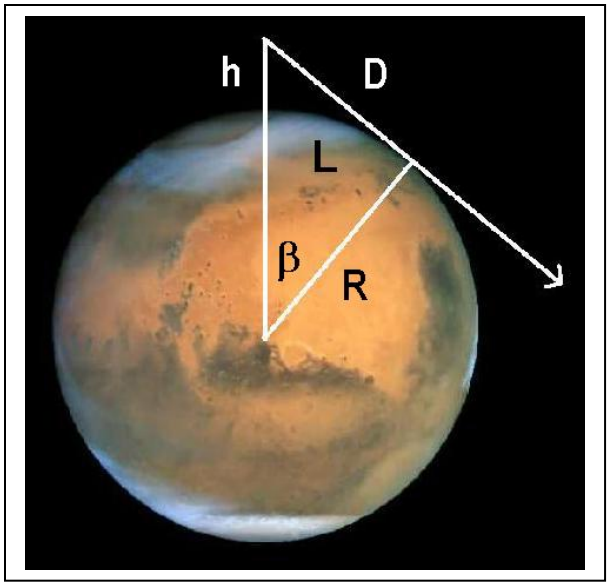

An important quantity in planetary exploration is the distance to the horizon. This will, naturally, depend on the diameter of the planet (or asteroid!) and the height of the observer above the ground.

Another application of this geometry is in determining the height of a transmission antenna in order to insure proper reception out to a specified distance.

Teachers: *Problems 1-4 can be successfully accomplished by algebra students. Problems 5 and 6 require a knowledge of derivatives and can be assigned to calculus students after they have completed Problem 1 and 2.*

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure above to derive the formula for the line-of-sight horizon distance, D, to the horizon tangent point.

Problem 2: Derive the distance along the planet, L, to the tangent point.

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6378 km); B) Mars (3,374 km); C) The Moon (1,738 km); Mar's moon Diemos (6 km)

Problem 4: A radio station has an antenna tower 50 meters tall. A) What is the maximum line-of-sight (LOS) reception distance in the Moon? B) On Mars?

Problem 5) What is the rate of change of the lunar LOS radius, D, for each additional meter of antenna height in Problem 4?

Problem 6) What is the rate-of-change of the distance to the lunar radio tower, L, at the LOS position in Problem 4?

Answer Key:

Problem 1: If the radius of the planet is given by R, and the height above the surface is given by h, use the figure to the left to derive the formula for the line-of-sight horizon distance, D.

Answer: By the Pythagorean Theorem $D^2 = (R+h)^2 - R^2$

so $D = (R^2 + 2Rh + h^2 - R^2)^{1/2}$ and so the answer is $D = (h^2 + 2Rh)^{1/2}$

Problem 2: Derive the distance along the planet, l, to the tangent point.

Answer: From the diagram, $\cos(\beta) = R/(R+h)$ and so $L = R \operatorname{Arccos}(R/(R+h))$

Problem 3: For a typical human height of 2 meters, what is the horizon distance on A) Earth (R=6,378 km); B) Mars (3,374 km); C) The Moon (1,738 km); Asteroid Dactyl (1.4 km)

Answer: Use the equation from Problem 1. A) R=6378 km and h=2 meters so
$D = ((2\text{ meters})^2 + 2 \times 2\text{ meters} \times 6.378 \times 10^6\text{ meters}))^{1/2}$ = 5051 meters or 5.1 kilometers.
B) For Mars, R=3374 km so D = 3,674 meters or 3.7 kilometers.
C) For the Moon, R=1,738 km so D = 2.6 kilometers
D) For Deimos, R = 6 km so D = 155 meters.

Problem 4: A radio station has an antenna tower 50 meters tall. A) What is the maximum line-of-sight (LOS) reception distance on the Moon? B) On Mars?

Answer: A) h = 50 meters, R=1,738 km so D = 13,183 meters or 13.2 kilometers.
B) h = 50 meters, R=3,374 km so D = 18,368 meters or 18.4 kilometers.

Problem 5: What is the rate of change of the lunar LOS radius, D, for each additional meter of antenna height in Problem 4?

Answer: Use the chain rule to take the derivative with respect to h of the equation for d in Problem 1. Evaluate dD/dh at h=50 meters for R=1,738 km.

Let $U = h^2 + 2Rh$ then $D = U^{1/2}$ so $dU/dh = (dD/dU)(dU/dh)$
Then $dD/dh = +1/2\, U^{-1/2}\, dU/dh = +1/2\,(2h + 2R)(h^2 + 2Rh)^{-1/2}$

For h=50 meters and R = 1,738 km,
$dD/dh = +0.5 \times (100 + 3476000)(2500 + 2 \times 50 \times 1738000)^{-1/2}$
= +131.8 meters in LOS distance per meter of height.

Problem 6: What is the rate-of-change of the distance, L, along the planet's surface to the lunar radio tower at the LOS position in Problem 4?

Answer: Let $U = R/(R+h)$, then $L = R \cos^{-1}(U)$. By the chain rule $dL/dh = (dL/dU) \times (dU/dh)$. Since $dL/dU = R \times (-1)(1 - u2)^{1/2}$ and $dU/dh = R \times (-1) \times (R + h)^{-2}$ then
$dL/dh = R^2 (R + h)^{-2} (R + h)^{1/2} /((R+ h)^2 - R^2)^{1/2}$ $dL/dh = R^2 (R+h)^{-1} (h^2 + 2Rh)^{-1/2}$
Since R >> h, $dL/dh = R/(2Rh)^{1/2}$
Evaluating this for R = 1,738 km and h = 50 meters gives dL/dh = +131.8 meters per kilometer.