The Robotic Arm

Background

This problem applies mathematical principles in NASA's human spaceflight.

In April 2001, the International Space Station (ISS) was equipped with a robotic arm built by the Canadian Space Agency. The robotic arm, known as Canadarm2, was installed by Canadian astronaut Chris Hadfield with the help of NASA astronaut Scott Parazynski. The Canadarm2 is the larger successor to the space shuttle's robotic arm.

Canadarm2 has been instrumental in the assembly of the ISS modules as well as movement of supplies and equipment—even astronauts. From 2001 to 2011, it was used in over a hundred spacewalks. It has been used to grapple (catch or grab hold of) and berth (attach) unmanned supply spacecraft for docking, and then to separate and release the spacecraft from the ISS. More recently, Canadarm2 was used in the same manner when the unmanned SpaceX Dragon delivered provisions to the ISS and returned to Earth with used equipment and scientific experiment payload.

The newly retired space shuttle, which carried astronauts and large payloads to the ISS from 1981 to 2011, did not need assistance from Canadarm2 to dock to the ISS. However, reusable commercial spacecraft, such as the SpaceX Dragon, will now be used to take supplies and equipment to the ISS, and Canadarm2 will be used in the grappling, berthing, separating, and releasing processes for all such missions.

Figure 1: Canadarm2 with a Japanese HTV in its grasp

Figure 2: Canadarm2 assisting on a spacewalk

The robotic arm measures 16.94 meters in linear length, and is comprised mostly by its own two middle sections. The two end sections of the robotic arm, known as the Latching End Effectors (LEEs), allow the arm to grapple the ISS and other objects. The LEE that grapples to the ISS is called the base (or base LEE), and the LEE that grapples with other objects is called the tip (or tip LEE). Each LEE has three joints that connect it to one of the middle sections of the arm and allow movement and rotation in three directions: pitch (y-axis rotation), yaw (z-axis rotation), and roll (x-axis rotation). A seventh joint connects the two middle sections of the robotic arm. These seven joints allow an unlimited number of arm orientations for any given position in three-dimensional space.

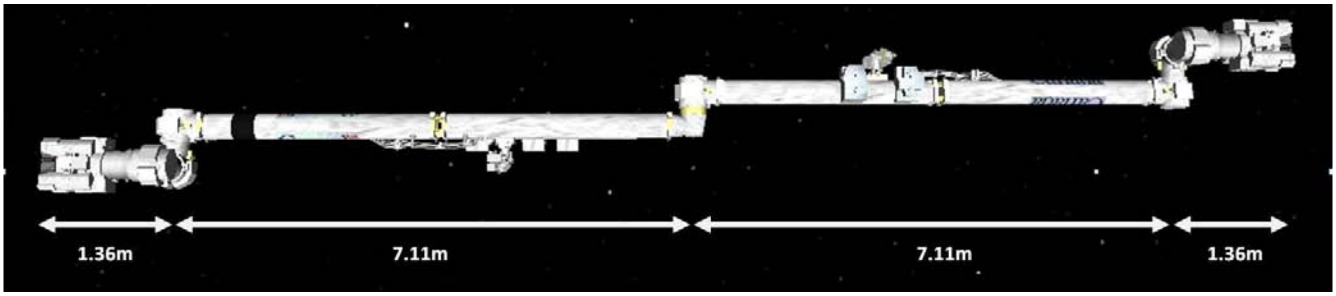

Figure 3: Canadarm2 components

Instructional Objectives

You will

- manipulate a two-dimensional simulation of the robotic arm to capture an astronaut;
- use trigonometric functions to determine distances spanned; and
- determine the relationship between angles of deflection and angles of rotation of the robotic arm.

Directions: On the TI-Nspire™ handheld, open the document, Precal-ST_RobotArm. Read through the problem set-up (pages 1.2–1.3), and complete the simulation on page 1.4. Complete the remaining simulations and embedded questions with your partner. Round all answers to the nearest thousandth, and label with appropriate units.

Problem

Simulation City is a training lab at NASA Johnson Space Center that uses computer simulations to train astronauts on many of the functions astronauts perform during spacewalks, including manipulating the robotic arm. The Simulation City engineers use forward kinetics, the process of using angles of deflection of the joints to determine the location of the arm's tip, and compare it to the object location the robotic arm is trying to capture.

By changing only the angles of deflection of the robotic arm, the astronaut can determine the position of the manipulator. In this activity, we will look at changing the angles of the robotic arm in only two dimensions.

Embedded Questions

1.5 Compare your angles to those of your partner. Are they the same? How many solutions do you think exist? Discuss this with your partner.

The angles you manipulated on page 1.4 were the angles of rotation for each section. Each section also has what is called an angle of deflection. This is the angle formed from the section and its original location on the *x*-axis.

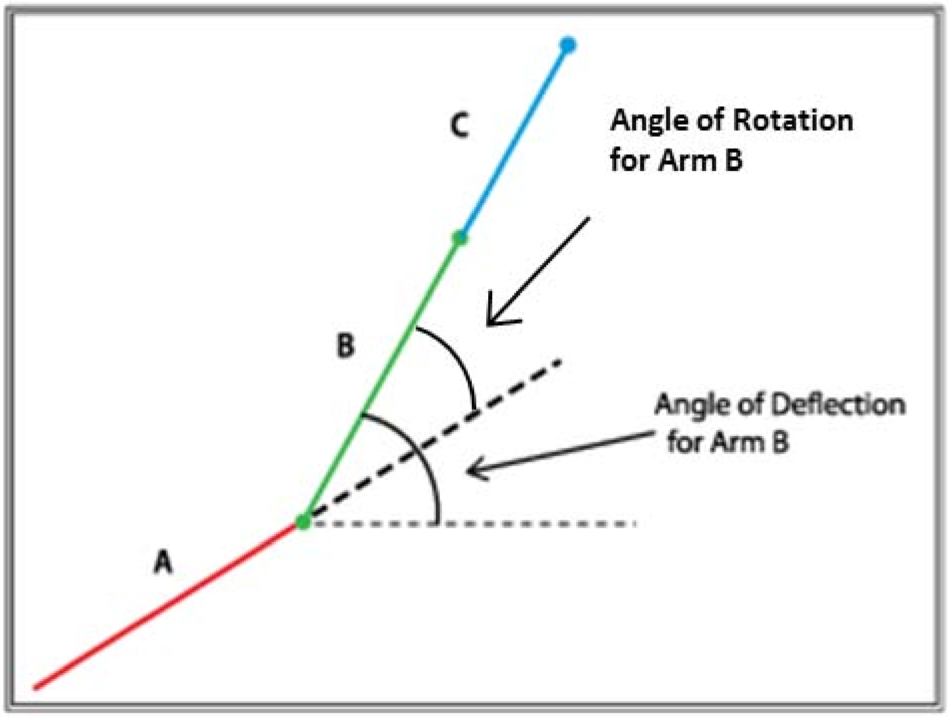

Figure 4: Diagram of angles

1.7 Look at page 1.4 and make a conjecture about the relationship of the angle of rotation and the angle of deflection for each arm.

Page 1.9 shows the first section of the robotic arm in the position that was set on page 1.4, as well as $\overline{AD}$, which represents the direct displacement to the astronaut. Use right triangle trigonometry to answer the questions that follow.

1.10 Determine the horizontal displacement spanned by the first section of the robotic arm (AP).

1.11 Determine the vertical displacement spanned by the first section of the robotic arm (BP).

On page 1.13, both the first and second sections of the robotic arm are shown in the diagram. Use right triangle trigonometry to answer the questions that follow.

1.14 Find the angle of deflection of the second section (section B). Justify your answer. (Remember the previous process if the angle is difficult to see.)

1.15 Determine the horizontal displacement spanned by the second section of the robotic arm (BQ').

1.16 Determine the vertical displacement spanned by the second section of the robotic arm (CQ').

On page 1.18, all three sections of the arm are shown. Use right triangle trigonometry and geometry to answer the questions that follow.

1.19 Find the angle of deflection of the third arm. Justify your answer.

1.20 What is the relationship between the angle of deflection of the third section and $\angle R'CD$?

1.21 Determine the horizontal displacement spanned by the third section of the robotic arm (CR').

1.22 Determine the vertical displacement spanned by the third section of the robotic arm (DR').

1.23 What are the final coordinates of the end of the robotic arm (D)? Round to the nearest whole number for each coordinate.

1.24 How do the vertical and horizontal displacement values of each section compare with your partner's? How do your coordinates for the end of the robotic arm compare with your partner's? Explain the similarities and/or differences in your answers.

1.25 There can be many angle combinations that will successfully capture the astronaut. Can you generate a solution with a negative angle? Go back to page 1.4 to explore and determine which, if any, of the three angles can be negative.

1.26 Why do you think the robotic arm was designed to have three movable sections?