PORTFOLIO OPTIMIZATION

BRING INTELLIGENCE AND INSIGHT TO THE DECISION MAKING PROCESS

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MOTIVATION

- The analytical capabilities of cost and schedule risk tools are advancing steadily.
- Most of these tools can serve as a powerful analytical platform to bring intelligence and insight to the decision making process.
- There are other Management Science Models that have been widely used by various industries that can be brought to our community.
- Mathematical programming is a technique that has been widely applied to many management science problems such as logistics, queuing and resource planning.
- An important pillar of project management is resource allocation and optimization.
- Like to introduce the concept and framework of optimization to the cost/schedule community to stimulate thoughts and efforts in further advancing the capability of the current tools.

MANAGEMENT SCIENCE MODELS

- There are generally two types of models:
  - ***Descriptive Model*** – A model that represents relationship but does not indicate any course of actions.
    - Useful in predicting the behavior of a system but have no capability to identify the "***bes***t" course of action. For example:
      - Most statistical models
      - Regression Models – Cost estimate models
      - Schedule Models, JCL
    - Model Structure – Dependent and Independent Variables
  - ***Prescriptive Models*** – A model prescribes the course of action that the decision maker should take to achieve a defined objective.
    - It implies that the objective is embedded in the model and it is possible to identify the effect of different courses of action on the objective. It may include a descriptive sub model.
    - Model Structure
      - Objective function(s)
      - Decision variables
      - Constraints – Equality or Inequality
      - Decision Variable Bounds

WHAT IS MATHEMATICAL PROGRAMMING

- Mathematical Programming is a collection of methods for solving optimization problems:
  - If f(x), g(x),h(x) is linear, then it is Linear Programming (LP)
  - If any of f(x), g(x), h(x) are non-linear, then it is Non-Linear Programming (NLP)
  - If Decision Variables *x* are integers, then it is Integer Programming
  - If Decision Variables contain both integers and non-integers, then it is Mix-Integer Programming
  - If the optimization is based on a sequence of optimal decisions, then it is called Dynamic Programming (for instance, the game of Nim)

General mathematical programming formulation:

$Optimize: f(x)$

Subjected to:

$$g(x) \leq b,$$

$$h(x) = b_{eq},$$

$$l \leq x \leq u$$

Where,

f(x) is an objective function(s) to be optimized

X is the decision variable

g(x) is a function of inequality constraint

h(x) is a function of equality constraint

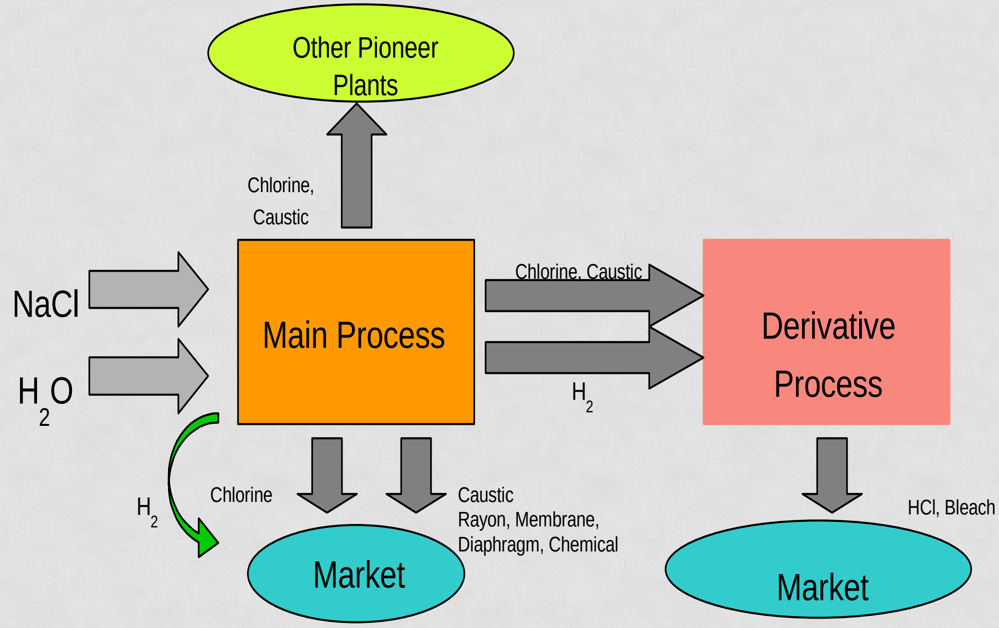
l,u are upper and lower bounds of the decision variables

APPLICATION OF MATHEMATICAL PROGRAMMING

- Mathematical Programming has been applied in virtually any industry that seeks optimization solutions:
  - Revenue optimization and crew rotation for the Airline industry (NLP)
  - The famous travelling salesman problem can be formulated in mathematical programming (Integer Linear Programming)
  - Valuation of financial derivatives (Dynamic Programming)
  - Portfolio replication of Exchange Traded Funds (ETFs)(Quadratic Programming)
  - Critical Path Method/schedule crashing (LP)
  - Minimum variance portfolio (NLP)
  - Resource allocation (LP or NLP)

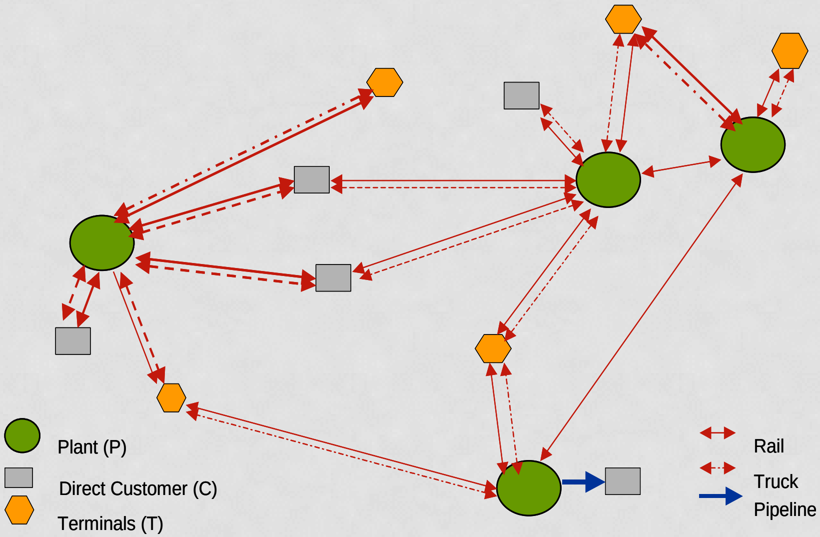
EXAMPLES AND FORMULATIONS
(MAXIMIZE THE PROFIT OF A CHLORINE PLANT)

- Company owns several manufacturing facilities through out North America
- Production of different material is strictly balanced by chemical equations
- There is derivative process and markets for derivative products
- Power is about 60% of variables cost and power prices varies by location and season
- Company lease a number of railcars for logistic
- Model maximizes profits subjected to the constraints of resource, logistics and demands



PLANT OPERATION AND LOGISTICS MANAGEMENT

- They have two questions:
  - What is the optimal production level at each plant based on regional demand pattern?
  - What is the optimal sales volume based on price and location, and what is the optimal means of delivery ?



$$\max imize : profit = \sum_i p_i d_i - \sum_j c_j d_j - \sum_k f_k d_k - C_{fix}$$

$$subject\ to : Ax \le b$$

$$x \ge 0$$

where

p is the price of the product

d is the demand

c is the production cost

f is the freight cost

C_{fix} is the fixed cost

A is the matrix of constraint coefficients

x is a collection of decision variables

b is vector of constraints

EXAMPLES AND FORMULATIONS
(CONSTRUCTING A MINIMUM VARIANCE PORTFOLIO)

Minimize : $\sigma_p^2 = W'\Sigma W$

Subjected to: $W'I = 1$

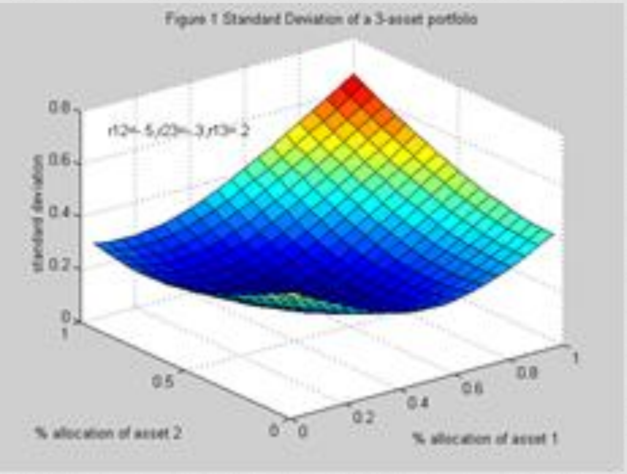
Solution:

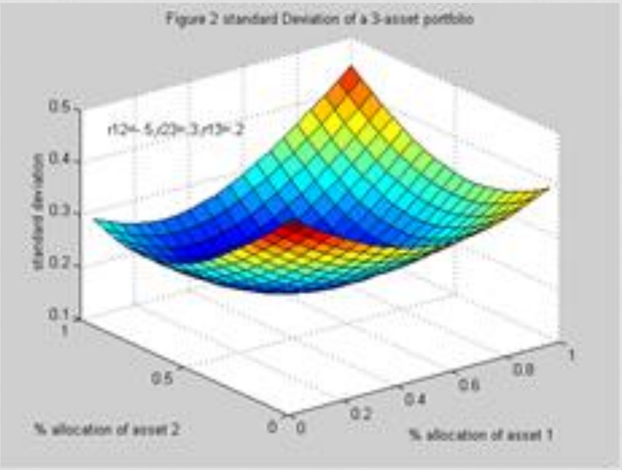
$$W_{\min} = \frac{\Sigma^{-1} I}{I'\Sigma^{-1} I}$$

$$\sigma_{\min}^2 = \frac{1}{I'\Sigma^{-1} I}$$

For a 3 asset Portfolio:

$$\sigma_1 = .3, \sigma_2 = .4, \sigma_3 = .5;$$
$$\rho_{12} = -.5, \rho_{23} = -.3, \rho_{13} = .2;$$
$$r = [.2 \quad .3 \quad .4];$$
$$W_{\min} = [.484 \quad .384 \quad .132];$$
$$\sigma_{\min} = .163;$$
$$r_{\min}^p = .264$$





APPLICATION OF LP IN SCHEDULE ANALYSIS

A simple example for calculating the critical path for a schedule.

Minimize: T_{finish}

Subjected To:

$T_{Finish} \geq T_E + 21$

$T_{Finish} \geq T_H + 28$

$T_{Finish} \geq T_J + 45$

$T_E \geq T_D + 20$

$T_H \geq T_D + 20$

$T_J \geq T_D + 20$

$T_J \geq T_I + 30$

$T_G \geq T_C + 5$

$T_G \geq T_F + 25$

$T_I \geq T_A + 30$

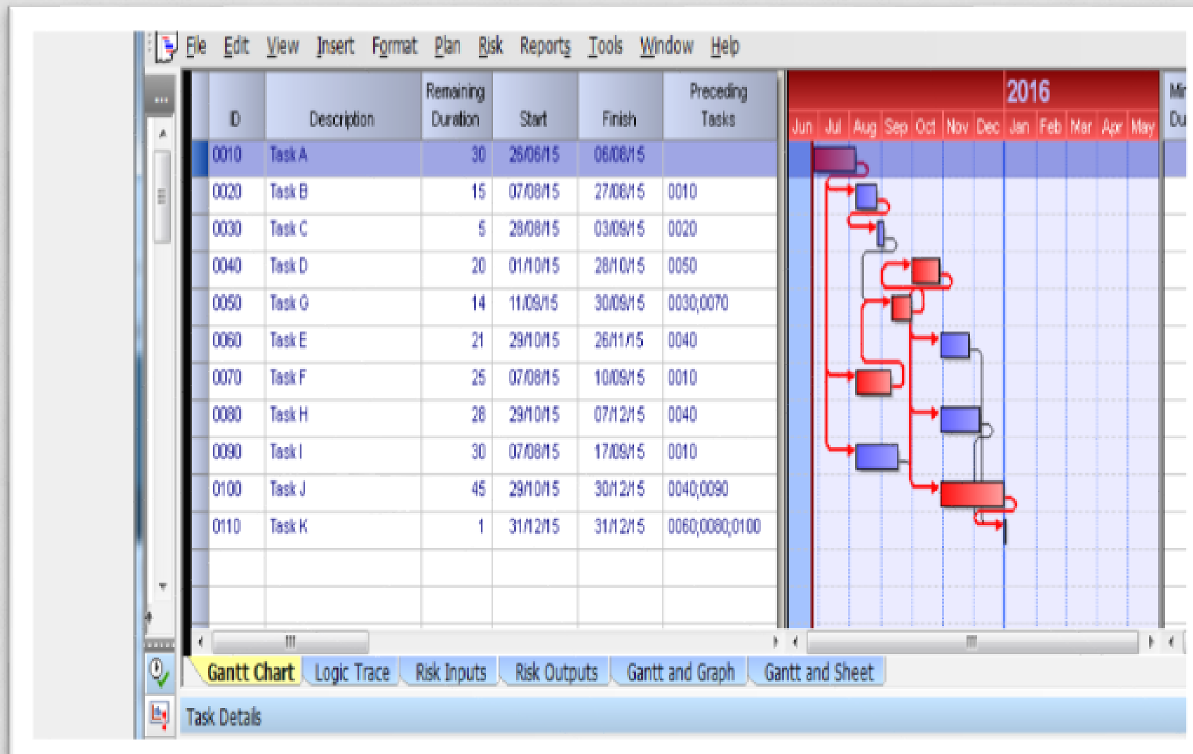
$T_F \geq T_A + 30$

$T_C \geq T_B + 15$

$T_D \geq T_G + 14$

$T_B \geq T_A + 30$

$All\ T_s \geq 0$



ID	Description	Remaining Duration	Start	Finish	Preceding Tasks
0010	Task A	30	26/06/15	06/08/15	
0020	Task B	15	07/08/15	27/08/15	0010
0030	Task C	5	28/08/15	03/09/15	0020
0040	Task D	20	01/10/15	28/10/15	0050
0050	Task G	14	11/09/15	30/09/15	0030;0070
0060	Task E	21	29/10/15	26/11/15	0040
0070	Task F	25	07/08/15	10/09/15	0010
0080	Task H	28	29/10/15	07/12/15	0040
0090	Task I	30	07/08/15	17/09/15	0010
0100	Task J	45	29/10/15	30/12/15	0040;0090
0110	Task K	1	31/12/15	31/12/15	0060;0080;0100

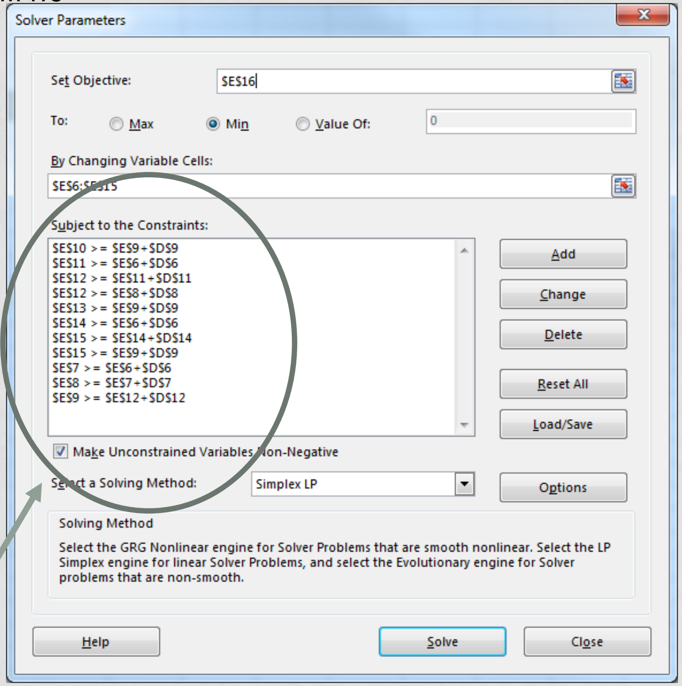
APPLICATION OF LP IN SCHEDULE ANALYSIS

We want to minimize finished time T_finish
By setting predecessor constraints as inequality constraints

Activity	Duration	T
A	90	0
B	15	95
C	5	110
D	20	129
E	21	149
F	25	90
G	14	115
H	28	149
I	30	119
J	45	149
Finish		194

Change these
(Decision Variables)

Minimize this
(Objective Function)



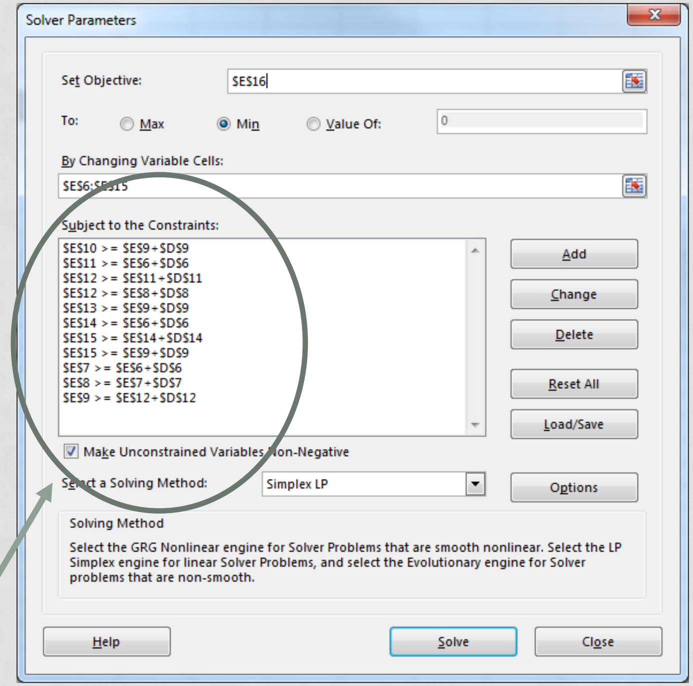
Subjected to these constraints
(Constraint equations)

APPLICATION OF LP IN SCHEDULE ANALYSIS

Using equality constraint to calculate slack

Activity	Duration	T	Slack
A	90	0	0
B	15	90	5
C	5	105	5
D	20	130	0
E	21	150	24
F	25	91	0
G	14	116	0
H	28	150	17
I	30	120	29
J	45	150	0
Finish		195	

Change these
(Decision Variables)



Minimize this
(Objective Function)

Subjected to these constraints
(Constraint equations)

LP AND SCHEDULE CRASHING

- For the same problem, now we want to crash the schedule by 20 days.
- Due to different type of tasks, there is a different set of crash costs.
- There is also a limit to how many days of duration you can shrink since you can not reduce a task to zero even at infinite cost.
- Now we want to crash the schedule by 20 days with a minimum of cost.
- The new objective function to be minimized is the sum of the cost of crashed days

Activity	Duration	Crashed Duration	Crash Cost/Day
A	90	10	200
B	15	5	100
C	5	0	150
D	20	5	150
E	21	6	150
F	25	5	200
G	14	4	200
H	28	5	150
I	30	7	170
J	45	10	200

Crashed Duration = Max. # of days that can be crashed

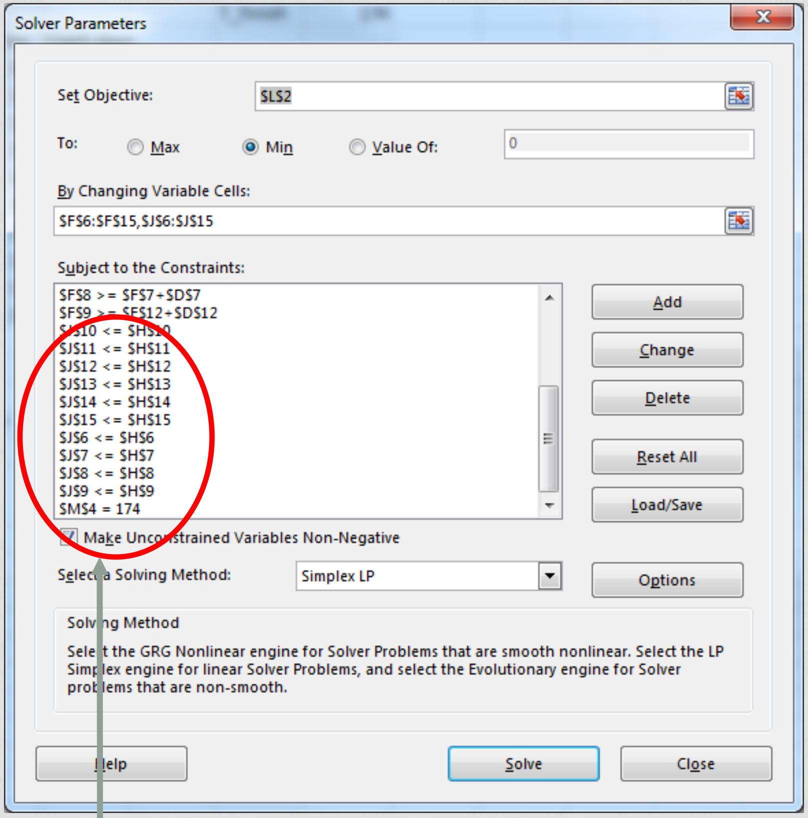
LP AND SCHEDULE CRASHING

What if I want to reduce total duration to 174 days?

Activity	Duration	Crashed Duration	Crash Cost/Day	Slack	Crash days
A	90	10	200	0	10
B	15	5	100	5	0
C	5	0	150	5	0
D	20	5	150	0	5
E	21	6	150	24	0
F	25	5	200	0	0
G	14	4	200	0	0
H	28	5	150	17	0
I	30	7	170	29	0
J	45	10	200	0	5
Finish	194		3750		174

Minimize this
(Sum of Cost * Crashed Days)

To achieve this



Adding a new set of constraints:
Crashed days <= Crashed Duration
T_{finish} = 174 Days

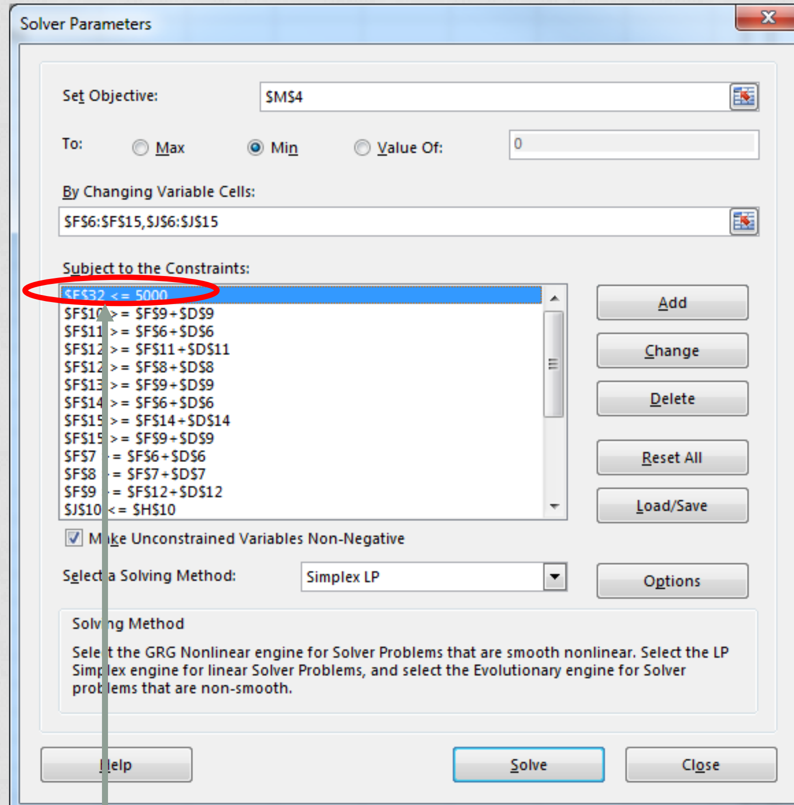
LP AND SCHEDULE CRASHING

What is the new total duration given $5000?

Activity	Duration	Crashed Duration	Crash Cost/Day	Slack	Crash days
A	90	10	200	0	10
B	15	5	100	5	0
C	5	0	150	5	0
D	20	5	150	0	5
E	21	6	150	24	0
F	25	5	200	0	5
G	14	4	200	0	4
H	28	5	150	17	0
I	30	7	170	29	0
J	45	10	200	0	2
Finish	194		4950		168

Final Cost

Minimize this



Adding a new set of constraints:
Crashed days <= Crashed Duration
Total Cost <= $5000

RISK MITIGATION/RESOURCE ALLOCATION
(EXAMPLE)

- A portfolio of 10 risks. They could be either cost or schedule risks.
- Each risk has a mitigation strategy to reduce the amount of risks, in terms of amount reduction in Mean and Standard Deviation.
- There is also a cost associated with each risk reduction strategy
- Risks are either mitigated or not, there is no partial risk mitigation.
- If there is a certain amount of resource ($) available, what is the optimal way to allocate which risk to mitigated?
- So the Objective Function can be:
  - Overall Mean?
  - Portfolio Standard Deviation?
  - A combination of the two?

	Before Mitigation		Mitigation Reduction		
Risk No.	**Mean**	**SD**	**Mean**	**SD**	**Cost**
1	10	10	8	7	5
2	30	10	24	7	24
3	20	10	16	7	12
4	15	5	12	3.5	12
5	60	20	48	14	48
6	50	20	40	14	40
7	35	15	28	11	22
8	45	15	36	10	30
9	75	20	60	14	60
10	90	25	72	18	50

RISK MITIGATION/RESOURCE ALLOCATION
(EXAMPLE)

Formulation: $Max: \sum_{i=1}^{10} \mu_i D_i$

Subjected To: $\sum_{i=1}^{10} C_i D_i \leq F$

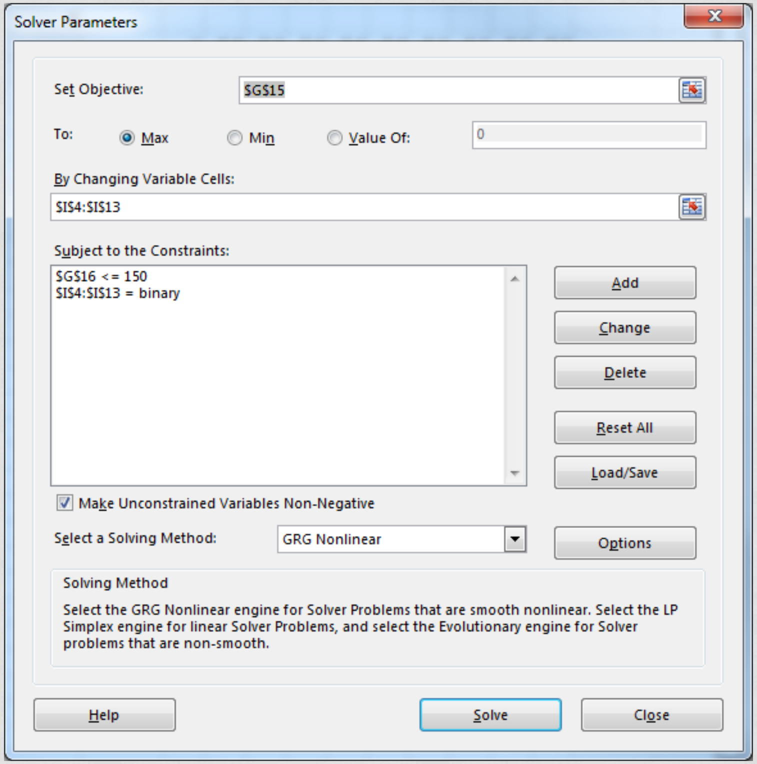
$D_i = [0,1]$

	Before Mitigation		Mitigation Reduction			
Risk No.	**Mean**	**SD**	**Mean**	**SD**	**Cost**	**DV**
1	10	10	8	7	5	0
2	30	10	24	7	24	1
3	20	10	16	7	12	1
4	15	5	12	3.5	12	1
5	60	20	48	14	48	0
6	50	20	40	14	40	0
7	35	15	28	11	22	1
8	45	15	36	10	30	1
9	75	20	60	14	60	0
10	90	25	72	18	50	1

Mean Reduction	188
Cost	150
Total Fund	150

Maximize this, by changing these

With this available fund



SOME THOUGHTS ON POTENTIAL APPLICATION

- Resource load schedule
  - Optimize resource allocation, or
  - Optimize schedule duration, or
  - A combination of both
  - Operational constraints such as skill set mix can be modelled as constraint equations
- Budget constraint scenario
  - Ideally suited for mathematical programming model
  - Operational constraints such as how to move work or content, budget etc. can be modelled by constraint equations
  - Objective functions to be optimized can be schedule delays, resource allocations, or any other project objectives or goals
- Risk Mitigation
  - One can define a risk metrics as a objective function to be optimized
  - Constrained by resource or information availability
- You may think of many others

SUMMARY AND CONCLUSION

- The cost and schedule community has been mainly been perfecting different descriptive models:
  - Better regression model for cost estimates
  - Increasing capability and efficiency in simulations
  - Better visualization of results
- The next natural step in the evolution of analytics should include Prescriptive Models to bring more intelligence out of the model for decision makers.
- This paper has briefly introduced the concept of optimization, objective functions, and decision variables. The tool for optimization is mathematical programming methods.
- Through some simple examples, this paper demonstrated that mathematical programming can:
  - Calculate critical path
  - Optimally crashing schedule
  - Optimally allocating resource for risk mitigation
- *The utility of Prescriptive models are only limited by our own creativity.*