Solar Flares

This image was taken by NASA’s Solar Dynamics Observatory on July 6, 2012 and shows a brilliant X-ray solar flare erupting from the sun.

Solar flares are not all the same. Some produce less energy than others, and so astronomers classify them by their X-ray energy using four different letters: B, C, M and X. C-class flares produce 10 times more X-ray energy than B-class flares. M-class flares produce 10 times more energy than C-class flares, and X-class flares produce 10 times more energy than M-class flares. One B-class flare can produce more energy than 240,000 million tons of TNT!

The table below lists all of the M and X-class flares detected between January 1, 2013 and August 15, 2013 at a time when solar activity was near its maximum. This period of time spans the first 227 days of 2013. Also during this time, there were about 690 C-class flares and 440 B-class flares. All of these flares were seen on the side of the sun facing Earth, which represents ½ of the total surface area of the sun.

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**Problem 1** - What were the total number of M and X-class flares during this period of time?

**Problem 2** – What were the total number of B, C, M and X-class flares detected during this period?

**Problem 3** – What percentage of all flares were B, C, M and X?

**Problem 4** – What was the average number of B and C-class flares seen each day?

**Problem 5** – An astronaut wants to do a spacewalk on a particular day during this period. What are the odds that she will see an M or X-class flare?
The flare data was obtained from
http://www.swpc.noaa.gov/ftpmenu/warehouse/2013/2013_events.html

**Problem 1** - What were the total number of M and X-class flares during this period of time?

Answer: By counting Ms in the table, there were **31 M-class and 4 X-class flares**.

**Problem 2** – What were the total number of B, C, M and X-class flares detected during this period?

Answer: \[690 + 440 + 31 + 4 = 1165 \text{ flares} .\]

**Problem 3** – What percentage of all flares were B, C, M and X?

Answer:

\[
\text{B} = 100\% \times \frac{440}{1165} = 38\% \\
\text{C} = 100\% \times \frac{690}{1165} = 59\% \\
\text{M} = 100\% \times \frac{31}{1165} = 3\% \\
\text{X} = 100\% \times \frac{4}{1165} = 0.3\% \\
\]

**Problem 4** – What was the average number of B and C-class flares seen each day?

Answer: B: \(440 \text{ flares}/227 \text{ days} = \text{about 2 flares}\)  \\
C: \(690 \text{ flares}/227 \text{ days} = \text{about 3 flares}\)

**Problem 5** – An astronaut wants to do a spacewalk on a particular day during this period. What is the probability that she will see each an M or X-class flare?

Answer: There are 227 days in the sample and 35 M or X-class flares were seen, so the probability is \(35/227 = 0.15\) which is also stated as 15%. The low probability means that it is not likely that on a random day the astronaut will see anything. In order to have a 50/50 chance, she would have to observe for at least 4 days \((4 \times 0.15 = 0.60 \text{ which is greater than 0.50 or 50%})\).
Every once in a while, the sun ejects huge clouds of heated gas, called plasma, which can contain billions of tons of matter and travel at speeds of millions of miles per hour. Occasionally these are directed at earth, and when they arrive they cause brilliant aurora. They can also cause problems for electrical systems on the ground and satellite systems in space.

The top image is a composite that shows the surface of the sun and one of these ‘coronal mass ejections’ being released. This one is directed away from earth and is harmless to us. When we spot a CME directed towards earth, the cloud seems to form a temporary ‘halo’ around the edge of the sun. These Halo CMEs are ejected from the sun, and can arrive at earth about 2 to 4 days later.

Soon after a Halo CME is ejected, satellites may detect a rain storm of radiation particles that were ejected from the sun at the same time. These travel so fast that they arrive at earth in only an hour or so. Also called Solar Proton Events (SPEs), these radiation storms are very harmful to astronauts in space and to sensitive satellite electronics. Predicting when SPEs will occur is an important goal of Space Weather Research.

The table below gives the dates for the CMEs detected during the 227 days from January 1, 2013 and August 15, 2013 during the peak of our sun’s current storm cycle. Yellow shading indicates that a SPE occurred on the same date.

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**Problem 1** – What is the average number of days between all of the CMEs in this sample?

**Problem 2** – What percentage of CMEs are of the Halo-type?

**Problem 3** – What percentage of Halo CMEs seem to produce Solar Proton Events?

**Problem 4** – If you observed a CME, what is the probability that it may produce a harmful solar proton event?
**Problem 1** – What is the average number of days between all of the CMEs in this sample?

Answer: There are a total of 20 CMEs in 227 days so the average interval is about $\frac{227}{20} = 11$ days.

**Problem 2** – What percentage of CMEs are of the Halo-type?

Answer: Of the 20 CMEs, 12 were Halo-type so the percentage is $100\% \times \frac{12}{20} = 60\%$

**Problem 3** – What percentage of Halo CMEs seem to produce Solar Proton Events?

Answer: Of the 12 Halo-type events, 4 produced SPEs so $100\% \times \frac{4}{12} = 33\%$

**Problem 4** – If you observed a CME, what is the probability that it may produce a harmful solar proton event?

Answer: There were 20 CMEs total of which 4 produced SPEs, so $100\% \times \frac{4}{20} = 20\%$

Note: Of the 4 SPEs, three occurred with Halo CMEs so this means that $\frac{3}{4}$ or 75\% of all SPEs coincide with Halo-type CMEs, however it is also true that only 33\% of Halo-type CMEs produce SPEs. Not all Halo events produce solar proton events, so using halo events to predict whether an SPE will occur will lead to a large number of false-positives by about 8 false to 4 positives (8+4 = 12 halo events).
When a solar coronal mass ejection collides with earth’s magnetic field, it can produce intense aurora that can be seen from the ground. Geophysicists who study magnetic disturbances have created a 9-point scale that indicates how intense the storm is. Kp=9 is the most intense, and aurora can be seen near Earth’s equator for many of these. Kp=7 and 8 are strong storms that can still cause aurora and upset electrical power systems. Kp=5 and 6 are mild storms that may or may not produce intense aurora.

The table below gives the dates when magnetic storms were detected which exceeded Kp=4. The multiple entries each day indicate consecutive 3-hour measurements of the storm intensity. The data are given for the 227 days from January 1, 2013 to August 15, 2013. Highlighted boxes in yellow indicate days when Halo CMEs were detected.

<table>
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**Problem 1** – What percentage of the days during this time period had magnetic storm events?

**Problem 2** – What percentage of the magnetic storm days were more intense than Kp=5?

**Problem 3** – What is the probability that of the 12 Halo CME events that occurred during this 227-day period, that these events produce a magnetic storm more intense than Kp=5?

**Problem 4** – The Kp index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds Kp=5 during its entire duration?
Problem 1 – What percentage of the days during this time period had magnetic storm events?

Answer: 17 days out of 227 so \( 100\% \times \frac{17}{227} = 7\% \)

Problem 2 – What percentage of the magnetic storm days were more intense than Kp=5?

Answer: 4 out of 17 or \( 100\% \times \frac{4}{17} = 23\% \).

Problem 3 – What is the probability that of the 12 Halo CME events that occurred during this 227-day period, that these events produce a magnetic storm more intense than Kp=5?

Answer: Only 4 of the 12, so \( P = 100\% \times \frac{4}{12} = 33\% \).

Problem 4 – The Kp index is measured every 3 hours. From the table, what is the average duration of a storm that exceeds Kp=5 during its entire duration?

Answer: There were 4 storms that exceeded Kp=5. These occurred on the dates: 3-17, 6-1, 6-7 and 6-29. The total hours for each storm were 6x3 = 18 hours (for 3-17), 5x3 = 15 hours (for 6-1); 3x3 = 9 (for 6-7) and 4x3 = 12 (for 6-29). The average for these hours is \( \frac{18+15+9+12}{4} = 14 \) hours.

So, once a strong magnetic storm begins, it takes about one-half a day for it to reach its maximum intensity and then fade away. If this happens in the summertime during the day, you will not see an aurora borealis because of the daylight brightness.
The following essay describes solar storms. The words that belong in the missing blanks can be found by solving the ten number sentences.

Look in the Word Bank and match the calculated number to the correct answer.

Fill-in the blank with the word you found to complete the essay and answer a question about solar storms!

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1)__________ and incredible explosions of 2)__________ almost every day. Sometimes, its entire surface is speckled by 3)__________ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4)__________ light that can cause problems for radio communication on Earth. They also heat up the 5)__________ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6)__________ called 7)_______________________ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8)_________. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9)_________ in the northern and southern skies. Most CMEs are not directed towards earth and are completely 10)____________.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!

Word Bank

-5 asteroids +48 heat -17 X-rays
-44 aurora +1 ocean +3 harmless
-15 ultraviolet +5 days +44 rainbows
-27 energy 0 atmosphere -5 months
-40 rocks -48 gas +24 sunspots
-6 harmful +3 plasma -3 coronal mass ejections
-2 flares +20 comets -7 prominences

Solve these problems to get the Word Bank number key.

1) 1+(1-3)-(5-8)+(-6+2) = 6) -2(+2(-3(+2(-3+1)))) =
2) 8(3-2)-2(3-8)+5(-6-3) = 7) -8/2 +(3+2)/(8-3) =
3) (1-3)(-5+2)(8-6)(3-1) = 8) -7+23-6(-10)+(-3)(4+1) =
4) 3(2-6)+(-8+4)-(4-3) = 9) 12/(-1/4) + (-36)/(-9) =
5) 5(-3+2)-2(6-2)+(-7-20) = 10) (-4)2 + 21/3 + 4 =

Space Math
Solve these problems to get the Word Bank number key.

1) \(1+(1-3)-(5-8)+(-6+2) = -2\)
2) \(8(3-2)-(2(3-8)+5(-6-3)) = -27\)
3) \((1-3)(-5+2)(8-6)(3-1) = +24\)
4) \(3(2-6)+(-8+4)-(4-3) = -17\)
5) \(5(-3+2)-2(6-2)+(-7-20) = 0\)
6) \(-2+2(-3+2(-3+1))) = -48\)
7) \(-8/2 + (3+2)/(8-3) = -3\)
8) \(-7+23-6(-10)+(-3)(4+1) = +5\)
9) \(12/(-1/4) + (-36)/(-9) = -44\)
10) \((-4)2 + 21/3 + 4 = +3\)

The words are

- Line 1 = -2 = flares
- Line 2 = -27 = energy
- Line 3 = +24 = sunspots
- Line 4 = -17 = X-ray
- Line 5 = 0 = atmosphere
- Line 6 = -48 = gas
- Line 7 = -3 = coronal mass ejections
- Line 8 = +5 = days
- Line 9 = -44 = aurora
- Line 10 = +3 = harmless

Our sun is a very predictable star. Each day it rises and sets as the world turns upon its axis, and warms the Earth making life possible. But the sun is also a stormy star. It produces 1) _____flares____ and incredible explosions of 2) _____energy____ almost every day. Sometimes, its entire surface is speckled by 3) _____sunspots_____ that come and go every 11 years. In 2013, the sun was at the peak of its maximum stormy activity. This means that many more flares and explosions of gas were happening compared to other times in the 11-year cycle. Solar flares are bursts of intense 4) _____X-ray____ light that can cause problems for radio communication on Earth. They also heat up the 5) _____atmosphere_____ and cause it to expand into space. About 1000 of these flares were detected during the first 8 months of 2013.

Occasionally the sun ejects billion-ton clouds of 6) _____gas_____ called 7) _____coronal mass ejections_____ or CMEs. Traveling at over a million miles an hour, they can reach Earth in only a few 8) _____days_____. When they arrive, they cause problems for satellites and our electric power grid, but they also cause beautiful 9) _____aurora_____ in the northern and southern skies. Most CMEs are not directed towards earth and are completely 10) _____harmless_____.

So, even though the sun looks the same every day, it really is a very stormy star that can sometimes create unpleasant surprises for us here on Earth!
One of the most basic activities that scientists perform with their data is to look for correlations between different kinds of events or measurements in order to see if a pattern exists that could suggest that some new 'law' of nature might be operating. Many different observations of the Sun and Earth provide information on some basic phenomena that are frequently observed. The question is whether these phenomena are related to each other in some way. Can we use the sighting of one phenomenon as a prediction of whether another kind of phenomenon will happen?

During most of the previous sunspot cycle (January-1996 to June-2006), astronomers detected 11,031 coronal mass ejections, (CME: Top image) of these 1186 were 'halo' events. Half of these were directed towards Earth.

During the same period of time, 95 solar proton events (streaks in the bottom image were caused by a single event) were recorded by the GOES satellite network orbiting Earth. Of these SPEs, 61 coincided with Halo CME events.

Solar flares (middle image) were also recorded by the GOES satellites. During this time period, 21,886 flares were detected, of which 122 were X-class flares. Of the X-class flares, 96 coincided with Halo CMEs, and 22 X-class flares also coincided with 22 combined SPE+Halo CME events. There were 6 X-flares associated with SPEs but not associated with Halo CMEs. A total of 28 SPEs were not associated with either Halo CMEs or with X-class solar flares.

From this statistical information, construct a Venn Diagram to interrelate the numbers in the above findings based on recent NASA satellite observations, then answer the questions below.

1 - What are the odds that a CME is directed towards Earth?

2 - What fraction of the time does the sun produce X-class flares?

3 - How many X-class flares are not involved with CMEs or SPEs?

4 - If a satellite spotted both a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur?

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both?

6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed?

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed?

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE?

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena?
Answer Key:

Venn Diagram Construction.
1. There are 593 Halo CMEs directed to Earth so $593 = 74$ with flares + 39 with SPEs + 22 both SPEs and Flares + 458 with no SPEs or Flares.

2. There are 95 SPEs. $95 = 39$ with CMEs + 6 with flares + 22 with both flares and CMEs + 28 with no flares or CMEs

3. There are 122 X-class flares. $122 = 74$ With CMEs only + 6 with SPEs only + 22 both CMEs and SPEs + 20 with no CMEs or SPEs.

1 - What are the odds that a CME is directed towards Earth? $\frac{593}{11031} = 0.054$ odds = 1 in 19

2 - What fraction of the time does the sun produce X-class flares? $\frac{122}{21886} = 0.006$

3 - How many X-class flares are not involved with CMEs or SPEs? $122 - 74 - 22 - 6 = 20$.

4 - If a satellite spotted BOTH a halo coronal mass ejection and an X-class solar flare, what is the probability that a solar proton event will occur? $\frac{22}{74+22} = 0.23$

5 - What percentage of the time are SPEs involved with Halo CMEs, X-class flares or both? $100\% \times \frac{39+22+6}{95} = 70.1\%$

6 - If a satellite just spots a Halo CME, what are the odds that an X-class flare or an SPE or both will be observed? $\frac{39+22+74}{593} = 0.227$ so the odds are 1/0.227 or about 1 in 4.

7 - Is it more likely to detect an SPE if a halo CME is observed, or if an X-class flare is observed? 
   $\frac{6+22}{95} = 0.295$ or 1 out of 3 times for X-flares
   $\frac{39+22}{95} = 0.642$ or 2 out of 3 for Halo CMEs

It is more likely to detect an SPE if a Halo CME occurs by 2 to 1.

8 - If you see either a Halo CME or an X-class flare, but not both, what are the odds you will also see an SPE? 
   $\frac{39+6}{95} = 0.50$ so the odds are 1/0.50 or 2 to 1.

9 - If you observed 100 CMEs, X-class flares and SPEs, how many times might you expect to see all three phenomena? 
   $100 \times \frac{22}{95+122+593} = 3$ times
The Sun is an active star, which produces solar flares (F) and explosions of gas (C). Astronomers keep watch for these events because they can harm satellites and astronauts in space. Predicting when the next storm will happen is not easy to do. The problems below are solved by writing out all of the possibilities, then calculating the probability of the particular outcome!

*Solar flare photo courtesy TRACE/NASA*

1 – During a week of observing the sun, astronomers detected 1 solar flare (F). What was the probability (as a fraction) that it happened on Wednesday?

2 – During the same week, two gas clouds were ejected (C), but not on the same days. What is the probability (as a fraction) that a gas cloud was ejected on Wednesday?

3 – Suppose that the flares and the gas clouds had nothing to do with each other, and that they occurred randomly. What is the probability (as a fraction) that both a flare and a gas cloud were spotted on Wednesday? (Astronomers would say that these phenomena are uncorrelated because the occurrence of one does not mean that the other is likely to happen too).
1 – Answer: There are only 7 possibilities:

F X X X X X X  X X X F X X X  X X X X X F
X F X X X X X  X X X F X X X  X X X X F X
X X F X X X X  X X X X F X

So the probability for any one day is \( \frac{1}{7} \).

2 – Here we have to distribute 2 storms among 7 days. For advanced students, there are \( \frac{7!}{2! \cdot 5!} = \frac{7 	imes 6}{2} = 21 \) possibilities which the students will work out by hand:

C C X X X X X  X C C X X X  X X C C X X  X X C C X X
C X C X X X X  X C C X X X  X X C X C X  X X C X C X
C X X C X X X  X C X C X X  X X C X C X  X X C X C X
C X X X C X X  X C X X C X  X X C X X C  X X X C C X
C X X X C X X  X C X X X C  X X C X X C  X X X C C X
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C X X X X X C  X C X X X C  X X C X X C  X X X C C X
C X X X X X C  X C X X X C  X X C X X C  X X X C C X

There are 6 possibilities (in red) for a cloud appearing on Wednesday (Day 3), so the probability is \( \frac{6}{21} \).

3 – We have already tabulated the possibilities for each flare and gas cloud to appear separately on a given day. Because these events are independent of each other, the probability that on a given day you will spot a flare and a gas cloud is just \( \frac{1}{7} \times \frac{6}{21} = \frac{6}{147} \). This is because for every possibility for a flare from the answer to Problem 1, there is one possibility for the gas clouds.

There are a total of \( 7 \times 21 = 147 \) outcomes for both events taken together. Because there are a total of \( 1 \times 6 \) outcomes where there is a flare and a cloud on a particular day, the fraction becomes \( \frac{1 \times 6}{147} = \frac{6}{147} \).
How Common are X-Class Solar Flares?

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X-Class solar flares are among the most powerful, explosive events on the solar surface. They can cause short-wave radio interference, satellite malfunctions and can even cause the premature re-entry of satellites into the atmosphere.

The table above lists the number of X-class flares detected on the sun during the last sunspot cycle which lasted from 1996 to about 2008. The second column also gives the year from the start of the 11-year sunspot cycle in 1996. The counts are listed by year (rows) and by month (columns). Study this table and answer the following questions to learn more about how common these flares are.

**Problem 1** – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain.


**Problem 3** – For each group, what is the median number of flares that occurs in the months that have flares?

**Problem 4** – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle?

**Problem 5** – We are currently in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year?
**Problem 1** – For the years and months considered, is the distribution of months with flares a uniform distribution? Explain. Answer; If you shaded in all the months with flares you would see that most occur between 1999-2002 so the distribution is not random, and is not uniform.

**Problem 2** - The sunspot cycle can be grouped into pre-maximum (1997,1998, 1999), maximum (2000,2001,2002) and post-maximum (2003,2004,2005). For each group, calculate A) the percentage of months with no flares, and B) The average number of weeks between flares. Answer: A) Pre-Maximum, N = 28 months so P = 100% x 28/36 = 78%. Maximum; N = 17 months so P = 100% x 17/36 = 47%; post-maximum N= 24 months so P = 100% x 24/36 = 67%. B) pre-maximum N = 22 flares so T = 36 months/22 flares = 1.6 months. Maximum: N = 50 flares so T = 36 mo/50 = 0.7 months; post-maximum: N = 48 flares so T = 36 mo/48 = 0.8 months.

**Problem 3** – For each group, what is the median number of flares that occurs in the months that have flares? Answer: Pre-maximum: 1,1,2,2,2,3,5,5 median = 3. Maximum: 1,1,1,1,1,1,1,2,3,3,3,4,4,4,5,8 median = 4; post-maximum 1,1,2,2,2,3,4,6,6,7 median = 6

**Problem 4** – Taken as a whole, what is the average number of flares per month during the entire 11-year sunspot cycle? Answer: There were 122 flares detected during the 132 months of the sunspot cycle, so the average is 122 flares/132 months = 0.9, which can be rounded to 1 flare/month.

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**Problem 5** – We are in Year-3 of the current sunspot cycle, which began in 2007. About how many X-class flares would you predict for this year using the tabulated flares from the previous sunspot cycle as a guide, and what is the average number of weeks between these flares for this year? Answer: From the table we find for Year 3 that there were 14, X-class flares. Since there are 12 months in a year, this means that the average time between flares is about 14/12 = 1.2 months.
Mars has virtually no atmosphere, and this means that, unlike Earth, its surface is not protected from solar and cosmic radiation. On Earth, the annual dosage on the ground is about 0.35 Rem/year, but can vary from 0.10 to 0.80 Rem/year depending on your geographic location, altitude, and lifestyle.

This figure, created with the NASA, MARIE instrument on the Odyssey spacecraft orbiting Mars, shows the unshielded surface radiation dosages, ranging from a maximum of 20 Rem/year (brown) to a minimum of 10 Rem/year (deep blue).

Astronauts landing on Mars will want to minimize their total radiation exposure during the 540 days they will stay on the surface. The Apollo astronauts used spacesuits that provided 0.15 gm/cm$^2$ of shielding. The Lunar Excursion Module provided 0.2 gm/cm$^2$ of shielding, and the orbiting Command Module provided 2.4 gm/cm$^2$. The reduction in radiation exposure for each of these was about 1/4, 1/10 and 1/50 respectively. Assume that the Mars astronauts used improved spacesuit technology providing a reduction of 1/8, and that the Mars Excursion Vehicle provided a 1/20 radiation reduction.

The line segments on the Mars radiation map represent some imaginary, 1,000 km exploration tracks that ambitious astronauts might attempt with fast-moving rovers, and not a lot of food! Imagine a schedule where they would travel 100 kilometers each day. Suppose they spend 20 hours a day within a shielded rover, and they study their surroundings in spacesuits for 4 hours each day.

1) Convert 10 Rem/year into milliRem/day.

2) What is the astronauts radiation dosage per day in a region (brown) where the ambient background produces 20 Rem/year?

3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the approximate total dosage to an astronaut in milliRems (mRems), given the exposure times and shielding information provided.

4) Which track has the highest total dosage in milliRems? The least total dosage? What is the annual dosage that is equivalent to these 20-day trips? How do these compare with the 350 milliRems they would receive if they remained on Earth?
1) Convert 10 Rem/year into milliRem/hour.

Answer: \[(10 \text{ Rem/yr}) \times (1 \text{ year/365 days}) \times (1 \text{ day/24 hr}) = 1.1 \text{ milliRem/hour}\]

2) What is the astronaut's radiation dosage per day in a region (brown) where the background is 20 Rem/year?

Answer: From Problem 1, 20 Rem/year = 2.2 milliRem/hour.

\[20 \text{ hours} \times (1/20) \times 1.1 \text{ milliRem/hr} + 4 \text{ hours} \times (1/8) \times 1.1 \text{ milliRem/hr} = 1.1 + 0.55 = 1.65 \text{ milliRem/day}\]

3) For each of the tracks on the map, plot a dosage history timeline for the 10 days of each journey. From the scaling relationship defined for one day in Problem 3, calculate the total dosage in milliRems to an astronaut, given the exposure times and shielding information provided. The scaling relationship is that for each 20 Rem/year, the daily astronaut dosage is 0.66 milliRem/day (e.g., 0.66/20). The factor of 2 in the answers accounts for the round-trip.

Track A dosage:
\[2x(12 \text{ Rems/yr} \times 10 \text{ days} \times (1.65/20)) = 2x(9.9) = 19.8 \text{ mRem}.\]

Track B dosage:
\[2x(16 \text{ Rems/yr} \times 3.3 \text{ days} + 18 \text{ Rems/yr} \times 3.3 \text{ days} + 20 \text{ Rems/yr} \times 3.3 \text{ days})(1.65/20) = 2x(14.8) = 29.6 \text{ mRem}\]

Track C dosage:
\[2x(12 \text{ Rems/yr} \times 5 \text{ days} \times (1.65/20) + 14 \text{ Rems/yr} \times 5 \text{ days} \times (1.65/20)) = 2x(5.0 + 5.8) = 21.6 \text{ mRem}\]

Track D dosage:
\[2x(18 \text{ Rems/yr} \times 5 \text{ days} \times (1.65/20) + 20 \text{ Rems/yr} \times 5 \text{ days} \times (1.65/20)) = 2x(7.5 + 8.25) = 31.5 \text{ mRem}\]

4) For this 20-day excursion, Track D has the highest dosage and Track A has the lowest. The equivalent annual dosage for the lowest-dosage track is 19.8 milliRem x 365 days/10 days = 722 milliRem, which is about twice the annual dosage they would receive if they remained on Earth. For the highest-dosage trip, the annualized dosage is 1,149 milliRems which is about 3 times the dosage on Earth.
Mathematical Model of Magnetic Field Lines

Magnets have a north and a south pole. If you make this magnet small enough so that it looks like a point, all you will see are the looping lines of force mapped out by iron fillings or by using a compass.

Physicists call these patterns of lines, magnetic lines of force, and they can describe them mathematically!

**Problem 1** - Create a standard Cartesian 'X-Y' graph with all four quadrants shown. Select a domain [-5.0, + 5.0] and a range [-2.0, +2.0] and include tic marks every 0.1 along each axis.

**Problem 2** - Plot the following points in the order given and connect them with a smooth curve.

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**Problem 2** - Reflect the curve you drew into Quadrant 4, then reflect the curve in Quadrant 1 and 4 into Quadrants 2 and 3 to complete a single magnetic line of force for a magnet located at the origin!

**Problem 3** - Add two additional lines of force to your picture by re-scaling the figure you drew so that the X-Y coordinates are now A) 1/4 as large and B) 1.5 times larger.
**Problem 1** - Create a standard Cartesian 'X-Y' graph with all four quadrants shown. Select a domain [-5.0, +5.0] and a range [-2.0, +2.0].

**Problem 2** - Plot the following points in the order given and connect them with a smooth curve.

![Graph Image]

**Problem 2** - Reflect the curve you drew into Quadrant 4, and then reflect the curve in Quadrant 1 and 4 into Quadrants 2 and 3 to complete a single magnetic line of force for a magnet located at the origin!

![Graph Image]

**Problem 3** - Add two additional lines of force to your picture by re-scaling (dilating or contracting) the figure you drew so that the X-Y coordinates are now A) 1/4 as large (contraction) and B) 1.5 times larger (dilation).
Mathematically, every point in space near a magnet can be represented by a vector, \( \mathbf{B} \). Because the field exists in 3-dimensional space, it has three ‘components’. The equations for the coordinates of \( \mathbf{B} \) in 2-dimensions looks like this:

\[
B_r = -\frac{2M \sin \theta}{r^3} \quad B_\theta = \frac{M \cos \theta}{r^3}
\]

It is convenient to graph a magnetic field on a 2-dimensional piece of paper to show its shape. The lines that are drawn are called ‘magnetic field lines’, and if you placed a compass at a particular point on the field line, the direction of the line points to 'north' or 'south'.

The slope of the magnetic field at any point \((R, \theta)\) is defined by \( \frac{B_\theta}{B_r} \).

From calculus, in a polar coordinate system, the slope of a line is defined by \( \frac{rd\theta}{dr} \).

**Problem 1** – What is the differential equation that relates \( \frac{B_\theta}{B_r} \) to \( \frac{rd\theta}{dr} \)?

**Problem 2** – Integrate your answer to Problem 1 to find the polar coordinate equation of a magnetic field line.
Problem 1 -

\[ \frac{rd\theta}{dr} = \frac{M \cos \theta}{-2M \sin \theta} \quad \text{so} \quad \frac{rd\theta}{dr} = -\frac{\cos \theta}{2\sin \theta} \]

Problem 2 -

Rearrange the terms into two integrands:

\[ \frac{dr}{r} = -\frac{2 \sin \theta}{\cos \theta} \, d\theta \]

The integrals become

\[ \int \frac{dr}{r} = -2 \int \frac{\sin \theta}{\cos \theta} \, d\theta \]

These are both logarithmic integrals that yield the solution:

\[ \ln(r) + C = 2 \ln(\cos \theta) + C \]

\[ R_0 \] is the distance from the center of the magnetic field to the point where the field line crosses the equatorial plane of the magnet at \( \theta = 0 \). Each field line is specified by a unique crossing point distance. In other words, the constants of integration are specified by the condition that \( r = r_0 \) for \( \theta = 0 \), which then gives us the final form of the equation:

\[ r = r_0 \cos^2 \theta \]
Do Fast CMEs produce intense SPEs?

The sun produces two basic kinds of storms; coronal mass ejections (SOHO satellite: top left) and solar flares (SOHO satellite: bottom left). These are spectacular events in which billions of tons of matter are launched into space (CMEs) and vast amounts of electromagnetic energy are emitted (Flares). A third type of 'space weather storm' can also occur.

Solar Proton Events (SPEs) are invisible, but intense, showers of high-energy particles near Earth that can invade satellite electronics and cause serious problems, even malfunctions and failures. Some of the most powerful solar flares can emit these particles, which streak to Earth within an hour of the flare event. Other SPE events, however, do not seem to arrive at Earth until several days latter.

Here is a complete list of Solar Proton Events between 1976-2005: http://umbra.nascom.nasa.gov/SEP/
Here is a complete list of coronal mass ejections 1996 - 2006: http://cdaw.gsfc.nasa.gov/CME_list/

Between January 1, 1996 and June 30, 2006 there were 11,031 CMEs reported by the SOHO satellite. Of these, 1186 were halo events. Only half of the halo events are actually directed towards Earth. The other half are produced on the far side of the sun and directed away from Earth. During this same period of time, 90 SPE events were recorded by GOES satellite sensors orbiting Earth. On the next page, is a list of all the SPE events and Halo CMEs that corresponded to the SPE events. There were 65 SPEs that coincided with Halo CMEs. Also included is the calculated speed of the CME event.

From the information above, and the accompanying table, draw a Venn Diagram to represent the data, then answer the questions below.

Question 1:  A) What percentage of CMEs detected by the SOHO satellite were identified as Halo Events?  
B) What are the odds of seeing a halo Event?  
C) How many of these Halo events are directed towards Earth?

Question 2:  A) What fraction of SPEs were identified as coinciding with Halo Events?  
B) What are the odds that an SPE occurred with a Halo CME?  
C) What fraction of all halo events directed towards earth coincided with SPEs?

Question 3:  A) What percentage of SPEs coinciding with Halo CMEs are more intense than 900 PFUs?  
B) What are the odds that, if you detect a 'Halo-SPE', it will be more intense than 900 PFUs?

Question 4:  A) What percentage of Halo-SPEs have speeds greater than 1000 km/sec?  
B) What are the odds that a Halo-SPE in this sample has a speed of > 1000 km/sec?

Question 5: From what you have calculated as your answers above, what might you conclude about Solar Proton Events and CMEs? How would you use this information as a satellite owner and operator?
### Data Tables showing dates and properties of Halo CMEs and Solar Proton Events.

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<td>2360</td>
<td>Note: Solar Proton Event strengths are measured in the number of particles that pass through a square centimeter every second, and is given in units called Particle Flux Units or PFUs.</td>
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**Space Math**
Question 1:  
A) What percentage of CMEs detected by the SOHO satellite were identified as Halo Events?
\[ \frac{1186}{11031} = 11\% \]
B) What are the odds of seeing a halo Event?
\[ \frac{1}{0.11} = 1 \text{ chance in 9} \]
C) How many of these Halo events are directed towards Earth?
From the text, only half are directed to Earth so \[ \frac{1186}{2} = 593 \text{ Halos}. \]

Question 2:  
A) What fraction of SPEs were identified as coinciding with Halo Events?
\[ \frac{65 \text{ table entries}}{90 \text{ SPEs}} = 72\% \]
B) What are the odds that an SPE occurred with a Halo CME?
\[ \frac{1}{0.72} = 1 \text{ chance in 1.38 or about 2 chances in 3} \]
C) What fraction of all halo events directed towards Earth coincided with SPEs?
\[ \frac{65 \text{ in Table}}{(528+65) \text{ Halos}} = 11\% \]

Question 3:  
A) What percentage of SPEs coinciding with Halo CMEs are more intense than 900 PFUs?
From the table, there are 12 SPEs out of 65 in this list or \[ \frac{12}{65} = 18\% \]
B) What are the odds that, if you detect a 'Halo- SPE', it will be more intense than 900 PFUs?
\[ \frac{1}{0.18} = 1 \text{ chance in 5} \]

Question 4:  
A) What percentage of Halo-SPEs have speeds greater than 1000 km/sec?
There are 50 out of 65 or \[ \frac{50}{65} = 77\% \]
B) What are the odds that a Halo-SPE in this sample has a speed of > 1000 km/sec?
\[ \frac{1}{0.77} = 1 \text{ chance in 1.3 or 2 chances in 3} \]

Question 5: From what you have calculated as your answers above, what might you conclude about Solar Proton Events and CMEs? How would you use this information as a satellite owner and operator?

A reasonable student response is that Halo CMEs occur only 11% of the time, and of the ones directed towards Earth only 1 out of 9 coincide with SPEs. However, in terms of SPEs, virtually all of the SPEs coincide with Halo events (2 out of 3) and SPEs are especially common when the CME speed is above 1000 km/sec. As a satellite owner, I would be particularly concerned if scientists told me there was a halo CME headed towards Earth AND that it had a speed of over 1000 km/sec. Because the odds are now 2 chances out of 3 that an SPE might occur that could seriously affect my satellite. I would try to put my satellite in a safe condition to protect it from showers of high-energy particles that might damage it.

Space Math
On November 8, 2000, the sun ejected a blast of plasma called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite. On December 8, the Hubble Space Telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

**Problem 1** – How many days did the disturbance take to reach Earth and Saturn?

**Problem 2** – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour?

**Problem 3** – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour?

**Problem 4** – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn?

**Problem 5** – How long would the disturbance have taken to reach Jupiter as it passed Earth’s orbit?

**Problem 6** – On what date would you have expected to see aurora on Jupiter?
On November 8, 2000, the sun ejected a blast of plasma called a coronal mass ejection or CME. On November 12, the CME collided with Earth and produced a brilliant aurora detected from space by the IMAGE satellite. On December 8, the Hubble Space telescope detected an aurora on Saturn. During the period from November to December, 2000, Earth, Jupiter and Saturn were almost lined-up with each other. Assuming that the three planets were located on a straight line drawn from the sun to Saturn, with distances from the sun of 150 million, 778 million and 1.43 billion kilometers respectively, answer the questions below:

1 – How many days did the disturbance take to reach Earth and Saturn?  
Answer:  Earth = 4 days; Saturn = 30 days.

2 – What was the average speed of the CME in its journey between the Sun and Earth in millions of km per hour?  
Answer:  Sun to Earth = 150 million km. Time = 4 days x 24 hrs = 96 hrs so Speed = 150 million km/96hr = 1.5 million km/hr.

3 – What was the average speed of the CME in its journey between Earth and Saturn in millions of km per hour?  
Answer:  Distance = 1,430 – 150 = 1,280 million km. Time = 30 days x 24h = 720 hrs so Speed = 1,280 million km/720hrs = 1.8 million km/hr.

4 – Did the CME accelerate or decelerate as it traveled from the Sun to Saturn?  
Answer:  The CME accelerated from 1.5 million km/hr to 1.8 million km/hr.

5 – How long would the disturbance have taken to reach Jupiter as it passed Earth’s orbit?  
Answer:  Jupiter is located 778 million km from the Sun or (778 – 150 = ) 628 million km from Earth. Because the CME is accelerating, it is important that students realize that it is more accurate to use the average speed of the CME between Earth and Saturn which is (1.8 + 1.5)/2 = 1.7 million km/hr. The travel time to Jupiter is then 628/1.7 = 369 hours.

6 – On what date would you have expected to see aurora on Jupiter?  
Answer:  Add 369 hours (~ 15 days) to the date of arrival at Earth to get November 23. According to radio observations of Jupiter, the actual date of the aurora was November 20. Note: If we had used the Sun-Earth average speed of 1.5 million km/hr to get a travel time of 628/1.5 = 418 hours, the arrival date would have been November 29, which is 9 days later than the actual storm. This points out that the CME was accelerating after passing Earth, and its speed was between 1.5 and 1.8 million km/hr.

For more details about this interesting research, read the article by Renee Prange et al. “An Interplanetary Shock Traced by Planetary Auroral Storms from the Sun to Saturn” published in the journal Nature on November 4, 2004, vol. 432, p. 78. Also visit the Physics Web online article “Saturn gets a shock” at http://www.physicsweb.org/articles/news/8/11/2/1
Scientists have discovered a massive particle accelerator in the heart of one of the Van Allen radiation belts. Scientists knew that something in space accelerated particles in the radiation belts to more than 99 percent the speed of light but they didn't know what that something was. New results from NASA’s Van Allen Probes now show that the particles inside the belts are sped up by local kicks of energy, buffeting the particles to ever faster speeds, much like a perfectly timed push on a moving swing.

To see how this happens, imagine a ball bouncing down a long staircase as shown in the diagram to the left. Each step it falls, adds a small amount of energy to the ball so it bounces a bit higher each time. This is because gravity pulls on the ball and increases its kinetic energy after each step. As the kinetic energy increases, the ball’s speed and height increases.

The formula for Kinetic Energy is $K.E. = \frac{1}{2} m v^2$ where $m$ is the mass of the particle in kilograms, $v$ is its speed in meters/sec and $K.E.$ is measured in units of Joules.

**Problem 1** – A small ball has a mass of 0.1 kilograms and a kinetic energy of 5 Joules, what is its speed in meters/sec?

**Problem 2** – A 0.1 kilogram ball bounces down a long staircase that has 100 steps. If it gains 0.3 Joules after each step, how much kinetic energy will it have at the bottom of the staircase, and how fast will it be moving?

**Problem 3** – An electron in the Van Allen belts has a mass of $9.1 \times 10^{-31}$ kg. It starts out with a speed of 10,000 km/sec and reaches a speed of 150,000 km/sec after 12 hours. About how much kinetic energy does it gain every hour as it travels around the Van Allen Belts?
**Problem 1** – A small ball has a mass of 0.1 kilograms and a kinetic energy of 5 Joules, what is its speed in meters/sec?

**Answer:** \(5.0 = 0.5 \times 0.1 \times V^2\), so \(V^2 = 100\) and \(V = 10\) meters/sec.

**Problem 2** – A 0.1 kilogram ball bounces down a long staircase that has 100 steps. If it gains 0.3 Joules after each step, how much Kinetic Energy will it have at the bottom of the staircase, and how fast will it be moving?

**Answer:** \(0.3 \times 100 = 30\) Joules, then \(30 = \frac{1}{2} (0.1) V^2\), and \(V = 24\) meters/sec.

Note: The potential energy of the ball at the top of its bounce is given my \(E = m \times g \times h\), where \(g = 9.8\) m/sec\(^2\) and \(h\) is the height in meters. So for this ball, \(m = 0.1\) kg, \(E = 30\) Joules and so its maximum bounce height is \(h = 30\) meters.

**Problem 3** – An electron in the Van Allen belts has a mass of \(9.1 \times 10^{-31}\) kg. It starts out with a speed of 10,000 km/sec and reaches a speed of 150,000 km/sec after 12 hours. About how much kinetic energy does it gain every hour as it travels around the Van Allen Belts?

**Answer:** The initial kinetic energy of the electron is \(E = \frac{1}{2} (9.1 \times 10^{-31})(10,000,000)^2 = 4.6 \times 10^{-17}\) Joules.

The final kinetic energy is \(E = \frac{1}{2}(9.1 \times 10^{-31})(150,000,000)^2 = 1.0 \times 10^{-14}\) Joules,

so the electron gained \(1.0 \times 10^{-14} - 4.6 \times 10^{-17} = 1.0 \times 10^{-14}\) Joules of energy.

If this was done equally over 12 hours, then the energy gained per hour was \(1.0 \times 10^{-14} / 12 = 8.5 \times 10^{-16}\) **Joules each hour**.