

On July 4, 2005, the Deep Impact spacecraft flew within 500 km of the nucleus of comet Tempel 1. This composite image of the surface of the nucleus was put together from images taken by the Impactor probe as it plummeted towards the comet before finally hitting it and excavating its own crater. The width of this picture is 8.0 kilometers.

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus by the arrow? D) What is the size of some of the smallest details you can see in the picture?

Problem 2 - The white streak identified by Arrow A is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of Arrow B. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Problem 1 - By using a millimeter ruler: A) what is the scale of this image in meters per millimeter? B) What is the approximate size of the nucleus of this comet in kilometers? C) How big are the two craters near the right-hand edge of the nucleus? D) What is the size of some of the smallest details you can see in the picture?

Answer: By using a millimeter ruler, what is the scale of this image in meters per millimeter? Answer: A) Width = 153 millimeters, so the scale is 8000 meters/153 mm = **52 meters/mm**. B) Width = 147 mm x 110 mm or **7.6 km x 5.7 km**. C) Although the craters are foreshortened, the maximum size gives a better indication of their 'round' diameters of about 7mm or **360 meters**. D) **Students may find features about 1 millimeter across or 50 meters**.

Note to Teacher: Depending on the quality of your printer, the linear scale of the image in millimeters may differ slightly from the 153 mm stated in the answer to Problem 1. Students may use their measured value as a replacement for the '153 mm' stated in the problem.

Problem 2 - The white streak near the center of the picture is a cliff face. What is the height of the cliff in meters, (the width of the white line) and the length of the cliff wall in meters?

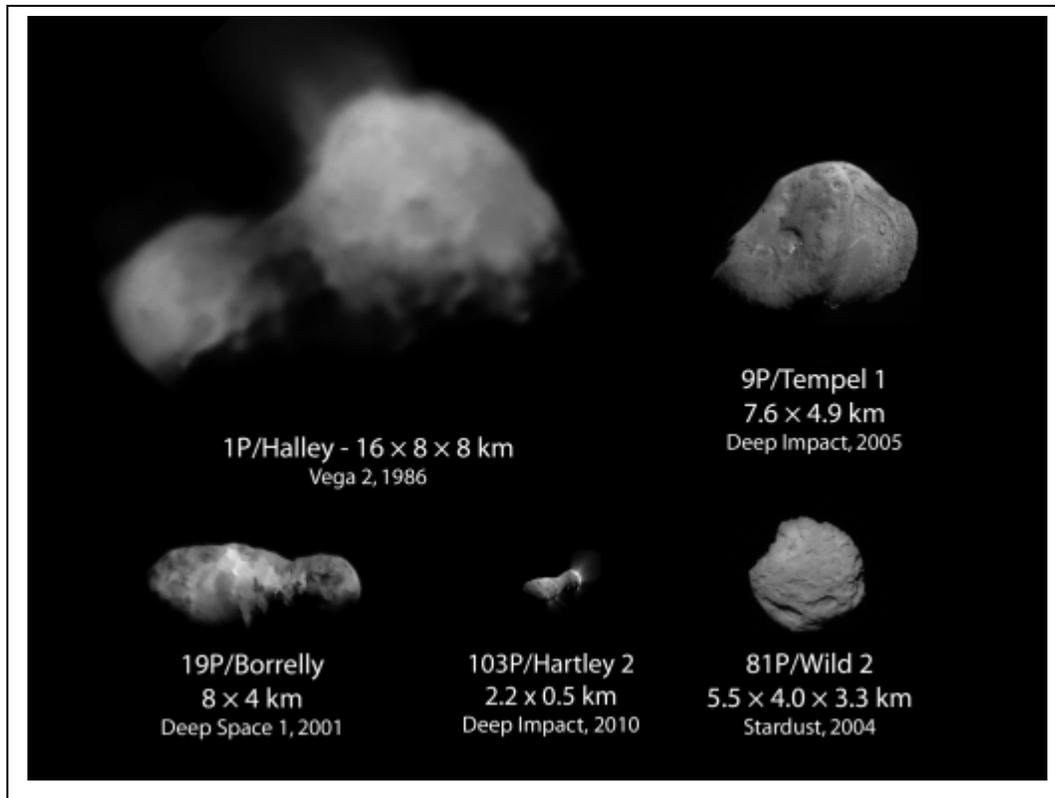
Answer: The width of the irregular white feature is about 0.5 millimeters or 26 meters. The length is about 15 millimeters or $15 \times 52 =$ **780 meters**.

Problem 3 - The Deep Impact Impactor probe collided with the comet at the point marked by the tip of the arrow. If there had been any uncertainty in the accuracy of the navigation, by how many meters might the probe have missed the nucleus altogether?

Answer: The picture shows that the shortest distance to the edge of the nucleus is about 20 millimeters to the right, so this is a distance of about $20 \times 52 =$ **1 kilometer!**

Note to Teacher:

Since the distance to the Earth was about 100 million kilometers, the spacecraft orbit had to be calculated to better than 1 part in 100 million over this distance in order for the probe to hit Tempel-1 as planned.



Spacecraft have flown-by five comets to study the dense object which produces the dramatic head and tails of these objects as seen from Earth. The figure above shows images of the nuclear objects to the same scale.

Problem 1 – What percentage of comet nuclei are:

- A) round
- B) potatoe-shaped?

Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Problem 1 – What percentage of comet nuclei are:

A) Round - answer $100\% \times (2/5) = 40\%$

B) potatoe-shaped? - answer $100\% \times (3/5) = 60\%$

Problem 2 - What is the range of size, in kilometers, for the dimensions of these nuclei?

Answer: The smallest dimension is for Comet Hartley 2 at 0.5 km. The largest dimension is for Halley's Comet at 16 km. **So the range is from 0.5 to 16 kilometers.**

Problem 3 – If the range in size represented one side of a cube, what is the range in volumes of the nuclei in cubic kilometers?

Answer: $S = 0.5$ km so volume = $0.5 \times 0.5 \times 0.5 = 0.125$ km³
 $S = 16$ km, so volume = $16 \times 16 \times 16 = 4096$ km³.

The range of volumes is 0.125 to 4096 km³.



On July 4, 2005 at 5:45 UT the 362-kilogram Impactor from NASA's Deep Impact mission, collided with the nucleus of the comet Tempel 1, causing a bright flash of light and a plume of ejected gas (see photo).

Traveling at 10.3 km/sec, the Impactor created a crater on the nucleus and ejected about 10,000 tons of material.

The average density of the comet nucleus is 400 kg/m^3 and its size can be approximated as a sphere with a radius of 3 kilometers.

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Problem 2 - If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Problem 3 – How far, in kilometers, will the comet nucleus have drifted 'sideways' to its orbit after 1 million years?

Problem 4 – Suppose that the comet had been headed toward Earth, and it was predicted that in 50 years it would collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5×10^8 kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Problem 1 – From the information given, what was the approximate mass of the comet nucleus in kilograms?

Answer: The spherical volume was $V = 4/3 \pi (3000 \text{ meters})^3 = 1.1 \times 10^{11} \text{ meters}^3$. The density was 400 kg/m^3 , so $\text{Mass} = \text{Density} \times \text{Volume} = 400 \times 1.1 \times 10^{11} = \mathbf{45 \text{ trillion kilograms}}$.

Problem 2 -- If the Impactor's path was perpendicular to the path taken by the Comet's nucleus, conservation of momentum requires that the product of the mass of the Impactor and its speed perpendicular to the orbit must equal the product of the comet's mass and the comet's speed perpendicular to the orbit after the impact assuming no mass loss. Although the impact ejected 10,000,000 kilograms of comet material, we will ignore this effect since the comet's mass was over 45 trillion kilograms! From the information, what is the final speed of the comet nucleus perpendicular to its orbit in A) kilometers/sec? B) meters/year?

Answer: A) $V_c = m_i V_i / M_c = (362 \text{ kg}) \times (10.3 \text{ km/sec}) / 45 \text{ trillion kg} = \mathbf{8 \times 10^{-11} \text{ kilometers/sec}}$.

B) $8 \times 10^{-11} \text{ km/s} \times (1000 \text{ m/km}) \times (3600 \text{ s/hr}) \times (24 \text{ hr/day}) \times (365 \text{ d/yr}) = \mathbf{2.5 \text{ meters/year}}$.

Problem 3 – How far, in kilometers, will the comet nucleus have drifted sideways to its orbit after 1 million years?

Answer: From Problem 2, the drift is 2.5 meters/year, so after 1 million years the nucleus will have drifted about 2,500,000 meters or **2,500 kilometers**.

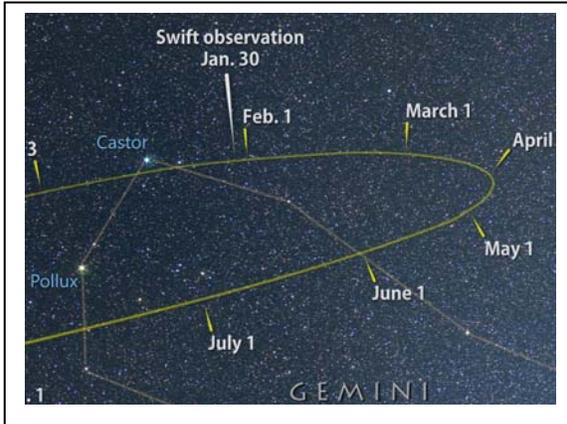
Problem 4 – Suppose that the comet had been headed towards Earth, and it is predicted that in 50 years it will collide with Earth. A nuclear bomb with an explosive yield equal to 10 million tons of TNT is launched to intercept the comet nucleus and deliver a blast, whose energy is equal to that of a 7.5×10^8 kilogram Impactor traveling at 10.3 km/sec. Assuming that the nucleus is not pulverized, A) about how far, in kilometers, will the nucleus drift after 20 years? B) Is this enough to avoid hitting Earth (diameter = 12,000 kilometers)?

Answer: Using $m_i V_i = m_c V_c$, we get

$$\begin{aligned} V_c &= (7.5 \times 10^8 \text{ kilograms}) \times (10.3 \text{ km/sec}) / 45 \text{ trillion kg} \\ &= 0.00017 \text{ kilometers/sec.} \end{aligned}$$

A) In 20 years ($20 \times 3.1 \times 10^7$ seconds) it travels **100,000 kilometers**.

B) **Yes**, since it only needs to travel 12,000 kilometers sideways to avoid hitting Earth, the detonation did help to avoid the collision...assuming the comet wasn't fragmented into a large cloud of debris!



Comet ISON will be carefully watched as it makes its closest approach to the sun in November, 2013. Some astronomers predict that it may break up into smaller comets because of the Sun's enormous gravity. The comet will travel to within 800,000 km of the center of the sun, or about 110,000 km from the hot solar surface! As it travels, it will also get very close to Mars and the asteroid 3362 Khufu, though no impacts are predicted!

A portion of its track across the sky is shown in the figure for January-July, 2013.

The table below gives the location of Comet ISON as it approaches the sun. The sun is located at the point (-0.5, +18.7) where all of the coordinate units are in millions of kilometers.

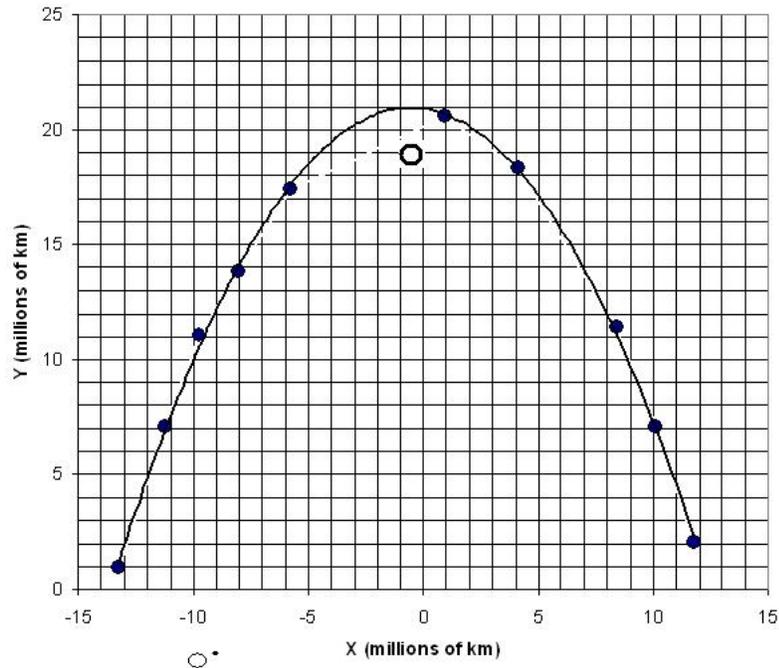
Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-13.3	+0.9	
November 27, 13:00 UT	-11.2	+7.1	
November 28, 01:00 UT	-9.7	+11.0	
November 28, 08:00 UT	-8.0	+13.8	
November 28, 14:00 UT	-5.8	+17.4	
November 28, 20:00 UT	+0.9	+20.6	
November 28, 23:00 UT	+4.1	+18.3	
November 29, 10:00 UT	+8.4	+11.4	
November 29, 19:00 UT	+10.1	+7.1	
November 30, 10:00 UT	+11.8	+2.1	

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve.

Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table.

Problem 3 – What is your prediction for the exact time when Comet ISON is at its closest point in its orbit to the sun?

Problem 1 - Plot these points on an X-Y graph and connect the points with a smooth parabolic curve. **Note: These coordinates are valid for the orbit as known on July 29, 2013 but may change as a more precise orbit is eventually determined.**



Problem 2 – Using either a millimeter ruler and the scale of the graph, or the Two Point Distance Formula, calculate the distance from each comet position to the sun in the table.

Answer: For advanced students using the distance formula: $D = ((x_2-x_1)^2 + (y_2-y_1)^2)^{1/2}$. For the first point x_1 at (-13.3, +0.9) and x_2 at (-0.5,+18.7) so $D = 21.9$ million km.

Date and Universal Time	X (million km)	Y (million km)	Distance to the sun (million km)
November 26, 18:00 UT	-13.3	+0.9	21.9
November 27, 13:00 UT	-11.2	+7.1	15.8
November 28, 01:00 UT	-9.7	+11.0	12.0
November 28, 08:00 UT	-8.0	+13.8	9.0
November 28, 14:00 UT	-5.8	+17.4	5.5
November 28, 20:00 UT	+0.9	+20.6	2.3
November 28, 23:00 UT	+4.1	+18.3	4.6
November 29, 10:00 UT	+8.4	+11.4	11.5
November 29, 19:00 UT	+10.1	+7.1	15.7
November 30, 10:00 UT	+11.8	+2.1	20.7

Problem 3 – What is your prediction for the exact time when Comet ISON is at its closest point in its orbit to the sun? Answer: Students may interpolate between Points 5, 6 and 7 using any convenient method. The answers should be close to **November 28 at 14:00 UT and a distance of 725.000 km.**

Year	Total	Amateurs	Spacecraft	Observatories
2012	60	27	1	32
2011	49	28	2	19
2010	161	29	119	13
2009	227	35	188	4
2008	220	34	182	4
2007	223	35	170	18
2006	206	31	152	23
2005	221	23	169	29
2004	223	8	172	43
2003	193	7	149	37
2002	182	9	131	42
2001	149	4	107	38
2000	135	9	99	27
1999	129	18	87	24

Every year, professional astronomers and dedicated amateur astronomers use everything from simple binoculars to sophisticated computer-driven telescopes to discover new comets. The table to the left gives a count of the number of comets detected between 1999 and 2012.

Comet hunters carefully compare images of the same part of the sky over a period of days or weeks. Although stars remain fixed, comets appear as fuzzy spots of light that change their positions.

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

The tabulated data is based upon the Catalog of Comet Discoveries archive at <http://www.comethunter.doc>

Problem 1 – During 2010, what percentage of comet discoveries were made by amateur astronomers, spacecraft and ground-based observatories?

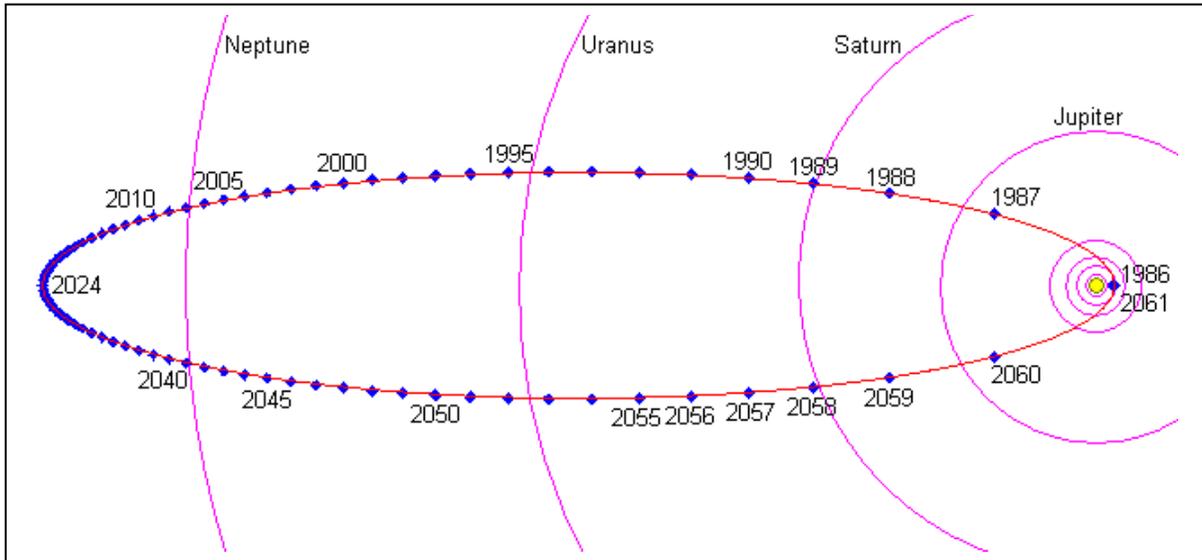
Answer: Amateur astronomers: $100\% \times (29/161) = \mathbf{18\%}$
 Spacecraft: $100\% \times (119/161) = \mathbf{74\%}$
 Observatories : $100\% \times (13/161) = \mathbf{8\%}$

Problem 2 – During the years 1999 to 2012, what is the average number of comets discovered by amateur astronomers and by ground-based observatories?

Answer: Total for amateurs = 297 over 14 years so the average was **21 comets/year**.
 For observatories: 353 over 14 years so the average is **25 comets/year**.

Problem 3 – What percentage of new comets would have been lost in 2012 had there not been any amateur astronomers searching the skies?

Answer: In 2012 the total number of comets was 60, and of these 27 were detected by amateur astronomers, so $60 - 27 = 33$ detected by other means, then $100\% \times (33/60) = \mathbf{55\%}$ of the comets would have remained undetected.



Comets are giant icebergs in space, sometimes over 50 miles across, that shed water vapor as they are heated while approaching the sun. Some comets only come around once and are never seen again. Others travel on elliptical paths called orbits, that take them far beyond the orbit of Jupiter before they, once again, loop back towards the sun.

Halley's Comet, which made a pass near the sun in 1986 is one of the most famous Periodic Comets, and will return to Earth's skies in the year 2061. This figure shows the orbit of Halley's Comet to the same scale as the orbits of the planets. Each dot is the position after one Earth year has elapsed.

Problem 1 – What is the period of Halley's Comet in Years?

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

Problem 3 – The distance between Saturn's orbit and the orbit of Venus is 1.3 billion km. About how fast is Halley's Comet traveling in km/year as it travels the Venus-Saturn distance?

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halley's Comet during this time in km/year?

Problem 1 – What is the period of Halley's Comet in Years?

Answer: $2061 - 1986 = 75$ years.

Problem 2 – What is the longest diameter of the elliptical orbit in kilometers if the distance between the orbits of Jupiter and Saturn is 650 million km?

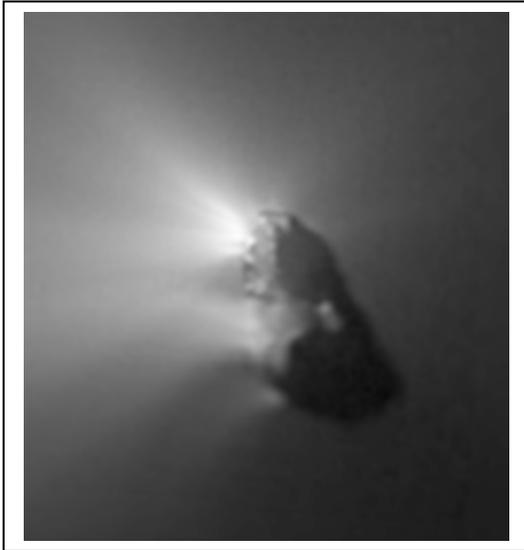
Answer: First determine the scale of this figure using a millimeter ruler and the actual Jupiter-Saturn distance. When printed on standard $8\frac{1}{2} \times 11$ paper, the separation should be about 19 millimeters, so the scale is $650 \text{ million km} / 19 \text{ mm} = 34 \text{ million km/mm}$. The length of the ellipse is 143 mm, so the actual distance is $143 \times 34 \text{ million} = 4.86 \text{ billion kilometers}$.

Problem 3 – The distance between Saturn's orbit and the orbit of Venus is 1.3 billion km. About how fast is Halley's Comet traveling in km/year as it travels the Venus-Saturn distance?

Answer: Counting the number of years, it takes 3 years to travel this distance, so the speed is $1.3 \text{ billion km} / 3 \text{ years} = 433 \text{ million km/year}$.

Problem 4 – The distance between the orbits of Uranus and Neptune is 1.6 billion km. From the diagram, about how many years does it take to travel this distance, and what is the average speed of Halley's Comet during this time in km/year?

Answer: Counting the year marks, it takes 12.5 years, so the speed is about $1.6 \text{ billion km} / 12.5 \text{ years} = 128 \text{ million km/year}$.



This historic image of the nucleus of Halley's Comet by the spacecraft Giotto in 1986 reveals the gases leaving the icy body to form the tail of the comet.

Once astronomers discover a new comet, a series of measurements of its location allows them to calculate the orbit of the comet and predict when it will be closest to the Sun and Earth.

Problem 1 – Astronomers measured two positions of Halley's Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which is a unit equal to the distance from Earth to the sun (150 million km). The positions are $(+10, +4)$ and $(+14, +3)$. What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b , which are the lengths of the semi-major and semi-minor axis of the ellipse?

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b .

Problem 3 – What is the orbit period of Halley's Comet from Kepler's Third Law if $P^2 = a^3$ where a is in Astronomical Units and P is in years?

Problem 4 – The perihelion of the comet is defined as $d = a - c$ where c is the distance between the focus of the ellipse and its center. How close does Halley's Comet come to the sun in this orbit in kilometers?

Problem 1 – Astronomers measured two positions of Halley’s Comet along its orbit. The x and y locations in its orbital plane are given in units of the Astronomical Unit, which equals 150 million km. The positions are (+10, +4) and (+14, +3). What are the two equations for the elliptical orbit based on these two points, written as quadratic equations in a and b, which are the lengths of the semimajor and semiminor axis of the ellipse?

Answer: The standard formula for an ellipse is $x^2/a^2 + y^2/b^2 = 1$ so we can re-write this as $b^2x^2 + a^2y^2 = a^2b^2$.

Then for Point 1 we have

$$10^2b^2 + 4^2a^2 = (ab)^2 \text{ so } 100b^2 + 16a^2 = (ab)^2. \text{ Similarly for Point 2 we have}$$

$$14^2b^2 + 3^2a^2 = (ab)^2 \text{ so } 196b^2 + 9a^2 = (ab)^2$$

Problem 2 – Solve the system of two quadratic equations for the ellipse parameters a and b.

Answer:

$$100b^2 + 16a^2 = (ab)^2$$

$$196b^2 + 9a^2 = (ab)^2$$

Difference the pair to get $7a^2 = 96b^2$ so $a^2 = (96/7)b^2$.

Substitute this into the first equation to eliminate b^2 to get

$$(700/96) + 16 = (7/96)a^2 \text{ or } a^2 = 2236/7 \text{ and so } a = 17.8 \text{ AU.}$$

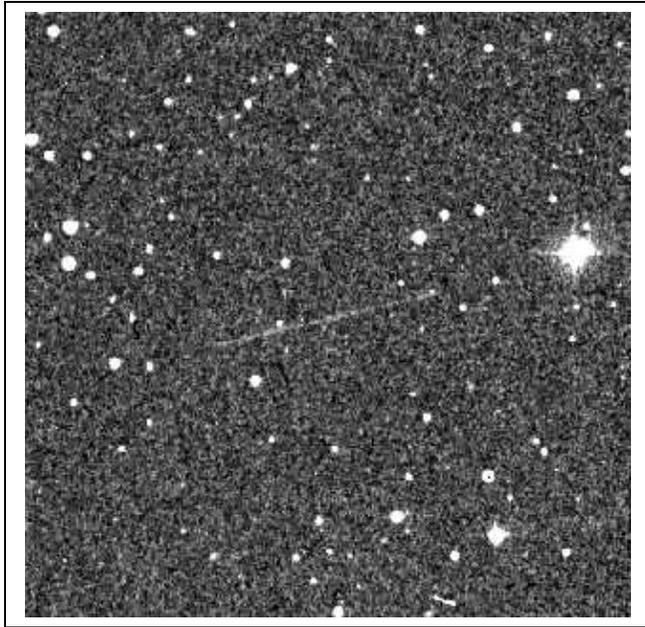
Then substitute this value for a into the first equation to get

$$5069 = 217 b^2 \text{ and so } b = 4.8 \text{ AU.}$$

Problem 3 – What is the orbit period of Halley’s Comet from Kepler’s Third Law is $P^2 = a^3$ where a is in Astronomical Units and P is in years? Answer: $P = a^{3/2}$ so for a = 17.8 AU we have **P = 75.1 years**.

Problem 4 – The perihelion of the comet is defined as $d = a - c$ where c is the distance between the focus of the ellipse and its center. How close does Halley’s Comet come to the Sun in this orbit in kilometers?

Answer: From the definition for c as $c = (a^2 - b^2)^{1/2}$ we have $c = 17.1$ AU and so the perihelion distance is just $d = 17.8 - 17.1 = 0.7$ AU. Since 1 AU = 150 million km, it comes to within **105 million km** of the Sun. This is near the orbit of Venus.



An asteroid, or comet, viewed from Earth will be either bright or faint depending on many quantifiable factors. Of course the size of the body and its reflectivity make a big difference. So does its distance from the sun and earth at the time you see it. The brightness also depends on whether, from Earth, it is fully-illuminated like the full moon, or only partly-illuminated like the crescent moon.

Astronomers can put all of these variables together into one single equation which works pretty well to predict a body's brightness just about anywhere inside the solar system!

The streak in the photo above is the asteroid 1999AN10 (Courtesy Palomar Digital Sky Survey). Orbit data suggest that on August 7, 2027 it will pass within 37,000 kilometers of Earth. The formula for the brightness of the asteroid is given by:

$$R = 0.011 d 10^{-\frac{1}{5}(m)}$$

where: R is the asteroid radius in meters, d is the distance to Earth in kilometers, and m is the apparent brightness of the asteroid viewed from Earth. Note, the faintest star you can see with the naked eye is about $m = +6.5$. The photograph above shows stars as faint as $m = +20$. The asteroid is assumed to have a reflectivity similar to lunar rock.

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be $d = 37,000$ kilometers, what is the formula $R(m)$ for the asteroid?

Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses?

Problem 1 - If the distance to the asteroid at the time of closest approach in 2027 will be $d = 37,000$ kilometers, what is the formula $R(m)$ for the asteroid? Answer: Substitute the given values into the equation and simplify. The formula will give the radius of the asteroid in meters as a function of its apparent brightness (called apparent magnitude by astronomers) given by m .

$$R(m) = 0.011 (37000)10^{-0.2m}$$

$$R(m) = 407 10^{-0.2m}$$

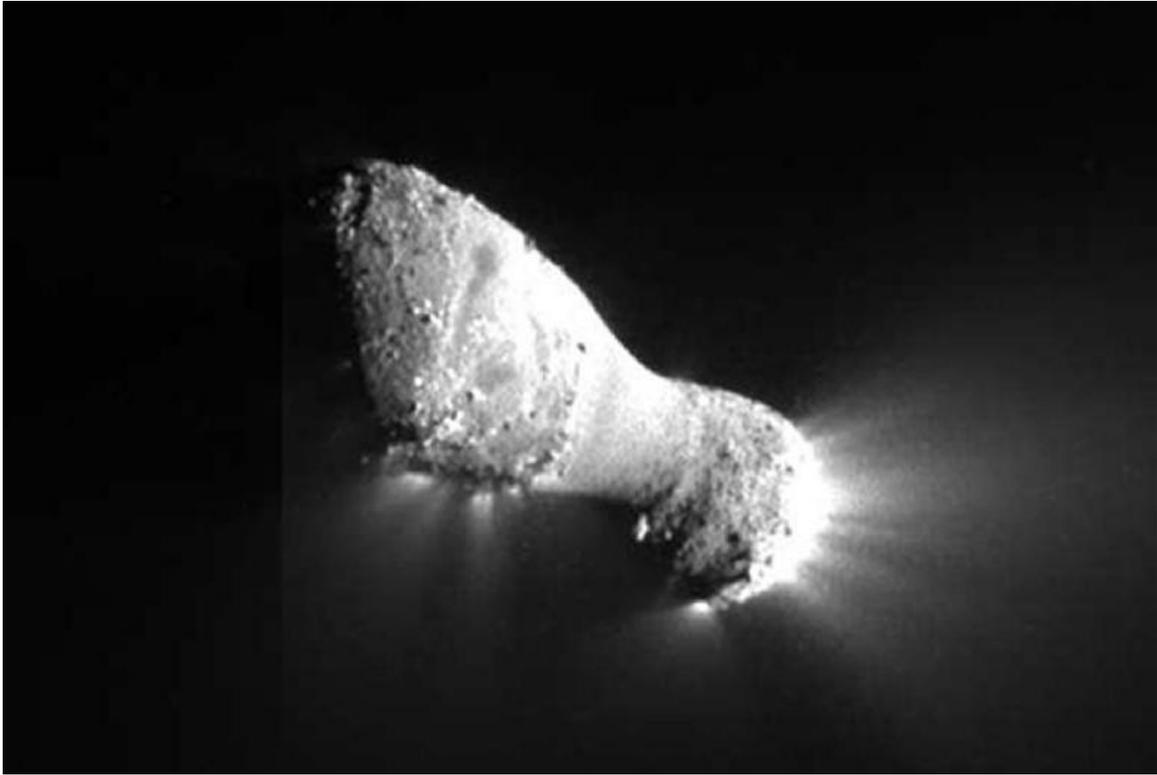
Problem 2 - If the radius of the asteroid is in the domain between 200 meters and 1000 meters, what is the range of apparent brightnesses? Answer: Solve the formula for $R(m)$ for $m(R)$ and evaluate for $R = 200$ meters and $R = 1000$ meters to obtain the range of the function.

$$m(R) = -5 \log_{10}(R/4224)$$

$$\begin{aligned} \text{so } m(R) \text{ for } R = 200 \text{ yields } m(200) = -5 (-11.3) & \quad \text{so } m(200) = +1.5 \\ \text{and } m(1000) = -5(0.39) & \quad \text{so } m(1000) = -2.0 \end{aligned}$$

so Domain R : [200,1000]
and Range m : [+1.5, -2.0]

Note: The planet Venus can be as bright as $m = -2.5$ so this asteroid should be easily visible if it is in this size domain.



Comet Hartley 2 is seen in this spectacular image taken by the Deep Impact/EPOXI Medium-Resolution Instrument on November 4, 2010 as it flew by the nucleus at a distance of 700 kilometers. The pitted surface, free of large craters, shows a complex texture in regions where gas plumes are actively ejecting gas. The potato-shaped nucleus is 2 kilometers long and 0.4 kilometers wide at its narrowest location. (Credit: NASA/JPL-Caltech/UMD).

Problem 1 - Suppose that the shape of the comet nucleus can be approximated by the following function

$$y(x) = -1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03$$

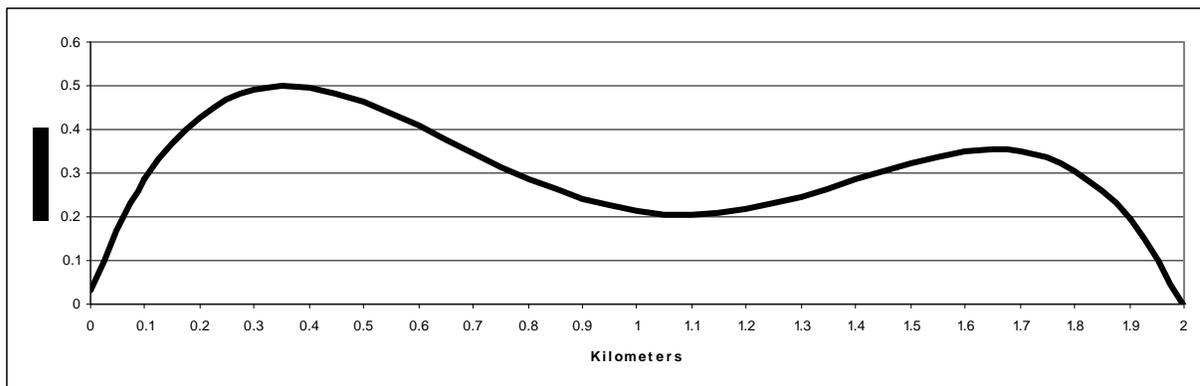
rotated about the x-axis between $x=0$ and $x=2.0$, where all units are in kilometers.

- A) Graph this function;
- B) Perform the required volume integration by using the method of circular disks.
- C) To two significant figures, what is the total volume of the nucleus in cubic meters?

Problem 2 - Assuming that the density of Comet Hartley-2 is 0.6 grams/cm^3 , what is your estimate for the mass of Comet Hartley-2 in megatons? (Note: $1000 \text{ kg} = 1 \text{ metric ton}$)

Problem 1 - Answer:

A) Graph:



B)

$$V = \int_0^2 \pi y(x)^2 dx \quad \text{then} \quad V = \pi \int_0^2 (-1.22x^4 + 5.04x^3 - 6.78x^2 + 3.14x + 0.03)^2 dx$$

Expand integrand and collect terms (be careful!):

$$V = \pi \int_0^2 (1.49x^8 - 12.30x^7 + 41.94x^6 - 76.00x^5 + 77.55x^4 - 42.28x^3 + 9.46x^2 + 0.18x + 0.0009) dx$$

Integrate each term:

$$V = \pi \left[0.17x^9 - 1.54x^8 + 5.99x^7 - 12.67x^6 + 15.51x^5 - 10.57x^4 + 3.15x^3 + 0.09x^2 + 0.0009x + c \right]_0^2$$

Now evaluate V(x) at the two limits to get V = V(2) - V(0): Note that the answer for V will be sensitive to the accuracy of the polynomial coefficients, here given to 4 decimal place accuracy:

$$V = (3.14)[0.1655(2)^9 - 1.5375(2)^8 + 5.9914(2)^7 - 12.6667(2)^6 + 15.51(2)^5 - 10.57(2)^4 + 3.1533(2)^3 + 0.09(2)^2 + 0.0009(2)]$$

$$V = 3.14[0.157]$$

So **V = 0.49 cubic kilometers.**

Problem 2 - Mass = Density x Volume; First convert the volume to cubic centimeters from cubic kilometers: $V = 0.49 \text{ km}^3 \times (10^3 \text{ meters}/1 \text{ km})^3 \times (100 \text{ cm}/1 \text{ meter})^3 = 4.9 \times 10^{14} \text{ cm}^3$.
Then, Mass = $0.6 \text{ gm}/\text{cm}^3 \times 4.9 \times 10^{14} \text{ cm}^3 = 2.9 \times 10^{14} \text{ gm}$. **Convert grams to megatons:**
Mass = $2.9 \times 10^{14} \text{ gm} \times (1 \text{ kg}/1000 \text{ gm}) \times (1 \text{ ton}/1000 \text{ kg}) = 2.9 \times 10^8 \text{ tons}$ or **290 megatons.**