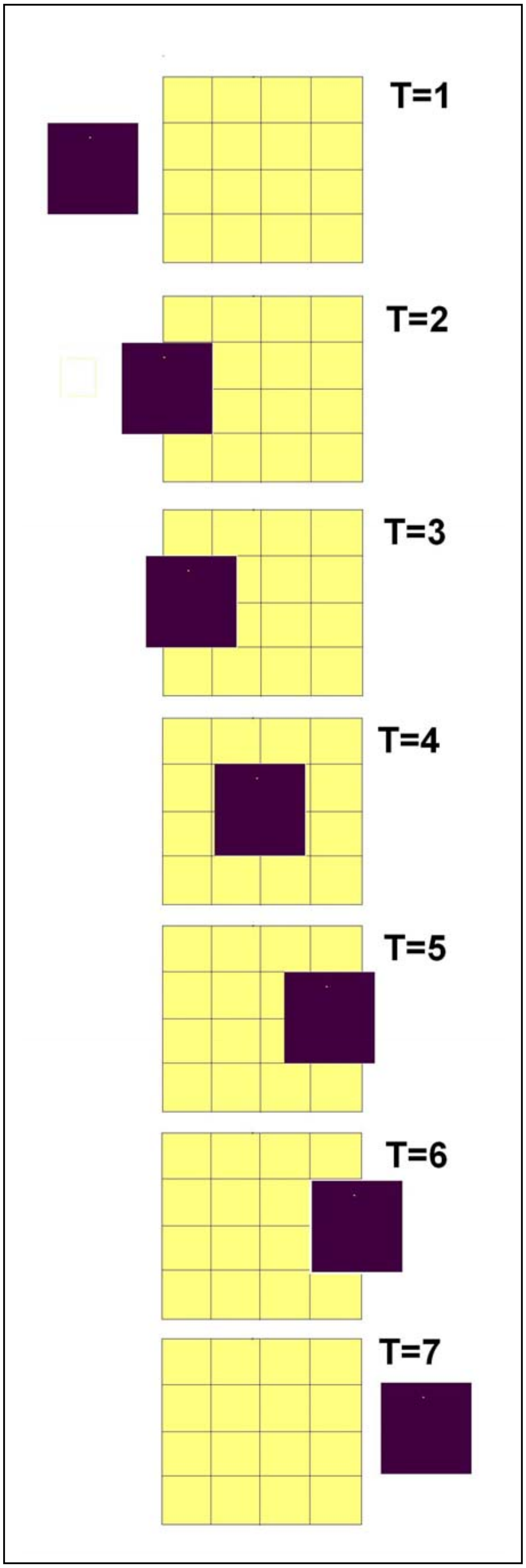
Transits and Brightness Change

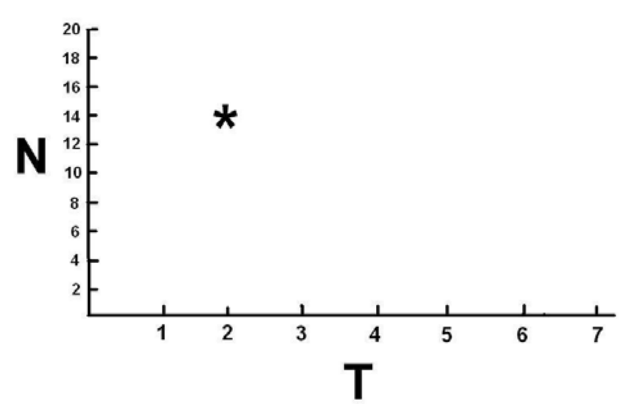


NASA's Kepler spacecraft recently announced the discovery of five new planets orbiting distant stars. The satellite measures the dimming of the light from these stars as planets pass across the face of the star as viewed from Earth. To see how this works, lets look at a simple model.

In the Bizarro Universe, stars and planets are cubical, hot spherical. Bizarro astronomers search for distant planets around other stars by watching planets pass across the face of the stars and cause the light to dim.

Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?



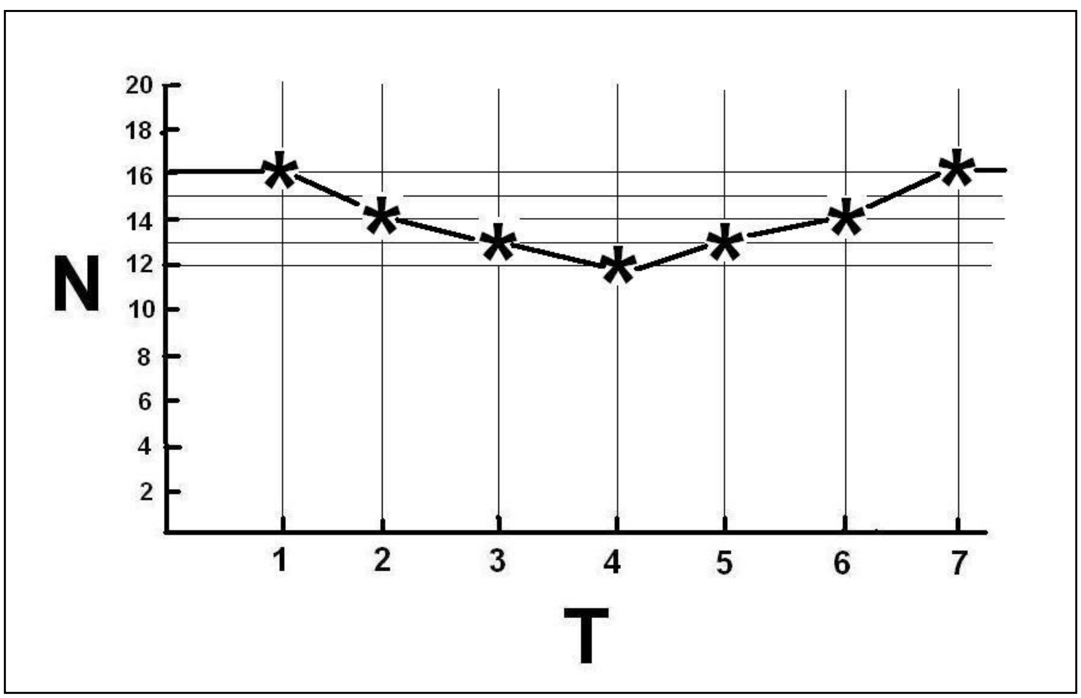
Problem 1 - The sequence of figures shows the transit of one such planet, Osiris (black). Complete the 'light curve' for this star by counting the number of exposed 'star squares' not shaded by the planet. At each time, T, create a graph of the number of star brightness squares. The panel for T=2 has been completed and plotted on the graph below.

Answer: **Count the number of yellow squares in the star and plot these for each value of T in the graph as shown below. Note, for T= 3 and 5, the black square of the planet occupies 2 full squares and 2 half squares for a total of 2 + 1/2 + 1/2 = 3 squares covered, so there are 16 - 3 = 13 squares remaining that are yellow.**

Problem 2 - If you knew that the width of the star was 1 million kilometers, how could you use the data in the figure to estimate the width of the planet?

Answer: The light curve shows that the planet caused the light from the star to decrease from 16 units to 12 units because the planet blocked 16-12 = 4 units of the stars surface area. That means that the planet squares occupy 4/16 of the stars area as seen by the astronomers. The area of the star is just the area of a square, so the area of the square planet is 4/16 of the stars area or Ap = 4/16 x Astar. Since the star as a width of Wstar = 1 million kilometers, the planet will have a width of $Wp = Wstar\sqrt{\frac{4}{16}}$ or **500,000 kilometers**.

The amount of star light dimming is proportional to the ratio of the area of the planet and the star facing the observer. The Kepler satellite can detect changes by as little as 0.0001 in the light from a star, so the smallest planets it can detect have diameters about 1/100 the size of the stars that they orbit. For a star with a diameter of the sun, 1.4 million kilometers, the smallest planet detectable by the Transit Method has a diameter about equal to 14,000 kilometers or about the size of Earth.



Kepler's First Look at Transiting Planets

Period (days)	F	G	K
0-10	11	138	20
11-20	7	53	16
21-30	4	25	6
31-40	2	13	0
41-50	1	7	0
51-60	1	1	0
61-70	0	1	0
71-80	0	1	0
> 81	0	3	2
Total:	26	242	44

On June 16, 2010 the Kepler mission scientists released their first list of stars that showed evidence for planets passing across the faces of their stars. Out of the 156,097 target stars that were available for study, 52,496 were studied during the first 33 days of the mission. Their brightness was recorded every 30 minutes during this time, resulting in over 83 million high-precision measurements.

700 stars had patterns of fading and brightening expected for planet transits. Of these, data were released to the public for 306 of the stars out of a sample of about 88,000 target stars.

The surveyed stars for this study were distributed by spectral class according to F = 8000, G = 55,000 and K = 25,000. For the 306 stars, 43 were K-type, 240 were G-type, and 23 were K-type. Among the 312 transits detected from this sample of 306 stars, the table above gives the number of transits detected for each stellar type along with the period of the transit.

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?

Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Problem 1 - Comparing the F, G and K stars, how did the frequency of the stars with transits compare with the expected frequency of these stars in the general population?
Answer: Of the 88,000 stars F = 8000/88000 = 9%; G = 55000/88000 = 63% and K=25000/88000 = 28%.

For the 306 stars: F = 43/306 = 14%; G = 240/306 = 78% and K = 23/306 = 8%....so there were significantly fewer transits detected for K-type stars (8%) compared to the general population (28%).

Note: Sampling error accounts for $s = (306)^{1/2}$ = +/-17 stars or a +/- 6% uncertainty which is not enough to account for this difference in the K-type stars.

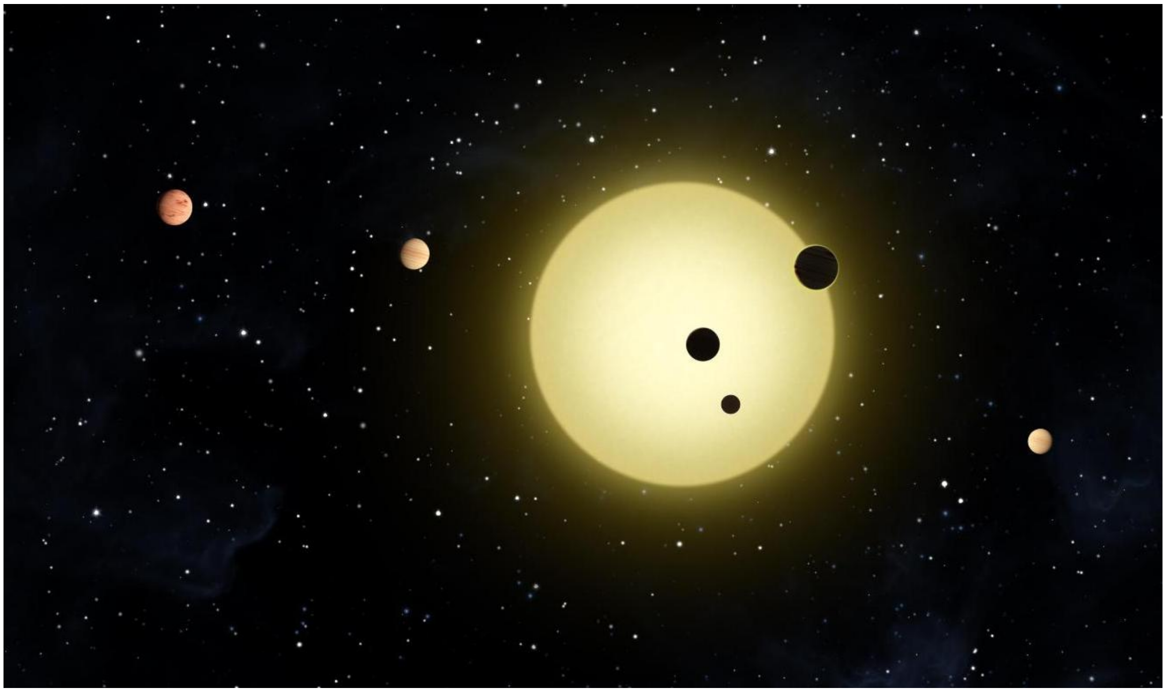
Problem 2 - The distance of the planet from its star can be estimated in terms of the orbital distance of Earth from our sun as $D^3 = T^2$ where D = 1.0 is the distance of Earth from the sun, and T is in multiples of 1 Earth Year. A) What is the distance of Mercury from our sun if its orbit period is 88 days? B) What is the range of orbit distances for the transiting planets in multiples of the orbit of Mercury if the orbit times range from 5 days to 80 days?

Answer: A) T = 88 days/365 days = 0.24 Earth Years, than $D^3 = 0.24^2$, $D^3 = 0.058$, so $D = (0.058)^{1/3}$ and so D = **0.39 times Earth's orbit distance**.
B) The time range is 0.014 to 0.22 Earth Years, and so D is in the range from 0.058 to 0.36 Earth distances. Since Mercury has D = 0.39, in terms of the orbit distance of Mercury, the transiting planets span a range from 0.058/0.39 = **0.15** to 0.36/0.39 = **0.92 Mercury orbits**.

Problem 3 - As a planet passes across the star's disk, the star's brightness dims by a factor of 0.001 in brightness. If the radius of the star is 500,000 km, and both the planet and star are approximated as circles, what is the radius of the planet A) in kilometers? B) In multiples of Earth's diameter (13,000 km)?

Answer: The amount of dimming is equal to the ratio of the areas of the planet's disk to the star's disk, so $0.001 = \pi R^2/\pi(500{,}000)^2$ so **R = 15,800 kilometers**, which equals a diameter of 31,600 kilometers. Since Earth's diameter = 13,000 km, the transiting planet is about **2.4 times the diameter of Earth**.

Earth-sized Planets by the Score!



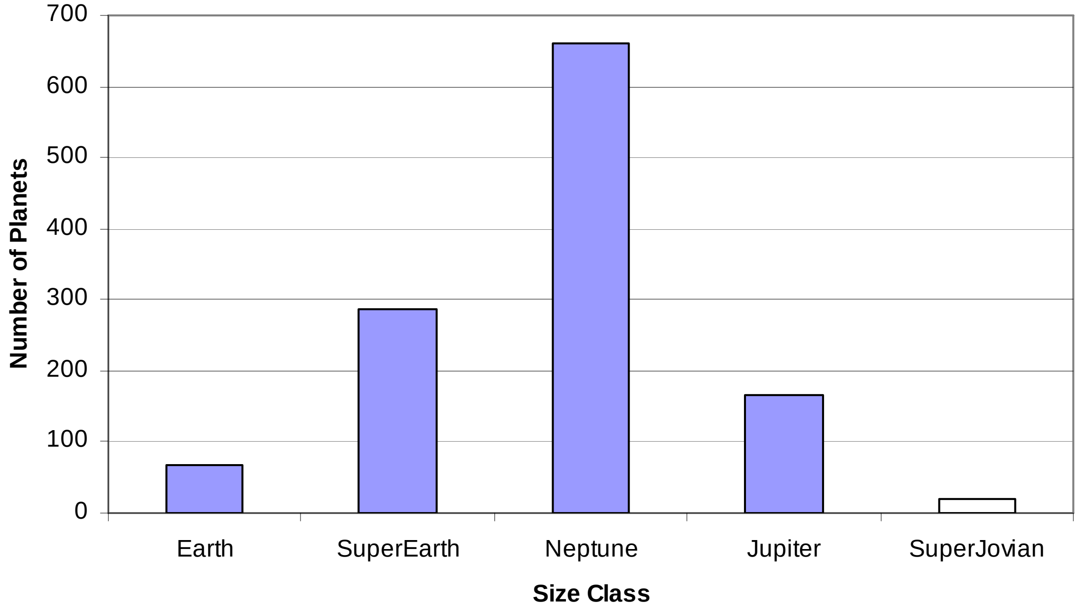
NASA's Kepler Space Observatory recently announced the results of its continuing survey of 156,453 stars in the search for planet transits. Their survey, in progress for just under one year, has now turned up 1,235 transits from among this sample of stars of which 33 were eliminated because they were too big to be true planets. About 30 percent of the remaining candidates belong to multiple-planet systems in which several planets orbit the same star. Among the other important findings are the numbers of planet candidates among the various planet types summarized as follows: Earth-sized = 68; superEarths = 288; Neptune-sized = 662; Jupiter-sized=165; superJovians=19.

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes.

Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized?

Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by NASA's Kepler Space Observatory?

Problem 1 - Create a histogram that shows the number of candidate planets among the 5 different size classes. Answer:



Problem 2 - What percentage of all the planets detected by Kepler were found to be Earth-sized? Answer: P = 100% x (68/1202) = **5.7 %**

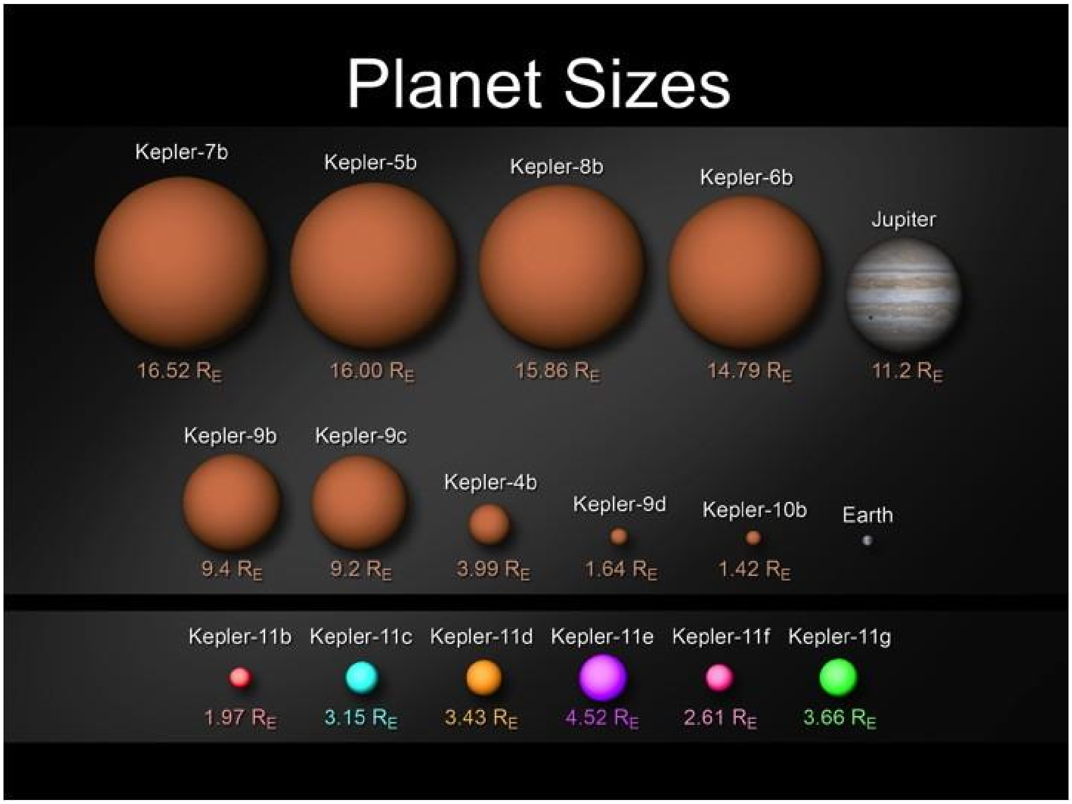
Problem 3 - Extrapolating from the Kepler findings, which was based on a search of 156,453 stars, about how many Earth-sized planets would you expect to find if the Milky Way contains about 40 billion stars similar to the ones surveyed by Kepler?

Answer: There are 40 billion candidate stars in the Milky Way, so by using simple proportions and re-scaling the survey to the larger sample size

$$\frac{68}{157,453} = \frac{X}{40billion}$$

we get about (40 billion/156,453) x 68 planets or about **17 million Earth-sized planets**.

Note: *The Kepler survey has not been conducted long enough to detect planets much beyond the orbit of Venus in our own solar system, so in time many more earth-sized candidates farther away from their stars will be reported in the years to come. This means that there may well be considerably more than 17 million Earth-sized planets orbiting stars in the Milky Way similar to our own sun.*



This diagram shows disks representing the planets discovered in orbit around 8 different stars all drawn to the same scale. Earth and Jupiter are also shown so that you can see how big they are in comparison. Solve the problems below using fraction arithmetic to find out how big these new planets are compared to Earth and Jupiter.

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b. Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Problem 2 – The planet Kepler-9c is 9./11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b. If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

Problem 1 – Kepler-5b is 8 times the diameter of Kepler-11b .Kepler-11b is twice the diameter of Earth. How big is the planet Kepler-5b compared to Earth?

Answer: It helps to set up these kinds of problems as though they were unit conversion problems, and then cancel the planet names to get the desired ratio:

$$\frac{1 \times \text{Kepler5b}}{8 \times \text{Kepler11b}} \times \frac{1 \times \text{Kepler11b}}{2 \times \text{Earth}} = \frac{1\ \text{Kepler5b}}{8 \times \text{Earth}} \quad \text{so Kepler 5b is } \mathbf{16 \times Earth}$$

Note how the 'units' for Kepler-11b have canceled out.

Problem 2 – The planet Kepler-9c is 9/11 the diameter of Jupiter, and Kepler-11e is 1/2 the diameter of Kepler-9c. How big is the planet Kepler-11e compared to Jupiter?

$$\frac{11 \times \text{Kepler9c}}{9 \times \text{Jupiter}} \times \frac{2 \times \text{Kepler11e}}{1 \times \text{Kepler9c}} = \frac{22\text{x Kepler9c}}{9 \times \text{Jupiter}} \quad \text{so Kepler 11e = } \mathbf{9/22\ \ Jupiter}$$

From the figure we see Kepler11e = 4.52 Re and Jupiter = 11.2 Re so Kepler11e = 4.5/11 = 9/22 Jupiter.

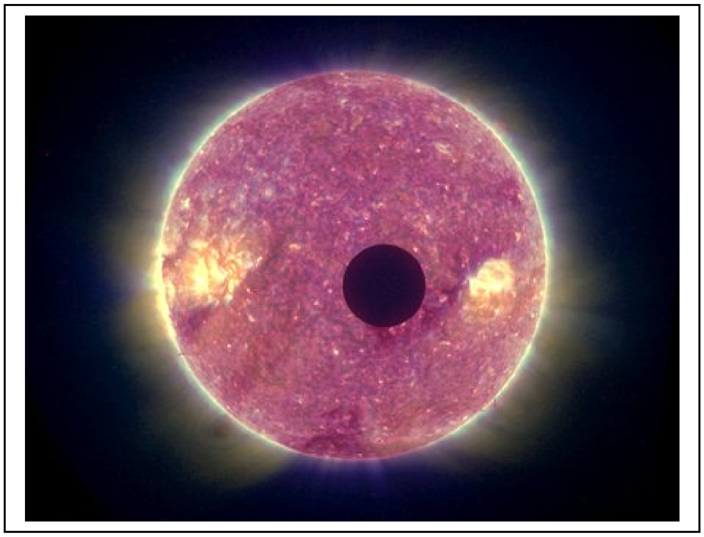
Problem 3 – The planet Kepler-10b is 1/10 the diameter of Kepler-6b, and Kepler-9b is 9/15 the diameter of Kepler-6b .If Kepler-11g is 4/9 the diameter of Kepler-9b, how big is Kepler-11g compared to Kepler-10b?

$$\frac{10\text{xKepler10b}}{1\text{xKepler6b}} \times \frac{9\ \text{Kepler6b}}{15\ \text{Kepler9b}} \times \frac{4\ \text{Kepler9b}}{9\ \text{Kepler11g}} = \frac{40\ \text{Kepler10b}}{15\ \text{Kepler11g}}$$

So 15 x the diameter of Kepler 11g is equal to 40x the diameter of Kepler 10b
And so, Kepler 11g is 40/15 times the diameter of Kepler 10b

From the figure we see that Kepler 11g is 3.66 Re and Kepler 10b is 1.4 Re which is in about the same ratio as our fractions. 3.66/1.4 = 2.6 and 40/15 = 2.7.

Kepler – The Hunt for Earth-like Planets



On March 11, 2009, NASA launched the Kepler satellite. Its 3-year mission is to search 100,000 stars in the constellation Cygnus and detect earth-sized planets. How can the satellite do this?

The image to the left shows what happens when a planet passes across the face of a distant star as viewed from Earth. In this case, this was the planet Mercury on February 25, 2007.

The picture was taken by the STEREO satellite. Notice that Mercury's black disk has reduced the area of the sun. This means that, on Earth, the light from the sun dimmed slightly during the Transit of Mercury. Because Mercury was closer to Earth than the Sun, Mercury's disk appears very large. If we replace Mercury with the Moon, the lunar disk would exactly cover the disk of the Sun and we would have a total solar eclipse.

Now imagine that the Sun was so far away that you couldn't see its disk at all. The light from the Sun would STILL be dimmed slightly. The Kepler satellite will carefully measure the brightness of more than 100,000 stars to detect the slight changes caused by 'transiting exoplanets'.

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the sun is 696,000 kilometers, what is the scale of your sun disk in kilometers/millimeter?

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)?

Problem 3 – What is the area of the Sun disk in square millimeters?

Problem 4 – What is the area of Earth and Jupiter in square millimeters?

Problem 5 – By what percent would the area of the Sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk?

Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer Key

Problem 1 – With a compass, draw a circle 160-millimeters in radius to represent the sun. If the radius of the Sun is 696,000 kilometers, what is the scale of your Sun disk in kilometers/millimeter? **Answer: 4,350 km/mm**

Problem 2 – At the scale of your drawing, what would be the radius of Earth (R = 6,378 km) and Jupiter (R = 71,500 km)? **Answer: 1.5 mm and 16.4 mm respectively.**

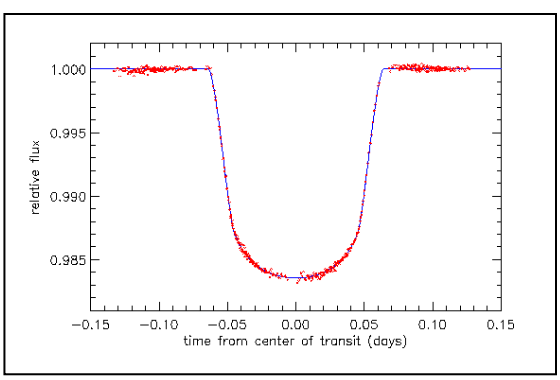
Problem 3 – What is the area of the sun disk in square millimeters? Answer; π x $(160)^2$ **= 80,400 mm^2.**

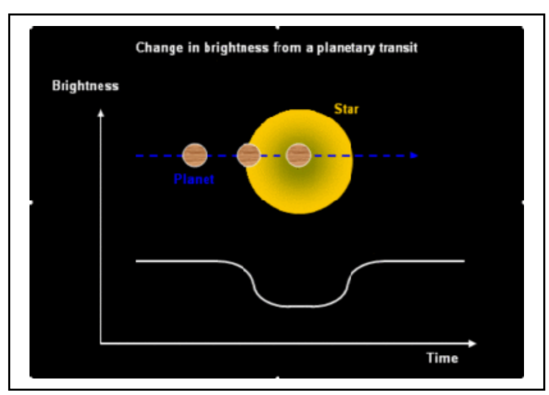
Problem 4 – What is the area of Earth and Jupiter in square millimeters? Answer: Earth = π x $(1.5)^2$ = **7.1 mm^2**. Jupiter = π x $(16.4)^2$ = **844.5 mm^2**.

Problem 5 – By what percent would the area of the sun be reduced if: A) Earth's disk were placed in front of the Sun disk? B) Jupiter's disk were placed in front of the Sun disk? Answer A) 100% x (7.1 mm^2 / 80400 mm^2) = 100% x 0.000088 **= 0.0088 %**
B) Jupiter: 100% x (844.5 mm^2 / 80400 mm^2) = 100% x 0.011 = **1.1%.**

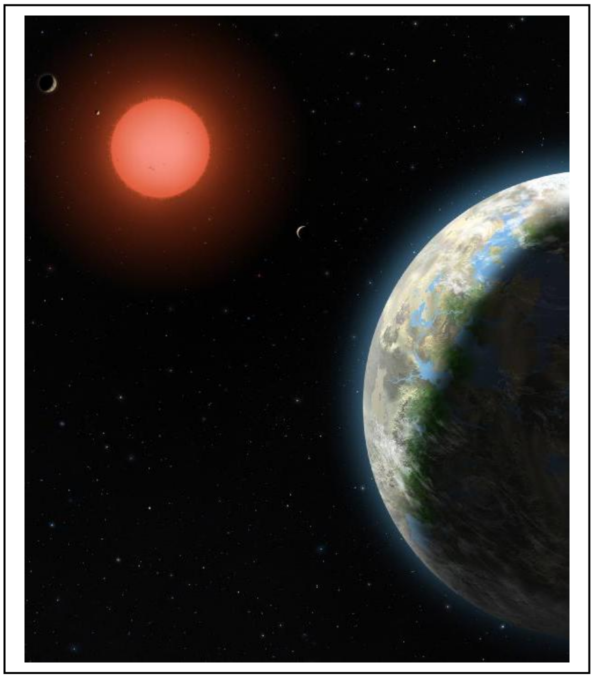
Problem 6 – For the transit of a large planet like Jupiter, draw a graph of the percentage brightness of the star (vertical axis) as it changes with time (horizontal axis) during the transit event. Assume that the entire transit takes about 1 day from start to finish.

Answer: Students should note from their answer to Problem 5 that when the planet disk is fully on the star disk, the star's brightness will dim from 100% to 100% - 1.1% = 98.9%. Students should also note that as the transit starts, the stars brightness will dim as more of the planet's disk begins to cover the star's disk. Similarly, as the planet's disk reaches the edge of the star's disk, the area covered by the planet decreases and so the star will gradually brighten to its former 100% level. The figures below give an idea of the kinds of graphs that should be produced. The left figure is from the Hubble Space Telescope study of the star HD209458 and its transiting Jupiter-sized planet.





The Earth-like Planet Gliese 581g



Professors Steven Vogt at UC Santa Cruz, and Paul Butler of the Carnegie Institution have just announced the discovery of a new planet orbiting the nearby red dwarf star Gliese 518. The star is located 20 light years from Earth in the constellation Libra. The planet joins five others in this crowded planetary system, and has a mass about three to four times Earth, making it in all likelihood a rocky planet, rather than a gas giant. The planet is tidally locked to its star which means that during its 37 day orbit, it always shows the same face to the star so that one hemisphere is always in daylight while the other is in permanent nighttime.

One of the most important aspects to new planets is whether they are in a distance zone where water can remain a liquid on the planets surface. The Habitable Zone (HZ) location around a star depends on the amount of light energy that the star produces. For the Sun, the HZ extends from about the orbit of Venus to the orbit of Mars. For stars that emit less energy, the HZ will be much closer to the star. Once an astronomer knows what kind of star a planet orbits, they can calculate over what distances the HZ will exist.

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system?

Problem 2 - One Astronomical Unit (AU) is the distance between Earth and the Sun (150 million kilometers). Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table below:

Planet	Discovery Year	Distance (AU)	Period (days)	Diameter (km)
Gliese 581 b	2005	0.04	5.4	50,000
Gliese 581 c	2007	0.07	13.0	20,000
Gliese 581 d	2007	0.22	66.8	25,000
Gliese 581 e	2009	0.03	3.1	15,000
Gliese 581 f	2010	0.76	433	25,000
Gliese 581 g	2010	0.15	36.6	20,000

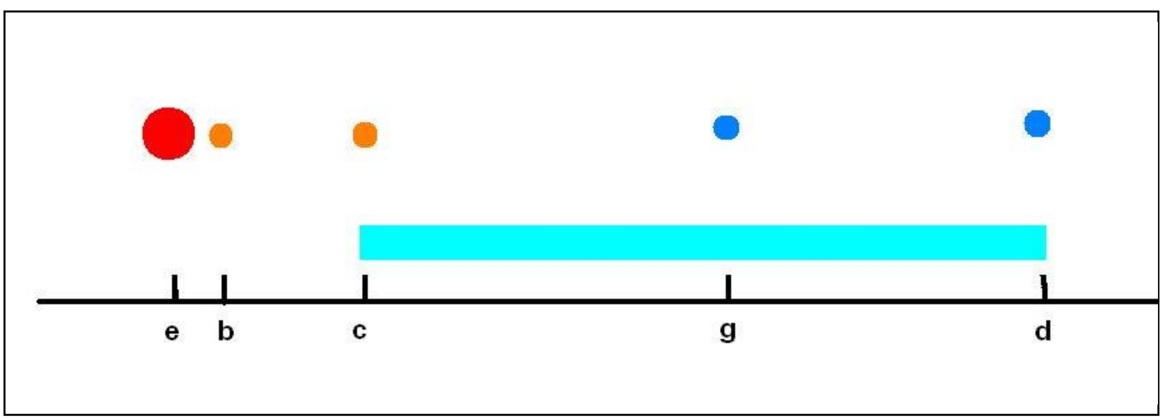
Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model diagram, shade-in the range of distances where the HZ exists for the Gliese 581 planetary system. Why do you think astronomers are excited about Gliese 581g?

Problem 1 - What is the pattern that astronomers use to name the discovered planets outside our solar system? Answer: **According to the order of discovery date.** Note: Gliese 581 A is the designation given to the star itself.

Problem 2 - Draw a model of the Gliese 581 planetary system with a scale of 0.01 AU per centimeter, and show each planet with a small circle drawn to a scale of 5,000 km/millimeter, based on the data in the table. Answer: **The table below gives the dimensions on the scaled diagram. See figure below for an approximate appearance. On the scale of the figure below, Gliese 581f would be located about 32 centimeters to the right of Gliese 581d.**

Planet	Discovery Year	Distance (cm)	Period (days)	Diameter (mm)
Gliese 581 b	2005	4	5.4	10
Gliese 581 c	2007	7	13.0	4
Gliese 581 d	2007	22	66.8	5
Gliese 581 e	2009	3	3.1	3
Gliese 581 f	2010	76	433	5
Gliese 581 g	2010	15	36.6	4

Problem 3 - The Habitable Zone for our solar system extends from 0.8 to 2.0 AU, while for Gliese 581 it extends from about 0.06 to 0.23 because the star shines with nearly 1/100 the amount of light energy as our sun. In the scale model, shade-in the range of distances where the HZ exists. Why do you think astronomers are excited about Gliese 581g? Answer: **See the bar spanning the given distances. Note that Gliese c, g and d are located in the HZ of Gliese 581. Because Gliese 581 g is located near the center of this zone and is very likely to be warm enough for there to be liquid water, which is an essential ingredient for life. Gliese 581c may be too hot and Gliese 581 d may be too cold.**



The Cometary Planet: HD 209458b

Artist view of planet G. Bacon (STScI/NASA)

Every 4 days, this planet orbits a sun-like star located 153 light years from Earth. Astronomers using NASA's Hubble Space Telescope have confirmed that this gas giant planet is orbiting so close to its star its heated atmosphere is escaping into space.

Observations taken with Hubble's Cosmic Origins Spectrograph (COS) suggest powerful stellar winds are sweeping the cast-off atmospheric material behind the scorched planet and shaping it into a comet-like tail. COS detected the heavy elements carbon and silicon in the planet's super-hot, $2{,}000^{o}$ F atmosphere.

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as 4×10^{11} grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3 times that of Jupiter. If the mass of Jupiter is 1.9×10^{27} kg, and its radius is 7.13×10^{7} meters, what is the density of A) Jupiter? B) HD209458b?

Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is 5.9×10^{24} kg, what is the mass of the atmosphere of HD209458b?

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Problem 1 - Based upon a study of the spectral lines of hydrogen, carbon and silicon, the estimated rate of atmosphere loss may be as high as $4\text{x}10^{11}$ grams/sec. How fast is it losing mass in: A) metric tons per day? B) metric tons per year?

Answer: A) $4\text{x}10^{11}$ grams/sec x (10^{-6} kg/gm) x (86,400 sec/day) x (1 ton / 1000 kg)
= **3.5×10^{10} tons/day**
B) 3.5×10^{10} tons/day x 365 days/year = **1.3×10^{13} tons/year**

Problem 2 - The mass of the planet is about 60% of Jupiter, and its radius is about 1.3 times that of Jupiter. If the mass of Jupiter is $1.9\text{x}10^{27}$ kg, and its radius is $7.13\text{x}10^{7}$ meters, what is the density of A) Jupiter? B) HD209458b?

Answer: A) $V = 4/3\ \pi\ R^3$ so V(Jupiter) = 1.5×10^{24} meters^3. Density = mass/volume so Density (Jupiter) = $1.9\text{x}10^{27}$ kg/1.5×10^{24} meters^3 = **1266 kg/meter^3**.
B) Mass = 0.6 M(Jupiter) and volume = $(1.3)^3$ V(Jupiter) so density = $0.6/(1.3)^3$ x 1266 kg/meter^3 =0.27x1266 = **342 kg/meter^3**.

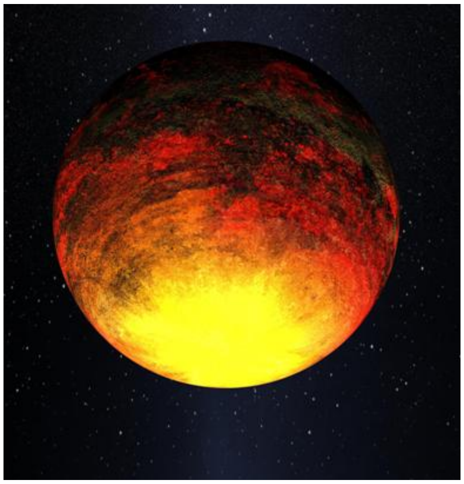
Problem 3 - Suppose that, like Jupiter, the planet has a rocky core with a mass of 18 times Earth. If Earth's mass is $5.9\text{x}10^{24}$ kg, what is the mass of the atmosphere of HD209458b? Answer: M(HD209458b) = 0.6x Jupiter = $1.1\text{x}10^{27}$ kg so M(atmosphere) = $1.1\text{x}10^{27}$ kg – 18x($5.9\text{x}10^{24}$ kg) = **9.9×10^{26} kg**.

Problem 4 - About how long would it take for HD209458b to completely lose its atmosphere at the measured mass-loss rate?

Answer: Time = Mass/rate, and the rate is 1.3×10^{13} tons/year. Since 1 metric ton = 1,000 kg, the rate is 1.3×10^{16} kg/year so that
= 9.9×10^{26} kg / (1.3×10^{16} kg/year)
= **7.6×10^{10} years**

The paper is Linsky et al., “Observations of Mass Loss from the Transiting Exoplanet HD 209458b,” *Astrophysical Journal* Vol. 717, No. 2 (10 July 2010), p. 1291. They estimate nearly a trillion years, so the planet is in no danger of disappearing!

Planet Kepler-10b: A matter of gravity



NASA Artist rendition of the sizzling-hot, Earth-like world: Kepler 10b.

The Kepler Space Observatory recently detected an Earth-sized planet orbiting the star Kepler-10. The more than 8 billion year old star, located in the constellation Draco, is 560 light years from Earth. The planet orbits its star at a distance of 2.5 million km with a period of 20 hours, so that its surface temperature exceeds 2,500 F.

Careful studies of the transit of this planet across the face of its star indicates a diameter 1.4 times that of Earth, and an estimated average density of 8.8 grams/cc, which is about that of solid iron, and 3-times the density of Earth's silicate-rich surface rocks.

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \quad \text{meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Problem 1 -Assume that Kepler-10b is a spherical planet, and that the radius of Earth is 6,378 kilometers. What is the total mass of this planet if its density is 8800 kg/meter^3?

Answer: The planet is 1.4 times the radius of Earth, so its radius is 1.4 x 6,378 km = 8,929 kilometers. Since we need to use units in terms of meters because we are given the density in cubic meters, the radius of the planet becomes 8,929,000 meters.

Volume = $\frac{4}{3}\pi R^3$

so V = 1.33 x (3.141) x (8,929,000 meters)^3

V = 2.98 x 10^{21} meter^3

Mass = Density x Volume

= 8,800 x 2.98 x 10^{21}

= **2.6x10^{25} kilograms**

Problem 2 - The acceleration of gravity on a planet's surface is given by the Newton's formula

$$a = 6.67x10^{-11}\frac{M}{R^2} \quad \text{meters/sec}^2$$

Where R is distance from the surface of the planet to the planet's center in meters, and M is the mass of the planet in kilograms. What is the acceleration of gravity at the surface of Kepler-10b?

Answer: a = 6.67 x 10^{-11} (2.6 x 10^{25})/(8.929x10^6)^2

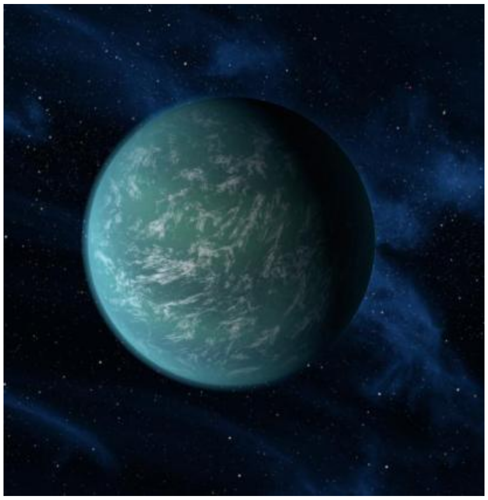
= **21.8 meters/sec^2**

Problem 3 - The acceleration of gravity at Earth's surface is 9.8 meters/sec^2. If this acceleration causes a 68 kg human to have a weight of 150 pounds, how much will the same 68 kg human weigh on the surface of Kepler-10b if the weight in pounds is directly proportional to surface acceleration?

Answer: The acceleration is 21.8/9.8 = 2.2 times Earth's gravity, and since weight is proportional to gravitational acceleration we have the proportion:

$\frac{21.8}{9.8} = \frac{X}{150lb}$ and so the human would weigh 150 x 2.2 = **330 pounds!**

Playing Baseball on the Earth-like Planet Kepler-22b!



An artists illustration of Kepler-22b
(Credit NASA/Ames/JPL-Caltech)

NASA's Kepler mission has confirmed its first planet in the "habitable zone," the region where liquid water could exist on a planet's surface.

The newly confirmed planet, Kepler-22b, is the smallest yet found to orbit in the middle of the habitable zone of a star similar to our sun.

The planet is about 2.4 times the radius of Earth. Scientists don't yet know if Kepler-22b has a rocky, gaseous or liquid composition, but its discovery is a step closer to finding Earth-like planets.

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97x1024 kg)?

Problem 2 - The acceleration of gravity on a planetary surface is given by the formula

$$a = \frac{GM}{R^2}$$

where M is in kilograms, R is in meters and G is the Newtonian Constant of Gravity with a value of 6.67×10^{-11} m^3 kg^{-1} sec^{-2} What is the surface acceleration of Kepler-22b A) In meters/sec^2? B) In multiples of Earth's surface gravity 9.8 meters/sec^2?

Problem 3 - The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b given the acceleration you calculated in Problem 2?

Problem 4 - The dimensions of a typical baseball park are determined by the farthest distance that an average batter can bat a home-run. This in turn depends on the acceleration of gravity, which is the force that pulls the ball back to the ground to shorten its travel distance. For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the minimum distance to the back-field fence from Home Plate along one of the two foul lines?

Problem 1 - Suppose Kepler-22b is a spherical, rocky planet like Earth with an average density similar to Earth (about 5,500 kg/meter^3). If the radius of Kepler-22b is 15,000 km, what is the mass of Kepler-22b in A) kilograms? B) multiples of Earth's mass (5.97×10^{24} kg)?

Answer: A) First find the volume of the spherical planet in cubic meters, then multiply by the density of the planet to get the total mass.

$$R = 15{,}000 \text{ km} \times (1000 \text{ m}/1 \text{ km}) = 1.5 \times 10^7 \text{ meters.}$$

$V = 4/3\, \pi\, R^3$

$= 1.33 \times 3.14 \times (1.5 \times 10^7 \text{ meters})^3 = 1.41 \times 10^{22} \text{ meters}^3$

Then M = density x volume

$= 5{,}500 \text{ kg/m}^3 \times (1.41 \times 10^{22})$

$= 7.75 \times 10^{25}$ kg

B) $M = 7.75 \times 10^{25} \text{ kg} / 5.97 \times 10^{24} \text{ kg}$ = **12.9 Earths**.

Problem 2 - A) In meters/sec^2? B) In multiples of Earth's surface gravity 9.8 meters/sec^2?

Answer: A) $a = 6.67 \times 10^{-11} (7.75 \times 10^{25})/(1.5 \times 10^7)^2$ = **23.0 meters/sec^2**

B) 23.0 / 9.8 = **2.3 times earth's surface gravity**

Note: From the formula for M and a, we see that the acceleration varies directly with the radius change, which is a factor of 2.4 times Earth, so a = 2.4xa(earth)

Problem 3 – The relationship between surface acceleration and your weight is a direct proportion. The surface acceleration of Earth is 9.8 meters/sec^2. If you weigh 150 pounds on the surface of Earth, how much will you weigh on the surface of Kepler-22b given the acceleration you calculated in Problem 2?

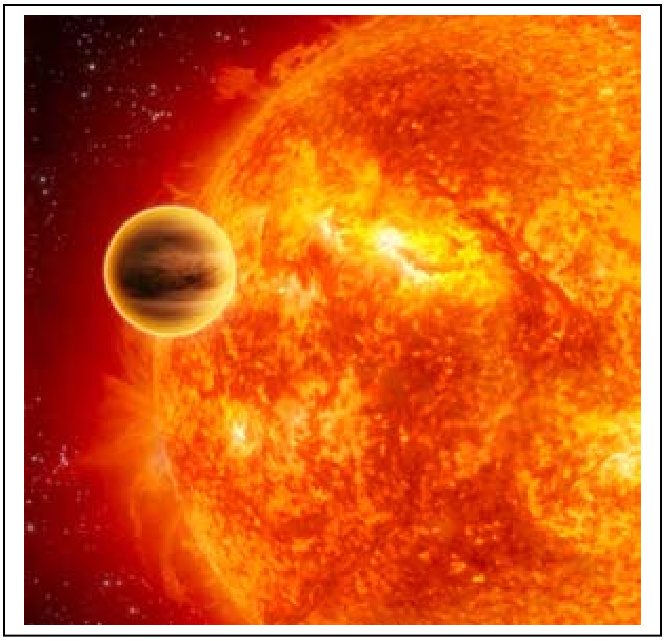
Answer: By a simple proportion: X/150 = 2.3/1.0 so x = 2.3x150 = **345 pounds**.

Problem 4 - For a standard baseball field, the distance to the back-field fence from Home Plate may not be less than 325 feet, and the baseball diamond must be exactly 90 feet on a side.

A) If the maximum travel distance of the baseball scales linearly with the acceleration of gravity, what is the distance to the back-field fence from Home Plate along one of the two foul lines? Answer: 325 / 2.3 = **141 feet**.

B) What are the dimensions of the baseball diamond? Answer: 90/2.3 = **39 feet** on a side.

Exoplanet Orbits and the Properties of Ellipses



By 2011, over 1700 planets have been discovered orbiting nearby stars since 1995. Called 'exoplanets' to distinguish them from the familiar 8 planets in our own solar system, they are planets similar to Jupiter in size, but orbiting their stars in mostly elliptical paths. In many cases, the planets come so close to their star that conditions for life to exist would be impossible.

Astronomers are continuing to search for smaller planets to find those that are more like our own Earth.

(Artist rendition: courtesy NASA)

Use the basic properties and formulae for ellipses to analyze the following approximate exoplanet orbits by first converting the indicated equations into standard form. Then determine for each planet the:

A) a= semi-major axis

B) b=semi-minor axis;

C) ellipticity $e = \frac{\sqrt{(a^2 - b^2)}}{a}$

D) 'perihelion' closest distance to star, defined as P=a(1-e);

E) 'aphelion' farthest distance from star, defined as A = a(1+e).

Problem 1: Planet: 61 Virginis-d Period=4 days $1 = 4x^2 + 5y^2$

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

Answer Key

Problem 1: 61 Virginis -d Period=4 days $1 = 4x^2 + 5y^2$

$$1 = \frac{x^2}{0.25} + \frac{y^2}{0.20}$$

a=0.5 b=0.45 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.43** P = (0.5)(1-0.43) = **0.28**, A=(0.5)(1+0.43) = **0.71**

Problem 2: Planet: HD100777-b Period=383 days $98 = 92x^2 + 106y^2$

$$1 = \frac{x^2}{1.06} + \frac{y^2}{0.92}$$

a=1.03 b=0.96 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.36** P = (1.03)(1-0.36) = **0.66,** A=(1.03)(1+0.36) = **1.40**

Problem 3: Planet: HD 106252-b Period=1500 days $35 = 5x^2 + 7y^2$

$$1 = \frac{x^2}{7.0} + \frac{y^2}{5.0}$$

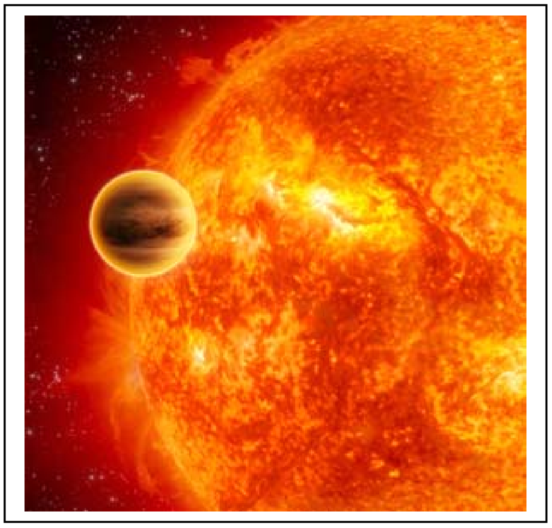
a=2.6 b=2.2 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.53** P = (2.6)(1-0.53) = **1.22**, A=(2.6)(1+0.53) = **4.0**

Problem 4: Planet: 47 UMa-c Period= 2190 days $132 = 11x^2 + 12y^2$

$$1 = \frac{x^2}{12.0} + \frac{y^2}{11.0}$$

a=3.5 b=3.3 $e = \frac{\sqrt{(a^2 - b^2)}}{a}$ = **0.33** P = (3.5)(1-0.33) = **2.35,** A=(3.5)(1+0.33) = **4.65**

Estimating the Temperatures of an Exoplanet



Because many exoplanets orbit their stars in elliptical paths, they experience large swings in temperature. Generally, organisms can not survive if water is frozen (0 C = 273 K) or near its boiling point (100 C or 373 K). Due to orbital conditions, this very narrow 'zone of life' may not be possible for many of the worlds detected so far.

Problem 1 - Complete the table below by calculating

A) The semi-minor axis distance B = A(1-e^2)
B) The perihelion distance Dp = A(1-e)
C) The aphelion distance, Da = B(1+e)

Problem 2 - Write the equation for the orbit of 61 Vir-d in Standard Form.

Problem 3 - The temperature of a planet similar to Jupiter can be approximated by the formula below, where T is the temperature in Kelvin degrees, and R is the distance to its star in Astronomical Units (AU), where 1 AU is the distance from Earth to the sun (150 million km). Complete the table entries for the estimated temperature of each planet at the farthest 'aphelion' distance Ta, and the closest 'perihelion;' distance Tp.

$$T(R) = \frac{250}{\sqrt{R}}$$

Problem 4 - Which planets would offer the most hospitable, or the most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39		0.22	2190				
61 Vir-d	0.47		0.35	123				
HD106252-b	2.61		0.54	1500				
HD100777-b	1.03		0.36	383				
HAT-P13c	1.2		0.70	428				

Problem 1 See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**		
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**		
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**		
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**		
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**		

Problem 2 Write the equation for the orbit of 61 Vir-d in Standard Form.
Answer: A = 0.47 and B = 0.4

So $1 = \frac{x^2}{0.47} + \frac{y^2}{0.40}$ and also $188 = 40x^2 + 47y^2$

Problem 3 - See table below:

Planet	A (AU)	B (AU)	e	Period (days)	Dp (AU)	Da (AU)	Ta (K)	Tp (K)
47 UMa-c	3.39	**3.3**	0.22	2190	**2.6**	**4.1**	**154**	**123**
61 Vir-d	0.47	**0.4**	0.35	123	**0.3**	**0.6**	**452**	**314**
HD106252-b	2.61	**2.2**	0.54	1500	**1.2**	**4.0**	**228**	**125**
HD100777-b	1.03	**1.0**	0.36	383	**0.7**	**1.4**	**308**	**211**
HAT-P13c	1.2	**0.9**	0.70	428	**0.4**	**2.0**	**417**	**175**

Problem 4 - Which planets would offer the most hospitable, or most hazardous, conditions for life to exist, and what would be the conditions be like during a complete 'year' for each world?
Answer: For the habitable 'water' range between 273K and 373K, none of these planets satisfy this minimum and maximum condition. They are either too hot at perihelion 'summer' or too cold at 'winter' aphelion.

Only HD100777-b during perihelion is in this temperature range during 'summer', at a temperature of 308 K (35 C). During 'winter' at aphelion, it is at -62 C which is below the freezing point of water, and similar to the most extreme temps in Antarctica.

Note: These temperature calculations are only approximate and may be considerably different with greenhouse heating by the planetary atmosphere included.