Introduction
Data Analysis
Regression Results
Summary and Further Research
Research funded by NASA/OE/CAD

Estimate annual funding for a mission
- Given a cost and schedule estimate
- Based on historical data … not “optimal”

Scope of PERs presented today:
- Time: System Requirements Review (SRR) to Launch
- Content:
  - Option 1: Total project excluding launch
  - Option 2: Spacecraft and instruments only
Applications:

- Support, assess, and/or defend budgets
- Starting point for analyzing cost & schedule ramifications

Keys to useful PERs:

- Clearly traceable to source data
  - Transparent and verifiable
  - Users can draw directly from analogy missions
- Logical drivers and functional form
  - Front/back-loading makes sense
  - Theoretical and empirical basis
- Differentiates between expenditures and obligation authority
- Useful accuracy metrics
  - Indexed to program events
  - Standard error vs. time
Functional Forms for Phasing

- **Rayleigh Curve**
  \[
  E(t) = 1 - e^{-t^2/2\sigma^2}
  \]

- **Norden-Rayleigh Curve**
  \[
  E(t) = d \left[ 1 - e^{-\alpha t^2} \right]
  \]

- **Weibull curve**
  \[
  E(t) = d \left[ 1 - e^{-\left(\frac{t-\gamma}{\delta}\right)^\beta} \right]
  \]

**John William Strutt, third Baron Rayleigh**
- Discovered Argon
- Won Nobel Prize for Physics, 1904
- Didn’t care about budget phasing

**Peter Norden, IBM, 1960s**
- Cared about phasing:
- Studied R&D projects
- Manpower build-up and phase-out follow distribution that happens to be Rayleigh’s\(^1\)

**Ernst Hjalmar Waloddi Weibull** (18 June 1887-12 October 1979)
- Swedish engineer, scientist, and mathematician.
- Proposed distribution as statistical model for life data (fatigue, reliability, etc.)
- Did not care about budget phasing

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Weibull: Better Empirical Results

- **Porter (2001):**
  - Used Weibull model to predict final costs when funding is curtailed
  - Claimed greater accuracy than Rayleigh due to additional parameters

- **Unger (2001):**
  - Showed that cost and schedule growth are correlated with poor initial phasing
  - Showed that Weibull distribution was a better fit to 37 DoD programs

- **Brown (2002):**
  - Use program characteristics to predict Weibull parameters (128 DoD programs)
  - Showed that Rayleigh curve was too inflexible

- **Burgess (2006):**
  - Compared Beta, Rayleigh, and Weibull for 26 space programs
  - Weibull performed better in every metric
  - Basis for DoD Space System Phasing Model

Weibull Distribution Has Theoretical and Empirical Bases
Beta Distribution

- Beta is from 9th Century BC: 1st consonant in Greek alphabet
- Beta distribution useful for Bayesian statistics (conditional)
- Also works for phasing!
  - Popular empirical curve for fitting manpower
  - Two parameters, BETADIST in Excel®
  - Very flexible, but no theoretical basis

\[
\frac{dW(t)}{dt} = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} t^{a-1} \cdot (1-t)^{b-1}, \quad 0 < t < 1
\]

Agenda

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CAD prioritized 99 potential projects → 37 used in final PERs

Normalization workbook created for each project
- All sources identified and/or linked
- Cost and schedule normalized on 0.0 to 1.0 scales

First tab in each workbook brought into regression model

Project Norm. Workbooks
- Traceable to CADRe and other data sources

Consolidated Workbook
- All data needed for regression
- May be useful for end-users

Phasing toolkit
- Implements the selected model
- Converts to NOA
Final Normalized Dataset
(Project-Level)

212 pooled data points from 37 projects

Baseline Fit (no independent variables)

Adding Project-specific Independent Variables Will Explain Front/Back-loading behavior
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### What We Expect to See

- **Weibull has two parameters, \( \alpha \) and \( \beta \)**
  - Plus a time shift if needed, \( \gamma \)

  \[
  E(t) \approx 1 - e^{-\alpha(t-\gamma)\beta}
  \]

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Affects</strong></td>
<td>Time of peak expenditures</td>
<td>Ramp-up rate</td>
</tr>
<tr>
<td><strong>Possible Drivers</strong></td>
<td>• Mission class</td>
<td>• Number of customers, primes, science organizations</td>
</tr>
<tr>
<td></td>
<td>• AO vs. Directed</td>
<td>• Total cost</td>
</tr>
<tr>
<td></td>
<td>• Total duration</td>
<td>• Total cost</td>
</tr>
<tr>
<td></td>
<td>• GFE payload</td>
<td>• % Time from SRR to PDR</td>
</tr>
<tr>
<td></td>
<td>• Competitive</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Instrument timelines</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• Percent new</td>
<td></td>
</tr>
</tbody>
</table>

- **We add a constant-rate term**
  - Reflects “standing army”
  - Usually higher on large, long projects

  \[
  E(t) \approx Rt + 1 - e^{-\alpha(t-\gamma)\beta}
  \]
Project-level Phasing Estimating Relationship (PER)

\[ E(t) = d \left[ Rt + 1 - e^{-\alpha(t-\gamma)^\beta} \right] \]

\[ d = \frac{\text{TOTAL COST}}{R + 1 - e^{-\alpha(1-\gamma)^\beta}} \]

\[ R = 0.329 + 0.381 \cdot (\text{Total Cost BY13}$B) \]

\[ \alpha = 3.387 - 0.190^{GFE} - 0.540^{AO} \]

\[ \beta = 2.31 - 4.64 \times \left( \frac{\text{months to PDR}}{\text{months to launch}} \right) \]

\[ \gamma = -0.2188 + 1.909 \times \left( \frac{\text{months to PDR}}{\text{months to launch}} \right) \]

Accuracy Metrics

<table>
<thead>
<tr>
<th>Metric</th>
<th>Value</th>
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<tbody>
<tr>
<td>SE of Cum Residuals</td>
<td>4.70%</td>
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<tr>
<td>R-squared Rate</td>
<td>0.63</td>
</tr>
<tr>
<td>Error @ 40% time</td>
<td>7.58%</td>
</tr>
</tbody>
</table>
Spacecraft-level Phasing Estimating Relationship (PER)

\[ E(t) = d \left[ Rt + 1 - e^{-\alpha(t-\gamma)}^\beta \right] \]

\[ d = \frac{\text{TOTAL COST}}{R + 1 - e^{-\alpha(1-\gamma)}^\beta} \]

\[ R = 0.299 + 0.154 \cdot (\text{Total Cost BY13}\$B) \]

\[ \alpha = 4.438 - 0.405^{GFE} - 0.616^{AO} \]

\[ \beta = 2.39 - 4.87 \ast \left( \frac{\text{months to PDR}}{\text{months to launch}} \right) \]

\[ \gamma = -0.211 + 1.88 \ast \left( \frac{\text{months to PDR}}{\text{months to launch}} \right) \]

**Accuracy Metrics**

<table>
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<tr>
<th>Metric</th>
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</thead>
<tbody>
<tr>
<td>SE of Cum Residuals</td>
<td>5.64%</td>
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<tr>
<td>R-squared Rate</td>
<td>0.66</td>
</tr>
<tr>
<td>Error @ 40% time</td>
<td>9.58%</td>
</tr>
</tbody>
</table>
A Powerful Accuracy Metric

This model has $\sigma = 7.58\%$

**Implications:**
- PER minus $1\sigma$ is a practical minimum
- Schedule slip or program restructure is defensible
Implementation in Phasing Toolkit

- Phasing estimating relationships are based on expenditures
- **Not the same as a budget profile (NOA)**
  - Obligation authority must account for total government liability
  - Difference between obligation authority and expenditures is the **annual outlay rate**
  - Toolkit allows user to specify outlay rates by year (default is 80/20)

- **Phasing toolkit computes expenditures and associated NOA**
  - Implements process published by Lee, Hogue, and Gallagher in 1997³
  - Allows quantitative evaluation of alternative profiles (e.g., the available budget!)

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Summary and Further Research

**Summary:**
- Two PERs are presented for NASA projects
- PERs reflect actual experience, consistent with data-driven cost and schedule models … not optimal
- Traceable to CADRe data
- Error metrics useful for formulating, assessing, or defending budgets

**Further research: Assess cost and schedule impacts of deviating from PERs**
- Do front-loaded programs cost less or more?
- How strong is the correlation between cost and phasing?
- What is the schedule impact of a funding cut in year $n$?


