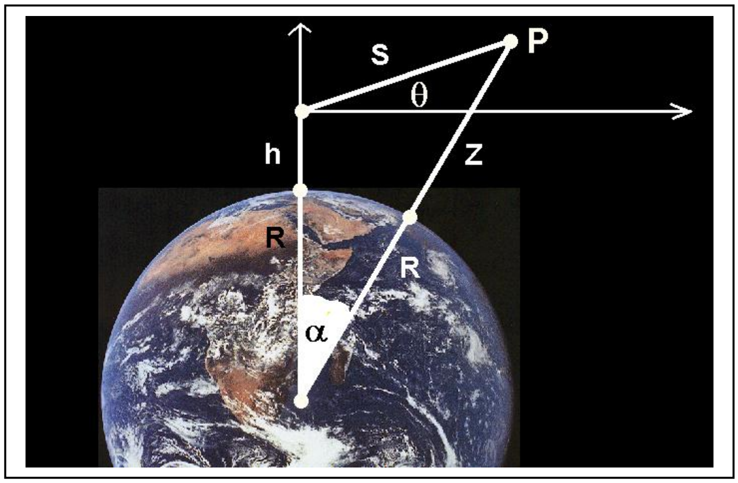
Atmospheric Shielding from Radiation I

The least expensive form of radiation shielding is a planetary atmosphere, but just how efficient is it? The walls of the International Space Station and the Space Shuttle provide substantial astronaut protection from space radiation, and have an equivalent thickness of 10 grams/cm^2 of aluminum, which has a density of 2.7 grams/cm^3. Compare this shielding to the spacesuits worn by Apollo astronauts of only 0.1 grams/cm^2. The atmosphere of Earth is a column of air with density of 0.0012 grams/cm^3, that is 100 kilometers tall. How much shielding does this provide at different altitudes above the ground?

In this three-part problem, we will begin the first step in constructing a mathematical model of the shielding from a planetary atmosphere. A similar calculation was published by Drs. Lisa Simonsen and John Nealy in February, 1993 in the article *"Mars Surface Radiation Exposure for Solar Maximum Conditions and 1989 Solar Proton Events"*, (NASA Technical Paper 3300)

The figure below right gives the necessary geometry and variable definitions.



The figure shows a radiation sampling point located 'h' above Earth's surface, and radiation from a source at point P, which is located at a distance 'S' from the sampling point. The distance from Earth's surface to point P is given by 'z'. Also, as seen from the sampling point, the vertical arrowed ray points to a point straight overhead, and the horizontal arrowed ray points to the horizon. The angle 'θ' is the elevation angle of the radiation source from the sampling point. So, a scientist would place a radiation detector at the sampling point located above Earth's surface, point the instrument at the radiation source at point P, and make a measurement of the amount of radiation coming from that particular direction in the sky.

Problem 1: From the information given in the figure, calculate the distance, S, in terms of h, R, z, and θ.

Problem 2: What is the form of S(R, h, z, θ) when;

A) If h is very much smaller than R? (h approaches zero)

B) $\theta = 90^o$?

C) If z is very much smaller than R? (z approaches zero)

Problem 1: From the information given in the figure, calculate the distance, S, in terms of h, R, z, and θ.
First, take three deep breaths, and play with the figure a bit. After some fascinating trial-and-error attempts, the simplest thing to realize is that the Law of Cosines can be used. There is only one of the three forms of this Law that do not involve the undesired angle, β , namely:

$$(R + Z)^2 = S^2 + (R + h)^2 - 2 (R + h) S \cos(\theta + 90^o)$$

Where we can use the angle addition theorem, cos (A + B) = Cos(A)cos(B) - sin(A)sin(B) to simplify it:

$$(R + Z)^2 = S^2 + (R + h)^2 + 2 (R + h) S \sin(\theta)$$

At first, it doesn't look like this pile of junk is useful because S doesn't appear by itself on one side of the equals sign. But by re-arranging, you see that it is really an equation with an interesting form:

$S^2 + [2(R+h) \sin(\theta)] + \left[(R + h)^2 - (R + Z)^2 \right] = 0$ which is a quadratic equation in which the coefficients are

$A = 1$ $B = 2(R+h) \sin(\theta)$ and $C = (R + h)^2 - (R + Z)^2$

We use the quadratic equation to solve for the positive root, because the negative root has no physical meaning. With a 'little' algebra we get:

Answer --- > $S(R,h,z,\theta) = \left((R + h)^2 \sin^2\theta + 2 R (z - h) + z^2 - h^2 \right)^{1/2} - (R + h) \sin \theta$

Problem 2: What is the form of $S(R, h, z, \theta)$ when;

A) $h << R$? Answer: Let $h = 0$

$$S(R,h,z,\theta) = \left(R^2 \sin^2\theta + 2 R z + z^2 \right)^{1/2} - R \sin \theta$$

B) $\theta = 90^o$? Answer:

$$S(R,h,z,\theta) = \left((R + h)^2 + 2 R (z - h) + z^2 - h^2 \right)^{1/2} - (R + h)$$

We can simplify this as

$$S(R,h,z,\theta) = \left((R + h)^2 + 2 R (z - h) + z^2 - h^2 \right)^{1/2} - (R + h)$$

$$S(R,h,z,\theta) = \left(R^2 + 2Rh + h^2 + 2Rz - 2Rh + z^2 - h^2 \right)^{1/2} - (R + h)$$

$$S(R,h,z,\theta) = \left(R^2 + 2Rz + z^2 \right)^{1/2} - R - h$$

$$S(R,h,z,\theta) = (R + z) - R - h$$

Answer --- > $S (R,h,z,\theta) = z - h$

C) $z << h$? Answer: Set $z = 0$ then

$$S(R,h,z,\theta) = \left((R + h)^2 \sin^2\theta + 2 R h - h^2 \right)^{1/2} - (R + h) \sin \theta$$