A Monograph on CER Slopes

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• Karl Marx wrote the *Communist Manifesto* while sitting in the Reading Room of the British Museum in London.
  – It was the winter of 1847-48.
  – He was surrounded by books but he didn’t use them.
  – He sat in the British Museum because it was heated.
  – The Communist Manifesto was “made of whole cloth”—that is, based only on theories spinning around in Marx’s head without any foundation in experiential fact.
  – His work led to the Russian Revolution, thence to two World Wars, Stalin and the Gulag, Communist China, the despot regime of North Korea, the dictatorship of Cuba and countless other atrocities of the past 150 years.
  – Conservatively, billions of people have suffered and hundreds of millions more have directly died as a result of Marx’s monograph.
Similarly, I wrote this monograph in the same way...

– Unencumbered by data and based only on theory (much of which I made up), sitting in my office but without consulting many reference sources.

– I hope it causes substantially less mayhem than Marx’s work.
Introduction

• Generally, as the physical size of a product increases, its cost (both development cost and unit production cost) also increases but at a slower rate than size.
  – For example, doubling the size of most products (from Wal-Mart trash cans to hydroelectric dams), all else held equal, will usually not double the cost.
  – This is sometimes referred to as economies of scale (but should not be confused with the economies of scale associated with larger quantities of production)
Some CER Shapes

- The slope of a cost estimating relationship (CER) reflects the economies of scale associated with “larger scale” (or going the other way, the diseconomies of smaller scale).
- Actual cost data that exhibits this trend often fits a $y = ax^b$ power curve well where “a” is the y intercept at $x = 1$ unit of weight (assuming weight is the independent variable) and “b” is the slope.
- Graphic shows various curve shapes that are used for CERs but the power curve with a slope less than 1.0 is very typical because this models the notion that cost increases at a slower pace as size continues to increase.
- In NAFCOM, “a” is referred to as the first pound cost.
- Conceptually equivalent of the MCMPLX factor in PRICE.

CER Function shapes on linear coordinates
CER Slopes

• Of course, lower b values (shallower slopes) imply more economies of scale.

• For example, doubling the weight of an item characterized by a CER with a slope of \( b = 0.5 \) would imply that the cost would increase by about a factor of 1.4 (the square root of 2).

• A slope of 1.0 would imply no economies of scale and when weight doubles, cost doubles.
Many cost analysts have observed that the slope $b$, tends to be very roughly around $b = 0.5$ for development cost and very roughly $b = 0.7$ for unit production cost when using weight as the independent variable.

Some cost analysts have formed quick one data point CERs by assuming such a relationship.

For example, given a system that weights 100 kg and has a development cost of $75M, the cost of a similar system that is expected to weight 350 kg could be estimated by:

$$y = 75 \cdot \left( \frac{350}{100} \right)^{0.5} = 75 \cdot (3.5)^{0.5} = 140M$$

Notice that without any economies of scale, a straight linear ratio would have given

$$y = 75 \cdot \left( \frac{350}{100} \right) = 75 \cdot 3.5 = 263M$$

So in this case, the economy of scale assumption reflects the logical belief that a system that is 3.5 times larger than the one we are familiar with can be developed, all else being equal, for less than double the cost of the original system.
The development cost $y = ax^b$ CER can be more generally and conveniently written out by:

$$y = ax^b$$

$$\ln y = \ln a + b \ln x$$

$$\ln a = \ln y - b \ln x$$

$$\ln a = \ln (75) - 0.5 \ln (100)$$

$$\ln a = 2.01$$

$$a = 7.50$$

Plugging in our one data point of $75M and 100 kg$

Thus,

$$y = 7.50 \text{ (Weight)}^{0.5} \quad \text{Equation 1}$$

Now armed with this more general CER, the 350 kg system can be estimated by:

$$y = 7.50 \text{ (Weight)}^{0.5} = 7.5 \times (350)^{0.5} = $140M$$

And if the designer suddenly scales the system up to 400 kg its expected cost would be:

$$y = 7.50 \text{ (Weight)}^{0.5} = 7.5 \times (400)^{0.5} = $150M$$
The Concept of “First Pound” Cost

• So the “a-value” calculated in Equation 1 can be thought of as the “first pound cost.” While the math is trivial, the concept is powerful and useful.

• This is essentially the core basis for the PRICE Model developed by Frank Freiman in the late 1960s while he was with the RCA Corporation.
  – The MCMPLX (Manufacturing Complexity Factor) in the PRICE model is the natural logarithm of the first pound production cost of systems expressed in about 1969 price levels (in millions of dollars) and with certain other assumptions that are proprietary to PRICE Systems.
  – Freiman’s focus in the PRICE model was more production cost centric.
  – In the case of our example here, assume that the production cost of the system that cost $75M to develop had been $15M in today’s dollars. That might convert to something like $2M in 1969 dollars. Then the MCMPLX would be roughly:
    – \( \ln a = \ln (2,000,000) - 0.7 \ln(100) \Rightarrow \ln a = 11.285 \) (using 0.7 for the slope)
    – The value 11.285 should be reminiscent of MCMPLX values for PRICE users (and is a little high because Freiman didn’t include wraps such as fee, G&A and some other things in the costs he used to derive MCMPLX values

• NAFCOM first pound methodology echoes the PRICE Model MCMPLX notion but is development cost centric and the first pound costs are not expressed in log terms and are normally in more current year prices. But the fundamental concept is the same.
NAFCOM “Assumed Slopes”

- The engineering judgment aspect of NAFCOM assumed slopes is based on the structural/mechanical content of the system versus the electronics/software content of the system.
- Systems that are more structural/mechanical are expected to demonstrate more economies of scale (i.e. have a lower slope) than systems with more electronics and software content.
- Software for example, is well known in the cost community to show diseconomies of scale (i.e. a CER slope of $b > 1.0$)—the larger the software project (in for example, lines of code) the more the cost per line of code.
- Larger weights in electronics systems implies more complexity generally, more software per unit of weight and more cross strapping and integration costs—all of which dampens out the economies of scale as the systems get larger.
- Therefore in the table, the assumed slopes are driven by considerations of how much structural/mechanical content each system has as compared to the system’s electronics/software content.

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>DDT&amp;E</th>
<th>FU</th>
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</thead>
<tbody>
<tr>
<td>Antennas Subsystem</td>
<td>0.85</td>
<td>0.80</td>
</tr>
<tr>
<td>ASE</td>
<td>0.55</td>
<td>0.70</td>
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<tr>
<td>Attitude Control/Guidance and Navigation Subsystem</td>
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<td>0.85</td>
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<tr>
<td>Avionics Group</td>
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<td>0.80</td>
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<tr>
<td>CC&amp;DH Group</td>
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<td>Communication Subsystem</td>
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<tr>
<td>Crew Accommodations</td>
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<td>Data Management Subsystem</td>
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<tr>
<td>ECLS Subsystem</td>
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<tr>
<td>Electrical Power and Distribution Group</td>
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<tr>
<td>Electrical Power Subsystem</td>
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<td>0.75</td>
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<tr>
<td>Instrumentation Display &amp; Control Subsystem</td>
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<td>0.80</td>
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<tr>
<td>Launch &amp; Landing Safety</td>
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<td>0.70</td>
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<tr>
<td>Liquid Rocket Engines Subsystem</td>
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<tr>
<td>Mechanisms Subsystem</td>
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<td>Miscellaneous</td>
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<td>Propulsion Subsystem</td>
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<td>Range Safety</td>
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<td>Reaction Control Subsystem</td>
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<td>Separation</td>
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<td>Structure Subsystem</td>
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<tr>
<td>Thrust Vector Control Subsystem</td>
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<td>0.60</td>
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</tbody>
</table>

See accompanying paper for further information on NAFCOM assumed slopes in the table above.
Other Factors That Logically Should Drive CER Slopes

- In addition to the consideration of the electronics/software content of a system, other factors that come to mind that might be expected to drive slope, all else being equal are:
  - Level of design repeat or replication:
    - A system having many repetitive parts (e.g. a solar array made up of multiple identical sub-arrays) would be expected to “scale up more efficiently” in development cost and therefore have lower slopes. That is, as the size of such a system is increased, the basic sub-module design can be used with little modification.
    - Production cost, on the other hand, might exhibit a more linear trend (higher slope) since larger scale would be obtained by adding additional sub-modules all with approximately equal cost.
  - The design heritage (or inversely the “percent new design) and/or Technology Readiness Level (TRL):
    - Systems with a high heritage (low new design factor) and/or a high TRL should scale more efficiently in development (have a lower slope) than systems with less heritage because the additional scale of a system that is more “off the shelf” should be more cost effective to achieve.
    - The production cost slope should also be inversely proportionally to the heritage/TRL because the production of familiar systems should be more easily scaled up (thus having a lower slope).
  - Level of automation:
    - Systems which have a high degree of automation would be expected to have less economies of scale (i.e. higher slopes) for development cost because additional size would imply a large re-investment in changing the hard tooling (assumed here to be part of the development cost).
    - On the production side, once the tooling cost is paid by development, systems which have a high degree of automation might be expected to have more economies of scale (i.e. lower slopes) for production cost because additional size wouldn’t affect cost so much. This is assuming that the level of automation is optimized for the production quantity.
### Summary of Previous Slide

<table>
<thead>
<tr>
<th>Consideration</th>
<th>Development CER Slope</th>
<th>Unit Production CER Slope</th>
</tr>
</thead>
<tbody>
<tr>
<td>As the content of electronics and software increases…</td>
<td>Higher slope</td>
<td>Higher slope</td>
</tr>
<tr>
<td>As the level of design repeat or replication increases…</td>
<td>Lower slope</td>
<td>Higher slope</td>
</tr>
<tr>
<td>As the level of design heritage and/or Technology Readiness Level (TRL) increases…</td>
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<td>Lower slope</td>
</tr>
<tr>
<td>As the level of automation increases…</td>
<td>Higher slope</td>
<td>Lower slope</td>
</tr>
</tbody>
</table>

Remember, I made all this crap up without data (just like Karl Marx)
Conclusion

• There are undoubtedly other opinions and considerations in estimating or assuming a scaling slope for CERs.
• Maybe even a statistical approach using data!
• I will leave all this up top you...
  – or to someone sitting in the British Museum.