

For decades, astronomers thought that the only geologically active object in our entire solar system was Earth. During the last 50 years of spacecraft studies, we now know that there are many locations where recent and even current volcanism can be found.

This image, taken by the Cassini spacecraft, shows plumes of gas and dust ejected from cracks in the surface of Saturn's moon Enceladus. Similar plumes have been found in Jupiter's satellite Io, and Neptune's moon Triton. They are called cryovolcanos because the temperatures are so low, only 100 kelvins (173 Celsius), and instead of rocky lava they eject water, methane or other frozen gases.

A simple 'square-root' formula relates the height of a plum, h, and its ejection speed, V, to the surface gravity of the body, g :

$$V = (2gh)^{1/2}$$

where h is in meters, V is in meters/sec and g is the acceleration of gravity at the surface in meters/sec².

Problem 1 – Complete the following table to estimate the ejection speeds and heights of volcanic plumes on the indicated bodies.

Object	Туре	G (m/s²)	V (m/s)	H (m)
Venus	Volcano	8.8	100	
Earth	Volcano	9.8		8,000
Earth	Geyser	9.8	35	
Mars	Volcano	3.8	100	
lo	Volcano	1.8		200,000
Enceladus	Geyser	0.1	60	
Titan	Volcano	1.4	100	
Triton	Geyser	0.8		5,000

Problem 2 – If 1 meter/sec = 2.2 miles/hr, which objects have the fastest and slowest ejection speeds in mph?

Problem 3 - Calculate the average ejection speeds for volcanos and geysers. What do you notice about the kind of event and its ejection speed?

Answer Key

Problem 1 – Complete the following table to estimate the ejection speeds and heights of volcanic plumes on the indicated bodies.

Object	Туре	G (m/s²)	V (m/s)	H (m)
Venus	Volcano	8.8	100	570
Earth	Volcano	9.8	400	8,000
Earth	Geyser	9.8	35	63
Mars	Volcano	3.8	100	1,300
lo	Volcano	1.8	850	200,000
Enceladus	Geyser	0.1	60	18,000
Titan	Volcano	1.4	100	3600
Triton	Geyser	0.8	89	5,000

Problem 2 – If 1 meter/sec = 2.2 miles/hr, which objects have the fastest and slowest ejection speeds in mph?

Answer: Fastest is lo at 850 m/s or 1,870 mph. Slowest is Earth geyser at 35 m/s or 77 mph.

Problem 3 - Calculate the average ejection speeds for volcanos and geysers. What do you notice about the kind of event and its ejection speed?

Answer: Volcanos: (100+400+100+850+100)/5 = 310 m/sec (682 mph). Geysers: (35+60+89)/3 = 61 m/sec (134 mph.)

Volcanos have the highest speeds. This is because they are produced by intense pressure of magma trapped in rocky 'pipes' that reach hundreds of kilometers below the surface, while geysers are created by more modest pressures in the surface crust.



This 1997 image taken by NASA's Galileo spacecraft shows the complex surface of Io. Sulfur dioxide frost appears in white and grey hues while yellowish and brownish hues are from other sulfurous materials. The new dark spot 400 km in diameter, surrounds a volcanic center named Pillan Patera. The spot did not exist 5 months earlier, and is the source of a 120 km high plume that has been seen erupting from this location.

Although no impact craters have been found, over 420 calderas and active vents have been mapped. About 15 are actively spewing fresh material within 175 km of each vent. This means that Io quickly resurfaces itself, covering over all of the impact craters within a million years or less.

Problem 1 – Assume Io is a sphere with a radius of 1820 km, and is covered to a depth of 1 kilometers to cover any new craters. What volume of fresh material must be produced in cubic meters?

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Problem 5 - What is the total resurfacing rate each year in centimeters/year?

Answer Key

Answer: Area = $4 \pi R^2$, and R = 1820,000 meters, so Area = $4 \times 3.14 \times (1820000)^2 = 4.2 \times 10^{13} m^2$. The volume of the surface shell 1km thick is that V = A x 1km = **4.2 \times 10^{16} m^3**.

Problem 2 – If the present surface was produced by the 420 calderas, what is the volume produced by each caldera?

Answer: $4.2 \times 10^{16} \text{ m}^3 / 420 = 1.0 \times 10^{14} \text{ m}^3 \text{ per caldera.}$

Problem 3 – The typical time between large meteor impacts is about 500,000 years. How much material would have to be produced by each caldera each year to cover the surface between impacts?

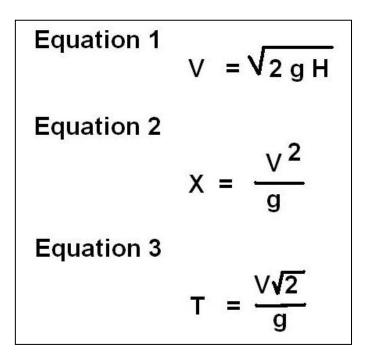
Answer: $1.0 \times 10^{14} \text{ m}^3 / 500000 \text{ yrs} = 2.0 \times 10^8 \text{ m}^3 / \text{year}.$

Problem 4 – If any given caldera is only active for 1% of its life, what does the resurfacing rate have to be for each caldera?

Answer: $2.0x10^8$ m³/year would be the rate if each caldera continuously operated for 500,000 years. If they only are active for 1% of this time, then the average rate has to be 100x higher or **2.0x10¹⁰** m³/yr.

Problem 5 – What is the total resurfacing rate each year in centimeters/year?

Answer: If the 1km depth is generated over 500,000 years, then each year the depth added is 100000 centimeters/500000 yr = 0.2 centimeters/year.



There are three equations that describe projectile motion on a planet:

Equation 1: Maximum velocity, V, needed to reach a height, H:

Equation 2: Maximum horizontal distance, X:

Equation 3: Time, **T**, required to reach maximum horizontal distance:

In all three equations, \mathbf{g} is a constant and is the acceleration of gravity at the surface of the planet, and all units are in meters or seconds.

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of H=17 miles (26 kilometers) what was the speed of the gas, V, that was ejected, in A) kilometers/hour? B) miles/hour? C) What was farthest horizontal distance, X, in kilometers that the ejecta reached? D) How long, T, did it take for the ejecta to travel the maximum horizontal distance? E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? (Note: $\mathbf{g} = 9.8$ meters/sec² for Earth.)

Problem 2 - An asteroid collides with the lunar surface and ejects lunar material at a speed of V=3,200 kilometers/hr. A) How high up, H, does it travel before falling back to the surface? B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: g = 1.6 meters/sec² for the Moon.)

Problem 3 - Plumes of gas are ejected by geysers on the surface of the satellite of Saturn called Enceladus. If g = 0.1 meters/sec², and H = 750 km, what is the speed of the gas, V, in the ejection in kilometers/hr?

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Space Math

Answer Key

Problem 1 - The volcano, Krakatoa, exploded on August 26, 1883 and obliterated an entire island. The detonation was heard over 2000 kilometers away in Australia, and was the loudest sound created by Nature in recorded human history! If the plume of gas and rock reached an altitude of 17 miles (26 kilometers) what was the speed of the gas that was ejected, in

A) Use Equation 1 with H = 26,000 meters; g = 9.8 m/s^2 and get H = $(2 \times 26000 \times 9.8)^{1/2}$ = 714 m/sec, but since the input numbers are only good to two significant figures, the answer is 710 meters/sec. Then converting to km/hr we get 710 m/s x (3600 sec/hr) x(1 km/1000meters) = 2,556 km/hr but again we only report to 2 significant figures so the answer is **2,600 km/hr**.

B) 2,600 km/hr x (0.62 miles/km) = 1,600 miles/hour to 2 significant figures

C) Use Equation 2: $X = (710 \text{ meters/sec})^2/9.8 = 51,439 \text{ meters}$, which to 2 significant figures becomes 51,000 meters or **51 kilometers**.

D) Use Equation 3: $T = 1.414 \times (710)/9.8 = 102.4$, but to 2 significant figures is **100** seconds.

E) About 30,000 people were killed in the explosion. Why do you think there were there so many casualties? Answer: **They had less than 100 seconds to flee from the advancing ejecta cloud!** You can also ask the students to calculate the sound travel time to cross 51 kilometers (sound speed = 340 m/sec) which would take 51,000 meters/340 = $150 \text{ seconds to reach someone at 51 kilometers...so the eject would strike them BEFORE they even heard the detonation.$

Problem 2 - Answer: 3,200 km/hr = 0.9 km/sec = **900 meters/sec**. to 2 significant figures. A) Solve Equation 1 for H... H = V2/2g so H = $(900)^2/(2 \times 1.6) = 250$ kilometers (2 SigFig). B) The escape speed from the lunar surface is 8,500 km/hr. From your answer to Problem 1, would a 'Krakatoa' explosion on the moon's surface have been able to launch lunar rock into orbit? (Note: g = 1.6 meters/sec² for the Moon.) Answer: Yes!

Problem 3 - Answer: From Equation $V = (2 \times 0.1 \times 750000)^{1/2} = 390$ meters/sec = **1,400** km/hr (2 SigFig)

Inquiry Problem: Program an Excel Spreadsheet to calculate the various quantities in the three equations given input data about the planet and ejecta. How does the maximum ejection velocity and height change with the value of **g** used for a variety of bodies in the solar system?

Answer: There are many ways for students to program each column in a spreadsheet to calculate the variables in the equations. Students should, for instance, notice that as the surface gravity, g, increases, the maximum speed, V, changes as the square-root of g, and the values for X and T vary inversely with g.