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# Phasing Estimation Relationships

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- Los Angeles ■ Washington, D.C. ■ Boston ■ Chantilly ■ Huntsville ■ Dayton ■ Santa Barbara
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# Agenda

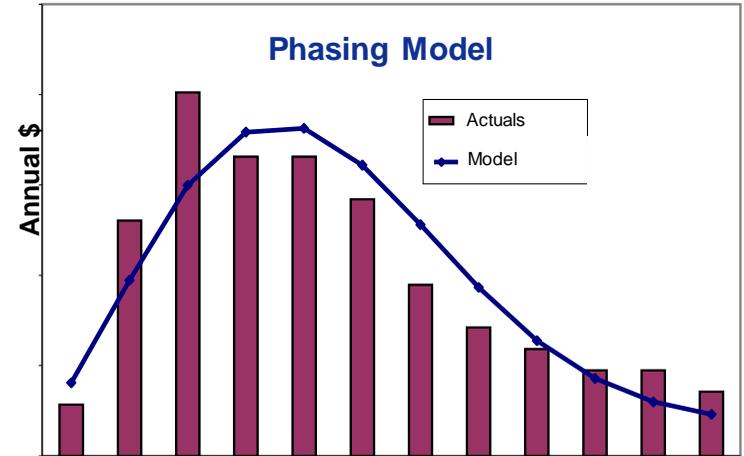
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- Introduction
- Data Analysis
- Regression Results
- Summary and Further Research



# Phasing Estimating Relationships (PERs)

- Research funded by NASA/OE/CAD
- Estimate annual funding for a mission
  - Given a cost and schedule estimate
  - Based on historical data ... not “optimal”



- Scope of PERs presented today:
  - Time: System Requirements Review (SRR) to Launch
  - Content:
    - Option 1: Total project excluding launch
    - Option 2: Spacecraft and instruments only



## Applications:

- Support, assess, and/or defend budgets
- Starting point for analyzing cost & schedule ramifications

## Keys to useful PERs:

- ✓ **Clearly traceable to source data**
  - ✓ Transparent and verifiable
  - ✓ Users can draw directly from analogy missions
- ✓ **Logical drivers and functional form**
  - ✓ Front/back-loading makes sense
  - ✓ Theoretical and empirical basis
- ✓ **Differentiates between expenditures and obligation authority**
- ✓ **Useful accuracy metrics**
  - ✓ Indexed to program events
  - ✓ Standard error vs. time



# Functional Forms for Phasing

## ■ Rayleigh Curve

$$E(t) = 1 - e^{-t^2 / 2\sigma^2}$$



### John William Strutt, third Baron Rayleigh

- Discovered Argon
- Won Nobel Prize for Physics, 1904
- Didn't care about budget phasing

## ■ Norden-Rayleigh Curve

$$E(t) = d \left[ 1 - e^{-\alpha t^2} \right]$$



### Peter Norden, IBM, 1960s

- Cared about phasing:
- Studied R&D projects
- Manpower build-up and phase-out follow distribution that happens to be Rayleigh's<sup>1</sup>

## ■ Weibull curve

$$E(t) = d \left[ 1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^\beta} \right]$$



### Ernst Hjalmar Waloddi Weibull (18 June 1887-12 October 1979)

- Swedish engineer, scientist, and mathematician.
- Proposed distribution as statistical model for life data (fatigue, reliability, etc.)
- Did not care about budget phasing



# Weibull: Better Empirical Results

## ■ Porter (2001):

- Used Weibull model to predict final costs when funding is curtailed
- Claimed greater accuracy than Rayleigh due to additional parameters

## ■ Unger (2001):

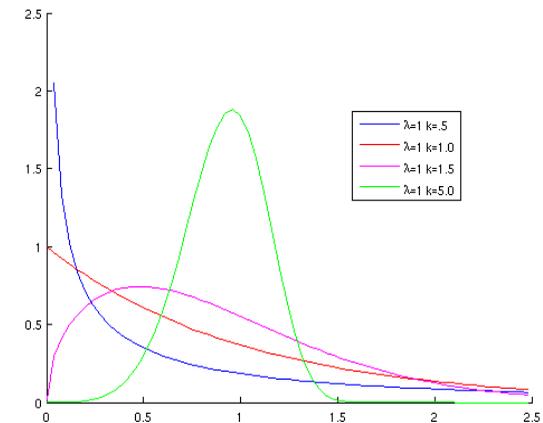
- Showed that cost and schedule growth are correlated with poor initial phasing
- Showed that Weibull distribution was a better fit to 37 DoD programs

## ■ Brown (2002)

- Use program characteristics to predict Weibull parameters (128 DoD programs)
- Showed that Rayleigh curve was too inflexible

## ■ Burgess (2006):

- Compared Beta, Rayleigh, and Weibull for 26 space programs
- Weibull performed better in every metric
- Basis for DoD Space System Phasing Model



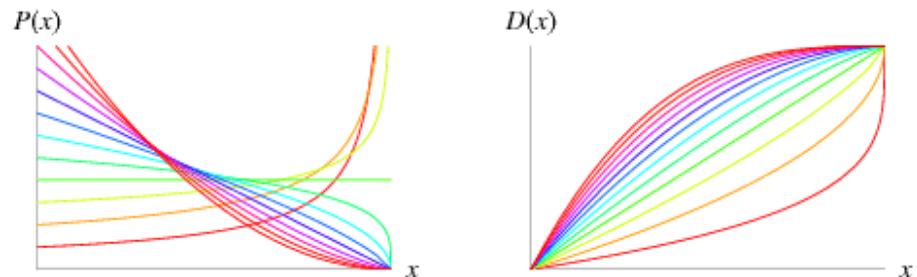
**Weibull Distribution Has Theoretical and Empirical Bases**



# Beta Distribution

- Beta is from 9<sup>th</sup> Century BC: 1<sup>st</sup> consonant in Greek alphabet
- Beta distribution useful for Bayesian statistics (conditional)
- Also works for phasing!
  - Popular empirical curve for fitting manpower
  - Two parameters, BETADIST in Excel<sup>©</sup>
  - Very flexible, but no theoretical basis

$$\frac{dW(t)}{dt} = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} t^{a-1} \cdot (1-t)^{b-1} \quad 0 < t < 1$$



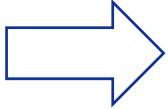
Weisstein, Eric W. "Beta Distribution." From MathWorld--A Wolfram Web Resource.  
<http://mathworld.wolfram.com/BetaDistribution.html>



# Agenda

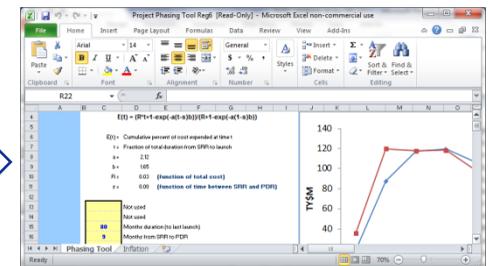
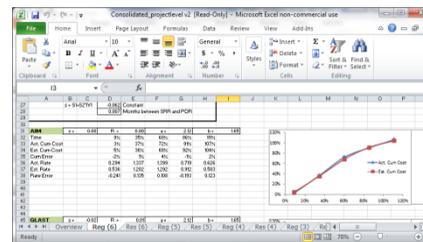
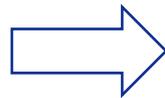
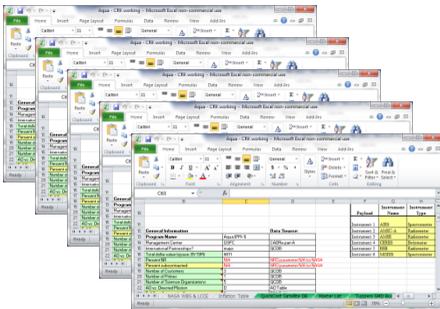
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# Data Analysis

- CAD prioritized 99 potential projects → 37 used in final PERs
- Normalization workbook created for each project
  - All sources identified and/or linked
  - Cost and schedule normalized on 0.0 to 1.0 scales
- First tab in each workbook brought into regression model



## Project Norm. Workbooks

- Traceable to CADRe and other data sources

## Consolidated Workbook

- All data needed for regression
- May be useful for end-users

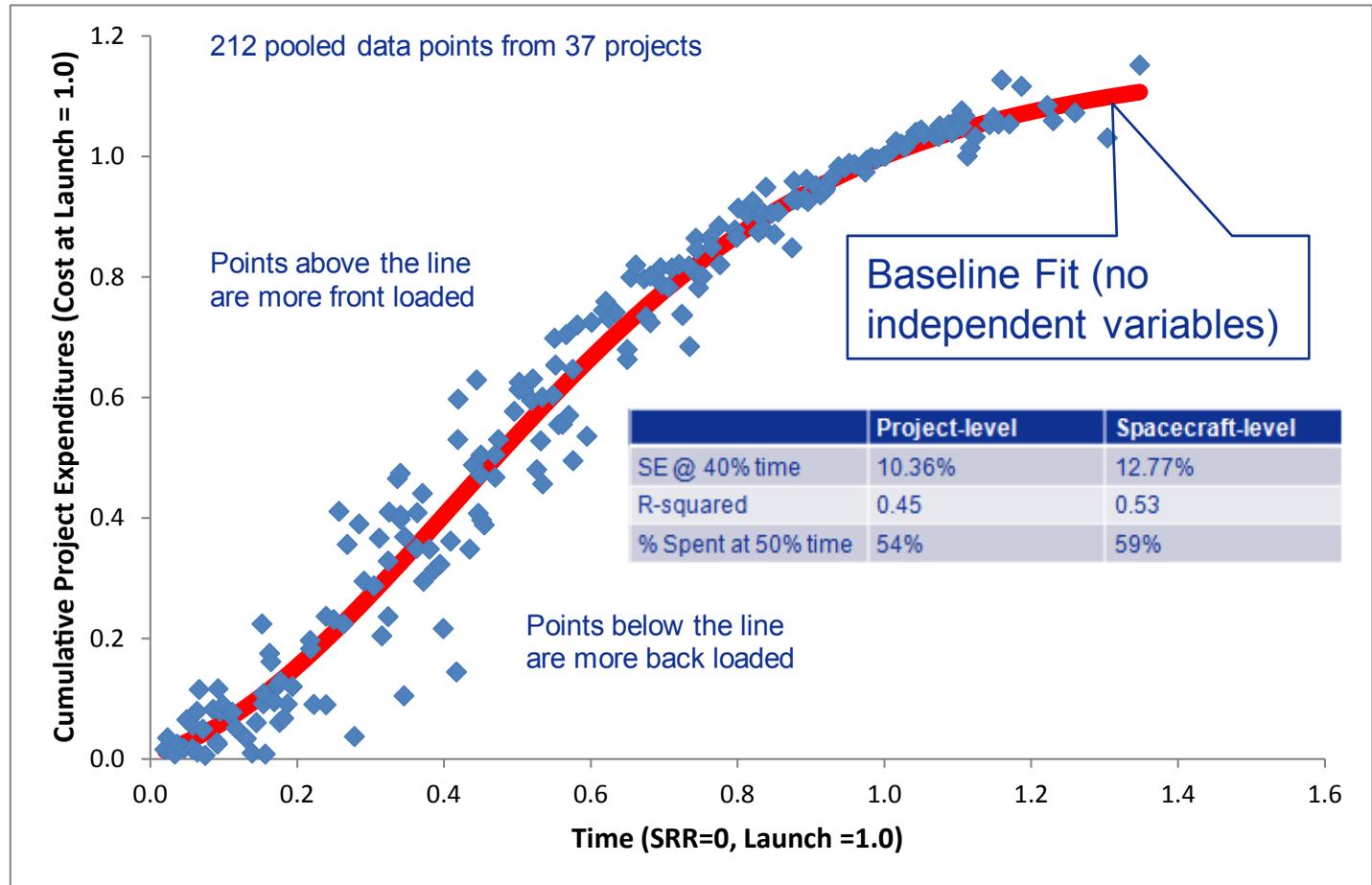
## Phasing toolkit

- Implements the selected model
- Converts to NOA



# Final Normalized Dataset (Project-Level)

AIM	MAP
GLAST/Fermi	Mars Odyssey
IBEX	MER
Contour	MGS
Stardust	TIMED
Dawn	Mars Pathfinder
Genesis	Kepler
THEMIS	OCO
CLOUDSAT	MSL
GALEX	Juno
GRACE	NuSTAR
LRO	SDO
MRO	COBE
New Horizons	ICESat
Phoenix	TRMM
SIRTF	NEAR
STEREO	Aqua (PM-1)
GRAIL	Aura (Chem-1)
Glory	



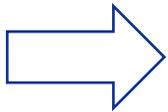
Adding Project-specific Independent Variables Will Explain Front/Back-loading behavior



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# What We Expect to See

■ Weibull has two parameters,  $\alpha$  and  $\beta$

- Plus a time shift if needed,  $\gamma$

$$E(t) \cong 1 - e^{-\alpha(t-\gamma)^\beta}$$

	$\alpha$	$\beta$	$\gamma$
Affects	Time of peak expenditures	Ramp-up rate	Shifts curve left or right
Possible Drivers	<ul style="list-style-type: none"> <li>• Mission class</li> <li>• AO vs. Directed</li> <li>• Total cost</li> <li>• Total duration</li> <li>• GFE payload</li> <li>• Competitive</li> <li>• Instrument timelines</li> <li>• Percent new</li> </ul>	<ul style="list-style-type: none"> <li>• Number of customers, primes, science organizations</li> <li>• Total cost</li> <li>• % Time from SRR to PDR</li> </ul>	<ul style="list-style-type: none"> <li>• % Time from SRR to PDR</li> </ul>

■ We add a constant-rate term

- Reflects “standing army”
- Usually higher on large, long projects

$$E(t) \cong Rt + 1 - e^{-\alpha(t-\gamma)^\beta}$$



# Project-level Phasing Estimating Relationship (PER)

$$E(t) = d \left[ Rt + 1 - e^{-\alpha(t-\gamma)^\beta} \right]$$

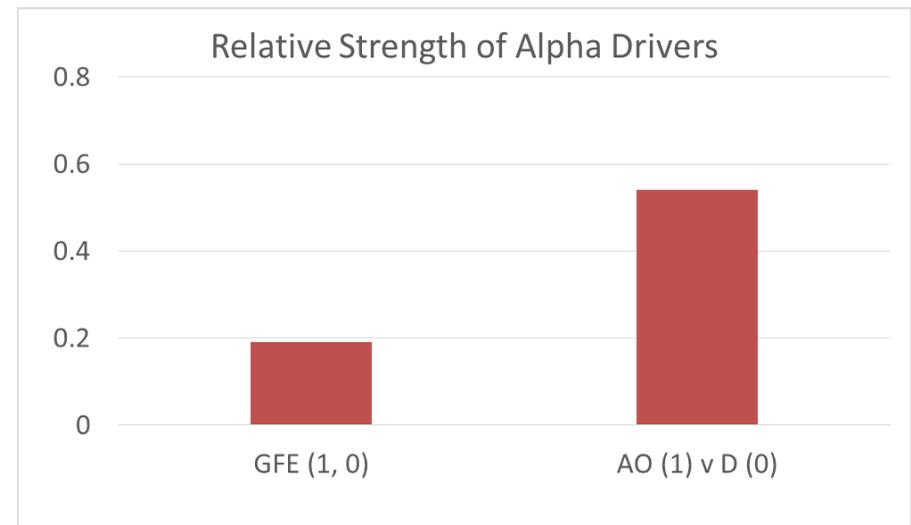
$$d = \frac{\text{TOTAL COST}}{R+1-e^{-\alpha(1-\gamma)^\beta}}$$

$$R = 0.329 + 0.381 \cdot (\text{Total Cost BY13\$B})$$

$$\alpha = 3.387 - 0.190^{GFE} - 0.540^{AO}$$

$$\beta = 2.31 - 4.64 * \left( \frac{\text{months to PDR}}{\text{months to launch}} \right)$$

$$\gamma = -.2188 + 1.909 * \left( \frac{\text{months to PDR}}{\text{months to launch}} \right)$$



## Accuracy Metrics

SE of Cum Residuals	4.70%
R-squared Rate	0.63
Error @ 40% time	7.58%



# Spacecraft-level Phasing Estimating Relationship (PER)

$$E(t) = d \left[ Rt + 1 - e^{-\alpha(t-\gamma)^\beta} \right]$$

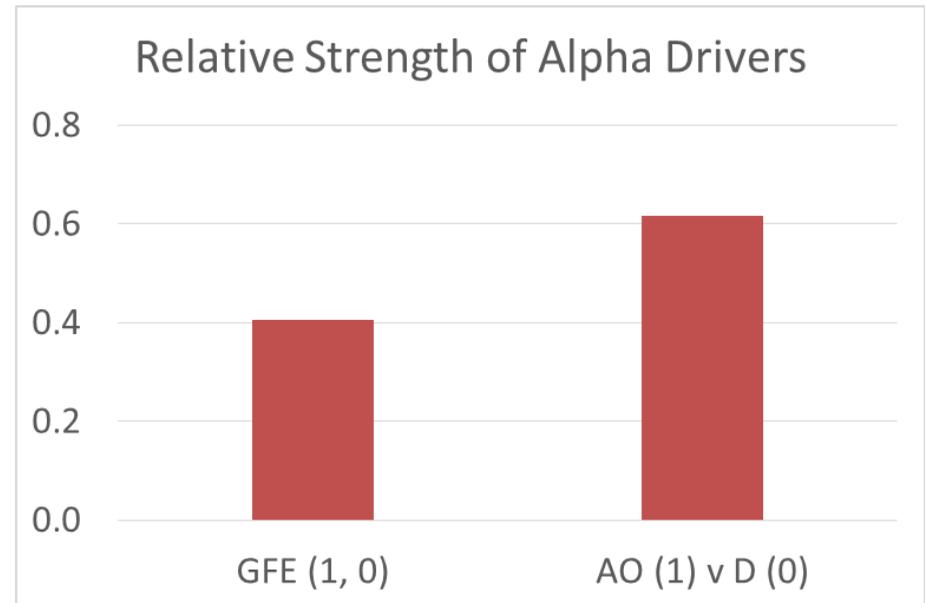
$$d = \frac{\text{TOTAL COST}}{R+1-e^{-\alpha(1-\gamma)^\beta}}$$

$$R = 0.299 + 0.154 \cdot (\text{Total Cost BY13\$B})$$

$$\alpha = 4.438 - 0.405^{GFE} - 0.616^{AO}$$

$$\beta = 2.39 - 4.87 * \left( \frac{\text{months to PDR}}{\text{months to launch}} \right)$$

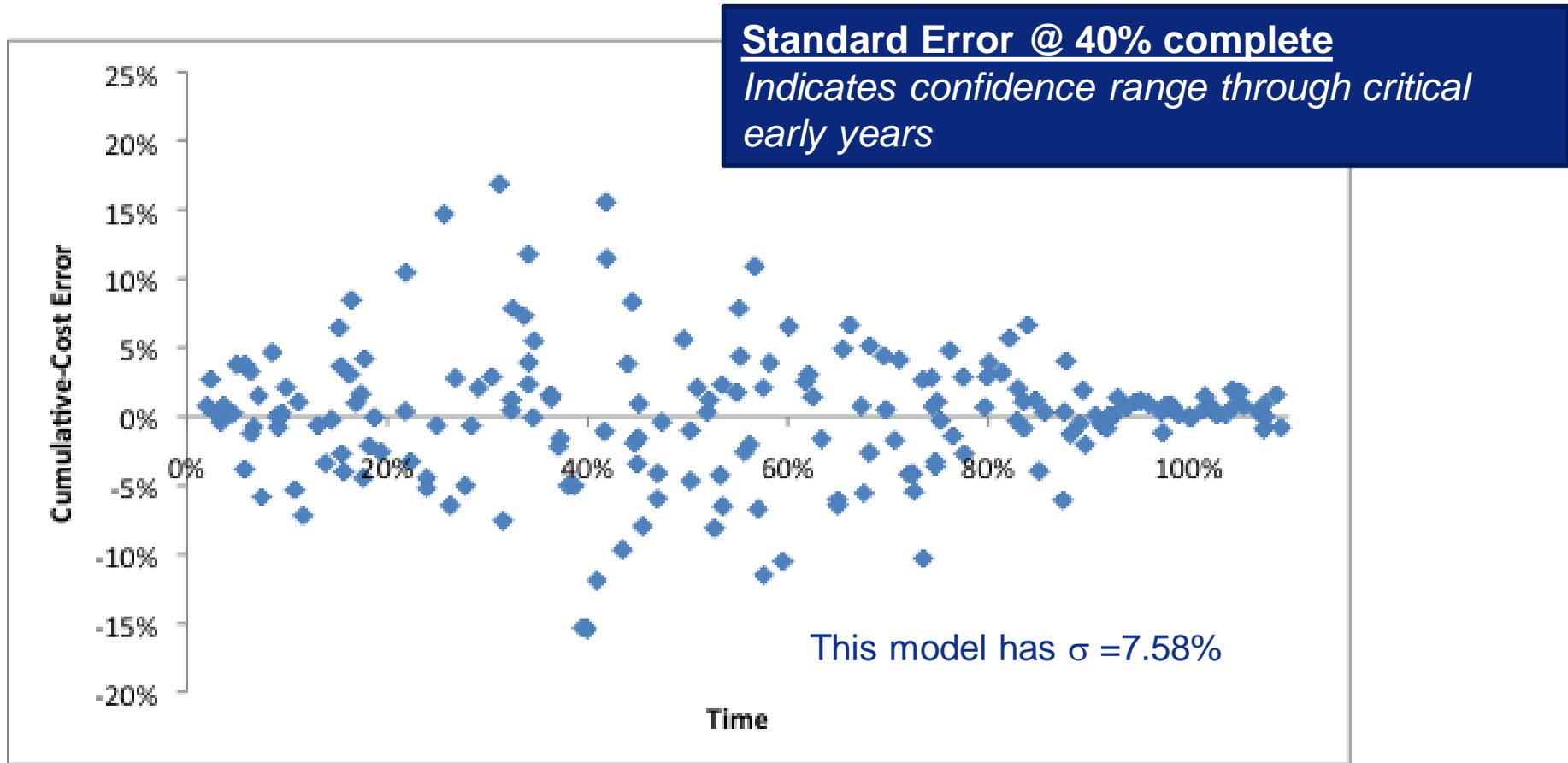
$$\gamma = -.211 + 1.88 * \left( \frac{\text{months to PDR}}{\text{months to launch}} \right)$$



Accuracy Metrics	
SE of Cum Residuals	5.64%
R-squared Rate	0.66
Error @ 40% time	9.58%



# A Powerful Accuracy Metric



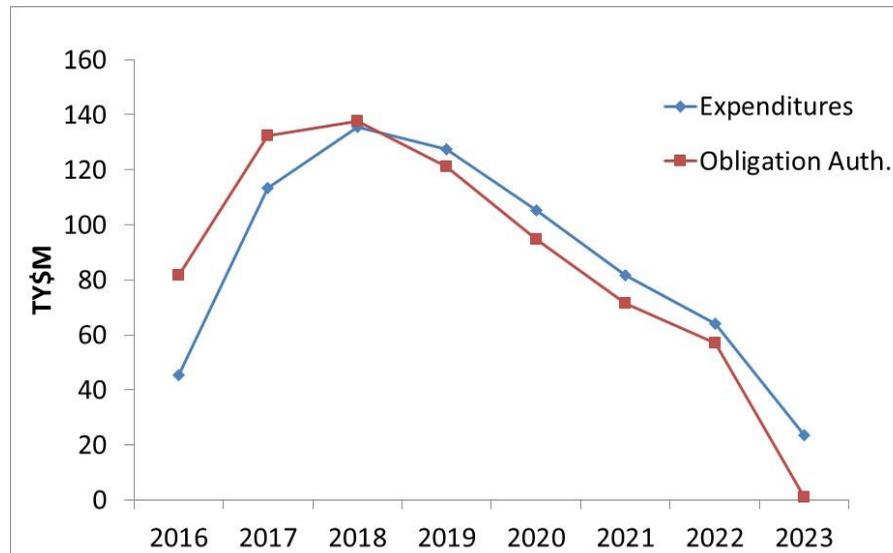
## Implications:

- PER minus  $1\sigma$  is a practical minimum
- Schedule slip or program restructure is defensible



# Implementation in Phasing Toolkit

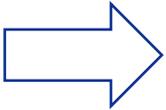
- Phasing estimating relationships are based on expenditures
- Not the same as a budget profile (NOA)
  - Obligation authority must account for total government liability
  - Difference between obligation authority and expenditures is the annual outlay rate
  - Toolkit allows user to specify outlay rates by year (default is 80/20)
- Phasing toolkit computes expenditures and associated NOA
  - Implements process published by Lee, Hogue, and Gallagher in 1997<sup>3</sup>
  - Allows quantitative evaluation of alternative profiles (e.g., the available budget!)



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# Summary and Further Research

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## ■ Summary:

- Two PERs are presented for NASA projects
- PERs reflect actual experience, consistent with data-driven cost and schedule models ... not optimal
- Traceable to CADRe data
- Error metrics useful for formulating, assessing, or defending budgets

## ■ Further research: Assess cost and schedule impacts of deviating from PERs

- Do front-loaded programs cost less or more?
- How strong is the correlation between cost and phasing?
- What is the schedule impact of a funding cut in year  $n$ ?



# References

- Abernathy, T. "An Application of the Rayleigh Distribution to Contract Cost Data." Master's Thesis, Naval Postgraduate School, Monterey, California, 1984.
- Brown, Thomas W., White, Edward D., and Gallagher, Mark A. "Weibull-based Forecasting of R&D Program Budgets." Presented to Military Operations Research Society Symposium (MORSS), Ft. Leavenworth, Kansas, 18 June 2002.
- Burgess, Erik. (2006). "R&D Budget Profiles and Metrics." *Journal of Parametrics* , 11-30.
- Burgess, Erik. (2012) "Modeling R&D Budget Profiles." SCEA/ISPA Joint Annual Conference, Orlando, FL, June 2012.
- Elrod, S.M. "Engineering and Manufacturing Development Cost Estimation: An Analysis of Combined Time-Phased and Classical Techniques." Master's Thesis, Naval Postgraduate School, Monterey, California, 1993.
- Gallagher, Mark A., and Lee, David A. "Final-Cost Estimates for Research and Development Programs Conditioned on Realized Costs," Military Operations Research, V2 N2, 1996.
- Garcia, Rick. "Ground Radar Expenditure Phasing Analysis," presented at the International Cost Estimating and Analysis Association Annual Symposium, New Orleans, LA, 18 June 2013.
- Lee, D., Hogue, M., and Hoffman, D. "Time Histories of Expenditures for Defense Acquisition Programs in the Development Phase," presented at the 1993 Annual Meeting of the International Society for Parametric Analysis.
- Lee, David A., Hogue, Michael R., and Gallagher, Mark A. "Determining a Budget Profile from a R&D Cost Estimate," Journal of Cost Analysis, 1997.
- Lee, David. "Norden-Rayleigh Analysis: A Useful Tool for EVM in Development Projects," presented to the CPM Spring Conference, April 2002.
- National Aeronautics and Space Administration (NASA). NASA Cost-Estimating Handbook, 2002, 2008.
- Naval Center for Cost Analysis (NCCA). "Inflation Indices & Outlay Profile Factors." March 2000.
- Norden, Peter V. "Useful Tools for Project Management," Management of Production, M.K. Starr, Editor. Penguin, Baltimore, Maryland, 1970.
- Porter, Paul H. "Revising R&D Program Budgets when Considering Funding Curtailment with a Weibull Model," Master's Thesis, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, March 2001.
- Unger, Eric J. "Relating Initial Budget to Program Growth with Rayleigh and Weibull Models," Master's Thesis, Air Force Institute of Technology, Wright-Patterson Air Force Base, Ohio, March 2001.
- Watkins, Harry. "An Application of Rayleigh Curve Theory to Contract Cost Estimation and Control," Master's Thesis, Naval Postgraduate School, Monterey, California, March, 1982.

