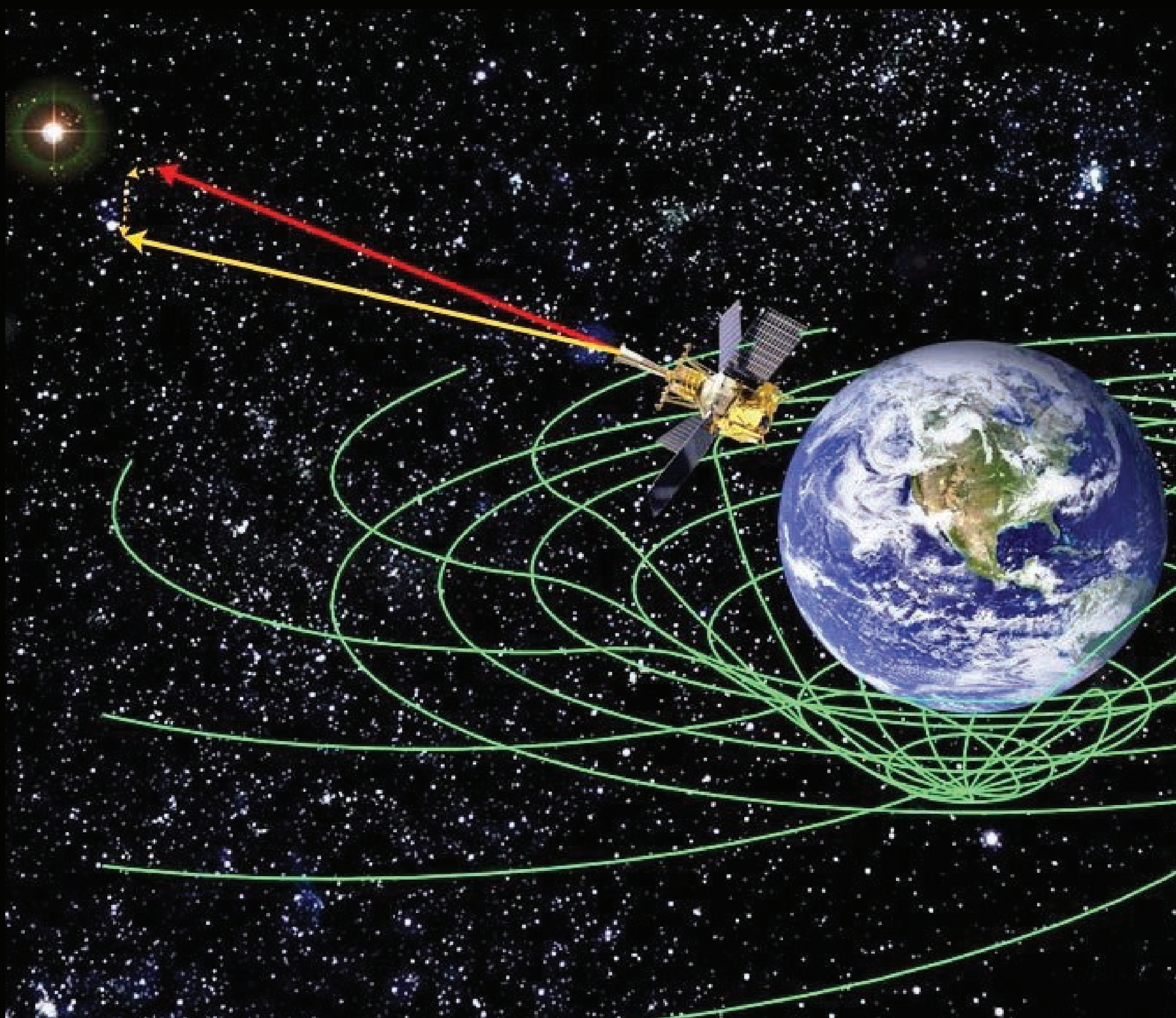
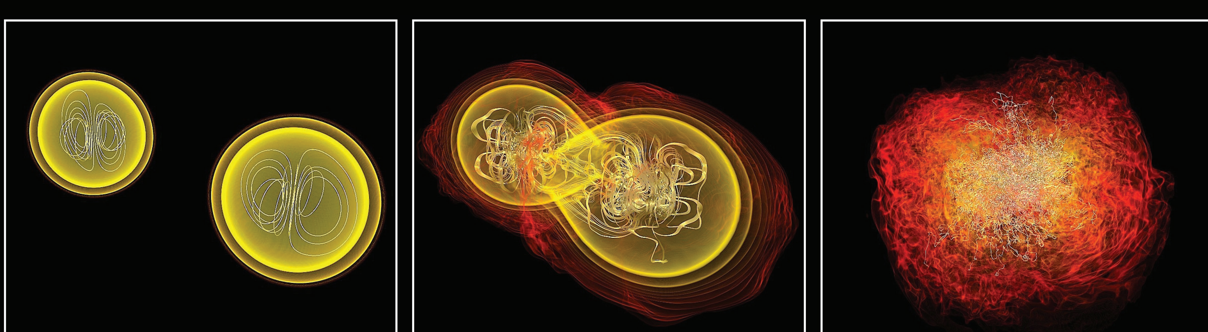
National Aeronautics and Space Administration

Space Math VII



$$\boldsymbol{\Omega} = \frac{3GM}{2c^2R^3}(\boldsymbol{R}\times\boldsymbol{v}) + \frac{GI}{c^2R^3}\left[\frac{3\boldsymbol{R}}{R^2}(\boldsymbol{\omega}\cdot\boldsymbol{R}) - \boldsymbol{\omega}\right]$$



This collection of activities is based on a weekly series of space science problems distributed to thousands of teachers during the 2010-2011 school year. They were intended for students looking for additional challenges in the math and physical science curriculum in grades 5 through 12. The problems were created to be authentic glimpses of modern science and engineering issues, often involving actual research data.

The problems were designed to be 'one-pagers' with a Teacher's Guide and Answer Key as a second page. This compact form was deemed very popular by participating teachers.

For more weekly classroom activities about astronomy and space visit the NASA website,

http://spacemath.gsfc.nasa.gov

Add your email address to our mailing list by contacting Dr. Sten Odenwald at

Sten.F.Odenwald@nasa.gov

Front and back cover credits: Front) Top - Gravity Probe-B sketch of warped space near Earth, and the equation from General Relativity that predicts the precession and 'frame-dragging' effects detected by the satellite. Illustration courtesy NASA/GP-B/Stanford University. Bottom - Three moments in the supercomputer calculation of colliding neutron stars showing the rearrangements in the magnetic field 'ribbons' during the collision and merger. **Courtesy….** Back) A combined image of the galaxy NGC-5128 also called Centarus-A; A galaxy containing a supermaassive black hole at its core, and ejecting jets of matter. The image is a superposition of X-ray (blue) data from the NASA Chandra Observatory, radio (orange) and visible light data. Credit: ESO/WFI (visible) ,MPIfR/ESO/APEX/A. Weiss (mictrowave); and NASA/CXC/CfA/R. Kraft et al (x-ray).

Interior Illustrations: 1) Black hole art (Anon Internet); 2)Black hole art (Anon Internet); 4) ISS and Sun (John Stetson); 11) Black hole (David Aguilar, CfA); 12) NASA/Artemis 13) Bone loss (MenaQ7.com); 16) Exoplanet art (G. Bacon ,STScI/NASA); 17) Black hole (Dana Berry, NASA/SkyWorks Digital); 20) Esquel Pallasite (Anon Internet); 22) Line Spectrum (Anon Internet); 23) Supercomputer (Anon Internet); 32) Gliese 581g Artwork (Lynette Cook/NASA); 38) Dreath Star (NASA/G. Bacon); 43) Transiting planets (NASA/Tim Pyle); 45) Habitable Zones (NASA/Kepler); 61) Solar Probe (JHU/APL); 73) Bacterium (NASA/Jodi Blum); 81) Warped space (Pancho Eekers/GPB); 78) Dosimeter (AP/Kyodo News)

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Note to Teachers:

The problems in this math guide use metric units throughout, but occasional problems (15, 27, 47, 55) use English measurements such as acres, miles or pounds to be consistent with American engineering practice. Students will be responsible as adults working in the STEM environment of the aerospace industry for being intimately familiar with metric conversions to English units. Since this book is a collection of real-world math problems it is appropriate that a mixture of math problems in metric and English units appear.

Although we do use metric units in all scientific discussions, we do not use SI units (meters-kilograms-seconds) exclusively because this particular metric standard common in K12 instruction is not always convenient for scientific research. For example, we do not state the diameter of Earth in meters but use kilometers. We do not determine the scale of a photographic image by using meters, but use millimeters, etc. Students need to learn that in the real world of day-to-day science, a variety of metric units are used in each discipline depending on the particular system being studied. Astronomers for example do not state the masses of stars in kilograms, but use the 'solar mass' as 1-unit of stellar mass. This is clearly not an SI unit, but it greatly serves to improve comprehension, which is always the point of scientific research and communication. It is not clear to the Author how stating stellar masses in the SI units of kilograms rather than solar units would improve student comprehension of astronomical scales.

Mathematics Topic Matrix

Topic	Problem Numbers																													
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31
Inquiry											X																				
Technology, rulers	X	X							X	X		X			X									X			X			X	
Numbers, patterns, percentages	X	X	X			X	X	X	X	X	X	X		X																	
Averages			X																												
Time, distance, speed												X															X				
Mass, density, volume																X				X											
Areas and volumes															X										X		X	X	X		
Scale drawings									X	X					X				X					X			X	X	X	X	
Geometry				X					X															X				X	X		
Scientific Notation							X									X					X				X						
Unit Conversions																															
Significant Figures																					X										
Fractions																															
Graph or Table Analysis					X								X	X					X							X					X
Solving for X				X																		X									
Evaluating Fns																	X	X	X		X		X		X	X			X		
Modeling					X																										
Probability																										X				X	
Rates/Slopes													X			X															
Linear Equations					X																		X								
Logarithmic Fns																															
Polynomials																															
Power Fns																															
Exponential Fns																															
Conics																															
Piecewise Fns																															
Trigonometry																															
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																													
	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62
Inquiry																															
Technology, rulers	X		X	X	X			X		X										X											
Numbers, patterns, percentages											X				X	X								X							
Averages																	X					X	X								
Time, distance,speed				X	X			X									X				X	X	X								
Mass, density, volume																				X						X					
Areas and volumes																															
Scale drawings	X		X	X	X			X		X					X	X				X	X		X						X		
Geometry															X	X					X								X		X
Scientific Notation				X		X	X																			X	X	X		X	
Unit Conversions										X						X		X	X												
Significant Figures																															
Fractions																															
Graph or Table Analysis	X	X							X		X	X	X	X				X	X					X	X						
Solving for X																															
Evaluating Fns		X				X																				X	X	X		X	
Modeling																															
Probability							X					X	X	X																	
Rates/Slopes									X									X	X												
Linear Equations									X																						
Logarithmic Fns																												X			
Polynomials																									X						
Power Fns																									X						
Exponential Fns																											X				
Conics																															
Piecewise Fns																															
Trigonometry																															X
Integration																															
Differentiation																															
Limits																															

Mathematics Topic Matrix (cont'd)

Topic	Problem Numbers																							
	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86
Inquiry																								
Technology, rulers																								
Numbers, patterns, percentages											X		X											
Averages																								
Time, distance,speed														X	X									
Mass, density, volume		X		X		X			X			X												
Areas and volumes		X		X		X																		X
Scale drawings	X			X		X																		
Geometry	X		X	X						X								X	X		X	X		
Scientific Notation		X		X		X	X	X	X			X		X				X	X	X			X	X
Unit Conversions						X											X							
Significant Figures																		X	X					
Fractions																								
Graph or Table Analysis		X							X		X		X			X				X				
Solving for X								X																
Evaluating Fns			X				X	X	X			X		X	X			X	X			X	X	X
Modeling					X		X	X																
Probability											X		X											
Rates/Slopes					X											X	X							
Linear Equations																								
Logarithmic Fns																X				X				
Polynomials																								X
Power Fns																								
Exponential Fns															X								X	
Conics																					X	X		
Piecewise Fns																								
Trigonometry	X									X														
Integration																							X	X
Differentiation																								
Limits																								

How to use this book

Teachers continue to look for ways to make math meaningful by providing students with problems and examples demonstrating its applications in everyday life. Space Math offers math applications through one of the strongest motivators-Space. Technology makes it possible for students to *experience* the value of math, instead of just reading about it. Technology is essential to mathematics and science for such purposes as “access to outer space and other remote locations, sample collection and treatment, measurement, data collection and storage, computation, and communication of information.” 3A/M2 authentic assessment tools and examples. The NCTM standards include the statement that "Similarity also can be related to such real-world contexts as photographs, models, projections of pictures" which can be an excellent application for all of the Space Math applications.

This book is designed to be used as a supplement for teaching mathematical topics. The problems can be used to enhance understanding of the mathematical concept, or as a good assessment of student mastery.

An integrated classroom technique provides a challenge in math and science classrooms, through a more intricate method for using **Space Math VII**. Read the scenario that follows:

Ms. Green decided to pose a new activity using Space Math for her students. She challenged each student team with math problems from the **Space Math VI**I book. She copied each problem for student teams to work on. She decided to have the students develop a factious space craft. Each team was to develop a set of criteria that included reasons for the research, timeline and budget. The student teams had to present their findings and compete for the necessary funding for their space craft. The students were to use the facts available in the Space Math VII book and images taken from the Space Weather Media Viewer, http://sunearth.gsfc.nasa.gov/spaceweather/FlexApp/bin-debug/index.html#

Space Math VII can be used as a classroom challenge activity, assessment tool, enrichment activity or in a more dynamic method as is explained in the above scenario. It is completely up to the teacher, their preference and allotted time. What it does provide, regardless of how it is used in the classroom, is the need to be proficient in math. It is needed especially in our world of advancing technology and physical science.

Alignment with Standards

AAAS: Project:2061 Benchmarks

(3-5) - Quantities and shapes can be used to describe objects and events in the world around us. 2C/E1 --- Mathematics is the study of quantity and shape and is useful for describing events and solving practical problems. 2A/E1 **(6-8)** Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The "things" from which they abstract can be ideas themselves; for example, a proposition about "all equal-sided triangles" or "all odd numbers". 2C/M1 **(9-12)** - Mathematical modeling aids in technological design by simulating how a proposed system might behave. 2B/H1 ---- Mathematics provides a precise language to describe objects and events and the relationships among them. In addition, mathematics provides tools for solving problems, analyzing data, and making logical arguments. 2B/H3 ----- Much of the work of mathematicians involves a modeling cycle, consisting of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. The actual thinking need not follow this order. 2C/H2

NCTM: Principles and Standards for School Mathematics

Grades 6–8 :

- work flexibly with fractions, decimals, and percents to solve problems;
- understand and use ratios and proportions to represent quantitative relationships;
- develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation; .
- understand the meaning and effects of arithmetic operations with fractions, decimals, and integers;
- develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.
- represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules;
- model and solve contextualized problems using various representations, such as graphs, tables, and equations.
- use graphs to analyze the nature of changes in quantities in linear relationships.
- understand both metric and customary systems of measurement;
- understand relationships among units and convert from one unit to another within the same system.

Grades 9–12 :

- judge the reasonableness of numerical computations and their results.
- generalize patterns using explicitly defined and recursively defined functions;
- analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior;
- understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions;
- draw reasonable conclusions about a situation being modeled.

Teacher Comments

"Your problems are great fillers as well as sources of interesting questions. I have even given one or two of your problems on a test! You certainly have made the problems a valuable resource!" (Chugiak High School, Alaska)

"I love your problems, and thanks so much for offering them! I have used them for two years, and not only do I love the images, but the content and level of questioning is so appropriate for my high school students, they love it too. I have shared them with our math and science teachers, and they have told me that their students like how they apply what is being taught in their classes to real problems that professionals work on." (Wade Hampton High School ,SC)

"I recently found the Space Math problems website and I must tell you it is wonderful! I teach 8th grade science and this is a blessed resource for me. We do a lot of math and I love how you have taken real information and created reinforcing problems with them. I have shared the website with many of my middle and high school colleagues and we are all so excited. The skills summary allows any of us to skim the listing and know exactly what would work for our classes and what will not. I cannot thank you enough. I know that the science teachers I work with and I love the graphing and conversion questions. The "Are U Nuts" conversion worksheet was wonderful! One student told me that it took doing that activity (using the unusual units) for her to finally understand the conversion process completely. Thank you!" (Saint Mary's Hall MS, Texas)

"I know I’m not your usual clientele with the Space Math problems but I actually use them in a number of my physics classes. I get ideas for real-world problems from these in intro physics classes and in my astrophysics classes. I may take what you have and add calculus or whatever other complications happen, and then they see something other than “Consider a particle of mass ‘m’ and speed ‘v’ that…” (Associate Professor of Physics)

Exploring Black Holes

All bodies produce gravity. The more mass an object has, the more gravity it creates. It is also true that the smaller you make an object by compressing it, the more intense its gravity is at its surface.

Suppose you made an object that had such an intense gravity that even light could not escape from it.

That object would be called a **black hole**, because anything falling into it, even light, could never escape from it again.

Black holes can come in all imaginable sizes. Suppose that some aliens could turn the planets and moons in our solar system into black holes. How big would they be?

On a black piece of paper, use a ruler and a compass to make circles that are as large as the black holes mentioned in each of the following problems.

Cut these circles out, and make a black hole mobile of the smaller bodies in the solar system!

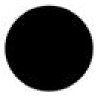
Problem 1 - Mercury is a black hole with a radius of 0.5 millimeters.
Problem 2 - Venus is a black hole with a radius of 7 millimeters
Problem 3 - Earth is a black hole with a radius of 9 millimeters
Problem 4 - The moon is a black hole with a radius of 0.1 millimeters
Problem 5 - Mars is a black hole with a radius of 1.0 millimeter
Problem 6 - Pluto is a black hole with a radius of 0.02 millimeters

The giant planets will need black circles that are much bigger!

Problem 7 - Jupiter is a black hole with a radius of 280 centimeters
Problem 8 - Saturn is a black hole with a radius of 83 centimeters
Problem 9 - Uranus is a black hole with a radius of 13 centimeter
Problem 10 - Neptune is a black hole with a radius of 15 centimeter

Answer Key (approximate sizes)

Mercury
Diameter = 1 mm



Venus
Diameter= 14 mm

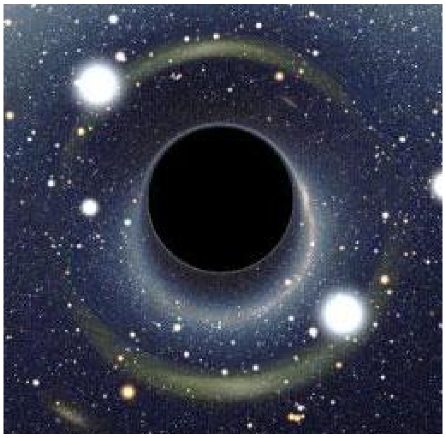
Earth
Diameter= 18 mm

Moon
Diameter= 0.2 mm

Mars
Diameter=2 mm

Pluto
Diameter= 0.04 mm

The Moon as a Black Hole!



Suppose that a group of hostile aliens passed through our solar system and decided to convert our moon into a black hole!

A body with the mass of our moon (about 7 million trillion tons!) would be compressed into a black hole with a diameter of only 0.2 millimeters!

Problem 1 – In the space below, draw a black disk 0.2 millimeters in diameter to represent the size of Black Hole Moon.

Problem 2 - The Earth as a black hole would have a radius of 8.7 millimeters. In the space below, draw a circle the size of Black Hole Earth.

Problem 3 - If the distance to the moon is 356,000 kilometers, how far from our Black Hole Earth would the new Black Hole Moon be located if its diameter were only 0.2 millimeters?

Problem 1 - Moon black hole shown as the following dot .

Problem 2 - Earth black hole shown above. It has a diameter of 17 millimeters, or about the same diameter as a dime.

Problem 3 - Answer: It would still be 356,000 kilometers because this is NOT a scaled drawing of the black holes sizes, but an illustration of their actual sizes, so the distance between the black disks above would be 356,000 kilometers!

The Oldest Lunar Rocks

Apollo astronauts recovered over 380 kg (840 pounds) of lunar rocks, and during the last 40 years, these have been carefully studied to find out which features came first, and the ancient history of the lunar surface including its formation. The table below shows the ages (in millions of years: Myr) of some of the mineral specimens determined by geologist using various radioisotope methods applied to the different rock samples.

Location	Mission	Rock Type	Age (Myr)
Mare Tranquillitatis	Apollo-11	Basalt	3,500
Oceanus Procellarum	Apollo-12	Basalt	3,200
Fra Mauro Formation	Apollo-14	Basalt	4,150
	Apollo-14	Impact melts	3,850
	Apollo-14	KREEP	4,420
Mare Imbrium	Apollo-15	Basalt	3,250
	Apollo-15	Anorthosite	4,090
Descartes Mountains	Apollo-16	Breccia	3,980
	Apollo-16	Basalt	3,790
	Apollo-16	Anorthosite	4,470
	Apollo-16	Plagioclase-old	4,560
	Apollo-16	Plagioclase-young	4,290
Taurus-Littrow Mountains	Apollo-17	Zircon -old	4,418
	Apollo-17	Zircon -Young	4,331
	Apollo-17	Basalt	3,750
	Apollo-17	Impact melts	3,900

Problem 1 - What is the average age of all of the samples in the table in millions of years?

Problem 2 - What is the average age of all of the samples in the table to the nearest 0.1 billion years?

Problem 3 - Order the samples from oldest to youngest age. About how many different groups can you identify in terms of their similar ages, and what is the average age of each group of samples in billions of years?

Problem 4 - To the nearest 100 million years, about how old are the lunar mare (dark regions on moon - basalts) compared to the mountainous highland regions (KREEP, anorthosites, zircons and plagioclase) in billions of years?

Answer Key

Problem 1 - Answer: Add the 16 numbers in the last column together and divide by 16 to get 63,949 million years/16 = **3,997 million years.**

Problem 2 -Answer: Add the 16 numbers in the last column together and divide by 16 to get 63,949/16 = 3,997 million years, which equals 3.997 billion years which needs to be rounded to **4.0 billion years.**

Problem 3 - The samples ordered from oldest to youngest age:

Descartes Mountains	Apollo-16	Plagioclase-old	4,560
Descartes Mountains	Apollo-16	Anorthosite	4,470
Fra Mauro Formation	Apollo-14	KREEP	4,420
Taurus-Littrow Mountains	Apollo-17	Zircon -old	4,418
Taurus-Littrow Mountains	Apollo-17	Zircon -Young	4,331
Descartes Mountains	Apollo-16	Plagioclase-young	4,290
Fra Mauro Formation	Apollo-14	Basalt	4,150
Mare Imbrium	Apollo-15	Anorthosite	4,090
Descartes Mountains	Apollo-16	Breccia	3,980
Taurus-Littrow Mountains	Apollo-17	Impact melts	3,900
Fra Mauro Formation	Apollo-14	Impact melts	3,850
Descartes Mountains	Apollo-16	Basalt	3,790
Taurus-Littrow Mountains	Apollo-17	Basalt	3,750
Mare Tranquillitatis	Apollo-11	Basalt	3,500
Mare Imbrium	Apollo-15	Basalt	3,250
Oceanus Procellarum	Apollo-12	Basalt	3,200

About how many different groups can you identify in terms of their similar ages, and what is the average age of each group of samples in billions of years? (One possibility is shaded)

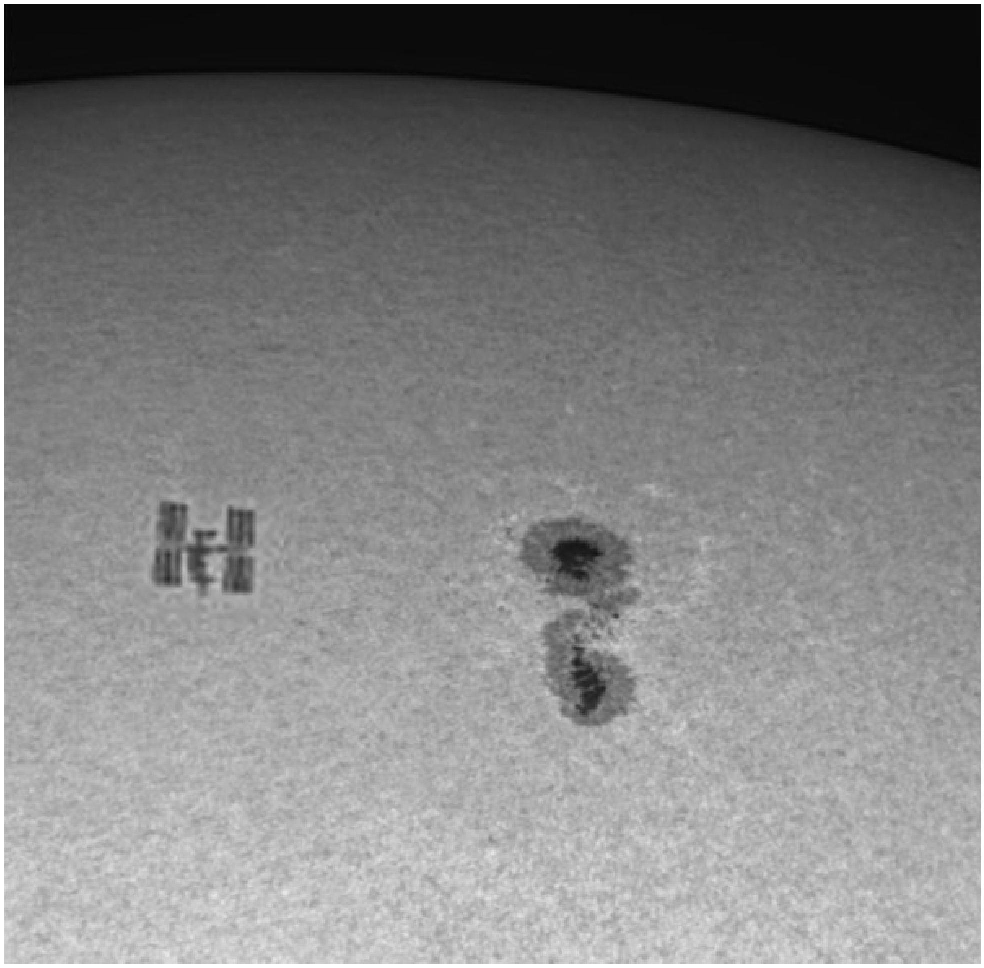
Oldest Group: (4.56+4.47+4.42+4.418+4.331)/5 = **4.4 billion years.**
Middle Group: (4.29+4.15+4.09+3.98+3.9+3.85)/6 = **4.0 billion years**
Young Group: (3.79+3.75+3.5+3.25+3.2)/5 = **3.5 billion years.**

Problem 4 - To the nearest 100 million years, about how old are the lunar mare (dark regions on moon - basalts) compared to the mountainous highland regions (KREEP, anorthosites, zircons and plagioclase) in billions of years?

Answer: The lunar basalts are mostly in the younger group with an age of **3.5 billion** years. The highland samples have ages (4.56 + 4.47 + 4.42 + 4.418 + 4.331 + 4.290 + 4.090)/7 = 4.37 billion years, or rounded to the nearest 0.1 you get **4.4 billion years**.

Note: The highland materials sampled by the Apollo astronauts are older than the mare samples by nearly 1 billion years, and represent the ancient, and very old, original crust of the moon before volcanism filled the mare with lava during the first billion years after lunar formation.

ISS and a Sunspot - Angular Scales



Photographer John Stetson took this photo on March 3, 2010 by carefully tracking his telescope at the right moment as the International Space Station passed across the disk of the sun.

The angular size, θ, in arcseconds of an object with a length of L meters at a distance of D meters is given by

$$\theta = 206265 \frac{L}{D}$$

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Problem 1 - The ISS is 108 meters wide, and was at an altitude of 350 km when this photo was taken. If the sun is at a distance of 150 million kilometers, how large is the sunspot in A) kilometers? B) compared to the size of Earth if the diameter of Earth is 13,000 km?

Answer: As viewed from the ground, the ISS subtends an angle of
Angle = 206265 x (108 meters/350,000 meters) so
Angle = 63 arcseconds.

At the distance of the sun, which is 150 million kilometers, the angular size of the ISS corresponds to a physical length of
L = 150 million kilometers x (63/206265) so
L = 46,000 kilometers.

The sunspot is comparable in width to that of the ISS and has a length about twice that of the ISS so its size is about **46,000 km x 92,000 km**.

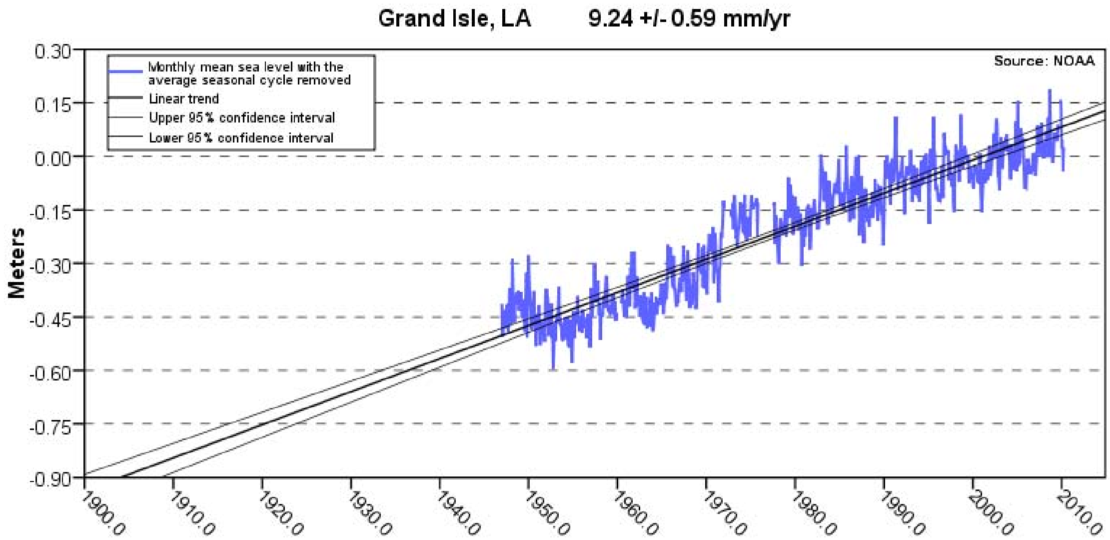
As a comparison, Earth has a diameter of 13,000 km so the sunspot is about **3 times the diameter of Earth in width, and 6 times the diameter of Earth in length.**

Problem 2 - The sun has an angular diameter of 0.5 degrees. If the speed of the ISS in its orbit is 10 km/sec, how long did it take for the ISS to cross the face of the sun as viewed from the ground on Earth?

Answer: From the ground, convert the speed of the ISS in km/sec to an angular speed in arcseconds/sec.

In one second, the ISS travels 10 km along its orbit. From the ground this corresponds to an angular distance of
Angle = 206265 x (10 km/350 km)
= 5900 arcseconds.

The speed is then 5900 arcseconds/sec. The diameter of the sun is 0.5 degrees which is 30 arcminutes or 1800 arcseconds. To cover this angular distance, the ISS will take
T = 1800 arcseconds / (5900 arcseconds/s) so
T = 0.3 seconds!



The NOAA 'Tides and Currents' website also provides an archive of sea level measurements for hundreds of places around the world based on tide gauges and satellite data for the past 100 years.

http://tidesandcurrents.noaa.gov/sltrends/sltrends_station.shtml?stnid=8761724

The plot above shows the sea level rise since 1950 for Grand Isle, Louisiana, which has an average elevation of 2-meters, and a population of 1,500 people. A storm surge from Hurricane Katrina on August 28, 2005 had an elevation of 1.5 meters, and did severe damage to hundreds of homes.

Problem 1 - Assume a linear sea level change of the form h = mt + b where h is the sea level height in meters and t is the number of years since 1950. What is the best-fit linear equation for the sea level rise?

Problem 2 - In what year will the sea level equal the storm surge from Hurricane Katrina?

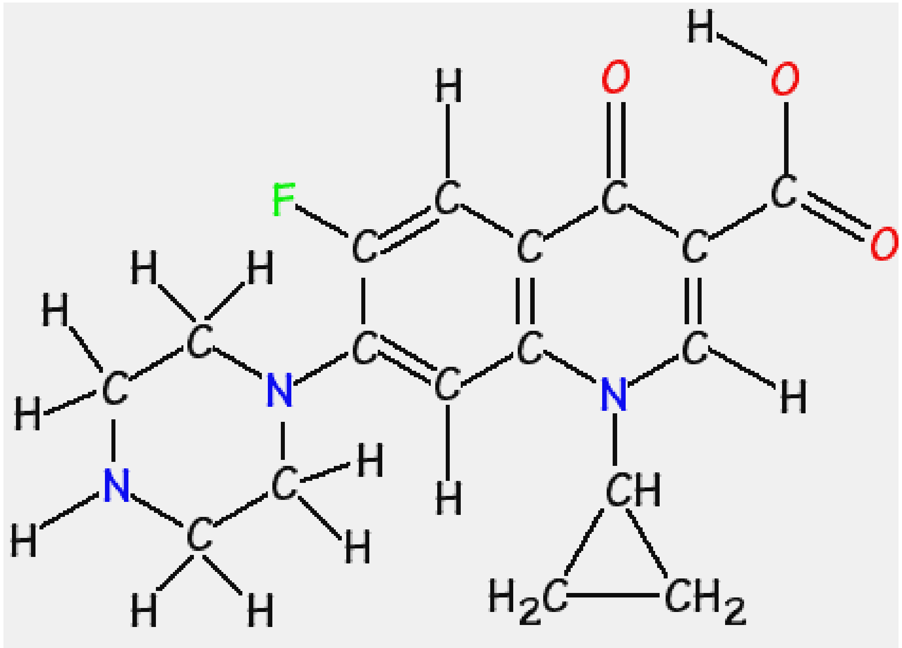
Problem 1 - Assume a linear change of the form h = mt + b where h is the sea level height in meters and t is the number of years since 1950. What is the best-fit linear equation for the sea level rise?

Answer: Point 1 = (1950, -0.45) Point 2 = (2010, +0.10) then
M = (0.10 + 0.45)/(2010-1950) = +0.0092 meters/year

H = 0.0092(t-1950) - 0.45

Problem 2 - In what year will the seal level equal the storm surge from Hurricane Katrina?

Answer: 1.5 = 0.0092 (T-1950) - 0.45 so **T = 2162**



This is a figure showing the locations of hydrogen (H), oxygen (O), carbon (C), nitrogen (N) and fluorine (F) atoms in one molecule of ciprofloxacin. This man-made compound kills bacteria such as anthrax by interfering with the enzymes that cause DNA in the anthrax bacterium to rewind after being copied, which stops DNA and protein synthesis.

Problem 1 - How many atoms of each element are present in one molecule of ciprofloxacin? (Note H_2 means 2 atoms of H)

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

$$C\_\_ \; H\_\_ \; FN_3O\_\_$$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in ciprofloxacin are H = 1 AMU, C=12 AMU, O=16 AMU, N = 14 AMU and F = 19 AMU, what is the total mass of the ciprofloxacin molecule in units of AMUs?

Problem 1 - How many atoms of each element are present in one molecule of ciprofloxacin?

Answer: **Carbon (C) = 17 atoms**
Hydrogen (H) = 18 atoms
Oxygen (O) = 3 atoms
Fluorine (F) = 1
Nitrogen (N) = 3

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

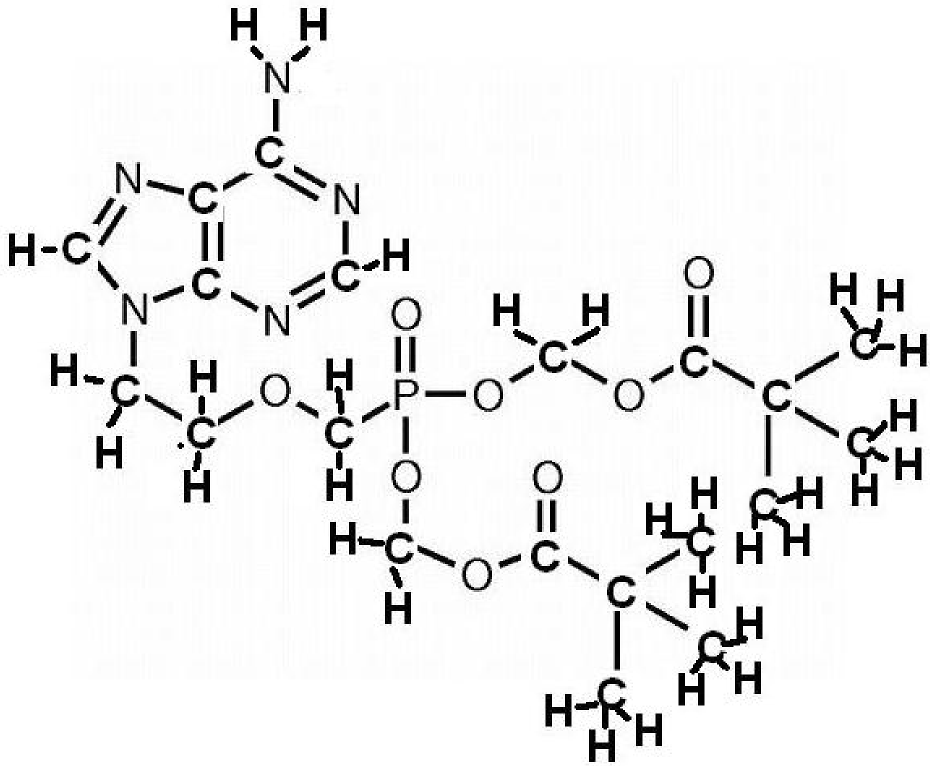
$\mathbf{C_{17} H_{18} FN_3O_3}$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in ciprofloxacin are H = 1 AMU, C=12 AMU, O=16 AMU, N = 14 AMU and F = 19 AMU, what is the total mass of the ciprofloxacin molecule in units of AMUs?

Answer: From the count of the numbers of atoms of each kind:

M = 17 (12) + 18(1) + 1(19) + 3(14) + 3 (16)

M **= 331 AMU**.



This is a figure showing the locations of hydrogen (H), oxygen (O), carbon (C) , nitrogen (N) and phosphorus (P) atoms in one molecule of adefovir dipivoxil, which is a drug designed to treat hepatitis B.

Problem 1 - How many atoms of each element are present in one molecule of adefovir dipivoxil?

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

$$C_{20}\,H\_\_\;N\_\_O\_\_P\_\_$$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in adefovir dipivoxil are H = 1 AMU, C=12 AMU, N= 14 AMU, O=16 AMU, and P = 31 AMU, what is the total mass of a single molecule in AMUs?

Problem 4 - If 1 AMU equals 1.7×10^{-27} kilograms, how many molecules are present in a sample with a mass of 1 microgram?

Problem 1 - How many atoms of each element are present in one molecule of adefovir dipivoxil?

Answer: **Carbon (C) = 20**
Oxygen (O) = 8
Hydrogen (H) = 32
Nitrogen (N) = 5
Phosporus (P) = 1

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

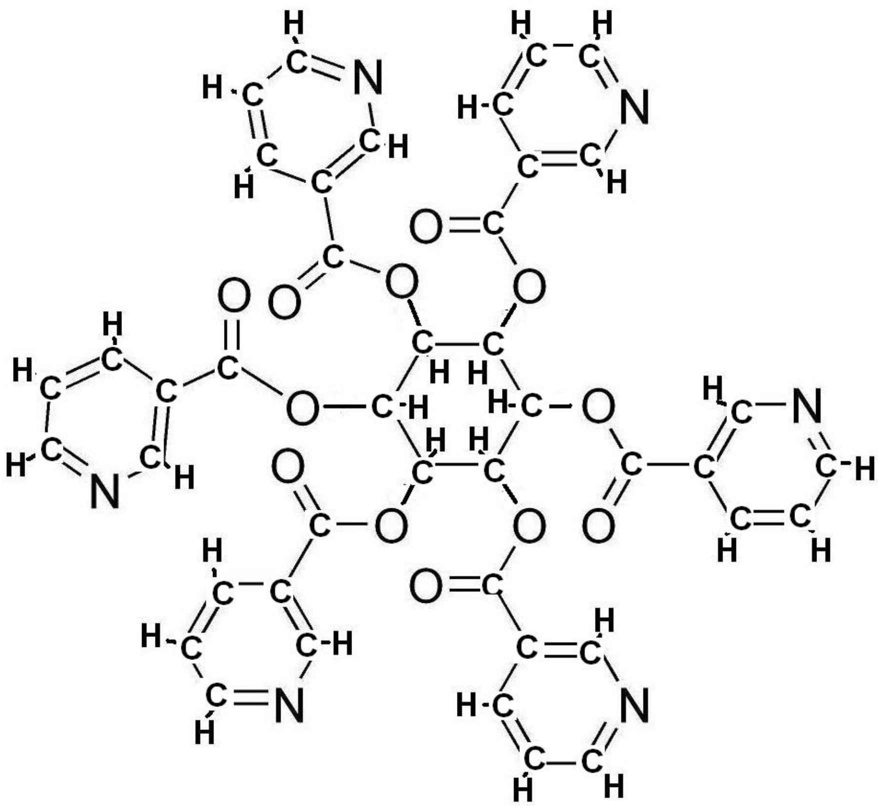
$$C_{20} H_{32} N_5 O_8 P$$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in adefovir dipivoxil are H = 1 AMU, C=12 AMU, N= 14 AMU, O=16 AMU, and P = 31 AMU, what is the total mass of a single molecule in AMUs?

Answer: M = 20(12) + 32(1) + 5(14) + 8(16) + 1(31) = **501 AMU.**

Problem 4 - If 1 AMU equals 1.7×10^{-27} kilograms, how many molecules are present in a sample with a mass of 1 microgram?

Answer: One molecule has a mass of 501 AMU x (1.7×10^{-27} kg/1 AMU) = 8.5×10^{-25} kg. The sample has a total mass of 1.0×10^{-6} grams which equals 1.0×10^{-9} kilograms. So the number of molecules is N = 1.0×10^{-9} /8.5×10^{-25} = **1.2×10^{15} molecules**.



This is a figure showing the locations of hydrogen (H), oxygen (O), carbon (C), and nitrogen (N) atoms in one molecule of inositol nicotinate, which is a synthetic drug used to treat coronary heart disease and arteriosclerosis.

Problem 1 - How many atoms of each element are present in one molecule of inositol nicotinate?

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

C__ H__ N__O__

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in inositol nicotinate are H = 1 AMU, C=12 AMU, N= 14 AMU, and O=16 AMU, what is the total mass of one molecule in AMUs?

Problem 1 - How many atoms of each element are present in one molecule of inositol nicotinate?

Answer: **Carbon (C) = 42**
Oxygen (O) = 12
Hydrogen (H) = 30
Nitrogen (N) = 6

Problem 2 - Write the molecular formula of this molecule by filling-in the blanks with the number of counted atoms in the following:

$$C_{42}\ H_{30}\ N_6\ O_{12}$$

Problem 3 – The mass of each element is given in terms of Atomic Mass Units (AMUs). If the masses of the atoms in inositol nicotinate are H = 1 AMU, C=12 AMU, N = 14 AMU, and O=16 AMU, what is the total mass of a molecule in AMUs?

Answer: M = 42(12) + 30(1) + 6(14) + 12(16) = 504 + 30 + 84 + 192 = **810 AMU.**

How many stars are there?

On a clear night in the city you might be able to see a few hundred stars. In the country, far away from city lights, perhaps 5000 can be seen. Telescopes can see literally millions of stars. But how do we accurately count them? This exercise will show you the basic method!

This image was taken by the 2MASS sky survey. It is a field that measures 9.0 arcminutes on a side. (One arcminute is 1/60 of a degree)

Problem 1 – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells.

Problem 2 – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell.

Problem 3 – A square degree measures 60 arcminutes x 60 arcminutes in area. The full sky has an area of 41,253 square degrees. What are the total number of stars in A) one square degree of the sky; B) the number of stars in the entire sky.

Problem 4 – Why do you think we needed to average the numbers in Problem 2?

Problem 1 – By using a millimeter ruler, divide this star field into an equally-spaced grid that is 3 x 3 cells. Answer: An example is shown below.

Problem 2 – Select 3 of these cells and count the number of star images you can see in each cell. Calculate the average number of stars in a cell. Answer: Using the cells in the top row you may get: 159, 154 and 168. The average is 481/3 = **160 stars.**

Problem 3 – Use the information in the text to convert your answer into the total number of stars in one square degree of the sky.

Answer: A) The answer from 3 is the number of stars in one cell. The area of that cell is 3 x 3 = 9 square arcminutes. One degree contains 60 arcminutes, so a square degree contains 60 x 60 = 3600 square arcminutes. Your estimated number of stars in one square degree is then 3600/9 = 40 times the number of stars you counted in one average one cell. For the answer to Problem 2, the number in a square degree would be 160 X 40 = **6,400 stars.**

B) The text says that there are 41,253 square degrees in the full sky, so from your answer to Problem 3, you can convert this into the total number of stars in the sky by multiplying the answer by 41,253 to get 6,400 x 41,253 = **264,000,000 stars!**

Problem 4 – Why do you think we needed to average the numbers in Problem 2? Answer: Because stars are not evenly spread across the sky, so you need to figure out the average number of stars.

Yes…It moves!!!

This is an image of a star field in the constellation Centaurus taken by the Hubble Space Telescope in 1994. In addition to the bright stars, the streak of a single asteroid can also be seen. The Hubble has 'accidentally' detected over 100 asteroids as its cameras have been looking at other targets. Many of the asteroids are new discoveries. The curvature of the asteroid's trail as it moved across the sky was caused by the slight viewing angle change (parallax) as the telescope orbited Earth during the 40-minute exposures. The field is 2.7 arcminutes on a side, and the distance to the asteroid was estimated to be 140 million kilometers from Earth. Based on the faintness of the asteroid at this distance, it was probably only 2 kilometers across!

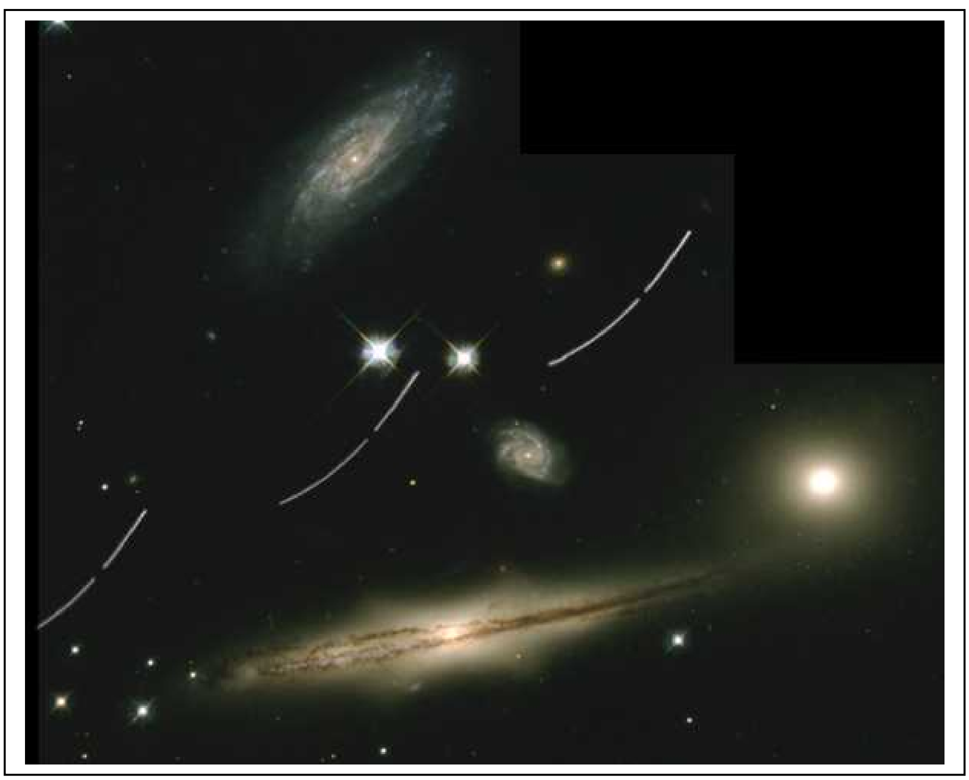
Problem 1 - At the distance of the asteroid, this field would measure about 110,000 kilometers across. How many kilometers did the asteroid travel during the time of the exposure?

Problem 2 - What was the approximate speed of the asteroid in kilometers/hour from the beginning to the end of the trail?

Problem 1 - At the distance of the asteroid, this field would measure about 110,000 kilometers across. How many kilometers did the asteroid travel during the time of the exposure?
Answer: Students will have to convert the length of the streak into kilometers using the scale of the image. Use a millimeter ruler to determine the scale of the image by first measuring the width of the image to get 118 millimeters. This physical length is equal to 110,000 kilometers, so the image scale is just 110,000 km/ 118 millimeters = 932 km/mm. The length of the asteroid streak is 20 millimeters, so its length is 20 x (932 km/mm) = **18,640 kilometers.**

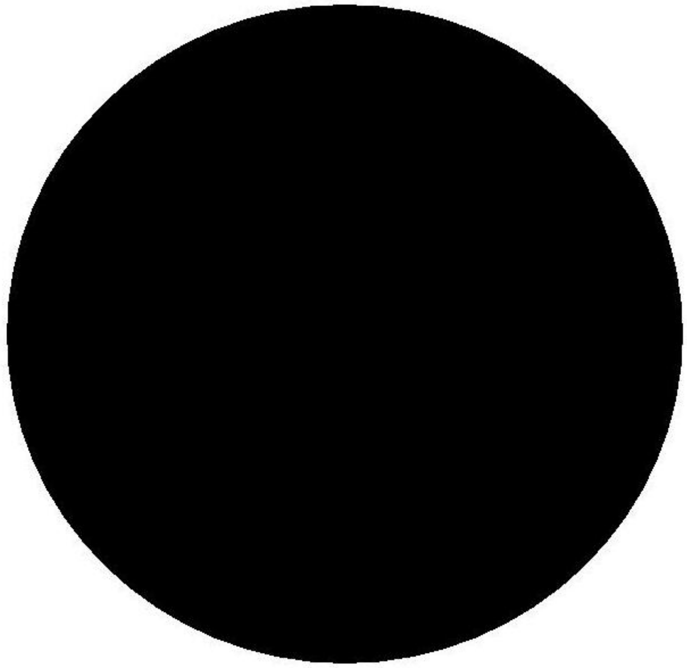
Problem 2 - What was the approximate speed of the asteroid in kilometers/hour from the beginning to the end of the trail?
Answer: The paragraph says that the exposure took 40 minutes, so during that time the asteroid moved the distance indicated in Problem 1. The speed is then 18,640 Kilometers/0.66 hours = **28,200 km/hr.**

The photograph below shows another asteroid streak across a picture taken of the Hickson Galaxy Group # 87 in the constellation Capricornus. The size of the field is 132,000 km at the distance of the asteroid, and the exposure was about 6.6 hours. During this time the asteroid traveled 166,000 kilometers for a speed of about 25,000 km/hr.



Exploring a Full-Sized Black Hole - Honest! 11

This black ball shown below is the <u>exact size</u> of a black hole with a diameter of 9.0 centimeters. Such a black hole would have a mass of **5 times** the mass of our Earth. All of this mass would be INSIDE the ball below.



Although it looks pretty harmless, if this black hole were at arms-length, you would already be dead. In fact, if you were closer to it than the distance from New York to San Francisco, a 150-pound person would weigh 3 tons and would be crushed by their own weight!

Suppose that you could survive being crushed to death as you got closer to the black hole shown above. To stay in an orbit around the black hole so that you did not fall in, you have to be traveling at a specific speed V, in kilometers per second, that depends on your distance R, in meters from the center of the black hole, is given below:

$$V = \frac{44,700}{\sqrt{R}}$$

Problem 1 - If you were orbiting at the distance of the Space Shuttle (R=6,800 km) from the center of this black hole, what would your orbital speed be in A) kilometers/sec? B) kilometers/hour? C) miles per hour (1 mile = 1.6 km).

Problem 2 - If a small satellite were orbiting 20 centimeters away from the center of the black hole shown above, how fast would it be traveling in A) km/second? B) percentage of the speed of light? (The speed of light = 300,000 km/sec).

Problem 3) If the orbit is a circle, how long: A) would the Space Shuttle in Problem 1 take to go once around in its orbit? B) would it take the satellite in Problem 2 to go once around in its orbit?

Problem 1 - If you were orbiting at the distance of the Space Shuttle (R=6,800 km) from the center of this black hole, what would your orbital speed be in A) kilometers/sec? B) kilometers/hour? C) miles per hour (1 mile = 1.6 km).

Answer; A) The formula says that for R = 6,800,000 meters, **V = 17 km/sec**.
B) 1 hour = 3600 seconds, so V = 17 km/sec x (3600 sec/1 hour) = **61,200 km/hour.**
C) V = 61,200 km/sec x (1 mile / 1.6 km) = **38,250 miles/hr**

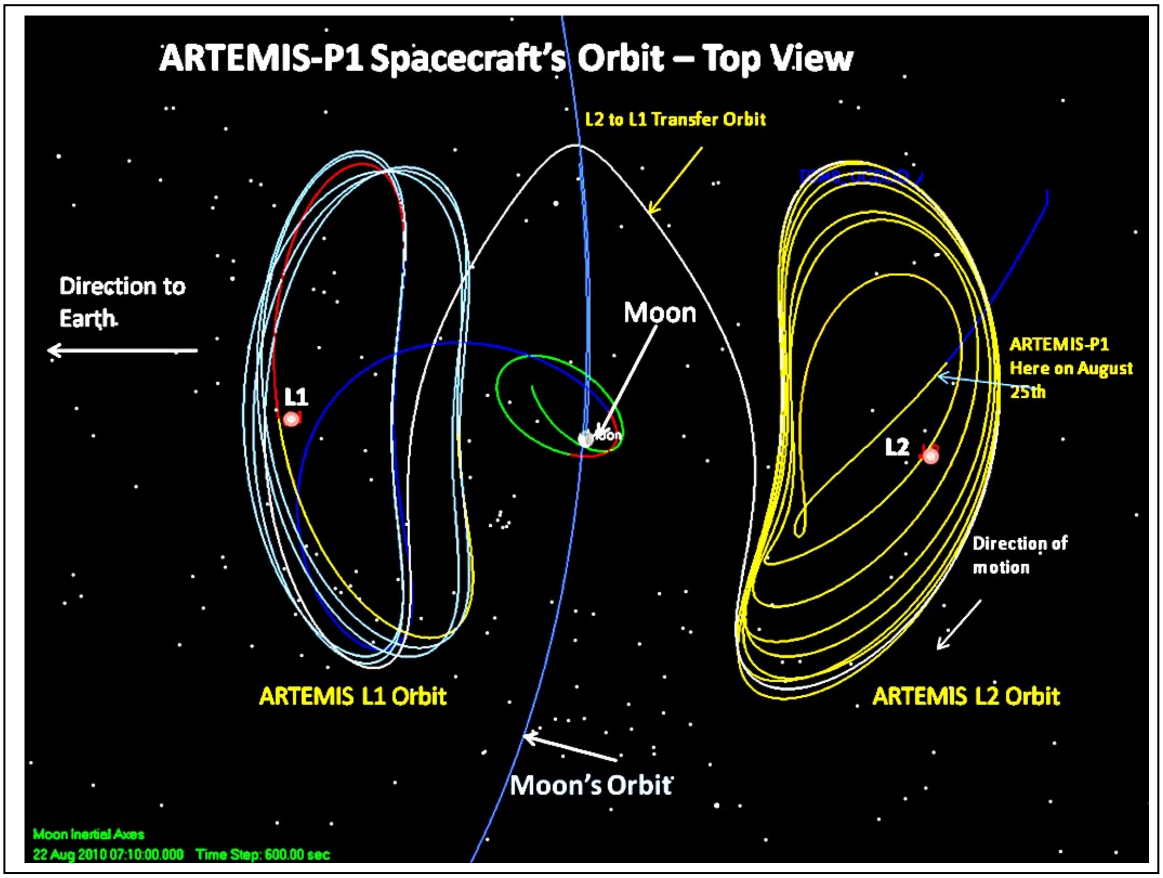
Problem 2 - If a small satellite were orbiting 20 centimeters away from the center of the black hole shown above, how fast would it be traveling in A) km/second? B) percentage of the speed of light? (The speed of light = 300,000 km/sec).

Answer; A) R = 0.2 meters, so from the formula V = **100,000 km/sec**
B) Speed = 100% x (100000/300000) so speed = **33% the speed of light**.

Problem 3) If the orbit is a circle, how long: A) would the Space Shuttle in Problem 1 take to go once around in its orbit? B) would it take the satellite in Problem 2 to go once around in its orbit?

A) Orbit circumference, C = $2\pi R$ so for R = 6,800 km, C = 40,000 kilometers. The Shuttle speed is V=17 km/sec, so the time is T = C/V or **2,353 seconds. This equals about 39 minutes.**

B) A) Orbit circumference, C = $2\pi R$ so for R = 0.2 meters, C = 1.25 meters. The satellite speed is V=100,000 km/sec. Converting this to meters we get 100,000,000 meters/sec, so the time is T = C/V or **0.000000013 seconds. This is 13 billionths of a second!**



The Artemis spacecraft, formerly P1 of the THEMIS constellation in Earth orbit, has been reprogrammed to journey to lunar orbit to continue making measurements of Earth's magnetic field. Once there, it will perform a complicated orbit between two 'invisible' points in space called the L1 and L2 Lagrange Points. These are locations within the combined gravitational forces of the Earth and Moon and spacecraft centrifugal forces are in near-equilibrium. The spacecraft can remain more or less at the same location with only minor use of its maneuvering rockets.

Problem 1 – The distance between L1 and L2 in the diagram above is 120,000 kilometers. Using a millimeter ruler, what is the scale of this figure in kilometers/millimeter?

Problem 2 – Using a piece of string and measuring to the nearest meter in length, about what is the total length of the spacecraft orbit shown in the figure in millions of kilometers?

Problem 3 – Assume that the average speed of the spacecraft in its travels is about 200 meters/sec. How long, in days, will it take the spacecraft to 'fly' the orbit pattern A) Around one of the Lagrange Points? B) Around the full orbital circuit shown above?

Problem 1 – The distance between L1 and L2 in the diagram above is 120,000 kilometers. Using a millimeter ruler, what is the scale of this figure in kilometers/millimeter?

Answer: The L1 and L2 points are 80 mm apart on the figure, so the scale is 120,000 km/80 mm = **1,500 km/mm**.

Problem 2 – Using a piece of string and measuring to the nearest meter in length, about what is the total length of the spacecraft orbit shown in the figure in millions of kilometers?

Answer: Depending on how students lay down the string along the indicated path the length should be about 2 meters. At the scale of the diagram, this is equivalent to **3 million kilometers.**

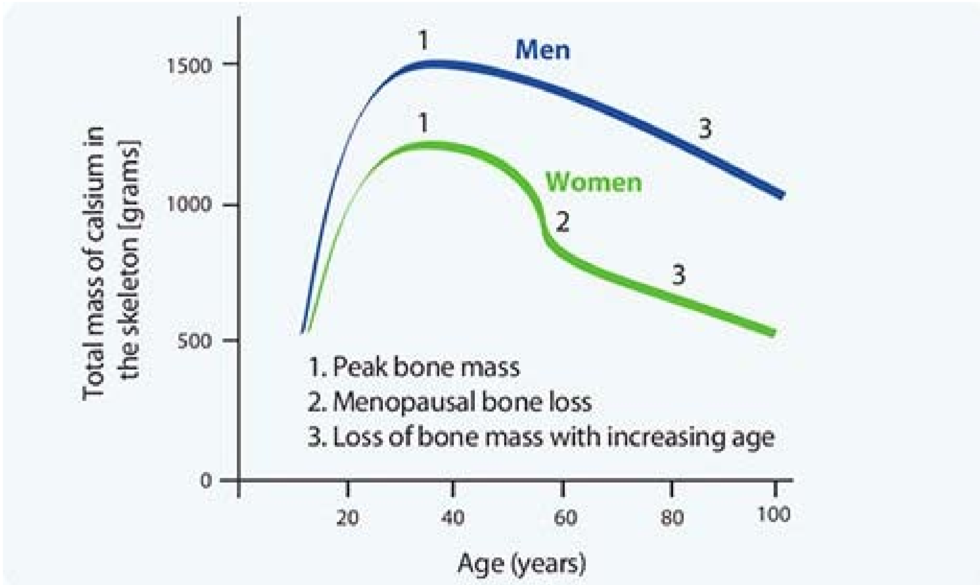
Problem 3 – Assume that the average speed of the spacecraft in its travels is about 200 meter/sec. How long, in days, will it take the spacecraft to 'fly' the orbit pattern A) Around one of the Lagrange points? B) Around the full orbital circuit shown above?

Answer:

A) The loop around L1 is about 150 mm, so the distance is about 225,000 kilometers. At a speed of 0.2 km/sec, a full loop will take about 1,100,000 seconds or since there are 86,400 seconds in 1 day, about **13 days**.

B) To travel 3 million kilometers at 0.2 km/sec will take about **170 days**.

Astronaut Bone Loss in Space



Under low-gravity conditions, human bones respond by losing mass. The longer an astronaut remains in space, the more bone loss occurs. This is considered one of the greatest challenges to humans operating in space for extended periods of time, such as journeys to Mars and living in 'space stations'. The above graph shows normal bone loss with age for humans living on Earth. The average bone loss for some astronauts is 1.9 percent per month.

Problem 1 – From the graph above, what is the average rate of bone loss between age 40 and age 70 for A) Men and B) Women?

Problem 2 – Suppose a 40 year old male astronaut spent 6 months aboard the International Space Station. If he started out with 1500 grams of bone calcium, how much calcium would remain in his bones when he returned to Earth?

Problem 3 - Suppose a 40 year old female astronaut spent 6 months aboard the International Space Station. If she started out with 1200 grams of bone calcium, how much calcium would remain in her bones when she returned to Earth?

Problem 4 – From the graph, how old would both astronauts be in order to have the same amount of calcium as they did after returning to Earth?

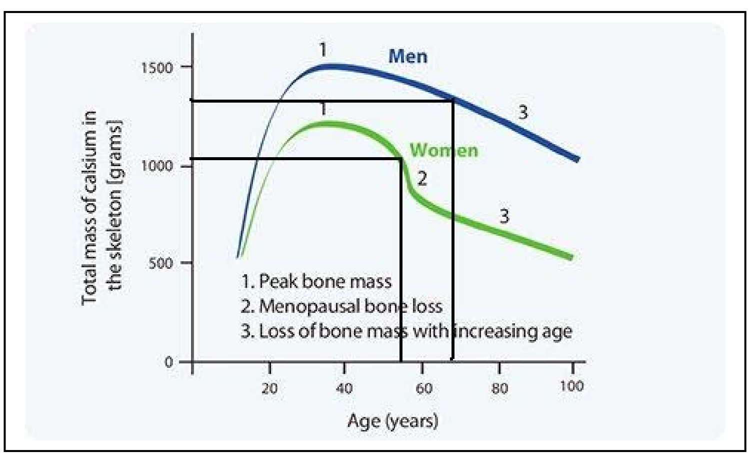
Problem 1 – From the graph above, what is the average rate of bone loss between age 40 and age 70 for A) Men and B)Women? Answer: A) rate = (change in mass) / (change in time) = (1100 – 1500) / (100 – 40) = **- 6.7 grams/year.** B) rate = (500 – 1200)/(100 – 40) = **- 11.7 grams/year.**

Note: Because the signs are negative, the rate is a mass loss. As a percentage, for men the rate is 100% x (6.7/1500) = 0.4% per year. For women it is 100% x (11.7/1200) = 1.0% per year…more than twice as fast as for men.

Problem 2 – Suppose a 40 year old male astronaut spent 6 months aboard the International Space Station. If he started out with 1500 grams of bone calcium, how much calcium would remain in his bones when he returned to Earth? Answer: The astronaut will lose 0.019 x 6 months x 1500 grams = **171 grams lost**

Problem 3 - Suppose a 40 year old female astronaut spent 6 months aboard the International Space Station. If she started out with 1200 grams of bone calcium, how much calcium would remain in her bones when she returned to Earth? Answer: The astronaut will lose 0.019 x 6 months x 1200 grams = **137 grams lost**

Problem 4 – From the graph, how old would both astronauts be in order to have the same amount of calcium as they did after returning to Earth? Answer: For the male astronaut, his final bone mass is 1500 grams – 171 grams = 1329 grams. From the graph below, this bone mass is reached when a typical male reaches the age of **about 67 years.** For the female astronaut, her final bone mass is 1200 grams – 137 grams = 1063 grams. From the graph below, this bone mass is reached when a typical male reaches the age of **about 55 years.**



Year	Month	X (meters)	Y (meters)
2008	1	-3	+8
2008	2	-4	+10
2008	3	-4	+13
2008	4	-2	+15
2008	5	+1	+16
2008	6	+4	+16
2008	7	+7	+15
2008	8	+9	+13
2008	9	+9	+10
2008	10	+8	+7
2008	11	+6	+5
2008	12	+3	+4
2009	1	0	+4
2009	2	-3	+6
2009	3	-4	+9
2009	4	-4	+12
2009	5	-2	+15
2009	6	+1	+16
2009	7	+4	+16
2009	8	+7	+15

The Earth rotates on its axis once every 24 hours (23 hours 56 minutes and 4 seconds more accurately!), but like a bobbing, spinning top, the direction doesn't point exactly at an angle of 23 1/2 degrees.

For over 200 years, careful measurements of the rotation axis have shown that it moves slightly. The data in the table to the left gives the axis location of the North Pole as it passes through Earth's surface. The X-axis runs East-West and the Y-axis runs North-South. The units are in meters measured on the ground.

Problem 1 - On a Cartesian Graph, plot the location of the North Pole during the time span indicated by the table.

Problem 2 - About how long, in days, does it take the North Pole to return to its starting position based on this data?

Problem 3 - About what is the average speed of this Polar Wander in meters per day?

Problem 4 - Based on your plot, does it look like the motion will exactly repeat itself in space during the next cycle?

Problem 5 - About what is the X and Y location of the center of the movement pattern?

Problem 6 - What would you use as the best location of the North Pole?

Problem 1 - Answer; **See below.**

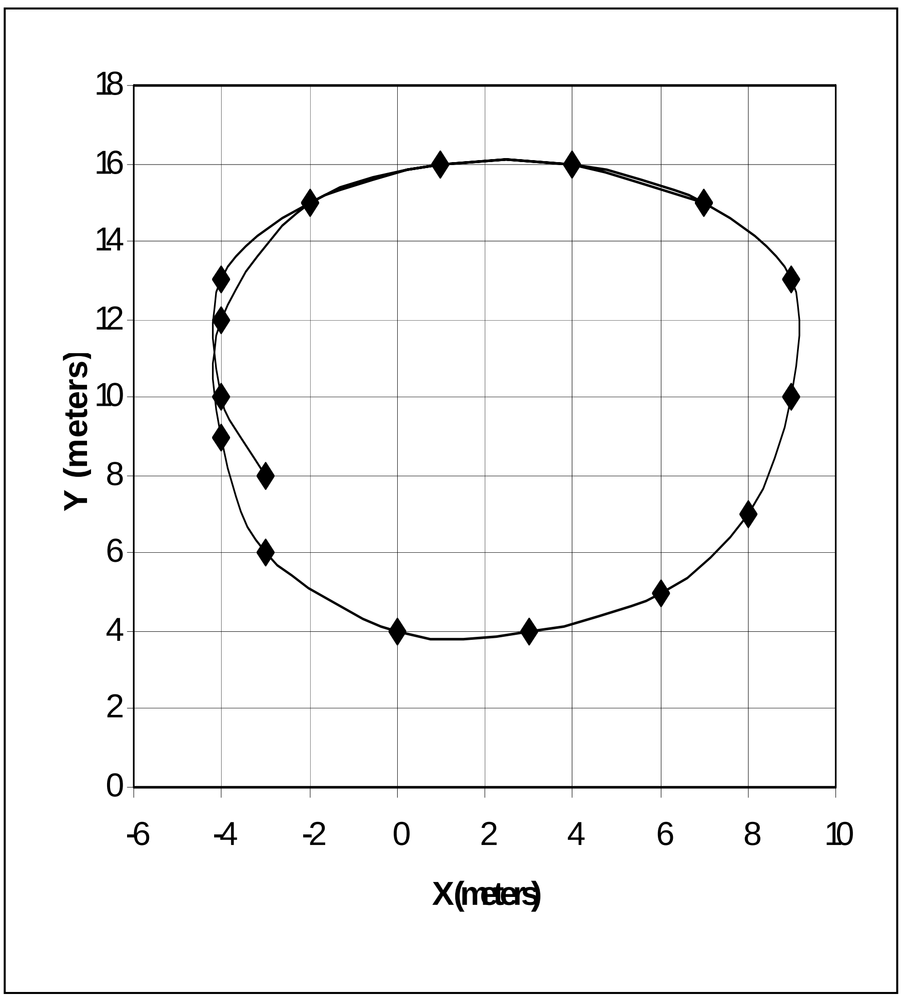
Problem 2 - Answer; **About 14 months** or 420 days. This is called the Chandler Period.

Problem 3 - Answer: In one month the point moves on the circle about 3 meters, so the speed is about 3 meters/30 days or **0.1 meter/day.**

Problem 4 - Answer; Not exactly. **The new cycle points do not follow the circle of the older cycle**. This motion is called the Chandler Wobble.

Problem 5 - Answer; **X = + 2 meters, Y = +10 meters**.

Problem 6 - Answer: **Probably the point at the center of the circle, because it is about equidistant from all of the other points.**



Growing Grapes in the Middle of the Desert

The Advanced Land Imager (ALI) on NASA's Earth Observing-1 (EO-1) satellite captured this true-color image on February 15, 2010. A network of bright rectangles of varying shades of green contrasts with surroundings of gray, beige, tan, and rust typical of the arid conditions prevailing in Namibia. The Orange River serves as part of the border between Namibia and the Republic of South Africa. Along the banks of this river, roughly 100 kilometers (60 miles) inland from where the river empties into the Atlantic Ocean, irrigation projects take advantage of water from the river and soils from the floodplains to grow produce, mostly grapes, turning parts of a normally earth-toned landscape emerald green. (North is indicated by arrow)

Problem 1 - Assume that each one of the square cultivated areas is about 300 meters on a side. How large is a single cultivated area in square meters and in acres? (Note: 1 acre = 4,047 square meters)

Problem 2 - What is the total area under cultivation in this region in square kilometers and in acres?

Problem 3 - If a single grape plant covers 4 square meter of land and requires about 5 gallons of water per day, how many grape plants are being supported in this cultivated area, and how many gallons of water per day do they require from the Orange River?

Problem 1 - Each one of the square cultivated areas is about 300 meters on a side. What is the total area of a single cultivated area in square meters and in acres if 1 acre equals 4,047 square meters?

Answer: Since each plot of land is assumed to be square, the area of a single plot is just A = (300 m) x (300 m) = **90,000 square meters**. Then since 1 acre equals 4,067 square meters, we have:

90,000 $meters^2$ x (1 acre / 4,067 m^2) = **22.1 acres**.

Problem 2 - What is the total area under cultivation in this region in square kilometers and in acres?

Answer: Carefully count the number of green squares in the picture. Count the partial square areas as best as possible. Answers should be near 103. Then multiply by the area of one square determined from Problem 1 to get A = 90,000 m^2 x (103) = 9.3 million square meters. In terms of square kilometers, 1 km^2 = 1 million square meters, so the area is **9.3 km^2**.

In terms of acres, 22.1 acres/square x (103 squares) = **2276.3 acres**. To three significant figures, the answer is also **2280 acres**.

Problem 3 - If a single grape plant covers 4 square meter of land and requires about 5 gallons of water per day, how many grape plants are being supported in this cultivated area, and how many gallons of water per day do they require from the Orange River?

Answer: The total cultivated area is 9.3 million square meters, which represents about N = 9.3 million $meters^2$ x (1 plant/4 $meters^2$) = 2.3 million plants. Since each plant requires 5 gallons per day, the total daily irrigation draw from the Orange River would be about 2.3 million plants x (5 gallons / 1 plant) = **11.5 million gallons per day.**

The Cometary Planet: HD 209458b

Artist view of planet G. Bacon (STScI/NASA)

Every 4 days, this planet orbits a sun-like star located 153 light years from Earth. Astronomers using NASA's Hubble Space Telescope have confirmed that this gas giant planet is orbiting so close to its star its heated atmosphere is escaping into space.

Observations taken with Hubble's Cosmic Origins Spectrograph (COS) suggest powerful stellar winds are sweeping the cast-off atmospheric material behind the scorched planet and shaping it into a comet-like tail. COS detected the heavy elements carbon and silicon in the planet's super-hot, $2,000^{o}$ F atmosphere.

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3 times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b?

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Answer: A) 4×10^{11} grams/sec x (10^{-6} kg/gm) x (86,400 sec/day) x (1 ton / 1000 kg)
= **3.5×10^{10} tons/day**
B) 3.5×10^{10} tons/day x 365 days/year = **1.3×10^{13} tons/year**

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3 times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

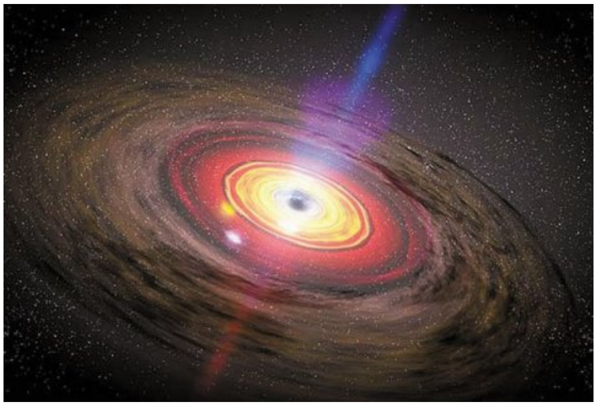
Answer: A) $V = 4/3 \pi R^3$ so V(Jupiter) = 1.5×10^{24} meters^3. Density = mass/volume so Density (Jupiter) = 1.9×10^{27} kg/1.5×10^{24} meters^3 = **1266 kg/meter^3**.
B) Mass = 0.6 M(Jupiter) and volume = $(1.3)^3$ V(Jupiter) so density = $0.6/(1.3)^3$ x 1266 kg/meter^3 =0.27x1266 = **342 kg/meter^3**.

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b? Answer: M(HD209458b) = 0.6x Jupiter = 1.1×10^{27} kg so M(atmosphere) = 1.1×10^{27} kg – 18x(5.9×10^{24} kg) = **9.9×10^{26} kg**.

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

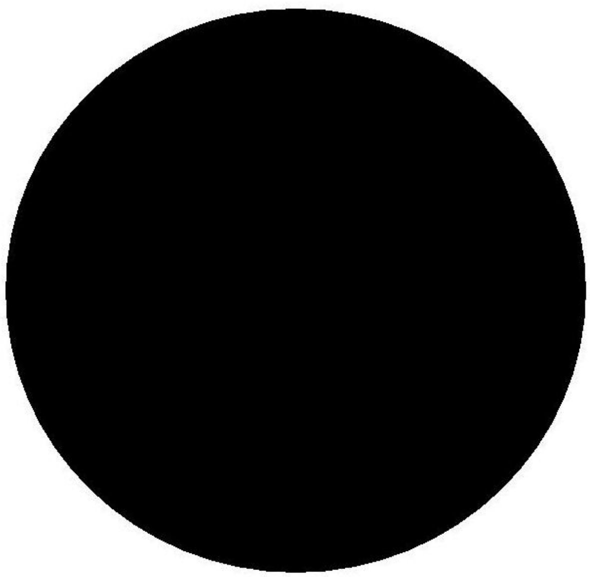
Answer: Time = Mass/rate, and the rate is 1.3×10^{13} tons/year. Since 1 metric ton = 1,000 kg, the rate is 1.3×10^{16} kg/year so that
= 9.9×10^{26} kg / (1.3×10^{16} kg/year)
= **7.6×10^{10} years**

The paper is Linsky et al., “Observations of Mass Loss from the Transiting Exoplanet HD 209458b,” *Astrophysical Journal* Vol. 717, No. 2 (10 July 2010), p. 1291. They estimate nearly a trillion years, so the planet is in no danger of disappearing!


Artist's impression of gas falling into a black hole
Image credit: NASA / Dana Berry, SkyWorks Digital

When gas flows into a black hole, it gets very hot and emits light. The gas is heated because the atoms collide with each other as they fall into the black hole. Far away from the black hole, the atoms do not travel very fast so the gas is cool. But close to the black hole, the atoms can be moving at millions of kilometers/hour and the gas can be thousands of degrees hot!

The circle below represents the spherical shape of a black hole with a mass of about 5 times our Earth. Its diameter is 9 centimeters.



The formula that gives the gas temperature, T in Kelvins, at a distance of R in meters from the center of the black hole, is given by:

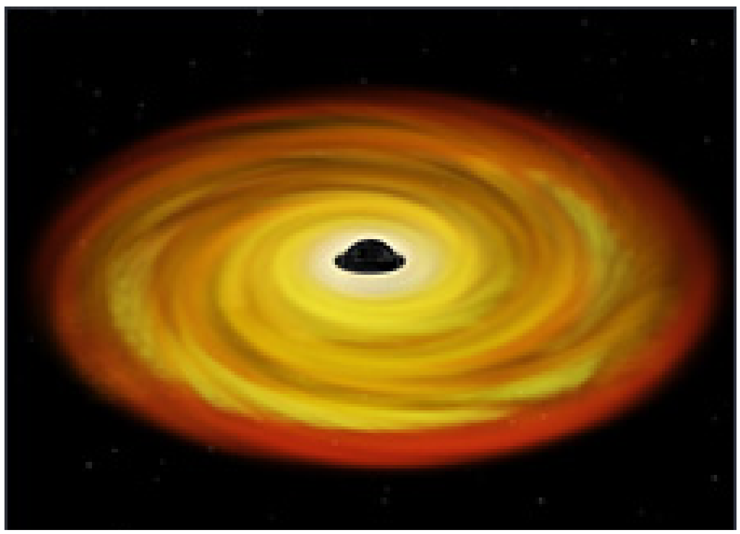
$$T = \frac{35,000}{R^{\frac{3}{4}}}$$

Problem 1 - Sketch a life-sized illustration of the gas surrounding the above black hole and give the temperature at a distance of 1 meter, 50 centimeters and 5 centimeters from the center of the black hole.

Problem 1 - Sketch a life-sized illustration of the gas surrounding the above black hole and give the temperature at a distance of 1 meter, 50 centimeters and 5 centimeters from the center of the black hole.

Answer: At 1 meter, **T = 35,000 K,** which is 6 times the surface temperature of our sun.
At 50 cm or 0.5 meters, **T = 59,000 K**.
At 5 centimeters or 0.05 meters, **T = 331,000 K**.

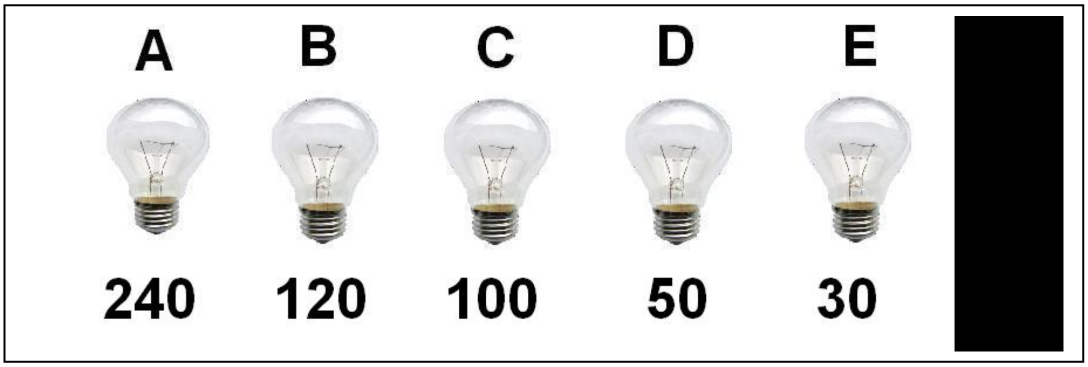
Students may color many different versions, but they should all show that the most distant gas is cooler (35,000 K) than the gas near the black hole (331,000 K) which could be temperature coded using some plausible scheme like the 'yellow to white' scheme below.



A Black Hole - Up Close!

The sketch below shows the edge of a black hole on the right hand-side. The distance in centimeters from the edge of the black hole, called the **event horizon**, increases from right to left to a maximum distance of 240 centimeters from the event horizon (Bulb A). In this figure, the radius of the black hole is about 1 meter. This corresponds to a black hole with a mass equal to 120 times the mass of our Earth.

Although all of the light bulbs would be destroyed this close to an actual event horizon, we will pretend, for simplicity, that they can survive.



Someone far away from a black hole will see things very differently than someone close to a black hole. Because of the intense gravitational forces, ordinary light emitted close to a black hole will have its wavelength stretched as viewed by someone far away. The closer the light source, the more wavelength -stretching will be seen by the distant observer.

Suppose all of these light bulbs in the above figure are emitting light at a wavelength of 570 nanometers (nm), and are shining brightly with a color near yellow like our sun. A distant observer will see this light stretched to longer wavelengths. The wavelength they will observe, W, in nanometers depends on the distance of the light source, R to the center of the black hole in centimeters according to the formula:

$$W = \frac{450}{\sqrt{1 - \frac{100}{R}}}$$

For example, the event horizon for this black hole is at R=100 centimeters. If the light bulb is 50 centimeters to the left of the horizon, R = 150 centimeters, and so W = 780 nanometers. The middle of the Visible Band is at about 500 nm, so instead of yellow light, you would see this light bulb emitting very deep red color!

Problem 1 - Suppose the bulbs were located at the distances from the event horizon shown in the figure above. What would be the wavelengths you would observe for Bulbs B, C, D and E?

Problem 2 - The human eye can only detect light at a wavelength shorter than about 650 nm. Which of the light bulbs would appear to be invisible to you and 'black'?

Problem 3 - How close to the event horizon would the light bulb have to be in order for you to only detect it as an invisible heat source emitting at a wavelength of just 14,000 nm?

Problem 1 - Suppose the bulbs were located at the distances indicated above. What would be the wavelengths you would observe for Bulbs B, C, D and E?

Bulb A	240 cm	R = 340 cm	**W = 535 nm**
Bulb B	120 cm	R = 220 cm	**W = 609 cm**
Bulb C	100 cm	R = 200 cm	**W = 636 cm**
Bulb D	50 cm	R = 180 cm	**W = 780 cm**
Bulb E	30 cm	R = 130 cm	**W = 937 cm**

Note: In the visible spectrum
Bulb A = yellow
Bulb B = orange
Bulb C = red
Bulb D = deep crimson or dull red and nearly invisible
Bulb E = infrared and invisible to the eye.

Problem 2 - The human eye can only detect light at a wavelength shorter than about 650 nm. Which of the light bulbs would appear to be invisible to you and 'black'?

Answer: From your location far from the black hole, you see that the Bulbs D and E are not visible to your eyes. **Note: With the proper light detectors you could still see them shining at these longer wavelengths!**

Problem 3 - How close to the event horizon would the light bulb have to be in order for you to only detect it as an invisible heat source emitting at a wavelength of just 14,000 nm?

Answer: We need to solve for R the equation:

$$14,000 = \frac{450}{\sqrt{1 - \frac{100}{R}}}$$ from this we get $$1 - \frac{100}{R} = \left(\frac{450}{14,000}\right)^2$$

$$1 - 0.001 = \frac{100}{R}$$ so R = 100.1 centimeters.

This means that the bulb is located **0.1 centimeters or 1 millimeter** just outside the event horizon!

Note: A black hole with a radius of 100 centimeters would have a mass of about 120 times that of Earth, or a little bit more than the planet Saturn.

The Most Massive Stars Known

Star	Mass (Sun=1)	Diameter (Sun=1)	Luminosity (Sun=1)
R136a1	350	35	10,000,000
Eta Carina	250	195	5,000,000
Peony Nebula	175	100	3,200,000
Pistol Star	150	340	6,300,000
HD269810	150	19	2,200,000
LBV1806	130	200	10,000,000
HD93129a	120	25	5,500,000
HD93250	118	18	5,000,000
S Doradus	100	380	600,000

The picture above shows the Tarantula Nebula in the Large Magellanic Cloud, located 165,000 light years from Earth. Astronomers using data from the Hubble Space Telescope and the European Space Observatory's Very Large Telescope in Chile have recently determined that it harbors the most massive known star in our 'corner' of the universe which they call R136a1. The table lists some of the most massive known stars as of July, 2010.

Problem 1 - From the indicated sizes relative to the Sun, create a scale model that shows the relative sizes of these stars compared to the Sun, assuming that in the scaled model the solar disk has a diameter of 1 millimeter.

Problem 2 - The predicted lifespan of a star depends on its luminosity according to the formula

$$T = \frac{10 \text{ billion years}}{M^{2.5}}$$

For the Sun, M = 1 and so its lifespan is about 10 billion years. A star with 10 times the mass of our Sun, M=10, will last about 30 million years! From the table above, what are the predicted life spans for these 'hypergiant' stars to two significant figures?

Problem 3 - How many generations of a 100 solar-mass hypergiant could pass during the life span of a single star like our own Sun?

Problem 1 - From the indicated sizes relative to the Sun, create a scale model that shows the relative sizes of these stars compared to the Sun, assuming that in the scaled model the solar disk has a diameter of 1 millimeter.

Answer: **Example of calculation: R136a1 diameter = 700x sun, so 700 x 1 mm = 0.7 meters in diameter!**

Problem 2 - From the table above, what are the predicted lifespans for these 'hypergiant' stars?

Answer:

Star	Mass (Sun=1)	Diameter (Sun=1)	Luminosity (Sun=1)	Lifespan (years)
R136a1	350	35	10,000,000	**4,400**
Eta Carina	250	195	5,000,000	**10,000**
Peony Nebula	175	100	3,200,000	**25,000**
Pistol Star	150	340	6,300,000	**36,000**
HD269810	150	19	2,200,000	**36,000**
LBV1806	130	200	10,000,000	**52,000**
HD93129a	120	25	5,500,000	**63,000**
HD93250	118	18	5,000,000	**66,000**
S Doradus	100	380	600,000	**100,000**

Problem 3 - How many generations of a 100 solar-mass hypergiant could pass during the life span of a single star like our own Sun?

Answer: Life span of our sun is 10 billion years. Life span of a 100 solar-mass hypergiant is 100,000 years, so about 10 billion/100,000 = **100,000 generations** could come and go.

Most people have heard about meteorites, and have seen meteors streaking across the night sky. These 'rocks' travel through space at thousands of kilometers per hour and can strike any other object in their way.

Astronomers have studied these recovered meteorites for over a century and have learned about their ages, compositions and in some cases their origins in the solar system.

The above meteorite sample is called the Esquel Pallasite, and was part of a 1000 kilogram 'fall' that occurred over Esquel, Argentine in 1951. The sample has a mass of 350 grams and was sliced, polished and stained to reveal the details of its composition. Pallasites contain small stone 'inclusions' of the mineral olivine, that are embedded in an iron-nickel alloy called a matrix.

It is thought that pallesites are small pieces of rock left over from the destruction of a large asteroid, which was not large enough for the iron-nickel and stony materials to segregate. Earth, by comparison, was massive enough for its iron and nickel to form a dense core surrounded by the lighter, stony materials.

Meteorite samples can be classified into several dozen different families that seem to indicate there were several dozen ancient bodies that disintegrated during collisions to form these remnants that we can now collect when they strike Earth.

Problem 1 – The density of the iron-nickel matrix is 8.2 grams/cm^3 and the olivine density is 3.3 grams/cm^3. If the total mass of the Esquel sample is 350 grams, and 20% of the sample is composed of olivine, what was the mass of each of the two ingredients to the pallasite?

Problem 2 - Meteorite collectors find and sell samples by the gram. The price of a gram of the Esquel pallasite is about $106. What was the price of the sample in Problem 1?

Problem 1 – The density of the iron-nickel matrix is 8.2 grams/cm^3 and the olivine density is 3.3 grams/cm^3. If the total mass of the Esquel sample is 350 grams, and 20% of the sample is composed of olivine, what was the mass of each of the two ingredients to the pallasite?

Answer: This is a very typical problem that also appears in chemistry classes in various different guises.

First, what are the volumes of the two ingredients?
Olivine = 20%
and so Iron/nickel = 80%.

Now, the product of density and volume = mass, so we can write the equation for the mass as:

8.2 x0.8V + 3.3 x 0.2V = 350 where V is the total volume of the sample.

Simplify and solve for V to get (6.56 + 0.66) V = 350 so V = 48.5 cm^3.

Then for the **olivine** we have 3.3 x 0.2 x (48.5) = **32 grams**
And for the **iron/nickel** we have 8.2 x 0.8x(48.5) = **318 grams**
for a total of 350 grams

Problem 2 - Meteorite collectors find and sell samples by the gram. The price of a gram of the Esquel pallasite is about $106. What was the price of the sample in Problem 1?

Answer: 350 grams x ($106 / 1 gram) = **$37,100**.

Rule 1: All digits from 1 to 9 are significant
1.234 grams has 4 significant figures,

Rule 2: All zeros bracketed by numbers are significant
30.07 Liters has 4 significant figures.

Rule 3: Ignore leading 'placeholder' zeros, but not stated trailing zeros
0.0**12** grams has 2 significant figures.
0.**20** meters has 2 significant figures.
0.0**230** liters has 3 significant figures,

Rule 4: Final answer only has as many SFs as the least number used.

3.56 x 1.3456 = 4.79
3 SF x 5 SF = 3 SF

For problems involving pure mathematics, one seldom takes the point of view that some digits in a stated number are more important than others, however in science where numbers represent measurements, not all digits are created equally!

The examples to the left show the four basic rules of how to count 'significant figures'. The basic rule is that you never state more digits in a number than the precision of your measurement. For example, if you can only measure to an accuracy of one meter, you should never state a measurement as 102.56 meters!

Problem 1 - For the numbers below, indicate how many SFs are involved:

A) 450.12 B) 450.120 C) 1.234×10^{-11} D) 0.00234500 E) 0.002345

Problem 2 - When multiplying or dividing numbers, the answer must have the same number of significant figures. Pure numbers such as '5' can be written with as many decimal places as needed. Evaluate the following expressions to the correct number of SF.

A) M = 3.275 x 1.3 grams B) T = 5 (356.19)/8 + 24.0347 degrees

C) S = π $(1.0850)^2$ meters^2 D) Y = [1.03 x 3.98720] / 1.1087 kilograms

Problem 3 - The Rydberg Constant is an import number in atomic physics and is given by the formula

$$R_\infty = \frac{2\pi^2 m e^4}{c h^3}$$

Evaluate the formula to the correct number of SF where:

Electron mass: $m = 9.10956 \times 10^{-28}$ grams
Speed of light: $c = 2.997925 \times 10^{10}$ cm/sec
Electron charge: $e = 4.80325 \times 10^{-10}$ ESU
Planck's Constant: $h = 6.62620 \times 10^{-27}$ erg sec

Problem 1 - For the numbers below, indicate how many SFs are involved:

A) 5 SF   B) 6 SF   C) 4 SF   D) 6 SF   E) 4 SF

Problem 2 - When multiplying or dividing numbers, the answer must have the same number of significant figures. Pure numbers such as '5' can be written with as many decimal places as needed. Evaluate the following expressions to the correct number of SF.

A) M = 3.3 grams   B) T = 246.65 degrees
C) S = 3.6984 meters^2 D) Y = 3.70 kilograms

Problem 3 - The Rydberg Constant is an import number in atomic physics and is given by the formula

$$R_\infty = \frac{2\pi^2 m e^4}{c h^3}$$

Evaluate the formula to the correct number of SF where:

Electron mass: $m = 9.10956 \times 10^{-28}$ grams
Speed of light: $c = 2.997925 \times 10^{10}$ cm/sec
Electron charge: $e = 4.80325 \times 10^{-10}$ ESU
Planck's Constant: $h = 6.62620 \times 10^{-27}$ erg sec

Answer: First evaluate the function using only the exponents:

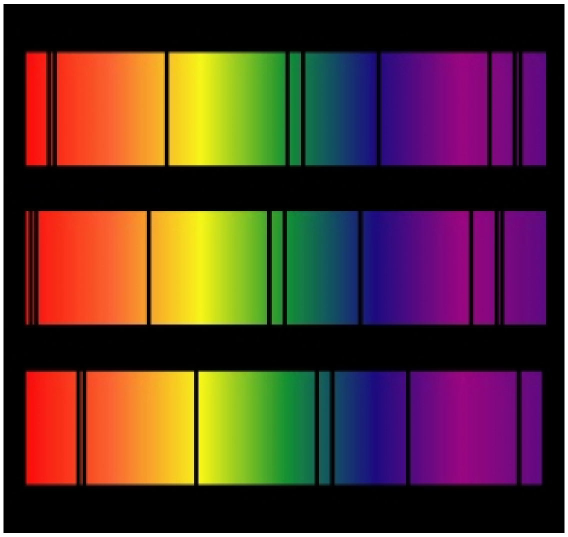
$R = (10^{-28})(10^{-10})^4 / (10^{10})(10^{-27})^3 = 10^3$. Now evaluate all the measured constants:

$$R = \frac{2\,(3.141592654)^2\,(9.10956)(4.80325)^4}{(2.997925)(6.62620)^3}$$

Using a calculator, one obtains the 'answer' R = 109.7371112 which has to be combined with the previous calculation for the exponents of 10^3 to get

$R = 109.7371112 \times 10^3$ or in scientific notation $R = 1.097371112 \times 10^5$

The smallest number of SFs occurs for the constants m, e and h which have 6 SF, so we round the answer to $\mathbf{R = 1.09737 \times 10^5}$.



Top: Normal 'dark' spectral line positions at rest.
Middle: Source moving away from observer.
Bottom: Source moving towards observer.

We have all heard the sound of an ambulance siren as it passes-by. The high-pitch as it approaches is replaced by a low-pitch as it passes by. This is an example of the Doppler Shift, which is a phenomenon found in many astronomical settings as well, except that instead of the frequency of sound waves, it's the frequency of light waves that is affected.

The figure to the left shows how the wavelength of various atomic spectral lines, normally found in the top locations, are shifted to the red (long wavelength; lower pitch) for a receding source, and to the blue (short wavelength; higher pitch) for an approaching source of light.

For very distant galaxies, the effects of curved space causes the wavelengths of light to be increasingly red-shifted as the distance to Earth increases. This is a Doppler-like effect, but it has nothing to do with the speed of the galaxy or star, but on the changing geometry of space over cosmological distances.

Just as in the Doppler Effect, where we measure the size of the Doppler 'red'-shift in terms of the speed of the object emitting the sound waves, for distant galaxies we measure their redshifts in terms of the cosmological factor, z. Close-by galaxies have z-values much less than 1.0, but very distant galaxies can have z=6 or higher.

Observing distant galaxies is a challenge because the wavelengths where most of the light from the galaxy are emitted, are shifted from visible wavelengths near 500 nanometers (0.5 microns) to much longer wavelengths. This actually makes distant galaxies very dim in the visible spectrum, but very bright at longer infrared wavelengths. For example, for redshifts of z = 3, the maximum light from a normal galaxy is shifted to a wavelength of L = 0.5 microns x (1+z) = 2.0 microns!

Problem 1 - The Webb Space Telescope, Mid-Infrared Instrument (MIR) can detect galaxies between wavelengths of 5.0 and 25.0 microns. Over what redshift interval can it detect normal galaxies like our Milky Way?

Problem 2 - An astronomer wants to study an event called Reionization, which occurred between 5.0 < z < 7.0. What wavelength range does this correspond to in normal galaxies?

Problem 3 - The NIRcam is sensitive to radiation between 0.6-5.0 microns, the MIR instrument range is 5.0 to 25.0 microns, and the Fine Guidance Sensor-Tunable Filter Camera detects light between 1 to 5 microns. Which instruments can study the Reionization event in normal galaxies?

Problem 1 - The Webb Space Telescope, Mid-Infrared Instrument (MIR) can detect galaxies between wavelengths of 5.0 and 25.0 microns. Over what redshift interval can it detect normal galaxies like our Milky Way?

Answer: L = 5.0, so 5.0 = 0.5 x (1+z) and so z = 9.0
L = 25, so 25 = 0.5 x (1+z), and so z = 49.0
The redshift interval is then z = [9,49]

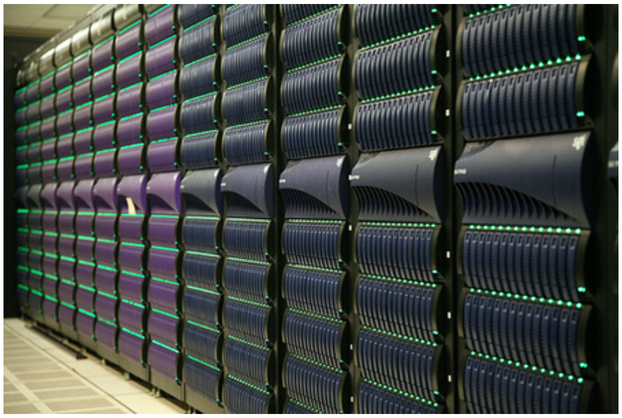
Problem 2 - An astronomer wants to study an event called Reionization, which occurred between 5.0 < z < 7.0. What wavelength range does this correspond to in normal galaxies?

Answer: Z=5.0 corresponds to L = 0.5 x (1 + 5) so L = **3.0 microns**
Z = 7.0 L = 0.5 x (1 + 7) = **4.0 microns**.

Problem 3 - The NIRcam is sensitive to radiation between 0.6-5.0 microns, the MIR instrument range is 5.0 to 25.0 microns, and the Fine Guidance Sensor-Tunable Filter Camera detects light between 1 to 5 microns. Which instruments can study the Reionization event in normal galaxies?

Answer: The Reionization event can be detected between 3.0 and 4.0 microns ,which is within the observing ranges of both the NIRcam and the FGS-TFC instruments, but not the MIR camera.

Note: The Reionization event is believed to be the result of an intense period of supernova activity in millions of galaxies across the universe, which caused intense amounts of ultraviolet radiation sufficient to ionize hydrogen gas clouds. The first time the universe was ionized to this degree ended about 400,000 years after the Big Bang when the universe cooled down during its expansion. The term 're' ionization indicates that the expanding cooled hydrogen gas clouds once again became ionized about 100 million years after the Big Bang in a second ionization event.


A supercomputer consists of millions of identical 'processors' that work in parallel to perform thousands of trillions of calculations each second.

A supercomputer can perform a large number of calculations very fast compared to ordinary computers such as your desk or lap-top computer. This makes them very important for mathematically modeling complex systems such as the movement of millions of particles though space, the folding of complicated proteins, or the sequencing of DNA.

Let's have a look at one simple application to see how this works!

Imagine that you are following the movement of three stars through space, and that their distance from our sun in light years is given by the following three equations, where T is the elapsed time in millions of years:

$$d_1 = 3T + 2$$
$$d_2 = 4T - 2$$
$$d_3 = 8T + 3$$

Problem 1 - Using a calculator, A) evaluate by hand the values for d_1, d_2 and d_3 for the three time steps T= 1, 2 and 3. B) How long did it take you, in seconds, to perform the calculations? C) How many key strokes were involved? D) How many mathematical operations were involved?

Problem 2 - Suppose you wanted to follow the movement of 300 stars in a star cluster for three time steps. A) How many equations would be involved? B) How many keystrokes would you have to execute? C) How many mathematical operation would you have to perform? D) How long would the calculation take for three time steps? D) If N is the number of particles in your simulation, what is the general formula that gives the computation time in seconds for N particles?

Problem 3 - Suppose that you wrote a computer program to perform the calculations on a high-end laptop computer using a programming language such as FORTRAN, or C. With a 'clock speed' of 1 billion operations per second, how long would the computer take to evaluate the three particle positions after three time steps?

Problem 4 - A supercomputer not only performs an individual mathematical operation very fast, (3 million billion 'floating point operations per second' which is called 3 petaFLOPS) but it can perform many operations simultaneously by using parallel processing. In the example above, for each time step, suppose the computer might be built to use three 'processors', (one for each particle), to evaluate all three values for d in one step! How long would it take a 3 petaFLOP supercomputer to calculate three time steps for 10 million particles?

Problem 1 - Using a calculator, A) evaluate by hand the values for d_1, d_2 and d_3 for the three time steps T= 1, 2 and 3. B) How long did it take you, in seconds, to perform the calculations? C) How many key strokes were involved? D) How many mathematical operations were involved?

Answer: A) d1 = 5, 8 and 11. d2 = 2, 6 and 10; d3 = 11, 19, 27.

B) It takes about 5 seconds per evaluation, or **45 seconds for all three particles for 3 time intervals**, not including the time to write down the answer.

C) For one value: 3 x 1 + 1 = each entry is a key stroke. There are 3 timesteps x 3 particles x 6 keystrokes = **54 keystrokes** total.

D) There are only 2 operations per evaluation 'x' and '+' or '-' so a total of **18 operations.**

Problem 2 - Suppose you wanted to follow the movement of 300 stars in a star cluster for three timesteps. A) How many equations would be involved? B) How many keystrokes would you have to execute? C) How many mathematical operation would you have to perform? D) How long would the calculation take for three time steps? D) If N is the number of particles in your simulation, what is the general formula that gives the computation time in seconds for N particles? E) How long, in hours ,will it take to calculate the position of 300 particles for three timesteps?

Answer: A) There would be **300 equations** for ; d_1, d_2, d_3, … d_{300}.

B) 6 keystrokes x 3 timesteps x 300 equations = **5400 keystrokes**

C) 2 operations x 3 timesteps x 300 equations = **1800 operations**

D) From your answer to Problem 1A it takes about 5 seconds to calculate one time step for one particle, so T = 3 timesteps x 5 seconds x N or **T = 15 N seconds**.

E) T = 15 x 300 = 4500 seconds or **1.25 hours**.

Problem 3 - Suppose that you wrote a computer program to perform the calculations on a high-end laptop computer using a programming language such as FORTRAN, or C. With a 'clock speed' of 1 billion operations per second, how long would the computer take to evaluate 10 million particle positions after three time steps?

Answer: T = 3 timesteps x (2 operations/timestep) x (1 second/1 billion operations) x 10 million particles

= **0.06 seconds**.

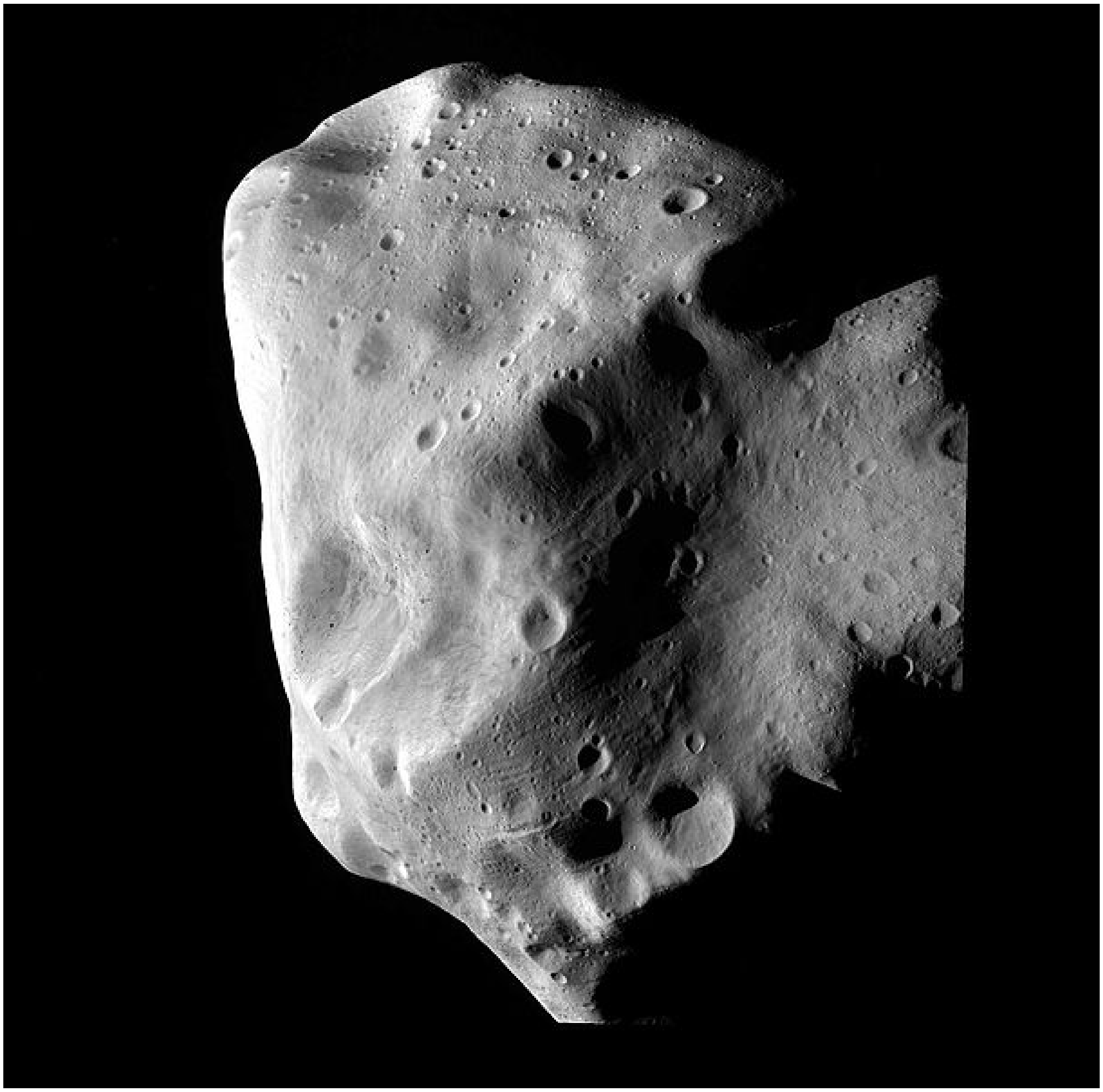
Problem 4 - A supercomputer not only performs an individual mathematical operation very fast, (3 million billion 'floating point operations per second' which is called 3 petaFLOPS) but it can perform many operations simultaneously by using parallel processing. In the example above, for each time step, suppose the computer might be built to use three 'processors', (one for each particle), to evaluate all three values for d in one step! How long would it take a 3 petaFLOP supercomputer to calculate three time steps for 10 million particles?

Answer: The supercomputer would use 10 million 'parallel processors' so that

T = 3 timesteps x (2 operations/timestep) x (1 second/3 x 10^{12} operations)

= **2 x 10^{-12} seconds**.

A Flyby of Asteroid Lutetia by Rosetta



Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

Problem 2 - At a distance of 3,162 km, A) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth (0.5 degrees), how much larger was Lutetia?

Problem 3 - If the Rosetta spacecraft traveled at a speed of 15 km/s on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Image credit: ESA 2010 MPS for OSIRIS Team MPS/UPD/LAM/IAA/RSSD/INTA/UPM/DASP/IDA. European Space Agency's Rosetta spacecraft, with NASA instruments aboard, flew past asteroid Lutetia on Saturday, July 10, 2010. Asteroid diameter about 130 km. This view is from a distance of 3,162 Km. The probe spent several hours shooting images of the irregular shaped space rock, circling more than 450 million km (280 million miles) out from the sun. The space agency says its OSIRIS camera was able to capture detail down to just a few dozen meters.

Problem 1 - What is the scale of this image in meters/mm if the width of the image is 130 kilometers?

Answer: 130 km / 152 mm = **855 meters/mm**.

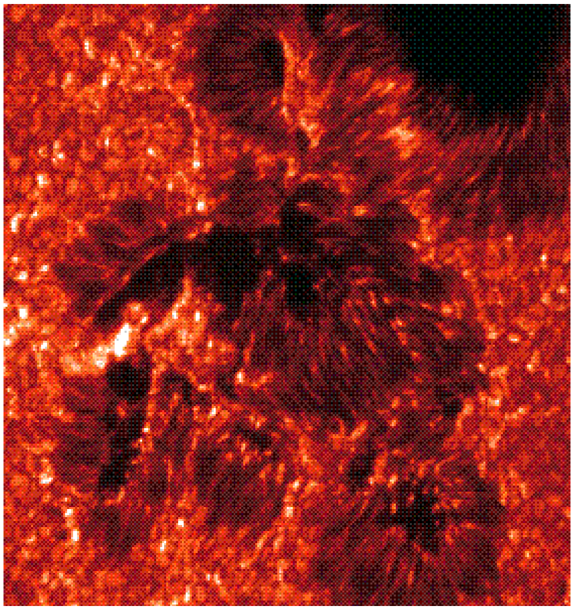
Problem 2 - At a distance of 3,162 km, A) what was the angular diameter of this asteroid? B) Compared to the full moon viewed from Earth (0.5 degrees), how much larger was Lutetia?

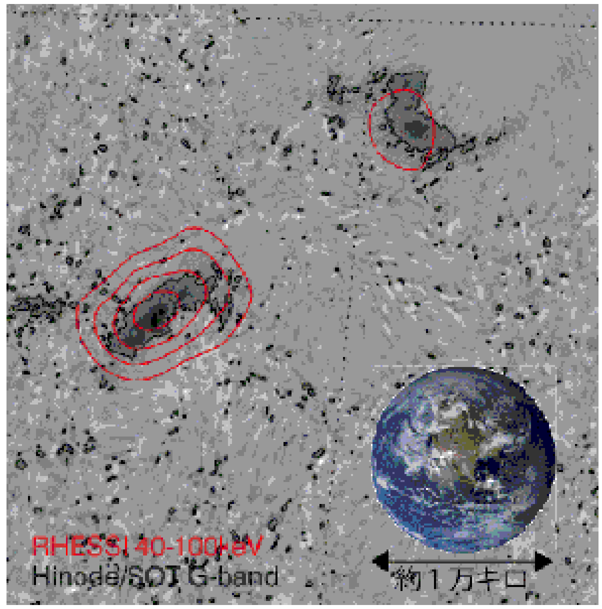
Answer: A) $\mathrm{Tan}(\theta) = 130/3162$ so θ = **2.3 degrees**.
B) Asteroid is 2.3/0.5 = **4.6 times** the diameter of the full moon.

Problem 3 - If the spacecraft traveled at a speed of 15 km/s on a path exactly tangent to the line connecting the center of the asteroid and spacecraft at closest approach, how long after closest approach would the asteroid have an angular diameter equal to the full moon?

Answer: To have an angular diameter of 0.5 degrees ,the spacecraft has to be 4.6 times farther away than at closest approach, or a distance of 4.6 x 3,162 = 14,545 km. From the Pythagorean theorem, the distance from the closest approach point is just d $= (14545^2 - 3162^2)^{1/2}$ = 14,197 km. To travel this distance takes T = 14197/15 = **946 seconds or 15.8 minutes.**

Hinode Discovers the Origin of White Light Flares





White light emissions were observed by the Hinode Solar Optical Telescope during an intense X-class flare on Dec. 14, 2006. The RHESSI satellite simultaneously recorded X-ray emissions (contours in right image) which are an indicator of electrons being boosted in energy during the flare event. The X-ray and optical emissions came from the same locations on the solar surface where electrons were being accelerated to over 40% the speed of light! This means that high-energy electrons are creating the white light flashes that are often seen in the most intense solar flares.

Problem 1 - From the size of Earth in the photo, the main flaring region has a length of 13,000 km and a width of 4,000 km. Assuming that it has a cylindrical shape, what is the volume of this region in cubic meters?

Problem 2 - The amount of magnetic energy stored in a volume of space of V cubic meters is given by the formula $E = 40,000\ B^2\ V$ where B is the magnetic field strength in units of Gauss, and where E is in units of ergs. What is the stored energy in this flaring region if B = 150 Gauss?

Problem 3 - A 1-megaton hydrogen bomb produces an energy release of about 4×10^{22} ergs. How much energy was released by this flaring region if all of the stored magnetic energy was involved in the flare?

Problem 1 - From the size of Earth in the photo, the main flaring region has a length of 13,000 km and a width of 4,000 km. Assuming that it has a cylindrical shape, what is the volume of this region in cubic meters?

Answer: $V = \pi R^2 H$ so $V = 3.14 (4,000,000)^2 (13,000,000)$ = **1.6×10^{20} meters^3**

Problem 2 - The amount of magnetic energy stored in a volume of space of V cubic meters is given by the formula $E = 40,000 B^2 V$ where B is the magnetic field strength in units of Gauss, and where E is in units of ergs. What is the stored energy in this flaring region if B = 150 Gauss?

$E = 40,000 (150)^2 (1.6 \times 10^{20})$ = **1.4×10^{29} ergs**

Problem 3 - A 1-megaton hydrogen bomb produces an energy release of about 4×10^{22} ergs. How much energy was released by this flaring region if all of the stored magnetic energy was involved in the flare?

Answer: 1.4×10^{29} ergs **/** 4×10^{22} ergs = **3.5 million one-megaton bombs!**

Period (days)	F	G	K
0-10	11	138	20
11-20	7	53	16
21-30	4	25	6
31-40	2	13	0
41-50	1	7	0
51-60	1	1	0
61-70	0	1	0
71-80	0	1	0
> 81	0	3	2
Total:	26	242	44

On June 16, 2010 the Kepler mission scientists released their first list of stars that showed evidence for planets passing across the faces of their stars. Out of the 156,097 target stars that were available for study, 52,496 were studied during the first 33 days of the mission. Their brightness was recorded every 30 minutes during this time, resulting in over 83 million high-precision measurements.

700 stars had patterns of fading and brightening expected for planet transits. Of these, data were released to the public for 306 of the stars out of a sample of about 88,000 target stars.

The surveyed stars for this study were distributed by spectral class according to F = 8000, G = 55,000 and K = 25,000. For the 306 stars, 43 were K-type, 240 were G-type, and 23 were K-type. Among the 312 transits detected from this sample of 306 stars, the table above gives the number of transits detected for each stellar type along with the period of the transit.

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?

Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?
Answer: Of the 88,000 stars F = 8000/88000 = 9%; G = 55000/88000 = 63% and K=25000/88000 = 28%.

For the 306 stars: F = 43/306 = 14%; G = 240/306 = 78% and K = 23/306 = 8%....so there were significantly fewer transits detected for K-type stars (8%) compared to the general population (28%).

Note: Sampling error accounts for s = $(306)^{1/2}$ = +/-17 stars or a +/- 6% uncertainty which is not enough to account for this difference in the K-type stars.

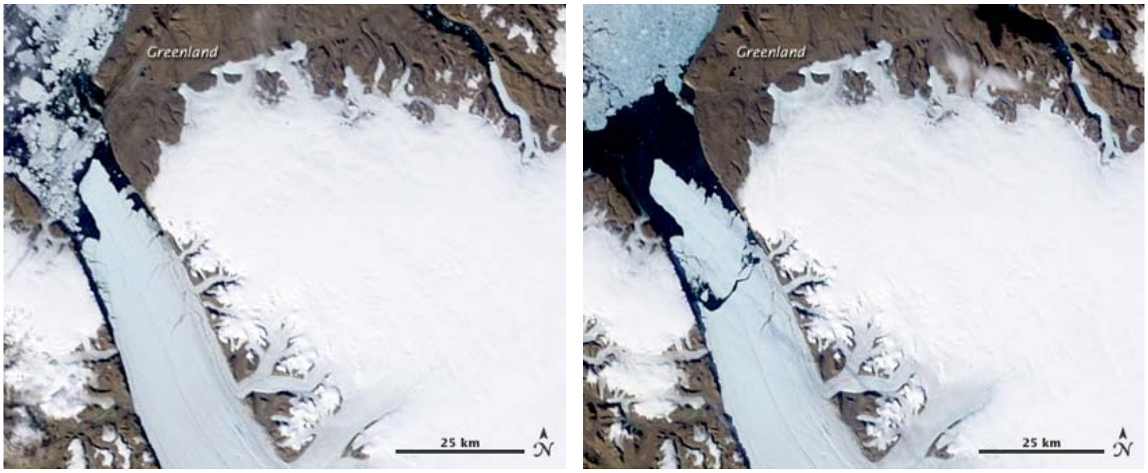
Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Answer: A) T = 88 days/365 days = 0.24 Earth Years, than $D^3 = 0.24^2$, $D^3 = 0.058$, so $D = (0.058)^{1/3}$ and so D = **0.39 times Earth's orbit distance**.
B) The time range is 0.014 to 0.22 Earth Years, and so D is in the range from 0.058 to 0.36 Earth distances. Since Mercury has D = 0.39, in terms of the orbit distance of Mercury, the transiting planets span a range from 0.058/0.39 = **0.15** to 0.36/0.39 = **0.92 Mercury orbits**.

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Answer: The amount of dimming is equal to the ratio of the areas of the planet's disk to the star's disk, so $0.001 = \pi R^2/\pi(500{,}000)^2$ so **R = 15,800 kilometers**, which equals a diameter of 31,600 kilometers. Since Earth's diameter = 13,000 km, the transiting planet is about **2.4 times the diameter of Earth**.

Terra Spies a Major Glacier Break-up



On August 5, 2010, an enormous chunk of ice broke off the Petermann Glacier along the northwestern coast of Greenland. The Moderate Resolution Imaging Spectroradiometer (MODIS) on NASA's Terra satellite captured these natural-color images of Petermann Glacier 18:05 UTC on July 28, 2010 (left) and August 5, 2010 (right) at 17:15 UTC. The Terra image of the Petermann Glacier on August 5 was acquired almost 10 hours after the Aqua satellite observation that first recorded the calving event. By the time Terra took this image, the oblong iceberg had broken free of the glacier and moved a short distance down the fjord.

Problem 1 - From the scale of the two images, what is the approximate surface area of the portion of the glacier that broke-off in A) square kilometers? B) square miles if 1 km = 0.62 miles?

Problem 2 - From the width of the break line in the August image, what was the speed of drift of the glacier fragment in kilometers/hour?

Problem 3 - Assuming that the fragment is 1000 meters thick, what is the total volume of the fragment in cubic meters?

Problem 4 - If one cubic meter of ice = 917 kilograms 1 gallon of water = 3.8 kg, what is the total amount of fresh water, in gallons, that will eventually be added to the ocean after it melts?

Problem 1 - From the scale of the two images, what is the approximate surface area of the portion of the glacier that broke-off in A) square kilometers? B) square miles if 1 km = 0.62 miles?

Answer: The length of the '25 km' line is about 17 mm so the scale is 1.5 km/mm. Students may approximate the area of the glacier as a triangular portion with sides of 10 and 15 mm (15 km and 22 km) with an area of 1/2 (15)(22) = 165 km^2, and a rectangular part that has a long side on the hypotenuse of the triangle with dimensions of 2 mm x 20 mm (3 km x 30 km) and an area of **90 km^2**. The total area is then approximately 255 km2 . B) the area in square miles is 255 km^2x (0.62 mi/km)(0.62 mi/km) = **98 square miles**.

Problem 2 - From the width of the break line in the August image, what was the speed of drift of the glacier fragment in kilometers/hour?
Answer: Students may have difficulty estimating the width of the 'hairline' fracture in the image, but answers near 0.2 mm are acceptable. Students may use a photocopy machine to increase the magnification of this image, and the resulting scale of the image to get a better estimate.

The information says that this image was taken 10 hours after the separation began, so the speed is 0.2 mm x 1.5 km/mm x (1/10 hours) = **30 meters/hour**.

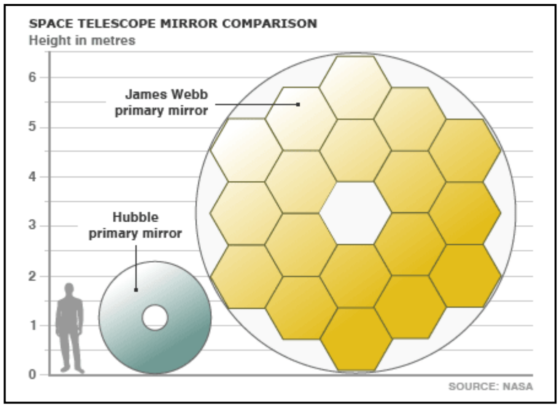
Problem 3 - Assuming that the fragment is 1000 meters thick, what is the total volume of the fragment in cubic meters?

Answer: 255 km^2 x 1 km = 255 km^3 since 1 km = 1000 meyters, we have 255 km^2 x $(1000\ meters/km)^3$ = **255 billion $meters^3$ of ice.**

Problem 4 - If one cubic meter of ice = 917 kilograms 1 gallon of water = 3.8 kg, what is the total amount of fresh water, in gallons, that will eventually be added to the ocean after it melts?

Answer: 255 billion m^3 x (917 kg/1 m^3) x (1 gallon/3.8 kg) = **62 trillion gallons**.

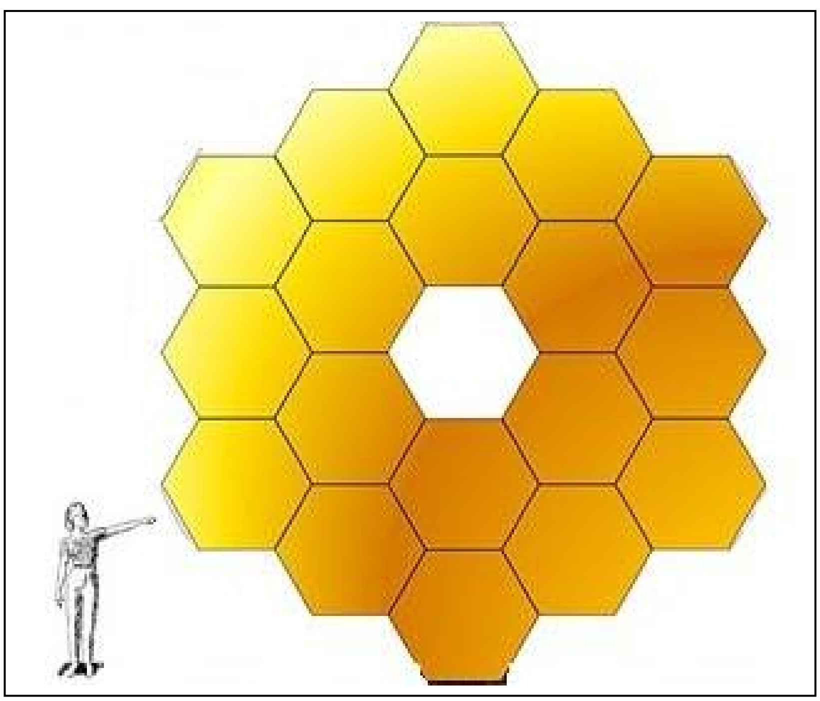
The Hexagonal Tiles in the Webb Telescope Mirror



The James Webb Space Telescope, to be launched by NASA in 2016, is a telescope designed to explore galaxies and stars that formed soon after the Big Bang.

Its unique design features a large mirror that consists of 18 hexagonal tiles; each tile is its own mirror. The 18 mirror tiles work together to form a single large mirror to collect faint star light.

An important feature of a telescope mirror is its surface area. The more surface area a mirror has, the more light it can collect. To make faint stars and galaxies appear bright enough to study in detail, mirrors with large collecting areas are needed.

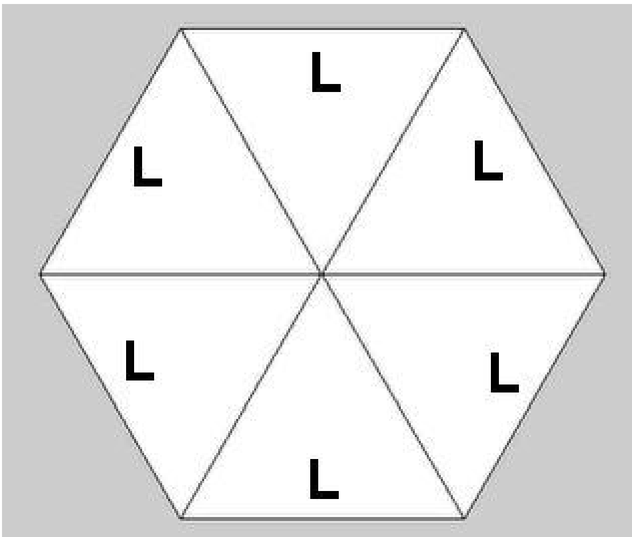


Problem 1 – For a regular hexagon with a side length of L, how many equilateral triangles with a side length of L can you fit into one hexagon?

Problem 2 – From the drawing of the Webb Space Telescope Mirror on the left, how many equilateral triangles are formed by the 18 hexagonal tiles?

Problem 1 – For a regular hexagon with a side length of L, how many equilateral triangles with a side length of L can you fit into one hexagon?

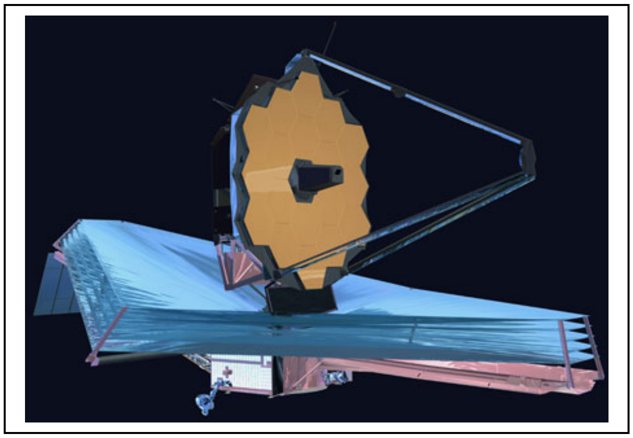
Answer: A total of six as shown below.



Problem 2 – From the drawing of the Webb Space Telescope Mirror on the left, how many equilateral triangles are formed by the 18 hexagonal tiles?

Answer. Students may draw the triangles inside each of the 18 hexagons and then count them, or can recognize that (6 triangles in each hexagon) x 18 hexagons = **108 equalateral triangles.**

Super-sizing the Webb Space Telescope Mirror

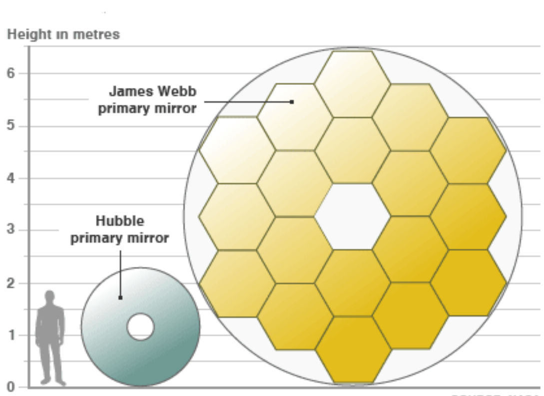


The James Webb Space Telescope, to be launched by NASA in 2016, is a telescope designed to explore galaxies and stars that formed soon after the Big Bang. Its unique design features a large mirror that consists of 18 hexagonal tiles; each tile is its own mirror.

Many of the largest telescope mirrors now being built for ground-based observatories use the hexagonal 'segmented' design. A single 1-meter wide hexagon, replicated dozens of times, is a lot easier to make than a single large mirror!

Suppose that in the problems below, the length of a side of the hexagon is L = 0.76 meters. New mirror designs are created from the Webb Space Telescope design by adding enough mirror tiles to complete a new outer ring. For example, the Webb Space Telescope mirror consists of two complete rings of hexagonal tiles.

Problem 1 - Using the sketch to the left as a guide, how many tiles will be in the assembled mirror if 1, 2 or 3 additional rings of hexagonal tiles are added?



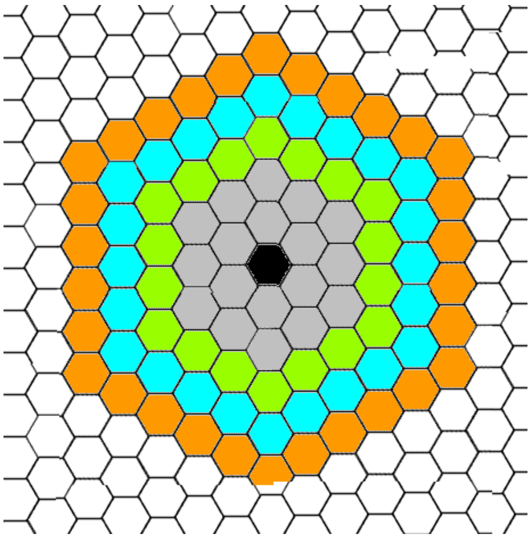
Problem 2 - What is the total area of each mirror design if the area of a single hexagon is

$$A = \frac{3}{2}\sqrt{3}L^2$$

Compared to the Webb Space Telescope design of 18 tiles, by what factor do the three new mirror designs exceed the Webb Space Telescope collecting area?

Problem 1 - Using the sketch to the left as a guide, how many tiles will be in the assembled mirror if 1, 2 or 3 additional rings of hexagonal tiles are added?

Answer: From the shaded rings indicated below: One additional ring (green) = 18 + 18 = **36 tiles**. Two rings = 36 + 24 = **60 tiles**. Three rings = 60 + 30 = **90 tiles**.



Problem 2 - What is the total area of each mirror design if the area of a single hexagon is

$$A = \frac{3}{2}\sqrt{3}L^2$$

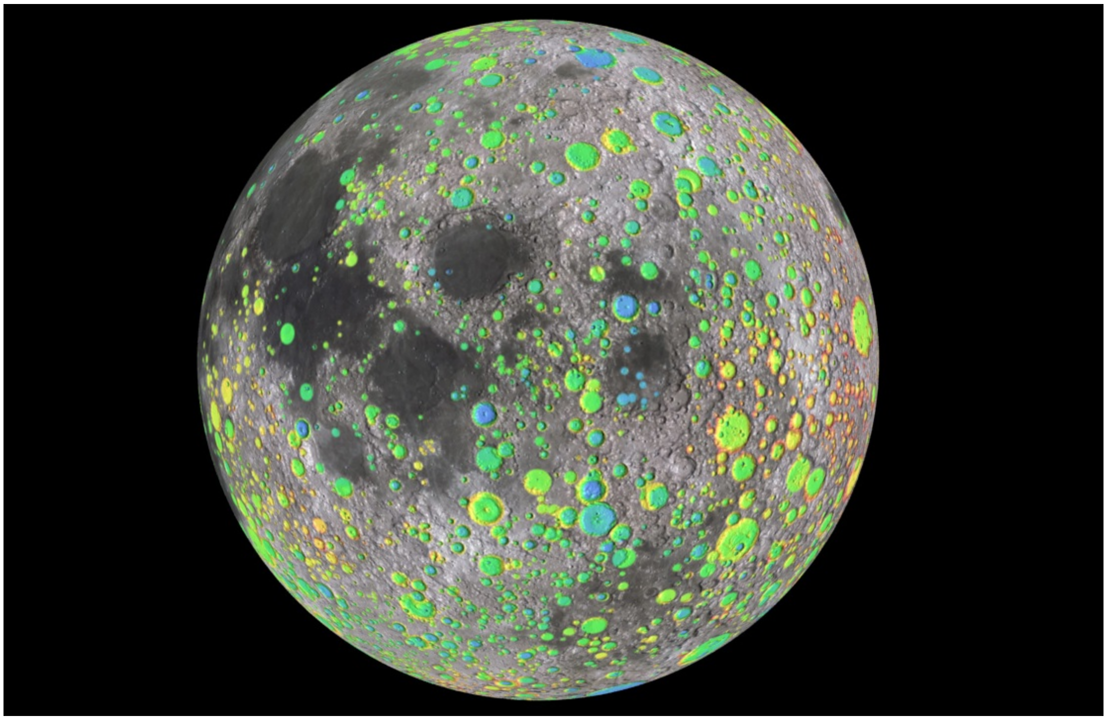
Compared to the Webb Space Telescope design of 18 tiles, by what factor do the three new mirror designs exceed the Webb Space Telescope collecting area?

Answer: The hexagon area is A = 1.5 (1.732) $(0.76)^2$ = 1.5 meters^2.
Webb Space Telescope Mirror: 18 tiles, Area = 18 x 1.5 = 27 meters^2.

One additional ring: Area = 36 x 1.5 = **54 meters^2.**
Two additional rings: Area= 60 x 1.5 = **90 meters^2.**
Three additional rings: Area = 90 x 1.5 = **135 meters^2**.

The areas increase by factors of **2.0, 3.3 and 5.0 times** the area of the Webb Space Telescope design.

LRO Determines Lunar Cratering History



The Lunar Reconnaissance Orbiter used millions of measurements of the lunar surface to establish the history of cratering on the surface.

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?

Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Problem 1 - The diameter of the moon is 3,400 kilometers. With a millimeter ruler determine the scale of the image above in kilometers/mm.
Answer: The image diameter is 90 millimeters so the scale is 3400 km/90 mm = **38 km/mm.**

Problem 2 - How many craters can you count that are larger than 70 kilometers in diameter?
Answer: 70 km equals 2 millimeters at this image scale. There are about **56 craters larger than 2 mm on the image. Students answers may vary from 40 to 60.**

Problem 3 - If the large impacts had happened randomly over the surface of the moon, about how many would you have expected to find in the 20% of the surface covered by the maria?
Answer: You would expect to find about 0.2 x 56 = **11 craters larger than 70 km.**

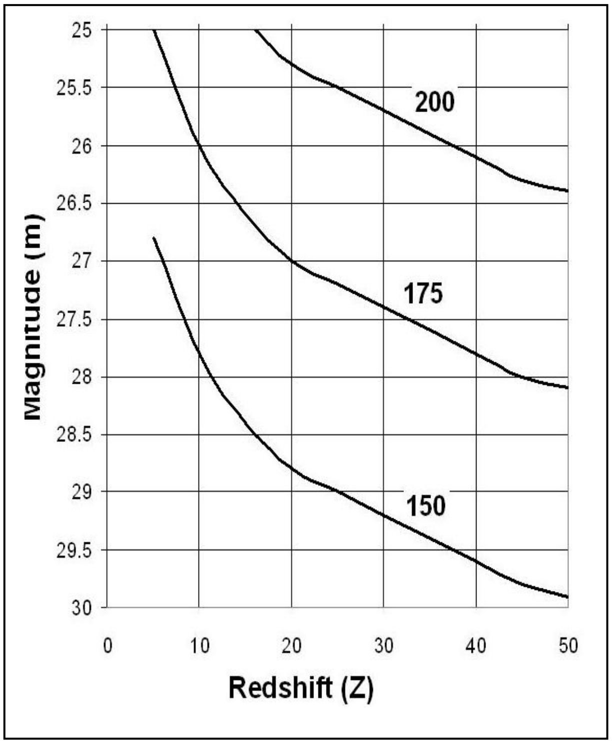
Problem 4 - From your answer to Problem 3, what can you conclude about the time that the impacts occurred compared to the time when the maria formed?

Answer: **The lunar highlands were present first and were impacted by asteroids until just before the maria formed. There are few/no craters in the maria regions larger than 70 km, so the maria formed after the episode of large impactors ended.**

For more information, see the LRO press release at:

"LRO Exposes Moon's Complex, Turbulent Youth"

http://www.nasa.gov/mission_pages/LRO/news/turbulent-youth.html



When the Webb Space Telescope begins operation in ca 2016, one of its first scientific goals will be to detect, if they exist, the most distant supernova in the universe.

Astronomers predict that soon after the Big Bang, stars more massive than our own sun were commonly formed - but exploded as supernova within a few million years after birth. This is a crucial phase in the evolution of our universe. Without them, the elements that now form planets and living systems would not exist.

These supernovae, surrounded by dense clouds of dust, will be very faint, and can only be observed at infrared wavelengths, and with very large telescopes like the Webb Space Telescope.

The graph to the left shows the predicted brightness of these massive 'Population III' supernovae for stars with different masses: 200 (top), 175 (middle) and 150 (bottom) times the mass of our sun.

The vertical scale is in terms of the stellar magnitude scale, which indicates the logarithmic brightness of a star. On this scale, our sun has a brightness of about -26.0. The faintest star visible with the human eye is about +6.0. Each magnitude step represents a factor of 2.512 in brightness change so that 5 magnitudes of change is exactly a factor of 100.0.

The horizontal scale gives the distance to the supernova in terms of its cosmological redshift. On this scale, the distance to the nearest galaxy, Andromeda, is about z = 0.001 and the most distant known quasar is at about z = 3.0. It is expected that infant stars will be detected by their light at a distance of z between 20 - 50, which corresponds to a time about 100 million years after the Big Bang. At a distance of z > 1000 the universe was opaque to starlight when the universe was less than 300,000 years old.

Problem 1 - For a 10,000-second exposure, the NIR Camera on the Webb Space Telescope can detect objects brighter than a magnitude of +29.0. Out to what redshift will the NIR camera be able to see supernova from stars about 150 times the mass of our sun?

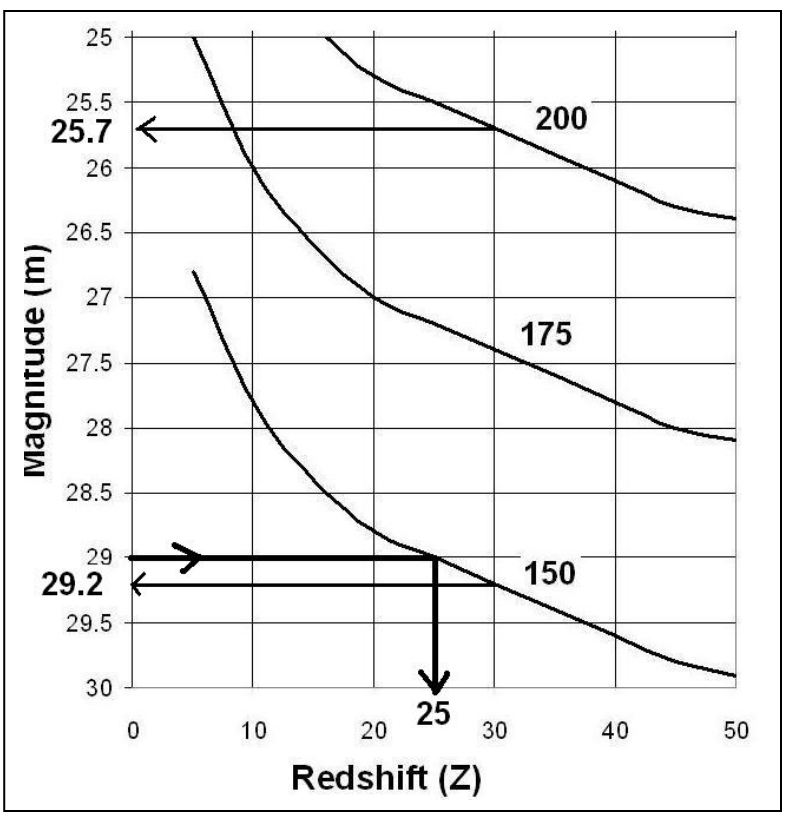
Problem 2 - At a redshift of z = 30, by what factor is the supernova of a star with a mass of 200 times the sun, brighter than the supernova of a star with a mass of 150 times the sun?

Problem 1 - For a 10,000-second exposure, the NIR Camera on the Webb Space Telescope can detect objects brighter than a magnitude of +29.0. Out to what redshift will it be able to see supernova from stars about 150 times the mass of our sun?
Answer: See graph below: **z = 25** is where the limit curve of m = +29.0 intersects the predicted supernova magnitude curve at this mass.

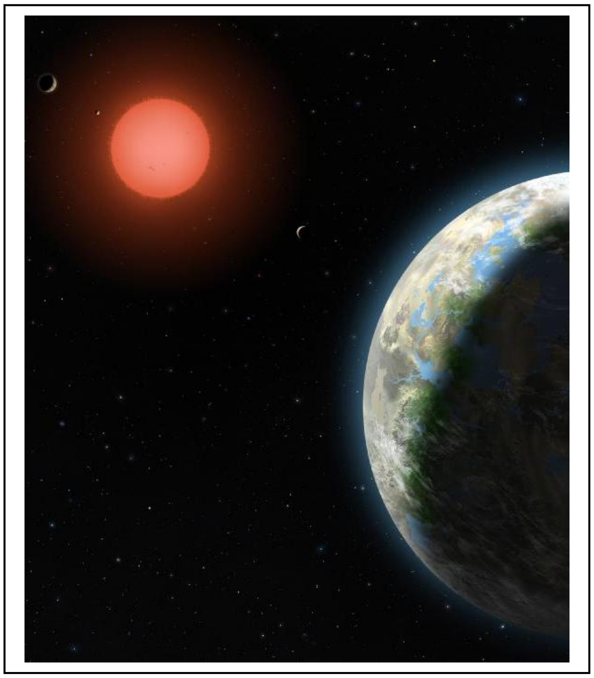
Problem 2 - At a redshift of z = 30, by what factor is the supernova of a star with a mass of 200 times the sun, brighter than the supernova of a star with a mass of 150 times the sun?

Answer: According to the graph below, at a redshift of Z = 30, the curve for a 150 solar mass star has a value of m = +29.2, and for a 200 solar mass star at the same Z we have m = +25.7. The difference in brightness is 29.2 - 25.7 = + 3.5. The brightness factor is between
(2.512)(2.512)(2.512) = 16 times and
(2.512)(2.512)(2.512)(2.512) = 40 times. (The actual factor, is 25 times)

At this redshift, the supernova from a 200 solar mass star is between 16 and 40 times brighter than the supernova from a 150 solar mass star.



The Earth-like Planet Gliese 581g



Professors Steven Vogt at UC Santa Cruz, and Paul Butler of the Carnegie Institution have just announced the discovery of a new planet orbiting the nearby red dwarf star Gliese 518. The star is located 20 light years from Earth in the constellation Libra. The planet joins five others in this crowded planetary system, and has a mass about three to four times Earth, making it in all likelihood a rocky planet, rather than a gas giant. The planet is tidally locked to its star which means that during its 37 day orbit, it always shows the same face to the star so that one hemisphere is always in daylight while the other is in permanent nighttime.

One of the most important aspects to new planets is whether they are in a distance zone where water can remain a liquid on the planets surface. The Habitable Zone (HZ) location around a star depends on the amount of light energy that the star produces. For the Sun, the HZ extends from about the orbit of Venus to the orbit of Mars. For stars that emit less energy, the HZ will be much closer to the star. Once an astronomer knows what kind of star a planet orbits, they can calculate over what distances the HZ will exist.

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system?

Problem 2 - One Astronomical Unit (AU) is the distance between Earth and the Sun (150 million kilometers). Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table below:

Planet	Discovery Year	Distance (AU)	Period (days)	Diameter (km)
Gliese 581 b	2005	0.04	5.4	50,000
Gliese 581 c	2007	0.07	13.0	20,000
Gliese 581 d	2007	0.22	66.8	25,000
Gliese 581 e	2009	0.03	3.1	15,000
Gliese 581 f	2010	0.76	433	25,000
Gliese 581 g	2010	0.15	36.6	20,000

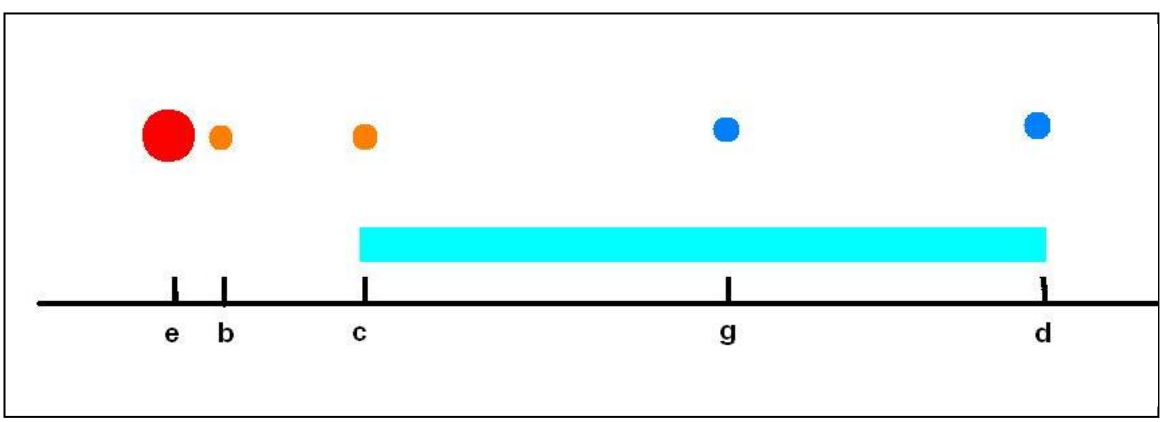
Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model diagram, shade-in the range of distances where the HZ exists for the Gliese 581 planetary system. Why do you think astronomers are excited about Gliese 581g?

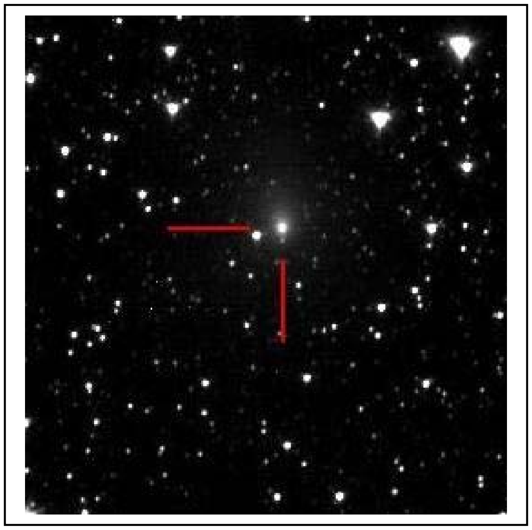
Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system? Answer: **According to the order of discovery date.** Note: Gliese 581 A is the designation given to the star itself.

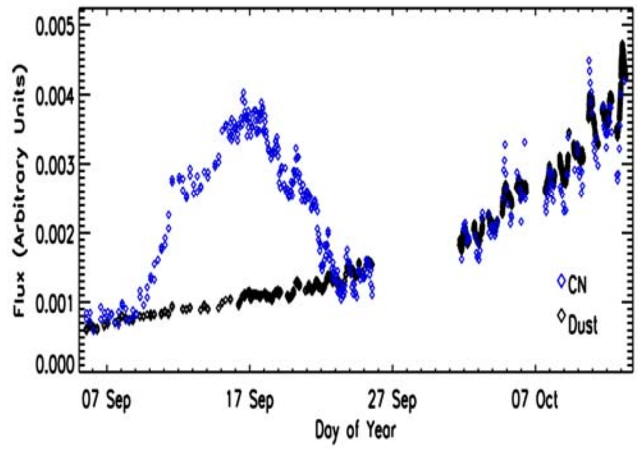
Problem 2 - Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table. Answer: **The table below gives the dimensions on the scaled diagram. See figure below for an approximate appearance. On the scale of the figure below, Gliese 581f would be located about 32 centimeters to the right of Gliese 581d.**

Planet	Discovery Year	Distance (cm)	Period (days)	Diameter (mm)
Gliese 581 b	2005	4	5.4	10
Gliese 581 c	2007	7	13.0	4
Gliese 581 d	2007	22	66.8	5
Gliese 581 e	2009	3	3.1	3
Gliese 581 f	2010	76	433	5
Gliese 581 g	2010	15	36.6	4

Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model, shade-in the range of distances where the HZ exists. Why do you think astronomers are excited about Gliese 581g? Answer: **See the bar spanning the given distances. Note that Gliese c, g and d are located in the HZ of Gliese 581. Because Gliese 581 g is located near the center of this zone and is very likely to be warm enough for there to be liquid water, which is an essential ingredient for life. Gliese 581c may be too hot and Gliese 581 d may be too cold.**







On September 25, 2010 at a distance of 41 million km, the spacecraft Deep Impact took this image of the comet Hartley 2 about 40 days before closest approach on November 4. The graph to the right indicates the brightness of the comet as measured by the spacecraft between 5 September and 13 October. There are two curves plotted, one that depicts dust (black) released from the nucleus and one taken with the cyanogen gas filter, sensitive to both the dust continuum and CN gas (blue), The gap in data from 25 September through 1 October was due to a scheduled break in the observations for calibration observations.

Problem 1 - From the graph, A) how long did the ejection burst of cyanogen gas last, B) on what date did it reach its maximum brightness, and C) in terms of its flux, by what factor did the burst increase the brightness of the comet compared to its dust intensity?

Problem 2 - The distance to the comet is given by the table below:

Date	Day Number	Distance (km)
September 5	0	60 million
September 20	15	46 million
September 25	20	41 million
September 29	24	37 million
October 27	52	9 million

Write a linear equation that gives the distance to the comet in terms of the day number. What is the average speed of the comet in millions of km/day?

Problem 3 - Assume that the brightness of the comet follows the inverse-square law so that if you halve the distance to the comet, its brightness will be 4-times greater. If the comet's brightness was B = 0.0006 units on September 5, and the physical conditions in the comet did not change, what would you predict its brightness would be on October 13?

Problem 1 - From the graph, A) how long did the ejection burst of cyanogen gas last, B) on what date did it reach its maximum brightness, and C) in terms of its flux, by what factor did the burst increase the brightness of the comet compared to its dust intensity?

Answer: A) The ejection lasted between September 9 and 23 for a total of **14 days**.
B) The maximum brightness occurred on about **September 17th**.
C) The dust brightness on Sept 17 was 0.0012 .The total brightness was 0.0039, so the cyanogen outburst increased the comet's brightness by a factor of 0.0039/0.0012 = **3.25 times**.

Problem 2 - The distance to the comet is given by the table below:

Date	Day Number	Distance (km)
September 5	0	60 million
September 20	15	46 million
September 25	20	41 million
September 29	24	37 million
October 27	52	9 million

Write a linear equation that gives the distance to the comet in terms of the day number. What is the average speed of the comet in millions of km/day?

Answer: From the tabulated data, the slope over the time interval is m = (9-60)/(52) = -1.0, so the linear equation would be **D(t) = -1.0(T) + 60** where D(t) is in millions of kilometers and T is the number of days since September 5. The average approach speed is **1 million km/day**.

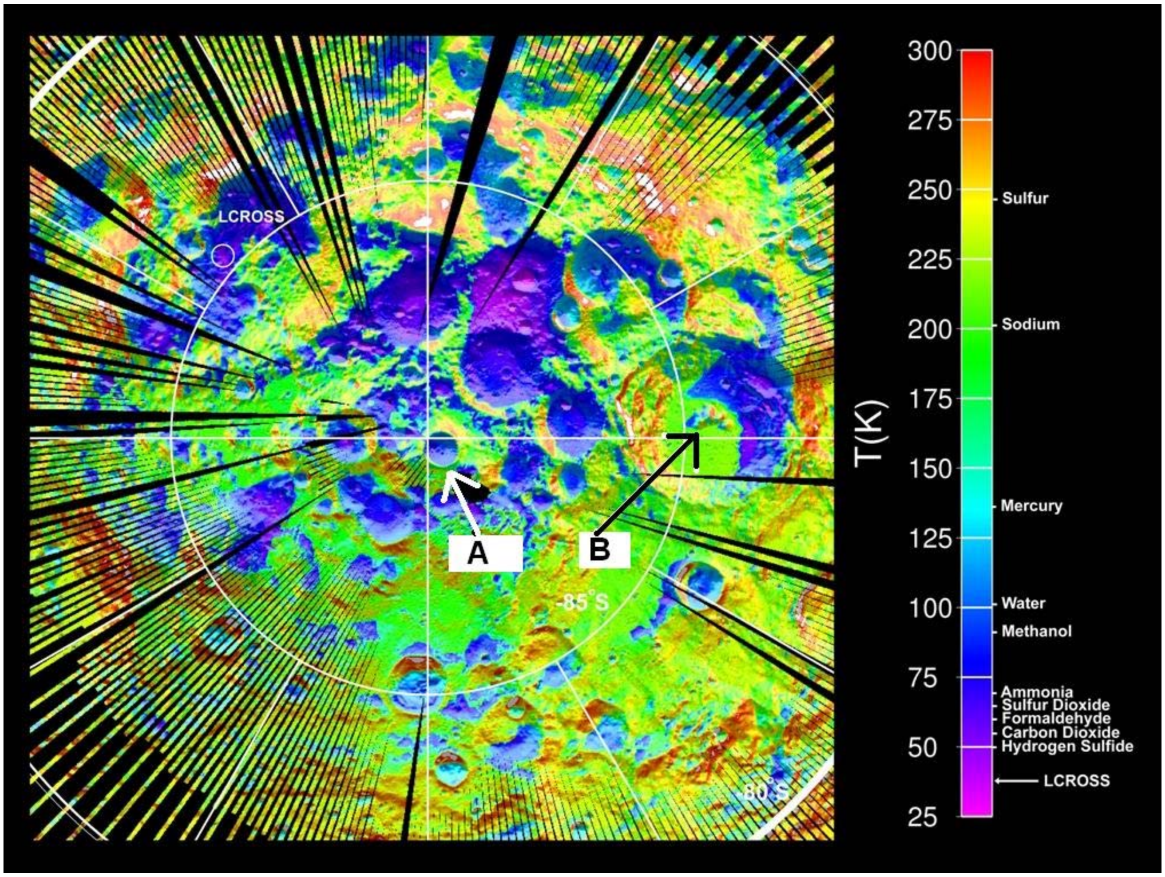
Problem 3 - Assume that the brightness of the comet follows the inverse-square law so that if you half the distance to the comet, its brightness will be 4-times greater. If the comet's brightness was B = 0.0006 units on September 5, and the physical conditions in the comet did not change, what would you predict its brightness would be on October 13?

Answer: October 13 occurs at T = 38 days, so its distance is D = -1.0(38) + 60 = 22 million km. On September 5 its distance was 60 million km, so the factor of its distance change is 38/60 = 0.63 .The comet's brightness from the inverse-square law is just

$$B = 0.0006\frac{1}{(0.63)^2}$$

so the brightness on October 13 is just **B=0.0015 flux units**.

Note: Students will notice from the light curve graph that the actual comet brightness on October 13 was B = 0.0042 units because the comet ejected a cloud of dust, which increased its brightness significantly - by 0.0042/0.0015 = 2.8 times its normal brightness without the dust and cyanogen eruption.



The Lunar Reconnaissance Orbiter (LRO) has recently created the first surface temperature map of the south polar region of the moon using date taken between September and October, 2009 when south polar temperatures were close to their annual <u>maximum</u> values. The colorized map shows the locations of several intensely cold impact craters that are potential cold traps for water ice as well as a range of other icy compounds commonly observed in comets. The approximate maximum temperatures at which these compounds would be frozen in place for more than a billion years is shown on the scale to the right. The LCROSS spacecraft was targeted to impact one of the coldest of these craters, and many of these compounds were observed in the ejecta plume. *(Courtesy: UCLA/NASA/JPL)*

Problem 1 - The width of this map is 500 km. What are the diameters of Crater A (Shackleton) and Crater B (Amundsen) in kilometers?

Problem 2 - In which colored areas might an astronaut expect to find conditions cold enough to recover all of the elements and molecules indicated in the vertical temperature scale to the right?

Problem 3 - The Shackleton Crater (Crater A) is cold enough to trap water and methanol. From Problem 1, and assuming that the thickness of the water deposit is 100 meters, and occupies 10% of the volume of the circular crater, how many cubic meters of water-ice might be present?

Problem 1 - The width of this map is 500 km. What are the diameters of Crater A (Shackleton) and Crater B (Amundsen) in kilometers?

Answer: If the page is printed as 8.5 x 11-inches, the width of the colorized image is 106 mm wide, which corresponds to 500 km, so the image scale is 500 km/106mm = 4.7 km/mm. Crater A has a diameter of 4.4 mm so its actual diameter is 4.4 mm x (4.7 km/mm) = **20 kilometers**. Crater B has a diameter of 34 mm, so its actual diameter is about 34 x 4.7 = **160 kilometers**.

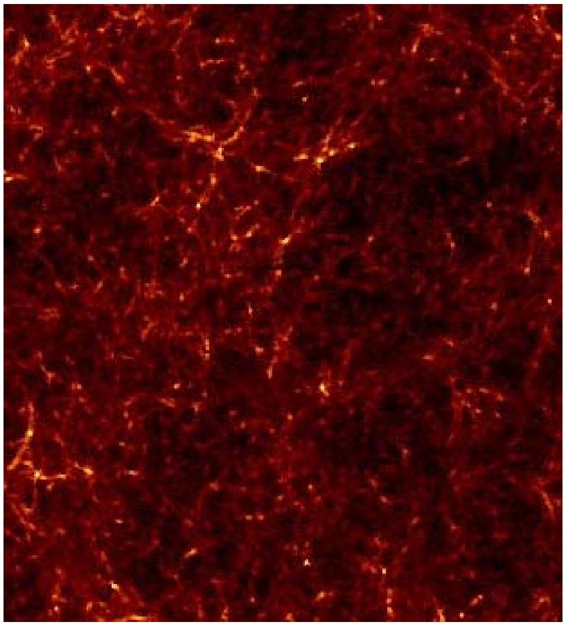
Problem 2 - In which colored areas might an astronaut expect to find conditions cold enough to recover all of the elements and molecules indicated in the vertical temperature scale to the right?

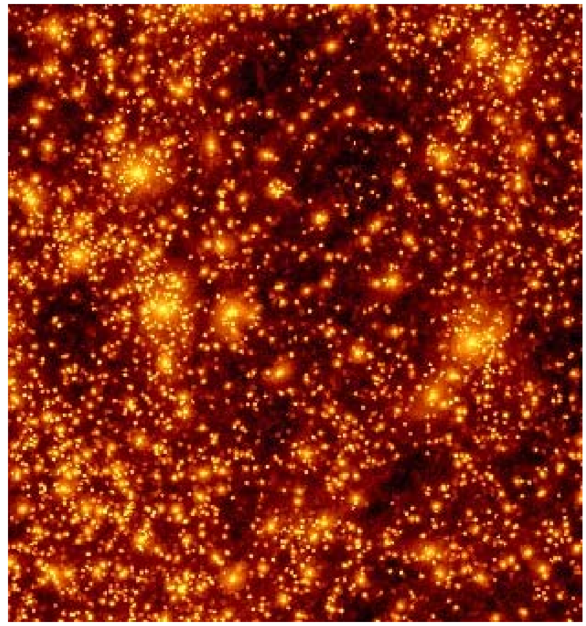
Answer: Students should note that all compounds above a given temperature will be present in conditions cold enough to 'trap' these compounds. **Areas where the temperature is below 50 K will be cold enough to trap all of the identified compounds. These are areas colored lavender at the bottom of the temperature scale, and in areas similar to where LCROSS impacted.**

Problem 3 - The Shackleton Crater (Crater A) is cold enough to trap water and methanol. From Problem 1, and assuming that the thickness of the water deposit is 100 meters, and occupies 10% of the volume of the circular crater, how many cubic meters of water-ice might be present?

Answer: The diameter of the circular crater is 20 km or 20,000 meters, so its radius is 10,000 meters. The area of the crater is then $A = \pi (10,000)^2 = 314,000,000$ meters^2. Since the thickness of the deposit is perhaps 100 meters, the volume of this disk of material is $V = 100$ meters x 314,000,000 meters^2 = 3.1×10^{10} meters^3. Since only 10% of this is hypothetically water-ice, the ice volume is just 0.1V or **3.1×10^9 meters^3.**

Note: This ice volume is similar to a small glacier!





Supercomputers can do trillions of calculations each second, and follow the movement of billions of particles over time. In 2008, scientists at the University of Chicago used supercomputer simulations to investigate dark matter. Dark matter is an invisible material of unknown composition that is estimated to be 5 times more abundant in our universe than ordinary matter. Astrophysicists believe that dark matter may have herded luminous matter in the universe from its initial smooth state into the cosmic web of galaxies and galaxy clusters that populate the universe today. These two images from the simulation of the evolution of the universe show a cubic volume of the universe measuring approximately 200 million light years wide. The images show how dark matter caused the distribution of the luminous matter to change from 470 million years after the big bang (left) to today, some 13.7 billion years after the big bang (right). (*Courtesy: University of Chicago: Andrey Kravtsov, Charlie Conroy and Risa Weschlser*).

Astronomers can use their catalogs of millions of distant galaxies to check these supercomputer calculations. The goal is to mathematically model the earliest epoch of galaxy formation and to use telescopes like the Webb Space Telescope to confirm the details of these models to within 20 million years after the big bang. Since no luminous stars existed then, this period from 1 million years to about 20 million years after the big bang is called the Dark Ages.

Problem 1 - Using a millimeter ruler; A) what is the scale of these two images in light years per millimeter? B) How large would the Milky Way be on the scale of these supercomputer images? C) The Local Group of galaxies, which is 10 million light years in diameter?

Problem 2 - The smallest mathematical feature that the supercomputer can follow in these calculations is about 100 light years across. If the entire volume modeled was 280 million light years on a side, how many cubic cells 100 light years wide are in this entire volume?

Problem 3 - If the time step between calculations was 1000 years, how many time steps did it take to complete a full 13.7 billion year calculation?

Problem 1 - Using a millimeter ruler; A) what is the scale of these two images in light years per millimeter? B) How large would the Milky Way be on the scale of these supercomputer images? C) The Local Group of galaxies, which is 10 million light years in diameter?

Answer: A) The width of each image is about 108 millimeters, so the scale is 200 million/108mm = **1.9 million light years/millimeter**.

B) The Milky Way would be about 100,000 light years x (1 mm/1.9 million) = **0.05 millimeters across!**

C) The Local Group = 10 million light years x (1 millimeter/1.9 million light years)

= **5.3 mm**.

Problem 2 - The smallest mathematical feature that the supercomputer can follow in these calculations is about 100 light years across. If the entire volume modeled was 280 million light years on a side, how many cubic cells 100 light years wide are in this entire volume?

Answer: The large volume has a width of 280,000,000 / 100 = 2,800,000 cells ,so the total number of cells in the volume is just N = $(2,800,000)^3$ = **2.2×10^{19} cells**

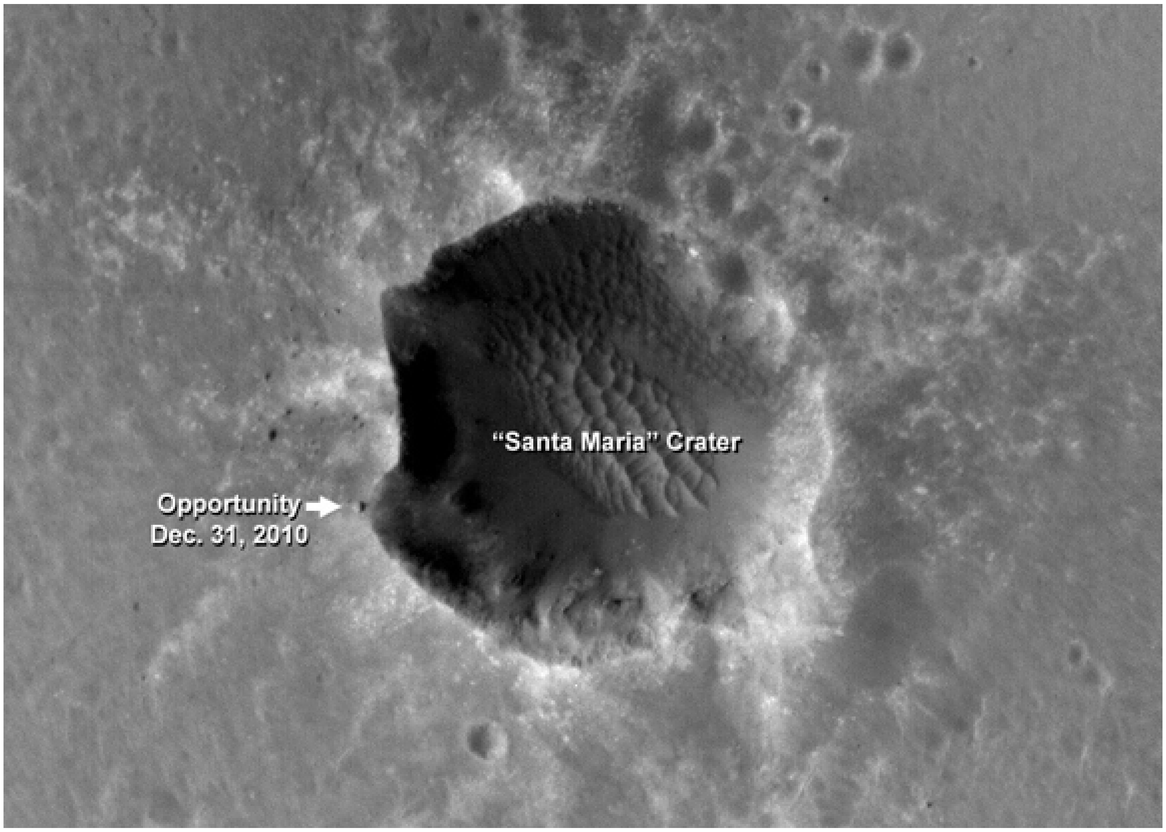
Problem 3 - If the time step between calculations was 1000 years, how many time steps did it take to complete a full 13.7 billion year calculation?

Answer: The total number of time steps is 13,700,000,000 / 1000 = **13.7 million**.

Note: The actual calculation used what is called a variable cell mesh so that for volumes of space where not much was happening, a large cell size many times larger than 100 light years was used. Only in regions where things were changing rapidly was a smaller cell size used, so the actual number of cells followed in the entire simulation was much less than 2.2×10^{19}. A similar variable mesh process was used for the time steps.

The Local Group of galaxies consists of the Milky Way, the Andromeda galaxy, and about 30 other smaller galaxies located within 5 million light years of the Milky Way.

Taking a Stroll around a Martian Crater



The High Resolution Imaging Science Experiment (HiRISE) camera on NASA's Mars Reconnaissance Orbiter acquired this image of the Opportunity rover on the southwest rim of "Santa Maria" crater on New Year's Eve 2010. Opportunity arrived at the western edge of Santa Maria crater in mid-December and will spend about two months investigating rocks there. That investigation will take Opportunity into the beginning of its eighth year on Mars. Opportunity is imaging the crater interior to better understand the geometry of rock layers and the meteor impact process on Mars. Santa Maria is a relatively young, 90 meter-diameter impact crater, but old enough to have collected sand dunes in its interior.

Problem 1 - Using a millimeter ruler, what is the scale of this image in meters per millimeter to one significant figure?

Problem 2 - To one significant figure, about what is the circumference of the rim of this crater in meters?

Problem 3 - The rover can travel about 100 meters in one day. To one significant figure, how long will it take the rover to travel once around this crater?

Problem 4 - A comfortable walking speed is about 100 meters per minute. To one significant figure, how long would it take a human to stroll around the edge of this crater?

Problem 1 - Using a millimeter ruler, what is the scale of this image in meters per millimeter to one significant figure?

Answer: The shape of the crater is irregular, but taking the average of several diameter measurements with a ruler gives a diameter of about 55 millimeters. Since this corresponds to 90 meters according to the text, the scale of this image is about 90 meters/55 mm = 1.8 meters/millimeter, which to one significant figure becomes **2 meters/millimeter.**

Problem 2 - To one significant figure, about what is the circumference of the rim of this crater in meters?

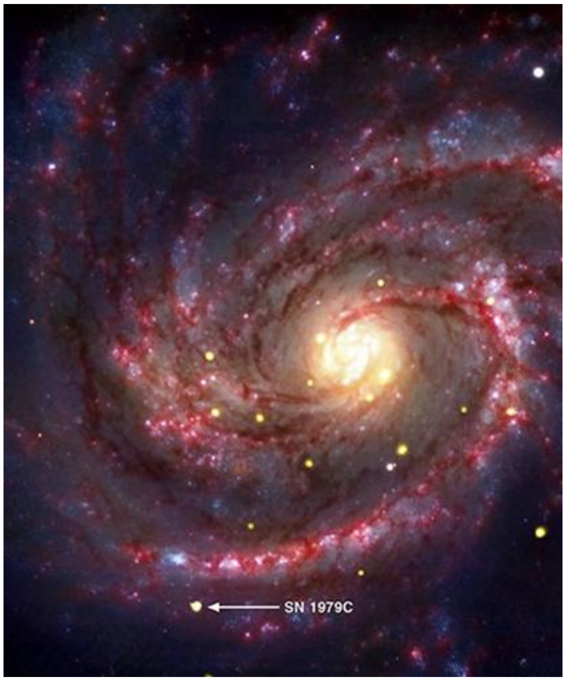
Answer: Students may use a piece of string and obtain an answer of about 200 millimeters. Using the scale of the image in Problem 1, the distance in meters is about 200 mm x 2 meters/mm = **400 meters**.

Problem 3 - The rover can travel about 100 meters in one day. To one significant figure, how long will it take the rover to travel once around this crater?

Answer: 400 meters x (1 day/100 meters) = **4 days**.

Problem 4 - A comfortable walking speed is about 100 meters per minute. To one significant figure, how long would it take a human to stroll around the edge of this crater?

Answer: 400 meters x (1 minute/100 meters) = **4 minutes**.



The Chandra X-Ray Observatory recently confirmed the discovery of an infant black hole in the nearby galaxy Messiar-100. The product of the supernova of a star with a mass of 20 times our sun, the resulting black hole may only involve about 8 times our sun's mass.

For black holes that do not rotate, called Schwarschild Black Holes, there are several different sizes for such black holes that all scale with the mass of the black hole. When referring to the size of a black hole, astronomers usually mention its mass, which is well defined, rather than its diameter, which depends on the specific kinds of physical processes involved.

The Schwarschild Radius – This is the distance from the center of the black hole at which an incoming person, or light signal, can enter the black hole interior, but cannot emerge back out into the universe. It is also called the Event Horizon. It is a perfectly spherical surface with a radius of Rs = 3.0 M kilometers, where M is the mass of the black hole in multiples of the sun's mass (1 M = 2.0×10^{30} kg). For the SN1979C black hole, with an estimated mass of 8 M, what is its Schwarschild Radius, Rs, in kilometers?

Last Photon Orbit – At this distance outside the Event Horizon, an incoming photon of light can enter into an exactly circular orbit, where it will stay until it is disturbed, at which time it will fall into the Event Horizon and never get back out. If Rp =1.5 Rs, what is the radius of the photon orbit for black hole SN1979C in kilometers?

Last Stable Particle Orbit – Inside this distance, a material particle cannot be in a stable circular orbit, but is relentlessly dragged to the Event Horizon and disappears. This occurs at a distance from the black hole center of Rl = 3.0 Rs. How close can a hydrogen atom, an asteroid or a planet remain in a stable circular orbit around the SN1979C black hole?

Problem – An asteroid is spotted at a distance of 700 km from a black hole with a mass of 120 solar masses. Can it escape or remain where it is?

NASA Press release 'Youngest Nearby Black Hole' November 15, 2010

"Data from Chandra, as well as NASA's Swift, the European Space Agency's XMM-Newton and the German ROSAT observatory revealed a bright source of X-rays that has remained steady for the 12 years from 1995 to 2007 over which it has been observed. This behavior and the X-ray spectrum, or distribution of X-rays with energy, support the idea that the object in SN 1979C is a black hole being fed either by material falling back into the black hole after the supernova, or from a binary companion.

The scientists think that SN 1979C formed when a star about 20 times more massive than the Sun collapsed. It was a particular type of supernova where the exploded star had ejected some, but not all of its outer, hydrogen-rich envelope before the explosion, so it is unlikely to have been associated with a gamma-ray burst (GRB). Supernovas have sometimes been associated with GRBs, but only where the exploded star had completely lost its hydrogen envelope. Since most black holes should form when the core of a star collapses and a gamma-ray burst is not produced, this may be the first time that the common way of making a black hole has been observed.

The very young age of about 30 years for the black hole is the observed value, that is the age of the remnant as it appears in the image. Astronomers quote ages in this way because of the observational nature of their field, where their knowledge of the Universe is based almost entirely on the electromagnetic radiation received by telescopes."
(**http://www.nasa.gov/mission_pages/chandra/multimedia/photoH-10-299.html)**

The Schwarschild Radius – This is the distance from the center of the black hole at which an incoming person, or light signal, can enter the black hole interior, but cannot emerge back out into the universe. It is also called the Event Horizon. It is a perfectly spherical surface with a radius of Rs = 3.0 M kilometers, where M is the mass of the black hole in multiples of the sun's mass (1 M = 2.0×10^{30} kg). For the SN1979C black hole, with an estimated mass of 8 M, what is its Schwarschild Radius, Rs, in kilometers?

Answer: M = 8, so Rs = 3.0 x 8 = **24 kilometers**.

Last Photon Orbit – At this distance outside the Event Horizon, an incoming photon of light can enter into an exactly circular orbit, where it will stay until it is disturbed, at which time it will fall into the Event Horizon and never get back out. If Rp =1.5 Rs, what is the radius of the photon orbit for black hole SN1979C in kilometers?

Answer: Rs = 24 kilometers so Rp = 1.5 x 24 km = **36 kilometers**.

Last Stable Particle Orbit – Inside this distance, a material particle cannot be in a stable circular orbit, but is relentlessly dragged to the Event Horizon and disappears. This occurs at a distance from the black hole center of Rl = 3.0 Rs. How close can a hydrogen atom, an asteroid or a planet remain in a stable circular orbit around the SN1979C black hole?

Answer: R = 3.0 x 24 km = **72 kilometers**.

Problem – An asteroid is spotted at a distance of 700 km from a black hole with a mass of 120 solar masses. Can it escape or remain where it is? Answer: Rs = 3.0 x 120 = 360 kilometers. Rp = 1.5 x 360 km = 540 km; Rl = 3.0 x 360 km = 1080 km. **Since the asteroid is at 700 km, it is inside the distance where it can remain in a stable orbit, so it is about to fall through the black hole's event horizon located some 700-360 = 340 kilometers inside its current position.**

Death Stars

Artist rendering courtesy NASA/G. Bacon (STScI)

Our sun is an active star that produces a variety of storms, such as solar flares and coronal mass ejections. Typically, these explosions of matter and radiation are harmless to Earth and its living systems, thanks to our great distance from the sun, a thick atmosphere, and a strong magnetic field. The most intense solar flares rarely exceed about 10^{21} Watts and last for an hour, which is small compared to the sun's luminosity of 3.8×10^{26} Watts.

A long-term survey with the Hubble Space Telescope of 215,000 red dwarf stars for 7 days each revealed 100 'solar' flares during this time. Red dwarf stars are about 1/20000 times as luminous as our sun. Average flare durations were about 15 minutes, and occasionally exceeded 2.0×10^{21} Watts.

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how many years would inhabitants on each planet have to wait between solar flares?

Problem 1 - By what percentage does a solar flare on our sun increase the brightness of our sun?

Answer: P = 100% x (1.0 x 10^{21} Watts)/(3.8 x 10^{26} Watts)
P = 0.00026 %

Problem 2 - By what percentage does a stellar flare on an average red dwarf increase the brightness of the red dwarf star?

Answer: The average luminosity of a red dwarf star is stated as 1/20000 times our sun's luminosity, which is 3.8 x 10^{26} Watts, so the red dwarf star luminosity is about 1.9 x 10^{22} Watts.

P = 100% x (2.0 x 10^{21} Watts)/(1.9 x 10^{22} Watts)
P = 10 %

Problem 3 - Suppose that searches for planets orbiting red dwarf stars have studied 1000 stars for a total of 480 hours each. How many flares should we expect to see in this survey?

Answer: We have two samples: N1 = 215,000 stars for 7 days each producing 100 flares. N2 = 1000 stars for 480 hours each, producing x flares.

From the first survey, we calculate a rate of flaring per star per day:
Rate = 100 flares x (1/215,000 stars) x (1 / 7days)
= 0.000066 flares/star/day

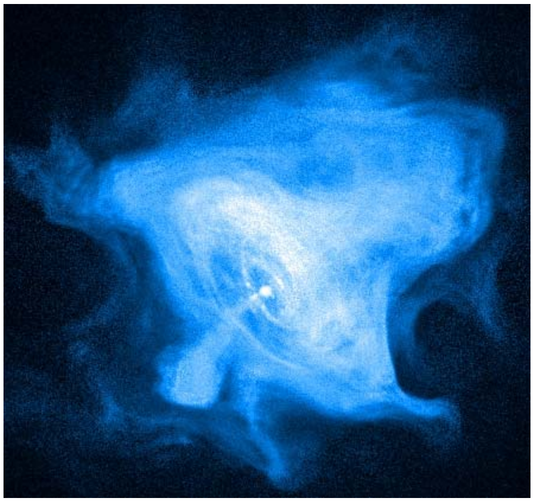
Now we multiply this rate by the size of our current sample to get the number of flares to be seen in the 480-hour (20-day) period.

N = 0.000066 flares/star/day) x (1000 stars) x (20 days)
= 1.32 or **1 flare event**.

Problem 4 - Suppose that during the course of the survey in Problem 3, 5 exoplanets were discovered orbiting 5 of the surveyed red dwarf stars. To two significant figures, about how long would inhabitants on each planet have to wait between solar flares?

Answer: The Flare rate was 0.000066 flares/star/day. For 1 star, we have T = 0.000066 flares/star/day x (1 star) = 0.000066 flares/day so the time between flares (days/flare) is just T = 1/0.000066 = 15,151 days. Since there are 365 days/year, this is about **42 years between flares on the average**.

The Crab Nebula - Exploring a pulsar up close!



The Crab Nebula is all that remains of a massive star that exploded as a supernova, and was seen by humans for the first time in 1054 AD. Located 6000 light years away, it is still being powered by a rapidly-spinning neutron star.

The electromagnetic energy from this spinning, magnetized object (also called a pulsar) produces the amazing high-energy cloud of particles now seen clearly by the NASA, Chandra Observatory in the image to the left.

This image gives the first clear view of the faint boundary of the Crab Nebula's X-ray-emitting pulsar wind nebula. The combination of rapid rotating and strong magnetic field generates an intense electromagnetic field that creates jets of matter and anti-matter moving away from the north and south poles of the pulsar, and an intense wind flowing out in the equatorial direction.

The inner X-ray ring is thought to be a shock wave that marks the boundary between the surrounding nebula and the flow of matter from the pulsar. Energetic electrons and positrons (a form of antimatter) move outward from this ring to produce an X-ray glow that Chandra sees as a ghostly cloud in the image above.

Problem 1 - The width of this image is about 5 light years. If the elliptical ring near the center is actually a circular ring seen at a tilted angle, what is the radius of this ring in: A) light years? B) kilometers? (Note: 1 light year = 5.9 trillion kilometers).

Problem 2 - The high-energy particles that make-up the ring were created near the neutron star at the center of the ring. If they are traveling at a speed of 95% the speed of light, to the nearest day, how many days did it take for the particles to reach the edge of the ring? (Speed of light = 300,000 km/s)

Problem 3 - Suppose the pulsar ejected the particles and was visible to astronomers on Earth as a burst of light from the central neutron star 'dot'. If the astronomers wanted to see the high-energy particles from this ejection reach the ring and change its shape, how long would they have to wait for the ring to change after seeing the burst of light?

Problem 1 - The width of this image is about 5 light years. If the elliptical ring near the center is actually a circular ring seen at a tilted angle, what is the radius of this ting in: A) light years? B) kilometers? (Note: 1 light year = 5.9 trillion kilometers).

Answer: the scale of this image can be found using a millimeter ruler. When printed, the image is about 70 mm. The scale is then 5 ly/70mm = 0.071 ly/mm. The radius of the ring will be the maximum radius of the elliptical ring, which you can see by drawing a circle on a piece of paper and tilting it so it looks like an ellipse. On the image, the length of the major axis of the ellipse is 10 mm, so the radius of the circle is 5 mm.
A) Using the scale of the image we get 5 mm x 0.071 ly/mm = **0.36 light years.**
B) The radius in kilometers is just 0.36 ly x 5.9 trillion km/1 ly = **2.1 trillion km.**

Problem 2 - The high-energy particles that make-up the ring were created near the neutron star at the center of the ring. If they are traveling at a speed of 95% the speed of light, to the nearest day, how many days did it take for the particles to reach the edge of the ring? (Speed of light = 300,000 km/s)

Answer: Time = distance/speed, so for s = 0.95x300,000 km/s = 285,000 km/s, and d = 2.1 trillion km, we get T = 2,100,000,000,000 / 285,000 = 7,368,421 seconds. Converting to days: 7,368,421 seconds x (1 hour/3600 sec) x (1 day/24 hours) = 85.28 days. To the nearest day, this is **85 days**.

Problem 3 - Suppose the pulsar ejected the particles and was visible to astronomers on Earth as a burst of light from the central neutron star 'dot'. If the astronomers wanted to see the high-energy particles from this ejection reach the ring and change its shape, how long would they have to wait for the ring to change after seeing the burst of light?

Answer; They would have to wait 85 days after seeing the burst of light because light travels faster than the matter in the particles.

Note: Another way to appreciate how much faster light travels, calculate the number of days it would take for the pulse of light to reach the ring, compared to the 85 days taken by the particles. The light pulse would take 2.1 trillion km/300,000 km/s = 7 million seconds or about 81 days. So astronomers would have to wait 81 days to see whether the light pulse affects the ring, and then another 4 days for the particles to arrive.

Table of Global Temperature Anomalies

Year	Temperature (degrees C)	Year	Temperature (degrees C)
1900	-0.20	1960	+0.05
1910	-0.35	1970	0.00
1920	-0.25	1980	+0.20
1930	-0.28	1990	+0.30
1940	+0.08	2000	+0.45
1950	-0.05	2010	+0.63

A new study by researchers at the Goddard Institute for Space Studies determined that 2010 tied with 2005 as the warmest year on record, and was part of the warmest decade on record since the 1800s. The analysis used data from over 1000 stations around the world, satellite observations, and ocean and polar measurements to draw this conclusion.

The table above gives the average 'temperature anomaly' for each decade from 1900 to 2010. The Temperature Anomaly is a measure of how much the global temperature differed from the average global temperature between 1951 to 1980. For example, a +1.0 C temperature anomaly in 2000 means that the world was +1.0 degree Celsius warmer in 2000 than the average global temperature between 1951-1980.

Problem 1 - By how much has the average global temperature changed between 1900 and 2000?

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/La Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature anomaly change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.

Problem 3 - From the data in the table, calculate the rate of change of the temperature anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900?

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Problem 1 - Answer: In 1900 it was -0.20 C and in 2000 it was +0.45, so it has changed by +0.45 - (-0.20) = **+0.65 C**.

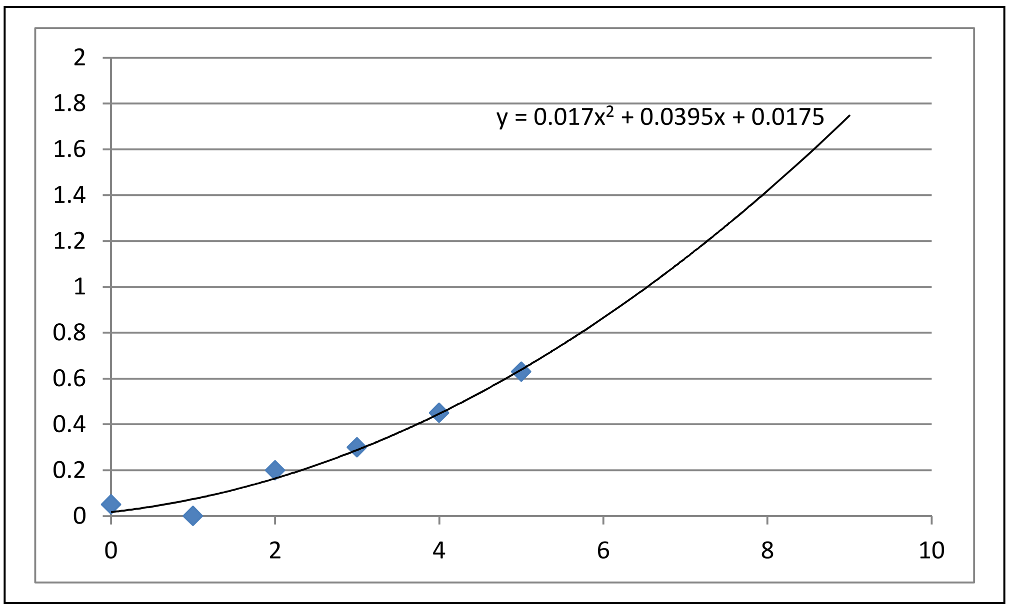
Problem 2 - Answer:
1900 to 1920: -0.25C - (-0.20C) = **-0.05 C a decrease (cooling) of 0.05 C**
1920 to 1950: - 0.05C - (-0.25C) = **+0.20 C an increase (warming) of 0.20 C**
1950 to 1980: +0.20C - (-0.05C) = **+0.25 C an increase (warming) of 0.25 C**
1980 to 2010: +0.63C - (+0.20C) = **+0.43 C an increase (warming) of 0.43 C**

Problem 3 - Answer:
1900 to 1920: -0.05 C/3 decades = **-0.017 C per decade**
1920 to 1950: +0.20 C/ 3 decades = **+0.067 C per decade**
1950 to 1980: +0.25 C/3 decades = **+0.083 C per decade**
1980 to 2010: +0.43 C/3 decades = **+0.143 C per decade**.

The pace of global temperature change is **increasing in time**. It is almost doubling every 10 years.

Problem 4 - **Answer:** Students may graph the data in the table, then use a ruler to draw a line on the graph between 1960 and 2000, to extrapolate to the temperature anomaly in 2050. A linear equation, T = mx + b, that models this data is b = +0.05C m = (+0.45 - 0.05)/4 decades so m = +0.10 C/decade. Then T = +0.10x + 0.05. For 2050, which is 9 decades after 1960, x=9 so T = +0.1(9) + 0.05 = **+0.95 C**. So, the world will be, on average, about **+1 C warmer** in 2050 compared to its average temperature between 1950 and 1980. This assumes a linear change in T with time.

Note to Teacher: From Problem 3 we see that the temperature anomaly change is accelerating because the value for the increase in each 3-decade interval continues to increase from +0.067 per decade to +0.143 per decade from 1920 to 2010. A linear trend would have only a constant value for the change in each interval. If we use Excel spreadsheets and enter the data for 1960-2010, we get a more accurate 'quadratic' fit to the data since 1960 (See figure below): T = $+0.017x^2 + 0.04x + 0.018$. For the year 2050, this quadratic prediction suggests T = $0.017(9)^2 + 0.04(9) + 0.018$ so T = +1.75 C.



Astronomers have recently discovered a massive cluster of young galaxies that formed when the universe was only about 1 billion years old. The light from these galaxies has taken over 12 billion years to reach Earth. The growing galactic metropolis, called COSMOS-AzTEC3 is the most distant known massive "proto-cluster" of galaxies known today. The circled red smudges in the above image are the individual galaxies that are a part of the cluster. In the time since the light started on its journey, these galaxies have probably fallen together under the influence of gravity to form a large galaxy about the size of our Milky Way. At the distance of the galaxy cluster, the width of this photograph, which was taken by the Japanese Subaru Telescope on Mauna Kea, is about 25 million light years across. (Image Credit: Subaru/ NASA / JPL-Caltech)

Problem 1 - Assuming all of the galaxies lie in the same plane, and at the same distance, to two significant figures: A) What is the greatest distance between the galaxies in this photograph? B) What is the smallest distance between galaxies in this photograph?

Problem 2 - The Milky Way galaxy and the Andromeda galaxy are separated by about 2,200,000 light years in diameter. How many millimeters apart would they appear if they were viewed at the same distance as this cluster?

Problem 3 - If the average speed of a galaxy in this cluster is about 1000 light years in 1 million years, how many years will it take for all the galaxies to fall to the center of the cluster?

Problem 1 - Assuming all of the galaxies lie in the same plane, and at the same distance. To two significant figures: A) What is the greatest distance between the galaxies in this photograph? B) What is the smallest distance between galaxies in this photograph?

Answer: With a millimeter ruler, the width of the image is about 153 millimeters, which corresponds to an actual distance of 25 million light years. The scale of the image is then 25 million/153 mm = 160,000 light years /mm, then:

A) The farthest distance between the 11 identified galaxies is about 96 mm, for a distance of 96 mm x (160,000 ly/1 mm) = **15,000,000 light years**. B) The closest pair of galaxies are about 2 mm apart, or 2 x 160,000 = **320,000 light years**.

Problem 2 - The Milky Way galaxy and the Andromeda galaxy are separated by about 2,200,000 light years in diameter. How many millimeters apart would they appear if they were viewed at the same distance as this cluster?

Answer: 2,200,000 light years x (1 mm/160,000 light years) = **14 millimeters**.

Problem 3 - If the average speed of a galaxy in this cluster is about 1000 light years in 1 million years, how many years will it take for all the galaxies to fall to the center of the cluster?

Answer: For the two most distant galaxies to fall to the center, a point half way between them in the photograph, they must fall a distance of 7,500,000 light years. Since the speed is stated as 1000 light years / 1 million years, the time it would take is about

T = 7,500,000 light years x (1 million years / 1000 light years)

= 7,500 million years or **7.5 billion years**.

Note that galaxies closer together will have fallen together much sooner, so the most distant pair of galaxies defines the approximate free-fall age of this cluster. Since the universe is 13.7 billion years, and the free-fall age of this cluster is only 7.5 billion years, it must have had enough time to collapse into a smaller size long ago...perhaps even before our own Earth was formed!

Discovery Year	Redshift Z	Age (millions of yrs)	Object Name or Catalog Number	Type
1930	Infinity	13,700	Big Bang	Creation
2011	10	13,200	UDFj-39546284	Galaxy fragment
2010	8.6	13,100	UDFy-38135539	Galaxy fragment
2009	8.2	13,070	GRB090423	Gamma ray burst
2008	7.6	13,000	A1689-zD1	Starburst Galaxy
2006	6.9	12,880	IOK-1	Galaxy
2007	6.4	12,700	CFHQS J2329-0301	Black Hole/Quasar
2011	5.3	12,600	COSMOS-AzTEC3	Galaxy Cluster

The universe was born 13.7 billion years ago in the 'Big Bang' and since then has been expanding to its present, enormous size. By using sensitive telescopes such as NASA's Hubble Space Telescope, astronomers can study the images of ancient galaxies, whose light has been traveling towards Earth for billions of years. For the most distant objects, their light has been traveling towards Earth for over 10 billion years, allowing astronomers to see what these distant galaxies looked like 10 billion years ago. The 'race' has been on to detect the most ancient images of galaxies so that astronomers can learn about the events that created the first galaxies and stars in the universe. The table above shows the light travel times for various known objects that have been identified by 2011. For the problems below, assume that these objects are common examples of their types at their estimated ages.

Problem 1 – Create a timeline that shows the age of each object since the Big Bang.

Problem 2 – How many million years after the Big Bang did the first galaxy fragments begin to form?

Problem 3 – About how many million years after the formation of the first galaxy fragments did the formation of the first galaxies begin?

Problem 4 – The quasar CFHQS J2329-0301 contains a black hole with a mass estimated to be 500 million times the mass of our sun. If this 'supermassive' black hole began to form when the first galaxy fragments appeared, about how many years did it take to form this black hole?

Problem 5 – About how many years elapsed between the appearance of the first galaxy fragments, and the first cluster of galaxies in the universe?

Problem 6 – In your timeline, what is the largest gap in time for which we, as yet, have not observed any candidate objects?

For more information online:

Cosmic Background Radiation http://wmap.gsfc.nasa.gov/
UDFj-39546284 **http://www.nasa.gov/mission_pages/hubble/science/farthest-galaxy.html**
UDFy-38135539 http://www.hubblephotoprints.com/blog/hubble-telescope-reveals-the-most-distant-galaxy-ever-found/
GRB090423 http://science.nasa.gov/science-news/science-at-nasa/2009/28apr_grbsmash/
A1689-zD1 http://www.spacetelescope.org/news/heic0805/
IOK-1 http://www.physorg.com/news176737523.html
CFHQS J2329-0301 http://www.gemini.edu/index.php?q=node/254
COSMOS-AzTEC3 http://www.nasa.gov/mission_pages/spitzer/news/spitzercluster20110112.html

Problem 1 – Create a timeline that shows the age of each object since the Big Bang. Answer:

Age (millions of yrs)	Years since the Big Bang	Object Type
13,700	0	Creation of time, space and matter
13,200	500 million	Galaxy fragments begin to form
13,100	600 million	Galaxy fragments still present
13,070	630 million	Gamma ray burst appears - supernovae
13,000	700 million	Starburst Galaxy and intense star formation
12,880	820 million	Galaxies begin to form
12,700	1 billion	Supermassive black holes appear
12,600	1.1 billion	Galaxy Clusters begin to form

Note: The Object Type can be used to indicate what kinds of events are taking place. Shown above is an example.

Problem 2 – How many million years after the Big Bang did the first galaxy fragments begin to form? Answer: that would be the time between the Big Bang (13.7 billion years ago) and UDFj-39546284 (13.2 billion years ago) or **500 million years.**

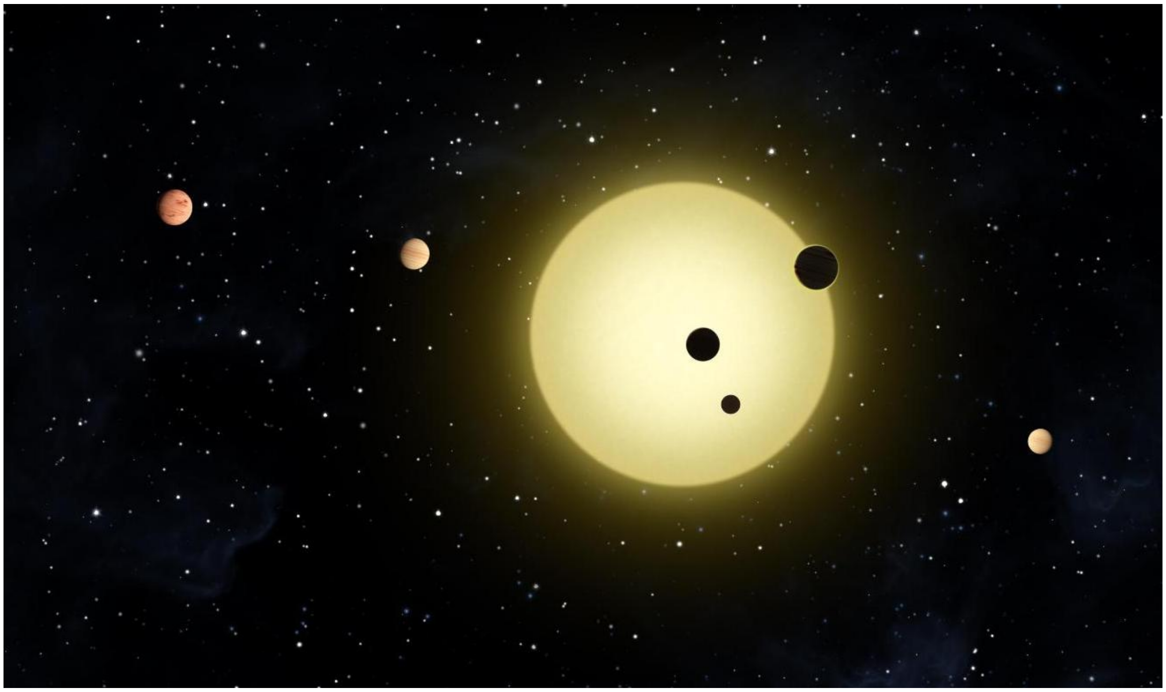
Problem 3 – About how many million years after the formation of the first galaxy fragments did the formation of the first galaxies begin? Answer: That would be the time between 13.2 billion years ago when we are seeing one of the first galaxies, UDFj-39546284, and the appearance of IOK-1 at 12.880 billion years ago or about **820 million years** after we first start seeing fragments of galaxies.

Problem 4 – The quasar CFHQS J2329-0301 contains a black hole with a mass estimated to be 500 million times the mass of our sun. If this 'supermassive' black hole began to form when the first galaxy fragments appeared, about how many years did it take to form this black hole? Answer: Galaxy fragments first appeared about 13.2 billion years ago (UDFj-39546284) and the quasar image is 12.7 billion years old, so the difference in time is about **1 billion years**.

Problem 5 – About how many millions of years elapsed between the appearance of the first galaxy fragments, and the first cluster of galaxies in the universe? Answer: Fragments formed about 13.2 billion years ago; clusters appeared about 12.6 billion years ago, so it took **about 600 million years**.

Problem 6 – In your timeline, what is the largest gap in time for which we, as yet, have not observed any candidate objects? Answer: **Between the Big Bang (13. 7 billion years ago) and the appearance of the first galaxy fragments (13.2 billion years ago) or 500 million years.**

Earth-sized Planets by the Score!



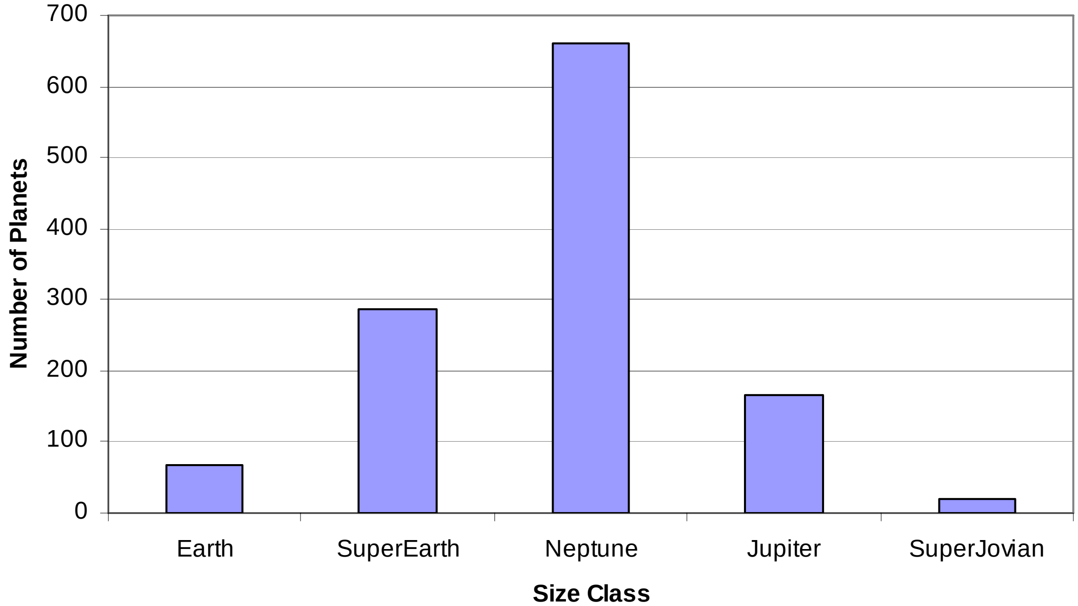
NASA's Kepler Space Observatory recently announced the results of its continuing survey of 156,453 stars in the search for planet transits. Their survey, in progress for just under one year, has now turned up 1,235 transits from among this sample of stars of which 33 were eliminated because they were too big to be true planets. About 30 percent of the remaining candidates belong to multiple-planet systems in which several planets orbit the same star. Among the other important findings are the numbers of planet candidates among the various planet types summarized as follows: Earth-sized = 68; superEarths = 288; Neptune-sized = 662; Jupiter-sized=165; superJovians=19.

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes.

Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized?

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by NASA's Kepler Space Observatory?

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes. Answer:



Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized? Answer: P = 100% x (68/1202) = **5.7 %**

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by Kepler?

Answer: There are 40 billion candidate stars in the Milky Way, so by using simple proportions and re-scaling the survey to the larger sample size

$$\frac{68}{157,453} = \frac{X}{40 billion}$$

we get about (40 billion/156,453) x 68 planets or about **17 million Earth-sized planets**.

Note: *The Kepler survey has not been conducted long enough to detect planets much beyond the orbit of Venus in our own solar system, so in time many more earth-sized candidates farther away from their stars will be reported in the years to come. This means that there may well be considerably more than 17 million Earth-sized planets orbiting stars in the Milky Way similar to our own sun.*

Earth-like Planets and Habitable Zones

Table of Candidate Planet Sizes

Size Class	Size (Earth Radius)	Number of candidates
Earths	R <1.25	68
Super-Earths	1.25 < R < 2.0	288
Neptunes	2.0 < R < 6.0	662
Jovians	6.0 < R < 15	165
Super-Jovians	15 < R < 22	19
Dwarf Stars, etc	R > 22	15

NASA's Kepler mission has just completed its first year of surveying 156,453 stars to detect the tell-tail signs of distant planets passing across the faces of their stars. This causes a slight dimming of the starlight, which can be detected by the satellite observatory. From the 1,235 transits detected so far, 33 were eliminated because the planets would have been far larger than Jupiter, and possibly dwarf stars. The table above shows the distribution of the remaining planet candidates among the interesting sizes ranges for planets in our own solar system.

About 30 percent of the candidates have been found to belong to multiple-planet systems, with several planets orbiting the same star. To find Earth-like planets where liquid water could be present, astronomers define a Habitable Zone (HZ) surrounding each star where planet surfaces could be warm enough for liquid water. This is roughly between temperatures of 270 to 300 Kelvin. A careful study of the orbits of the planetary candidates discovered 54 candidate planets in the HZs of their stars. Of these, five planets are roughly Earth-sized, and the other 49 planets range from twice the size of Earth to larger than Jupiter.

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

Problem 1 - What fraction of the candidate planets from the full Kepler survey were found within the HZs of their respective stars?

Answer: There are 1,202 candidate planets in the larger survey, and 54 found in their HZ so the percentage is P = 100% x (54/1202) = **4.5%**.

Note: *The essay says that 33 candidates were eliminated because they were probably not planets, so 1,235 - 33 = 1,202 planet candidates.*

Problem 2 - What percentage of Earth-sized planets in the full Kepler survey were found in the HZs of the respective stars?

Answer: There were 68 Earth-sized planets found from among 1,202 candidates, and 5 were Earth-sized, so the percentage of Earth-sized planets in their HZs is P = 100% x (5/68) = 7.4%.

So, if you find one Earth-sized planet orbiting a star in the Kepler survey, there is a 7.4% chance that it will be in its HZ so that liquid water can exist.

Problem 3 - If there are about 40 billion stars in the Milky Way that are similar to the stars in the Kepler survey, about how many Earth-sized planets would you expect to find in the HZs of there other stars?

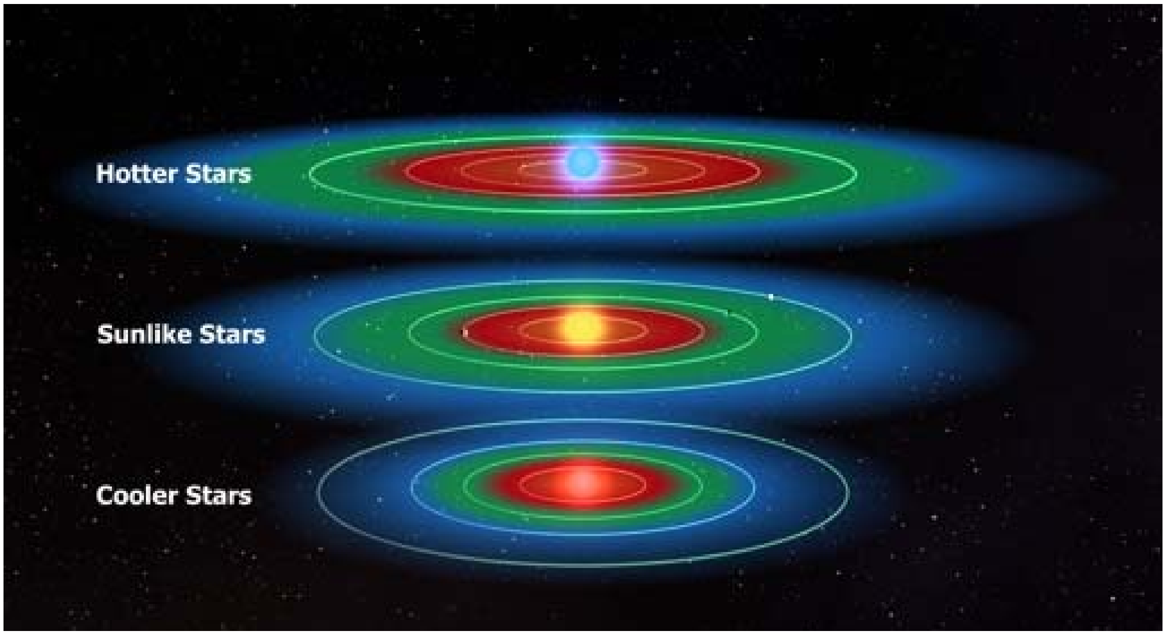
Answer: Students may use simple scaling. The Kepler star sample contains 156,453 stars. It resulted in the discovery, so far, of 5 Earth-sized planets in the Habitable Zones of their respective stars. So

$$\frac{5}{156,453} = \frac{N}{40 billion}$$ so

N = (40 billion / 156,453) x 5 planets

or about **1.3 million planets**.

Another way to think about this is that if you select 30,000 stars in the sky similar to our own sun, you could expect to find about 1 Earth-sized planet orbiting within the Habitable Zone of its star. (5 x 30,000/156453 = 0.96 or about 1.0 planets)



Once you have discovered a planet, you need to figure out whether liquid water might be present. In our solar system, Mercury and Venus are so close to the sun that water cannot remain in liquid form. It vaporizes! For planets beyond Mars, the sun is so far away that water will turn to ice. Only in what astronomers call the Habitable Zone (shown in green in the figure above) will a planet have a chance for being at the right temperature for liquid water to exist in large quantities (oceans) on its surface!

The Table on the following page lists the 54 planets that were discovered by NASA's Kepler Observatory in 2010. These planets come in many sizes as you can see by their radii. The planet radii are given in terms of the Earth, where '1.0' means a planet has a radius of exactly 1 Earth radius (1.0 Re) or 6,378 kilometers. The distance to each planet's star is given in multiples of our Earth-Sun distance, called an Astronomical Unit, so that '1.0 AU' means exactly 150 million kilometers.

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth?

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re?

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re?

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets?

Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

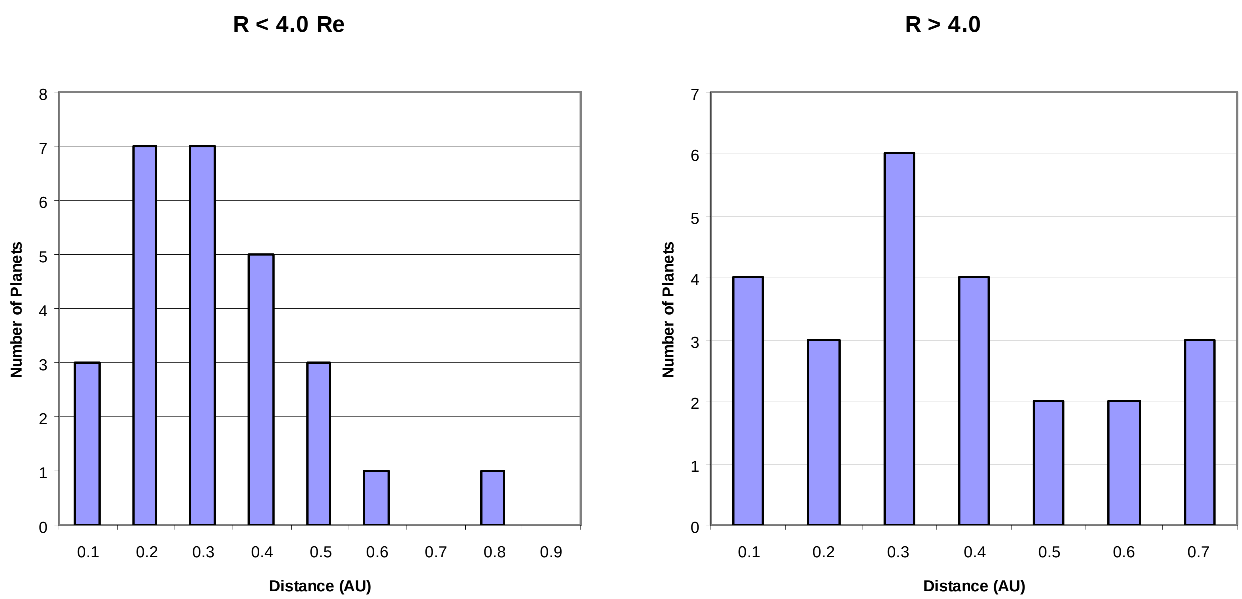
Answer Key

Problem 1 - For a planet discovered in its Habitable Zone, and to the nearest whole number, what percentage of planets are less than 4 times the radius of Earth? Answer: There are 28 planets for which R < 4.0 re, so P = 100% x (28/54) = **52%**

Problem 2 - About what is the average temperature of the planets for which R <4.0 Re? Answer; Students will identify the 28 planets in the table that have R < 4.0, and then average the planet's temperatures in Column 6. **Answer: 317 K**.

Problem 3 - About what is the average temperature of the planets for which R >4.0 Re? Students will identify the 26 planets in the table that have R > 4.0, and then average the planet's temperatures in Column 6. **Answer: 306 K**.

Problem 4 - Create two histograms of the number of planets in each distance zone between 0.1 and 1.0 AU using bins that are 0.1 AU wide. Histogram-1: for the planets with R > 4.0 Re. Histogram-2 for planets with R < 4.0 Re. Can you tell whether the smaller planets favor different parts of the Habitable Zone than the larger planets? Answer; They tend to be found slightly closer to their stars, which is why in Problem 2 their average temperatures were slightly hotter than the larger planets.

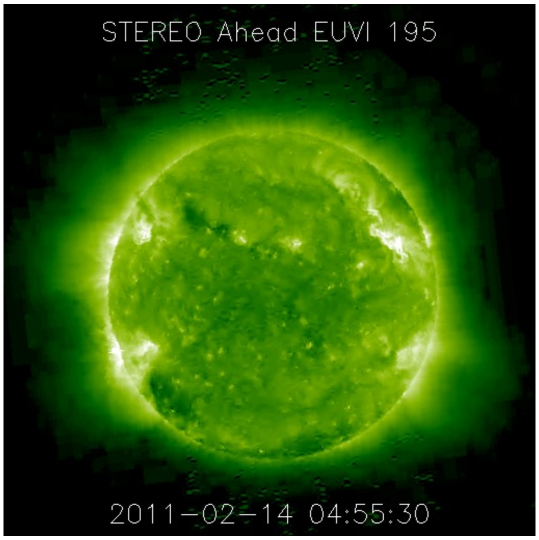


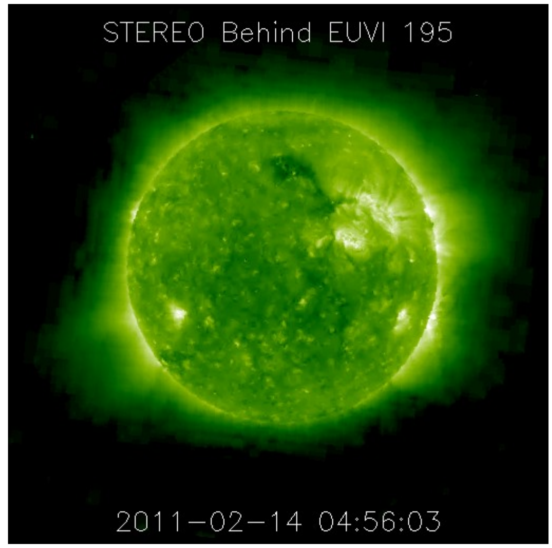
Problem 5 - If you were searching for Earth-like planets in our Milky Way galaxy, which contains 40 billion stars like the ones studies in the Kepler survey, how many do you think you might find in our Milky Way that are at about the same distance as Earth from its star, about the same size as Earth, and about the same temperature (270 - 290 K) if 157,453 stars were searched for the Kepler survey?

Answer: Students may come up with a number of different strategies and estimates. For example, they might create Venn Diagrams for the data in the table that meet the criteria given in the problem. Then, from the number of planets in the intersection, find their proportion in the full sample of 54 planets, then multiply this by the ratio of 40 billion to 157,453. Estimates near 1 million are in the right range.

Table of Habitable Zone Candidates

	Planet Name (KOI)	Orbit Period (days)	Distance To Star (AU)	Planet Radius (Re)	Planet Temp. (K)	Star Temp. (K)
1	683.01	278	0.84	4.1	239	5,624
2	1582.01	186	0.63	4.4	240	5,384
3	1026.01	94	0.33	1.8	242	3,802
4	1503.01	150	0.54	2.7	242	5,356
5	1099.01	162	0.57	3.7	244	5,665
6	854.01	56	0.22	1.9	248	3,743
7	433.02	328	0.94	13.4	249	5,237
8	1486.01	255	0.80	8.4	256	5,688
9	701.03	122	0.45	1.7	262	4,869
10	351.01	332	0.97	8.5	266	6,103
11	902.01	84	0.32	5.7	270	4,312
12	211.01	372	1.05	9.6	273	6,072
13	1423.01	124	0.47	4.3	274	5,288
14	1429.01	206	0.69	4.2	276	5,595
15	1361.01	60	0.24	2.2	279	4,050
16	87.01	290	0.88	2.4	282	5,606
17	139.01	225	0.74	5.7	288	5,921
18	268.01	110	0.41	1.8	295	4,808
19	1472.01	85	0.37	3.6	295	5,455
20	536.01	162	0.59	3.0	296	5,614
21	806.01	143	0.53	9.0	296	5,206
22	1375.01	321	0.96	17.9	300	6,169
23	812.03	46	0.21	2.1	301	4,097
24	865.01	119	0.47	5.9	306	5,560
25	351.02	210	0.71	6.0	309	6,103
26	51.01	10	0.06	4.8	314	3,240
27	1596.02	105	0.42	3.4	316	4,656
28	416.02	88	0.38	2.8	317	5,083
29	622.01	155	0.57	9.3	327	5171
30	555.02	86	0.38	2.3	331	5,218
31	1574.01	115	0.47	5.8	331	5,537
32	326.01	9	0.05	0.9	332	3,240
33	70.03	78	0.35	2.0	333	5,342
34	1261.01	133	0.52	6.3	335	5,760
35	1527.01	193	0.67	4.8	337	5,470
36	1328.01	81	0.36	4.8	338	5,425
37	564.02	128	0.51	5.0	340	5,686
38	1478.01	76	0.35	3.7	341	5,441
39	1355.01	52	0.27	2.8	342	5,529
40	372.01	126	0.50	8.4	344	5,638
41	711.03	125	0.49	2.6	345	5,488
42	448.02	44	0.21	3.8	346	4,264
43	415.01	167	0.61	7.7	352	5,823
44	947.01	29	0.15	2.7	353	3,829
45	174.01	56	0.27	2.5	355	4,654
46	401.02	160	0.59	6.6	357	5,264
47	1564.01	53	0.28	3.1	360	5,709
48	157.05	118	0.48	3.2	361	5,675
49	365.01	82	0.37	2.3	363	5,389
50	374.01	173	0.63	3.3	365	5,829
51	952.03	23	0.12	2.4	365	3,911
52	817.01	24	0.13	2.1	370	3,905
53	847.01	81	0.37	5.1	372	5,469
54	1159.01	65	0.30	5.3	372	4,886





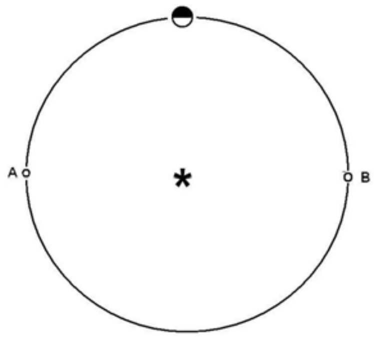
This remarkable pair of images of the sun was taken on February 14, 2011 by NASA's STEREO A (left) and B (right) spacecraft. The spacecraft are close to Earth's orbit, however as viewed looking down on Earth's orbit with the sun at the center, STEREO-A is 90-degrees counter-clockwise of Earth's orbit location, and STEREO-B is 90-degrees clockwise. This means that when the two images above are combined, the entire 360-degree span of the solar surface can be seen at the same time, making this an historical moment for Humanity.

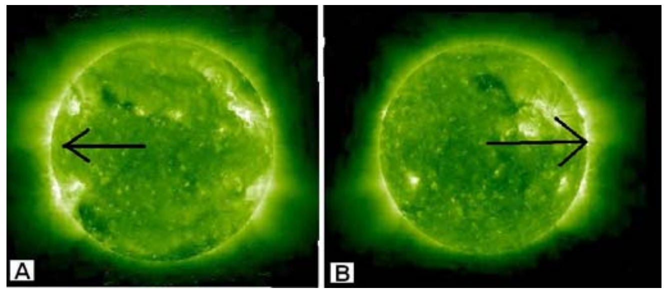
Problem 1 – From the information in the text, draw a diagram that shows the location of the sun, Earth's orbital path (assume it is a circle, whose plane passes through the equator of the sun) and the locations of the STEREO spacecraft. In each of the two images, draw an arrow that points in the direction of Earth.

Problem 2 – The images were taken in the light of iron atoms heated to 1.1 million degrees. The bright (white) features are solar 'active regions' that correspond to the locations of sunspots. At the time the images were taken, what active regions in the above images: A) Could be observed from Earth? B) Could not be observed from Earth?

Problem 3 – Relative to the location of Earth in its orbit, the STEREO-A and B spacecraft move 22 degrees farther from Earth each year. On your diagram from Problem 1, about what will be the positions of the spacecraft along Earth's orbit in February of A) 2013? B) 2015? C) 2017? and D) 2022?

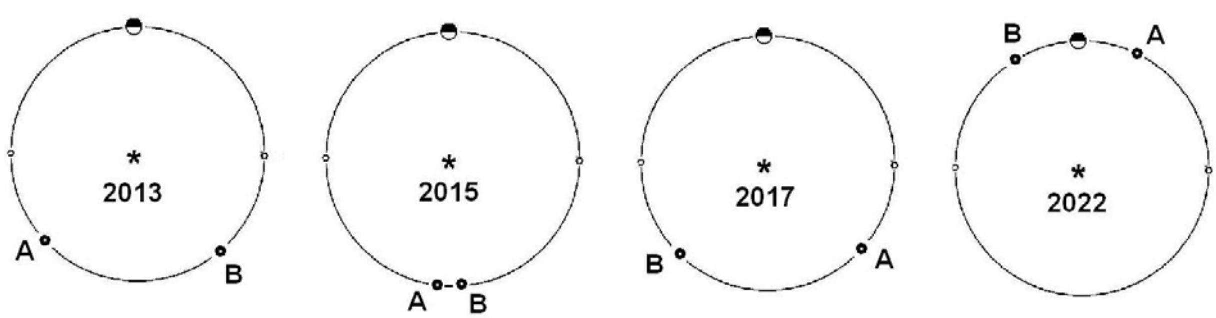
Problem 1 – From the information in the text, draw a diagram that shows the location of the sun, Earth's orbital path (assume it is a circle whose plane passes through the equator of the sun) and the locations of the STEREO spacecraft. In each of the two images, draw an arrow that points in the direction of Earth.



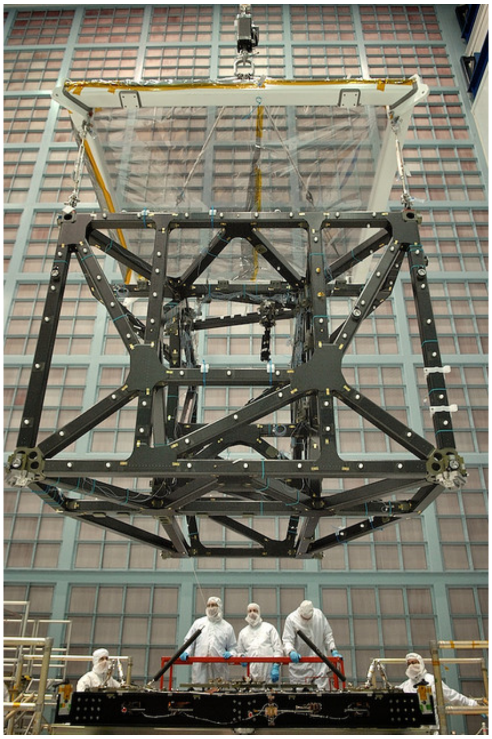


Problem 2 – The images were taken in the light of iron atoms heated to 1.1 million degrees. The bright (white) features are solar 'active regions' that correspond to the locations of sunspots. At the time the images were taken, what active regions in the above images: A) Could be observed from Earth? B) Could not be observed from Earth? **Answer A) Features on the left half of Image A and the right half of Image B. B) Features on the right half of Image A and the left half of Image B.**

Problem 3 – Relative to the location of Earth in its orbit, the STEREO-A and B spacecraft move 22 degrees farther from Earth each year. On your diagram from Problem 1, about what will be the positions of the spacecraft along Earth's orbit in February of A) 2013? B) 2015? C) 2017? and D) 2022? Answer: **If the satellites move 22 degrees along Earth's orbit each year, then for A) 2013, Spacecraft A will be (2013 - 2011) x 22 = 44 degrees counter-clockwise of its February 2011 position. Spacecraft B will be 44 degrees clockwise of its February 2011 position. B) For 2015, Spacecraft A = (2015 - 2011) x 22 = 88 degrees CCW; Spacecraft B = (2015 - 2011) x 22 = 88 degrees CW C) For 2017, Spacecraft A = (2017 - 2011) x 22 = 132 degrees CCW; Spacecraft B = (2017 - 2011) x 22 = 132 degrees CW; For 2022, Spacecraft A = (2022 - 2011) x 22 = 242 degrees CCW; Spacecraft B = (2022 - 2011) x 22 = 242 degrees CW. See diagram below**: - Angles approximate at this scale



Note: The spacecraft were launched in October 2006, so the time for the spacecraft to drift back to earth's vicinity will be 360/22 = 16 1/3 years after October 2006 or about February 2023.



The Webb Space Telescope, to be launched in 2016, will allow astronomers to see the first generations of stars that formed in the universe.

The ISIM, or the Integrated Science Instrument Module Flight Structure, will serve as the structural "heart" of the James Webb Space Telescope. It will serve as the cage in which the many sensitive instruments will analyze the ancient starlight from the distant universe.

NASA engineers recently tested the ISIM structure by exposing it to extreme cryogenic temperatures, proving that the structure will remain stable when exposed to the harsh environment of space.

The ISIM is 1.92 meters tall and has a mass of 193 kilograms. Engineers invented a new material so that the ISIM can support more than four times its own mass in sensitive scientific equipment. It also has to keep the geometry of this equipment exactly right under cold space conditions. The heating and cooling of the ISIM in space causes expansion and contraction, which can ruin the scientific measurements. The latest cold tests by NASA engineers proved that the ISIM does not change its shape by more than 150 microns as it is chilled from room-temperature (300 K) to near-absolute zero (20 K). This is very good news!

To better see how little the ISIM changed its dimensions, suppose that you created a scale model of this structure and made it the same size as a 10-story building 30 meters tall.

Problem 1 –On this scale, how much would your model of the ISIM have changed its dimensions?

Problem 2 – American engineers work in English units. If the density of the composite materials used in this structure is 0.06 pounds/inch^3, what is the density in metric units of kilograms/meter^3 if 1 pound = 0.45 kg, and 1 meter = 39.4 inches?

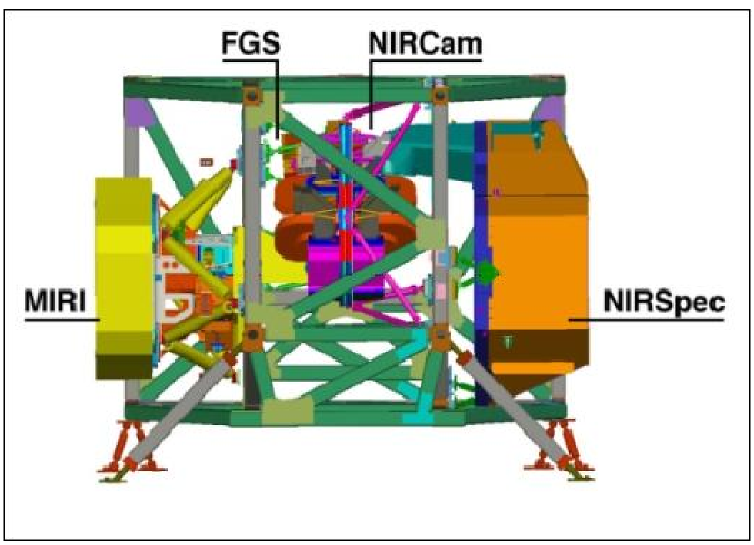
Problem 1 – On this scale, how much would your model of the ISIM have changed its dimensions?

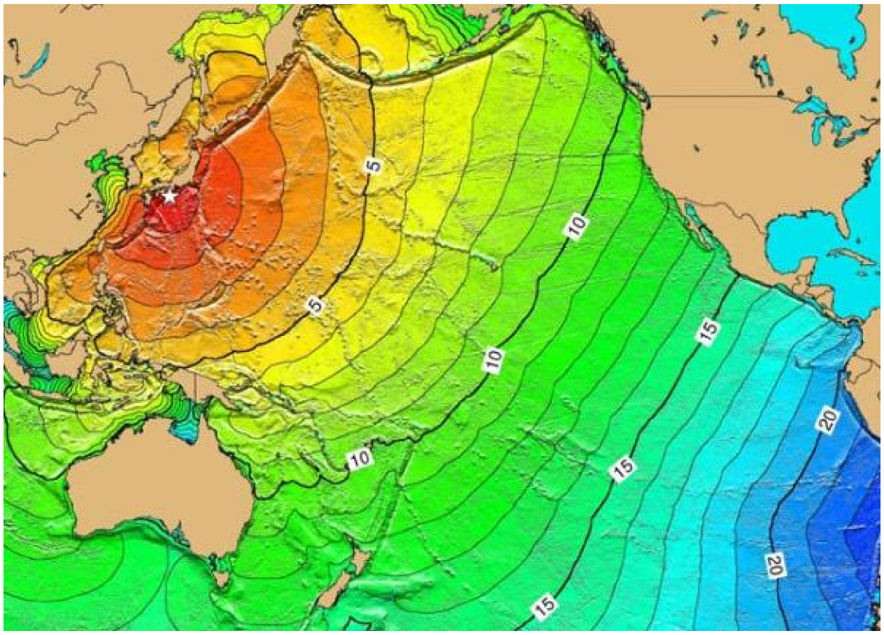
Answer: The true size of the ISIM is 1.92 meters tall, so the new scaled model is a factor of 30 meters / 1.92 meters = 15.6 times larger that the actual ISIM module. The original ISIM module changed its dimensions during cooling by 150 microns, so in the new scaled model the dimensions would have changed by 15.6 x 150 microns = 2340 microns. In terms of meters, this is just 2340 microns x (1 millimeter/1000 microns) = **2.3 millimeter**. On the scale of the 10-story building, the structure would have changed by about the thickness of two dimes!

Problem 2 – Engineers prefer to work in English units. If the density of the composite materials used in this structure is 0.06 pounds/inch^3, what is the density in metric units of kilograms/meter^3 if 1 pound = 0.45 kg, and 1 meter = 39.4 inches?

Answer: 0.06 pounds/inch^3 x (0.45 kg/1 pound) x (39.4 inches/1 meter)^3 = **1651 kg/m^3**
Note: For the correct number of significant figures (0.29 and 0.45 have 2 which is the minimum number) the answer would be **1700 kg/m^3** . By comparison, water has a density of 1000 kg/m^3.

The diagram below shows how the ISIM module will be loaded with scientific equipment to measure the faint starlight collected by the Webb Space Telescope in 2016. The instruments are very much like 'high-tech' digital cameras that can precisely measure the brightness of the light falling onto the millions of pixels in each camera system. Astronomers will use these images to discover the first generation of stars and galaxies born in our universe over 13 billion years ago.





A massive 8.9/9.0 magnitude earthquake hit the Pacific Ocean near Honshu, Japan at around 2:46 PM (Japan Standard Time) on March 11, 2011 causing damage with blackouts, fire and tsunami. The tsunami wave then raced across the Pacific Ocean and arrived at many cities around the Pacific Rim. The map above, produced by the National Oceanic and Atmospheric Administration, shows the predicted arrival times in hours for the tsunami wave.

At 7:46 AM Pacific Standard Time on March 11, it resulted in 2.5 meter (8-foot) waves in some locations in Crescent City, California. Waves as tall as 3.5 meters (11-feet) were recorded in Kealakekua Bay, Hawaii at 3:24 AM Local Hawaii Time on March 11. The distance from the epicenter near Honshu, Japan to Crescent City is 8,020 km; to Kealakekua Bay, Hawaii is 6,640 km.

Because these destinations are not all in the same time zone, we have to also include this information: 6:00 AM (Japan Standard Time) on March 11, is the same as 11:00 AM March 10 (Hawaii Standard Time) in Hawaii, and 1:00 PM, March 10 (Pacific Standard Time) in Crescent City.

Problem 1 - At what local time and date did the earthquake occur in A) Hawaii? B) Crescent City?

Problem 2 - How long did it take the tsunami to reach each of the landfalls?

Problem 3 - What was the average speed of the tsunami based on the two estimates?

Problem 1 - At what local time and date did the earthquake occur in A) Hawaii? B) Crescent City?

Answer: Working with time zone calculations, especially across the International Date Line, can be very tricky, but it is an essential skill in today's world where many people travel around the world for vacations and for their careers.
The earthquake began at 2:46 PM JST on March 11. Using the '24-hour clock' makes the calculations easier, so this becomes 14:46 JST.
Then from
06:00 March 11 (Japan Standard Time) in Japan is the same as
11:00 March 10 (Hawaii Standard Time) in Hawaii, and the same as
13:00 March 10 (Pacific Standard Time) in Crescent City,

The table below shows the relevant calculations and answers converted back to the local 12-hour clock.

Location	Distance	Difference from JST	Local Time	Date
Japan	0	0	**2:46 PM (JST)**	**March 11**
Hawaii	6,640 km	-19 hours	**7:46 PM (HST)**	**March 10**
Crescent City	8,020 km	-17 hours	**9:46 PM (PST)**	**March 10**

Problem 2 - How long did it take the tsunami to reach each of the landfalls?
Answer: Using the local times for the earthquake calculated in Problem 1, we just take the difference in local time between the landfall time and the earthquake time:

Hawaii: 03:24 March 11 - 19:46 March 10 = **7.6 hours.**
Crescent City: 07:46 March 11 - 21:46 March 10 = **10 hours**

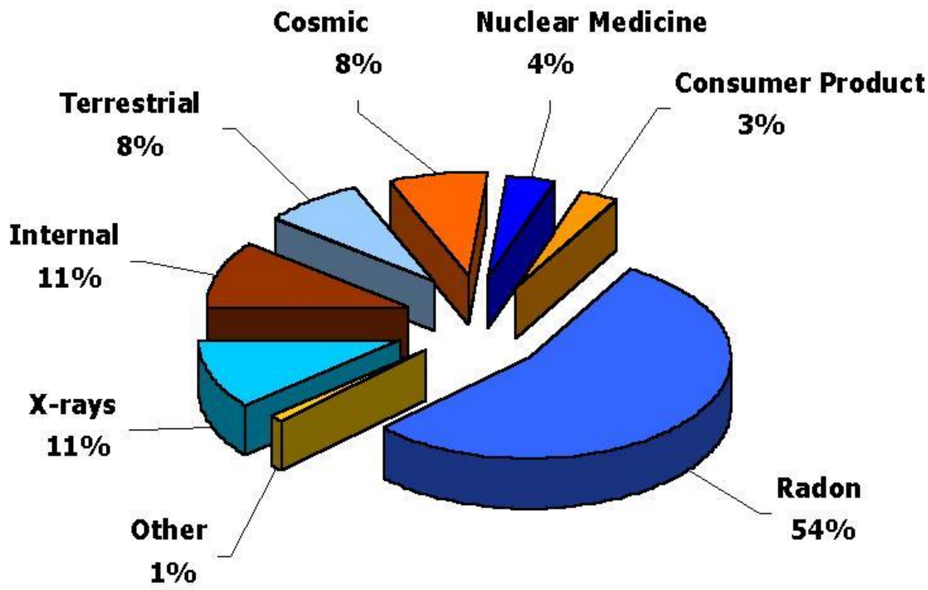
Problem 3 - What was the average speed of the tsunami based on the two estimates?

Answer:

Hawaii: 6,640 km / 7.6 hours = **873 km / hr.**
Crescent City: 8,020 km / 10 hours = **802 km/hr.**

The average speed is about (873 + 802)/2 = **837 km/hr**.

Note: The actual distance traveled by the tsunami wave is not exactly a straight line between the epicenter and the destinations. More accurate models by NOAA's Tsunami Forecasting Center predicted a wave speed closer to 800 km/hour.



Our exposure to many unavoidable sources of radiation is a fact of life, and one that can seldom be modified by simple lifestyle changes. Each year, on average, a single human is exposed to sources of radiation from the food supply, cosmic rays, the ground beneath your feet and various medical diagnostic and treatment regimens. Health physicists, professionals who monitor and determine the consequences of human radiation impacts, have estimated that the average human accumulates about 365 millirem/year (3.65 milliSeivert/year) as a typical radiation 'background' exposure. The pie graph above shows the breakdown of this exposure in terms of the known categories of sources.

Problem 1 - A health physicist wants to study the day-to-day exposure changes using a device called a dosimeter. What is the daily radiation exposure rate in A) milliRem? B) microSeiverts?

Problem 2 – On the pie graph above, three categories are natural and unavoidable (terrestrial, internal and cosmic) while four categories can be altered by lifestyle choices (Radon, X-rays, Nuclear medicine and consumer products). What is the total exposure rate from the natural, and the total exposure rate from the lifestyle contributions to your annual background exposure in A) milliRem/year B) microSeiverts/day?

Problem 3 - The Japan 2011 earthquake caused nuclear power plant exposures amounting to an additional 50 microSeiverts/hour for someone living 30 km from the Fukushima Nuclear Plant. A) How many microSeiverts is this per day? B) In terms of the three components 'Lifestyle', 'Natural' and 'Fukushima', how will the percentage contributions to the daily radiation dose change when the Fukushima source is included in terms of microSeiverts/day?

Problem 1 - A health physicist wants to study the day-to-day exposure changes using a device called a dosimeter. What is the daily radiation exposure rate in A) milliRem? B) microSeiverts?

Answer: 365 milliRem/year x (1 year/365 days) = **1 milliRem/day**. B) 3.65 milliSeiverts/year x (1 year/365 days) x (1,000 microSeiverts/1 milliSeivert) = (0.01)x (1000) = **10 microSeiverts/day**.

Problem 2 - On the pie graph, three categories are natural and unavoidable (terrestrial, internal and cosmic) while four categories can be altered by lifestyle choices (Radon, X-rays, Nuclear medicine and consumer products). What is the total exposure rate from the natural, and the total exposure rate from the lifestyle contributions to your annual background exposure in A) milliRem/year B) microSeiverts/day?

Answer:

Unavoidable	**Lifestyle**
Terrestrial = 8%	Radon = 54%
Internal = 11%	X-rays = 11%
Cosmic = 8%	Medicine = 4%
	Consumer = 3%
Total = 27%	**Total = 72%**

A) For 365 milliRem/year:

Unavoidable = 0.27 x 365 milliRem/year = **99 milliRem/year**

Lifestyle = 0.72 x 365 milliRem/year = **263 milliRem/year**

B) For 10 microSeiverts/day:

Unavoidable = 0.27 x 10 = **2.7 microSeiverts/day**

Lifestyle = 0.74 x 10 = **7.4 microSeiverts/day**.

Problem 3 - The Japan 2011 earthquake caused nuclear power plant exposures amounting to an additional 50 microSeiverts/hour for some one living 30 km from the Fukushima Nuclear Plant. A) How many microSeiverts is this per day? B) In terms of the three components 'Lifestyle', 'Unavoidable' and 'Fukushima', how will the percentage contributions to daily dosage change when the Fukushima source is included in terms of microSeiverts/day?
Answer: If the pie graph is based upon 10 microSeiverts/day total, of which 2.7 microSeiverts/day are unavoidable background radiation, and 7.4 microSeiverts/day are from various Lifestyle choices. Then:

A) The Fukushima radiation at 30 kilometers is 50 microSeiverts/hour or in terms of a daily dosage, 24 hours x 50 microSeiverts/hour = **1200 microSeiverts/day.**

B) The total dosage would be 2.7 + 7.4 + 1200 = 1210 microSeiverts/day. The contributions would then be
Unavoidable = 100% x (2.7/1210) so **Unavoidable = 0.2 %**
Lifestyle = 100% x (7.4/ 1210) so **Lifestyle = 0.6 %**
Fukushima = 100% (1200/1210 so **Fukushima = 99%**

Lifestyles and Radiation Dose

Factor	Dose Rate
Lives in Denver, Colorado	0.90 milliSv/yr
Air travel	5 microSv/hr
Live in a brick/stone house	0.70 milliSv/yr
One CT scan	7 milliSv/scan
Radon gas in basement	1.50 milliSv/yr
Eating one banana	0.1 microSv
Your body	1.5 microSv/day
Smoke 1 pack a day	10 milliSv/day
Lives in Boston, Mass.	0.20 milliSv/yr
Astronaut on Space Station	1.2 milliSv/day

The amount of radiation you receive each year depends on where you live and your lifestyle. Do you live near the coast or in the mountains? Do you travel by jet a lot? Do you have frequent medical diagnostic tests? All of these factors will add together to change your annual radiation dose.

The chart to the left gives a few common contributors to your dose. These include both geographic and lifestyle factors. The units include both dose and dosage rates as appropriate to the factor.

Problem 1 - In 2011, Anders Olssen lived in Denver, Colorado for 6 months then moved to Boston, Massachusetts for the remainder of the year. He was a non-smoker, who lived in a quaint stone house on Beacon Hill. He telecommuted 8 hours a day from his basement office, and enjoyed a banana for lunch every day. What is the total radiation dose for this individual in an average year, in units of milliSeiverts?

Problem 2 - Create a more complicated history for Anders by including a stint on the International Space Station, or including possible medical diagnostic procedures!

Factor	Dosage
Lives in Denver, Colorado	0.90 milliSv/yr
Air travel	5 microSv/hr
Live in a brick/stone house	0.70 milliSv/yr
One CT scan	7 milliSv/scan
Radon gas in basement	1.50 milliSv/yr
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Smoke 1 pack a day	10 milliSv/day
Lives in Boston, Mass.	0.20 milliSv/yr
Astronaut on Space Station	1.2 milliSv/day

Problem 1 - In 2011, Anders Olssen lived in Denver, Colorado for 6 months then moved to Boston, Massachusetts for the remainder of the year. He was a non-smoker, who lived in a quaint stone house on Beacon Hill. He telecommuted 8 hours a day from is basement office, and enjoyed a banana for lunch every day. What is the total radiation dose for this individual in an average year, in units of Seiverts?

Answer: Denver for 6 months = 0.90 milliSv/yr x 0.5 yr = 0.45 milliSeiverts.
Boston for 6 months = 0.20 milliSV/yr x 0.5 years = 0.1 milliSeiverts
Stone house living = 0.70 milliSv/yr x 1 year = 0.70 milliSeiverts
8 hr/day exposure to Radon gas: 1.50 milliSV/yr x 1/3 = 0.50 milliSeiverts
1 banana a day = 0.1 microSv/banana x 365 banana = 36.5 microSeiverts

We also have to add the radiation from his own body!
= 1.5 microSV/day x 365 days = 0.55 milliSeiverts

The total is 0.45 + 0.1 + 0.7 + 0.5 + 0.036 + 0.55 = **2.3 milliSeiverts for a full year**.

Note: typical values on the surface of Earth are between 2.0 and 4.0 milliSeiverts/year, however some inhabited locations on Earth have total rates as high as 7.0 milliSeiverts/year because communities are built on uranium-rich sands, soils and granite deposits.

Problem 2 - Remember that when the astronaut is on the space station, you do not include his exposure to ground-level sources such as radon gas, or living in a different geographic location, during the in-space portion of his life!

The image of Mercury's surface on the left was taken by the MESSENGER spacecraft on March 30, 2011 of the region near crater Camoes near Mercury's south pole. In an historic event, the spacecraft became the first artificial satellite of Mercury on March 17, 2011. The image on the right is a similar-sized area of our own Moon near the crater King, photographed by Apollo 16 astronauts.

The Mercury image is 100 km wide and the lunar image is 115 km wide.

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Problem 1 – Using a millimeter ruler, what is the scale of each image in meters/millimeter?

Answer: Mercury image width is 80 mm, scale is 100 km/80mm = **1.3 km/mm**
Moon image width = 68 mm, scale is 115 km/68mm = **1.7 km/mm**

Problem 2 – What is the width of the smallest crater, in meters, you can find in each image?

Answer: Mercury = 0.5 mm x (1.3 km/mm) = **700 meters**.
Moon = 1.0 mm x (1.7 km/mm) = **1,700 meters**.

Problem 3 – The escape velocity for Mercury is 4.3 km/s and for the Moon it is 2.4 km/s. Why do you suppose there are more details in the surface of Mercury than on the Moon?

Answer: **On Mercury, less of the material ejected by the impact gets away, and so more of it falls back to the surface near the crater. For the Moon, the escape velocity is so low that ejected material can travel great distances, or even into orbit and beyond, so less of it falls back to the surface to make additional craters**.

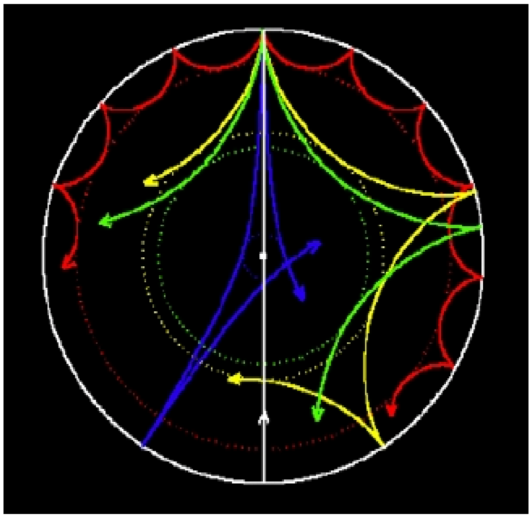
Problem 4 – The diameter of Mercury is 1.4 times the diameter of the Moon. From the equation for the volume of a sphere, by what factor is the volume of Mercury larger than the volume of the Moon?

Answer: The volume of a sphere is given by $4/3 \pi R^3$, so if you increase the radius of a sphere by a factor of 1.4, you will increase its volume by a factor of 1.4^3 = 2.7 times, so the volume of Mercury is **2.7 times** larger than the volume of the Moon.

Problem 5 – If mass equals density times volume, and the average density of Mercury is 5400 kg/m^3 while for the Moon it is 3400 kg/m^3, by what factor is Mercury more massive than the Moon?

Answer: The density of Mercury is a factor of 5400/3400 = 1.6 times that of the Moon. Since the volume of Mercury is 2.7 times larger than the Moon, the mass of Mercury will be density x volume or 1.6 x 2.7 = **4.3 times than of the Moon**.

Note: Actual masses for Mercury and the Moon are 3.3×10^{23} kg and 7.3×10^{22} kg respectively, so that numerically, Mercury is 4.5 times the Moon's mass...which is close to our average estimate.



Sound waves from turbulent activity on the star's Surface are reflected through the interior along a variety of paths. The surface layer, producing the light we see, moves up and down in complex ways that cause brightness changes over many different time scales from seconds to minutes.

One of the hallmarks of a truly successful space mission is that, not only does it meet all of its planned scientific goals for which it was designed, but that it contributes to advancements in science well-beyond the perimeter of its own specialized research areas.

Recently, the Kepler satellite, which was designed to detect Earth-sized planets orbiting other stars, has made a series of discoveries that have revolutionized the subject of stellar structure and evolution.

Using the minute brightness changes that indicate sound waves traveling in the interiors of giant stars, Kepler data has now confirmed a fundamental hypothesis of stellar evolution proposed by physicist George Gamow nearly 75 years ago!

As stars no longer have enough hydrogen fuel in their cores to sustain them, they switch to fusing hydrogen into helium in a thin shell just outside the core. This shell-burning phase is the prelude to the star becoming a red giant or red supergiant star, depending on its mass. By measuring the brightness changes in many of these red giant stars, Kepler data has been used to probe the interior structure of these stars, revealing just such a shell-burning zone! The sound waves that bounce from the surface to the inner shell are reflected back to the surface where that cause minute brightness changes over time. These 'seismic waves' can be modeled to reveal the location, thickness and depth of the shell-burning zones.

Problem 1 - Draw a circle to represent a star's surface. Draw a series of semi-circles of different radii similar to the ones in the diagram above, but where the sequence of circles along the circumference have the same ending point as its starting point. This represents a 'standing wave' on the surface. The point on the surface where the arc meets the surface is called the 'node'. How many nodes do you count for circles of the different radii that you create?

Problem 2 - If the location of the shell-burning zone outer edge is half-way to the center of the star, for the arc that just touches the shell-zone, how far apart will the nodes be on the surface of the star, if the radius of the star is 5 million kilometers?

Problem 3 - If the speed of the wave is 1,000 km/sec, how many seconds elapse in the travel between the nodes, and what would you see in the light from this star?

Problem 1 - Draw a circle to represent a star's surface. Draw a series of semi-circles of different radii similar to the ones in the diagram above, but where the sequence of circles along the circumference have the same ending point as its starting point. This represents a 'standing wave' on the surface. The point on the surface where the arc meets the surface is called the 'node'. How many nodes do you count for circles of the different radii that you create?

Answer: For example, divide the circle into four equal quadrants. Place the compass in the upper right quadrant at a position on the circumference that is half-way between the two points where the horizontal and vertical lines intersect the circumference. Set your compass radius so that it touches one of these points, then draw the interior arc. Repeat this for the other three quadrants. Divide the circle into five equal parts and repeat this construction process. The number of nodes for each compass radius length are shown in the table below. The arc radius is given in terms of the length of the radius of the circle taken as R=1.0.

Nodes	Number of arcs	Radius of arc
2	**2**	**1.414**
3	**3**	**1.000**
4	**4**	**0.765**
5	**5**	**0.618**
6	**6**	**0.518**
7	**7**	**0.445**
8	**8**	**0.390**

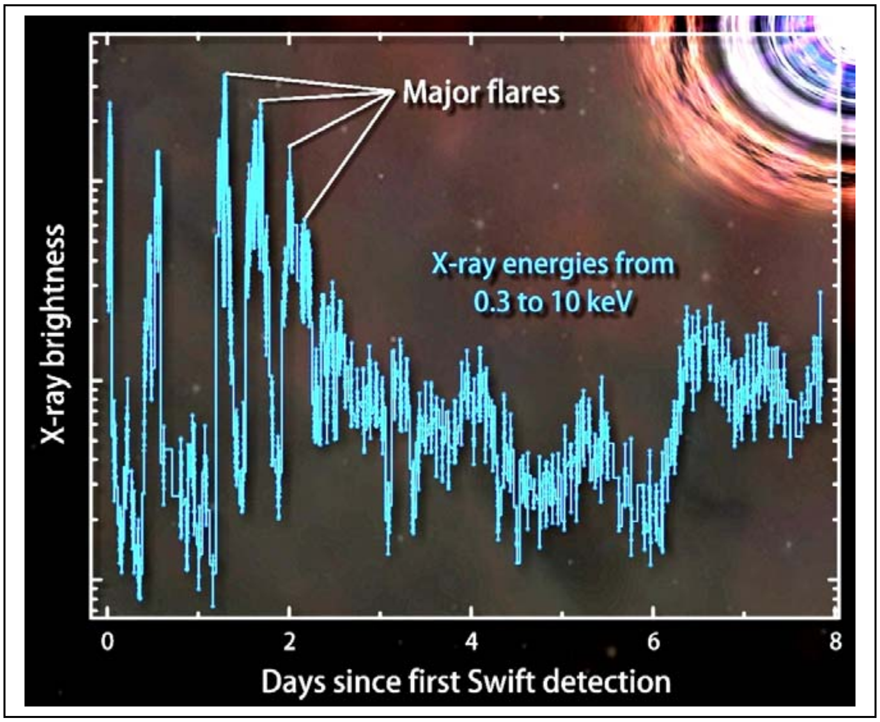
Students may prove that, if the radius of the circle is defined as R, and the number of nodes on the circumference is N, the general formula for the arc radius is just

$$d = R\sqrt{2 - 2\cos\left(\frac{\pi}{N}\right)}$$

Problem 2 - If the location of the shell-burning zone outer edge is half-way to the center of the star, for the arc that just touches the shell-zone, how far apart will the nodes be on the surface of the star, if the radius of the star is 5 million kilometers?

Answer: If the shells outer edge is half way to the center of the star, its radial location is 0.500 in units of the star's radius. The closest wave arc that come close to this is for the Node=6 arc with R = 0.518. The full circumference of the star's circle is 2π, so the nodes are located along the circumference a distance of $2\pi/N = 2\pi/6$ or $\pi/3$ apart in multiples of the radius of the star. Since the star's radius is 5 million km, the nodes are located (3.141) x (0.333) x (5million km) = **5.22 million km apart**.

Problem 3 - If the speed of the wave is 1,000 km/sec, how many seconds elapse in the travel between the nodes, and what would you see in the light from this star? Answer: The wave travels along the arc between the node points, whose radius is 5.2 million km, and length is 3.141 x (5.22 million km) = 16.4 million km.
T = 16.4 million km/(1000 km/s) = 16,400 seconds or about 4.6 hours.
The light from the star may 'flicker' slightly in brightness with a period of 4.6 hours.



On March 28, 2011 Swift's Burst Alert Telescope discovered the source in the constellation Draco when it erupted with the first in a series of powerful X-ray blasts. The satellite determined a position for the explosion, now cataloged as gamma-ray burst (GRB) 110328A, and informed astronomers worldwide. As dozens of telescopes turned to study the spot, astronomers quickly noticed that a small, distant galaxy appeared very near the Swift position. A deep image taken by Hubble on April 4 pinpoints the source of the explosion at the center of this galaxy, which lies 3.8 billion light-years away.

That same day, astronomers used NASA's Chandra X-ray Observatory to make a four-hour-long exposure of the puzzling source. The Chandra image shows that it lies at the center of the galaxy that Hubble imaged.

The duration of the x-ray bursts tells astronomers approximately how large the emitting region is. It cannot be larger than the time it takes light to cross its width, otherwise you will not see a well-formed pulse of light. This also lets us estimate the maximum diameter of the black hole and the mass of the black hole. The radius of a black hole is related to its mass by the simple formula R = 3 M, where M is the mass of the black hole in units of the sun's mass, and R is the radius of the Event Horizon in kilometers.

Problem 1 - What is the average duration of the four flare events seen in the X-ray plot above?

Problem 2 - Light travels at a speed of 300,000 km/s. How many kilometers across is the x-ray emitting region based on the average time of the three x-ray flares?

Problem 3 - The size of the x-ray emitting region from Problem 2 is a crude estimate for the diameter of the black hole. For reasons having to do with relativity, a better black hole size estimate will be 100 times smaller than your answer for Problem 2. From this better-estimate, about what is the mass of the black hole GRB110328A in solar masses?

Problem 1 - What is the average duration of the three flare events seen in the X-ray plot above?

Answer: There were about 3 flares in one day, so the average flare duration is about **8 hours**.

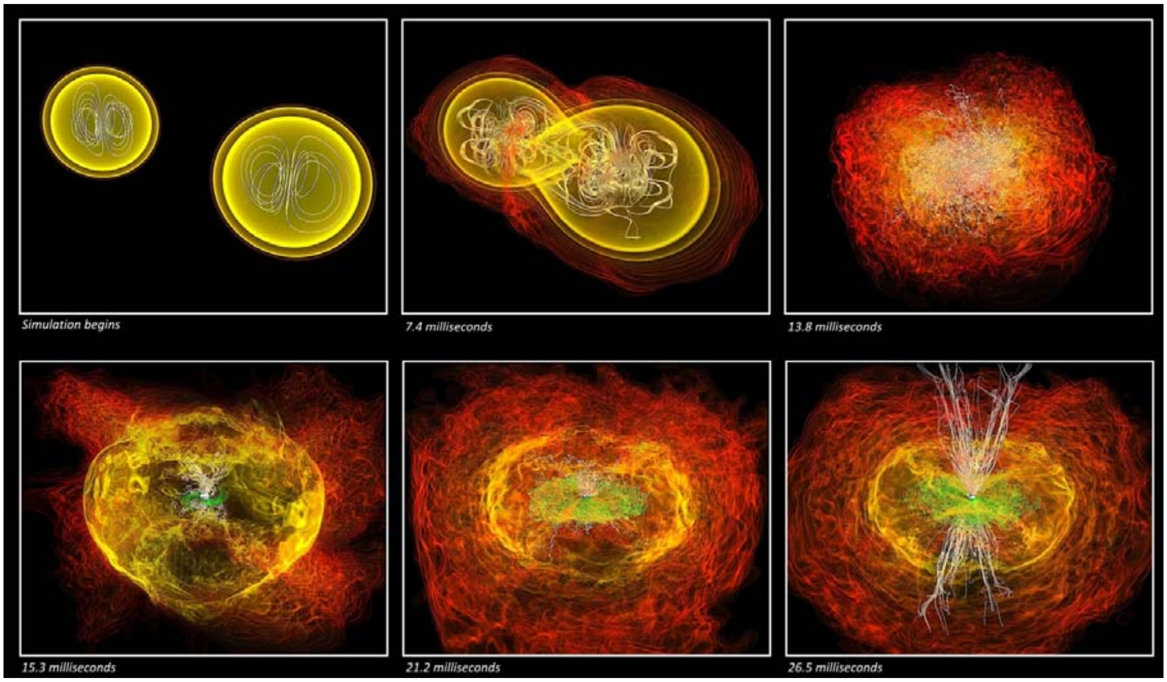
Problem 2 - Light travels at a speed of 300,000 km/s. How many kilometers across is the x-ray emitting region based on the average time of the three x-ray flares?

Answer: Distance = speed x time, so D = 300,000 km/s x 8 hours x (3600 sec/1 hour) = **8.6 billion kilometers.**

Problem 3 - The size of the x-ray emitting region from Problem 2 is a crude estimate for the diameter of the black hole. For reasons having to do with relativity, a better black hole size estimate will be 100 times smaller than your answer for Problem 2. From this better-estimate, about what is mass of the black hole in solar masses?

Answer: If 8.6 billion kilometers is the width of the emitting region, then the radius of the region is about 4.3 billion kilometers, and the estimated radius of the black hole is about 100 times smaller than this or 43 million kilometers. Since the radius of a black hole is R = 3 x M, the mass of the black hole is 43 million = 3 x M, or M = **14 million solar masses**.

Note: Astrophysicists have studied and modeled these kinds of events for decades, and it is generally agreed that gamma-ray bursts are probably caused by beams of particles and radiation leaving the vicinity of the black hole. Because of this, the estimated light-travel size of the emitting region from the changes in the gamma ray or x-ray brightness will greatly over-estimate the actual size of the emitting region. The 'factor of 100' is added to this calculation to account for this 'beaming' effect. Actual astrophysical models of these regions that take into account relativity physics are still in progress and will eventually lead to much better estimates for the black hole size and mass. Also, the relationship between black hole radius and mass that we used only works for black holes that do not rotate, called 'Schwarschild Black Holes'. In actuality, we expect most black holes to be rotation, at speeds that are perhaps even near the speed of light, and these will be significantly larger in size. These are called Kerr Black Holes.



Once astrophysicists understand the physics involved, they can use supercomputers to model events that can never be observed first-hand. The panel of images above was created by Drs Koppits and Rezolla, and shows what happens to two neutron stars when they collide and merge. They wanted to find out whether the magnetic fields re-arrange themselves to allow jets of matter to be 'beamed out' of the collision area. Each image, from the upper left to the lower right, represents snapshots of the event calculated by the supercomputer at 0.0, 0.007, 0.014, 0.015, 0.021 and 0.026 seconds. The width of each image is about 48 kilometers. Each neutron star, the dense core of a star after a supernova, has a mass of about 1.5 times our own sun.

Problem 1 - How fast were the two neutron stars approaching each other between 0.007 and 0.014 seconds after the start of the calculation?

Problem 2 - The radius of a black hole with a mass of M solar masses is given by the formula R = 3.0 M, where R is the radius of the event horizon in kilometers. If the last calculated image at 0.026 seconds represents the final size of the neutron star merger, will it become a black hole?

Problem 3 - The double cone formed by the neutron star's magnetic field is shown in the last computed image at 0.026 seconds after the start of the collision. Astrophysicists predict that high-speed particles and radiation will flow out of the neutron stars within this 'beaming channel' formed by these concentrated magnetic fields. From the measured angle, and by using any method, what will be the width of this beam by the time it leaves its galaxy and reaches our Milky Way located 5 billion light years away in A) light years? B) Milky Way diameters if 1 MW = 100,000 light years?

Problem 1 - How fast were the two neutron stars approaching each other between 0.007 and 0.014 seconds after the star of the calculation?

Answer: The width of each image is 48 kilometers, and using a millimeter ruler, the width is about 48 millimeters, so the scale is 1 kilometer/mm. The center to center separation between the neutron stars was 18 millimeters or 18 kilometers at 0.007 seconds, and 0 millimeters at 0.014 seconds, so the distance traveled was 18 kilometers in (0.014 – 0.007) = 0.007 seconds. The speed is then S = 18 km/0.007 sec or **2600 kilometers/sec**.

Problem 2 - The radius of a black hole with a mass of M solar masses is given by the formula R = 3.0 M, where R is the radius of the event horizon in kilometers. If the last calculated image at 0.026 seconds represents the final size of the neutron star merger, will it become a black hole?

Answer; If no mass is lost, the final mass is 3.0 solar masses, so the radius of the black hole will be R = 9 kilometers. For a black hole to form, the mass must be located inside this radius. The last image shows that the matter in the collision has an extent of about 20 km, so a black home will **probably not form**.

Problem 3 - The double cone formed by the neutron star's magnetic field is shown in the last computed image at 0.026 seconds. Astrophysicists predict that high-speed particles and radiation will flow out of the neutron stars within this 'beaming channel' formed by these concentrated magnetic fields. From the measured angle, and by using any method, what will be the width of this beam by the time it leaves its galaxy and reaches our Milky Way located 5 billion light years away in: A) light years? B) Milky Way diameters if 1 MW = 100,000 light years?

Answer: Students may use a protractor to measure this angle. Answers near 30 degrees are acceptable.

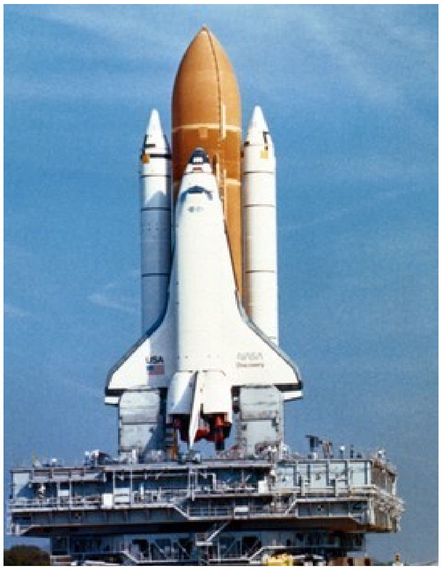
A) Method 1: Students may draw a scaled drawing of this event.

Method 2 : The full circumference of the circle is 360 degrees, so a 30-degree segment represents about (30/360) = 1/12 of the full circumference. The circumference is 2 p d = 6.24 x 5 billion = 31 billion light years, so 1/12 of this is **about 2.6 billion light years**.

Method 3: The tangent of one-half the angle is equal to the ½ of the width of the beam divided by the distance to the Milky Way, so w = 2 d x Tan(θ/2). For θ = 30 and d = 5 billion light years, w = **2.7 billion light years**.

B) Width = 2.7 billion x (1 MW/100,000 ly) = **27,000 Milky Way diameters!**

Note: The outflowing energy tends to be even more tightly beamed than this rather large angle. Estimates of about 0.6 degrees are not uncommon when actual beaming events are observed and analyzed. The width of this beam at a distance of 5 billion light years is more like (0.6/30)x2.7 billion light years or about 50 million light years. This is about 500 times the width of the Milky Way!



A common misconception shared by many students, and perhaps some members of the public, is that the Space Shuttle could have been used to travel to the Moon.

A few simple math problems will show that this could never have been a possibility, even if engineers had decided to 'upgrade' the Shuttle. Here are some basic facts:

Lunar distance : 384,000 km

Shuttle Orbital Speed : 28,000 km/h (17,500 mph)

Maximum cargo mass : 25,000 kg (55,000 Lb)

Astrodynamicists are physicists and engineers who calculate the trajectories and orbits for spacecraft throughout the solar system. Near Earth, we can think of successive orbits at increasing distance from Earth as representing a ladder. A spacecraft needs to expend more energy the higher up the ladder its orbit is from the surface of earth. This means that more distant orbits require larger launch vehicles and longer 'burn' times to get to them, than orbits close to Earth. Astrodynamicists think of the process of changing from one orbit to the other as a series of speed (velocity) changes. For example, the orbit of the Space Shuttle at an altitude of 380 km has an orbital speed of 7.68 km/s. The more distant orbits of the commercial geostationary communications satellites, located 35,800 km from Earth's surface, represent an orbital speed of 3.07 km/sec.

You might think that, since the GEO satellite orbit speed is slower than the Space Shuttle, all you have to do is 'slow down' the Space Shuttle by decreasing its kinetic energy, and it will move out to GEO satellite orbits. In fact, because of the way that kinetic energy changes in a gravitational field, you actually have to increase the kinetic energy of the Space Shuttle to make this orbit change. This is done by turning on its rockets for enough time to move it outwards from Earth. Once its orbit speed has dropped 4.61 km/s to the new value of 3.07 km/s, it will find itself at GEO orbit altitude.

Problem 1 – The Moon is 'located' at an orbit speed of 1.0 km/sec. What must be the Space Shuttle speed change to reach lunar orbit?

Problem 2 – When the Shuttle Orbital Maneuvering System (OMS) is turned on, it can cause a speed change of 0.6 meter/s for every second that the engines are burning. How many seconds would the OMS have to remain on in order for the Space Shuttle to build up the necessary velocity change to reach the Moon?

Problem 3 – The OMS can produce a maximum delta-V of 1,000 m/s before consuming all of its 4,400 kg (9,700 pounds) of fuel. How many pounds of fuel will the OMS have to expend to get to the Moon?

Problem 4 – Does the Space Shuttle have the payload capacity to carry enough extra fuel for this one-way trip?

Answer Key

Problem 1 – The Moon is 'located' at an orbit speed of 1.0 km/sec. What must be the Space Shuttle speed change to reach lunar orbit?

Answer: Delta-V = 1.0 – 7.68 = **- 6.68 km/sec**.

Problem 2 – When the Shuttle Orbital Maneuvering System (OMS) is turned on, it can cause a speed change of 0.6 meter/s for every second that the engines are burning. How many seconds would the OMS have to remain on in order for the Space Shuttle to build up the necessary velocity change to reach the Moon?

Answer: The delta-V to get to the moon is 6,680 meters/sec. The OMS can produce 0.6 m/s every second, so the total burn time would have to be T = 6,680 / 0.6 or about **11,000 seconds or 3 hours of continuous thrust.**

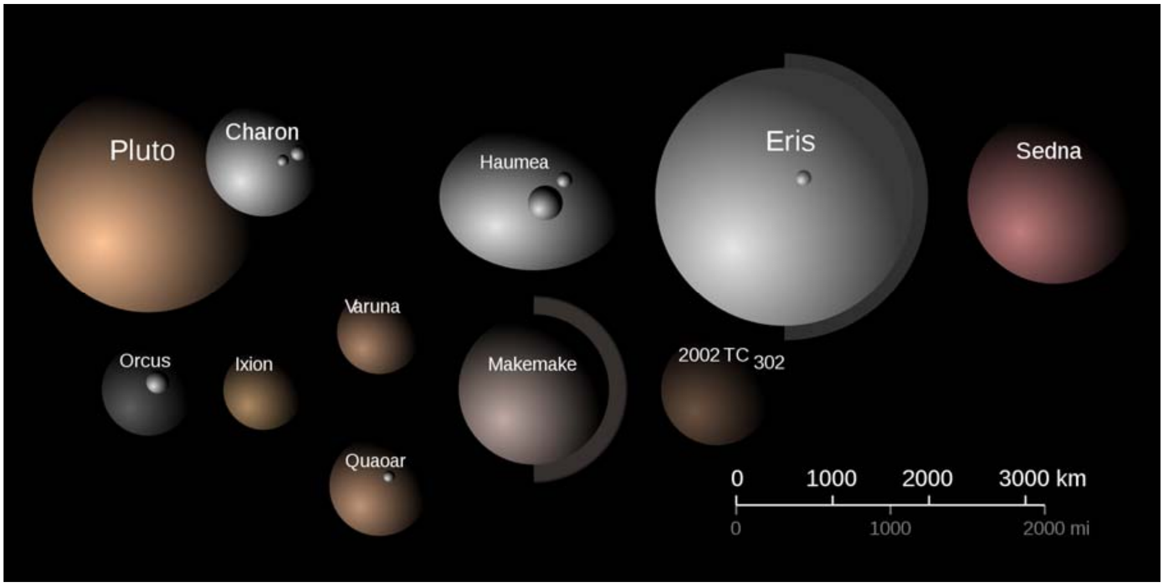
Problem 3 – The OMS can produce a maximum delta-V of 1,000 m/s before consuming all of its 9,700 pounds of fuel. How many pounds of fuel will the OMS have to expend to get to the Moon?

Answer: We need a total delta-V of 6,680 m/s, so 9,700 pounds x (6,680/1,000) = **64,796 pounds of fuel.**

Problem 4 – Does the Space Shuttle have the payload capacity to carry enough extra fuel for this one-way trip?

Answer: The maximum cargo mass is 55,000 pounds, so the OMS fuel would require 64,796 / 55,000 = 1.2 times the maximum load of the Space Shuttle cargo bay. **No, the Shuttle does not have enough capacity to lift all of the required OMS fuel to Earth orbit.**

Dwarf Planets and Kepler's Third Law



Object	Distance (AU)	Period (years)
Mercury	0.4	0.24
Venus	0.7	0.61
Earth	1.0	1.0
Mars	1.5	1.88
Ceres	2.8	4.6
Jupiter	5.2	11.9
Saturn	9.5	29.5
Uranus	19.2	84.0
Neptune	30.1	164.8
Pluto	39.4	247.7
Ixion	39.7	
Huya	39.8	
Varuna	42.9	
Haumea	43.3	285
Quaoar	43.6	
Makemake	45.8	310
Eris	67.7	557
1996-TL66	82.9	
Sedna	486.0	

Note: Distances are given in Astronomical Units (AU) where 1 AU = earth-sun distance of 150 million km.

Astronomers have detected over 500 bodies orbiting the sun well beyond the orbit of Neptune. Among these 'Trans-Neptunian Objects (TNOs) are a growing number that rival Pluto in size. This caused astronomers to rethink how they should define the term 'planet'.

In 2006 Pluto was demoted from a planet to a dwarf planet, joining the large asteroid Ceres in that new group. Several other TNOs also joined that group, which now includes five bodies shown highlighted in the table. A number of other large objects, called Plutoids, are also listed.

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit: A) Polynomial function? B) Power-law function?

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers?

Problem 3 - Using your best-fit model, what would you predict for the periods of the TNOs in the table?

Problem 1 - From the tabulated data, graph the distance as a function of period on a calculator or Excel spreadsheet. What is the best-fit:

A) Polynomial function? **The N=3 polynomial $D(x) = -0.0005x^3 + 0.1239x^2 +2.24x - 1.7$**

B) Power-law function? **The N=1.5 powerlaw: $D(x) = 1.0x^{1.5}$**

Problem 2 - Which of the two possibilities can be eliminated because it gives unphysical answers? The two predictions are shown in the table:

Object	Distance	Period	N=3	N=1.5
Mercury	0.4	0.24	-0.79	0.25
Venus	0.7	0.61	-0.08	0.59
Earth	1	1	0.66	1.00
Mars	1.5	1.88	1.93	1.84
Ceres	2.8	4.6	5.53	4.69
Jupiter	5.2	11.9	13.22	11.86
Saturn	9.5	29.5	30.33	29.28
Uranus	19.2	84	83.44	84.13
Neptune	30.1	164.8	164.34	165.14
Pluto	39.4	247.7	248.31	247.31
Ixion	39.7		251.21	250.14
Huya	39.8		252.19	251.09
Varuna	42.9		282.94	280.99
Haumea	43.3	285	286.99	284.93
Quaoar	43.6		290.05	287.89
Makemake	45.8	310	312.75	309.95
Eris	67.7	557	562.67	557.04
1996-TL66	82.9		750.62	754.80
Sedna	486		-27044.01	10714.07

Answer: The N=3 polynomial gives negative periods for Mercury, Venus and Sedna, and poor answers for Earth, Mars, Ceres and Jupiter compared to the N=3/2 power-law fit. The N=3/2 power-law fit is the result of Kepler's Third Law for planetary motion which states that the cube of the distance is proportional to the square of the period so that when all periods and distances are scaled to Earth's orbit, $\text{Period} = \text{Distance}^{3/2}$

Problem 3 - See the table above for shaded entries

Celestial Fireworks Near Cluster NGC-3603

This young star cluster, barely 1 million years old, is still surrounded by the clouds of interstellar gas and dust from which it formed. The nebula, located 20,000 light-years away in the constellation Carina, contains a central cluster of 50, huge, hot stars.

The image to the left was taken by the Hubble Space Telescope, and is 20 light years across.

The massive, hot stars in this cluster emit nearly 1/3 of their light at ultraviolet wavelengths and shorter. A single ultraviolet photon, when it encounters a hydrogen atom, can fully ionize the atom so that the lone electron is stripped away from the hydrogen's proton nucleus. These stars produce so many ultraviolet photons that they can ionize the entire gas cloud that surrounds them to a distance of many light years.

Problem 1 - A star with a temperature of 40,000 K emits about $6x10^{49}$ ultraviolet photons each second. The density of the hydrogen gas surrounding the star is about 100 atoms/cm^3. How many hydrogen atoms exist in a volume of space with a radius of 3 light years? (1 ly = 9.5×10^{17} centimeters.)

Problem 2 - The size of an ionized region is determined by the balance between the rate at which ultraviolet photons are ionizing the hydrogen atoms, and the rate at which the electrons 'recombine' with the protons to re-form the hydrogen atom. A formula that determines the radius of an ionization region is given by

$$R^3 = 2.6x10^{11} \frac{P}{n^2}$$

If P = $6.0x10^{49}$ photons/sec and n = 100 atoms/cm^3, what is the radius of this nebula in light years?

Image credit: NASA, ESA, R. O'Connell (University of Virginia), F. Paresce (National Institute for Astrophysics, Bologna, Italy), E. Young (Universities Space Research Association/Ames Research Center), the WFC3 Science Oversight Committee, and the Hubble Heritage Team (STScI/AURA)

Problem 1 - A star with a temperature of 40,000 K emits $6.0x10^{49}$ ultraviolet photons each second. The density of the hydrogen gas surrounding the star is about 100 atoms/cm^3. How many hydrogen atoms exist in a volume of space with a radius of 12 light years? (1 ly = 9.5 x 10^{17} centimeters.)

Answer: R = 12 x 9.5 x 10^{17} cm = 1.1 x 10^{19} centimeters. V = 4/3 π R^3, so V = 1.66(3.14) $(1.1x10^{19})^3$ = $5.6x10^{57}$ cm^3. The number of hydrogen atoms is just N = 100 atoms/cc x $5.6x10^{57}$ cm^3 = **$5.6x10^{59}$ atoms**.

Problem 2 - The size of an ionized region is determined by the balance between the rate at which ultraviolet photons are ionizing the hydrogen atoms, and the rate at which the electrons 'recombine' with the protons to re-form the hydrogen atom. A formula that determines the radius of an ionization region is given by

$$R^3 = 2.6x10^{11}\frac{P}{n^2}$$

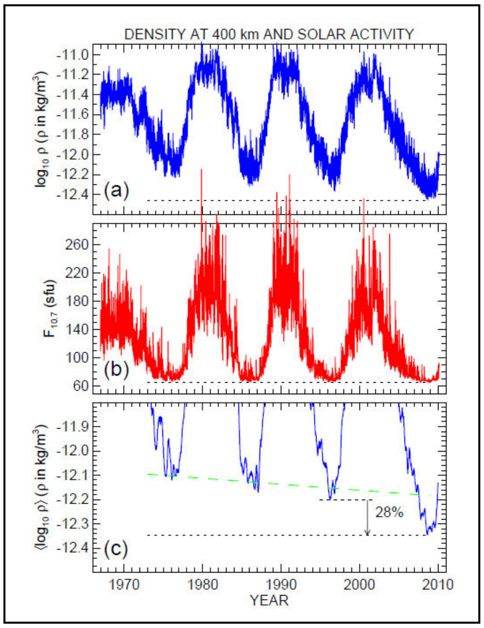
If P = $6.0x10^{49}$ photons/sec and n = 100 atoms/cm^3, what is the radius of this nebula in light years?

Answer: $$R^3 = 2.6x10^{11}\frac{(6.0x10^{49})}{100^2}$$ so

$R^3 = 1.6 \times 10^{57}$
R = 1.1 x 10^{19} cm.

Then R = 1.1 x 10^{19} cm x (1 ly/$9.5x10^{17}$ cm) = **12 light years**.

Note: NGC-3603 contains 50 of these stars. At the above gas density, this cluster can form a nebula with a radius of at least 12 light years x $50^{1/3}$ = 44 light years, which is quite a bit larger than the scale of the Hubble image.



The height of Earth's thermosphere is determined, in part, by the amount of solar ultraviolet radiation reaching it, and this varies during the sunspot cycle as the graphs show. According to recent work by Dr. John Emmett at the Naval Research Laboratory, the prolonged sunspot minimum we have just passed through caused an historic shrinkage in the thermosphere density. The temperatures can range from 700 K at sunspot minimum to 1,600 K at maximum. This causes the density to rise and fall as the atmosphere expands and contracts. A simple formula relates the 'scale height' of an atmosphere to its temperature:

$$H = \frac{kT}{mg}$$

In this formula, k is Boltzman's Constant, which has a value of 1.38 x10^{-23} Joules/K, g is the acceleration of gravity, which has a value of 9.8 meters/sec^2; m is the average mass of the gas particles in kilograms, and T is the temperature of the gas in Kelvins. The scale height, h, in meters represents the distance over which the gas will decrease in density by 2.7 times (a factor of e^{-1}).

Problem 1 - Suppose the gas consists of atoms of hydrogen for which m = 1.7 x 10^{-27} kg. What is the scale height, in meters for an atmosphere with a temperature of A) 700 K at sunspot minimum? B) 1,600 K at sunspot maximum?

Problem 2 - Suppose that during sunspot maximum, a satellite orbited at a constant altitude of 250 km, where the density of the gas was 1.0x10^{-12} kg/meter^3 and the temperature was 1,600 K. If the density, D, at a given altitude, z, is given by the formula

$$D(z) = D_0 e^{-\frac{Z}{H}}$$

what is the density of the atmosphere at 250 km when the temperature cools to 700 K at sunspot minimum?

Problem 1 - Suppose the gas consists of atoms of hydrogen for which m = 1.7 x 10^{-27} kg. What is the scale height, in meters for an atmosphere with a temperature of A) 700 K at sunspot minimum? B) 1,600 K at sunspot maximum?

Answer: At 700K, $H = \frac{(1.38x10^{-23})(700)}{(1.7x10^{-27})9.8}$ so **H = 580 km**

At 1,600K, H = 580 km x (1600/700) = **1,300 km**.

Problem 2 - Suppose that during sunspot maximum, a satellite orbited at a constant altitude of 250 km, where the density of the gas was 1.0x10^{-12} kg/meter^3 and the temperature was 1,600 K. If the density, D, at a given altitude, z, is given by the formula

$$D(z) = D_0 e^{-\frac{Z}{H}}$$

What is the density of the atmosphere at 250 km when the temperature cools to 700 K at sunspot minimum?

Answer: First you have to solve for D0 using H(1,600) = 1300 km and D(250) = 1.0x10^{-12} kg/meter^3 so

$$1.0x10^{-12} = D_0 e^{-\frac{250}{1300}}$$

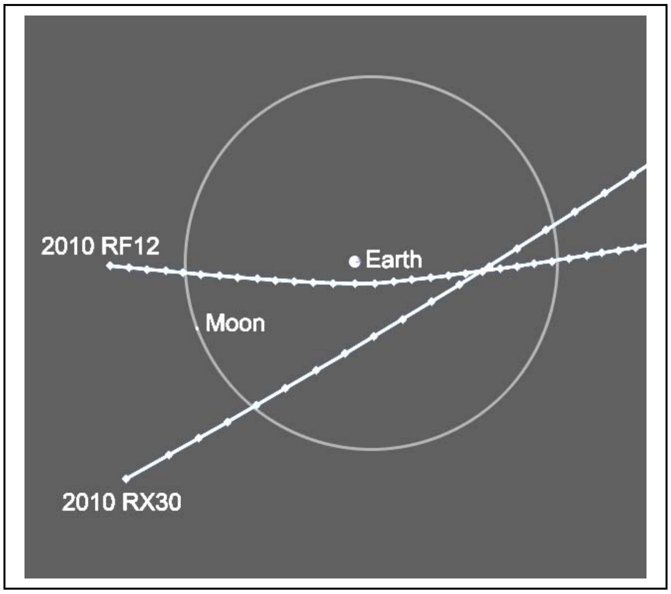
and so D0 = 1.2x10^{-12} kg/meter^3

Now use this equation with the new H value to calculate D(250) for T = 700 K using H(700) = 580 km.

$$D(z) = 1.2x10^{-12} e^{-\frac{250}{580}}$$

so D(250) = **7.8 x 10^{-13} kg/meter^3**

So at the orbit of the satellite, during sunspot minimum, the density is lower than it was during sunspot maximum by about 100% x (0.78/1.0) = 78%.



The Catalina Sky Survey near Tucson, Ariz., discovered two small asteroids on the morning of Sunday, September 5, 2010 during a routine monitoring of the skies.

Asteroid 2010RX30 is about 15 meters in diameter and will pass within 248,000 kilometers of Earth. Asteroid 2010RF12, about 10 meters in diameter, will pass within 79,000 kilometers of Earth.

Both asteroids should be observable near closest approach to Earth with moderate-sized amateur telescopes.

Neither of these asteroids has a chance of hitting Earth. A 10-meter- sized near-Earth asteroid from the undiscovered population of about 50 million would be expected to pass almost daily within a lunar distance, and one might strike Earth's atmosphere about every 10 years on average. The last asteroid that was observed to enter Earth's atmosphere in this size range was the *Great Daylight Fireball of 1972* which streaked above the Grand Tetons. It was about 5 meters in diameter but skipped out of the atmosphere and never struck ground.

Small asteroids appear very faint in the sky, not only because they are small in size, but because their surfaces are very dark and reflect very little sunlight. The formula for the brightness of a typical asteroid that is spotted within a few million kilometers of Earth is given by:

$$R = 0.011\, d\ 10^{-\frac{1}{5}(m)}$$

where: R is the asteroid radius in meters, d is the distance to the asteroid from Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about m = +6.5. The planet Venus when it is brightest in the evening sky has a magnitude of m = -2.5. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid (10 km/sec) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

Answer Key

Problem 1 - What does the formula estimate as the brightness of these two asteroids when they are closest to Earth on September 8, 2010?
Answer:

2010RX30, R=15 meters, d = 248,000 kilometers

$15 = 0.011\ (248{,}000)10^{-.2m}$ then $0.0055 = 10^{-.2(m)}$

Log(0.0055) = -0.2m so **m = +11.3 magnitudes**

2010RF12, R=10 meters, d= 79,000 kilometers

$10 = 0.011\ (79{,}000)10^{-.2m}$ then $0.011 = 10^{-.2(m)}$

Log(0.011) = -0.2m so **m = +9.7 magnitudes**

Problem 2 - Astronomers are anxious to catalog all asteroids that can potentially impact Earth and cause damage to cities. Suppose that at the typical speed of an asteroid (10 km/sec) it will take about 24 hours for it to travel 1 million kilometers (3 times lunar orbit distance). What is the astronomical brightness range for asteroids with diameters between 1 meter and 500 meters?

Answer: First evaluate the equation for d = 1 million km and solve for m(R)

$R = 0.011\ (1.0 \times 10^{6})\ 10^{-.2m}$
$R = 1.1 \times 10^{4}\ \ 10^{-.2m}$

so m(R) = - 5 log10(0.000091R)

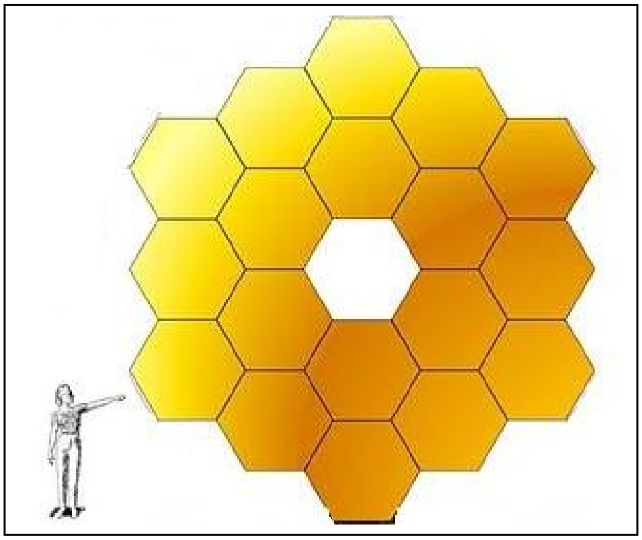
For R = 1 to 500 meters, **m = +20.2 to +6.7**

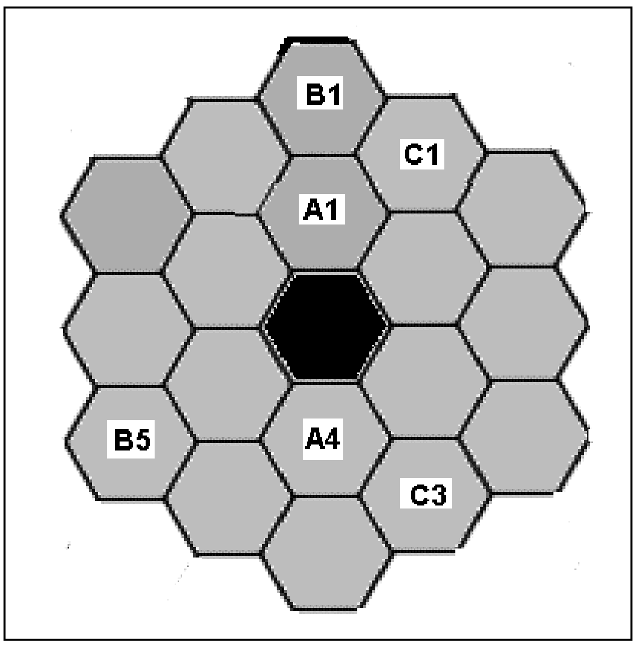
The most common asteroids have sizes between 1 meter and 50 meters, so the detection of such small, faint, and rapidly moving asteroids with ground based telescopes is a major challenge and may be a matter of luck in most cases.

For more information, read the NASA press release at

"Two Asteroids to Pass By Earth Wednesday"

http://www.nasa.gov/topics/solarsystem/features/asteroid20100907.html





The Webb Space Telescope segmented mirror consists of 18 hexagonal mirror tiles assembled to make a larger mirror just over 6 meters in diameter. The placement of these tiles is not random, however.

Tiles located at a specific distance from the center of the mirror are manufactured with exactly the same optical properties. For example, in the drawing to the left, each of the inner ring of 6 tiles has an identical 'twin' to each of the other tiles in this ring.

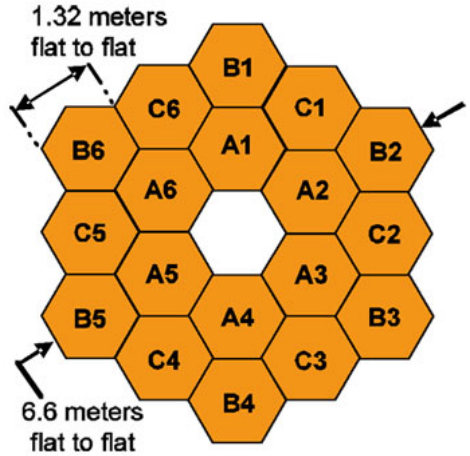
Another property of regular hexagons is that, when you rotate them by 60 degrees, the pattern looks identical to the one you started with. Suppose you labeled one of the six sides, Side A, and placed it at the top of the pattern. If you rotate the hexagon in steps of 60 degrees, it will take exactly 6 shifts to bring Side A back to the top of the pattern. The assembled Webb Space telescope mirror shows this same pattern among the tiles.

Problem 1 - Using the tile labeling shown above, find all other tiles that follow 6-fold symmetry and label them using the same scheme. How many classes can you identify, and how many tiles per class?

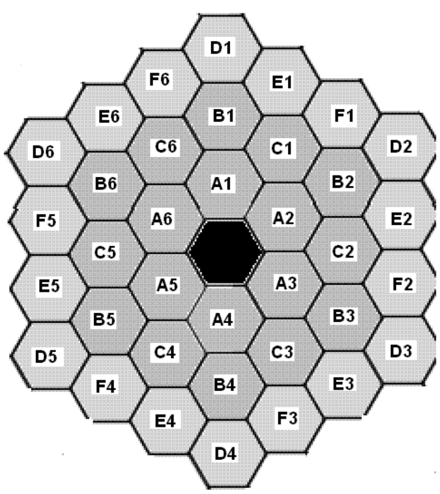
Problem 2 - If you were to add one more ring of hexagonal tiles to the outer edge of the Webb Space Telescope mirror, how many different kinds of mirror tiles would there be, and in each class, how many identical mirror tiles would be present with the same optical properties?

Problem 1 - Using the tile labeling shown above, find all other tiles that follow 6-fold symmetry and label them using the same scheme. How many classes can you identify, and how many tiles per class?

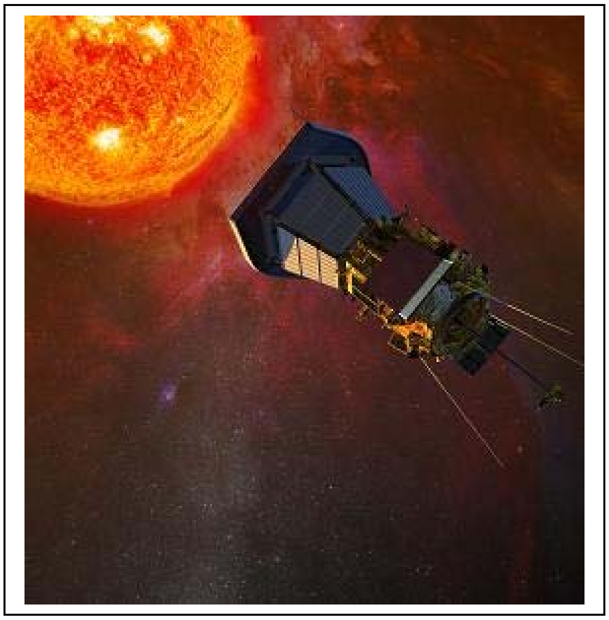
Answer: See figure below. **There are exactly three different tile classes, A, B and C, and 6 tiles per class.**



Problem 2 - If you were to add one more ring of hexagonal tiles to the outer edge of the Webb Space Telescope mirror, how many different kinds of mirror tiles would there be, and in each class, how many identical mirror tiles would be present with the same optical properties?



There are 6 unique mirror classes (A, B, C, D, E, F) and six identical mirror per class. Each mirror class is at a unique distance from the center of the mirror and has the same optical properties. This means that A1 can be interchanged with A5, but that A1 cannot be swapped for B1, C1 etc.



The Solar Probe Plus mission will be launched in 2018 for a rendezvous with the sun in 2024. To lose enough energy to reach the sun, the spacecraft will make seven fly-bys of Venus.

As the spacecraft approaches the sun, its heat shield must withstand temperatures exceeding 2500^{o} F and blasts of intense radiation.

The 480 kg spacecraft, costing $740 million, will approach the sun to a distance of 6 million km. Protected by the heat shield will be five instruments that will peak over the edge of the heat shield and measure the particles and radiation fields in the outer solar corona.

A simple formula that predicts the temperature, in Kelvins, of a surface exposed to solar radiation is given by

$$T = 396\frac{(1-A)^{\frac{1}{4}}}{\sqrt{R}}$$

where R is the distance to the solar surface in Astronomical Units, and A is the fraction of the incoming radiation that is reflected by the surface. Because only the amount of absorbed sunlight determines how hot a body becomes, the final temperature depends on the quantity (1 – A) rather than A alone. (Note 1 Astronomical Unit is the distance of Earth from the center of the sun; 147 million kilometers).

Problem 1 – An astronaut’s white spacesuit reflects 80% of the incoming radiation at Earth’s orbit (1 Astronomical Unit). From the formula, about what is the temperature of the surface of the spacesuit?

Problem 2 - The Solar Probe Plus spacecraft will use a heat shield facing the sun with a reflectivity of about A = 0.60. What will be the temperature of the heat shield, called the Thermal Protection System or TPS, at the distance of 5.9 million km (0.040 AU), A) In Kelvins? B) In degrees Celsius? C) In degrees Fahrenheit?

Problem 3 – Suppose that the TPS consisted of a highly-reflective mirrored coating with a reflectivity of 99%. What would be the temperature in Kelvins, of the back of the heat shield when the Solar Probe is closest to the sun at a distance of 0.04 AU?

Problem 1 – An astronaut's white spacesuit reflects 80% of the incoming radiation at Earth's orbit (1 Astronomical Unit). From the formula, about what is the temperature of the surface of the spacesuit?

Answer: $T = 396\frac{(1-0.8)^{\frac{1}{4}}}{\sqrt{1AU}}$ so **T = 265 Kelvin**.

Problem 2 - The Solar Probe Plus spacecraft will use a heat shield facing the sun with a reflectivity of about A = 0.60. What will be the temperature of the heat shield, called the Thermal Protection System or TPS, at the distance of 5.9 million km (0.040 AU), A) In Kelvins? B) In degrees Celsius? C) In degrees Fahrenheit?

Answer: A) **T = 1600 K**.
B) Tc = Tk – 273 so **Tc = 1,300^{o} C**
C) Tf = 9/5 (Tc) + 32 so **Tf = 2,400^{o} F**.

Problem 3 – Suppose that the TPS consisted of a highly-reflective mirrored coating with a reflectivity of 99%. What would be the temperature in Kelvins, of the back of the heat shield when the Solar Probe is closest to the sun at a distance of 0.04 AU?

Answer: A = 0.99 then

$T = 396\frac{(1-0.99)^{\frac{1}{4}}}{\sqrt{0.04AU}}$ so Tk = **630 Kelvins**.

For more mission details, visit:

http://solarprobe.jhuapl.edu/

http://www.nasa.gov/topics/solarsystem/sunearthsystem/main/solarprobeplus.html

The Solar Probe Plus mission will be launched in 2018 for a rendezvous with the sun in 2024. To lose enough energy to reach the sun, the spacecraft will make seven fly-bys of Venus.

The 480 kg spacecraft, costing $740 million, will approach the sun to a distance of 5.8 million km. Protected by the heat shield will be five instruments that will peak over the edge of the heat shield and measure the particles and radiation fields in the outer solar corona.

At the closest approach distance, the sun will occupy a much larger area of the sky than what it does at the distance of Earth.

The angular size of an object as it appears to a viewer depends on the actual physical size of the object, and its distance from the viewer. Although physical size and diameter are usually measured in terms of meters or kilometers, the angular diameter of an object is measured in terms of degrees or radians. The angular size, t, depends on its actual size d and its distance r according to the simple formula

$$\theta = 2arcTan\left(\frac{d}{2R}\right)$$

For example, the angular size of your thumb with your arm fully extended for d = 2 centimeters and r = 60 cm is just θ = 2 arcTan(0.016) so θ = 1.8 degrees.

Problem 1 – At the distance of Earth, 147 million kilometers, what is the angular diameter of the sun whose physical diameter is 1.4 million kilometers?

Problem 2 – At the distance of closest approach to the solar surface Solar Probe Plus will be at a distance of 5.8 million kilometers from the solar surface. What will be the angular diameter of the sun if its physical diameter is 1.4 million kilometers?

Problem 3 – A DVD disk measures 12 cm in diameter. How far from your eyes will you have to hold the DVD disk in order for it to subtend the same angular size as the disk of the sun viewed by Solar Probe Plus at its closest distance?

Problem 1 – At the distance of Earth, 147 million kilometers, what is the angular diameter of the sun whose physical diameter is 1.4 million kilometers?

Answer: $\theta = 2arcTan\left(\frac{1.4million}{2x147million}\right)$

θ = 2 arctan(0.0048)

so θ **= 0.55 degrees**.

Problem 2 – At the distance of closest approach to the solar surface Solar Probe Plus will be at a distance of 5.8 million kilometers from the solar surface. What will be the angular diameter of the sun if its physical diameter is 1.4 million kilometers?

Answer: θ = 2 arctan (1.4/(2 x 5.8)) so θ **= 14 degrees**.

Problem 3 – A DVD disk measures 12 cm in diameter. How far from your eyes will you have to hold the DVD disk in order for it to subtend the same angular size as the disk of the sun viewed by Solar Probe Plus at its closest distance?

Answer: The desired angular diameter of the sun is θ = 14 degrees, and for the DVD disk we have d = 12 cm, so we need to solve the formula for r.

Tan (θ/2) = d/2r so

2r = d / tan(θ/2) then

r = 6 cm / tan(7) yields

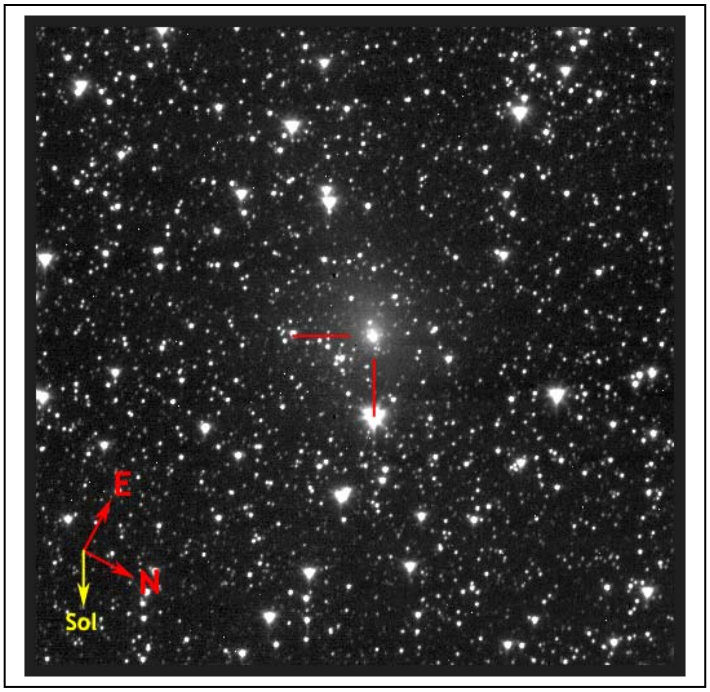
r = **49 cm. This is about 18 inches**.

Note: From the vantage point of the Solar Probe Plus spacecraft, the sun's disk at a temperature of 5570 K is much larger (14 degrees) than it appears in Earth's sky (0.5 degrees), and this results in the spacecraft heat shield being heated to over 1600 K.

For more mission details, visit:

http://solarprobe.jhuapl.edu/

http://www.nasa.gov/topics/solarsystem/sunearthsystem/main/solarprobeplus.html



This image was taken by the Deep Impact spacecraft on September 20 as it approaches Comet Hartley 2 from a distance of 46 million kilometers.

On November 4, the spacecraft will approach to within 700 kilometers of the surface of the comet.

The image area is 0.29 degrees on a side, and the nucleus of the comet is about 1 kilometer in diameter. The comet can be seen as the fuzzy spot near the center.

Whenever you look at a distant object, you see the object's angular size not its actual physical size. There is a simple formula that relates the actual size and distance to an object, to its apparent angular size:

$$Tan\left(\frac{\theta}{2}\right) = \frac{d}{2R}$$

where d is the object's width or diameter in kilometers, and R is its distance from the obeserver in kilometers. For example, a DVD disk (d=12 cm) held at arms-length (R=60 cm) will subtend an angle of θ = 11 degrees.

Problem 1 - Deep Impact will pass to within 700 km of the nucleus. How big will the comet nucleus appear to the Deep Impact spacecraft if its diameter is 1 kilometer?

Problem 2 - The Deep Impact High-Resolution Imager (HRI) has a format of 1024 x 1024 pixels and a field of view of 0.118 degrees. A single pixel sees an angular field of 0.118 deg/1024 pix = 0.000115 degrees. At a distance of 700 km, what linear distance will a pixel resolution of 0.000115 degrees represent?

Problem 3 - How many pixels across will the comet nucleus appear in the image taken at closest approach with the HRI?

Problem 1 - Deep Impact will pass to within 700 km of the nucleus. How big will the comet nucleus appear to the Deep Impact spacecraft if its diameter is 1 kilometer?

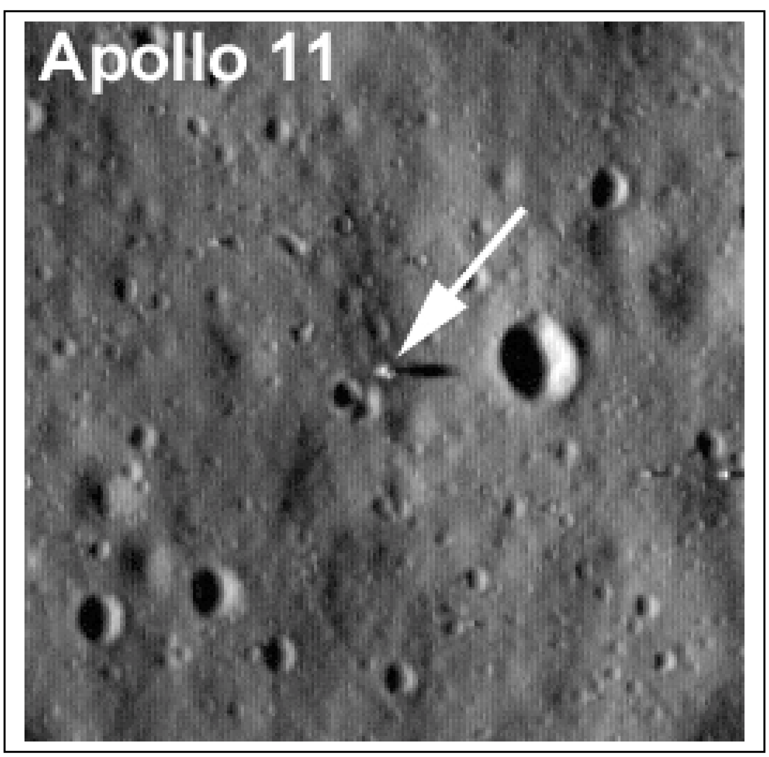
Answer: d = 1 kilometer, and R = 700 km, so solving for θ in the formula we have
$Tan(\theta/2) = \frac{d}{2R}$ so Tan $(\theta/2)$ = 1 kilometer / (2 x 700), and so θ = **0.08 degrees**

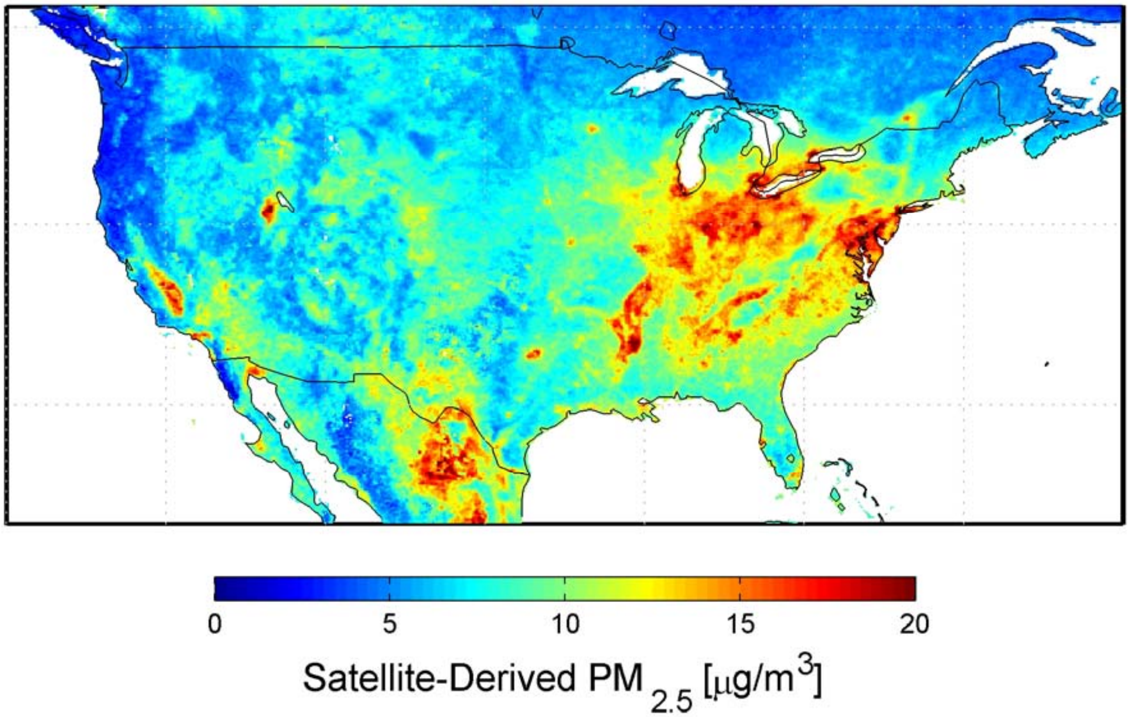
Problem 2 - The Deep Impact High-Resolution Imager (HRI) has a format of 1024 x 1024 pixels and a field of view of 0.118 degrees. A single pixel sees an angular field of 0.118 deg/1024 pix = 0.000115 degrees. At a distance of 700 km, what linear distance will a pixel resolution of 0.000115 degrees represent?

Answer: Tan(0.000115) = L/700 km so
L = 700 Tan(0.000115) km **L = 1.4 meters/pixel**.

Problem 3 - How many pixels across will the comet nucleus appear in the image taken at closest approach with the HRI?

Answer: The resolution of the imager is 1.4 meters/pixel. The diameter of the nucleus is believed to be about the 1 kilometer, so the comet nucleus will subtend about 1,000 meters x (1 pixel/1.4 meters) = **714 pixels**. Below is an image taken by the Lunar Reconnaissance Orbiter at a resolution of 1 meter showing the Apollo 11 landing area on the moon. A similar-resolution image will be possible for Comet Hartley 2 using HRI!





Dust particles can cause many serious diseases when inhaled, and lead to millions of premature deaths each year around the world. The dust particles are less that 2.5 micrometers in diameter (one tenth the diameter of human hair) and are present in the atmosphere in many geographic locations.

For the first time, the Terra Satellite's MISR and MODIS instruments were able to compile a map of the concentration of these deadly particles around the world. The figure above shows the concentration over the United States.

Problem 1 - The color scale gives the concentration of these dust particles in micrograms per cubic meter of air. Suppose these dust grains are perfect spheres with a density of 2000 kilograms per cubic meter. How many dust particles are present in a cubic meter of air at a concentration of 15 micrograms per cubic meter?

Problem 2 - A normal human breath is about 1 liter of gas volume. How many dust particles at a concentration of 15 micrograms/meter^3 are taken in with each breath?

Problem 1 - The color scale gives the concentration of these dust particles in micrograms per cubic meter of air. Suppose these dust grains are perfect spheres with a density of 2000 kilograms per cubic meter. How many dust particles are present in a cubic meter of air at a concentration of 15 micrograms per cubic meter?

Answer: First we have to determine the mass of a single dust particle. Since mass = density x volume, and for a sphere volume = $4/3 \pi R^3$,we have for a 2.5 micron diameter dust particle $R = 1.25 \times 10^{-6}$ meters and so:

$V = 1.33 (3.141) (1.25 \times 10^{-6} \text{ meters})^3$ so
$V = 8.2 \times 10^{-18} \text{ meter}^3$.

Density = 2000 kg/meter^3 so
Mass = 8.2×10^{-18} meter^3 x 2000 kg/meter^3
= 1.6×10^{-14} kilograms.

In terms of micrograms this becomes
M = 1.6×10^{-14} kg x (1000 gm/1kg) x (1 microgram/10^{-6} grams)
= 1.6×10^{-5} micrograms/particle

The concentration is 15 micrograms/meter^3 ,so the particulate density is then
N = 15 micrograms/meter^3 x (1 particle / 1.6×10^{-5} micrograms)
= 9.4×10^5 particles/meter^3.

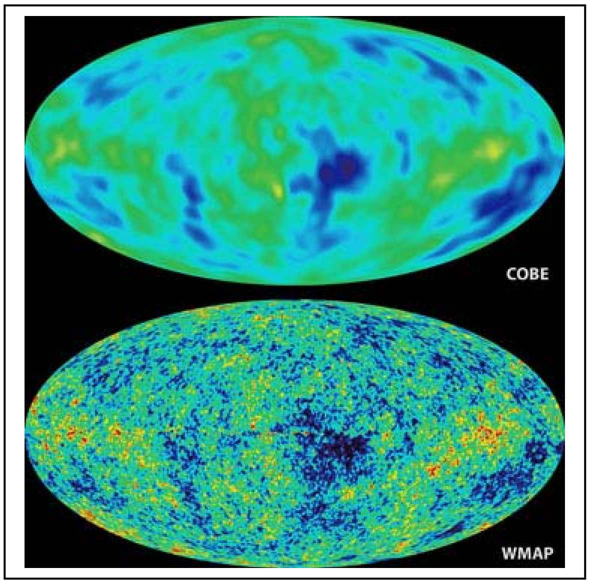
This means that one cubic meter can contain **nearly 1 million particles of dust**!

Problem 2 - A normal human breath is about 1 liter of gas volume. How many dust particles at a concentration of 15 micrograms/meter^3 are taken in with each breath?

Answer: From Problem 1, this concentration equals 940,000 dust particles per cubic meter. Since 1 liter is 0.001 cubic meters, a single breath takes in about **940 dust particles at this concentration!**

More details can be found at 'New Map Offers Global View of Health-Sapping Air Pollution'; **http://www.nasa.gov/topics/earth/features/health-sapping.html**

"Human-generated particles often predominate in urban air -- what most people actually breathe -- and these particles trouble medical experts the most, explained Arden Pope, an epidemiologist at Brigham Young University, Provo, Utah. That's because small particles can make their way deep into human lungs and some ultrafine particles can even enter the bloodstream. Once there, they can spark a whole range of diseases including asthma, cardiovascular disease, and bronchitis. The American Heart Association estimates that in the United States alone, $PM_{2.5}$ air pollution spark some 60,000 deaths a year."



The Webb Space Telescope uses a very large mirror to enable astronomers to see distant objects more clearly. This is a very important goal of this telescope since astronomers know so little about how objects look far from Earth. In the universe, viewing objects at a great distance also means that you are seeing them as they were long ago.

The scientific mission of the Webb Space Telescope is to study how galaxies like the Milky Way formed when the universe was only a few million years old compared to its current age of over 13 billion years.

The image above shows the sky imaged by the COBE satellite with a 7-degree resolution compared to the WMAP satellite with a 1/2-degree resolution. Note the increased detail with WMAP.

The sky is measured in angular units (degrees, minutes, seconds) but we would actually like to know how many kilometers a given angular measurements corresponds to. The simple formula below relates size to distance and angular width:

$$L = \frac{\theta}{206265} d$$

where θ is the angular diameter in arcseconds, d is the distance to the object in light years and L is the actual diameter of the object in light years. Note that for closer objects, L and d will both be in units kilometers.

The smallest feature that the human eye can see on the moon has an angular width of about 2 arcminutes or 120 arcseconds. The smallest feature that the Webb Space Telescope Near Infrared Camera (NIRcam) can see clearly has a width of about 0.032 arcseconds per pixel.

Problem 1 - What is the width of the smallest feature that the human eye can see on the moon at the distance of Earth, d = 384,000 kilometers?

Problem 2 - How far, d, would a planet the size of Earth (L = 12,800 km) have to be in order for the Webb Space Telescope to just see it (θ = 0.032 arcseconds)?

Problem 3 - Suppose that the most distant object that can be detected by the Webb Space Telescope is located 13 billion light years from Earth. What would be the minimum diameter of this object, L, at the maximum resolution of the telescope?

Problem 1 - What is the width of the smallest feature that the human eye can see on the moon at the distance of Earth, d = 384,000 kilometers?

Answer: $L = \frac{120}{206265} x 384,000 km$ so L = **220 kilometers**

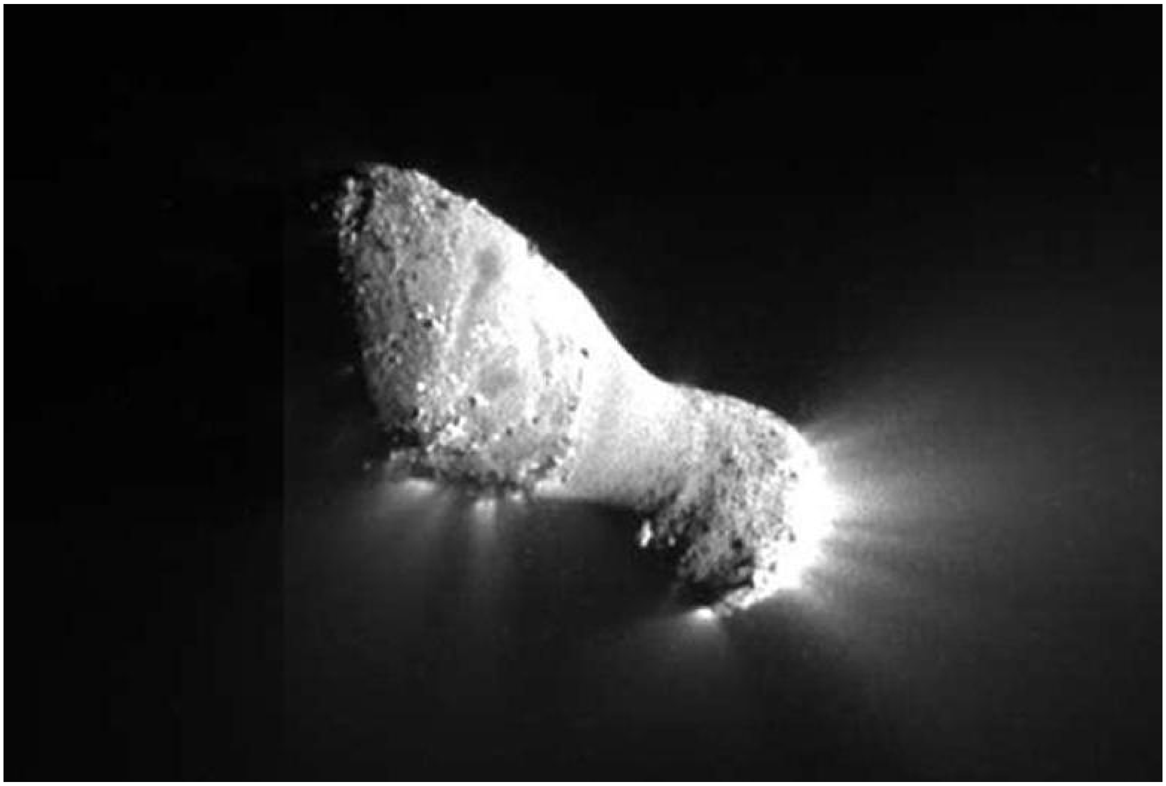
Problem 2 - How far, d, would a planet the size of Earth (L = 12,800 km) have to be in order for the Webb Space Telescope to just see it (θ = 0.032 arcseconds)?

Answer: $d = \frac{206265}{0.032} x 12,800 km$ so **d = 83 billion km**.

Problem 3 - Suppose that the most distant object that can be detected by the Webb Space Telescope is located 13 billion light years from Earth. What would be the minimum diameter of this object ,L, at the maximum resolution of the telescope?

$L = \frac{0.032}{206265} x 13 \text{ billion light years}$ so **L = 2,000 light years**

Note: In Problem 3, no correction has been made for the fact that over these great distances, the curvature of space causes the relationship between angular size and distance to be different than the formula used, which is only valid for the geometry of flat 'Euclidean' space.



Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The scale of this image is 25 meters/millimeter. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Assume that the volume of the nucleus can be approximated by a dumbbell-shaped model consisting of two spherical 'end-caps' connected by a cylindrical bar. To two significant figures, about what is the total volume of the nucleus in cubic meters? (Note: the spheres need not have the same diameters)

Problem 2 - The densities of only a few cometary nuclei have been determined from their mass and volume: Halley's Comet (0.6 gm/cm^3); Comet Tempel-1 (0.62 gm/cm^3); Comet Borrelly (0.3 gm/cm^3) and Comet Wild (0.6 gm/cm^3). The low density indicates large quantities of water and other ices make up the composition of these bodies. Assuming that the density of Comet Hartley-2 is similar to the median density of these comets, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Problem 1 - Assume that the volume of the nucleus can be approximated by a dumbbell-shaped model consisting of two spherical 'end-caps' connected by a cylindrical bar. To two significant figures, about what is the total volume of the nucleus in cubic meters? (Note: the spheres need not have the same diameters)

Answer: Although it appears that the nucleus was viewed at an oblique angle and not face-on, we will not include this perspective effect in the size estimates. Students may attempt to make this correction by, for example, assuming that the end-cap spheres are of equal diameter, with a diameter that is the average of the widths of the two ends of the nucleus.
Using a millimeter ruler, the diameter of the left end-cap is about 40mm and the right endcap is about 30 mm, so at a scale of 25 meters/mm, the actual diameters are 1,000 meters and 750 meters respectively. The diameter of the cylindrical bar is about 20 mm or 500 meters. The length of the cylinder is 2000 meters - 1000 meters - 750 meters = 250 meters based on the assumed length of 2 km and subtracting the two spheres.
The volume of a sphere is $4/3 \pi R^3$ and the volume of a cylinder is $\pi R^2 h$ so we have
Right endcap $V = 1.33 (3.14) (750/2)^3 = 2.2 \times 10^8$ meters^3
Left endcap $V = 1.33 (3.14) (1000/2)^3 = 5.2 \times 10^8$ meters^3
Cylinder $V = 3.14 (250)^2 (250) = 4.9 \times 10^7$ meters^3

So the total volume using this geometric model is just V = **7.9×10^8 meters^3**

Problem 2 - The densities of only a few cometary nuclei have been determined from their mass and volume: Halley's Comet (0.6 gm/cm^3); Comet Tempel-1 (0.62 gm/cm^3); Comet Borrelly (0.3 gm/cm^3) and Comet Wild (0.6 gm/cm^3). The low density indicates large quantities of water and other ices make up the composition of these bodies. Assuming that the density of Comet Hartley-2 is similar to the median density of these comets, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Answer: The median density is **0.6 gm/cm^3.**
Mass = Density x Volume

First convert the volume to cubic centimeters from cubic meters:

$V = 7.9 \times 10^8$ meters^3 x (100 cm/1 meter)^3 = 7.9×10^{14} cm^3.

Then, Mass = 0.6 gm/cm^3 x 7.9×10^{14} cm^3
= 4.7×10^{14} gm

Convert grams to megatons:

Mass = 4.7×10^{14} gm x (1 kg/1000 gm) x (1 ton/1000 kg) = 4.7×10^8 tons or **470 megatons**.

As a comet orbits the sun, it produces a long tail stretching millions of kilometers through space. The tail is produced by heated gases leaving the nucleus of the comet.

This image of the head of Comet Tempel-1 was taken by the Hubble Space Telescope on June 30, 2005. It shows the 'coma' formed by these escaping gases about 5 days before its closest approach to the sun (perihelion). The most interesting of these ingredients is ordinary water.

Problem 1 – The NASA spacecraft Deep Impact flew-by Temple-1 and measured the rate of loss of water from its nucleus. The simple quadratic function below gives the number of tons of water produced every minute, W, as Comet Tempel-1 orbited the sun, where T is the number of days since its closest approach to the sun, called perihelion.

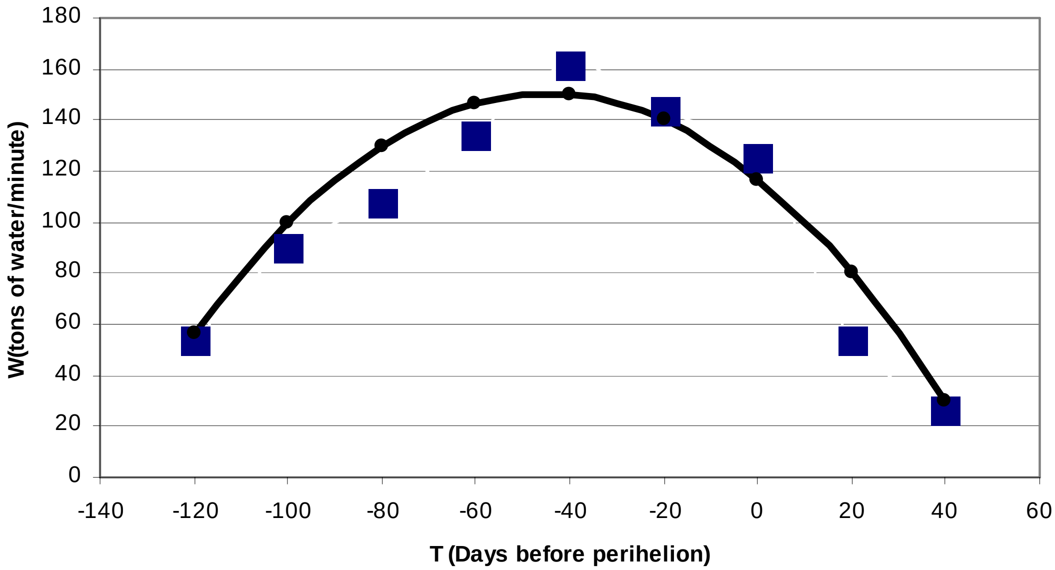
$$W(T) = \frac{(T+140)(60-T)}{60}$$

A) Graph the function W(T). B) For what days, T, will the water loss be zero? C) For what T did the comet eject its maximum amount of water each minute?

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Problem 1 – A) The graph below was created with Excel. The squares represent the actual measured data and are shown as an indicator of the quality of the quadratic model fit to the actual data.



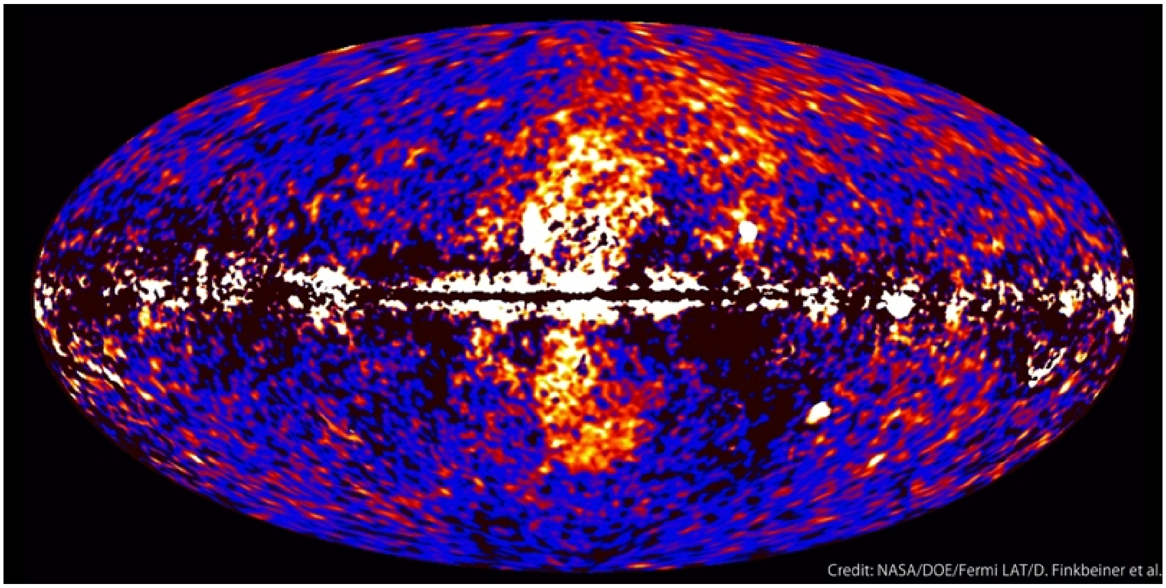
Answer: B) The roots of the quadratic equation, where W(T)=0 are for **T=-140** days and **T=+60** days after perihelion. C) The maximum (vertex of the parabola) occurs half-way between the two intercepts at T = (-140+60)/2 or **T = -40** which indicates 40 days before perihelion.

Problem 2 – To two significant figures, how many tons of water each minute were ejected by the comet 130 days before perihelion (T = -130)?

Answer: W(-130) = (-130+140)(60+130)/60 = **32 tons/minute**

Problem 3 - To two significant figures, determine how many tons of water each minute were ejected by the comet 70 days after perihelion (T = +70). Can you explain why this may be a reasonable prediction consistent with the mathematical fit, yet an implausible 'Real World' answer?

Answer: The fitting function W(T) predicts that W(+70) = (70+140)(60-70)/60 = -35 tons per minute. Although this value smoothly follows the prediction curve, it implies that instead of ejecting water (positive answer means a positive rate of change) the comet is absorbing water (negative answer means a negative rate of change), so the prediction is not realistic.



NASA's Fermi Gamma-ray Space Telescope has unveiled a previously unseen structure centered in the Milky Way. The feature spans 50,000 light-years and may be the remnant of an eruption from a supersized black hole at the center of our galaxy, or the gas ejected by a burst of star formation in the center of the galaxy several million years ago. The bubbles extend 25,000 light years above the plane of the Milky Way, and appear to be symmetric in shape.

Problem 1 - Approximate the volume of these two bubbles as two spheres with diameters of 25,000 light years. What is the total volume of the two bubbles in cubic light years?

Problem 2 - The average density of interstellar gas is about 1 hydrogen atom per cubic centimeter. If 1 light year = 9.5×10^{17} centimeters, and one hydrogen atom has a mass of 1.6×10^{-27} kg, how much mass may have been displaced by the formation of these bubbles A) in kilograms? B) in solar mass units if 1 SMU = 2.0×10^{30} kg?

Problem 3 - Suppose that the density of the gas ejected by the activity at the center of our Milky Way had a density of about 10% of the normal interstellar gas. If the process took 1,000,000 years to form the bubbles, at what rate would gas from the core of our Milky Way have to be produced in Earth mass units per year?

Problem 1 - Approximate the volume of these two bubbles as two spheres with diameters of 25,000 light years. What is the total volume of the two bubbles in cubic light years?

Answer: For one bubble: $V = 4/3\ \pi\ R^3$ so

$V = 1.33\ (3.14)\ (25{,}000/2)^3$ cubic light years

$V = 8.2 \times 10^{12}$ cubic light years.

For two bubbles $2V =$ **1.6×10^{13} cubic light years.**

Problem 2 - The average density of interstellar gas is about 1 hydrogen atom per cubic centimeter. If 1 light year = 9.5×10^{17} centimeters, and one hydrogen atom has a mass of 1.6×10^{-27} kg, how much mass may have been displaced by the formation of these bubbles A) in kilograms? B) in solar mass units if 1 SMU = 2.0×10^{30} kg?

Answer: A) Mass in one cubic light year = 1 atoms/cm^3 x $1.6\text{x}10^{-27}$ kg/atom x ($9.5e^{17}$ cm/light year)^3 = 1.4×10^{27} kg. The volume of the bubbles is 1.6×10^{13} ly^3, so the mass is $M=1.4\text{x}10^{27}$ x $(1.6\text{x}10^{13})$ and so M= **2.2×10^{40} kg**.

B) In solar mass units M= 1.4×10^{27} kg x (1 SMU/$2.0\text{x}\ 10^{30}$ kg)

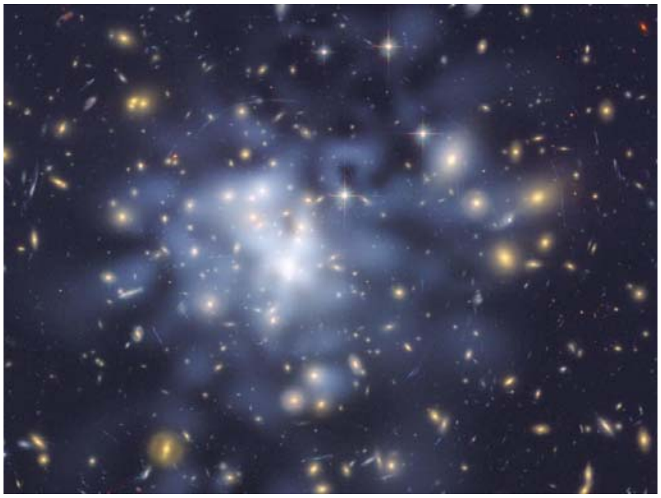
= **1.1×10^{10} solar masses.**

Problem 3 - Suppose that the density of the gas ejected by the activity at the center of our Milky Way had a density of about 10% of the normal interstellar gas. If the process took 1,000,000 years to form the bubbles, at what rate would gas from the core of our Milky Way have to be produced in Earth mass units per year?

Answer: The density is 0.10 x the normal interstellar gas, so from Problem 2(B) the amount of material in the bubble volume would be 0.1 x 1.1×10^{10} solar masses = $1.1\text{x}10^9$ solar masses. It took 1,000,000 years to inject this gas into the bubbles, so the rate is just

R =$1.1\text{x}10^9$ solar masses/1,000,000 years

= **1100 solar masses/year**.



This NASA Hubble Space Telescope image shows the distribution of dark matter in the center of the giant galaxy cluster Abell 1689. This cluster contains about 1,000 galaxies and trillions of stars, and is located 2.2 billion light-years from Earth.

Dark matter is an invisible form of matter that accounts for most of the universe's mass. Hubble cannot see the dark matter directly, but used the distorted images of background galaxies to infer its location.

Astronomers used the observed positions of 135 distorted images of 42 background galaxies to calculate the location and amount of dark matter in the cluster. They superimposed a map of these inferred dark matter concentrations, tinted blue, on an image of the cluster taken by Hubble's Advanced Camera for Surveys. If the cluster's gravity came only from the visible galaxies, the galaxy image distortions would be much weaker, and the corresponding area in the image above would be black. The map reveals that the densest concentration of dark matter is in the cluster's core.

Problem 1 - The escape speed of a body located R meters from a second body with a mass of M kilograms can be given by the basic 'escape velocity' equation:

$$V = \sqrt{\frac{2GM}{R}}$$

where the constant of gravity given by G = 6.66 x 10^{-11}. We can re-write this more conveniently for a cluster of N galaxies each with an average mass of 10 billion stars, and an average star mass of 2 x 10^{30} kg, and a cluster radius of R light years (1 light year = 9.4 x 10^{15} meters)

$$V = 17,000\sqrt{\frac{N}{R}} \quad \text{km/sec}$$

Show that the second equation is related to the first one as indicated.

Problem 2 - An astronomer counts the galaxies in two clusters and measures the average radius of each galaxy cluster. The astronomer also measures the average speeds of the galaxies in each cluster and finds that for Cluster A: R= 10 million, N= 1000, V= 300 km/s, and for Cluster B: R= 5 million light years, N= 350, V= 140 km/s. For which cluster does the measured average speed not match the expected speed based on the visible, countable, masses found in the galaxies?

Problem 3 - How many extra 'invisible' galaxies would the discrepant cluster have to have in order for the mass of the cluster to be consistent with the observed speeds of its visible galaxies? (This is the 'dark matter' problem, originally called the 'missing mass' problem, and indicates that extra gravitating mass is present that is not in the form of ordinary stars and gas within the visible galaxies.)

Problem 1 - Answer: The escape speed of a body located R meters from a second body with a mass of M kilograms can be given by the basic 'escape velocity' equation:

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$$V = \sqrt{\frac{2(6.66x10^{-11})(N)(10billion)(2x10^{30})}{9.4x10^{15}(R)}} \quad \text{so} \quad V = 17,000,000\sqrt{\frac{N}{R}} \text{ meters/sec}$$

And so in kilometers/sec we have:

$$V = 17,000\sqrt{\frac{N}{R}} \text{ km/sec}$$

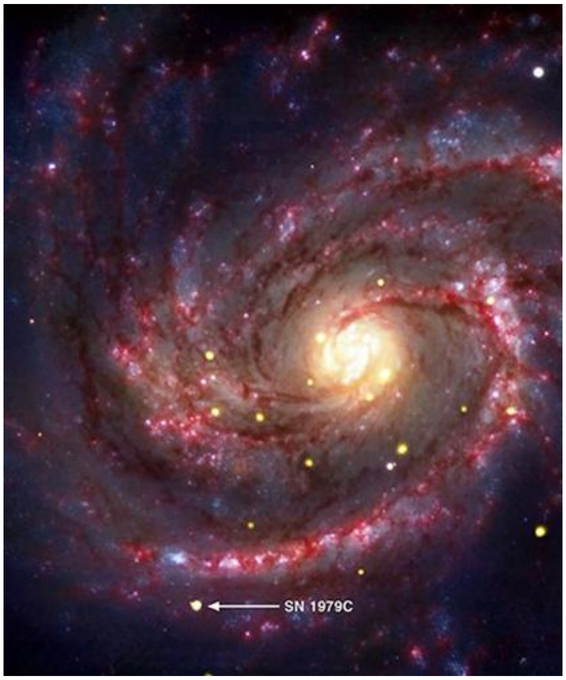
Problem 2 - An astronomer counts the galaxies in two clusters and measures the average radius of each galaxy cluster. The astronomer also measures the average speeds of the galaxies in each cluster and finds that for Cluster A: R= 10 million, N= 1000, V= 300 km/s, and for Cluster B: R= 5 million light years, N= 350, V= 140 km/s. For which cluster does the measured average speed not match the expected speed based on the visible, countable, masses found in the galaxies?

Answer: Cluster A: V(predicted) = 17,000 (1000/10 million)$^{1/2}$ = 170 km/s
Cluster B: V(predicted) = 17,000 (350/5 million)$^{1/2}$ = 140 km/s
Although the average measured speed of the galaxies in Cluster B are identical to the predicted speed given the size and mass of this cluster, for Cluster A, its measured galaxy speeds are much higher than the speed deduced from the clusters visible galaxies and size, so **Cluster A is the discrepant one**.

Problem 3 - How many extra 'invisible' galaxies would the discrepant cluster have to have in order for the mass of the cluster to be consistent with the observed speeds of its visible galaxies? (This is the 'dark matter' problem, originally called the 'missing mass' problem, and indicates that extra gravitating mass is present that is not in the form of ordinary stars and gas within the visible galaxies.)

Answer: In order to make the average speeds of the galaxies in Cluster A (300 km/s) match the expected speed for a cluster with this many visible galaxies (V=170 km/s) we have to add more mass in the form of Dark Matter.

$$300 = 17,000\sqrt{\frac{n}{10million}}$$ so n = 3114 galaxies-worth of equivalent mass. Since only N=1000 is accounted for, we need to add a **Dark Matter equivalent to about 2114 extra galaxies,** or about m(Dark) = 2114 galaxies x 10 billion stars/galaxy = 2.1 x 10^{13} stellar masses-worth of Dark Matter.



The Chandra X-Ray Observatory recently found evidence for an infant black hole in the nearby galaxy Messiar-100. The black hole is thought to have been produced when a star with a mass of about 20 times that of the sun exploded and left behind a black hole with a mass about 8 times the sun's mass.

The satellite observatory has detected x-rays from the gasses in the orbiting accretion disk that are falling into this young black hole. Infalling gas can be heated to over 100,000,000 K as atoms collide at higher and higher speed during the infall process. The temperature of this x-ray emitting gas is related to its distance from the black hole.

At a distance of R kilometers from a black hole with a mass of M times the sun, suppose that the two equations below relate the temperature of the gas, T, and the wavelength, L, at which the in-flowing gas emits most of its light:

Equation 1 - $T = 100,000,000\left(\frac{M}{R^3}\right)^{1/4}$ Kelvin

Equation 2 - $L = \frac{3,600,000}{T}$ nanometers

where M is in solar mass units, and R is in kilometers.

Problem 1 - Combining these equations using the method of substitution, what is the new formula L(R,M), for the wavelength, L, emitted by the gas as a function of its distance from the black hole center, R, and the mass of the black hole, M?

Problem 2 – X-rays are detected from the vicinity of the SN 1979C black hole at a wavelength of 0.53 nanometers (2,300 electronVolts). If the mass of the black hole is 8 times the sun, at what distance from the center of the black hole is the gas being detected?

Problem 3 – The Event Horizon of a black hole that is not rotating (called a Schwarschild black hole) is located at a distance of Rs = 3.0 M from the center of the black hole, where M is the mass of the black hole in units of our sun, and Rs is in units of kilometers. What is Rs for the SN 1979C black hole, and where is the x-ray emitting gas in relation to the Event Horizon?

NASA Press release 'Youngest Nearby Black Hole' November 15, 2010

"Data from Chandra, as well as NASA's Swift, the European Space Agency's XMM-Newton and the German ROSAT observatory revealed a bright source of X-rays that has remained steady for the 12 years from 1995 to 2007 over which it has been observed. This behavior and the X-ray spectrum, or distribution of X-rays with energy, support the idea that the object in SN 1979C is a black hole being fed either by material falling back into the black hole after the supernova, or from a binary companion.

The scientists think that SN 1979C formed when a star about 20 times more massive than the Sun collapsed. It was a particular type of supernova where the exploded star had ejected some, but not all of its outer, hydrogen-rich envelope before the explosion, so it is unlikely to have been associated with a gamma-ray burst (GRB). Supernovas have sometimes been associated with GRBs, but only where the exploded star had completely lost its hydrogen envelope. Since most black holes should form when the core of a star collapses and a gamma-ray burst is not produced, this may be the first time that the common way of making a black hole has been observed.

The very young age of about 30 years for the black hole is the observed value, that is the age of the remnant as it appears in the image. Astronomers quote ages in this way because of the observational nature of their field, where their knowledge of the Universe is based almost entirely on the electromagnetic radiation received by telescopes."
(**http://www.nasa.gov/mission_pages/chandra/multimedia/photoH-10-299.html)**

Problem 1 - Answer: Substitute Equation 1 into Equation 2 to eliminate T,

$$L(R,M) = \frac{3,600,000}{100,000,000}\left(\frac{R^3}{M}\right)^{1/4}$$

so $L(R,M) = 0.036\left(\frac{R^3}{M}\right)^{1/4}$ nanometers.

Problem 2 – Answer: $0.53 = 0.036\,(8^{-1/4})\,R^{3/4}$ so solve for R to get
$R = (24.8)^{4/3}$
and so R = **72 km**.

Problem 3 – Answer: The Event Horizon is at Rs = 3.0 x 8 = 24 kilometers. **The x-ray emitting gas is located at R = 72 km, just outside the Event Horizon at a distance of about 48 kilometers.**

Artist's rendition of GJ 1214b with a hypothetical moon.
(Courtesy: CfA/David Aguilar)

In December 2009, astronomers announced the discovery of the transiting super-Earth planet GJ 1214b located 42 light years from the sun, and orbits a red-dwarf star. Careful studies of this planet, which orbits a mere 2 million km from its star and takes 1.58 days to complete 'one year'. Its mass is known to be 6.5 times our Earth and a radius of about 2.7 times Earth's. Its day-side surface temperature is estimated to be 370^{o} F, and it is locked so that only one side permanently faces its star.

When a planet passes in front of its star, light from the star passes through any atmosphere the planet might contain and travels onwards to reach Earth observers. Although the disk of the planet will temporarily decrease the brightness of its star by a few percent, the addition of an atmosphere causes an additional brightness decrease. The amount depends on the thickness of the atmosphere, the presence of dust and clouds, and the chemical composition. By studying the light dimming at many different wavelengths, astronomers can distinguish between different atmospheric constituents by using specific spectral 'fingerprints'. They can also estimate the thickness of the atmosphere in relation to the diameter of the planet.

Problem 1 – Assuming the planet is a sphere, from the available information, to two significant figures, what is the average density of the planet in kg/meter^3? (Earth mass = 6.0×10^{24} kg; Diameter = 6378 km).

Problem 2 – The average density of Earth is 5,500 kg/m^3. Suppose that GJ 1214b has a rocky core with Earth's density and a radius of R, and a thin atmosphere with a density of D. Let R = 1.0 at the top of the atmosphere, and R=0 at the center of the planet, and assume the core is a sphere, and that the atmosphere is a spherical shell with inner radius R and outer radius R=1.0. The formula relating the atmosphere density, D, to the core radius, R, is given by:

$$1900 = (5500 - D)R^3 + D$$

A) Re-write this equation by solving for D.

B) Graph the function D(R) over the domain R:[0,1].

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

Problem 1 – Answer: Volume = 4/3 (3.141) $(2.7x6378000)^3$ = 2.1 x 10^{22} $meter^3$.
Density = Mass/Volume
= (6.5x6.0 x 10^{24} kg) / (2.1 x 10^{22} $meter^3$)
= **1,900 kg/meter^3**

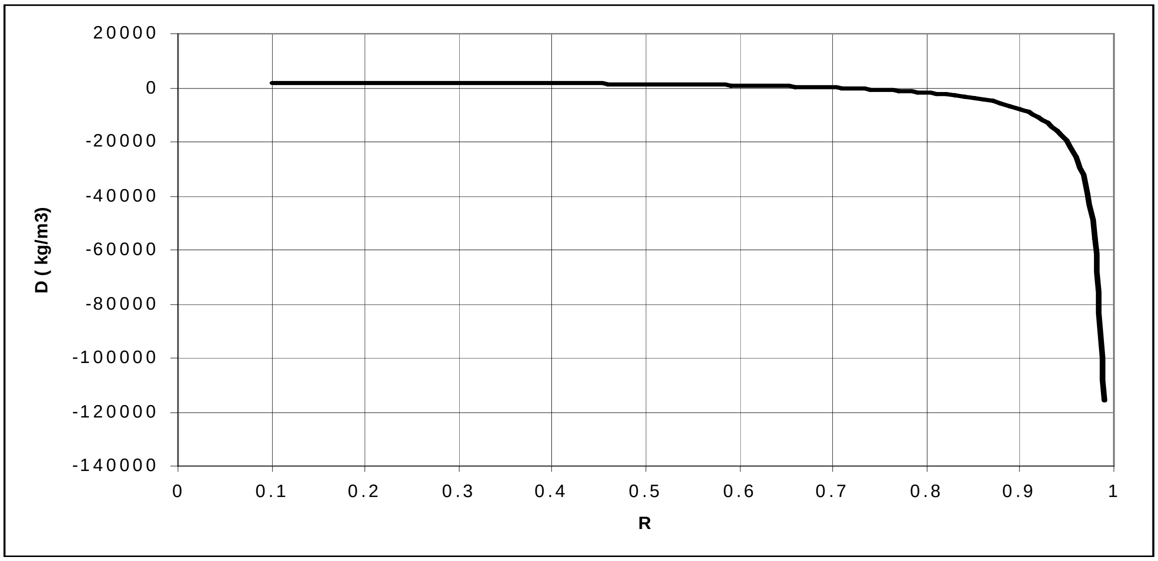
Problem 2 – A) Answer: $1900 = (5500 - D)R^3 + D$

$$1900 - D = 5500R^3 - DR^3$$

$$D\left(R^3 - 1\right) = 5500R^3 - 1900$$

$$D = \frac{5500R^3 - 1900}{R^3 - 1}$$

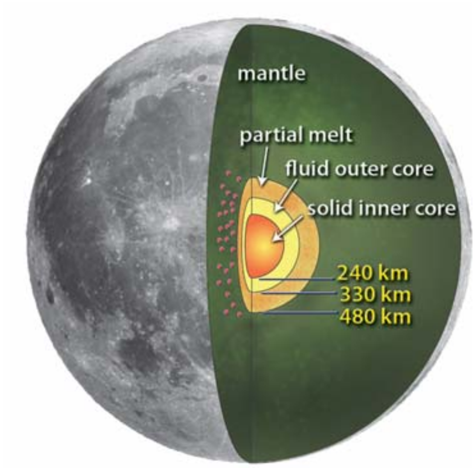
B) See below:



Note that for very thin atmospheres where D: [0, 100] the function predicts that the core has a radius of about R=0.7 or 70% of the radius of the planet. Since the planet's radius is 2.7 times Earth's radius, the core is about 0.7x2.7 = 1.9 x Earth's radius. Values for D < 0 are unphysical even though the function predicts numerical values. This is a good opportunity to discuss the limits of mathematical modeling for physical phenomena.

C) If the average density of the atmosphere is comparable to that of Venus's atmosphere for which D= 100 kg/m^3, what fraction of the radius of the planet is occupied by the Earth-like core, and what fraction is occupied by the atmosphere?

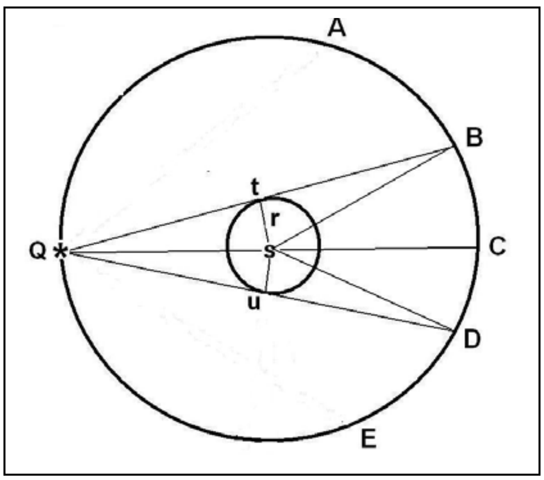
Answer: **For D=100, R = 0.71, so the core occupies the inner 71 % of the planet, and the surrounding atmospheric shell occupies the outer 29% of the planet's radius.**



Sometimes old data can uncover new secrets! Four seismometers were deployed by Apollo astronauts between 1969 and 1972. They were able to record continuous lunar seismic activity until late-1977.

A detailed mathematical analysis of this data reveals that the seismic data are consistent with a model in which the moon has a solid, iron-rich inner core and a fluid, primarily liquid-iron outer core. The core contains a small amount of elements such as sulfur, which is a composition similar to the core of our own Earth.

The analysis they used can be shown with a simple geometry problem:



Suppose a 'moonquake' occurs at Point Q in the figure. Its shock waves travel along the chords from Q to the various seismometers located at points A, B, C, D and E. In addition:

1 - The radius of the moon, R has a length of 1,738 km.

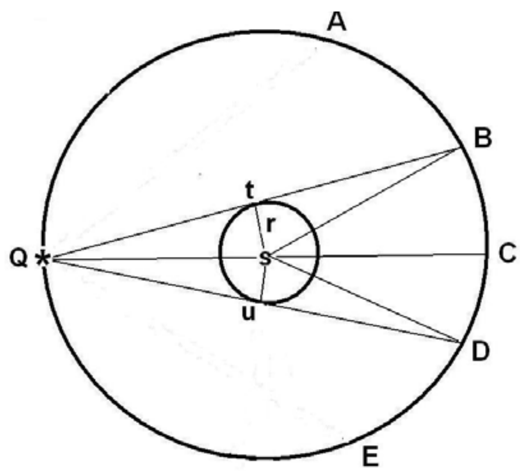
2 - The radius of the core, r, is defined by the segment **ts**.

3 - Angles **Qts** and **Qus** are right-angles.

Basic Seismology: An earthquake generates two kinds of shock wave signals called P-waves and S-waves. When rock is compressed like a sound wave, it produces a pressure wave called the P-wave along its direction of travel. When it moves from side-to-side perpendicular to its direction of travel, it is called a shear wave or S-wave. Although P-waves can travel through a liquid, S-waves are strongly reduced in strength, or sometimes absent all-together.

Suppose on the moon, Stations A and E record normal seismic S and P-wave signals, however, Stations B and D record signals in which the S-wave is slightly reduced in strength compared to Station's A and E. Station C records P-waves but no S-waves. Assume that Station C is in the shadow zone of the liquid lunar core, and that Stations B and D define seismic signals grazing the outer edge of a hypothetical lunar liquid core. Stations B and D are separated by 900 km along the lunar surface.

Problem 1 - From the figure above, and your knowledge of the properties of inscribed arcs what is the radius in kilometers of the core, r, based on this seismic date?



The arc, BCD = 900 km.
Qs = R = 1,738 km

Angle BsD = 360 x (900 km/ $2\pi R$)
= 360 x (900 km / 10918 km)
= 30 degrees

Angle BsC = 15 degrees

Angle BQC = 1/2 Angle BsC = 7.5 degrees

Angle BQC = Angle tQs

Segment st = r

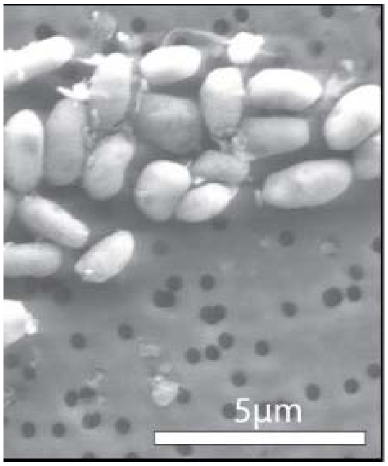
Then:

Sin (Angle tQs) = r/R

r = R sin(7.5 degrees)

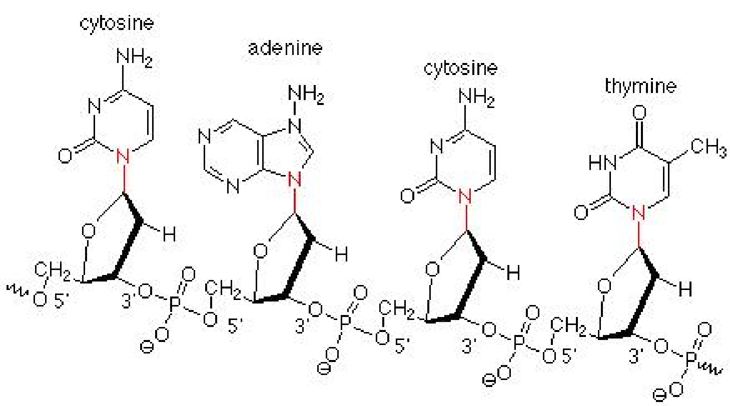
r = 1,738 (0.13)

r = 226 km.


Microphotograph of the new bacterium GFAJ-1 that subsists on the toxic element arsenic.

NASA researchers exploring extremophile bacteria in Mono Lake, California recently discovered a new strain of bacterium GFAJ-1 in the Gammaproteobacteria group, which not only feeds on the poisonous element arsenic, but incorporates this element in its DNA as a replacement for normal phosphorus. All other known life forms on Earth use 'standard' DNA chemistry based upon the common elements carbon, oxygen, nitrogen and phosphorus.

In the search for life on other worlds, knowing that 'life' can exist that is fundamentally different than Earth life now broadens the possible places to search for the chemistry of life in the universe.



This diagram shows the elements that make up a small section of normal DNA containing the four bases represented from top to bottom by the sequence 'CACT'. They are held together by a 'phosphate backbone' consisting of a phosphor atom, P, bonded to four oxygen atoms, O. Each phosphor group (called a phosphodiester) links together two sugar molecules (dioxyribose), which in turn bond to each of the bases by a nitrogen atom, N.

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4 ? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4?

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

Problem 1 - The atomic mass of phosphor P= 31 AMU, arsenic As= 75 AMU, hydrogen H=1 AMU and Oxygen O= 16 AMU. A) What is the total atomic mass of one phosphodiester molecule represented by the formula PO_4? B) For the new bacterium, what is the total atomic mass of one arsenate molecule represented by the formula AsO_4 ?

Answer: A) PO_4 = 1 Phosphorus + 4 Oxygen
= 1 x 31 AMU + 4 x 16 AMU
= **95 AMU**

B) AsO_4 = 1 Arsenic + 4 Oxygen
= 1 x 75 AMU + 4 x 16 AMU
= **139 AMU**

Problem 2 - The DNA for the smallest known bacterium, mycoplasma genetalium, has about 582,970 base pairs. Suppose that the 1,166,000 phosphodiester molecules contribute about 30% of the total mass of this organism's DNA. If arsenic were substituted for phosphorus to form a twin arsenic-based organism, by how much would the DNA of the new organism increase?

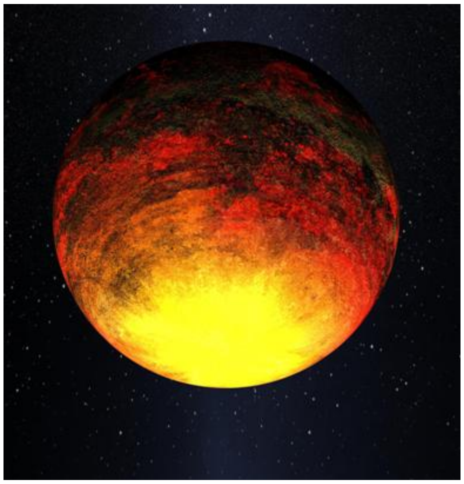
Answer: The arsenic-substituted ester has a mass of 139 AMU compared to the phosphorus-based ester with 95 AMU, so the new molecule AsO_4 is 100% x (95/139) = 68% more massive than PO_4.

Since in the normal DNA the PO_4 contributes 30% of the total DNA mass, the non-PO_4 molecules contribute 70% of the normal mass.

This is added to the new arsenic-based molecule mass for AsO_4 of 30% x1.68 = 50% to get a new mass that is 70% + 50% = **120% heavier** than the original, 'normal' DNA based on PO_4.

So we would predict that the DNA of the twin arsenic-based organism is only 20% more massive than the DNA of the original phosphate-based organism.

Note: Students may have a better sense of the calculation if they start with a concrete amount of 100 grams of normal DNA. Then 70 grams are in the non-PO_4 molecules and 30 grams is in the PO_4 molecules. Because AsO_4 is 68% more massive than PO4, its contribution would be 30 grams x 1.68 = 50 grams. Then adding this to the 70 grams you get 120 grams with is 20 grams more massive than normal DNA for a gain of 120%.


NASA Artist rendition of the sizzling-hot, Earth-like world: Kepler 10b.

The Kepler Space Observatory recently detected an Earth-sized planet orbiting the star Kepler-10. The more than 8 billion year old star, located in the constellation Draco, is 560 light years from Earth. The planet orbits its star at a distance of 2.5 million km with a period of 20 hours, so that its surface temperature exceeds 2,500 F.

Careful studies of the transit of this planet across the face of its star indicates a diameter 1.4 times that of Earth, and an estimated average density of 8.8 grams/cc, which is about that of solid iron, and 3-times the density of Earth's silicate-rich surface rocks.

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \quad \text{meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Answer: The planet is 1.4 times the radius of Earth, so its radius is 1.4 x 6,378 km = 8,929 kilometers. Since we need to use units in terms of meters because we are given the density in cubic meters, the radius of the planet becomes 8,929,000 meters.

Volume = $\frac{4}{3}\pi R^3$

so V = 1.33 x (3.141) x (8,929,000 meters)^3

V = 2.98 x 10^{21} meter^3

Mass = Density x Volume

= 8,800 x 2.98 x 10^{21}

= **2.6x10^{25} kilograms**

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \text{ meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

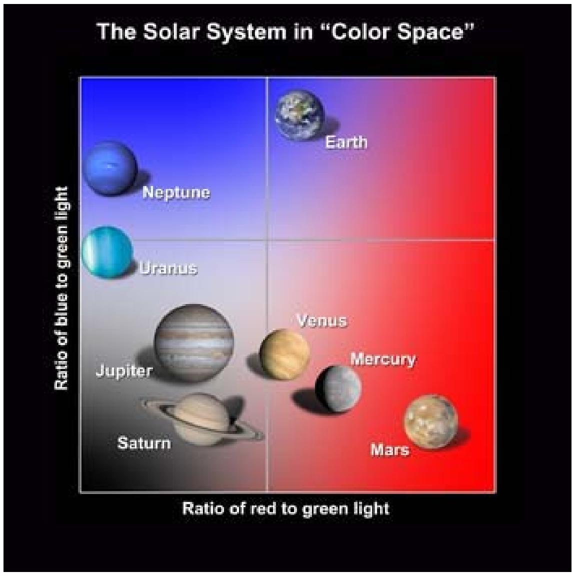
Answer: a = 6.67 x 10^{-11} (2.6 x 10^{25})/(8.929x10^6)^2

= **21.8 meters/sec^2**

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Answer: The acceleration is 21.8/9.8 = 2.2 times Earth's gravity, and since weight is proportional to gravitational acceleration we have the proportion:

$\frac{21.8}{9.8} = \frac{X}{150lb}$ and so the human would weigh 150 x 2.2 = **330 pounds!**



Earth is invitingly blue. Mars is angry red. Venus is brilliant white. NASA astronomer Lucy McFadden, and UCLA research assistant in geochemistry Carolyn Crow, have now discovered that a planet's "true colors" can reveal important details.

Mars is red because its soil contains rusty red stuff called iron oxide. Our planet, the "blue marble" has an atmosphere that scatters blue light rays more strongly than red ones. This suggests that astronomers could use color information to identify Earth-like worlds. Their colors will tell us which ones to study in more detail.

As NASA's Deep Impact spacecraft cruised through space, its High Resolution Instrument (HRI) measured the intensity of Earth's light. HRI is a 30-cm telescope that feeds light through seven different color filters. Each filter samples the incoming light at a different portion of the visible-light spectrum, from ultraviolet and blue to near-infrared. A table showing the reflectivity of each body is shown below. The numbers indicate the percentage of light reflected by the planet at 350, 550 and 850 nanometers (nm). For example, compared to the light that it reflects at 550 nm, Venus reflects 116% more light at 850 nm.

Object	350 nm	550 nm	850nm	Object	350nm	550nm	850nm
Mercury	47	100	142	Jupiter	60	100	64
Venus	58	100	109	Saturn	45	100	78
Earth	152	100	110	Titan	34	100	88
Moon	67	100	169	Uranus	98	100	15
Mars	34	100	203	Neptune	125	100	13

Note: Table based upon data published by the astronomers in the Astrophysical Journal (March 10, 2011).

Problem 1 – One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where X = R(850)/R(550) and X = R(350)/R(550). For example, for the Moon, where R(350) = 61% , R(550) = 100%, and R(850) = 155% we have X = 155/100 = 1.55, and Y = 61/100 = 0.61. Using this method, calculate X and Y for each object and then plot the (X,Y) points on a graph.

Problem 2 – The planetary data in the table can be written as an ordered triplet. For example, for Mercury the reflectivities in the table above would be written as (56, 100, 177). Using the definition for X and Y in Problem 1, which of the planets below would you classify as Earth-like, Jupiter-like, or Moon-like, if the planetary reflectivities are: Planet A (61, 82, 156), Planet B (45, 35, 56), Planet C (90, 120, 67).

Problem 3 – Can you create a different plot for the planets that makes their differences stand out even more?

Problem 1 – One way to plot this data so that the planets can be easily separated and identified is to plot the ratio of the reflectivities for each planet where X = R(850)/R(550) and X = R(350)/R(550). For example, for the Moon, where R(350) = 61% , R(550) = 100%, and R(850) = 155% we have X = 155/100 = 1.55, and Y = 61/100 = 0.61. Using this method, calculate X and Y for each object and then plot the (X,Y) points on a graph. See graph below.

Object	350 nm	550 nm	850nm	X	Y
Mercury	47	100	142	**1.42**	**0.47**
Venus	58	100	109	**1.09**	**0.58**
Earth	152	100	110	**1.10**	**1.52**
Moon	67	100	169	**1.69**	**0.67**
Mars	34	100	203	**2.03**	**0.34**
Jupiter	60	100	64	**0.64**	**0.60**
Saturn	45	100	78	**0.78**	**0.45**
Titan	34	100	88	**0.88**	**0.34**
Uranus	98	100	15	**0.15**	**0.98**
Neptune	125	100	13	**0.13**	**1.25**

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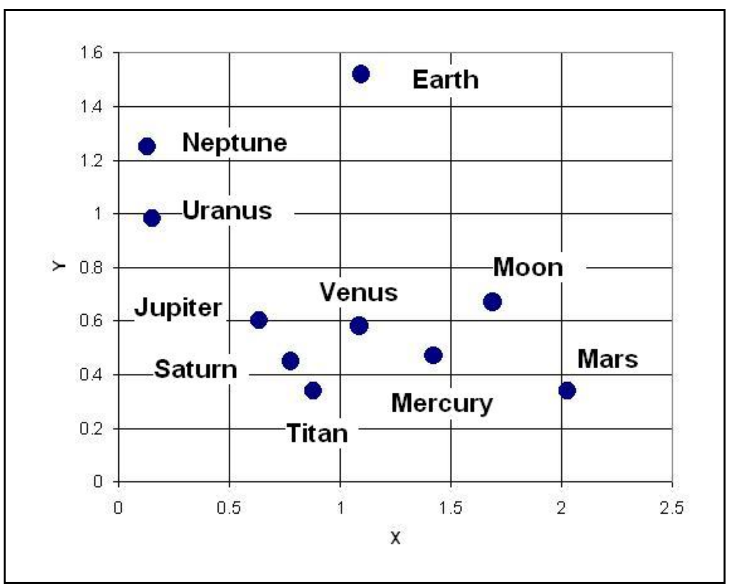
Problem 3 – Can you create a different plot for the planets that makes their differences stand out even more? Answer: **There are more than 100 different ways in which students may decide to create new definitions for X and Y such as X = R(350) – R(850); Y = R(850)/R(350) and so on. Some will not visually let you see a big difference between the planet 'colors' while other may. Astronomers try many different combinations, usually with some idea of the underlying physics and how to enhance what they are looking for. There is no right or wrong answer, only ones that make the analysis easier or harder!**



According to geophysicist Richard Gross at NASA's Jet Propulsion Laboratory in Pasadena, California, the March 11, 2011 Japan Quake caused the rotation of Earth to speed up by about 1.8 millionths of a second (that's 0.0000018 seconds). This doesn't sound like much, but in today's high-tech world where time is regularly measured in billionths of a second, this is a huge change!

The reason this happens is similar to a spinning ice skater pulling in her arms during a spin. The mass in her hands brought closer to her spin axis causes her to spin faster in order to conserve momentum. Earthquakes may move mass, like the island of Japan, slightly closer to the center of Earth and so it will also spin up to conserve momentum. Japan was moved around by nearly 4 meters, which changed the way matter was distributed in Earth's crust (the ice skater's hands) in a significant way. How does this work? Here is a simple model.

Problem 1 - The formula for the angular momentum of a uniform sphere is just

$$J = \frac{2}{5} M r^2 \omega$$

If the mass, M, is conserved and the angular momentum, J, is conserved, what is the formula for the initial radius, r, and angular velocity, ω, compared to the final radius and angular velocity?

Problem 2 - If the Earth rotates 2π radians in exactly 24 hours, with a radius of 6378.00 km, what will its final angular velocity be in radians/sec, after it has shrunk by 1.00 kilometer?

Problem 3 - What will be the difference in its rotation period in seconds?

Problem 1 - The angular momentum of a uniform sphere is just $J = \frac{2}{5}Mr^2\omega$

If the mass, M, is conserved and the angular momentum, J, is conserved, what is the formula for the initial radius, r, and angular velocity, ω, compared to the final radius and angular velocity?

Answer: $r_i^2\omega_i = r_f^{\ 2}\omega_f$

Problem 2 - If the Earth rotates 2π radians in exactly 24 hours, with a radius of 6378 km, what will its final angular velocity be after its has shrunk by 1 kilometer?

R_i = 6378 km ω_i = 6.28318/86400sec = 7.27220x10^{-5} radians/sec

R_f = 6377 km, then solve for w_f

$$\frac{(6378)^2(7.27220x10^{-5})}{(6377)^2} = \omega_f$$

ω_f = 7.274481 x 10^{-5} radians/sec.

Problem 3 - What will be the difference in its rotation period?

Answer: The difference will be

$$P = \frac{2\pi}{\omega_f} - \frac{2\pi}{\omega_i}$$

$$P = \frac{6.2831853}{\left(7.274481 \text{ x } 10^{-5}\right)} - \frac{6.2831853}{\left(7.27220\text{x}10^{-5}\right)}$$

P = 86372.970 seconds - 86400.000 seconds
P = -27.0 seconds.

So the 'toy' Earth will have a day that is 27 seconds shorter.

Note: The Japan quake is a more complex type of crustal motion, but the basic principle is the same.

The devastating earthquake that struck northern Japan on March 10, 2011 also caused several of the nuclear reactors at the Fukushima Nuclear Plant to explosively vent radioactive gas and dust clouds into the environment.

Although the initial radiation levels were extremely high, the natural decay of the radioactive compounds will cause the radiation levels at any given distance to steadily reduce in intensity.

The most common radioisotopes that are likely to be involved in the vented gases are Cesium-137 and Iodine-131. These will become incorporated into atmospheric dust grains and fall to the ground, contaminating the soil. Cesium-137 has a half-life of 30 years, while Iodine-131 has a half-life of 8 days. This means that, for example, if you measured the dosage you get from a sample of Iodine-131, after 8 days, the dosage will have dropped to 1/2 its original level on Day-1. After 16 days it will be 1/4 of its original level on Day-1, and so on.

The formula for half life and radiation dose rate is:

$$D(t) = D(0)e^{\left(-0.69\frac{t}{T}\right)}$$

where D(0) is the dose rate on Day-0, D(t) is the dose rate on Day-t, and T is the half-life in days.

Problem 1 - The natural background radiation dose rate is about 3.5 milliSeiverts/year. What is this natural radiation dose rate in microSeiverts/hour?

Problem 2 - On Friday, March 18, *NHK News*, the online news service for Japan ,reported that Japan's Science Ministry had posted new radiation measurements. The Ministry indicated that at a location 30 km northwest of the Fukushima Daiichi nuclear plant, the radiation dose rates were 170 microSeiverts/hour on March 17, and 150 microSeiverts/hour on March 18. Assuming that the decrease is entirely caused by the decay of a radioisotope, A) what is the half-life of this isotope in days? B) What is the likely candidate for the radioisotope that is causing most of the radiation at this location?

Problem 1 - The natural background radiation dose rate is about 3.5 milliSeiverts/year. What is this natural radiation dose rate in microSeiverts/hour?

Answer; 3.5 milliSeiverts/year x (1000 micros/1 milli) x (1 year/365 days) x (1 day/24 hours) = **0.4 microSeiverts/hour**.

Problem 2 - On Friday, March 18, *NHK News*, the online news service for Japan ,reported that Japan's Science Ministry had posted new radiation measurements. The Ministry indicated that at a location 30 km northwest of the Fukushima Daiichi nuclear plant, the radiation dose rates were 170 microSeiverts/hour on March 17, and 150 microSeiverts/hour on March 18. Assuming that the decrease is entirely caused by the decay of a radioisotope, A) what is the half-life of this isotope in days? B) What is the likely candidate for the radioisotope that is causing most of the radiation at this location?

Answer: A) From the information given, and the formula for D(t) we have D(0) = 170, t = 1 day, D(1) = 150, the we solve

$$150 = 170e^{\left(-0.69\frac{1day}{T}\right)}$$

$$\frac{150}{170} = e^{\left(-0.69\frac{1day}{T}\right)}$$

$$0.882 = e^{\left(-0.69\frac{1day}{T}\right)}$$ now take the natural-log of both sides

$$\ln(0.882) = -0.69\frac{1day}{T}$$ so $$-0.126 = -0.69\frac{1day}{T}$$

then $T = \frac{0.69}{0.126}$

so **T = 5.5 days is the half-life for the dosage decay.**

B) **The most likely candidate contributing to the radiation exposure at this location is Iodine-131, which has a half-life of 8 days.**

This iconic photo was taken at the Shibuya train station in Toyko on March 15, a few days after the Japan 2011 earthquake, which caused severe damage to the Fukushima Nuclear Plant in northern Japan located 250 km from Tokyo. The monitor indicates a reading of 0.6 microSeiverts per hour. The normal background dose rate is about 0.4 microSeiverts per hour.*(Courtesy Associated Press/Kyodo News)*

The devastating Japan 2011 earthquake damaged the nuclear reactors in Fukushima, which emitted clouds of radioactive gas and dust into the atmosphere. News reports indicated the radiation levels at many different locations and times through out the following week. Because radioactivity decays with time and distance, it is difficult to compare the many measurements to know if the dosages are declining as expected. The following table of measurements was collected from a variety of news reports:

Date	Distance (km)	Location	Dose Rate (microSeiverts/hr)
March 15	1 km	Fukushima #2 plant	8,200
March 15	20 km	Namai	330
March 16	30 km	Iwaki City	150
March 16	50 km	Koriyama City	2.7
March 16	70 km	Kitaibaraki City	1.2
March 16	160 km	Maebiashi City	4.0
March 15	250 km	Tokyo	0.9 maximum

Problem 1 - Graph the base-10 log of the radiation dose rate versus the base-10 log of distance from the Fukushima nuclear plant.

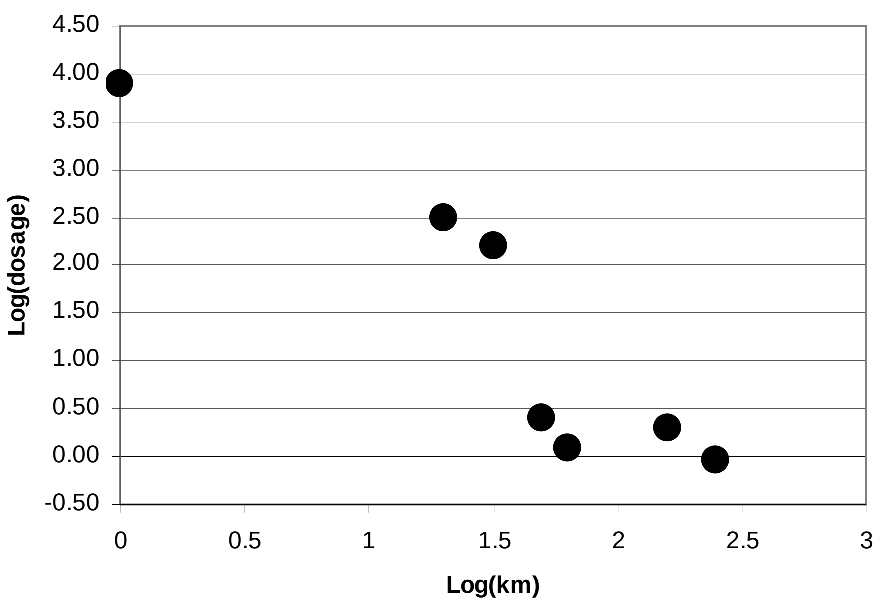
Problem 2 - On the Log-Log graph you just created, what is the approximate slope of the line that fits the data best? Show that an 'inverse-square law' has a slope of -2 on this graph.

Problem 3 - Using the graph, what would you predict for the dose rates near the city of Tamura, located 40 km from the nuclear reactors, and a position located at 10-kilometers, which is inside the limit to the evacuation zone?

Problem 1 - Graph the base-10 log of the radiation dose rate versus the base-10 log of distance from the Fukushima nuclear plant.

Distance	Dose Rate	Log(km)	Log(D)
1 km	8,200	0	3.9
20 km	330	1.3	2.5
30 km	150	1.5	2.2
50 km	2.7	1.7	0.4
70 km	1.2	1.8	0.08
160 km	4.0	2.2	0.3
250 km	0.9	2.4	-0.05

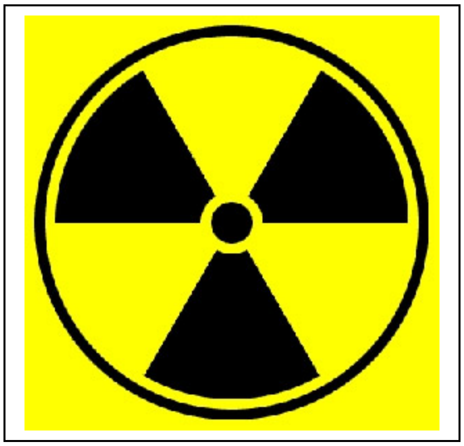
The graph of the Log(Dose Rate) and Log(distance) is as follows:



Problem 2 - On the Log-Log graph, what is the approximate slope of the line that fits the data best? Show that an 'inverse-square law' has a slope of -2 on this graph.
Answer: Using the first (0.0, 3.8) and last (2.4, -0.05) points, Slope = (-0.05 – 3.8)/(2.4 – 0.0), slope = -3.85/2.4); **Dose rate slope = -1.6**. An inverse square law is written as $y = x^{-2}$ Taking the log of both sides we get log(y) = -2 log(x). So the slope of an inverse-square law on a log-log graph would be exactly -2.0

Problem 3 – Using the graph, what would you predict for the radiation dose rates near the city of Tamura (d= 40 km), and a position located at 10-kilometers, which is inside the 20-kilometer limit to the evacuation zone?

Answer: Log(40km) = 1.6, so for x = 1.6, we can estimate from the graph that y = 1.4, and so Log(dosage) = +1.5 so that dosage = $10^{+1.5}$ = **32 microSeiverts/hour**. At a distance of 10-km, X = log(10) = 1.0 and so y = 2.0 and the dosage would be **1,000 microSeiverts/hour.**



Radiation is measured in two units. The first is a measure of the rate at which you are being exposed to a source of radioactivity, while the second is a measure of how much radiation you have accumulated over time. Radiation dose is measured in units of microSeiverts while dose rate is measured in terms of dose per unit time such as microSeiverts per hour or milliSeiverts per year.

Hasty reports about the devastating Japan 2011 nuclear power plant radiation leakages have occasionally confused these two concepts. When you consider the analogy of filling up a 1-liter kettle of water from the water tap, it is easy to keep the distinction between dose and dose rate clear. The volume of water in the kettle (the dose) depends on the rate of water flow (the dosage rate) times the filling time.

Problem 1 - The natural radiation background at sea level is about 0.4 microSeiverts/hour. In terms of milliSeiverts, what is your total radiation dose after A) 1 year? B) a 70-year lifetime?

Problem 2 - At the typical cruising altitude of a passenger jet, about 10,000 meters (33,000 feet), the dose rate is about 5 microSeiverts/hour. During its years of operation, the French Concorde jet traveled at altitudes of 18,000 meters (54,000 feet) where the cosmic radiation dose rate was about 15 microSeiverts/hour. If a flight from Paris to New York takes 8 hours by ordinary jet and 3.5 hours by Concorde, what are the total radiation doses for a passenger in each case?

Problem 3 - The Japan 2011 earthquake damaged several nuclear reactors, causing radiation leakage across northern Japan. On March 22, 2011 typical radiation levels across most of Japan are now below 50 microSeiverts/hour. The typical annual radiation dose from all forms of natural sources, medical tests, and food consumption is about 0.4 milliSeiverts. How many days will it take for a Japanese citizen to reach this annual dose level?

Problem 1 - The natural radiation background at sea level is about 0.4 microSeiverts/hour. In terms of milliSeiverts, what is your total dose after A) 1 year? B) a 70-year lifetime? Answer: A) 0.4 microSeiverts/hour x (24 hours/1 day) x (365 days/1 year) = 3500 microSeiverts. Converting this to milliServerts: 3500 microSeiverts x (1 milliSeivert/1000 microSeiverts) = **3.5 milliSeiverts**. B) In a 70-year lifetime ,your total dose will be 70 years x 3.5 milliSeiverts/year = **245 milliSeiverts**.

Problem 2 - At the typical cruising altitude of a passenger jet, about 10,000 meters (33,000 feet), the dose rate is about 5 microSeiverts/hour. During its years of operation, the French Concorde jet traveled at altitudes of 18,000 meters (54,000 feet) where the cosmic radiation dose rate was about 15 microSeiverts/hour. If a flight from Paris to New York takes 8 hours by ordinary jet and 3.5 hours by Concorde, what are the total doses for a passenger in each case?

Answer: Ordinary jet: 5 microSeiverts/hour x 8 hours = **40 microSeiverts**.
Concorde: 15 microSeiverts/hour x 3.5 hours = **53 microSeiverts**.

Problem 3 - The Japan 2011 earthquake damaged several nuclear reactors, causing radiation leakage across northern Japan. On March 22, 2011 typical radiation levels across most of Japan are now below 10 microSeiverts/hour. The typical annual radiation dose from all forms of natural sources, medical tests, and food consumption is about 0.4 milliSeiverts. How many days will it take for a Japanese citizen to reach this annual dose level?

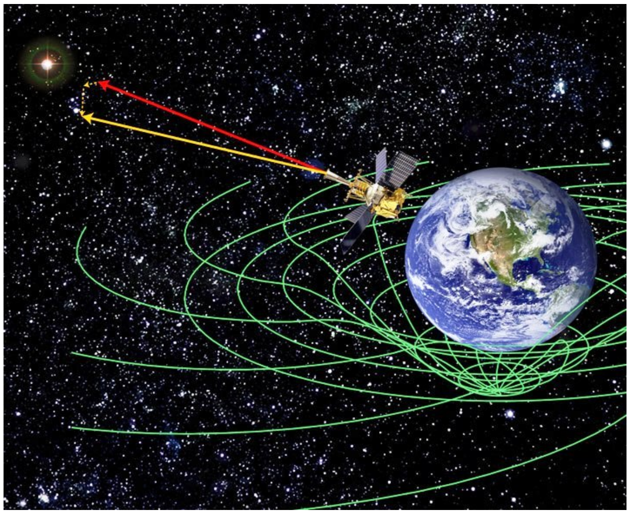
Answer: Time = Amount / Rate

$$Time = \frac{0.4 milliSeiverts}{10 microSeiverts / hr}$$

$$Time = \frac{400 microSeiverts}{10 microSeiverts / hr}$$ so Time = 40 hours.

This means that in one year they will accumulate over 88 milliSeiverts of radiation dose, which is 88/0.4 = 220 times the normal annual dosage.

Fortunately, the dose rates are declining each day as the radioactive isotopes decay, and are also diluted through wind action. The actual accumulated dose may only be 10% of this in most regions of the country. This is still, however, 20 times the natural pre-contamination dose rate. Other communities where the dose rates are higher than 10 microSeiverts/hr will have to be decontaminated by soil removal and other time-consuming and expensive methods.



Physicists at Stanford University have recently completed their analysis of data from the Gravity Probe-B (GP-B) satellite, launched in 2004, and have confirmed two predictions of Albert Einstein's relativistic theory of gravity called General Relativity.

The pointing direction of a high-precision gyroscope was measured for over 50 weeks as it orbited Earth. If Newton's theory of gravity were correct, the pointing direction should stay absolutely the same. If Einstein's theory was correct, it should point in a slightly different direction.

The effect is called 'frame dragging' and was first predicted in 1918 by Austrian physicists Josef Lense (1890-1985) and Hans Thirring (1888-1976) using Einstein's mathematical theory of gravity published in 1915. The rate at which the pointing angle will change is given by the formula for Ω, in degrees/sec, shown below:

$$\Omega = \frac{GJ}{2c^2a^3\left(1-e^2\right)^{3/2}}\left(\frac{360}{2\pi}\right)$$

c is the speed of light: $c = 300{,}000{,}000$ m/s

J is the angular momentum of Earth: $J = 5.861 \times 10^{33}\ m^2\ kg\ sec^{-1}$

G is the Newtonian Gravitational constant $G = 6.67 \times 10^{-11}\ m^3\ kg^{-1}\ s^{-2}$

a is the semi-major axis of the satellite orbit

e is the eccentricity of the satellite orbit

Problem 1- The GP-B satellite orbits at a distance from Earth's center of a = 7020 km, in a circular orbit for which e=0. To two significant figures, what is the value for Omega in A) degrees per second? B) arcseconds per year? (Note 1 degree = 3600 arcseconds and 1 year = 3.1×10^7 seconds)

Problem 2 - The GP-B spacecraft took observations for 50 weeks. About what would be the accumulated angular shift by the end of this time to two significant figures?

Problem 1 - The GP-B satellite orbits at a distance from Earth's center of a = 7020 km, in a circular orbit for which e=0. To two significant figures, what is the value for Omega in A) degrees per second? B) arcseconds per year? (Note 1 degree = 3600 arcseconds and 1 year = 3.1×10^7 seconds)

$$\Omega = \frac{GJ}{2c^2a^3\left(1-e^2\right)^{3/2}}\left(\frac{360}{2\pi}\right)$$ in degrees/sec

$$\Omega = \frac{(6.67x10^{-11})(5.861x10^{33})(360)}{4(3.141)\left(3.0x10^8\right)^2\left(7.02x10^6\right)^3\left(1-0^2\right)^{3/2}}$$ **3.66×10^{-13}** degrees/sec

Answer

A) Ω = **3.6×10^{-13} degrees/sec**

B) Ω = **3.6×10^{-13}** degrees/sec x (3600 arcsec/1 degree) x (3.1×10^7 sec/1 year)

= **0.04 arcseconds/year**

Problem 2 - The GP-B spacecraft took observations for 50 weeks. About what would be the accumulated angular shift by the end of this time to two significant figures?

Answer: 50 weeks is 50/52 = 0.96 years, so the total shift is just

$\Theta = \Omega$ x 0.96

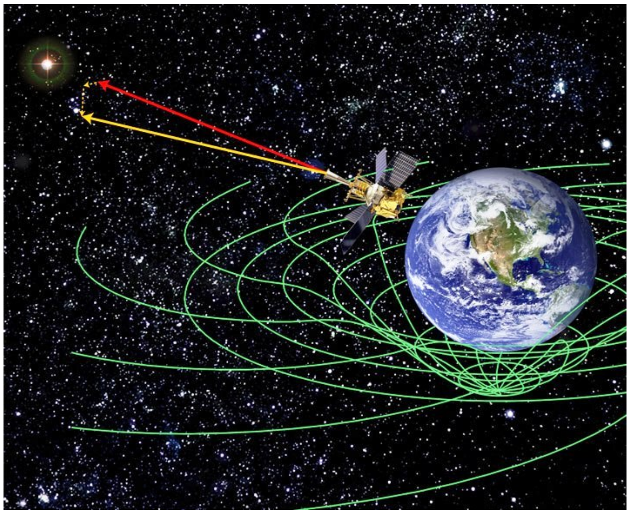
= 0.04 arcseconds/year x (0.96 years)

= 0.038 arcseconds.

Note: This calculation is an approximation to the actual models used to represent the complex spacecraft motion and Earth's gravitational field. Because a more detailed model for Earth and the satellite's motion was used by the GP-B science team, the actual shift detected by the Gravity Probe-B satellite was 0.041 arcseconds, in agreement to within 1% with refined calculations from Einstein's theory.

The equation used in this problem, which predicts the rate of advance of the right ascension of the ascending node of the spacecraft's orbit due to the Lens-Thirring Effect, was obtained from the article:

"*Gravitation, Relativity and Precise Experimentation*" by C.W. Everitt, Proceedings of the First Marcel Grossmann Meeting on General Relativity, pp. 545-615, North Holland, 1977 (p. 567, Equation 22). See the archive of scientific papers at the GP-B website **http://einstein.stanford.edu/content/sci_papers/index.html**



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A prediction of Albert Einstein's relativistic theory of gravity says that the pointing direction of a spinning gyroscope orbiting a massive body should slowly change over time. For Earth, this amount equals degrees/year, and this was recently confirmed by NASA's Gravity Probe-B satellite in 2011.

Einstein's theory predicts much larger shifts if the satellite orbits close to our sun, or to a dense body such as a neutron star.

The effect is called 'frame dragging' and was first predicted in 1918 by Austrian physicists Josef Lense (1890-1985) and Hans Thirring (1888-1976) using Einstein's mathematical theory of gravity published in 1915. The rate, in degrees per second, at which the gyroscope pointing angle will change is given by the formula for Ω, in degrees/sec, shown below:

$$\Omega = \frac{Rac}{r^3 + a^2 r + Ra^2}\left(\frac{360}{2\pi}\right) \quad \text{where} \quad R = \frac{2GM}{c^2} \quad \text{and} \quad a = \frac{2\,\mathrm{R}\,s^2}{5c}\left(\frac{2\pi}{T}\right)$$

and where c is the speed of light (300,000,000 m/s), Rs is the radius of the massive body in meters, M is its mass in kilograms, T is the satellite orbit period in seconds, and G is the Newtonian Gravitational constant 6.67 x 10^{-11} m^3 kg^{-1} s^{-2}. For the GP-B satellite orbiting near Earth at an altitude of 700 km, the measured value for Ω is about 1.2 x 10^{-5} degrees/year.

Problem 1 - In the future, physicists might like to verify this effect near the sun by placing a satellite in a circular orbit at a distance of 10 million kilometers (r = 10^{10} meters). If the radius of the sun is Rs = 6.96x10^{8} meters, and its rotation period is T = 24.5 days, and the mass of the sun is M = 2.0 x 10^{30} kg. To two significant figures, what is the value for the Lens-Thirring rate, Ω, in degrees/year? (Note: 1 degree = 3600 arcseconds)

Problem 2 - A neutron star is the compressed nuclear core of a massive star after it has become a supernova. Suppose the mass of a neutron star is equal to our sun, its radius is 12 kilometers, a gyroscope orbits the neutron star at a distance from its center of r = 6,000 kilometers, and its orbit period is T = 8 seconds. To two significant figures, what is Ω for such a dense, compact system in degrees/year?

Problem 1 - In the future, physicists would like to verify this effect near the sun by placing a satellite in a circular orbit at a distance of 10 million kilometers (r = 10^{10} meters). The radius of the sun is Rs = 6.96×10^8 meters, and its rotation period is T = 24.5 days, and the mass of the sun is M = 2.0×10^{30} kg. To two significant figures, what is the value for the Lens-Thirring rate, Ω, in degrees/year?

$$R = \frac{2(6.67x10^{-11})(2.0x10^{30})}{(300,000,000)^2} = 2{,}964 \text{ m} \qquad a = \frac{2(6.96x10^8)^2}{5(300,000,000)}\left(\frac{2(3.141)}{24.5(24)3600)}\right) = 1{,}883 \text{ m}$$

then

$$\Omega = \frac{(2964)(1883)(3x10^8)}{\left(10^{10}\right)^3 + 1883^2(10^{10}) + (2964)(1883)^2}\left(\frac{360}{2(3.14)}\right) = 9.60x10^{-14} \text{ degrees/sec}$$

Ω = 9.6×10^{-14} deg/sec x (365d/1yr) x (24h /1day) x (3600 s / 1 hr) = **3.0 x 10^{-7} deg/yr**

Note, for GP-B the effect near Earth was 1.2 x 10^{-5} degrees/year because GP-B was orbiting closer to the mass of Earth than our hypothetical satellite around the sun.

Problem 2 - A neutron star is the compressed nuclear core of a massive star after it has become a supernova. Suppose the mass of a neutron star is equal to our sun, its radius is 12 kilometers, a gyroscope orbits the neutron star at a distance from its center of r = 6,000 kilometers, and its orbit period is T = 8 seconds. To two significant figures, what is Ω for such a dense, compact system in degrees/year?

$$R = \frac{2(6.67x10^{-11})(2.0x10^{30})}{(300,000,000)^2} = 2{,}964 \text{ meters} \qquad a = \frac{2(12,000)^2}{5(300,000,000)}\left(\frac{2(3.141)}{8.0}\right) = 0.15 \text{ meters}$$

then

$$\Omega = \frac{(2964)(0.15)(3x10^8)}{\left(6.0x10^6\right)^3 + \left(0.15\right)^2\left(6.0x10^6\right) + (4150)\left(0.15\right)^2}\left(\frac{360}{2(3.141)}\right)$$

$$\Omega = \frac{(1.33x10^{11})}{\left(2.16x10^{20}\right) + \left(1.35x10^5\right) + (93.4)}\left(\frac{360}{(6.242)}\right) = 3.65 \text{ x } 10^{-8} \text{ degrees/sec}$$

Ω = 3.65 x10^{-8} deg/sec x (365d/1yr) x (24h /1day) x (3600 s / 1 hr) = **1.1 deg/yr**

Note this is nearly 100,000 times the corresponding Lens-Thirring rate near Earth.

For a detailed discussion of the derivation of the formula for Ω in the equatorial plane of a spinning body, see Wikipedia:

http://en.wikipedia.org/wiki/Frame-dragging

The universe is a BIG place…but it also has some very small ingredients! Astronomers and physicists often find linear plotting scales very cumbersome to use because the quantities you would most like to graph differ by many powers of 10 in size, temperature or mass. Log-Log graphs are commonly used to see the 'big picture'. Instead of a linear scale '1 kilometer, 2 kilometers 3 kilometers etc' a Logarithmic scale is used where '1' represents 10^1, '2' represents 10^2 …'20' represents 10^{20} etc. A calculator easily lets you determine the Log of any decimal number. Just enter the number, n, and hit the 'log' key to get m = log(n). Then just plot a point with 'm' as the coordinate number!

Below we will work with a Log(m) versus log(r) graph where m is the mass of an object in kilograms, and r is its size in meters.

Problem 1 - Plot some or all of the objects listed in the table below on a LogLog graph with the 'x' axis being Log(M) and 'y' being Log(R).

Problem 2 - Draw a line that represents all objects that have a density of A) nuclear matter (4×10^{17} kg/m^3), and B) water (1000 kg/m^3).

Problem 3 - Black holes are defined by the simple formula R = 3.0 M, where r is the radius in kilometers, and M is the mass in multiples of the sun's mass (1 M = 2.0×10^{30} kilograms). Shade-in the region of the LogLog plot that represents the condition that no object of a given mass may have a radius smaller than that of a black hole.

Problem 4 - The lowest density achievable in our universe is set by the density of the cosmic fireball radiation field of 4×10^{-31} kg/m^3. Draw a line that identifies the locus of objects with this density, and shade the region that excludes densities lower than this.

	Object	R (meters)	M (kg)
1	You	2.0	60
2	Mosquito	$2x10^{-3}$	$2x10^{-6}$
3	Proton	$2x10^{-15}$	$2x10^{-27}$
4	Electron	$4x10^{-18}$	$1x10^{-30}$
5	Z boson	$1x10^{-18}$	$2x10^{-25}$
6	Earth	$6x10^{6}$	$6x10^{24}$
7	Sun	$1x10^{9}$	$2x10^{30}$
8	Jupiter	$4x10^{8}$	$2x10^{27}$
9	Betelgeuse	$8x10^{11}$	$6x10^{31}$
10	Milky Way galaxy	$1x10^{21}$	$5x10^{41}$
11	Uranium atom	$2x10^{-14}$	$4x10^{-25}$
12	Solar system	$1x10^{13}$	$2x10^{30}$
13	Ameba	$6x10^{-5}$	$1x10^{-12}$
14	100-watt bulb	$5x10^{-2}$	$5x10^{-2}$
15	Sirius B white dwarf.	$6x10^{6}$	$2x10^{30}$
16	Orion nebula	$3x10^{18}$	$2x10^{34}$
17	Neutron star	$4x10^{4}$	$4x10^{30}$
18	Binary star system	$1x10^{13}$	$4x10^{30}$
19	Globular cluster M13	$1x10^{18}$	$2x10^{35}$
20	Cluster of galaxies	$5x10^{23}$	$5x10^{44}$
21	Entire visible universe	$2x10^{26}$	$2x10^{54}$

The figure below shows the various items plotted, and excluded regions cross-hatched. Students may color or shade-in the permitted region. This wedge represents all of the known objects and systems in our universe; a domain that spans a range of 85 orders of magnitude (10^{85}) in mass and 47 orders of magnitude (10^{47}) in size!

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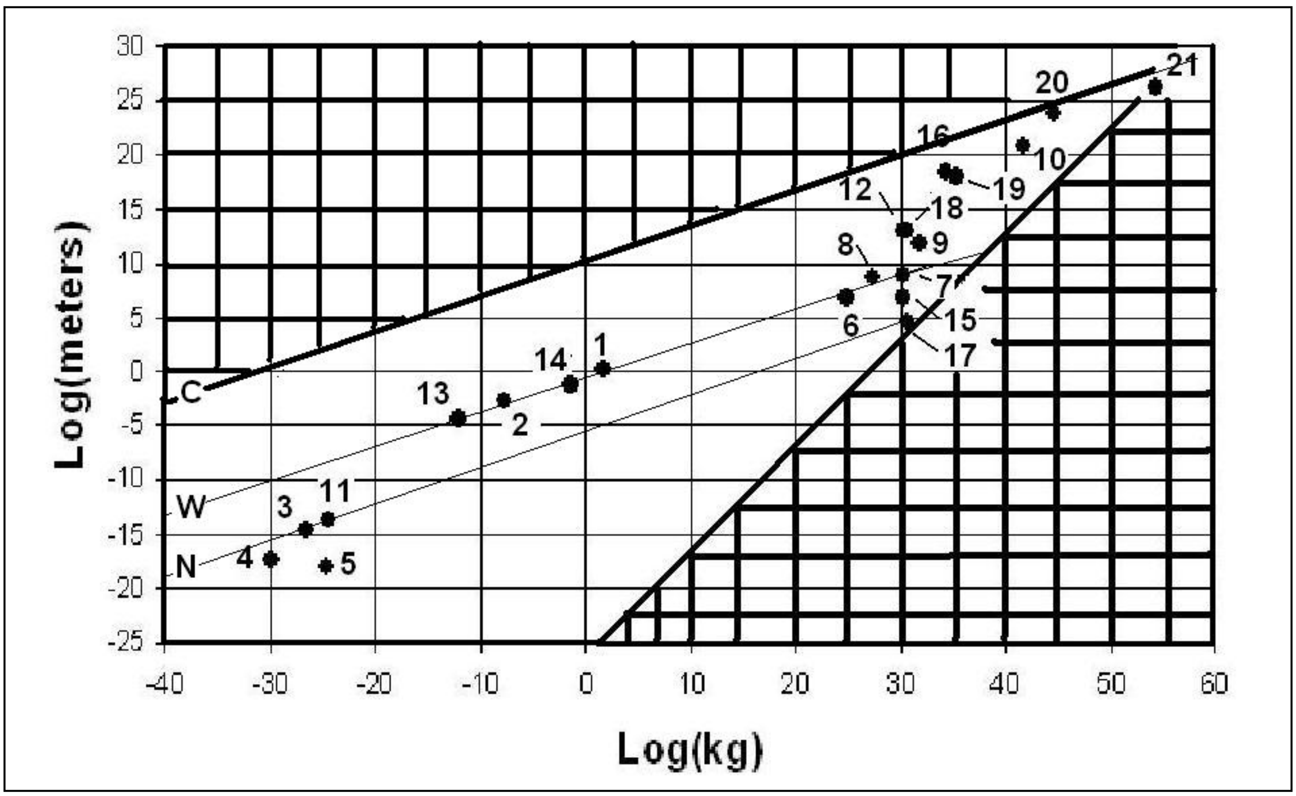
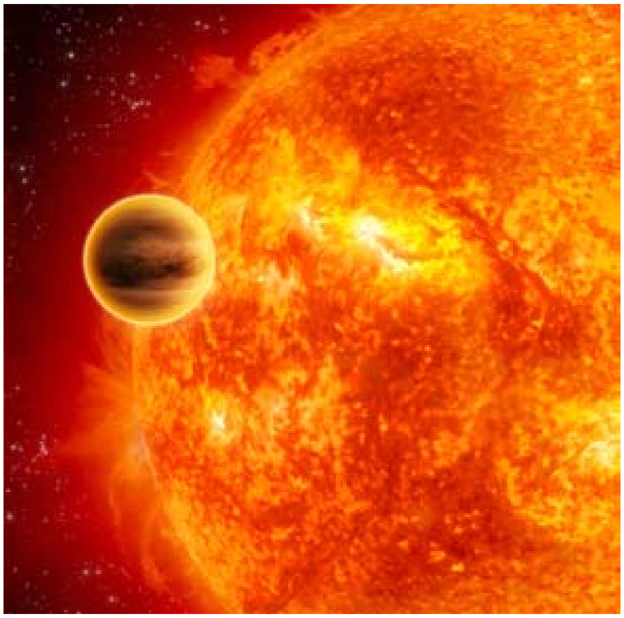
Inquiry: Can you or your students come up with more examples of objects or system that occupy some of the seemingly 'barren' regions of the permitted area?





By 2011, over 1700 planets have been discovered orbiting nearby stars since 1995. Called 'exoplanets' to distinguish them from the familiar 8 planets in our own solar system, they are planets similar to Jupiter in size, but orbiting their stars in mostly elliptical paths. In many cases, the planets come so close to their star that conditions for life to exist would be impossible.

Astronomers are continuing to search for smaller planets to find those that are more like our own Earth.

(Artist rendition: courtesy NASA)

Use the basic properties and formulae for ellipses to analyze the following approximate exoplanet orbits by first converting the indicated equations into standard form. Then determine for each planet the:

A) a= semi-major axis

B) b=semi-minor axis;

C) ellipticity $e = \frac{\sqrt{(a^2 - b^2)}}{a}$

D) 'perihelion' closest distance to star, defined as P=a(1-e);

E) 'aphelion' farthest distance from star, defined as A = a(1+e).

Problem 1: Planet: 61 Virginis-d Period=4 days $1 = 4x^2 + 5y^2$

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

Problem 1: 61 Virginis -d Period=4 days $1 = 4x^2 + 5y^2$

$$1 = \frac{x^2}{0.25} + \frac{y^2}{0.20}$$

a=0.5 b=0.45 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.43** P = (0.5)(1-0.43) = **0.28**, A=(0.5)(1+0.43) = **0.71**

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

$$1 = \frac{x^2}{1.06} + \frac{y^2}{0.92}$$

a=1.03 b=0.96 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.36** P = (1.03)(1-0.36) = **0.66,** A=(1.03)(1+0.36) = **1.40**

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

$$1 = \frac{x^2}{7.0} + \frac{y^2}{5.0}$$

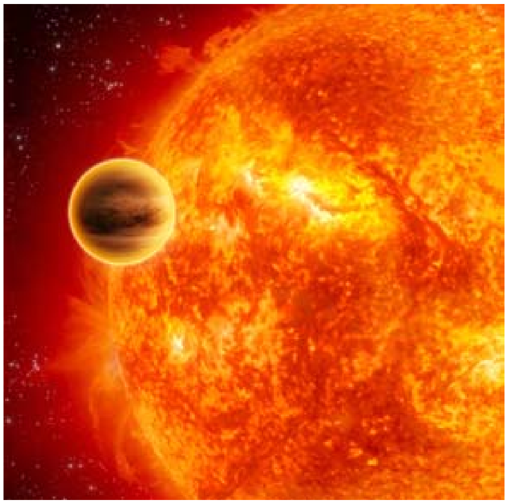
a=2.6 b=2.2 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.53** P = (2.6)(1-0.53) = **1.22**, A=(2.6)(1+0.53) = **4.0**

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

$$1 = \frac{x^2}{12.0} + \frac{y^2}{11.0}$$

a=3.5 b=3.3 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.33** P = (3.5)(1-0.33) = **2.35,** A=(3.5)(1+0.33) = **4.65**

Estimating the Temperatures of Exoplanets



Because many exoplanets orbit their stars in elliptical paths, they experience large swings in temperature. Generally, organisms can not survive if water is frozen (0 C = 273 K) or near its boiling point (100 C or 373 K). Due to orbital conditions, this very narrow 'zone of life' may not be possible for many of the worlds detected so far.

Problem 1 - Complete the table below by calculating
A) The semi-minor axis distance $B = A(1-e^2)$
B) The perihelion distance $Dp = A(1-e)$
C) The aphelion distance, $Da = B(1+e)$

Problem 2 - Write the equation for the orbit of 61 Vir-d in Standard Form.

Problem 3 - The temperature of a planet similar to Jupiter can be approximated by the formula below, where T is the temperature in Kelvin degrees, and R is the distance to its star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). Complete the table entries for the estimated temperature of each planet at the farthest 'aphelion' distance Ta, and the closest 'perihelion;' distance Tp.

$$T(R) = \frac{250}{\sqrt{R}}$$

Problem 4 - Which planets would offer the most hospitable, or the most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39		0.22	2190				
61 Vir-d	0.47		0.35	123				
HD106252-b	2.61		0.54	1500				
HD100777-b	1.03		0.36	383				
HAT-P13c	1.2		0.70	428				

Problem 1 See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**		
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**		
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**		
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**		
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**		

Problem 2 Write the equation for the orbit of 61 Vir-d in Standard Form.
Answer: A = 0.47 and B = 0.4

So $1 = \frac{x^2}{0.47} + \frac{y^2}{0.40}$ and also $188 = 40x^2 + 47y^2$

Problem 3 - See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**	**154**	**123**
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**	**452**	**314**
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**	**228**	**125**
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**	**308**	**211**
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**	**417**	**175**

Problem 4 - Which planets would offer the most hospitable, or most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?
Answer: For the habitable 'water' range between 273K and 373K, none of these planets satisfy this minimum and maximum condition. They are either too hot at perihelion 'summer' or too cold at 'winter' aphelion.

Only HD100777-b during perihelion is in this temperature range during 'summer', at a temperature of 308 K (35 C). During 'winter' at aphelion, it is at -62 C which is below the freezing point of water, and similar to the most extreme temps in Antarctica.

Note: These temperature calculations are only approximate and may be considerably different with greenhouse heating by the planetary atmosphere included.

A very common way to describe the atmosphere of a planet is by its 'scale height'. This quantity represents the vertical distance above the surface at which the density or pressure if the atmosphere decreases by exactly 1/e or $(2.718)^{-1}$ times (equal to 0.368).

The scale height, usually represented by the variable **H**, depends on the strength of the planet's gravity field, the temperature of the gases in the atmosphere, and the masses of the individual atoms in the atmosphere. The equation to the left shows how all of these factors are related in a simple atmosphere model for the density P. The variables are:

z: Vertical altitude in meters
T: Temperature in Kelvin degrees
m: Average mass of atoms in kilograms
g: Acceleration of gravity in meters/sec^2
k: Boltzmann's Constant 1.38×10^{-23} J/deg

$$P(z) = P_0 \, e^{-\frac{z}{H}} \quad \text{and} \quad H = \frac{kT}{mg}$$

Problem 1 - For Earth, g = 9.81 meters/sec^2, T = 290 K. The atmosphere consists of 22% O_2 (m= $2 \times 2.67 \times 10^{-26}$ kg) and 78% N_2 (m= $2 \times 2.3 \times 10^{-26}$ kg). What is the scale height, H?

Problem 2 - Mars has an atmosphere of nearly 100% CO_2 (m=7.3×10^{-26} kg) at a temperature of about 210 Kelvins. What is the scale height H if g= 3.7 meters/sec^2?

Problem 3 - The Moon has an atmosphere of nearly 100% sodium (m=6.6×10^{-26} kg). If the scale height deduced from satellite observations is 120 kilometers, what is the temperature of the atmosphere if g= 1.6 meters/sec^2?

Problem 4 - At what altitude on Earth would the density of the atmosphere P(z) be only 10% what it is at sea level, P_0?

Problem 5 - Calculate the total mass of the atmosphere in a column of air, below a height h with integral calculus. At what altitude, h, on Earth is half the atmosphere below you?

Problem 1 - Answer: First we have to calculate the average atomic mass. <m> = 0.22 ($2\times2.67\times10^{-26}$ kg) + 0.78 ($2\times2.3\times10^{-26}$ kg) = 4.76×10^{-26} kg. Then,

$$H = \frac{(1.38\times10^{-23})(290)}{(4.76\times10^{-26})(9.81)} = $$ **8,570 meters or about 8.6 kilometers.**

Problem 2 - Answer:

$$H = \frac{(1.38\times10^{-23})(210)}{(7.3\times10^{-26})(3.7)} = $$ **10,700 meters or about 10.7 kilometers.**

Problem 3 - Answer

$$T = \frac{(6.6\times10^{-26})(1.6)(120000)}{(1.38\times10^{-23})} = $$ **918 Kelvins.**

Problem 4 - Answer: $0.1 = e^{-(z/H)}$, Take ln of both sides, ln(0.1) = -z/H then z = 2.3 H so for H= 8.6 km, **z = 19.8 kilometers**.

Problem 5 - First calculate the total mass:

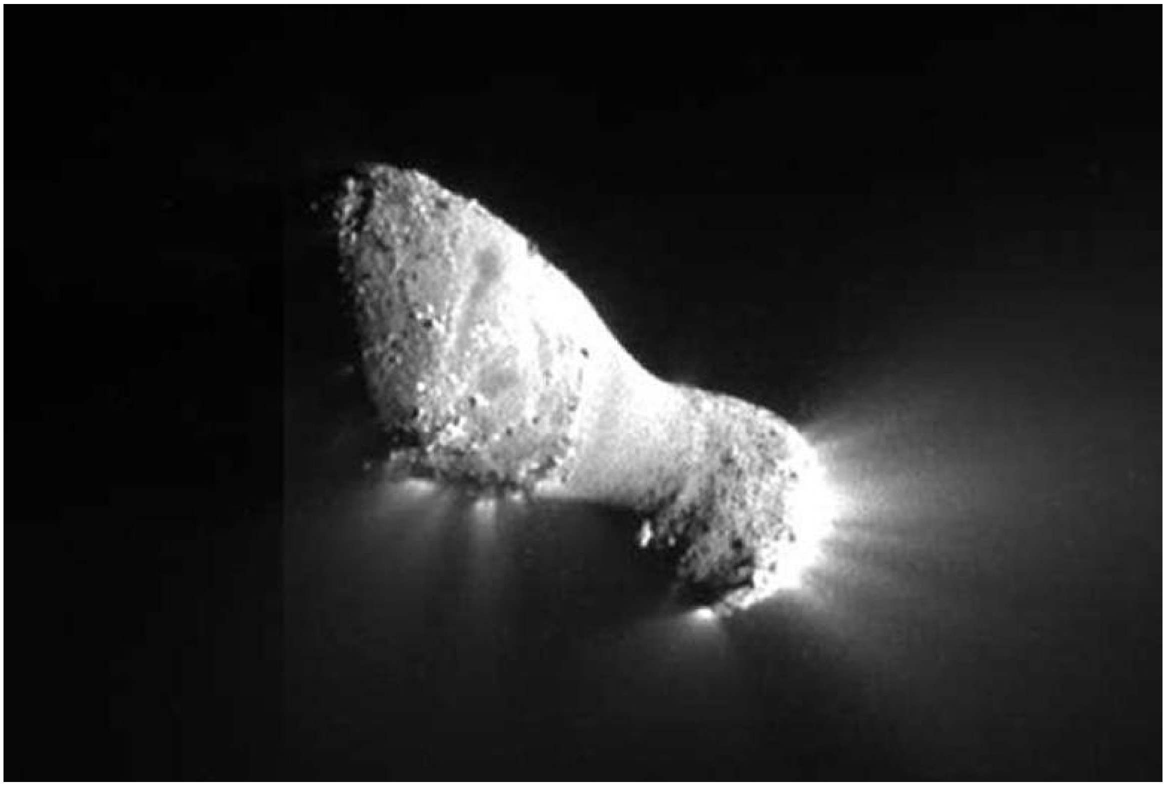
$$M = \int_0^\infty P(z)dz \qquad M = P_0\int_0^\infty e^{-\frac{z}{H}}dz \qquad M = P_0H\int_0^\infty e^{-x}dx \qquad M = P_0H$$

Then subtract the portion above you:

$$m = \int_h^\infty P(z)dz \qquad M = P_0\int_h^\infty e^{-\frac{z}{H}}dz \qquad M = P_0H\int_h^\infty e^{-x}dx \qquad M = P_0H\left[e^{-h} - 1\right]$$

To get: $dm(h) = P_0He^{-h}$

So $1/2 = e^{-h/H}$ and h = ln(2)(8.6 km) and so the height is **6 kilometers!**



Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Suppose that the shape of the comet nucleus can be approximated by the following function

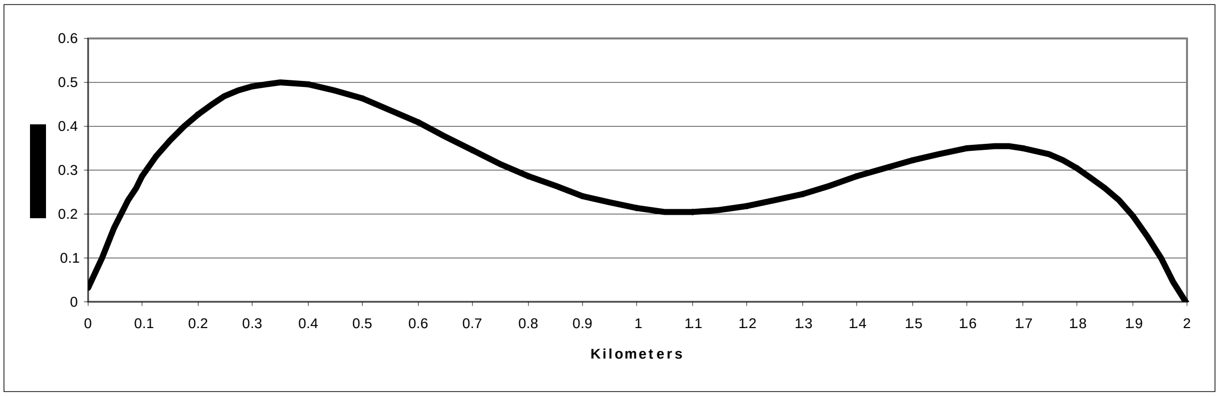
$$y(x) = -1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03$$

rotated about the x-axis between x=0 and x=2.0, where all units are in kilometers.
A) Graph this function;
B) Perform the required volume integration by using the method of circular disks.
C) To two significant figures, what is the total volume of the nucleus in cubic meters?

Problem 2 - Assuming that the density of Comet Hartley-2 is 0.6 grams/cm^3, what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: 1000 kg = 1 metric ton)

Problem 1 - Answer:

A) Graph:



B)

$$V = \int_0^2 \pi y(x)^2 dx \quad \text{then} \quad V = \pi \int_0^2 \left(-1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03\right)^2 dx$$

Expand integrand and collect terms (be careful!):

$$V = \pi \int_0^2 \left(1.49x^8 - 12.30x^7 + 41.94x^6 - 76.00x^5 + 77.55x^4 - 42.28x^3 + 9.46x^2 + 0.18x + 0.0009\right) dx$$

Integrate each term:

$$V = \pi \left[0.17x^9 - 1.54x^8 + 5.99x^7 - 12.67x^6 + 15.51x^5 - 10.57x^4 + 3.15x^3 + 0.09x^2 + 0.0009x + c\right]_0^2$$

Now evaluate V(x) at the two limits to get V = V(2) - V(0): Note that the answer for V will be sensitive to the accuracy of the polynomial coefficients, here given to 4 decimal place accuracy:

$V=(3.14)[0.1655(2)^9 - 1.5375(2)^8 + 5.9914(2)^7 - 12.6667(2)^6 + 15.51(2)^5 - 10.57(2)^4 + 3.1533(2)^3 + 0.09(2)^2 + 0.0009(2)]$

V = 3.14[0.157]

So **V = 0.49 cubic kilometers**.

Problem 2 - Mass = Density x Volume; First convert the volume to cubic centimeters from cubic kilometers: V = 0.49 km^3 x (10^3 meters/1 km)^3 x (100 cm/1 meter)^3 = 4.9 x 10^{14} cm^3. Then, Mass = 0.6 gm/cm^3 x 4.9 x 10^{14} cm^3 = 2.9 x 10^{14} gm**.** Convert grams to megatons: Mass = 2.9 x 10^{14} gm x (1 kg/1000 gm) x (1 ton/1000 kg) = 2.9 x 10^8 tons or **290 megatons**.

Useful Internet Resources

Space Math @ NASA

http://spacemath.gsfc.nasa.gov

A Math Refresher

http://istp.gsfc.nasa.gov/stargaze/Smath.htm

Developers Guide to Excelets

http://academic.pgcc.edu/~ssinex/excelets/

Interactive Science Simulations

http://phet.colorado.edu/en/simulations/category/physics

My Physics Lab

http://www.myphysicslab.com/

NASA Press Releases

http://www.nasa.gov/news/index.html

NCTM - Principles and Standards for School Mathematics

http://www.nctm.org/standards/content.aspx?id=16909

Practical Uses of Math and Science (PUMAS)

http://pumas.gsfc.nasa.gov

Teach Space Science

http://www.teachspacescience.org

A note from the Author:

July, 2011

Hi again!

Here is another collection of 'fun' problems based on NASA space missions across the solar system and the universe!   It has been an amazing year for new discoveries, and confronting the major changes in our manned space program.

The discovery of over 1200 new planets by the Kepler mission has revolutionized the search for earth-like planets in our Milky Way. Who would ever have thought that it would be so easy to turn up so many planets once the right technology was used. When I was starting out as an astronomer planets orbiting other stars was more of a belief than a scientific reality. Now we have so many 'exoplanets' to explore that the daily increase in numbers has led to the expectation that we will soon find one with an atmosphere resembling Earth's.

The verification by Gravity Probe-B of a relatively obscure prediction by Einstein's theory of gravity called general relativity, now makes it even more unlikely that this simple and elegant theory is somehow fatally wrong as so many detractors keep insisting. The consequence is that we now have to again change the way we look at space. Not only is it not an empty stage waiting for actors, but it behaves like a viscous fluid that can be dragged along as a body rotates. Can the universe get any stranger than this?

As the 2010-2011 school year draws to a close, we are also seeing the end of the Space Shuttle Era with the launch of the last Shuttle in July. There will be huge media attention focused on this event. We will all reflect upon the successes of our space program; the assembly of the International Space Station, the launch of the Hubble Space telescope, and other milestones.

But most importantly, we should reflect upon the dreams we have given to our children as they begin to choose their careers. One of them sitting in your classroom today might be the scientist that helps discover the nature of dark matter, or finds the first martian bacterial fossil.

All of these dreams are made possible with the help of a little mathematics!

Sincerely,
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