For decades, geologists and astronomers have studied the contents of our solar system. They have compared surface features on planets and moons across the solar system, the orbits of asteroids and comets, and the chemical composition and ages for recovered meteorites. From all this effort, and with constant checking of data against mathematical models, scientists have created a timeline for the formation of our solar system.

Our solar system began as a collapsing cloud of gas and dust over 4.6 billion years ago. Over the next 600 million years, called by geologists the Hadean Era, the sun and the planets were formed, and Earth’s oceans were probably created by cometary impacts. Comets are very rich in water ice.

The fossil record on Earth shows that the first bacterial life forms emerged about 600 million years after the formation of the solar system. Geologists call this the Archaen Era – The era of ancient life.

**Problem 1** – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

**Problem 2** – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

**Problem 3** – About how many years ago do the oldest fossils date from on Earth?

**Problem 4** – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

**Problem 5** – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Space Math
**Problem 1** – If the Pre-Solar Nebula Era occurred 4.6 billion years ago, how long ago did the Rocky Planet Era end?

Answer: On the Timeline ‘0.0’ represents a time 4.6 billion years ago, so the Rocky Planet Era ended 100 million years after this or 4.5 billion years ago.

**Problem 2** – How many years from the current time did the Late Heavy Bombardment Era end in the inner solar system?

Answer: LHB ended 600 million years after Time ‘0.0’ or 4.6 billion – 600 million = 4.0 billion years ago.

**Problem 3** – About how many years ago do the oldest fossils date from on Earth?

Answer: 4.6 billion – 800 million = 3.8 billion years ago.

**Problem 4** – How many years were there between the Planetessimal Era and the end of the Rocky Planet Era?

Answer: On the timeline the difference is 100 million – 51 million = 49 million years.

**Problem 5** – If 80 objects the size of the Moon collided to form Earth during the time period in Problem 4, about how many years elapsed between these impact events?

Answer: The time interval is 49 million years so the average time between impacts would have been 49 million years/80 impacts = 612,000 years.
The Changing Pace of Global Warming

<table>
<thead>
<tr>
<th>Year</th>
<th>Temperature (degrees C)</th>
<th>Year</th>
<th>Temperature (degrees C)</th>
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<tr>
<td>1900</td>
<td>-0.20</td>
<td>1960</td>
<td>+0.05</td>
</tr>
<tr>
<td>1910</td>
<td>-0.35</td>
<td>1970</td>
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</tr>
<tr>
<td>1920</td>
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<td>1980</td>
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<td>2000</td>
<td>+0.45</td>
</tr>
<tr>
<td>1950</td>
<td>-0.05</td>
<td>2010</td>
<td>+0.63</td>
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A new study by researchers at the Goddard Institute for Space Studies determined that 2010 tied with 2005 as the warmest year on record, and was part of the warmest decade on record since the 1800s. The analysis used data from over 1000 stations around the world, satellite observations, and ocean and polar measurements to draw this conclusion.

The table above gives the average ‘temperature anomaly’ for each decade from 1900 to 2010. The Temperature Anomaly is a measure of how much the global temperature differed from the average global temperature between 1951 to 1980. For example, a +1.0 C temperature anomaly in 2000 means that the world was +1.0 degree Centigrade warmer in 2000 than the average global temperature between 1951-1980.

**Problem 1** - By how much has the average global temperature changed between 1900 and 2000?

**Problem 2** - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/La Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature anomaly change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.

**Problem 3** - From the data in the table, calculate the rate of change of the temperature anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900?

**Problem 4** - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Space Math
Problem 1 - By how much has the average global temperature changed between 1900 and 2000? Answer: In 1900 it was -0.20 C and in 2000 it was +0.45, so it has changed by +0.45 - (-0.20) = +0.65 C.

Problem 2 - The various bumps and wiggles in the data are caused by global weather changes such as the El Nino/El Nina cycle, and year-to-year changes in other factors that are not well understood by climate experts. By how much did the global temperature change between: A) 1900 and 1920? B) 1920 to 1950? C) 1950 and 1980? D) 1980 to 2010? Describe each interval in terms of whether it was cooling or warming.

Answer: 
1900 to 1920: -0.25C - (-0.20C) = -0.05 C a decrease (cooling) of 0.05 C
1920 to 1950: - 0.05C - (-0.25C) = +0.20 C an increase (warming) of 0.20 C
1950 to 1980: +0.20C - (-0.05C) = +0.25 C an increase (warming) of 0.25 C
1980 to 2010: +0.63C - (+0.20C) = +0.43 C an increase (warming) of 0.43 C

Problem 3 - From the data in the table, calculate the rate of change of the Temperature Anomaly per decade by dividing the temperature change by the number of decades (3) in each time period. Is the pace of global temperature change increasing, decreasing, or staying about the same since 1900? Answer:

1900 to 1920: -0.05 C/3 decades = -0.017 C per decade
1920 to 1950: +0.20 C/ 3 decades = +0.067 C per decade
1950 to 1980: +0.25 C/3 decades = +0.083 C per decade
1980 to 2010: +0.43 C/3 decades = +0.143 C per decade.

The pace of global temperature change is increasing in time. It is almost doubling every 10 years.

Problem 4 - Based on the trends in the data from 1960 to 2000, what do you predict that the temperature anomaly will be in 2050? Explain what this means in terms of average global temperature in 2050.

Answer: Students may graph the data in the table, then use a ruler to draw a line on the graph between 1960 and 2000, to extrapolate to the temperature anomaly in 2050. A linear equation, T = mx + b, that models this data is b = +0.05C m = (+0.45 - 0.05)/4 decades so m = +0.10 C/decade. Then T = +0.10x + 0.05. For 2050, which is 9 decades after 1960, x=9 so T = +0.1(9) + 0.05 = +0.95 C. So, the world will be, on average, about +1 C warmer in 2050 compared to its average temperature between 1950 and 1980. This assumes a linear change in T with time.

However from Problem 3 we see that the temperature anomaly change is accelerating. The 'second order' differences are +0.033, +0.033, +0.06. If we take the average change as (0.033+0.033+0.06)/3 = +0.042 we get a more accurate 'quadratic' expression: T = 0.042 x^2 +0.1x + 0.05. For the year 2050, this quadratic prediction suggests T = 0.042(9)^2 +0.1(9) + 0.05 so T = +1.32 C.

For more information about this research, see the NASA Press Release at http://www.nasa.gov/topics/earth/features/2010-warmest-year.html

Space Math
New measurements from the Herschel Space Observatory show that comet Hartley 2, which comes from the distant Kuiper Belt, contains water with the same chemical signature as Earth’s oceans. This remote region of the solar system, some 30 to 50 times as far away as the distance between Earth and the sun, is home to icy, rocky bodies including Pluto, other dwarf planets and innumerable comets.

Herschel detected the signature of vaporized water in this coma and, to the surprise of the scientists, Hartley 2 possessed half as much "heavy water" as other comets analyzed to date. In heavy water, one of the two normal hydrogen atoms has been replaced by the heavy hydrogen isotope known as deuterium. The amount of deuterium is similar to the abundance of this isotope in Earth’s ocean water.

The deposition of Earth's oceans probably occurred between 4.2 and 3.8 billion years ago. Suppose that the comet nuclei consisted of three major types, each spherical in shape and made of pure water-ice: Type 1 consisting of 2km in diameter bodies arriving once every 6 months, Type-2 consisting of 20 km diameter bodies arriving once every 600 years and Type-3 consisting of 200 km diameter bodies arriving every one million years.

Problem 1 - What are the volumes of the three types of comet nuclei in km$^3$?

Problem 2 - The volume of Earth’s liquid water oceans is $1.33 \times 10^9$ cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth’s oceans between 4.2 and 3.8 billion years ago?

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?
Space Observatory Provides Clues to Creation of Earth's Oceans


Problem 1 - What are the volumes of the three types of comet nuclei in km³?

Answer: \( V = \frac{4}{3} \pi R^3 \) so

Type 1: Volume = \( 4.2 \text{ km}^3 \)

Type 2: Volume = \( 4,200 \text{ km}^3 \)

Type 3: Volume = \( 4.2 \times 10^6 \text{ km}^3 \)

Problem 2 - The volume of Earth's liquid water oceans is \( 1.33 \times 10^9 \) cubic kilometers. If solid ice has 6 times the volume of liquid water, what is the volume of cometary ice that must be delivered to Earth's surface every year to create Earth's oceans between 4.2 and 3.8 billion years ago?

Answer: \( 1.33 \times 10^9 \text{ km}^3 \) of water requires

\[ 6 \times (1.33 \times 10^9 \text{ km}^3) = 8.0 \times 10^9 \text{ km}^3 \] of ice.

The average delivery rate would be about

\[ R = \frac{8.0 \times 10^9 \text{ km}^3}{400 \text{ million years}} \]

= \( 20 \text{ km}^3 \) of ice per year.

Problem 3 - What is the annual ice deposition rate for each of the three types of cometary bodies?

Type 1: \( R_1 = 4.2 \text{ km}^3 / 0.5 \text{ yrs} = 8.4 \text{ km}^3 / \text{yr} \)

Type 2: \( R_2 = 4200 \text{ km}^3 / 600 \text{ yrs} = 7.0 \text{ km}^3 / \text{yr} \)

Type 3: \( R_3 = 4.2 \times 10^6 \text{ km}^3 / 1000000 \text{ yrs} = 4.2 \text{ km}^3 / \text{yr} \)

Problem 4 - How many years would it take to form the oceans at the rate that the three types of cometary bodies are delivering ice to Earth's surface?

Answer: The total deposition rate is \( R_1 + R_2 + R_3 = 20 \text{ km}^3 / \text{yr} \), so it would take

\[ T = \frac{8.0 \times 10^9 \text{ km}^3}{(20 \text{ km}^3 / \text{yr})} = 400 \text{ million years}. \]
Earth's Declining Magnetism

Not only do the magnetic poles of Earth drift over time, but the entire strength of Earth's magnetic field increases and decreases. The strength of this field is commonly measured in terms of a quantity called VADM with the units of Ampere meter$^2$ (Am$^2$). For example, a 1 Ampere current circulating in a closed circle with an area of 1 meter$^2$ has a VADM of 1 Am$^2$.

The top figure shows the variations in Earth's VADM since end of the last Ice Age about 12,000 years ago. The gray area represents the range of measurement uncertainty. The current era is to the far-right of the plot.

The lower figure shows the most recent changes since 1800 using a slightly different unit scale.

Problem 1 - During the last 12,000 years, what has been the range in the VADM dipole strength as indicated by the black line?

Problem 2 - In about how many years from the present would you predict that the VADM will reach the lower end of its range in the last 12,000 years?

Problem 3 - Based on the slope of the line in the lower figure, what is the current rate-of-change of the magnetic field in terms of percent per century?

Problem 4 - If the decline continues at this pace, by what year will the strength of Earth's main dipole field be near-zero?

Problem 5 - Comparing the trends displayed by the upper plot with the lower graph, do you think the current rate-of-change exceptional?

Teacher note: VADM is an acronym for "Virtual Axial Dipole Moment"

Problem 1 - During the last 12,000 years, what has been the range in the VADM dipole strength as indicated by the black line? Answer: Estimating from the lowest and highest values reached by the black line we get a range from $7.0\times10^{22}$ Am$^2$.

Problem 2 - In about how many years from the present would you predict that the VADM will reach the lower end of its range in the last 12,000 years? Answer: The 'current era' are the years to the far-right in the top graph. The trend shows a slope of 10.5 to 8.5 from about 1000 years ago to 250 years ago. The slope is then $(8.5 - 10.5)/(250 - 1000) = 0.003$/year. The lower limit of the range is at 7.0 which is 1.5 below the last plotted point that occurred 250 years ago, so $-250 + 1.5/0.003 = 500$ years from now. Students may also solve this problem graphically with a ruler by extending the line for 'VADM=7.0' to where it meets up with the trend line from the last 1000 years of data.

Problem 3 - Based on the slope of the line in the lower figure, what is the current rate-of-change of the magnetic field in terms of percent per century? Answer: The decrease was from 8.6 to 8.0 over 170 years. This is a percentage change of $0.6/8.6 \times 100\% = 6.9\%$ over 1.7 centuries and so the rate of decrease has been $6.9\%/1.7C = 4\%$ per century.

Problem 4 - If the decline continues at this pace, by what year will the strength of Earth's main dipole field be near-zero? Answer: To go from 8.0 to 0.0 at a rate of 4%/100 years will take $100\% / 4\% = 25$ centuries or 2500 years. Adding this to the current year, 2009 gives us the year 4509 AD. Students answer will vary depending on the actual current year.

Problem 5 - Comparing the trends displayed by the upper plot with the lower graph, do you think the current rate-of-change exceptional? Answer: This question asks whether there have been other times in the last 12,000 years when the SLOPE of the data has been at least as steep as the current slope (i.e. rate of change). Some of the line slopes around 9000 years ago seem, at least for a limited time, to be as rapid as the current era. The period between 2000 and 3000 years ago also shows a similar rapid decline. The current era does seem unique in terms of the duration of this decline which has lasted for 1,500 years.