
urban air mobility


STEM LEARIING:
Advanced Air Mobility: Flight Control Math 3 (Distance/Speed) Educator Guide

## OVERVIEW

This lesson provides a real-world scenario through which students get practice using the distance and speed formulas. Students calculate the distance between coordinates and use the speed formula to determine time, distance, or speed. Through this lesson, students are introduced to the concept of Advanced Air Mobility, or AAM, and air metro services.

## Objectives

Students will be able to:

- Use the distance formula to calculate the distance between two points
- Use the formula for speed to help develop a schedule for an air metro system


## Student Prerequisite Knowledge

Before beginning this lesson, students should be familiar with:

- Obtaining coordinates of points plotted on a graph
- Plotting coordinates on a graph
- Determining the distance between two points using the distance formula
- Using the speed formula


## Standards

CCS.MATH.CONTENT.6.NS.C.8: Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

CCS.MATH.CONTENT.6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems (for example, by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).

CCS.MATH.CONTENT.7.RP.A.1: Compute unit rates associated with ratios of fractions, including ratios of lengths, areas, and other quantities measured in like or different units.

CCS.MATH.CONTENT.8.G.B.8: Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

## Next Generation Science Standards

## Science and Engineering Practices

- Developing and using models
- Using mathematical and computational thinking


## Materials

- Student Guides (one per student)


## Lesson Modifications

This lesson can be modified depending on student needs. Coordinates can be changed to adjust the difficulty of the calculations. More stops can be added to the air metro routes, or additional loops may be added (or both). Skipping different parts of the assignment allows for further tailoring of the lesson.

The difficulty can be varied by allowing or prohibiting the use of calculators. If calculators are not used, answers will need to be expressed as square roots unless locations are changed so that the math leads to perfect squares.

## Grouping Students

This activity is designed to be completed independently. Grouping can, however, be used if necessary. Students working in pairs can independently solve the problems and then compare answers to validate their solutions or find errors.

## Steps

1. Use a warm-up or other method to introduce the students to the concept of AAM. Students can read through the information found in the student guide as an introduction to this topic.
2. Review the distance and speed formulas with students to ensure they understand how to use it.
3. Distribute the student guide(s) and allow students to begin working.
4. As students are working, their work should be checked to discover if they are making errors. If they are, they can be shown the correct method.

## ANSWERS

PART 1 : USING THE DISTANCE FORMULA
Begin by determining the distances between the various vertiports. Remember that each square is equal to $\mathbf{1}$ kilometer (km). Use the distance formula to fill in the missing information in the data tables:

## Blue Loop

|  | Distance (km) |
| :--- | :---: |
| Distance between vertiports 1 and 6 | 8.5 |
| Distance between vertiports 6 and 4 | 10.4 |
| Distance between vertiports 4 and 5 | 6.0 |
| Distance between vertiports 5 and 1 | 17.0 |

## Purple Loop

|  | Distance (km) |
| :--- | :--- |
| Distance between vertiports 3 and 8 | 13.5 |
| Distance between vertiports 8 and 10 | 10.0 |
| Distance between vertiports 10 and 9 | 10.6 |
| Distance between vertiports 9 and 3 | 10.8 |

Red Loop

|  | Distance (km) |
| :--- | :--- |
| Distance between vertiports 2 and 5 | 15.0 |
| Distance between vertiports 5 and 7 | 12.2 |
| Distance between vertiports 7 and 8 | 11.4 |
| Distance between vertiports 8 and 2 | 13.6 |

Green Loop

|  | Distance (km) |
| :---: | :---: |
| Distance between vertiports 1 and 2 | 4.1 |
| Distance between vertiports 2 and 3 | 2.8 |
| Distance between vertiports 3 and 1 | 3.6 |

## PART 2: USING THE SPEED FORMULA

1. To determine the speed the aircraft can fly for the suburban routes, a test was performed. In this test, the aircraft flew $1273 / 4 \mathrm{~km}$ in 1 hour and 4 minutes.

Calculate the speed (show your work):

$$
\text { speed }=\frac{\text { distance }}{\text { time }}=\frac{127.75 \mathrm{~km}}{64 \mathrm{~min}}=2 \frac{\mathrm{~km}}{\mathrm{~min}}
$$

2. Using the speed you calculated, calculate the time it takes to fly each leg of the Blue Loop. Show your work.
a. From vertiport 1 and vertiport 6:
time $=\frac{\text { distance }}{\text { speed }}=\frac{8.5 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\mathrm{~min}}}=4.3 \mathrm{~min}$
b. From vertiport 6 to vertiport 4:

$$
\text { time }=\frac{\text { distance }}{\text { speed }}=\frac{10.4 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\mathrm{~min}}}=5.2 \mathrm{~min}
$$

c. From vertiport 4 to vertiport 5:

$$
\text { time }=\frac{\text { distance }}{\text { speed }}=\frac{6.0 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\mathrm{~min}}}=3.0 \mathrm{~min}
$$

d. From vertiport 5 to vertiport 1:
time $=\frac{\text { distance }}{\text { speed }}=\frac{17.0 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\mathrm{~min}}}=8.5 \mathrm{~min}$
3. Houses continue to be built in the eastern suburb, so a new stop is being considered for the Purple Loop. The systems engineers are trying to determine a good location for a new stop.
a. The desired flight time to the new stop is between 2.5 minutes and 8 minutes. Using the speed you calculated previously, calculate how close and how far the new stop can be.

$$
\begin{aligned}
\text { distance } & =\text { speed } \times \text { time }=2 \frac{\mathrm{~km}}{\mathrm{~min}} \times 2.5 \mathrm{~min}=5 \mathrm{~km} \\
\text { distance } & =\text { speed } \times \text { time }=2 \frac{\mathrm{~km}}{\mathrm{~min}} \times 8 \mathrm{~min}=16 \mathrm{~km}
\end{aligned}
$$

b. The engineers have narrowed the search down to two possible spots, $(14,10)$ and $(17,12)$. Determine which one, or both, of these would be in the desired distance range from both vertiports 8 and 10.

Vertiport 10 to (14, 10):

$$
\begin{aligned}
\text { distance }= & \sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}=\sqrt{(14-20)^{2}+(10-6)^{2}} \\
& \text { distance }=\sqrt{(-6)^{2}+4^{2}}=\sqrt{52}=7.2 \mathrm{~km}
\end{aligned}
$$

Vertiport 10 to (17, 12):

$$
\begin{aligned}
\text { distance }= & \sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}=\sqrt{(17-20)^{2}+(12-6)^{2}} \\
& \text { distance }=\sqrt{(-3)^{2}+6^{2}}=\sqrt{45}=6.7 \mathrm{~km}
\end{aligned}
$$

Vertiport 8 to $(14,10)$ :

$$
\begin{gathered}
\text { distance }=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}=\sqrt{(14-12)^{2}+(10-12)^{2}} \\
\text { distance }=\sqrt{2^{2}+(-2)^{2}}=\sqrt{8}=2.8 \mathrm{~km}
\end{gathered}
$$

Vertiport 8 to (17, 12):

$$
\begin{gathered}
\text { distance }=\sqrt{\left(x_{2}-x_{1}\right)^{2}+\left(y_{2}-y_{1}\right)^{2}}=\sqrt{(17-12)^{2}+(12-12)^{2}} \\
\text { distance }=\sqrt{5^{2}+0^{2}}=\sqrt{25}=5.0 \mathrm{~km}
\end{gathered}
$$

Vertiport 10 is in range of both, but vertiport 8 is not.
4. The first schedule below was created for the Red Loop. Unfortunately, there is an error in the portion of the chart that contains the times. Assume that the aircraft must be on the ground at the vertiport for at least 1 minute, but no more than 2 minutes, making the departures time always 2 minutes later than the arrival time.

Find the error, correct it (as well as any other times that would change), and enter the correct times in the blank schedule.


| RED LOOP SCHEDULE <br> Chart shows first aircraft of the day. |  |
| :--- | :---: |
|  | Arrival |
|  | Departure |
| Vertiport 2 |  |
| Vertiport 5 |  |
| Vertiport 7 |  |
| Vertiport 8 |  |
| Vertiport 2 |  |
| Aircraft arrive every 10 minutes until 7:00 PM and then |  |
| every 20 minutes until 11:30 PM. |  |

Enter the correct times in the table above.

Vertiport 2 to Vertiport 5: time $=\frac{\text { distance }}{\text { speed }}=\frac{15 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\mathrm{~min}}}=7.5 \mathrm{~min} \longleftarrow$ Good on the schedule

Vertiport 5 to Vertiport 7: time $=\frac{\text { distance }}{\text { speed }}=\frac{12.2 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\mathrm{~min}}}=6.1 \mathrm{~min} \longleftarrow \sim \begin{aligned} & \text { Error on the schedule } \\ & \text { (only } 5 \text { minutes allotted) }\end{aligned}$

Vertiport 7 to Vertiport 8: time $=\frac{\text { distance }}{\text { speed }}=\frac{11.4 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\min }}=5.7 \mathrm{~min} \longleftarrow$ Good on the schedule

Vertiport 8 to Vertiport 2: time $=\frac{\text { distance }}{\text { speed }}=\frac{13.6 \mathrm{~km}}{2 \frac{\mathrm{~km}}{\min }}=6.8 \mathrm{~min} \longleftarrow$ Good on the schedule

5. For the Green Loop, aircraft fly in the downtown portion of the city. To lower the noise and ensure safety, they fly slower than their capacity. To calculate the average speed, a researcher collects the following data.

## Data Collected

|  | Arrival Time | Time on the Ground <br> (minutes:seconds) | Departure Time |
| :--- | :---: | :---: | :---: |
| Vertiport 1 | $3: 28$ PM | $1: 55$ | $3: 30 \mathrm{PM}$ |
| Vertiport 2 | $3: 32 \mathrm{PM}$ | $1: 02$ | $3: 34 \mathrm{PM}$ |
| Vertiport 3 | $3: 35 \mathrm{PM}$ | $1: 54$ | $3: 37 \mathrm{PM}$ |
| Vertiport 1 | $3: 40 \mathrm{PM}$ | $1: 55$ | $3: 42 \mathrm{PM}$ |

Calculate the average speed, in $\frac{k m}{\min }$, of the aircraft. Only count the time spent flying, not the time on the ground. Show your work. (Hint: To calculate the total time spent flying, subtract the sum of the times on the ground in each vertiport from the time between arriving in vertiport 1 the first time and arriving there the second time.)

$$
\text { Distance }=2.7 \mathrm{~km}+2.6 \mathrm{~km}+4.1 \mathrm{~km}=9.4 \mathrm{~km}
$$

Time on the ground $=1: 55+1: 02+1: 54=4: 51=4.85 \mathrm{~min}$
Time flying $=12 \mathrm{~min}-4.85 \mathrm{~min}=7.15 \mathrm{~min}$

$$
\text { speed }=\frac{\text { distance }}{\text { time }}=\frac{9.4 \mathrm{~km}}{7.15 \mathrm{~min}}=1.3 \frac{\mathrm{~km}}{\mathrm{~min}}
$$



Figure 1. Satellite image of Indianapolis, Indiana. Credit: USGS


Figure 2. Map of Indianapolis, Indiana. Credit: USGS


Figure 3. Map of Indianapolis, Indiana and the surrounding area with a grid overlaid on it. Credit: USGS


Figure 4. Graph showing air metro routes.

National Aeronautics and Space Administration
Headquarters
300 E Street SW
Washington, DC 20546
www.nasa.gov

