Exploring Space Through MATH

Applications in Precalculus

EDUCATOR EDITION

The Robotic Arm

Instructional Objectives

The 5-E's Instructional Model (Engage, Explore, Explain, Extend, and Evaluate) will be used to accomplish the following objectives.

Students will

- manipulate a two-dimensional simulation of the robotic arm to capture an astronaut;
- use trigonometric functions to determine distances spanned; and
- determine the relationship between angles of deflection and angles of rotation of the robotic arm.

Prerequisites

Students should have a good understanding of right triangle trigonometry.

Background

This problem applies mathematical principles in NASA's human spaceflight.

In April 2001, the International Space Station (ISS) was equipped with a robotic arm built by the Canadian Space Agency. The robotic arm, known as Canadarm2, was installed by Canadian astronaut Chris Hadfield with the help of NASA astronaut Scott Parazynski. The Canadarm2 is the larger successor to the space shuttle's robotic arm.

Canadarm2 has been instrumental in the assembly of the ISS modules as well as movement of supplies and equipment—even astronauts. From 2001 to 2011, it was used in over a hundred spacewalks. It has been used to grapple (catch or grab hold of) and berth (attach) unmanned supply spacecraft for docking, and then to separate and release the spacecraft from the ISS. More recently, Canadarm2 was used in the same manner when the unmanned SpaceX Dragon delivered provisions to the ISS and returned to Earth with used equipment and scientific experiment payload.

The newly retired space shuttle, which carried astronauts and large payloads to the ISS from 1981 to 2011, did not need assistance from Canadarm2 to dock to the ISS. However, reusable commercial spacecraft, such as the SpaceX Dragon, will now be used to take supplies and equipment to the ISS, and Canadarm2 will be used in the grappling, berthing, separating, and releasing processes for all such missions.

Key Concepts
Trigonometric functions

Problem Duration
65 minutes

Teacher Prep Time
15 minutes

Technology
Computer with Internet access and projector, TI-Nspire™ handhelds

Materials
- TI-Nspire Student Edition: *Precal-ST_ Nspire_RobotArm.pdf*
- TI-Nspire Student document: *Precal-ST_RobotArm.tns*
- Video: *ISS Update: Capturing a Dragon*

Skills
Operations with trigonometric functions, translating angles

NCTM Principles
- Geometry
- Problem Solving
- Connections

Common Core Standards
- Geometry

Figure 1: Canadarm2 with a Japanese HTV in its grasp

Figure 2: Canadarm2 assisting on a spacewalk

The robotic arm measures 16.94 meters in linear length, and is comprised mostly by its own two middle sections. The two end sections of the robotic arm, known as the Latching End Effectors (LEEs), allow the arm to grapple the ISS and other objects. The LEE that grapples to the ISS is called the base (or base LEE), and the LEE that grapples with other objects is called the tip (or tip LEE). Each LEE has three joints that connect it to one of the middle sections of the arm and allow movement and rotation in three directions: pitch (y-axis rotation), yaw (z-axis rotation), and roll (x-axis rotation). A seventh joint connects the two middle sections of the robotic arm. These seven joints allow an unlimited number of arm orientations for any given position in three-dimensional space.

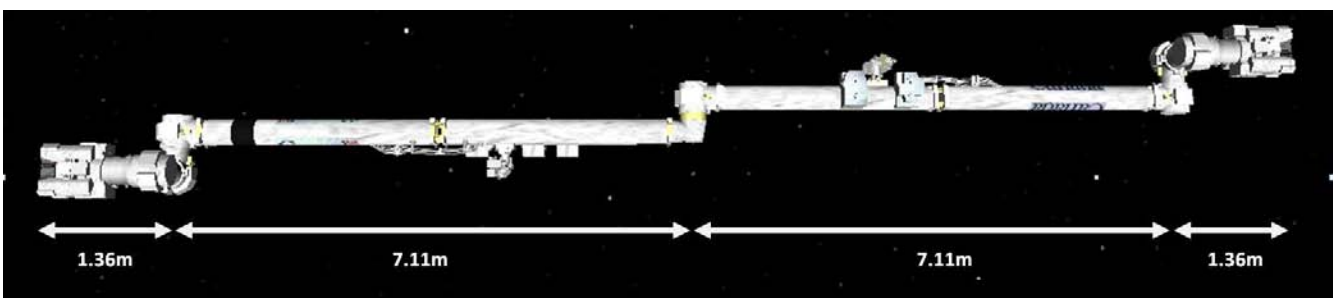


Figure 3: Canadarm2 components

NCTM Principles and Standards

Geometry

- Use trigonometric relationships to determine lengths and angle measures.
- Understand and represent translations, reflections, rotations, and dilations of objects in the plane by using sketches, coordinates, vectors, function notation, and matrices.

Problem Solving

- Build new mathematical knowledge through problem solving.
- Apply and adapt a variety of appropriate strategies to solve problems.

Connections

- Recognize and use connections among mathematical ideas.
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

Common Core Standards

Geometry

- Similarity, Right Triangles, and Trigonometry.
- Modeling with Geometry

Lesson Development

Following are the phases of the 5-E's model in which students can construct new learning based on prior knowledge and experiences. The time allotted for each activity is approximate. Depending on class length, the lesson may be broken into multiple class periods.

The questions in this activity are embedded in the TI-Nspire document, *Precal-ST_RobotArm.tns,* which should be distributed to students' handhelds. Some screenshots have been provided in the solution key to show what the students will see on their handhelds.

1 – Engage (10 minutes)

- Read the background as a class. Encourage student discussion about the background by asking some key questions. Sample questions:
    - Name some tasks for which the robotic arm has been used.

      Spacewalks, the completion of the International Space Station, and moving supplies
    - Now that the space shuttle has retired, how will the robotic arm be used when other vehicles visit the International Space Station?

      The robotic arm will capture spacecraft instead of the Space Shuttle docking with the International Space Station.
- Play the video, *ISS Update: Capturing a Dragon* (8:32 minutes), which is an interview with two space station training instructors who explain how the Canadarm2 will be used to grapple and berth the SpaceX Dragon with the ISS. They also show the simulation software that is used to train the astronauts to perform these functions. The video can be downloaded or streamed from the listed link. After they have viewed the video, discuss with students the advantage of having multiple degrees of freedom to perform these functions.

  http://www.nasa.gov/multimedia/videogallery/index.html?media_id=142266251

2 – Explore (10 minutes)

- Have students open the TI-Nspire document, *Precal-ST_RobotArm*, on their handhelds.
- Group students in pairs to read the contents of pages 1.2–1.3. Then, on page 1.5 of the TI-Nspire document, have them individually change the angles on their handhelds to capture the astronaut.
- Have students work with their partners as they read page 1.6 and answer the question on page 1.7.
- Educators should spend a few minutes discussing angles of deflection and rotation if these terms are unfamiliar to students. Use the figure on the next page to demonstrate.

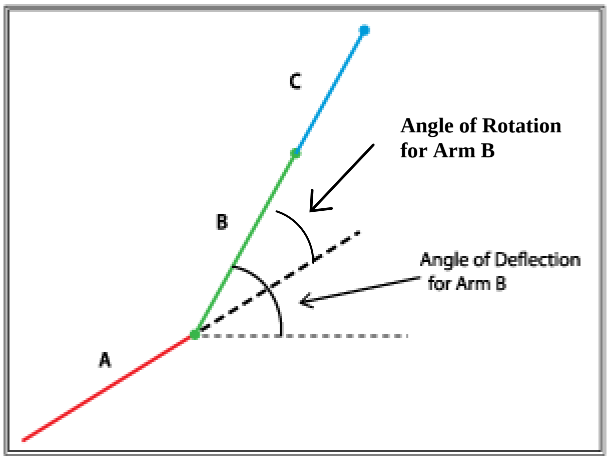


Figure 4: Diagram of angles

- Ask for students to share some of the conjectures they made on page 1.7. You may record some of these on the board to refer to at the end of the activity.

3 – Explain (35 minutes)

- Have students continue to work with their partner as they read and answer questions on pages 1.8–1.14. Encourage them to discuss why they may get some answers that are different from each other.
- Ask a few students to explain their justifications to the question on page 1.14. Using a projector, show page 1.13 so they can illustrate their justifications. Make sure at least two different cases are highlighted—one in which the angles of rotation are both positive and one in which the angle of rotation for section B is negative. Examples of both are provided in the solution key.
- Have students continue to work through pages 1.15–1.19 with their partners.
- Ask a couple of students to explain their justifications to the question on page 1.19.
- As the students explain their justifications, refer back to the conjectures students made on page 1.7 to be sure students see the relationship between the angles of deflection and the angles of rotation.
- Have students continue to work through pages 1.20–1.24.

4 – Extend (5 minutes)

- On page 1.25, students are encouraged to go back to page 1.6 and find an angle combination that involves a negative angle rotation and to determine which of the angles could be negative.
- Encourage student discussion and ask if there are any questions.

5 – Evaluate (5 minutes)

- On page 1.26, students are asked why the robotic arm has three movable sections. You might do this as a class discussion or as a reflective writing as time permits. This may be done in class or assigned as homework.

The Robotic Arm

Solution Key

Note to teacher: Instruct students to open TI-Nspire document, Precal-ST_RobotArm.tns on their handhelds. You may choose to have students record their work directly on their handhelds or on their worksheets in the provided spaces. A solution key is provided below as well as in the educator's version of the TI-Nspire document, Precal-ED_RobotArm.tns.

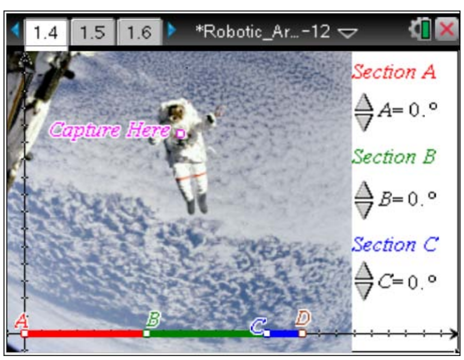
Directions: On the TI-Nspire handheld, open the document, Precal-ST_RobotArm. Read through the problem set-up (pages 1.2–1.3), and complete the simulation on page 1.4. Complete the remaining simulations and embedded questions with your partner. Round all answers to the nearest thousandth, and label with appropriate units.

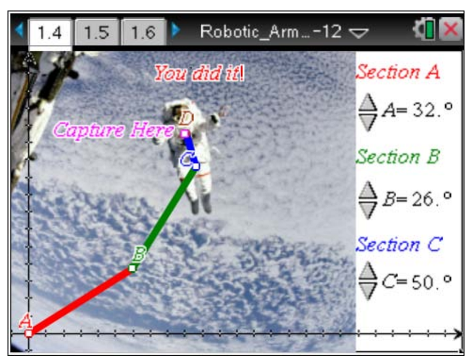
Problem

Simulation City is a training lab at NASA Johnson Space Center that uses computer simulations to train astronauts on many of the functions astronauts perform during spacewalks, including manipulating the robotic arm. The Simulation City engineers use forward kinetics, the process of using angles of deflection of the joints to determine the location of the arm's tip, and compare it to the object location the robotic arm is trying to capture.

By changing only the angles of deflection of the robotic arm, the astronaut can determine the position of the manipulator. In this activity, we will look at changing the angles of the robotic arm in only two dimensions.

On page 1.4, move the end of the mechanical arm to capture the astronaut by rotating each of the three joints. Change the angle of rotation of each section by clicking the sliders along the right side of the screen. Move the end of the robotic arm to the "Capture Here" point for success. When you have succeeded, you will be notified on the screen.





1.5 Compare your angles to those of your partner. Are they the same? How many solutions do you think exist? Discuss this with your partner.

Students' angles chosen will most likely be different. Answers will vary, but students should see that there are many solutions that exist.

The angles you manipulated on page 1.4 were the angles of rotation for each section. Each section also has what is called an angle of deflection. This is the angle formed from the section and its original location on the *x*-axis.

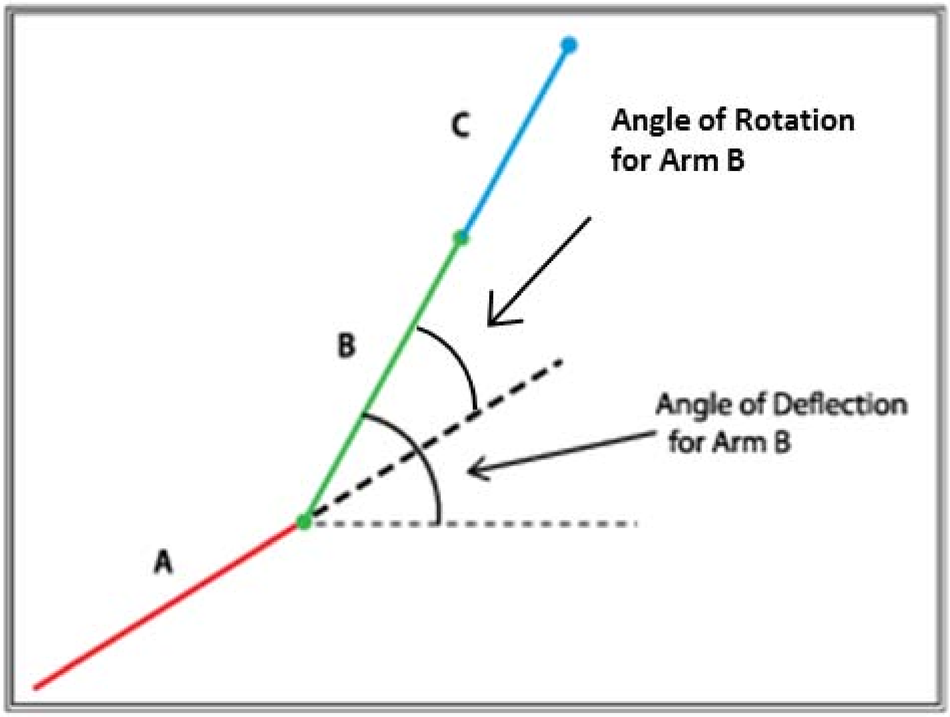
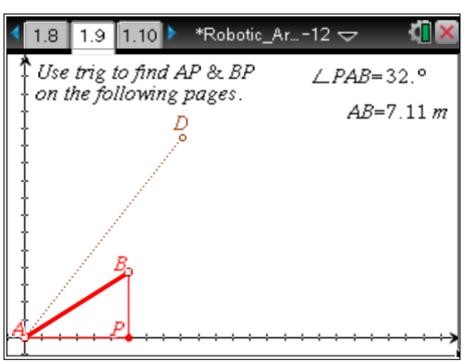


Figure 4: Diagram of angles

1.7 Look at page 1.4 and make a conjecture about the relationship of the angle of rotation and the angle of deflection for each arm.

Students' answers will vary. As students work through the activity, they should discover the following: For section A, the angle of deflection is the same as the angle of rotation. For section B, the angle of deflection is the sum of the angle of rotation for section A and the angle of rotation for section B. For section C, the angle of deflection is the sum of the angles of rotation of all three sections.

Page 1.9 shows the first section of the robotic arm in the position that was set on page 1.4, as well as $\overline{AD}$, which represents the direct displacement to the astronaut. Use right triangle trigonometry to answer the questions that follow.



1.10 Determine the horizontal displacement spanned by the first section of the robotic arm (AP).

Answers will vary, but will be between 0 and 7.11 m. Based on the above example, the correct answer is calculated below.

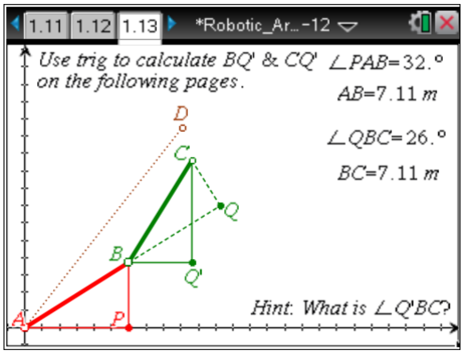
$$\cos(32°) = \frac{AP}{7.11\,m}$$
$$(7.11\,m)\cdot\cos(32°) = AP$$
$$6.030\,m \approx AP$$

1.11 Determine the vertical displacement spanned by the first section of the robotic arm (BP).

Answers will vary, but will be between 0 and 7.11 m. Based on the above example, the correct answer is calculated below.

$$\sin(32°) = \frac{BP}{7.11\,m}$$
$$(7.11\,m)\cdot\sin(32°) = BP$$
$$3.768\,m \approx BP$$

On page 1.13, both the first and second sections of the robotic arm are shown in the diagram. Use right triangle trigonometry to answer the questions that follow.

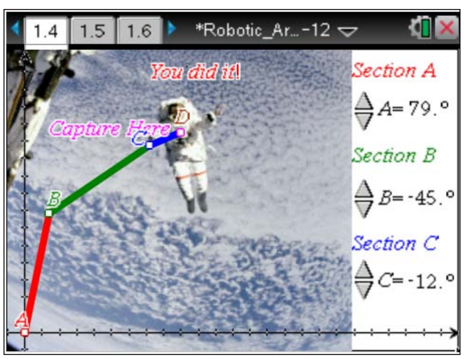


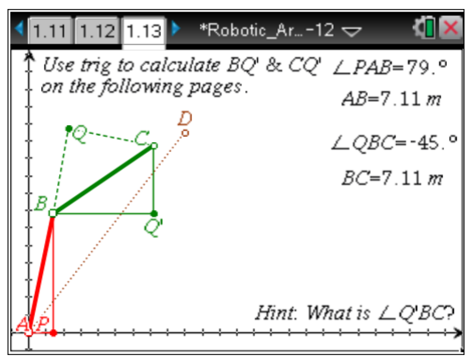
1.14 Find the angle of deflection of the second section (section B). Justify your answer.

Numeric answers will vary. For section B, the angle of deflection is $\angle Q'BC$. In the above example, $m\angle Q'BC = 58°$.

Since $\angle Q'BQ$ is congruent to $\angle PAB$, $m\angle Q'BC = m\angle QBC + M\angle PAB$.

Note to teacher: While it is easier for students to see if the angles of rotation are both positive, it holds true if one of them is negative. If needed, spend some time making sure students understand this. The example screenshots shows angle combinations that could be used to explain/illustrate this to your students.





1.15 Determine the horizontal displacement spanned by the second section of the robotic arm (BQ').

Answers will vary, but will be between 0 and 7.11 m. Based on the above example, the correct answer is calculated below.

$$\cos(58°) = \frac{BQ'}{7.11\,m}$$

$$(7.11\,m) \cdot \cos(58°) = BQ'$$

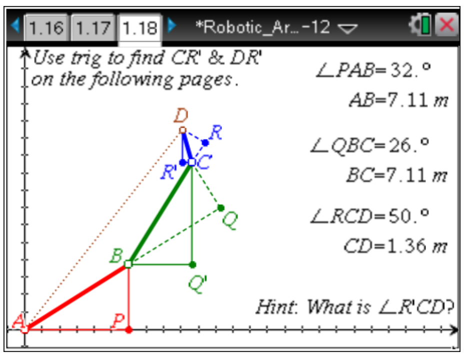
$$3.768\,m \approx BQ'$$

1.16 Determine the vertical displacement spanned by the second section of the robotic arm (CQ').

Answers will vary, but will be between 0 and 7.11 m. Based on the above example, the correct answer is calculated below.

$$\sin(58^\circ) = \frac{CQ'}{7.11\,m}$$
$$(7.11\,m)\cdot\sin(58^\circ) = CQ'$$
$$6.030\,m \approx CQ'$$

On page 1.18, all three sections of the arm are shown. Use right triangle trigonometry and geometry to answer the questions that follow.



1.19 Find the angle of deflection of the third arm. Justify your answer. (Remember the previous process if the angle is difficult to see.)

Answers will vary. The angle of deflection equals the sum of all three angles of rotation. ($m\angle PAB + m\angle QBC + m\angle RCD$). In the above example, the angle of deflection = 108°.

1.20 What is the relationship between the angle of deflection of the third section and $\angle R'CD$?

If the angle of deflection found is less than 90°, then it is $\angle R'CD$. If the angle of deflection is greater than 90°, then it is supplementary to $\angle R'CD$. This is the case in the above example.

1.21 Determine the horizontal displacement spanned by the third section of the robotic arm (CR').

Answers will vary, but will be between magnitude of 0 and 1.36 m. Based on the above example, the correct answer is calculated below. The total deflection of $\overline{CD}$ is equal to the sum of all three angles.

$$\cos(108^\circ) = \frac{CR'}{1.36\,m}$$
$$(1.36\,m)\cdot\cos(108^\circ) = CR'$$
$$-0.420\,m \approx CR'$$

1.22 Determine the vertical displacement spanned by the third section of the robotic arm (DR').

Answers will vary, but will be between magnitude of 0 and 1.36 m. Based on the above example, the correct answer is calculated below.

$$\sin(108^\circ) = \frac{DR'}{1.36\,m}$$
$$(1.36\,m)\cdot\sin(108^\circ) = DR'$$
$$1.293\,m \approx DR'$$

1.23 What are the final coordinates of the end of the robotic arm (D)? Round to the nearest whole number for each coordinate.

x = AP + BQ' +CR' and y = BP + CQ'+DR'

In the above example, the coordinates are D(9, 11).

1.24 How do the vertical and horizontal displacement values of each section compare with your partner's? How do your coordinates for the end of the robotic arm compare with your partner's? Explain the similarities and/or differences in your answers.

The horizontal and vertical displacement values of the sections will be different because your partner chose different angles for each section. The coordinates of the end of the arm will be the same (or very close to the same) because they arrived at the same location.

1.25 There can be many angle combinations that will successfully capture the astronaut. Can you generate a solution with a negative angle? Go back to page 1.4 to explore and determine which, if any, of the three angles can be negative.

Only angles B and C can be negative in order to reach the astronaut.

1.26 Why do you think the robotic arm was designed to have three movable sections?

Answers will vary. Students should understand that the three sections allow the arm to move in several directions so the robotic arm can complete complex tasks without becoming bound.

Contributors

This problem was developed by the Human Research Program Education and Outreach (HRPEO) team with the help of NASA subject matter experts and high school mathematics educators.

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