



One of the best ways to explore the effects of gravity on different bodies in the solar system is to calculate what your weight would be if you were standing on their surfaces!

Scientists use kilograms to indicate the mass of an object, and it is common for Americans to use pounds as a measure of weight. On Earth, the force that one kilogram of mass has on the bathroom scale is equal to 9.8 Newtons or a weight of 2.2 pounds.

The surface gravity of a planet or other body is what determines your weight by the simple formula  $W = Mg$  where  $W$  is the weight in Newtons,  $M$  is the mass in kilograms, and  $g$  is the acceleration of gravity at the surface in  $\text{meters/sec}^2$ . For example, on Earth,  $g = 9.8 \text{ m/sec}^2$ , and for a person with a mass of 64 kg, the weight will be  $W = 64 \times 9.8 = 627 \text{ Newtons}$ . Since 9.8 Newtons equals 2.2 pounds, this person weighs  $627 \times (2.2/9.8) = 140 \text{ pounds}$ .

**Problem 1** - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G ( $\text{m/sec}^2$ )	Weight (pounds)
Earth	Planet	9.8	110
Mercury	Planet	3.7	
Mars	Planet	3.7	
Io	Jupiter moon	1.8	
Moon	Earth moon	1.6	
Titan	Saturn moon	1.4	
Europa	Jupiter moon	1.3	
Pluto	Planet	0.58	
Charon	Pluto moon	0.28	
Vesta	Asteroid	0.22	
Enceladus	Saturn moon	0.11	
Miranda	Uranus moon	0.08	
Deimos	Mars moon	0.003	

**Problem 1** - Using proportional math, complete the following table to estimate the weight of a 110-pound (50 kg) person on the various bodies that have solid surfaces.

Object	Location	G (m/sec <sup>2</sup> )	Weight (pounds)
Earth	Planet	9.8	<b>110</b>
Mercury	Planet	3.7	<b>41.5</b>
Mars	Planet	3.7	<b>41.5</b>
Io	Jupiter moon	1.8	<b>20.2</b>
Moon	Earth moon	1.6	<b>18.0</b>
Titan	Saturn moon	1.4	<b>15.7</b>
Europa	Jupiter moon	1.3	<b>14.6</b>
Pluto	Planet	0.58	<b>6.5</b>
Charon	Pluto moon	0.28	<b>3.1</b>
Vesta	Asteroid	0.22	<b>2.5</b>
Enceladus	Saturn moon	0.11	<b>1.2</b>
Miranda	Uranus moon	0.08	<b>0.9</b>
Deimos	Mars moon	0.003	<b>0.03</b>

Note: For the Mars moon Deimos, which is a rocky body only 12 km (7.5 miles) in diameter, your weight would be 0.03 pounds or just ½ ounce! Astronauts that visit this moon would not 'land' on its surface but 'dock' with the moon the way that they do with visits to the International Space Station!



One of the simplest kinds of motion that were first studied carefully is that of falling. Unless supported, a body will fall to the ground under the influence of gravity. But the falling does not happen smoothly. Instead, the speed of the body increases in proportion to the elapsed time. This is called acceleration.

Near Earth's surface, the speed of a body increases 32 feet/sec (9.8 meters/sec) for every elapsed second. This is usually written as an acceleration of 32 feet/sec/sec or 32 feet/sec<sup>2</sup>. (also 9.8 meters/sec<sup>2</sup>) A simple formula gives you the speed of the object after an elapsed time of T seconds:

$$S = 32 T \quad \text{in feet/sec}$$

If instead of just dropping the object, you threw it downwards at a speed of 12 feet/sec (3 meters/sec) you could write the formula as:

$$S = 12 + 32T \quad \text{feet/sec}$$

In general you could also write this by replacing the selected speed of 12 feet/sec, with a fill-in speed of  $S_0$  to get

$$S = S_0 + 32T \quad \text{feet/sec.}$$

**Problem 1** – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

**Problem 2** – On Mars, the acceleration of gravity is only 12 feet/sec<sup>2</sup>. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

**Problem 3** – Two astronauts standing on the surface of two different objects in the solar system want to decide which object is the largest in mass. The first astronaut drops a hammer off a cliff that is exactly 5000 feet tall and measures the impact speed with a radar gun to get 100 feet/sec. It takes 8 seconds for the hammer to hit the bottom. The second astronaut drops an identical hammer off a cliff that is only 500 feet tall and also measures the impact speed at 150 feet/sec, but he accidentally gave the object a release speed of 5 feet/sec. It takes 29 seconds for the hammer to reach the ground. They can't re-do the experiments, but given this information what are the accelerations of gravity on the two bodies and which one has the highest mass?

**Problem 1** – On Earth, a ball is dropped from an airplane. If the initial speed was 0 feet/sec, how many seconds did it take to reach a speed of 130 miles per hour, which is called the Terminal Velocity? (130 mph = 192 feet/sec)

Answer:  $192 = 32T$ , so  $T = \mathbf{6 \text{ seconds}}$ .

**Problem 2** – On Mars, the acceleration of gravity is only 12 feet/sec<sup>2</sup>. A rock is dropped from the edge of the huge canyon called Valles Marineris and falls 20,000 to the canyon floor. If the impact speed was measured to be 700 feet/sec (480 mph) how long did it take to impact the canyon floor?

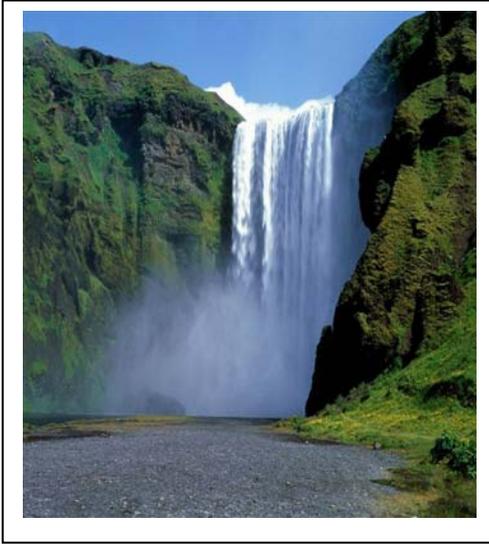
Answer:  $700 = 12T$ , so  $T = \mathbf{58 \text{ seconds}}$ .

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Answer: Astronaut 1.  $100 = A_1 \times T_1$   $T_1 = 8 \text{ seconds}$ , so  $A_1 = 100/8 = 12 \text{ feet/sec}^2$ .

Astronaut 2:  $150 = 5 + A_2 T_2$   $T_2 = 29 \text{ sec}$ , so  $A_2 = (150-5)/29 = 5 \text{ feet/sec}^2$

The acceleration measured by the second astronaut is much lower than for the first astronaut, so the first astronaut is standing on the more-massive objects. In fact, Astronaut 1 is on Mars and Astronaut 2 is on the moon!



Energy can be changed from one form to another. When you peddle a bike, your body uses up stored food energy (in calories) and converts this into kinetic energy of motion measured in joules. When you connect an electric motor to a battery, electrical energy stored in the battery is converted into rotational kinetic energy causing the motor shaft to turn.

A millstone paddle wheel uses the gravitational energy of falling water to turn the millstone wheel and perform work by grinding wheat, or even running simple machinery to cut wood in a lumber mill.

The energy in Joules of an object falling from a height near the surface of Earth can be calculated from

$$E = mgh$$

where  $m$  is the mass of the falling body in kilograms,  $g$  is the acceleration of gravity ( $9.8 \text{ meters/sec}^2$ ) and  $h$  is the distance of the fall in meters.

**Problem 1** – Nevada Falls in Yosemite Valley California has a height of 180 meters. Every second, 500 cubic feet of water goes over the edge of the falls. If 1 cubic foot of water has a mass of 28 kilograms, how much energy does this waterfall generate every day?

**Problem 2** – For a science fair project, a student wants to build a water hose powered hydroelectric plant to run a light bulb. Every second, the light bulb needs 60 Joules to operate at full brightness. If the water hose produces a steady flow of 0.2 kilograms every second, how high off the ground does the water hose have to be to turn a paddle wheel to generate the required electrical energy?

**Problem 3** – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If  $g = 0.1 \text{ meters/sec}^2$ , and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

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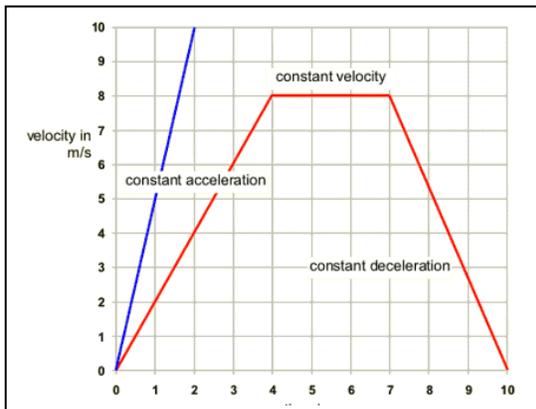
Answer: 1 day =  $24 \times 60 \times 60 = 86,400$  seconds, then the total mass is  $500 \times 28 \times 86400 = 1.2$  billion kilograms.  $E = 1.2$  billion kg  $\times 9.8 \times 180 =$  **2.1 trillion joules** per day. Note: since 1 watt = 1 Joule/second, this waterfall has a wattage of  $500 \times 28 \times 9.8 \times 180 = 25$  megawatts.

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Answer:  $60 = 0.2 \times 9.8 \times h$  so  $h =$  **30.6 meters** (or 90 feet!).

**Problem 3** – A geyser on Saturn's moon Enceladus ejects water from its caldera with an energy of 1 million Joules. If  $g = 0.1$  meters/sec<sup>2</sup>, and the mass moved is 2000 kilograms, how high can the geyser stream travel above the surface of Enceladus?

Answer:  $1,000,000 = 2000 \times 0.1 \times h$ , so  $h =$  **5,000 meters or 5 kilometers**.



In the same way that speed = distance divided by time, we can also look at acceleration as the change in speed over the time that the change occurred. Both of these quantities can be thought of as rates of change or 'slopes' on a graph like the one to the left.

When the final speed is, larger than the initial speed, the slope of the line is positive (upward) and we say that the object is accelerating. When the final speed is less than the initial speed, the slope is negative (downward) and we say that the object is decelerating.

**Problem 1** – A car leaves its parking spot and accelerates to 30 mph (13 m/s) in 10 seconds. It travels on a road at a constant speed of 30 mph for another 30 seconds and enters the onramp of a highway where it accelerates from 30 mph to a speed of 60 mph ( 26 m/s) after 6 seconds. It stays at this speed for another 2 minutes, then the car exits an off ramp, slowing to a speed of zero after 2 seconds. It then accelerates to 30 mph after 3 seconds as it merges into the local street traffic. After 1 minute at this speed the car approaches a gas station and decelerates to zero after 4 seconds. Draw a speed versus time graph in metric units that represents the car's journey.

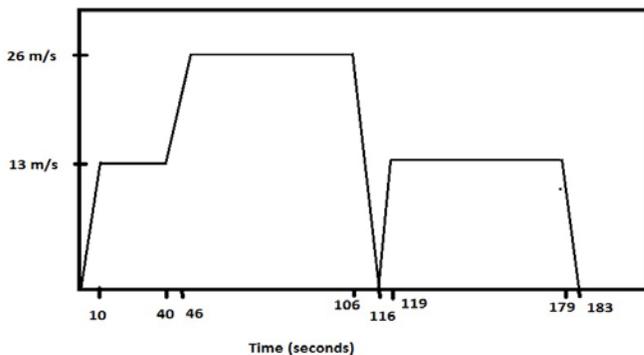
Time (Sec)	Speed (m/s)
0	0
1	3
2	4
3	7
4	10
5	12
6	15
7	16
8	20
9	23
10	25
11	27
12	31
13	34
14	36
15	39
16	43
17	46
18	49
19	53
20	56

**Problem 2** – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

### Saturn V Rocket Launch Speed vs Time

**Problem 3** – The table to the left shows the speed of the Saturn V rocket during a launch from the Kennedy Space Center on July 16, 1969 at 9:32:00 a.m. (EDT). What was the average acceleration of the Saturn V rocket during its first 20 seconds of constant thrust? How far did it travel during this time?

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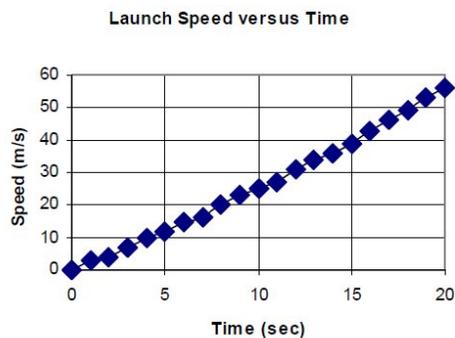


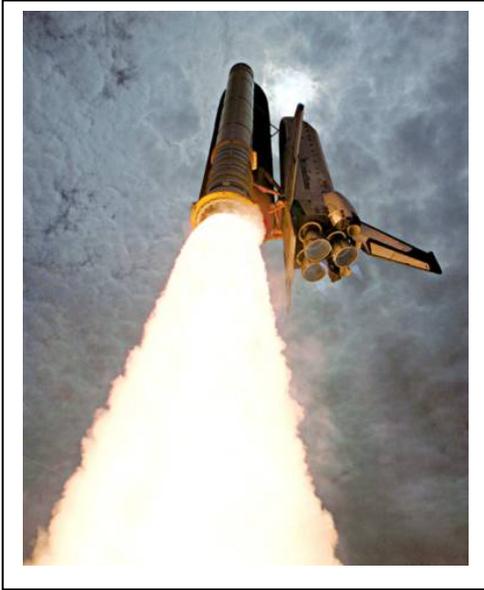
**Problem 2** – Explain how the area under a speed vs time graph gives the distance traveled, and use this to calculate the total distance traveled by the car using a combination of rectangles and triangles to calculate the total area.

Answer: **Area = speed x time = meters/sec x seconds = meters. So the area under the graph has the units of distance in meters.** This figure has five triangular areas as the car is accelerating and decelerating, and three rectangular areas as the car is traveling at constant speed. The sum of the triangular areas is  $A = \frac{1}{2} \text{ time interval} \times \text{speed} = \frac{1}{2} [ 10 \times 13 + 6 \times (26-13) + 10 \times 26 + 3 \times 13 + 4 \times 13 ] = 279$  meters. The sum of the rectangular areas is  $A = \text{time} \times \text{speed} = (30 \times 13 + (106-46) \times 26 + (179-119) \times 13) = 2730$  meters, so the total sum is  $2730 + 279 = 3009$  meters or **3 kilometers of total travel.**

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Answer: Acceleration =  $(56 \text{ m/s} - 0 \text{ m/s}) / 20 \text{ sec} = 2.8 \text{ m/s}^2$ . (Note this is about 2.8 times the acceleration of gravity at earth's surface so the astronauts would have felt about 2.8 times heavier). The triangular area is  $\frac{1}{2}(20 \text{ sec})(56 \text{ m/s}) = 560$  meters.





It would be nice if we could just jump real hard and we would suddenly be in space orbiting Earth. Fortunately it is not that easy as any Olympic High Jumper will tell you!

Because of the pull of gravity, every planet, asteroid or other object in the universe has its own speed limit. If you move slower than this speed you will stay on the body. If you move faster than this speed you will escape into space. Scientists call this the **escape speed** or **escape velocity**.

It's not just a number you guess at. It depends exactly on how much mass the planet or moon has, and how far from its center you are located. That means you can predict what this speed will be as you travel to other planets. That's very handy if you are an astronaut!

For Earth, the escape speed  $V$  in kilometers/second (km/s) at a distance  $R$  from Earth's center in kilometers, is given by

$$V = \frac{894}{\sqrt{R}}$$

**Problem 1** - What is the escape speed for a rocket located on Earth's surface where  $R = 6378$  km?

**Problem 2** – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

**Problem 3** – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

**Problem 4** – To enter a circular orbit at a distance of  $R$  from the center of Earth, you only need to reach a speed that is  $2^{1/2}$  smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

**Problem 1** - What is the escape speed for a rocket located on Earth's surface where  $R = 6378$  km?

Answer:  $V = 894/(6378)^{1/2} = 11.19 \text{ km/s}$

**Problem 2** – An Engineer proposes to launch a rocket from the top of Mt Everest (altitude 8.9 km) because its summit is farther from the center of Earth. Is this a good plan?

Answer:  $V = 894/(6378+8.9)^{1/2} = 11.18 \text{ km/s}$ . **This does not change the required escape speed by very much considering the effort to build such a launch facility at this location.**

**Problem 3** – A spacecraft is in a parking orbit around Earth at an altitude of 35,786 km. What is the escape speed from this location?

Answer:  $V = 894/(6378+35,786)^{1/2} = 4.35 \text{ km/s}$

**Problem 4** – To enter a circular orbit at a distance of  $R$  from the center of earth, you only need to reach a speed that is  $2^{1/2}$  smaller than the escape speed at that distance. What is the orbit speed of a satellite at an altitude of 35,786 km?

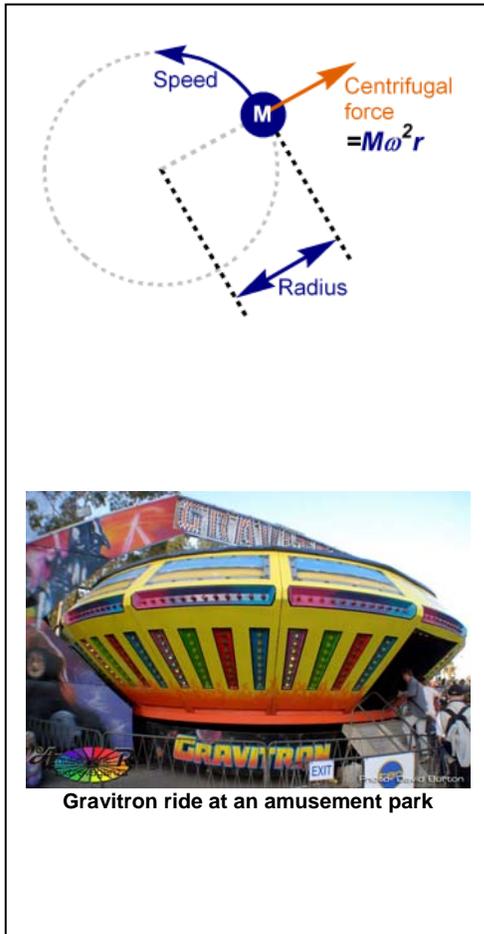
Answer:  $4.35 \text{ km/s} / (1.414) = 3.079 \text{ km/sec}$ .

Note: Satellites at an altitude of 35,800 orbit earth exactly once every day and are called geosynchronous satellites because they remain over the same geographic spot on Earth above the equator.

Circumference of the orbit =  $2 \pi R = 2 \times 3.141 \times (42164 \text{ km}) = 264,924 \text{ km}$ .

Speed = 3.079 km/sec so

$T = 264924/3.079 = 86,042$  seconds or 23.9 hours – Earths rotation period.



Most science fiction stories require some form of artificial gravity to keep spaceship passengers operating in a normal earth-like environment. As it turns out, weightlessness is a very bad condition for astronauts to work in on long-term flights. It causes bones to lose about 1% of their mass every month. A 30 year old traveler to Mars will come back with the bones of a 60 year old!

The only known way to create artificial gravity is to supply a force on an astronaut that produces the same acceleration as on the surface of earth: 9.8 meters/sec<sup>2</sup> or 32 feet/sec<sup>2</sup>. This can be done with bungee chords, body restraints or by spinning the spacecraft fast enough to create enough centrifugal acceleration.

Centrifugal acceleration is what you feel when your car 'takes a curve' and you are shoved sideways into the car door, or what you feel on a roller coaster as it travels a sharp curve in the tracks. Mathematically we can calculate centrifugal acceleration using the formula:

$$A = \frac{V^2}{R}$$

where  $V$  is in meters/sec,  $R$  is the radius of the turn in meters, and  $A$  is the acceleration in meters/sec<sup>2</sup>.

Let's see how this works for some common situations!

**Problem 1** - The Gravitron is a popular amusement park ride. The radius of the wall from the center is 7 meters, and at its maximum speed, it rotates at 24 rotations per minute. What is the speed of rotation in meters/sec, and what is the acceleration that you feel?

**Problem 2** - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity ( $1\text{ g} = 9.8\text{ meters/sec}^2$ )?

**Problem 3** – Another way to create a 1-G force is for the rocket to continuously fire during its journey so that its speed increases by 9.8 meters/sec every second. Suppose that NASA could design such a rocket system for a 60 million km trip to Mars. The idea is that it will accelerate during the first half of its trip, then make a 180-degree flip and decelerate for the remainder of the trip. If

$$\begin{aligned} \text{distance} &= 1/2aT^2 \\ \text{speed} &= aT, \end{aligned}$$

where  $T$  is in seconds,  $a$  is in meters/sec<sup>2</sup>, and distance is in meters, how long will the trip take in hours, and what will be the maximum speed of the spacecraft at the turn-around point?

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Answer: For circular motion, the distance traveled is the circumference of the circle  $C = 2 \pi (7 \text{ meters}) = 44 \text{ meters}$ . At 24 rpm, it makes one revolution every  $60 \text{ seconds}/24 = 2.5 \text{ seconds}$ , so the rotation speed is  $44 \text{ meters}/2.5 \text{ sec} = 17.6 \text{ meters/sec}$ .

The acceleration is then  $A = (17.6)^2/7 = \mathbf{44.3 \text{ meters/sec}^2}$ . Since 1 earth gravity = 9.8 meters/sec<sup>2</sup>, the 'G-Force' you feel is  $44.3/9.8 = 4.5 \text{ Gs}$ . That means that you feel 4.5 times heavier than you would be just standing in line outside!

**Problem 2** - On a journey to Mars, one design is to have a section of the spacecraft rotate to simulate gravity. If the radius of this section is 30 meters, how many RPMs must it rotate to simulate one Earth gravity ( $1 \text{ g} = 9.8 \text{ meters/sec}^2$ )?

Answer: The circumference is  $C = 2 \pi (30) = 188 \text{ meters}$ . 1 RPM is equal to rotating one full circumference every minute, for a speed of  $188/60 \text{ sec} = 3.1 \text{ meters/second}$ . So  $V = 3.1 \text{ meters/sec} \times \text{RPM}$ . Then  $A = (3.1\text{RPM})^2 / 188 = 0.05 \times \text{RPM}^2$ . We need  $9.8 = 0.05\text{RPM}^2$  so **RPM = 14**.

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Answer: The turn-around point happens at the midway point 30 million km from Earth, so  $d = 3.0 \times 10^{10} \text{ meters}$ ,  $a = 9.8 \text{ meters/sec}^2$ , and so solving for T,

$$3.0 \times 10^{10} = \frac{1}{2} (9.8) T^2 \quad \text{so}$$

$$T = 78,246 \text{ seconds or}$$

$$\text{for a full trip} = 2 \times 78246 \text{ seconds} = \mathbf{43 \text{ hours!}}$$

$$\text{Speed} = 9.8 \times 78246 = 767,000 \text{ meters/sec} = \mathbf{767 \text{ kilometers/sec.}}$$



Bungee jumping has become a popular but dangerous sport. It also shows how the acceleration of gravity is connected to the total distance traveled during the fall. The distance traveled is given by the formula

$$D = \frac{1}{2} g T^2$$

Where  $g$  is the acceleration of gravity in meters/sec<sup>2</sup>,  $D$  is the distance in meters, and  $T$  is the elapsed time in seconds. For locations near the surface of Earth,  $g = 9.8$  meters/sec<sup>2</sup> (32 feet/sec<sup>2</sup>)

**Problem 1** - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

**Problem 2** – How fast would the Daredevil be traveling at the moment of impact if  $S = 32T$ ?

**Problem 3** – Once he reaches 130 mph (190 feet/sec), called the terminal velocity, his free-fall speed stops increasing. How soon after he jumps does he reach terminal velocity, and how far has he fallen from the plane?

**Problem 4** - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

**Problem 5** – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec<sup>2</sup>, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

**Problem 1** - A confused Daredevil jumps from a plane at an altitude of 15,000 feet. How long does it take for the Daredevil to land if there is no air friction to slow him down?

Answer:  $15,000 = \frac{1}{2} (32) T^2$ , so  $T^2 = 937$  and so  $T = \mathbf{31 \text{ seconds}}$ .

**Problem 2** – How fast would the Daredevil be traveling at the moment of impact if  $S = 32T$ ?

Answer:  $S = 32 \times 31 = 992$  feet/second or 676 miles/hour!

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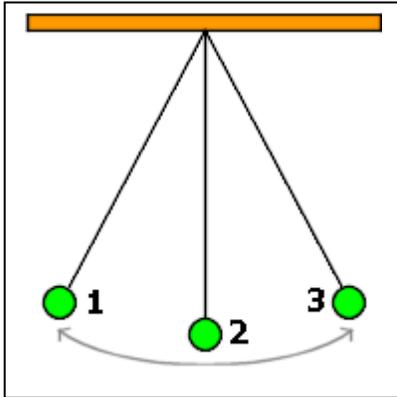
Answer:  $190 = 32 \times T$  so  $T = \mathbf{6 \text{ seconds}}$ . He has fallen  $d = \frac{1}{2} (32)(6)^2 = \mathbf{576 \text{ feet}}$ .

**Problem 4** - In 2012, Felix Baumgartner jumped from a high-altitude balloon at an altitude of 24 miles (127,000 feet), landing safely on the ground after 4 minutes and 19 seconds. With little atmosphere friction, he reached a maximum speed of 844 mph (1240 feet/sec). How long after he jumped did he reach this speed, and how high above the ground was he at that time?

Answer:  $1240 = 32 T$  so  $T = \mathbf{39 \text{ seconds}}$ .  
 $D = \frac{1}{2} (32) (39)^2 = 24,336$  feet,  
 so  $127,000 - 24336 = \mathbf{102,700 \text{ feet from the ground}}$ .

**Problem 5** – On Mars, the Valles Marineris canyon is 23,000 feet deep. If the acceleration of gravity is 12 feet/sec<sup>2</sup>, how long would it take a rock to fall into the canyon and how fast is it traveling when it hits bottom?

Answer:  $23,000 = \frac{1}{2}(12)T^2$  so  $T = \mathbf{62 \text{ seconds}}$ .  
 Speed =  $12 \times 62 = \mathbf{744 \text{ feet/sec or } 507 \text{ mph}}$ .



A pendulum is a very simple toy, but you can actually use it to measure gravity! The beat of a pendulum called its period,  $P$ , depends on the length of the pendulum,  $L$ , and the acceleration of gravity,  $g$ , according to:

$$P = 2\pi \sqrt{\frac{L}{g}}$$

If you measure  $P$  in seconds, and know the length of the pendulum,  $L$ , in meters, you can figure out how strong the acceleration of gravity is,  $g$ , in meters/sec<sup>2</sup>. Let's see how this works for explorers working on different planets and moons in the solar system!

**Problem 1** – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is 3.8 m/sec<sup>2</sup>, how long will the pendulum have to be in meters?

**Problem 2** - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

**Problem 3** – On Earth, prospectors are looking for a deposit of iron ore beneath the ground. They decide to use the acceleration of gravity to find where the iron is located because the additional iron mass should change the acceleration of gravity. They use a carefully-made pendulum with a length of 2.00000 meters and measure the period of the swing as they walk around the area where they think the deposit is located. To the nearest millionth of a second, how much will the period change if the acceleration of gravity between two spots changes from 9.80000 meters/sec<sup>2</sup> to 9.80010 meters/sec<sup>2</sup>? (use  $\pi = 3.14159$ )

**Problem 1** – A mars colonist wants to make a pendulum that has a beat of 4 seconds. If the acceleration of gravity on mars is  $3.8 \text{ m/sec}^2$ , how long will the pendulum have to be in meters?

Answer:  $4 = 2\pi(L/3.8)^{1/2}$  then  $L = (16/4\pi^2)*3.8 = \mathbf{1.54 \text{ meters}}$ .

**Problem 2** - A pendulum clock on the moon has a length of 2 meters, and its period is carefully measured to be 7.00 seconds. What is the acceleration of gravity on the moon?

Answer:  $7.00 = 2\pi(2/g)^{1/2}$  so  $g = 8\pi^2/49$  so  $g = \mathbf{1.61 \text{ meters/sec}^2}$ .

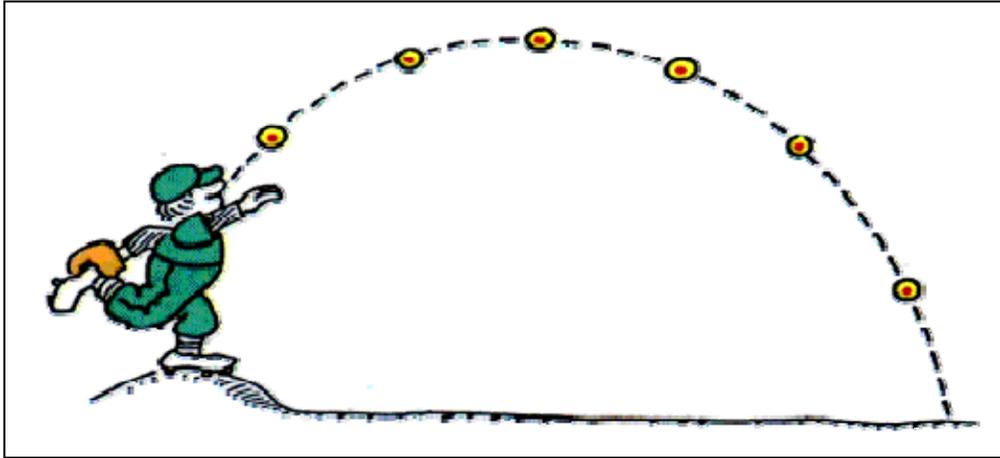
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Answer:  $P = 2\pi(2.00000/9.80000)^{1/2} = \mathbf{2.838451 \text{ seconds}}$ .  
 And  $P2 = 2\pi(2.00000/9.80010)^{1/2} = \mathbf{2.838437 \text{ seconds}}$

So the difference in period will be **0.000014 seconds or 14 microseconds**.

$$T = 2\pi\sqrt{\frac{L}{g}} \left( 1 + \frac{1}{16}\theta_0^2 + \frac{11}{3072}\theta_0^4 + \dots \right)$$

Where theta is the start angle from verticle.



The horizontal motion of a rock (projectile) is given by the formula:

$$X = V_h T$$

Independently, the vertical motion is given by the formula

$$Y = H_0 + V_v T - \frac{1}{2} g T^2$$

The speed of the projectile has been described in terms of its vertical ( $V_v$ ) and horizontal ( $V_h$ ) speeds so that the total speed is given by the Pythagorean Theorem  $S = (V_h^2 + V_v^2)^{1/2}$ .

**Problem 1** – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land? ( $g = 32 \text{ feet/sec}^2$ )

**Problem 2** – On Mars ( $g = 12 \text{ feet/sec}^2$ ) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

**Problem 1** – A rock is tossed horizontally from the top of the Eiffel Tower at a speed of 60 mph (40 feet/sec). The Eiffel Tower stands 1,063 feet above the street. How far from the centerline of the tower does the rock land?

Answer:  $H_0 = 1063$  feet,  $V_h = 40$  feet/sec,  $V_v = 0.0$ ,  $g = 32$  feet/sec<sup>2</sup>. The vertical equation give us the time to reach the ground ( $y=0$ ):  $0 = 1063 - 16 T^2$ , so  $1063/16 = T^2$  and  $T = 8.1$  seconds. From the horizontal motion, it travels  $d = 40 \times 8.1 = \mathbf{324}$  feet.

**Problem 2** – On Mars ( $g = 12$  feet/sec<sup>2</sup>) an astronaut throws a rock up in the air so that its vertical speed is 30 feet/sec and its horizontal speed is 10 feet/sec. The rock starts at a shoulder height of 5 feet. How high does the rock travel, and how far from the astronaut does it finally reach the ground?

Answer: The two equations are  $X = 10 T$  and  $Y = 5.0 + 30T - 6T^2$

Write Y in terms of X:  $Y = 5.0 + 30(X/10) - 6 (X/10)^2$  so  $Y = \mathbf{5.0 + 3.0X - 0.06X^2}$

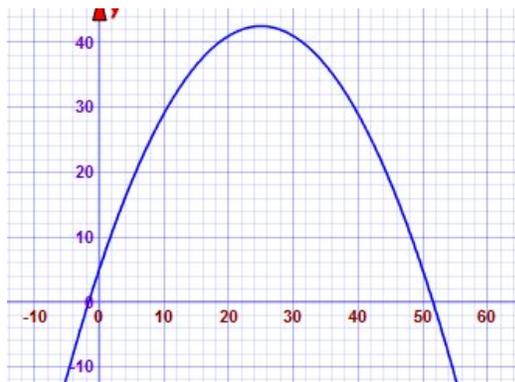
Solve for the roots of  $Y(X) = -0.06X^2 + 3.0X + 5.0$  with coefficients  $a = -0.06$ ,  $b=3.0$  and  $c = 5.0$ , to get the ground intercept points using the Quadratic Formula:

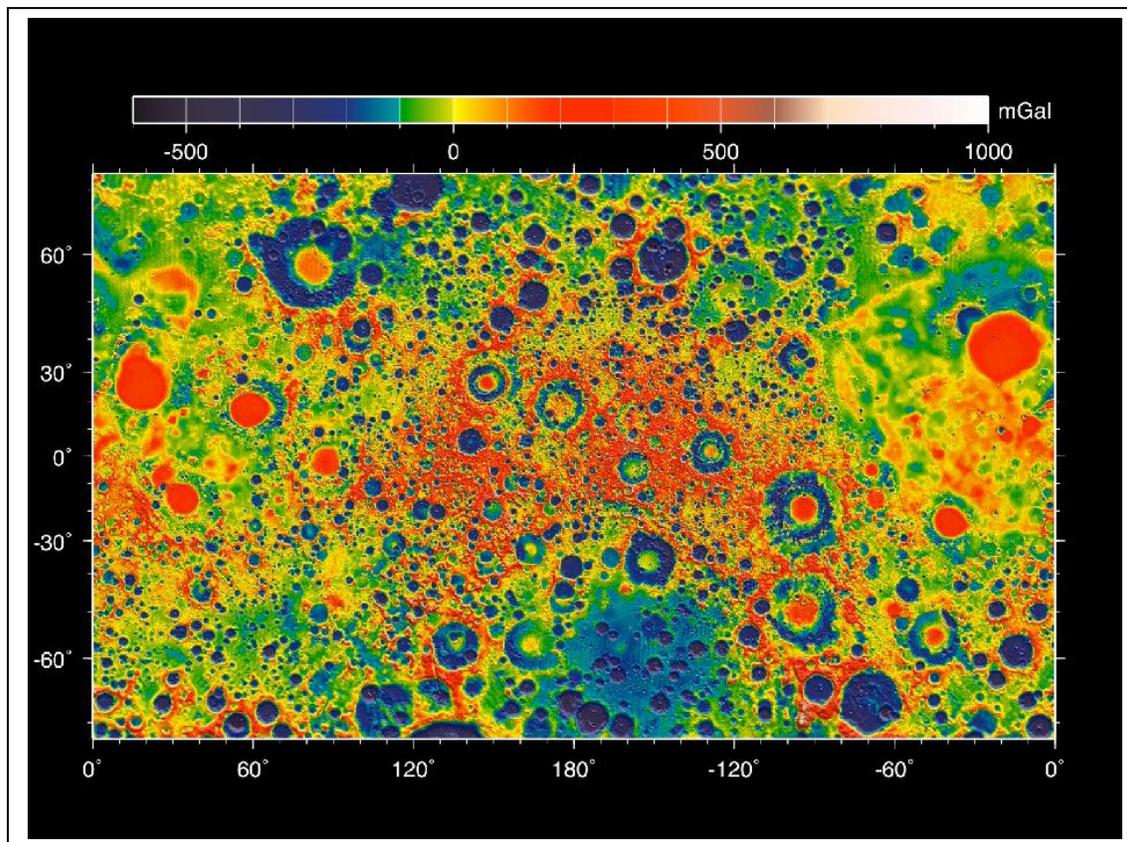
$$X = \frac{-3.0 \pm \sqrt{9 - 4(-0.06)(5.0)}}{2(-0.06)} \text{ so}$$

$$X = \frac{-3 + 3.19}{-0.12} = -1.6 \text{ feet, and the second root is}$$

$$X = \frac{-3 - 3.19}{-0.12} = +51.6 \text{ feet. see graph below.}$$

The peak of the parabola is  $\frac{1}{2}$  way between the x-intercepts at  $x = (51.6 - 1.6)/2 = +25.0$  feet  
 And since  $X = 10T$ , we have  $25 = 10T$  so  $T = 2.5$  seconds. From  $Y(T)$ , the altitude of the peak is  $Y = 5.0 + 30(2.5) - 6(2.5)^2 = \mathbf{42.5}$  feet. From the x-intercept, it reaches a distance of  $\mathbf{51.6}$  feet from the astronaut.





**During 2012**, NASA's twin Grail satellites orbited the moon at altitudes of only 30 km. As they traveled, minute changes in their speeds tracked from Earth revealed changes in the gravitational field of the moon. These changes could be mapped, and revealed density changes in the lunar surface below them. In this way, scientists could look hundreds of kilometers beneath the lunar surface and explore how the surface was formed billions of years ago! On Earth, the acceleration of gravity is  $9,807 \text{ cm/sec}^2$ . The normal acceleration of gravity on the average lunar surface is  $1620 \text{ cm/sec}^2$ , but in the blue regions of the map this is as low as  $1520 \text{ cm/sec}^2$ , and in the red regions it is as high as  $1920 \text{ cm/sec}^2$ . A pendulum clock has a swinging period,  $T$  in seconds, given by the formula  $T = 2\pi\sqrt{\frac{L}{g}}$  where  $L$  is the length of the pendulum in centimeters, and  $g$  is the acceleration of gravity in  $\text{cm/sec}^2$ .

**Problem 1** - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length  $L = 100 \text{ cm}$ . What is the swing period? (use  $\pi = 3.141$ )

**Problem 2** - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period? (use  $\pi = 3.141$ )

**Problem 3** - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

**Problem 4** - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

**Problem 1** - A lunar colony in a lunar 'blue' area has a Blue Clock with a pendulum length  $L = 100$  cm. What is the swing period?

Answer:  $T = 2 (3.141) (100/1520)^{1/2} = \mathbf{1.61 \text{ seconds}}$ .

**Problem 2** - A lunar colony in a lunar 'red' area has an identical Red Clock. What is the swing period?

Answer;  $T = 2 (3.141) (100/1920)^{1/2} = \mathbf{1.43 \text{ seconds}}$ .

**Problem 3** - After how many swings on the Blue Clock will the clocks differ in time by 1 hour?

Answer: Each swing on the slower Blue Clock pendulum is behind the faster Red Clock by  $1.61 - 1.43 = 0.18$  seconds. We want this difference to be 3600 seconds in 1 hour, which will take  $N = 3600/0.18 = \mathbf{20,000 \text{ swings}}$  on the Blue Clock.

**Problem 4** - If both clocks were synchronized to 1:00:00 am local time, what will the time on the Blue Clock and the Red Clock be when the two colony clocks are off by 1 hour relative to each other?

Answer: On the Blue Clock, 20,000 swings have to pass, each taking 1.61 seconds for a total time of 32,200 seconds or 8 hours, 56 minutes, 40 seconds. So the time on the Blue Clock will read **09:56:40 am local time**.

On the Red Clock, because after 20,000 swings it is exactly 1 hour behind the Blue Clock, its time will read 08:56:40 am local time. Another 'long way' to see this is that we still need 20,000 swings to add up to a 1 hour time difference, but on the Red Clock each swing is only 1.43 seconds long and so this takes 28,600 seconds or 7 hours, 56 minutes, 40 seconds. The time on the Red Clock will be **08:56:40 am local time**.

**This is why colonists will NOT be using pendulum clocks on the moon!!**

**Note:** Devices that act like pendulum clocks were once used by prospectors on Earth to search for oil and other valuable materials below ground before the advent of more accurate magnetometer-based technology. Minute changes in the pendulum period indicate changes in the density of rock below ground and these can be used to identify high-gravity, density regions (like iron ore) or low-gravity regions (like caverns). Another way to measure minute gravity changes is by the shape of a satellite orbit, or by the subtle changes in speed between two satellites on the same orbit. Lunar scientists used this orbit method with the two Grail spacecraft only 200 kilometers apart.