



Exploring Space Through MATH

Applications in Precalculus



STUDENT
EDITION

Simulating Weightlessness

Background

This problem applies mathematical principles in NASA's human spaceflight.

Exploration provides the foundation of our knowledge, technology, resources, and inspiration. Through exploration we seek answers to fundamental questions about our existence, to discover and to respond to new environments, to put in place revolutionary techniques and capabilities to inspire our nation, the world, and the next generation. Through NASA, we touch the unknown; we learn and we understand. As we take the first steps toward sustaining a human presence in the solar system, we can look forward to far-off visions of the past becoming realities of the future.

In our quest to explore, humans will have to adapt to functioning in a variety of gravitational environments. The Earth, Moon, Mars and space all have different gravitational characteristics. Earth's gravitational force is referred to as one Earth gravity, or 1 g . Since the Moon has less mass than the Earth, its gravitational force is only one sixth that of Earth's, or 0.17 g . The gravitational force on Mars is equivalent to about 38% of Earth's gravity, or 0.38 g . The gravitational force in space is called microgravity and is very close to zero- g .

When astronauts are in orbit, either in the space shuttle or on the International Space Station, they are still affected by Earth's gravitational force. However, astronauts maintain a feeling of weightlessness, since both the vehicle and crewmembers are in a constant state of free-fall. Even though they are falling towards the Earth, they are traveling fast enough around the Earth to stay in orbit. During orbit, the gravitational force on the astronauts relative to the vehicle is close to zero- g .



Figure 1: C-9 jet going into a parabolic maneuver



Figure 2: Astronaut crew training for an upcoming mission on the C-9 jet



The C-9 jet is one of the tools utilized by NASA to simulate the gravity, or reduced gravity, that astronauts feel once they leave Earth (Figure 1). The C-9 jet flies a special parabolic pattern that creates several brief periods of reduced gravity. A typical NASA C-9 flight leaves from Houston, Texas, and goes out over the Gulf of Mexico. It lasts about two hours and completes between 40 and 60 parabolas. These reduced gravity flights are performed so astronauts, as well as researchers and their experiments, can experience the gravitational forces of the Moon and Mars and the microgravity of space.

By using the C-9 jet as a reduced gravity research laboratory, astronauts can simulate different stages of spaceflight; allowing crew members to practice what might occur during a real mission. These reduced gravity flights provide the capability for the development and verification of space hardware, scientific experiments, and other types of research (Figure 2). NASA scientists can also use these flights for crew training, including exercising in reduced gravity, administering medical care, performing experiments, and many other aspects of spaceflight that will be necessary for an exploration mission.

Instructional Objectives

You will

- understand and write parametric equations that model the path of NASA's C-9 jet used to simulate microgravity;
- determine the distance and height of a projectile after t seconds;
- analyze given parameters to solve this real-life problem situation;
- find the maximum time and distance of one parabolic maneuver and interpret their significance; and
- use time as a parameter in parametric equations.

Directions: *On the TI-Nspire™ handheld, open the document, Precal-ST_SimWt. Read through the problem set-up (pages 1.2–1.8) and complete the embedded questions.*

Problem

To prepare for an upcoming mission, an astronaut participates in a C-9 flight simulating microgravity. The pilot flies out over the Gulf of Mexico, dives down to increase to a maximum speed, then climbs up until the nose of the C-9 is at a 45° angle with the ground. At this point, the velocity of the jet is 123.333 meters per second (about 283 mph) and the altitude is 9,144 meters (about 30,000 ft). To go into a parabolic maneuver, the pilot cuts the thrust of the engine, letting the nose of the jet rise, and then comes back down at a -45° angle with the ground. Ending the parabolic maneuver, the pilot throttles the engine back up and begins another dive to prepare for the next parabola. The pilot is able to complete 50 parabolas during the 2-hour flight.

Figure 3 shows the movement of the jet during a typical flight. The parabolic maneuver, where microgravity is felt, is highlighted.

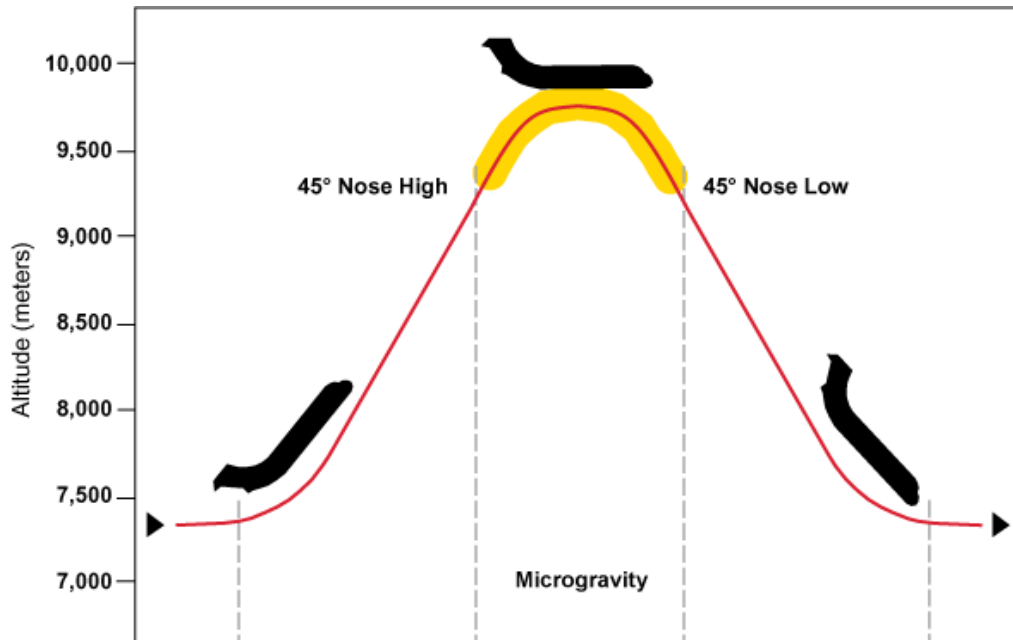


Figure 3: A typical microgravity maneuver

Parametric equations describing projectile motion are of the form:

$$x_{1t} = v_0 \cos(\theta)t$$

$$y_{1t} = -\frac{1}{2}gt^2 + v_0 \sin(\theta)t + h_0$$

The variable, g , is the acceleration due to gravity; v_0 is the initial velocity; t is time; θ is the angle of ascent; and h_0 is the initial altitude of the C-9 jet when it enters the parabolic maneuver.

On Earth, acceleration due to gravity is approximately 9.8 m/s^2 . For this problem, other environmental influences (such as air resistance) should be disregarded.

Directions: Answer questions 1.9–2.3 in your group. Discuss answers to be sure everyone understands and agrees on the solutions. Round all answers to the nearest thousandth, and label with the appropriate units.

The graph on page 1.8 represents the graph of a C-9 parabolic maneuver as previously defined. Grab the point that lies on the curve and observe the direction and magnitude of the arrows (velocity vectors) as you drag the point along the flight path. Write your answers to the questions in the space provided below.

- 1.9 What is happening to the angle of the jet's ascent/descent as it moves along its parabolic path?



- 1.10 What is happening to the vertical velocity vector as the jet moves along its parabolic path?
- 1.11 What is happening to the horizontal velocity vector as the jet moves along its parabolic path?
- 1.12 Explain the reasons for the changes in the vertical and horizontal velocity vectors.
- 1.13 Determine the parametric equations to model the problem situation and graph them on page 1.14.
- 1.15 How many seconds does it take the C-9 jet to reach its maximum height?
- 1.16 What is the maximum time the C-9 is in one parabolic maneuver experiencing microgravity?
- 1.17 Use your answer from the previous question to calculate the total horizontal distance (in meters) that the jet has traveled during the maneuver.
- 1.18 What is the maximum height that the C-9 jet reaches?



1.19 What is the height and horizontal distance traveled of the C-9 after 5 seconds?

1.20 What is the height and distance traveled of the C-9 after 15 seconds?

2.1 Explore how the parabolic maneuver changes by keeping the same initial velocity, but changing the jet's angle of ascent to 30° and 60° by using the graph and slider on page 2.2. How does each angle affect the parabolic maneuver?

2.3 Now that you have explored several angles of ascent, calculate the time spent in microgravity for the 60° angle explored on page 2.2. Why do you think the mission planner chose 45° as the jet's angle of ascent?