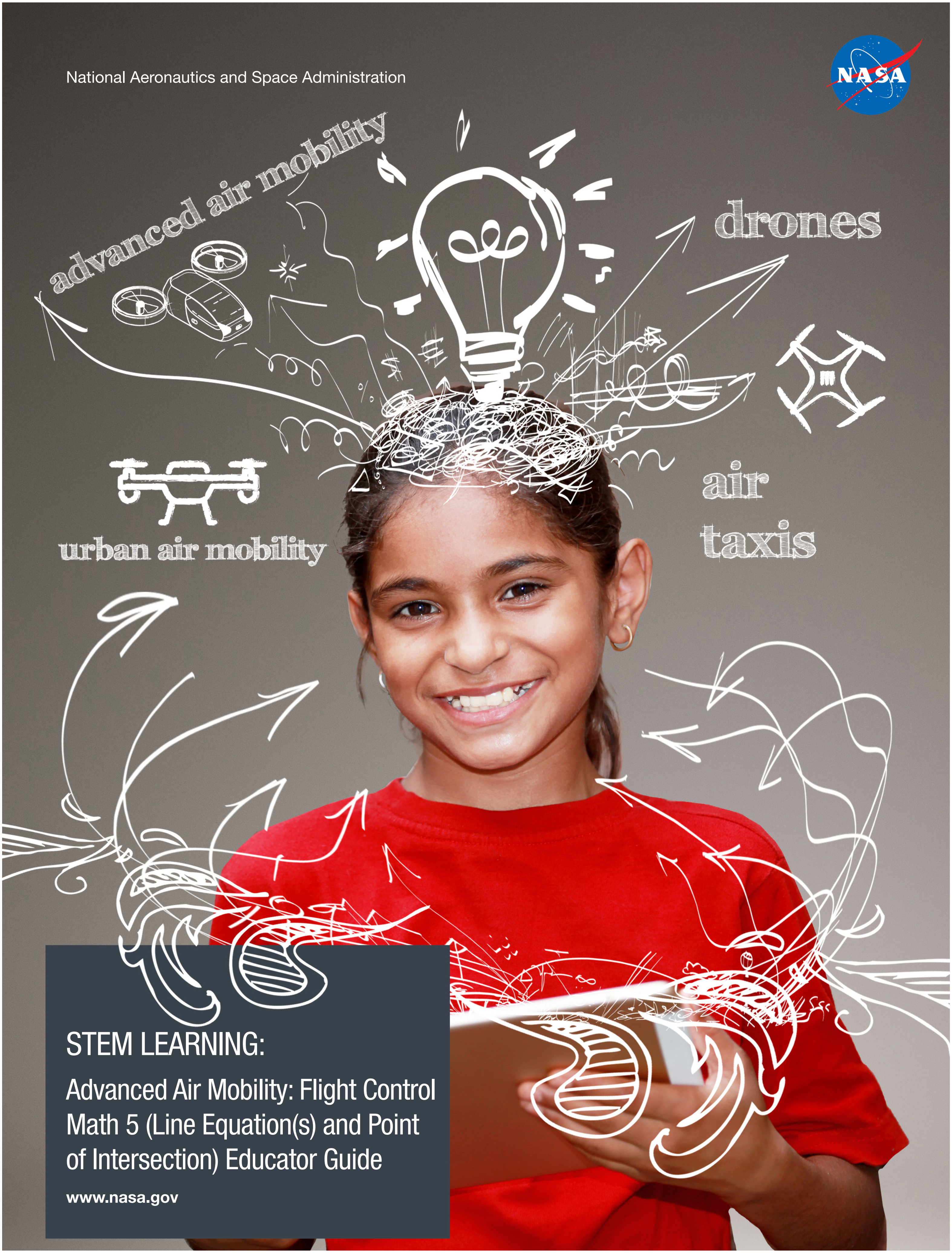
National Aeronautics and Space Administration



STEM LEARNING:

Advanced Air Mobility: Flight Control Math 5 (Line Equation(s) and Point of Intersection) Educator Guide

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OVERVIEW

This lesson provides a real-world scenario through which students use a series of steps to calculate the intercept point of two lines whose endpoints are defined by coordinates. Students find: (in slope-intercept form) the equation of lines, where these lines intersect, and the distance between coordinates. Students also use the speed formula to determine the time to the intersect point. Through this lesson, students are introduced to the concept of Detect and Avoid (DAA) which is an important aspect of Advanced Air Mobility (AAM).

Objectives

Students will be able to:

- Determine the equation, in slope-intersect form, of a line defined by two sets of coordinates
- Find the point of intersection for two lines
- Use the distance formula to calculate the distance between two points
- Use the speed formula to calculate the time it takes to move between two points

Student Prerequisite Knowledge

Before beginning this lesson, students should be familiar with:

- Plotting coordinates on a graph
- Using the distance formula for determining the distance between two points
- Using the speed formula

Standards

CCS.MATH.CONTENT.8.EE.C.7: Solve linear equations in one variable.

CCS.MATH.CONTENT.8.EE.C.8: Analyze and solve pairs of simultaneous linear equations.

CCS.MATH.CONTENT.8.F.A.3: Interpret the equation ($y = mx + b$) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

CCS.MATH.CONTENT.8.GB.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Next Generation Science Standards

Science and Engineering Practices

- Developing and using models
- Using mathematical and computational thinking

Materials

- Student Guides (one per student)

Lesson Modifications

This lesson can be modified depending on student needs. Coordinates can be changed to adjust the difficulty of the calculations. Additional points can be added to provide more opportunities for practice.

The activity is set up in parts, each of which builds on the previous part. If students haven't learned the concepts contained in the later parts, the earlier parts can still be completed. Allowing students to check answers as they complete a part can help find errors that affect later problems.

The difficulty can be varied by allowing or prohibiting the use of calculators. If calculators are not used, answers will need to be expressed as square roots unless locations are added that lead to perfect squares.

Grouping Students

This activity is designed to be completed independently. Grouping can, however, be used if necessary. Students working in pairs can independently solve the problems and then compare answers to validate their solutions or find errors.

Steps

1. Use a warm-up or other method to introduce the students to the concept of AAM. Students can read through the information found in the student guide as an introduction to this topic.

2. To save copies, print sections as needed. Pages 5-8 of the student guide contain the graph and all the problems for this activity. Pages 1-4 provide background information and a review of the mathematical concepts included in the activity.

3. Review the major concepts listed here with students to ensure they understand how to use them:
    - Slope-intercept form
    - Solving systems of two linear equations
    - Distance formula
    - Speed formula

4. Distribute the student guides and allow students to begin working.

5. As students are working, their work should be checked to discover if they are making errors. If they are, they can be shown the correct method.

ANSWERS

PART 1: FIND THE EQUATIONS FOR EACH LINE

Calculate the equation, in slope-intercept format, for each of the three UAVs. Show your work.

1. UAV 1: (-10, -5) → (12, 6)

$$m = \frac{y2 - y1}{x2 - x1} = \frac{6 - (-5)}{12 - (-10)} = \frac{11}{22} = 0.5$$

$$y = mx + b = \ 0.5x + b$$

$$6 = (0.5)(12) + b = 6 + b$$

$$b = 0$$

$$y = 0.5x$$

2. UAV 2: (1, -4) → (7, 8)

$$m = \frac{y2 - y1}{x2 - x1} = \frac{8 - (-4)}{7 - 1} = \frac{12}{2} = 6$$

$$y = mx + b = \ 2x + b$$

$$8 = (2)(7) + b = 14 + b$$

$$b = -6$$

$$y = 0.5x - 6$$

3. UAV 2: (6, 7) → (-18, -5)

$$m = \frac{y2 - y1}{x2 - x1} = \frac{-5 - 7}{-18 - 6} = \frac{-12}{-24} = 0.5$$

$$y = mx + b = \ 0.5x + b$$

$$7 = (0.5)(6) + b = 3 + b$$

$$b = 4$$

$$y = 0.5x + 4$$

PART 2: FIND THE POINTS OF INTERSECTION

You are in charge of UAV 1. Determine where the points of intersection with each of the other two UAVs are. Show your work.

1. Point of intersection for UAV 1 and UAV 2:

   UAV 1: $y = 0.5x$

   UAV 2: $y = 2x - 6$

$$0.5x = 2x - 6$$
$$-1.5x = -6$$
$$x = 4$$
$$y = 0.5x = (0.5)(4) = 2$$

   Point of intersection: (4, 2)

2. Point of intersection for UAV 1 and UAV 3:

   UAV 1: $y = 0.5x$

   UAV 2: $y = 0.5x + 4$

$$0.5x = 0.5x + 4$$

$0 = 4 \leftarrow$ NO SOLUTIONS

   There is no point of intersection (parallel lines).

PART 3: CALCULATING TIME TO INTERSECT

Given that each UAV travels at 1.2 km/min, calculate the time until each UAV reaches the point of intersection. You will first need to calculate the distance from the starting point to the point of intersection and then calculate the time it takes to arrive.

Based on your calculations, determine if there is a collision risk. Remember that there is a risk if the UAVs arrive at the point of intersection **within 30 seconds of each other**.

Remember to convert your answers to km (1 square = 0.5 km).

1. Time to the point of intersection for UAV 1 and UAV 2:

   UAV 1:

$$distance = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{\left(4-(-10)\right)^2 + \left(2-(-5)\right)^2} = \sqrt{14^2 + 7^2}$$
$$= \sqrt{245} = 15.7\ squares * \ 0.5 = 8.9\ km$$

$$time = \frac{distance}{speed} = \frac{8.9}{1.2} = 7.4\ min$$

   UAV 2:

$$distance = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(4-1)^2 + \left(2-(-4)\right)^2} = \sqrt{3^2 + 6^2} = \sqrt{45}$$
$$= 6.7\ squares * \ 0.5 = 3.4\ km$$

$$time = \frac{distance}{speed} = \frac{3.4}{1.2} = 2.8\ min$$

There is no collision risk.

2. Time to the point of intersection of UAV 1 and UAV 3:

There is no point of intersection, so there is no collision risk.

PART 4: EXTRA PRACTICE

For each of the following, calculate:

- The equation, in slope-intercept format, of each UAV
- The point of intersection for the two UAVs
- The distance and time until reaching the point of intersection
- Assess whether there is a collision risk

1. UAV 1 flies from (-2, -2) to (2, 6) and UAV 2 flies from (2, 0) to (5, 9).

UAV 1:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-2)}{2 - (-2)} = \frac{8}{4} = 2$$

$$y = mx + b \rightarrow 6 = (2)(2) + b \rightarrow b = 2$$

$$y = 2x + 2$$

UAV 2:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 0}{5 - 2} = \frac{9}{3} = 3$$

$$y = mx + b \rightarrow 9 = (3)(5) + b \rightarrow b = -6$$

$$y = 3x - 6$$

Point of intersection:

$$2x + 2 = 3x - 6 \rightarrow \; x = 8$$

$$y = 2x + 2 = (2)(8) + 2 = 18$$

Point of intersection: (8, 18)

The (x) and (y) coordinates of the point of intersection are out of range for the flights. No collision risk.

2. UAV 1 flies from (-7, 0) to (3, 10) and UAV 2 flies from (-2, 0) to (-8, 9).

   UAV 1:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{10 - 0}{3 - (-7)} = \frac{10}{10} = 1$$

$$y = mx + b \rightarrow 10 = (1)(3) + b \rightarrow b = 7$$

$$y = x + 7$$

   UAV 2:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 0}{-8 - (-2)} = \frac{9}{-6} = -1.5$$

$$y = mx + b \rightarrow 9 = (-1.5)(-8) + b \rightarrow b = -3$$

$$y = -1.5x - 3$$

   Point of intersection:

$$\mathrm{x} + 7 = -1.5\mathrm{x} - 3 \rightarrow 2.5x = -10 \rightarrow \mathrm{x} = -4$$

$$y = \mathrm{x} + 7 = -4 + 7 = 3$$

   Point of intersection: (-4, 3)

   UAV 1 time to the point of intersection:

$$distance = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(0-3)^2 + \left(-7 - (-4)\right)^2} = \sqrt{3^2 + (-3)^2} = \sqrt{18}$$
$$= 4.2\ squares * 0.5 = 2.1\ km$$

$$time = \frac{distance}{speed} = \frac{2.1}{1.2} = 1.8\ min$$

UAV 2 time to the point of intersection:

$$distance = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2} = \sqrt{(0-3)^2 + \left(-2-(-4)\right)^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$$
$$= 3.6\ squares * \ 0.5 = 1.8\ km$$

$$time = \frac{distance}{speed} = \frac{1.8}{1.2} = 1.5\ min$$

Difference in time: 1.8 – 1.5 = 0.3 min × 60 = 18 seconds

18 seconds < 30 seconds, so there is a collision risk.

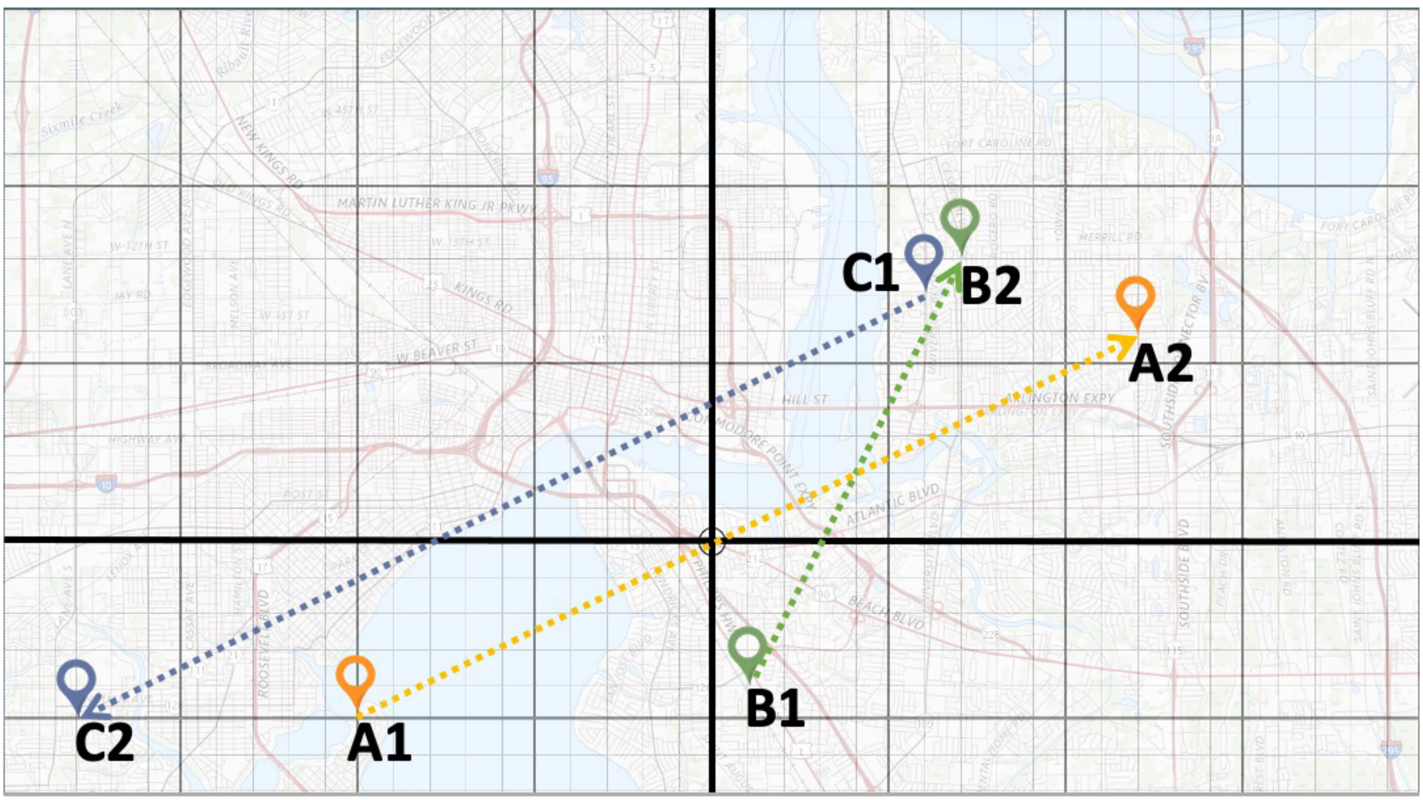

Figure 1. Map with courses A, B, and C shown. Credit: USGS

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