$$ becomes

$$T = 0.00175\int_{6}^{16} m^{7}e^{-(2.3)0.4m}dm$$

Changing variables to y=0.92x so dy = 0.92 dx we have

$$T = 0.0034\int_{5.5}^{14.7} y^{7}e^{-y}dy$$

This integral can be evaluated by approximation, as we did in Problem 3 using large rectangles with a base size of 1.0. Improved approximations can be created with base sizes of 1/2m, 1/4m, 1/8m…etc until a limit is reached for a desired degree of accuracy. Note: This integral can actually 'looked up' by advanced students, and its solution will be found to involve a recursive integral of $x^m e^{ax}$ where m=7 and a = -1. With computers, it is actually <u>faster</u> to evaluate it by successive approximation!

The Wide-field Infrared Survey Experiment (WISE) recently took its first photo of a test field in the constellation Carina to check out its instruments.

WISE is based on a 40-cm (16-inch) telescope designed to detect radiation at four wavelengths: 3.4, 4.6, 12 and 22 microns. The telescope is kept cold using solid hydrogen at a temperature of -438 F (12 Kelvin), and will be able to function for about 10 months in space until the hydrogen evaporates. In this time, WISE will take over a million pictures of the whole sky, revealing hundreds of millions of stars, galaxies, asteroids and other objects that shine brightly in infrared light.

The image to the left, measures 47arcminutes x 23 arcminutes (1 arcminute is 1/60 of one angular degree). It is a portion of the WISE 'First Light' image near the bright star V482 Carinae seen to the right.

Astronomers not only study individual stars in a field like this, but also count the stars in each brightness interval in order to mathematically model how stars are distributed in the Milky Way. Such models help us understand the shape and history of our galaxy.

Problem 1 - The bright star seen in this field is V482 Carinae. It has a stellar brightness of +6.0. If the area of this field has dimensions of 0.8 degrees x 0.4 degrees, how many stars as bright as V482 are present, on average, A) per square degree of the sky? B) Across the entire sky if its area is 41,253 deg^2?

Problem 2 - The function A(m) represents the total number of stars per deg^2 counted in the stellar brightness (called 'magnitude') bin from m-1/2 to m+1/2 centered on m. The function has the units of 'stars/deg^2/magnitude'. If $A(m)= 3m^{+3.5}$, how many stars would be counted, on average, in: A) a field that is the size of the full moon, (area = 0.19 deg^2) at a magnitude of m=+12.0? B) the same field and with a magnitude bin +2.3 in size?

Problem 3 - Near the wavelength of 3.4 microns being explored by WISE, an astronomer estimated from the Spitzer Infrared Observatory 'First Light' survey that at this wavelength, $A(m) = 2.4 \times 10^{-6}\ m^{+7.4}$ stars/magnitude/deg^2. Suppose that the faintest star detectable by WISE has a magnitude of m = +15. Using the method of integration, how many total number of stars would WISE be able to detect in a field equal to the WISE survey area of 0.64 deg^2, and that are fainter than V482 Carina?

Problem 1 - The bright star seen in this field is V482 Carinae. It has a stellar brightness of +6.0. If the area of this field has dimensions of 0.8 degrees x 0.4 degrees, how many stars as bright as V482 are present, on average, A) per square degree of the sky? B) Across the entire sky if its area is 41,253 deg^2?

Answer: A) The area of the field is 0.8 x 0.4 = 0.32 deg^2 and since there is only one star with a brightness of +6 in this area, A(+6.0) = 1 star / 0.32 deg^2 = **3.1 stars/deg^2**. B) Across the entire sky, N = 3.1 stars/deg^2 x 41,253 deg^2 gives **N = 128,000 stars**.

Problem 2 - The function A(m) represents the total number of stars per deg^2 counted in the stellar brightness (called 'magnitude') bin from m-1/2 to m+1/2 centered on m. The function has the units of 'stars/deg^2/magnitude'. If A(m)= $3m^{+3.5}$ stars/deg^2/magnitude how many stars would be counted, on average, in A) a field that is the size of the full moon, (area = 0.19 deg^2) at a magnitude of m=+12.0? B) the same field and with a magnitude bin that is +2.3 in size?

Answer: A) First A(12.0) = 3 $(12)^{+3.5}$ = 18,000 stars/deg^2/magnitude. Then for an area of 0.19 deg^2 ,we have N(m) = A(m) x (0.19 deg^2) = 18,000 x 0.19 = **3,400 stars/magnitude**.
B) The bin width in the histogram is now 2.0, so N = 3,400 stars/magnitude x (2.3 magnitudes) = **7820 stars**.

Problem 3 - Near the wavelength of 3.4 microns being explored by WISE, an astronomer estimated from the Spitzer Infrared Observatory 'First Light' survey that at this wavelength, A(m) = 2.4 $x10^{-6}$ $m^{+7.4}$ stars/magnitude/deg^2. Suppose that the faintest star detectable by WISE has a magnitude of m = +15. Using the method of integration, how many total number of stars would WISE be able to detect in a field equal to the WISE survey area of 0.64 deg^2, and that are fainter than V482 Carina?

Answer: The integral to perform is:

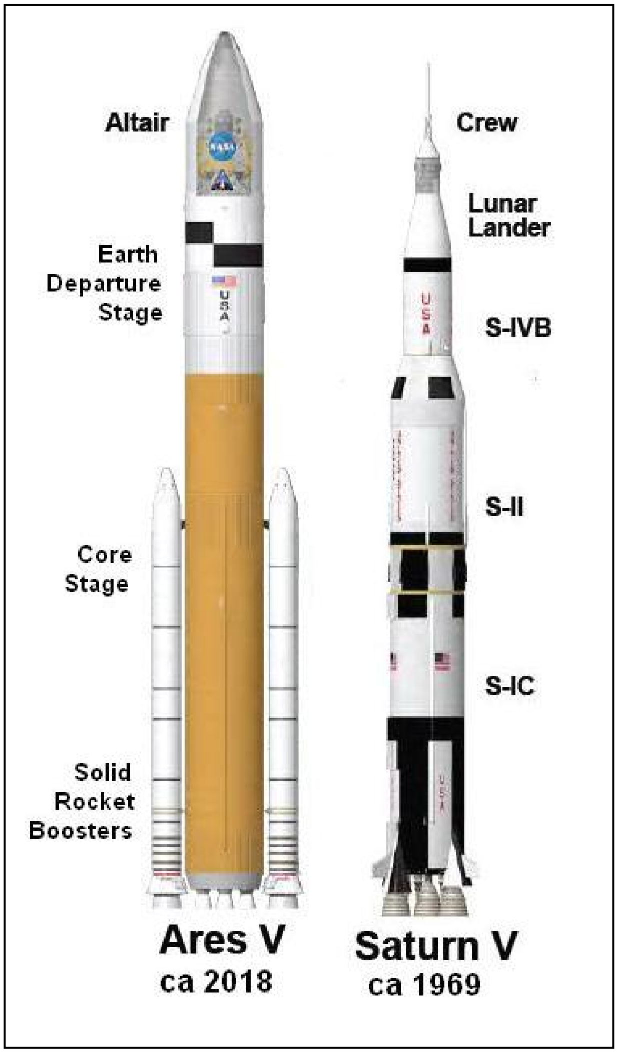
$$N = \int_{+6}^{+15} 2.4x10^{-6} m^{+7.4} dm$$ which yields: $$N = \frac{2.4x10^{-6}}{8.4}\left[(15)^{8.4} - (6)^{8.4}\right]$$

so N = 2,162 stars/deg^2.

Since the field in question has an area of 0.64 deg^2, the prediction is that there would be N = 2,164 stars/deg^2 x (0.64 deg^2) = 1,385 stars in an average star field of this size.

Note: The WISE satellite's First Light star field near V482 Carinae was close to the plane of the Milky Way, so the estimated 3,000 stars reportedly seen in its field is higher than what one may estimate from functions that model average numbers. Students may experiment with other choices for the power-law exponent, n, to see how sensitive the predictions are to the choice made, and the number of stars seen in the WISE image. Challenge them to find a means for excluding 0 < n < 3, or N > 8 as possibilities.

The Ares-V Cargo Rocket



The Ares-V rocket, now being developed by NASA, will weigh 3,700 tons at lift-off, and be able to ferry 75 tons of supplies, equipment and up to 4 astronauts to the moon. As a multi-purpose launch vehicle, it will also be able to launch complex, and very heavy, scientific payloads to Mars and beyond. To do this, the rockets on the Core Stage and Solid Rocket Boosters (SRBs) deliver a combined thrust of 47 million Newtons (11 million pounds). For the rocket, let's define:

T(t) = thrust at time-t
m(t) = mass at time-t
a(t) = acceleration at time-t

so that:

$$a(t) = \frac{T(t)}{m(t)}$$

The launch takes 200 seconds. Suppose that over the time interval [0,200], T(t) and m(t) are approximately given as follows:

$$T(x) = 8x^3 - 16x^2 - x^4 + 47$$

$$m(x) = 35 - x^2$$

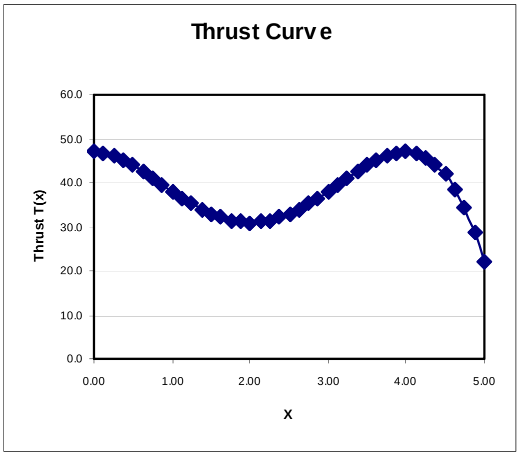
where t = 40 x

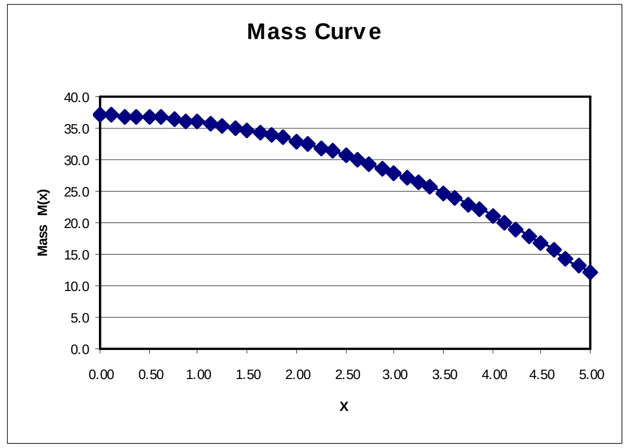
Problem 1 - Graph the thrust curve T(x), and the mass curve m(x) and find all minima, maxima inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 2 - Graph the acceleration curve a(x) and find all maxima, minima, inflection points in the interval [0,5]. You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? How many seconds will this be after launch? You may use a graphing calculator, or Excel spreadsheet, or differential calculus.

Answer Key

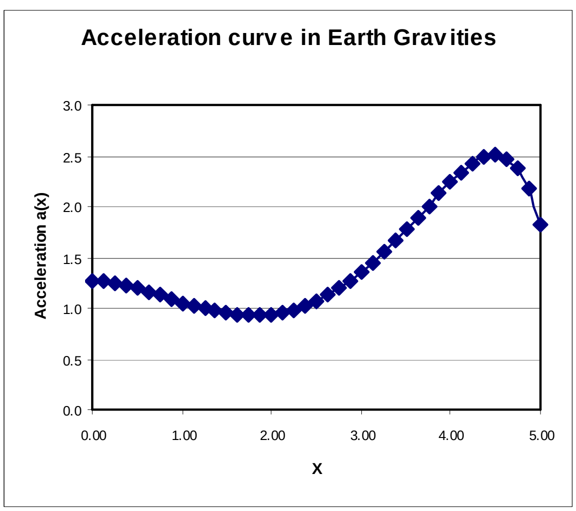




Problem 1 - The above graphs show T(x) and m(x) graphed with Excel. Similar graphs will be rendered using a graphing calculator. For the thrust curve, T(x), the relative maxima are at (0, 47) and (4,47). The relative minimum is at (2,31).

Problem 2 - For the mass curve, M(x), the absolute maximum is at (0,37).

Problem 3 - For what value of x will the acceleration of the rocket be at its absolute maximum in the interval [0,5]? You may use a graphing calculator, or Excel spreadsheet, or differential calculus. How many seconds will this be after launch?

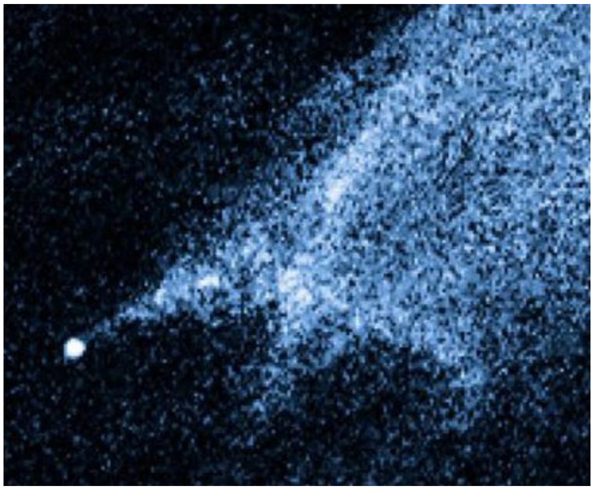


Answer: The curve reaches its maximum acceleration near (4.5,2.5). Because t = 40 X, this occurs about 40 x 4.5 = 180 seconds after launch. ***Note to teacher:*** *The units for acceleration are in Earth Gravities (1 G = 9.8 meters/sec^2) so astronauts will feel approximately 2.5 times their normal weight at this point in the curve.* U sing the Quotient Rule in differential calculus, and setting da(x)/dx = 0 we get:

$$0 = \frac{2x^5 - 8x^4 - 140x^3 + 840x^2 - 1026x}{(35 - x^2)^2}$$

Although after factoring out 'x' from the numerator we see that X=0 is a trivial solution, the locations of the remaining two extrema near x=1.5-2.0 and x=4.0-5.0 have to be found by solving a 4th-order equation. A graphing calculator can be used to find these two points at x ~ 1.8 and x ~ 4.56 where da/dt ~ 0. Since t = 40x, we see that the absolute maximum acceleration occurs near t = 4.56 x 40 = 182 seconds after launch.

Hubble Spies Colliding Asteroids!



It doesn't look like much, but this picture taken by the Hubble Space Telescope on January 25, 2010 shows all that remains of two asteroids that collided! The object, called P/2010 A2, was discovered in the asteroid belt 290 million kilometers from the sun, by the Lincoln Near-Earth Asteroid Research sky survey on January 6, 2010.

Hubble shows the main nucleus of P/2010 A2, about 150 meters in diameter, lies outside its own halo of dust. This led scientists to the interpretation that it is the result of a collision.

How often do asteroids collide in the asteroid belt? The collision time can be estimated by using the formula:

$$T = \frac{1}{NAV}$$

Where N is the number of bodies per cubic kilometer, v is the speed of the bodies relative to each other in kilometers/sec and A is the cross-sectional area of the body in square-kilometers. The answer, T, will be in the average number of seconds between collisions.

Estimating A: Assume that the bodies are spherical and 100 meters in diameter .What will be A, the area of a cross-section through the body?

Estimating the asteroid speed V: At the orbit of the asteroids, they travel once around the sun in about 3 years. What is the average speed of the asteroid in kilometers/sec at a distance of 290 million kilometers?

Estimating the density of asteroids N: - This quantity is the number of asteroids in the asteroid belt, divided by the volume of the belt in cubic kilometers. A) Assume that the asteroid belt is a thin disk 1 million kilometers thick, with an inner radius of 1.6 AU and an outer radius of 2.5 AU. If 1 AU = 150 million kilometers, want is the volume? B) Based on telescopic observations, an estimate for the number of asteroids in the belt that are larger than 100 meters across is about 30 billion. From this information, and your volume estimate, what is the average density of asteroids in the asteroid belt?

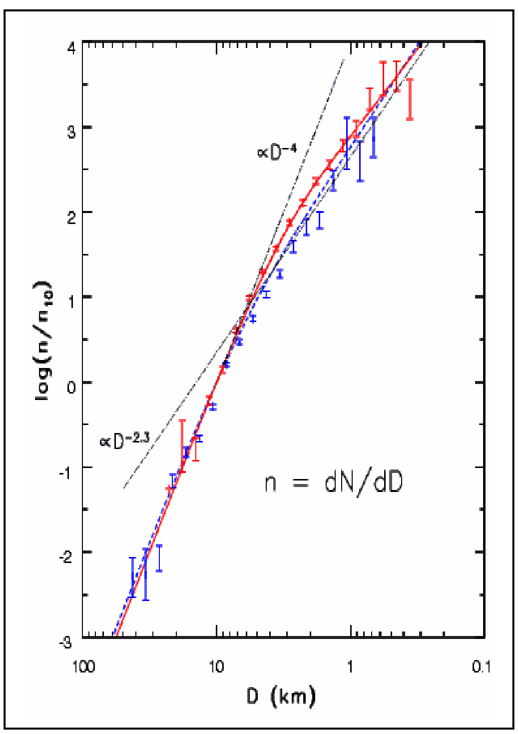
Estimating the collision time T: From the formula, A) what do your estimates for N, V and A imply for the average time between collisions in years? B) What are the uncertainties in your estimate?

Estimating A: Answer: The cross-section of a sphere is a circle, so $A = \pi R^2$ or for R = 100 meters, we have $A = 3.14 (0.05)^2 = 8\times10^{-3}$ km^2.

Estimating the asteroid speed V: Answer: The circumference of the orbit is C = 2 π (290 million km) = 1.8 x 10^9 kilometers, and the time in seconds is 3.0 years x (3.1 x 10^7 seconds/1 year) = 9.3 x 10^7 seconds, so the average speed V= C/T = 19 kilometers/sec. But what we want is the relative speed. If all the asteroids were going around in their orbits at a speed of 19 km/sec, their relative speeds would be zero. Example, although two cars are on the freeway traveling at 65 mph, their relative speeds are zero since they are not passing or falling behind each other.

Estimating the density of asteroids N: - Answer: R(inner) = 2.4 x 10^8 km .R(outer)= 3.4 x 10^8 km, so the volume of the disk is V = disk area x thickness = [$\pi(3.4\times10^8)^2$ - $\pi(2.4 \times 10^8)^2$] x 10^6 so the volume is **1.8 x 10^{23} km^3**. Then the average asteroid density is N= 3x10^{10} asteroids/1.8 x 10^{23} km^3 = **1.7 x 10^{-13} asteroids/km^3.**

Estimating the collision time T: Answer: A) T = 1/NAV so T = 1/(1.7 x 10^{-13} x 8x10^{-3} x 19) = 3.8 x 10^{13} seconds. Since 1 year = 3.1 x 10^7 seconds, we have about **1,200,000 years between collisions**. B) Students can explore a number of places where large uncertainties might occur such as: 1) the thickness of the asteroid belt; 2) the number of asteroids; 3) their actual relative speeds. 4) The non-uniform distribution of the asteroids…not smoothly distributed all over the volume of the asteroid belt, but may be in clumps or rings that occupy a smaller actual volume. Encourage them to come up with their own estimates and see how that affects the average time between collisions. For advanced students see the problem below.



Extra Credit with Calculus: Data from the Sloan Digital Sky Survey suggests that the number of asteroids in specific size ranges follow a piecewise power-law distribution:

$$N(D) = \begin{cases} 1.8x10^9 D^{-2.3} & \text{for D <70 km} \\ 2.4x10^{12} D^{-4.0} & \text{for D>70 km} \end{cases}$$

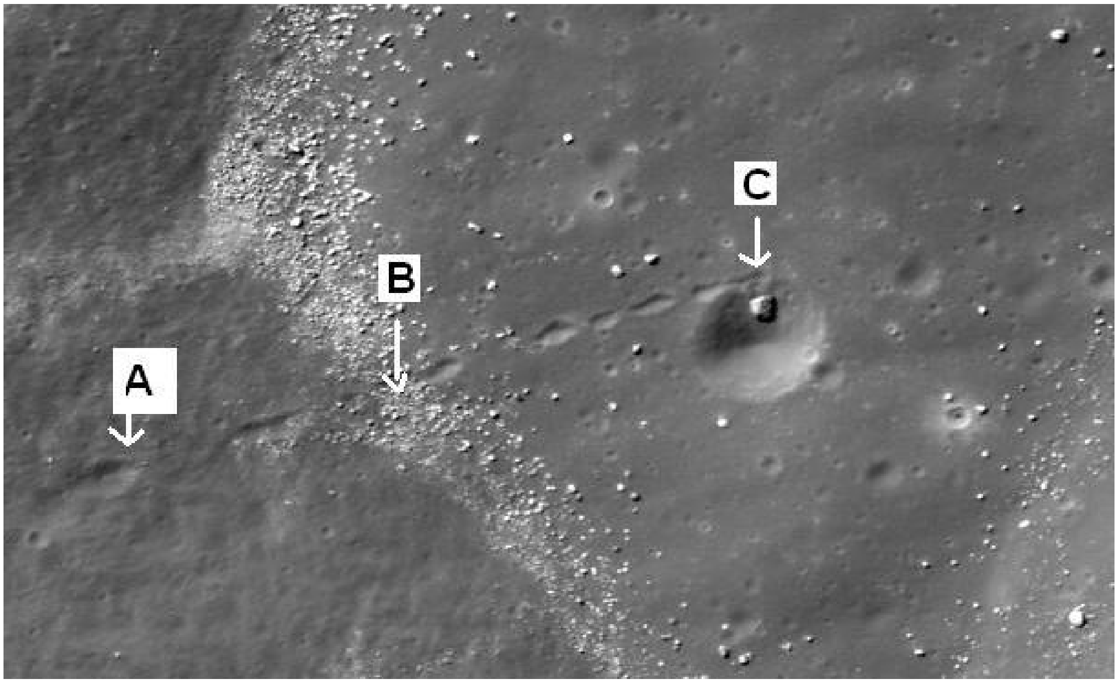
Integrate N(D) from 100 meters to infinity to determine the number of asteroids larger than 100 meters.

Answer: $n = \int_{0.1}^{70} N(D)dD + \int_{70}^{\infty} N(D)dD$

= 2.7x10^{10} + 2.32x10^6 = 2.7x10^{10} asteroids.

Note: the figure is scaled to the number of 10-km asteroids (n_{10}) which we take to be 10,000.

LRO Spots a Rolling Lunar Boulder with No Moss!



This image (LROC MAC M122597190L), taken by the Lunar Reconnaissance Orbiter shows a boulder that has rolled and skipped down hill from the left-hand edge of the image to a 'hole-in-one' location in a small crater. The width of the image is 510 meters. To two significant figure accuracy in your answers:

Problem 1 – Mark those portions of the path where the boulder must have A) rolled and B) skipped, in order to cover the distance.

Problem 2 – What is the scale of this image in meters/km?

Problem 3 – Assuming that the boulder is roughly a sphere in shape with a density of D=3000 kg/m^3, what is A) The diameter of the boulder? B) The mass of the boulder in tons?

Problem 4 – How far did the boulder skip and roll from A) Point A to B? B) From Point B to C?

Problem 5 – Suppose that the crater wall slope from Point A to B is 60 degrees to the horizontal, and the settled regolith from B to C has a 'repose' slope of 30 degrees to the horizontal. What is the total distance traveled A) Horizontally? B) Vertically?

Problem 6 – If the average vertical speed from Point A to B is 7 m/s and from Point B to C is 13 m/s, how long did the boulder take to travel from Point A to Point C?

Problem 1 – Mark those portions of the path where the boulder must have A) rolled and B) skipped, in order to cover the distance. Answer: Wherever you see a track is where the boulder was rolling in contact with the lunar regolith. Wherever you see gaps in the track is where the boulder was skipping through the air.

Problem 2 – What is the scale of this image in meters/km? Answer: The width of the image is about 142 mm, so the scale is 510 meters/142mm = **3.6 meters/mm**.

Problem 3 – Assuming that the boulder is roughly a sphere in shape, with a density of D=3000 kg/m^3, what is A) The diameter of the boulder? B) The mass of the boulder in tons? Answer; A) The measured diameter is 3mm so the physical size is 3 mm x 3.6 meters/mm = **11 meters**. B) The volume is $V= 4/3 (3.14)(5.5)^3$ = 700 $meter^3$. The mass is then M = V x D so M = 700 x 3000 ; M = 2,100,000 kg ; M = **2,100 tons**.

Problem 4 – How far did the boulder skip and roll from A) Point A to B? B) From Point B to C? Answer: A) L(AB) = 35 mm x (3.6m/mm) = **130 meters**. B) L(BC) = 50 mm x (3.6 m/mm) = **180 meters**.

Problem 5 – Suppose that the crater wall slope from Point A to B is 60 degrees to the horizontal, and the settled regolith from B to C has a 'repose' slope of 30 degrees to the horizontal. What is the total distance traveled A) Horizontally? B) Vertically? Answer: A) D1 = 130cos(60) = 65 meters. D2 = 180cos(30) = 160 meters so D=d1+d2 and **D=220 meters**. B) h1 = 130 sin(60) = 65 meters; h2 = 180sin(30)=90 meters so H=h1+h2 and so **H= 160 meters.**

Problem 6 – If the average vertical speed from Point A to B is 7 m/s and from Point B to C is 13 m/s, how long did the boulder take to travel from Point A to Point C? Answer: T = 65m/7 m/s + 90m/13m/s so T = 9.3s + 6.9s so **T = 16 seconds.**

Note to Teacher: Assuming that our estimates for the slopes are correct, the terminal velocity of a falling body in vacuum is given by equating the boulder's potential energy to the kinetic energy; $mhg = 1/2mV^2$ where g = 1.6 $meters/sec^2$ for the moon. So the total vertical speed gain during 65-meter free fall from Point A to B would be V = 14 m/s for an average speed of (0+14)/2 = 7 m/sec. This assumes minimal energy loss from friction against the wall of the crater during its skidding contacts. This is probably an OK assumption because the amount of regolith moved during these contacts is a small part of the mass of the boulder. The total drop on the 30-degree slope is 90 meters, so the total 'free fall' vertical speed change on this segment is V = 17 m/s. There was far more regolith excavation on this segment so if we assume ½ of the kinetic energy is lost in this way, the actual speed change (Vfinal-Vinitial)= $17/(2)^{1/2}$ = 12 m/sec. The average vertical speed between Point B to C is then (7 m/sec + 19 m/sec)/2 = 13 m/sec.

Useful Internet Resources

Space Math @ NASA

http://spacemath.gsfc.nasa.gov

Practical Uses of Math and Science (PUMAS)

http://pumas.gsfc.nasa.gov

Teach Space Science

http://www.teachspacescience.org

Space Weather Action Center

http://sunearthday.nasa.gov/swac

THEMIS Classroom guides on Magnetism

http://ds9.ssl.berkeley.edu/themis/classroom.html

The Stanford Solar Center

http://solar-center.stanford.edu/solar-math/

A Math Refresher

http://istp.gsfc.nasa.gov/stargaze/Smath.htm

A note from the Author:

June, 2010

Hi again!

Here is another collection of 'fun' problems based on NASA space missions across the solar system and the universe. Thanks to the relentless march of current events, the problems range from the spectacular launch of the Ares 1X in the Fall of 2009, to the devastating crisis of the BP oil leak in the Spring of 2010.

Mathematics underlies many of the dramatic stories from the Space Age, but unless you know how to look beyond the verbiage of press releases, it is hard to see the connections between scientific ideas and discoveries, and the mathematics that give them life and certainty. My goal, with Space Math @ NASA is to try to show how exciting ideas in science connect with mathematics. As an astronomer, it is second-nature to think of celestial events and objects in mathematical terms, revealing their inner-beauty. I hope that by working through many of the problems in this book you and your students will better appreciate the many subtle interconnections that underlie the physical world.

This has been a very complex year for NASA as it launches its last Shuttle missions in 2010. With only one more launch to go in November, I am also a bit worried about what lies in store for us in the future. Our astronauts will have to travel to launch facilities in Russia to be ferried up to the International Space Station for the next few years. It may seem like an international embarrassment to us, but in fact it demonstrates that we are now willing to join the international community in a shared vision for space exploration among manned missions. The successful launch of the SpaceX Falcon-9 rocket in June also underscores the fact that our commercial space systems are now up to speed in taking the next step into the manned space arena.

President Obama made an historic decision in his vision for NASA. The new Vision that was offered will open up NASA to using domestic, commercial launch systems to ferry astronauts to the ISS, and prolong the ISS operation through 2020. In the vision offered by the previous administration, which was never funded to succeed, ISS was scheduled for de-orbit in ca 2016 only a few years after achieving full operation as a science research facility following 1 years of construction.

The historic launch of Falcon-9 will be followed in the next few years by the return of our astronauts to the ISS using our own launch vehicles again, but this time at nearly 10-times lower cost that the Space Shuttle. The commercialization of launch systems in the USA will also mean that many non-astronaut sightseers will now have access to space, with thousands of people already having paid for the first tickets!

The next 10 years will be a very exciting period for space travel!

Sincerely,
Dr. Sten Odenwald
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