When the solar system was young, planets were built as huge numbers of smaller bodies like asteroids collided with each other. Over millions of years of collisions, planets like Earth grew to their present sizes.

To see how this happened, we will model the process using a ball of clay!

Problem 1 - Take one-half pound of clay and form it into a round ball. Slice this round ball exactly in half and measure its diameter using a millimeter ruler.

Problem 2 – Take the two halves of the clay ball and re-form them into a round ball again. Now divide this ball into ten equal pieces of clay and roll each of these into round balls of about equal sizes. Slice one of these balls in half and measure its diameter.

Problem 3 – On a piece of graph paper, mark the horizontal axis with the number of small balls used from one to ten and the vertical axis with the diameter of the finished ball from 1 centimeter to 20 centimeters. Plot the diameter of one of these small balls on the graph.

Problem 4 – Take two of the small balls and roll them together into a single ball. Cut this new ball in half and measure its diameter. Mark the finished ball on the graph by its size in centimeters and the number of small balls '2'. Continue this process until you have collected all ten balls into one large ball and plot the diameter of the large ball and the number of small balls used '10'.

Problem 5 – Connect the ten points on the graph. What do you notice about the curve you plotted?

Problem 6 – If the large ball represented the final size of our Earth with a diameter of 12,000 kilometers, on this scale of the clay balls, how big would each of the ten 'planetessimals' have been that collided to form the final planet?

Problem 7 – Suppose you started with 100 equal-sized small clay balls. Would you get the same kind of plotted curve?

Problem 1 - Take one-half pound of clay and form it into a round ball. Slice this round ball exactly in half and measure its diameter using a millimeter ruler. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 2 – Take the two halves of the clay ball and re-form them into a round ball again. Now divide this ball into ten equal pieces of clay and roll each of these into round balls of about equal sizes. Slice one of these balls in half and measure its diameter. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 3 – On a piece of graph paper, mark the horizontal axis with the number of small balls used from one to ten and the vertical axis with the diameter of the finished ball from 1 centimeter to 20 centimeters. Plot the diameter of one of these small balls on the graph. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 4 – Take two of the small balls and roll them together into a single ball. Cut this new ball in half and measure its diameter. Mark the finished ball on the graph by its size in centimeters and the number of small balls '2'. Continue this process until you have collected all ten balls into one large ball and plot the diameter of the large ball and the number of small balls used '10'. **Answer will depend on how the students make the ball and how much clay is used.**

Problem 5 – Connect the ten points on the graph. What do you notice about the curve you plotted? **Answer: The curve should show an increasing diameter from left to right.**
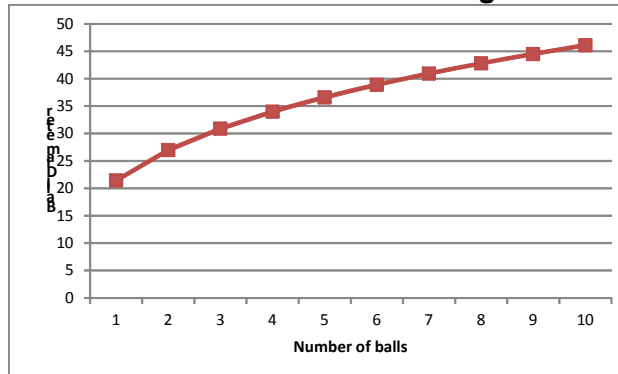
Problem 6 – If the large ball represented the final size of our Earth with a diameter of 12,000 kilometers, on this scale of the clay balls, how big would each of the ten 'planetessimals' have been that collided to form the final planet? Answer: Suppose the big balls final diameter is 45 millimeters and the small balls diameter is 20 mm. Use a simple proportion to find   12,000/45 = X/20 and so **X = 5300 km**. This is a bit larger than the diameter of our moon (3,500 km).

Problem 7 – Suppose you started with 100 equal-sized small clay balls. Would you get the same kind of plotted curve? **Answer: The curve would look the same but the scale on the horizontal axis would be changed. Each of the small balls would have a diameter of about 10 mm and represent a planetessimal with a diameter of about 12000/45 = X/10 so X = 2700 kilometers or slightly smaller than the diameter of our moon!**

Space Math

Between 4.1 and 3.8 billion years ago, the surfaces of all the planets were being bombarded by asteroids and other large bodies called <u>impactors</u> that had formed in the solar system by this time. Astronomers call this the Late Heavy Bombardment Era, and it is the era which finalized the formation of the planets at their present sizes.

The surface of our moon shows many large round basins called mare that are all that remains of this era. Similar scars on Earth have long since vanished due to erosion, volcanism and plate tectonic activity.

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

Space Math

Problem 1 – Using the large crater and impact basin record on the lunar surface, astronomers can estimate that Earth had about 20,000 craters over 20 km across, about 50 impact basins with diameters of 1,000 kilometers, and perhaps 5 large basins with diameters of 5,000 kilometers. If the Late Heavy Bombardment Era lasted about 300 million years, how many years elapsed between the impacts of each of the three kinds of objects during this era?

Answer: Small: 300 million/20,000 = **15,000 years**. Medium: 300 million/50 = **6 million years**, Large: 300 million/5 = **60 million years**.

Problem 2 – A Rule-of-Thumb says that the actual diameter of an impacting body is about 1/6 the diameter of the crater it forms. What were the average sizes of the three kinds of impactors during this Era?

Answer: Small = 20 km/6 = **3 km**. Medium: 1000 km/6 = **166 km**. Large: 5000km/6 = **833 km**.

Problem 3 – Use the formula for the volume of a sphere to calculate A) the total volume added to Earth of the small impactors in cubic kilometers. B) the total volume added to Earth of the medium-sized impactors in cubic kilometers. C) the total volume added to Earth of the large impactors in cubic kilometers.

Answer: $V = 4/3 \, \pi \, R^3$ and there were 20,000 of these so
Small = 20,000 x 4/3 x 3.141 x (3 km/2)^3 = **282,000 km^3**
Medium: There were 50 of these so V = 50 x 4/3 x 3.141 x (166 km/2)^3 = **120 million km^3**
Large: There were 5 of these so V = 5 x 4/3 x 3.141 x (833 km/2)^3 = **1.5 billion km^3**

Problem 4 - If the radius of Earth is 6,378 km, what percentage of Earth's volume was added by each of the three kinds of impactors?

Answer: The total volume of a spherical Earth is V = 4/3 x 3.141 x (6378)^3 = 1.1 trillion km^3

So the three kinds of impactors contributed:

Small = 100% x (282000/1.1 trillion) = **0.00003 % of the final volume**.
Medium = 100% x (120 million/1.1 trillion) = **0.01 % of the final volume**.
Large = 100% x (1.5 billion/1.1 trillion) = **0.13% of the final volume**.

So the infrequent (every 60 million years) but largest impactors changed Earth's size the most rapidly during the Late Heavy Bombardment Era!

Space Math

Era	Time (years)	Description
Pre-solar Nebula Era	0.0	Collapse of cloud to form flattened disk
Asteroid Era	3 million	Formation of large asteroids up to 200 km across ends
Gas Giant Era	10 million	Rapid formation of Jupiter and Saturn ends
Solar Birth Era	50 million	Sun's nuclear reactions start to produce energy in core
Planetessimal Era	51 million	Formation of numerous small planet-sized bodies ends
T-Tauri Era	80 million	Solar winds sweep through inner solar system and strip off primordial atmospheres
Ice Giant Era	90 million	Formation of Uranus and Neptune
Rocky Planet Era	100 million	Formation of rocky planets by mergers of 50-100 smaller bodies
Late Heavy Bombardment Era	600 million	Migration of Jupiter disrupts asteroid belt sending large asteroids to impact planetary surfaces in the inner solar system.
Ocean Era	600 million	LHB transports comets rich in water to Earth to form oceans
Life Era	800 million	First traces of life found in fossils on Earth

For decades, geologists and astronomers have studied the contents of our solar system. They have compared surface features on planets and moons across the solar system, the orbits of asteroids and comets, and the chemical composition and ages for recovered meteorites. From all this effort, and with constant checking of data against mathematical models, scientists have created a timeline for the formation of our solar system.

Our solar system began as a collapsing cloud of gas and dust over 4.6 billion years ago. Over the next 600 million years, called by geologists the Hadean Era, the sun and the planets were formed, and Earth's oceans were probably created by cometary impacts. Comets are very rich in water ice.

The fossil record on Earth shows that the first bacterial life forms emerged about 600 million years after the formation of the solar system. Geologists call this the Archaen Era – The era of ancient life.

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Space Math

Problem 1 – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Answer: On the Timeline '0.0' represents a time 4.6 billion years ago, so the Rocky Planet Era ended 100 million years after this or **4.5 billion years ago**.

Problem 2 – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Answer: LHB ended 600 million years after Time '0.0' or 4.6 billion – 600 million = **4.0 billion years ago.**

Problem 3 – About how many years ago do the oldest fossils date from on Earth?

Answer: 4.6 billion – 800 million = **3.8 billion years ago**.

Problem 4 – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Answer: On the timeline the difference is 100 million – 51 million = **49 million years**.

Problem 5 – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Answer: The time interval is 49 million years so the average time between impacts would have been 49 million years/80 impacts = **612,000 years**.

As they are forming, planets and raindrops grow by accreting matter (water or asteroids) at their surface.

The basic shape of a planet or a raindrop is that of a sphere. As the sphere increases in size, there is more surface area for matter to be accreted onto it, and so the growth rate increases.

In the following series of problems, we are going to follow a step-by-step logical process that will result in a simple mathematical model for predicting how rapidly a planet or a raindrop forms.

Problem 1 - The differential equation for the growth of the mass of a body by accretion is given by Equation 1 and the mass of the body is given by Equation 2

Equation 1) $\dfrac{dM}{dt} = 4\pi\ \rho\ V\ R(t)^2$ Equation 2) $M(t) = \dfrac{4}{3}\pi\ D\ R(t)^3$

where R is the radius of the body at time t, V is the speed of the infalling material, ρ is the density of the infalling material, and D is the density of the body.

Solve Equation 2 for R(t), substitute this into Equation 1 and simplify.

Problem 2 - Integrate your answer to Problem 1 to derive the formula for M(t).

Raindrop Condensation -   A typical raindrop might form so that its final mass is about 100 milligrams and D = 1000 kg/m**3**, under atmospheric conditions where ρ = 1 kg/m**3** and V = 1 m/sec. How long would it take such a raindrop to condense?

Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9×10^{24} kg,   and D = 3000 kg/m^3, under conditions where ρ = 0.000001 kg/m^3 and V = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

Space Math

Answer Key

2.4

Figure from 'Planetary science: Building a planet in record time' by Alan Brandon, Nature 473,460–461(26 May 2011)

Answer 1:

$$R(t) = \left(\frac{3M(t)}{4\pi D}\right)^{\frac{1}{3}} \qquad \text{then} \qquad \frac{dM}{dt} = 4\pi\ V\rho\left(\frac{3M}{4\pi D}\right)^{\frac{2}{3}} \qquad \text{so} \qquad \frac{dM}{dt} = 4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}} M^{\frac{2}{3}}$$

Answer 2:   First rearrange the terms to form the integrands:

$$\frac{dM}{M^{\frac{2}{3}}} = 4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}} dt \qquad \text{Integrate both sides:} \qquad 3M^{\frac{1}{3}} = 4\pi V\rho\left(\frac{3}{4\pi D}\right)^{\frac{2}{3}} t$$

Now solve for M(t) to get the answer: $$M(t) = \left(\frac{4\pi V\rho}{3}\right)^{3}\left(\frac{3}{4\pi D}\right)^{2} t^{3}$$

Raindrop Condensation -   A typical raindrop might form so that its final mass is about 100 milligrams and **D** = 1000 kg/m**³**, under atmospheric conditions where ρ = 1 kg/m**³** and **V** = 2.0 m/sec (about 5 miles per hour). How long would it take such a raindrop to condense using this approximate mathematical model?

$$0.0001 = \left(\frac{4\pi(2.0)(1.0)}{3}\right)^{3}\left(\frac{3}{4\pi(1000)}\right)^{2} t^{3} \qquad \text{so} \qquad t^{3} = 2.98$$

and so it takes about **1.4 seconds**.

Planet Accretion - A typical rocky planet might form so that its final mass is about that of Earth or 5.9x10**24** kg,   and **D** = 3000 kg/m**³**, under conditions where ρ = 0.000001 kg/m**³** and **V** = 1 km/sec. How long would it take such a planet to accrete using this approximate mathematical model?

$$5.9x10^{24} = \left(\frac{4\pi(1000)(0.000001)}{3}\right)^{3}\left(\frac{3}{4\pi(3000)}\right)^{2} t^{3} \qquad \text{so} \qquad t^{3} = 1.27x10^{40}$$

and so it takes about   2.3x10**13** seconds or **750,000 years**.

Space Math
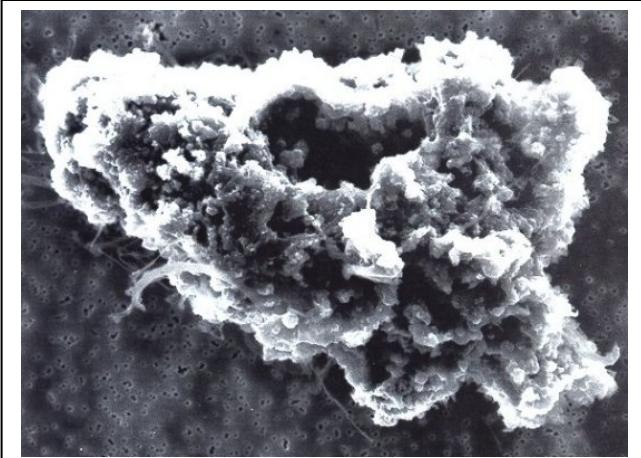
A cosmic dust grain about 0.1mm across captured by a NASA high-altitude aircraft. Probably debris from a passing comet

Planets are built in several stages. The first of these involves small, micron-sized interstellar dust grains that collide and stick together to eventually form centimeter-sized bodies. A simple model of this process can tell us about how long it takes to 'grow' a rock-sized body starting from microscopic dust. This process occurs in dense interstellar clouds, which are known to be the birth places for stars and planets.

Problem 1 – Assume that the forming rock is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the rock as a function of time, M(t)?

Problem 2 – The rock grows by absorbing incoming dust grains that have an average mass of m grams and a density of N particles per cubic centimeter in the dust cloud. The particles collide with the surface of the rock at a speed of V cm/sec, what is the equation that gives the rate of growth of the rock's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the rock when it reaches a diameter of 1 centimeter if its density is 3 grams/cc?

Problem 6 – The rock begins at t=0 with a mass of 1 dust grain, m = 8 x 10^{-12} grams. The cloud density N = 3.0 x 10^{-5} dust grains/cc and the speed of the dust grains striking the rock, without destroying the rock, is V=10 cm/sec. How many years will the growth phase have to last for the rock to reach a diameter of 1 centimeter?

Space Math

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3 \pi R^3 \rho$ and so **M(t) = 4/3 π ρ R(t)^3**

Problem 2 –Answer: The change in the mass, dM, occurs as a quantity of dust grains land on the surface area of the rock per unit time, dt.   The amount is proportional to the surface area of the rock, since the more surface area the rock has, the more dust particles will be absorbed.   Also, the rate at which dust grain mass is brought to the surface is proportional to the product of the dust grain density in the interstellar gas, times the speed of the grains landing on the surface. This leads to m x N x V where m is in grams per dust grain, N is in dust grains per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the rock, will then have the units of grams/sec representing the rate at which the rock mass is growing. The full formula for the growth of the rock mass is then
dM/dt = 4 π R^2 m N V

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3 M(t)/ 4 \pi \rho)^{1/3}$. Then substituting into dM/dt we have dM/dt = 4 π m N V (3 M(t)/4 π ρ)$^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV \left(\frac{3}{4\pi\rho} \right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand   M(t)$^{-2/3}$ dM = 4 π m N V (3/4 π ρ)$^{2/3}$ dt   Then integrate both sides to get:
3 M(t)$^{1/3}$ = 4 π m N V (3/4 π ρ)$^{2/3}$ t + c.   Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV \left(\frac{3}{4\pi\rho} \right)^{\frac{2}{3}} t + c \right]^3$$

Problem 5 –Answer: The radius will be 0.5 centimeters so, m = 4/3 π (0.5 cm)^3 x 3.0 gm/cc = **1.6 grams.**

Problem 6 – Answer: For t = 0, M(0) = m   so the constant of integration is c = m$^{1/3}$ so c = $(8x10^{-12})^{1/3}$ = 2 x 10^{-4}.
Then M(t) = (4/3 (3.14) (8 x 10^{-12})(3.0 x 10^{-5})(10.0)(3/(4(3.14) (3.0)))$^{2/3}$ t + 2 x 10^{-4})^3
    M(t) = (1.9 x 10^{-15} t + 2 x 10^{-4})^3

To reach M(t) = 1.6 grams,   t = 6.1 x 10^{14} seconds or about **19 million years!**

Space Math

Asteroid Gaspra about 15 km across. Image taken by the NASA Galileo spacecraft.

Planets are built in several stages. The first of these involves small, interstellar dust grains that collide and stick together to form centimeter-sized bodies. This can take millions of years. The second stage involves the formation of kilometer-sized asteroids from the centimeter-sized rocks. A simple model of this process can tell us about how long it takes to 'grow' an asteroid from rock-sized bodies.

Problem 1 – Assume that the forming asteroid is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the asteroid as a function of time, M(t)?

Problem 2 – The asteroid grows by absorbing incoming rocks that have an average mass of 5.0 grams and a density of N rocks per cubic centimeter in the cloud. The rocks collide with the surface of the forming asteroid at a speed of V cm/sec, what is the equation that gives the rate of growth of the asteroid's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to problem 3 so determine M(t).

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc?

Problem 6 – The asteroid begins at t=0 with a mass of m=5 grams. The cloud density $N = 1.0 \times 10^{-8}$ rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer?

Problem 1 – Answer: Because mass = density x volume, we have
M = 4/3 π R^3 ρ and so **M(t) = 4/3 π ρ R(t)^3**

Problem 2 –Answer: The change in the mass, dM, occurs as a quantity of rocks land on the surface area of the forming asteroid per unit time, dt.   The amount is proportional to the surface area of the asteroid, since the more surface area the asteroid has, the more rocks will be absorbed.   Also, the rate at which rock mass is brought to the surface of the asteroid is proportional to the product of the rock density in the solar nebula, times the speed of the rocks landing on the surface of the asteroid. This leads to m x N x V where m is in grams per rock, N is in rocks per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the asteroid, will then have the units of grams/sec representing the rate at which the asteroid mass is growing. The full formula for the growth of the asteroid mass is then

dM/dt = 4 p R^2 m N V

Problem 3 – Answer: From Problem 1 we see that R(t) = (3 M(t)/ 4 π ρ)$^{1/3}$. Then substituting into dM/dt we have dM/dt = 4 π m N V (3 M(t)/4 π ρ)$^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV \left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand   M(t)$^{-2/3}$ dM =   4 π m N V (3/4 π ρ)$^{2/3}$ dt   Then integrate both sides to get:
3 M(t)$^{1/3}$ = 4 π m N V (3/4 π ρ)$^{2/3}$ t + c.   Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3}\pi mNV \left(\frac{3}{4\pi\rho}\right)^{\frac{2}{3}} t + c\right]^3$$

Problem 5 – What is the mass of the asteroid when it reaches a diameter of 1 kilometer if its density is 3 grams/cc? Answer M = 4/3 π (50,000 cm)^3 x 3.0 gm/cc = **1.6 x 10^{15} grams.**

Problem 6 – The rock begins at t=0 with a mass of 1 rock, m = 5 grams. The cloud density N = 1.0 x 10^{-8} rocks/cc and the speed of the rocks striking the asteroid, without destroying the asteroid, is V=1 kilometer/sec. How many years will the growth phase have to last for the asteroid to reach a diameter of 1 kilometer? Answer: For t = 0, M(0) = m   so the constant of integration is c = m$^{1/3}$ so c = 1.7.
Then M(t) = (4/3 (3.14) (5)(1.0 x 10^{-8})(100,000)(3/(4(3.14) (3.0)))$^{2/3}$ t +1.7)^3
        M(t) = (0.0039 t + 1.7)^3

So to get M(t) = 1.6 x 10^{15} grams, solve for t to get t = 29,600,000 seconds or about **342 days!**

  Space Math

From Asteroids to Planets

2.7
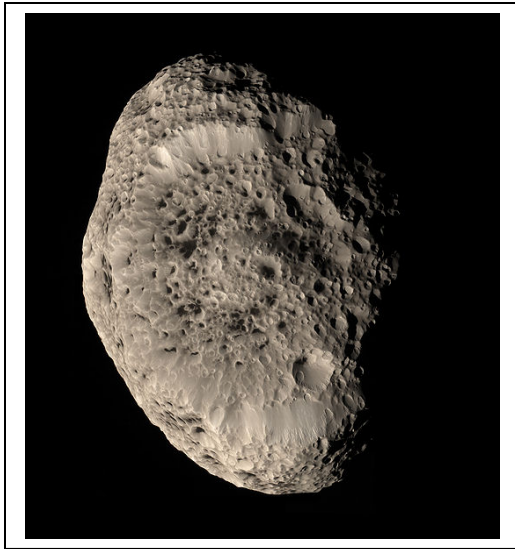
Planets are built in several stages. Dust grains grow to large rocks in a million years, then rocks accumulate to form asteroids in a few years or so. The third stage combines kilometer-wide asteroids to make rocky planets. A simple model of this process can tell us about how long it takes to 'grow' a planet by accumulating asteroid-sized bodies through collisions. Saturn's moon Hyperion (see image) is 300 km across and is an example of a 'small' planet-sized body called a planetoid.

Problem 1 – Assume that the forming planet is spherical with a density of 3 gm/cc, a radius R, and a mass M. If the radius is a function of time, R(t), what is the equation for the mass of the planet as a function of time, M(t)?

Problem 2 – The planet grows by absorbing incoming asteroids that have an average mass of 10^{15} grams and a density of N asteroids per cubic centimeter in the cloud. The asteroids collide with the surface of the forming planet at a speed of V cm/sec, what is the equation that gives the rate of growth of the planet's mass in time (dM/dt)?

Problem 3 – From your answer to Problem 1 and 2, re-write dM/dt in terms of M not R.

Problem 4 – Integrate your answer to Problem 3 so determine M(t).

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc?

Problem 6 – The planetoid begins at t=0 with a mass of m= 2 x 10^{15} grams. The cloud density N = 1.0 x 10^{-24} asteroids/cc (1 asteroid per 1000 cubic kilometers), and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers?

Space Math

Problem 1 – Answer: Because mass = density x volume, we have
$M = 4/3 \pi R^3 \rho$ and so **M(t) = 4/3 π ρ R(t)^3**

Problem 2 –Answer: The change in the mass, dM, occurs as a quantity of asteroids land on the surface area of the planet per unit time, dt. The amount is proportional to the surface area of the planet, since the more surface area the planet has, the more asteroids will be absorbed. Also, the rate at which asteroid mass is brought to the surface of the forming planet is proportional to the product of the asteroid density in the planetary nebula, times the speed of the asteroids landing on the surface of the planet. This leads to m x N x V where m is in grams per dust grain, N is in asteroids per cubic centimeter, and V is in centimeters/sec. The product of all three factors has the units of grams per square centimeter per second. The product of (m x N x V) with the surface area of the planet, will then have the units of grams/sec representing the rate at which the planet mass is growing. The full formula for the growth of the planet mass is then

dM/dt = 4 p R^2 m N V

Problem 3 – Answer: From Problem 1 we see that $R(t) = (3 M(t)/ 4 \pi \rho)^{1/3}$. Then substituting into dM/dt we have dM/dt = 4 π m N V (3 M(t)/4 π $\rho)^{2/3}$ so

$$\frac{dM(t)}{dt} = 4\pi mNV \left(\frac{3}{4\pi\rho} \right)^{\frac{2}{3}} M(t)^{\frac{2}{3}}$$

Problem 4 –Answer: Re-write the differentials and move M(t) to the side with dM to get the integrand M(t)$^{-2/3}$ dM = 4 π m N V (3/4 π $\rho)^{2/3}$ dt Then integrate both sides to get:
3 M(t)$^{1/3}$ = 4 π m N V (3/4 π $\rho)^{2/3}$ t + c. Solve for M(t) to get the final equation for M(t), and remember to include the integration constant, c:

$$M(t) = \left[\frac{4}{3} \pi mNV \left(\frac{3}{4\pi\rho} \right)^{\frac{2}{3}} t + c \right]^3$$

Problem 5 – What is the mass of the planet when it reaches a diameter of 5000 kilometers if its density is 3 grams/cc? Answer M = 4/3 π (2.5 x 10^8 cm)^3 x 3.0 gm/cc = **2.0 x 10^{26} grams.**

Problem 6 – The planetoid begins at t=0 with a mass of 1 asteroid, m = 2.0 x 10^{15} grams. The cloud density N = 1.0 x 10^{-24} asteroids/cc (This equals 1 asteroid per 1000 cubic kilometers) and the speed of the asteroids striking the planet, without destroying the planet, is V=1 kilometer/sec. How many years will the growth phase have to last for the planet to reach a diameter of 5000 kilometers? Answer: For t = 0, M(0) = m so the constant of integration is c = m$^{1/3}$ so c = (2.0 x 10^{15} g)$^{1/3}$ = 1.3 x 10^5.
Then M(t) = (4/3 (3.14) (2 x 10^{15})(1.0 x 10^{-24})(100,000)(3/(4(3.14) (3.0)))$^{2/3}$ t +1.3 x 10^5)^3
M(t) = (0.00015 t + 1.3 x 10^5)^3

To get M(t) = 2.0 x 10^{26} grams will take t = 3.9 x10^{12} seconds or about **126,000 years!**

Space Math